

$$\frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{r}^2 \right) = m \dot{r} \ddot{r}$$