$$\dot{\vec{e}}_\theta = \dot{\phi}\,\cos(\theta)\vec{e}_\phi - \dot{\theta}\,\vec{e}_r$$