$$a_\theta(t) = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} - r\,\sin(\theta)\,\cos(\theta)\,\dot{\phi}^2$$