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Question 1

$$\vec{V} = U\hat{i} + \frac{Ua^2}{(x^2+y^2)^2}\left[(y^2+x^2)\hat{i} - 2xy\hat{j}\right]$$

a) Is this flow incompressible?

A flow is incompressible if $\vec{\nabla}\cdot\vec{v} = 0$

$$\vec{V} = U\hat{i} + \frac{Ua^2}{(x^2+y^2)^2}(x^2+y^2)\hat{i} - 2xy\frac{Ua^2}{(x^2+y^2)^2}\hat{j}$$

$$= \left(U + \frac{Ua^2}{x^2+y^2}\right)\hat{i} + \left(-2xy\frac{Ua^2}{(x^2+y^2)^2}\right)\hat{j}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(U + Ua^2(x^2+y^2)^{-1}\right) = -Ua^2(x^2+y^2)^{-2}(2x) = -\frac{2Ua^2x}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(-2xyUa^2(x^2+y^2)^{-2}\right) = 4xyUa^2(x^2+y^2)^{-3}(2y) = \frac{4Ua^2xy^2}{(x^2+y^2)^3}$$

$$\nabla\cdot\vec{v} = -\frac{2Ua^2x}{(x^2+y^2)^2} + \frac{4Ua^2xy^2}{(x^2+y^2)^3} = -\frac{2Ua^2x(x^2+y^2)}{(x^2+y^2)^3} + \frac{4Ua^2xy^2}{(x^2+y^2)^3}$$

$$= \frac{-2Ua^2x(x^2+y^2) + 4Ua^2xy^2}{(x^2+y^2)^3}$$

$$= 2Ua^2x\,\frac{x^2+y^2+2y^2}{(x^2+y^2)^3}$$

$$= 2Ua^2x\,\frac{x^2+3y^2}{(x^2+y^2)^3} \neq 0$$

The flow is not incompressible.

b) Is the flow irrotational?

A flow is irrotational if $\vec{\nabla}\times\vec{v} = 0$

$$\vec{v} = \left(U + \frac{Ua^2}{x^2+y^2}\right)\hat{i} + \left(-2xy\frac{Ua^2}{(x^2+y^2)^2}\right)\hat{j}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

$$\vec{V} = \left(U + \frac{Ua^2}{x^2+y^2}\right)\hat{i} + \left(-2xy\frac{Ua^2}{(x^2+y^2)^2}\right)\hat{j}$$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U+\frac{Ua^2}{x^2+y^2} & -2xy\frac{Ua^2}{(x^2+y^2)^2} & 0 \end{vmatrix}$$

$$= \cancel{\left(0 - \frac{\partial}{\partial z}\left(-2xy\frac{Ua^2}{(x^2+y^2)^2}\right)\right)\hat{i}} - \cancel{\left(0 - \frac{\partial}{\partial z}\left(U + \frac{Ua^2}{x^2+y^2}\right)\right)\hat{j}} + \left(\frac{\partial}{\partial x}\left(-2xy\frac{Ua^2}{(x^2+y^2)^2}\right) - \frac{\partial}{\partial y}\left(U + \frac{Ua^2}{x^2+y^2}\right)\right)\hat{k}$$

$$\frac{\partial}{\partial x}\left(-2xyUa^2(x^2+y^2)^{-2}\right) = \left(-2yUa^2(x^2+y^2)^{-2}\right) + \left(-2xyUa^2\left(-2(x^2+y^2)^{-3}(2x)\right)\right)$$

$$= \frac{-2Ua^2y}{(x^2+y^2)^2} + \frac{8Ua^2x^2y}{(x^2+y^2)^3}$$

$$= 2Ua^2y\left(\frac{x^2+y^2}{(x^2+y^2)^3} + \frac{4x^2}{(x^2+y^2)^3}\right)$$

$$= 2Ua^2y\,\frac{5x^2+y^2}{(x^2+y^2)^3}$$

$$\frac{\partial}{\partial y}\left(U + Ua^2(x^2+y^2)^{-1}\right) = -Ua^2(x^2+y^2)^{-2}(2y) = \frac{-2Ua^2y}{(x^2+y^2)^2}$$

$$\vec{\nabla}\times V = \left(2Ua^2y\frac{5x^2+y^2}{(x^2+y^2)^3} + 2Ua^2y\frac{x^2+y^2}{(x^2+y^2)^3}\right)\hat{k}$$

$$= \frac{2Ua^2y}{(x^2+y^2)^3}\left(5x^2+y^2+x^2+y^2\right)\hat{k}$$

$$= \frac{2Ua^2y}{(x^2+y^2)^3}\left(6x^2+2y^2\right)\hat{k}$$

$$= \frac{4Ua^2y}{(x^2+y^2)^3}\left(3x^2+y^2\right)\hat{k} \neq 0$$

The flow is neither incompressible nor irrotational.

Question 2

$$\vec{V} = x(y+1)z^3\hat{j} \qquad \vec{\nabla}\cdot\vec{V} = \frac{\partial}{\partial y}\left(x(y+1)z^3\right) = xz^3$$

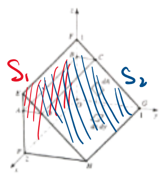
<u>Divergence Theorem</u>

$$\iiint_V \vec{\nabla}\cdot\vec{V}\,dV = \iint_S \hat{n}\cdot\vec{V}\,dA$$

$$\iiint_V \vec{\nabla}\cdot\vec{v}\, dV = \iint_S \hat{n}\cdot\vec{v}\, dA$$

$$\iiint_V xz^3\, dV = \frac{1}{2}\int_0^1\int_0^1\int_0^2 xz^3\, dx\, dy\, dz = \frac{1}{2}\int_0^1\int_0^1 \left(\frac{1}{2}x^2 z^3\right)\Big|_0^2 dy\, dz = \frac{1}{4}\int_0^1\int_0^1 \left(2^2 z^3\right) dy\, dz = \int_0^1 \left(yz^3\right)\Big|_0^1 dz$$

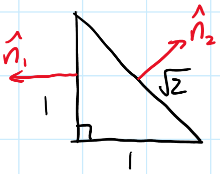
$$= \int_0^1 \left(z^3\right) dz = \left(\frac{1}{4}z^4\right)\Big|_0^1 = \boxed{\frac{1}{4}}$$



$$\iint_S \hat{n}\cdot\vec{v}\, dA = \iint_{S_1} \hat{n}_1\cdot\vec{v}\, dA_1 + \iint_{S_2} \hat{n}_2\cdot\vec{v}\, dA_2$$

$\hat{n}_1 = (-1)\hat{j}$ $\quad dA_1 = dx\, dz$

$\hat{n}_2 = \left(\frac{\sqrt{2}}{2}\right)\hat{j} + \left(\frac{\sqrt{2}}{2}\right)\hat{k}$ $\quad dA_2 = \sqrt{2}\, dx\, dy$



$n_2 = (1)\hat{\imath} + (1)\hat{\jmath}$

$|n_2| = \sqrt{2}$

$\hat{n}_2 = \left(\frac{\sqrt{2}}{2}\right)\hat{\imath} + \left(\frac{\sqrt{2}}{2}\right)\hat{\jmath}$

$\hat{n}_1\cdot\vec{v} = -x(y+1)z^3$

$\hat{n}_2\cdot\vec{v} = \frac{\sqrt{2}}{2}x(y+1)z^3$

$$\iint_{S_1} \hat{n}_1\cdot\vec{v}\, dA_1 + \iint_{S_2} \hat{n}_2\cdot\vec{v}\, dA_2 = \int_0^1\int_0^2 \left(-x(y+1)z^3\right) dx\, dz + \int_0^1\int_0^2 \left(\frac{\sqrt{2}}{2}x(y+1)z^3\right)\sqrt{2}\, dx\, dy$$

$$= -\int_0^1 \left(\frac{1}{2}x^2(y+1)z^3\right)\Big|_0^2 dz + \int_0^1 \left(\frac{1}{2}x^2(y+1)z^3\right)\Big|_0^2 dy$$

$$= -\frac{1}{2}\int_0^1 \left(4(y+1)z^3\right) dz + \frac{1}{2}\int_0^1 \left(4(y+1)z^3\right) dy$$

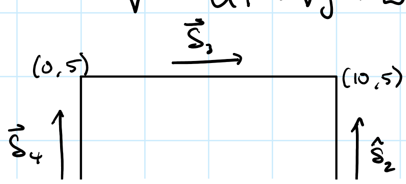
$$= -2\left((y+1)\frac{1}{4}z^4\right)\Big|_0^1 + 2\left(\frac{1}{2}y^2z^3 + yz^3\right)\Big|_0^1$$

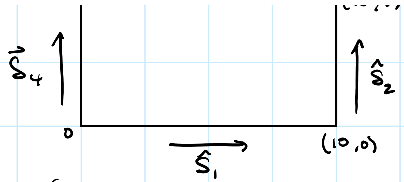
$$= -2\left(\frac{1}{4}(y+1)\right) + 2\left(\frac{1}{2}z^3 + z^3\right)$$

$$= -\frac{1}{2}y - \frac{1}{2} + 3z^3 \quad \boxed{= \frac{1}{4}}$$

Question 3

$$\Gamma = -\oint \vec{v}\cdot d\vec{s} \qquad \vec{v} = u\hat{\imath} + v\hat{\jmath} + w\hat{k} = \left(16x^2 + y\right)\hat{\imath} + (10)\hat{\jmath} + \left(yz^2\right)\hat{k}$$





$$\Gamma = -\oint \vec{V}\cdot d\vec{s} = -\left(\int \vec{V}\cdot d\vec{s}_1 + \int \vec{V}\cdot d\vec{s}_2 - \int \vec{V}\cdot d\vec{s}_3 - \int \vec{V}\cdot d\vec{s}_4\right)$$

$$= -\left(\int_0^{10} 16x^2\,dx + \int_0^{5} 10\,dy - \int_0^{10} (16x^2+5)\,dx - \int_0^{5} 10\,dy\right)$$

$$= -\left(16\left(\tfrac{1}{3}x^3\right)\Big|_0^{10} + 10(y)\Big|_0^{5} - \left(\tfrac{16}{3}x^3+5x\right)\Big|_0^{10} - 10(y)\Big|_0^{5}\right)$$

$$= -\left(\cancel{16\left(\tfrac{1}{3}1000\right) + 50 - \left(16\tfrac{1}{3}1000\right.} \cancel{+ 50}\right) - 50\Big)$$

a) $\boxed{\Gamma = 50\,m^2/s}$

$$\nabla\times\vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 16x^2+y & 10 & yz^2 \end{vmatrix} = \left(\frac{\partial}{\partial y}yz^2 - \cancel{\frac{\partial}{\partial z}10}\right)\hat{\imath} - \left(\cancel{\frac{\partial}{\partial x}yz^2 - \frac{\partial}{\partial z}(16x^2+y)}\right)\hat{\jmath} + \left(\cancel{\frac{\partial}{\partial x}10} - \frac{\partial}{\partial y}(16x^2+y)\right)\hat{k}$$

$$= \frac{\partial}{\partial y}(yz^2)\hat{\imath} - \frac{\partial}{\partial y}(16x^2+y)\hat{k}$$

$$\vec{\omega} = z^2\hat{\imath} - \hat{k}$$

$\Gamma = \iint_A \vec{\omega}\cdot\hat{n}\,dA \qquad \hat{n} = \hat{k} \qquad A = 5\cdot 10 = 50\,m^2$

b) $\boxed{\Gamma = (\vec{\omega}\cdot\hat{k})A = (-1)(50) = -50\,m^2/s}$

Both methods derive a circulation of equal magnitude but in opposite directions, indicating a difference in sign convention. It is worth noting that the direction of positive rotation in aerodynamics (CW) is opposite to the convention in mathematics (CCW).

Question 4

$$T(x,y) = \frac{C}{\sqrt{x^2+y^2}} \qquad \vec{V} = 1.0\hat{\imath} + 1.0\hat{\jmath} \qquad (x_0, y_0) = (1,1)$$

$$\frac{DT(x,y)}{Dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} \qquad \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(C(x^2+y^2)^{-1/2}\right) = C\left(-\tfrac{1}{2}(x^2+y^2)^{-3/2}(2x)\right) = \frac{-Cx}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial T}{\partial y} = \frac{-Cy}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial x}{\partial t} = V(x) = 1.0 \qquad \frac{\partial y}{\partial t} = V(y) = 1.0$$

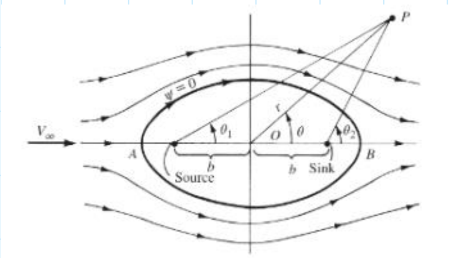
$$\frac{DT(x,y)}{Dt} = \frac{-Cx}{(x^2+y^2)^{3/2}}(1) + \frac{-Cy}{(x^2+y^2)^{3/2}}(1) = -C\,\frac{x+y}{(x^2+y^2)^{3/2}}$$

$$x(t) = x_0 + V(x)t = 1+t$$
$$y(t) = y_0 + V(y)t = 1+t$$

$$\frac{DT(x,y)}{Dt} = -C\,\frac{1+t+1+t}{((1+t)^2+(1+t)^2)^{3/2}} = -C\,\frac{2+2t}{(2(1+t)^2)^{3/2}} = -C\,\frac{2\cancel{(1+t)}}{\sqrt{8}\,(1+t)^3} = -C\,\frac{2}{2\sqrt{2}\,(1+t)^2}$$

$$\boxed{\frac{DT(t)}{Dt} = \frac{-C}{\sqrt{2}\,(1+t)^2}}$$

Question 5



$$\Psi = V_\infty y + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$
$$\tan\theta_1 = \frac{y}{x+b} \qquad \theta_1 = \tan^{-1}\left(\frac{y}{x+b}\right)$$
$$\tan\theta_2 = \frac{y}{x-b} \qquad \theta_2 = \tan^{-1}\left(\frac{y}{x-b}\right)$$

$$\Psi = V_\infty y + \frac{\Lambda}{2\pi}\left(\tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{y}{x-b}\right)\right)$$

Points A, B are stagnation points that lie on the x-axis ($y=0$, $v=0$)

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left(V_\infty y + \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{y}{x+b}\right) - \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{y}{x-b}\right)\right) = 0$$

$$= V_\infty + \frac{\Lambda}{2\pi}\,\frac{x+b}{y^2+(x+b)^2} - \frac{\Lambda}{2\pi}\,\frac{x-b}{y^2+(x-b)^2}$$

$$= V_\infty + \frac{\Lambda}{2\pi}\left(\frac{\cancel{x+b}}{(x+b)^{\cancel{2}}} - \frac{\cancel{x-b}}{(x-b)^{\cancel{2}}}\right)$$

$$= V_\infty + \frac{\Lambda}{2\pi}\left(\frac{1}{x+b} - \frac{1}{x-b}\right)$$

$$= V_\infty + \frac{\Lambda}{2\pi}\left(\frac{(x-b)-(x+b)}{(x+b)(x-b)}\right)$$

$$= V_\infty + \frac{\Lambda}{2\pi}\left(\frac{-2b}{x^2-b^2}\right) = 0$$

$$V_\infty = \frac{\Lambda}{\pi}\ \frac{b}{x^2-b^2}$$

$$x^2 - b^2 = \frac{\Lambda b}{\pi V_\infty}$$

$$x^2 = b^2 + \frac{\Lambda b}{\pi V_\infty}$$

$$\boxed{x = \pm\sqrt{b^2 + \frac{\Lambda b}{\pi V_\infty}}}$$

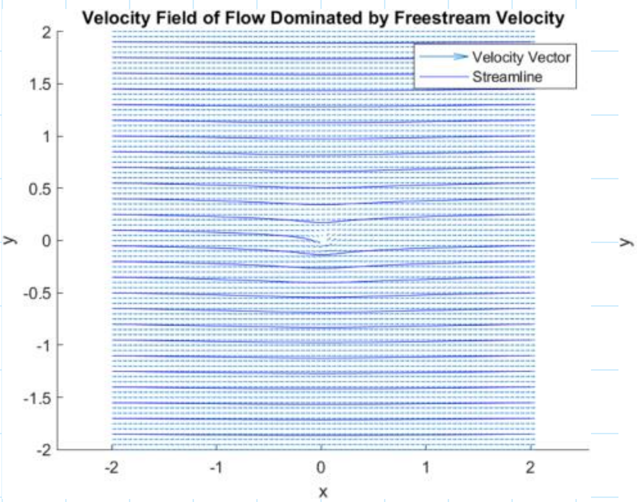
Question 6

a) $\phi = V_\infty x + \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)$ $\quad \psi = V_\infty y - \frac{\Lambda}{2\pi}\ln\sqrt{x^2+y^2}$

$$\boxed{u = \frac{\partial\psi}{\partial y} = V_\infty - \frac{\Lambda}{2\pi}\frac{y}{x^2+y^2} \qquad v = \frac{-\partial\psi}{\partial x} = \frac{\Lambda}{2\pi}\frac{x}{x^2+y^2}}$$

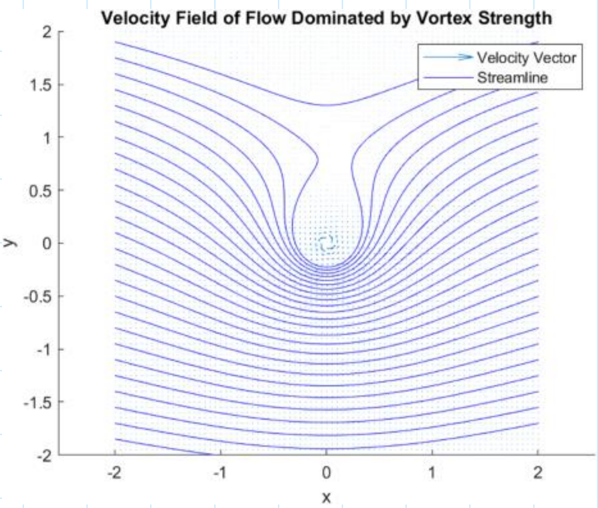
i) The velocity field around a point vortex of strength Λ is plotted in MATLAB. The freestream velocity and the vortex strength compete to be the dominant force near the centre. A large

Flow Dominated by Freestream



$V_\infty = 10\,m/s \qquad \Lambda = 2\,m^2/s$

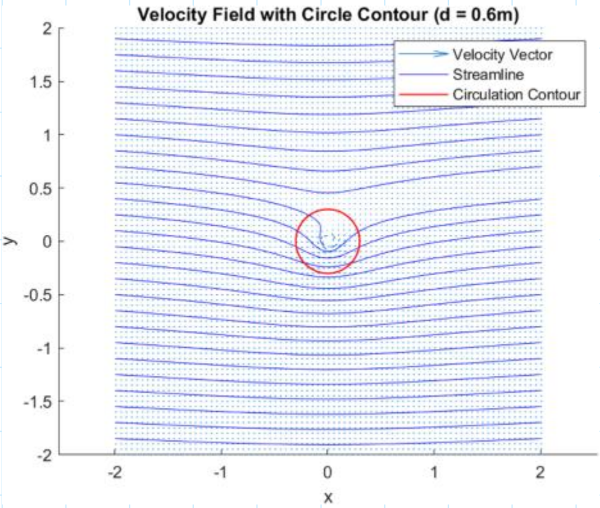
Flow Dominated by Vortex



$V_\infty = 2\,m/s \qquad \Lambda = 10\,m^2/s$

The vortex in the freestream-dominated flow acts as a minute disturbance to the overall velocity field. The vortex-dominated flow sees very dominant circulation, where the freestream only acts to exacerbate one side of the vortex acting in the same direction and hinder the other side.

ii) $\Gamma = -\oint \vec{V} \cdot d\vec{s}$



$V_\infty = 5 m/s$ $\quad \Lambda = 5 m^2/s$ $\quad d = 0.3 m$

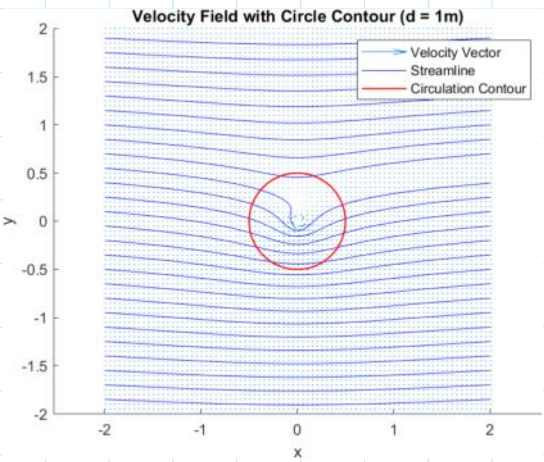
The circulation calculated at any contour around the vortex should equal the same vortex strength due to Stoke's Theorem.

The plot to the left takes a circular contour of diameter 0.6m around the point vortex. The tangential velocity vectors are taken around this contour and summed.

$\Gamma = -\oint_S \vec{V} \cdot d\vec{s} = \Sigma V_\theta \Delta s = 4.9969 m^2/s$

The slight discrepancy between the initial Γ_0 and the calculated Γ can be attributed to discretization error around the contour S, and/or rounding error from the floating point data type.

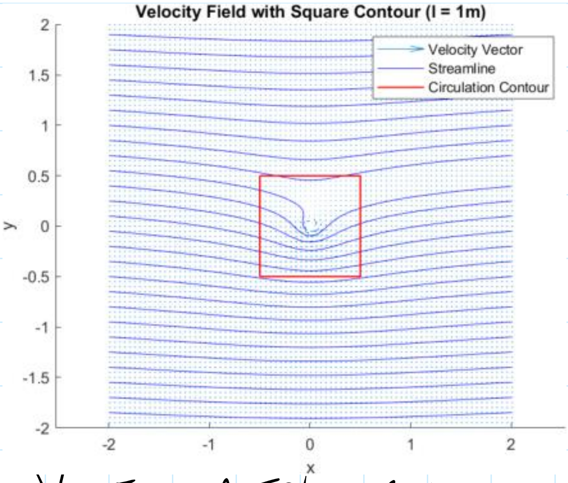
iii)



$V_\infty = 5 m/s$ $\quad \Lambda = 5 m^2/s$ $\quad d = 1 m$

Using the same method as part ii), the circulation is calculated along the circular contour of diameter 1m.

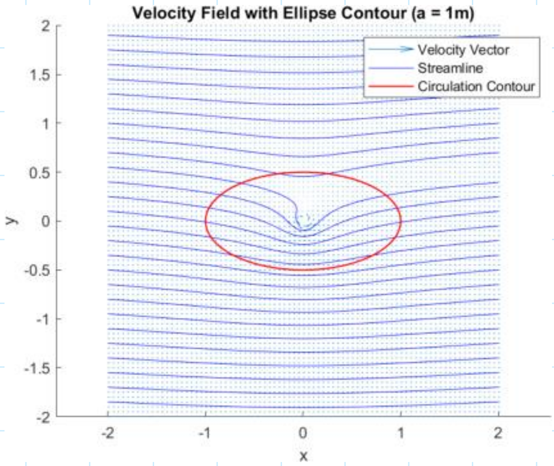
$\Gamma = 4.9991 \ m^2/s$



The same method is used again to calculate the circulation along a square contour.

$\Gamma = 4.9972\ m^2/s$

$V_\infty = 5 m/s \quad \Lambda = 5 m^2/s \quad l = 1m$



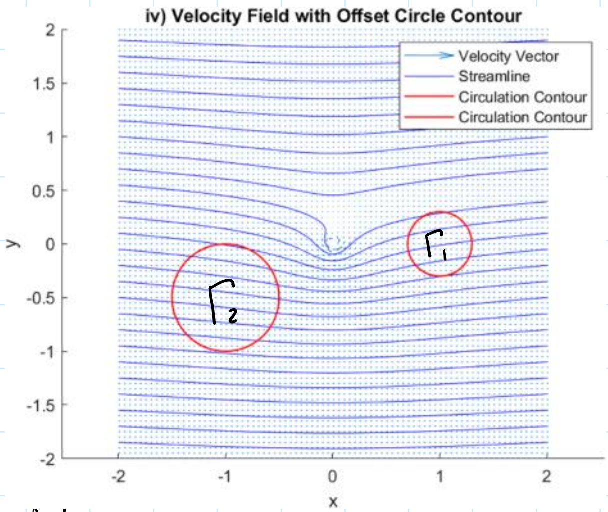
And again for a ellipse contour.

$\Gamma = 4.9955\ m^2/s$

$V_\infty = 5 m/s \quad \Lambda = 5 m^2/s \quad a = 1m$

The circulation is equivalent no matter the shape or size of the contour, as long as it encompasses the vortex.

iv)



With the circle offset from the vortex, the contour no longer encompasses it.

$\Gamma_1 = 0.011632\ m^2/s$

$\Gamma_2 = -0.025069\ m^2/s$

Therefore, the evaluation of the circulation about the respective contours no longer equals the vortex strength.

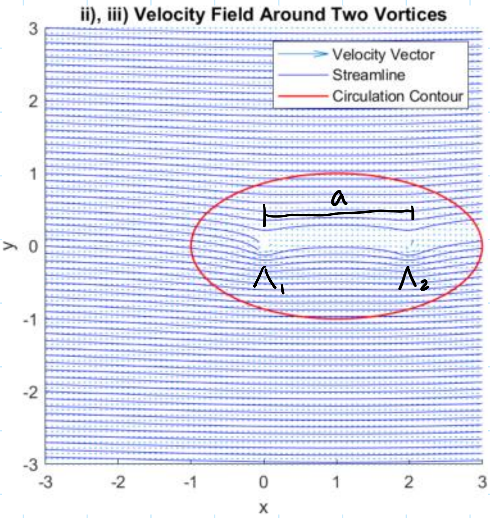
$V_\infty = 5 m/s \quad \Lambda = 5 m^2/s \quad d = 0.6m, 1m$

b) Second vortex at $(x, y) = (a, 0)$, $\Lambda_1 = \Lambda_2 = \frac{1}{2}\Lambda$

i) $$u = V_\infty - \frac{\Lambda_1}{2\pi}\frac{y}{x^2+y^2} - \frac{\Lambda_2}{2\pi}\frac{y}{(x-a)^2+y^2} = V_\infty - \frac{\Lambda y}{4\pi}\left(\frac{1}{x^2+y^2} - \frac{1}{(x-a)^2+y^2}\right)$$

$$v = \frac{\Lambda_1}{2\pi}\frac{x}{x^2+y^2} + \frac{\Lambda_2}{2\pi}\frac{x-a}{(x-a)^2+y^2} = \frac{\Lambda}{4\pi}\left(\frac{x}{x^2+y^2} + \frac{x-a}{(x-a)^2+y^2}\right)$$

ii)



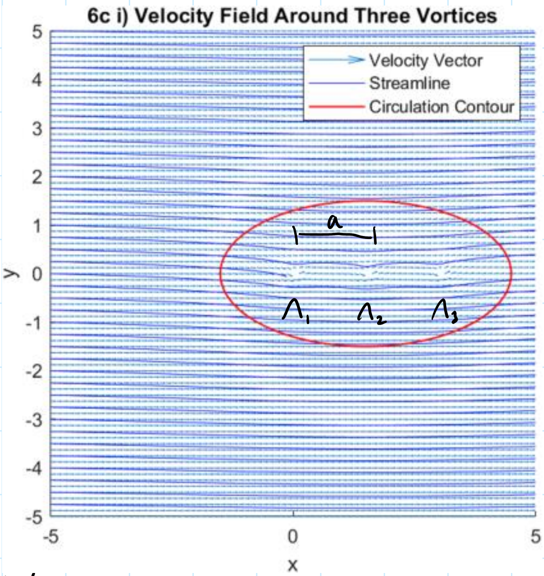
$V_\infty = 5\,m/s \quad \Lambda = 5\,m^2/s \quad a = 2m$

iii) The total circulation around the elliptical contour encompassing both vortices is $\Gamma = 4.9967\,m^2/s$.
This is equivalent to the sum of both vortex strengths $\Lambda = \Lambda_1 + \Lambda_2 = 5\,m^2/s$.
This is proof of Stokes' Theorem applying to circulation, where the tangential velocity around the contour is equal to the total circulation inside the contour.

$$\iint \vec{\Gamma}\, dA = -\oint \vec{v} \cdot d\vec{s}$$

c) $$u = V_\infty - \frac{\Lambda_1}{2\pi}\frac{y}{x^2+y^2} - \frac{\Lambda_2}{2\pi}\frac{y}{(x-a)^2+y^2} - \frac{\Lambda_3}{2\pi}\frac{y}{(x-2a)^2+y^2} = V_\infty - \frac{\Lambda y}{6\pi}\left(\frac{1}{x^2+y^2} + \frac{1}{(x-a)^2+y^2} + \frac{1}{(x-2a)^2+y^2}\right)$$

$$v = \frac{\Lambda_1}{2\pi}\frac{x}{x^2+y^2} + \frac{\Lambda_2}{2\pi}\frac{x-a}{(x-a)^2+y^2} + \frac{\Lambda_3}{2\pi}\frac{x-2a}{(x-2a)^2+y^2} = \frac{\Lambda}{6\pi}\left(\frac{x}{x^2+y^2} + \frac{x-a}{(x-a)^2+y^2} + \frac{x-2a}{(x-2a)^2+y^2}\right)$$



$V_\infty = 5\,m/s \quad \Lambda = 5\,m^2/s \quad a = 1.5m$

The same principles apply to this analysis, only now there are three vortices within the circulation contour. The initial vortex strengths sum to $\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3 = 5\,m^2/s$
The circulation about the contour is calculated to be
$\Gamma = 4.9964\,m^2/s$

d) $$u_3 = V_\infty - \frac{\Lambda_1}{2\pi}\frac{y}{x^2+y^2} - \frac{\Lambda_2}{2\pi}\frac{y}{(x-a)^2+y^2} - \frac{\Lambda_3}{2\pi}\frac{y}{(x-2a)^2+y^2} = V_\infty - \frac{\Lambda y}{6\pi}\left(\frac{1}{x^2+y^2} + \frac{1}{(x-a)^2+y^2} + \frac{1}{(x-2a)^2+y^2}\right)$$

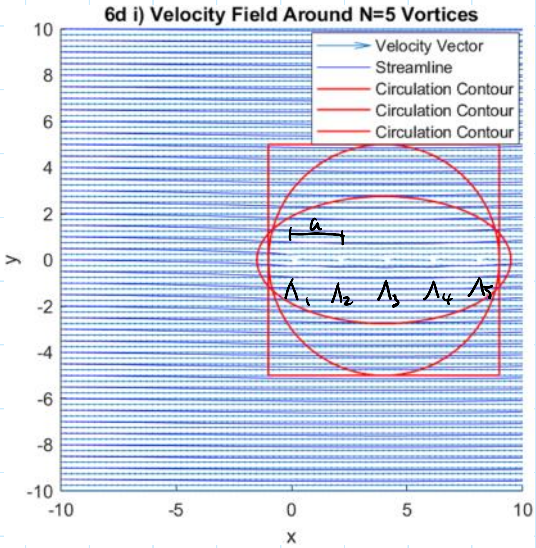
$$U_n = V_\infty - \frac{\Lambda y}{n2\pi}\left(\sum_1^n \frac{1}{(x-(n-1)a)^2+y^2}\right)$$

$$V_3 = \frac{\Lambda_1}{2\pi}\frac{x}{x^2+y^2} + \frac{\Lambda_2}{2\pi}\frac{x-a}{(x-a)^2+y^2} + \frac{\Lambda_3}{2\pi}\frac{x-2a}{(x-2a)^2+y^2} = \frac{\Lambda}{6\pi}\left(\frac{x}{x^2+y^2} + \frac{x-a}{(x-a)^2+y^2} + \frac{x-2a}{(x-2a)^2+y^2}\right)$$

$$V_n = \frac{\Lambda}{n2\pi}\left(\sum_1^n \frac{x-(n-1)a}{(x-(n-1)a)^2+y^2}\right)$$

where n = # of vortices

a = spacing between vortices



$\Gamma_{ellipse} = 4.9963\ m^2/s$

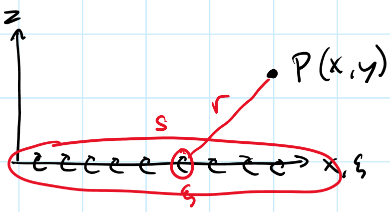
$\Gamma_{square} = 4.9995\ m^2/s$

$\Gamma_{circle} = 5.0372\ m^2/s$

$V_\infty = 5\ m/s$ $\Lambda = 5\ m^2/s$ $a = 1\ m$ $n = 5$ vortices

Each contour results in the sum of the vortex strengths within the contour, due to the same principle as discussed before. Slight discrepancies can be attributed to how well the contour encapsulates the flow affected by the vortices. The more encapsulating it is, the more circulation it will capture closer to the actual vortex strength.

e) (Bonus) $\gamma(\theta) = 2\alpha V_\infty\left(\frac{1+\cos\theta}{\sin\theta}\right)$



$\vec{r} = (x, y)$

$x = \frac{c}{2}(1-\cos\theta)$

$\cos\theta = 1 - 2\frac{x}{c}$

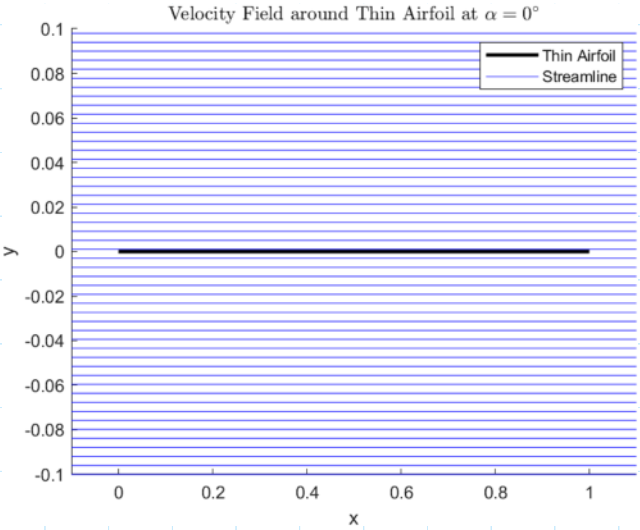
$\theta = \cos^{-1}\left(1 - 2\boxed{\frac{x}{c}}\right)$ normalized chord

Biot-Savart Law $\quad u_i = \frac{-\gamma_i y}{2\pi((x-\xi_i)^2+y^2)} \quad V_i = \frac{\gamma_i(x-\xi_i)}{2\pi((x-\xi_i)^2+y^2)}$

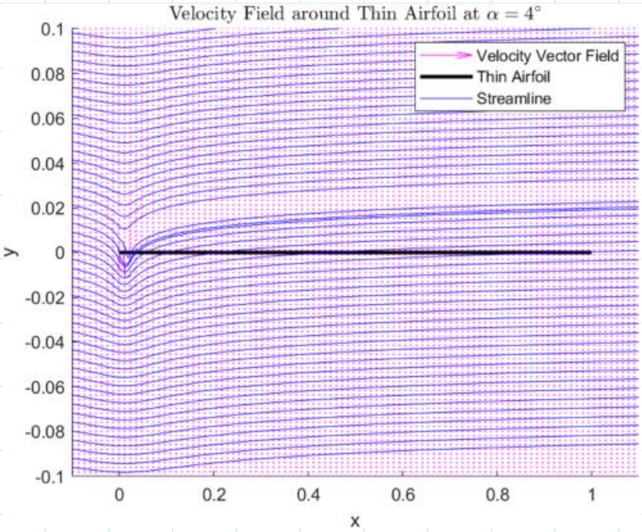
$|r|^2 = (x-\xi)^2 + y^2$

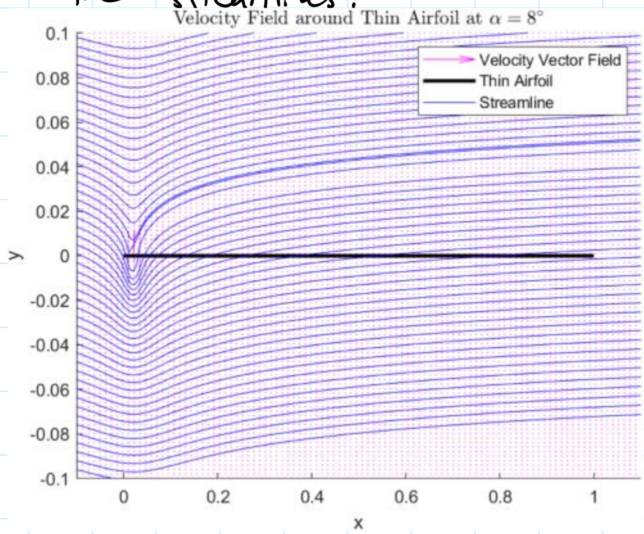
$$u_i = V_\infty + u_{i-1} - \frac{\gamma_i y}{2\pi|r|^2}\Delta x$$
$$V_i = V_{i-1} + \frac{\gamma(x-\xi)}{2\pi|r|^2}\Delta y$$

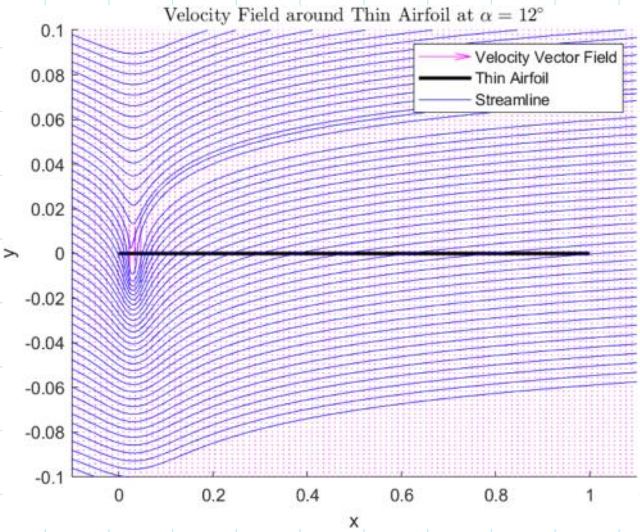
For all the figures below, the freestream velocity $V_\infty = 5\ m/s$



At $\alpha = 0°$, the thin airfoil enacts no perturbations to the freestream flow. The black, flat line represents the thin airfoil, and the blue, horizontal lines are streamlines.
The flow travels from left to right on all plots, and the velocity vectors are omitted for neatness, since they act parallel to the streamlines.







At $\alpha > 0°$, the flow everywhere is disrupted by the presence of the thin airfoil. Even in a simplified model, there is evidence of a lift generating pressure gradient due to the dense streamlines below the LE and the less dense streamlines above.