$$a_{\theta}(t) = r \ddot{\theta} + 2 \dot{r} \dot{\theta} - r \sin(\theta) \cos(\theta) \dot{\phi}^2$$