Homework Assignment 1

1 Readings

Read Chapters 5, 5.5, and 6 of Naked Stats. Read the "Regression to mean" and "Differential privacy" articles in the class website.

2 Senate Probabilities (Election 2014)

According to some analysis the control of the senate in the upcoming elections will be determined by the races in 3 states: Arkansas, Louisiana and North Carolina where 3 democratic incumbents face very competitive opponents. Based on predictions by experts at the NY Times, the Republicans have the following probabilities of wining each of these races: Arkansas 67%, Louisiana 61% and North Carolina 52%.

1. To win control of the senate, Republicans need to win at least two of these races. Based on the numbers above, what is the probability of the Republicans taking control of the senate?

2. The betting markets are currently trading at a 80% probability for the Republicans to control the senate. How does your answer from the question above compare to this number? Can you explain why you are seeing a difference? (*Hint:* Did you have to make any assumption to answer the first questions)?

3 Disease Testing - revisited

Suppose a person is randomly drawn from a large population and then tested for a disease.
Let $D = 1$ if the person has the disease and 0 otherwise.
Let $T = 1$ if the person tests positive and 0 otherwise.

Suppose
$P(D = 0) = .99$.
$P(T = 1 \mid D = 0) = .01$.
$P(T = 1 \mid D = 1) = .97$.

(a) Draw the diagram depicting the marginal of D and the conditional of $T \mid D$. (you know, the one that branches as you go left to right).

(b) Give the joint distribution of D and T in the two way table format.

(c) What is $P(D = 1 \mid T = 1)$?

4 Trump Election (From Jan 2016)

Based on betting markets the probability of Donald Trump being the Republican nominee is 25%. The same markets have the probability that the next President will be a Democrat at 62%.

Assume that if Trump is the nominee he has no chance of becoming the President... so, if the nominee is someone NOT Donald Trump, what is the probability of a Republican becoming the President?

5 Car or Goat

This problem is named after the host of the long running TV show *Let's make a deal.*
There has been a vigorous debate about what the correct answer is!!
A contestant must choose one of three closed doors.
There is a prize (say a car) behind one of the three doors.
Behind the other two doors, there is something worthless (traditionally a goat).
After the contestant chooses one of the three doors, Monty opens one of the other two, revealing a goat (never the car!!).
There are now two closed doors.
The contestant is asked whether he would like to switch from the door he intially chose, to the other closed door.

The contestant will get whatever is behind the door he has finally chosen.

Should he switch?

6 Spam Filter

Here's a simplified look at a spam filter algorithm...

We are worried about the term *"Nigerian general"* and our IT team has figured that $pr(\text{"Nigerian general"}|\text{junk mail}) = 0.20$ and $pr(\text{"Nigerian general"}|\text{NOT junk mail}) = 0.001$
In addition they figured that half of our emails is junk.

1. What is the marginal probability of seeing *"Nigerian general"* in a message?
   In other words, what is the $pr(\text{"Nigerian general"})$?

2. If the spam filter always classify a message containing *"Nigerian general"* as junk, how often will it make a mistake?
   In other words, what is the $pr(\text{NOT junk mail}|\text{"Nigerian general"})$?

7 To Drill or Not to Drill?

An oil company has purchased an option on land in Midland, TX. Preliminary geological studies have assigned the following probabilities of finding oil in the land:

$$Pr(\text{high quality oil}) = 0.5 \quad Pr(\text{medium quality oil}) = 0.2 \quad Pr(\text{NO oil}) = 0.3$$

After buying the option the company decided to perform a soil test. They found soil "type A". The probabilities of finding this particular type of soil are as follow:

$Pr(\text{soil} = \text{"type A"}|\text{high quality oil}) = 0.2$
$Pr(\text{soil} = \text{"type A"}|\text{medium quality oil}) = 0.8$
$Pr(\text{soil} = \text{"type A"}|\text{NO oil}) = 0.2$

1. Given the information from the soil test what is the probability the company will find oil in this land?

2. Before deciding to drill in the land the company has to perform a cost/benefit analysis of the project. They know it will cost \$1,000,000 to drill and start operating a well. In addition, under current oil prices, they access that if oil is found (any kind) the revenue stream will be of \$1,500,000. Should they exercise the option, ie, should they drill?

8 Normal Distribution Calculations

$X \sim N(5, 10)$ (Read X distributed Normal with mean 5 and var 10) Compute:

(i) Prob$(X > 5)$

(ii) Prob$(X > 5 + 2 \times \sqrt{10})$

(iii) Prob $(X = 8)$

(iv) Express Prob$(-2 \leq X \leq 6)$ in terms of Z, the standard normal random variable.

9 Downside Risk

A company can purchase raw material from either of two suppliers and is concerned about the amounts of impurity the material contains. A review of the records for each supplier indicates that the percentage impurity levels in consignments of the raw material follow normal distributions with the means and standard deviations given in the table below. The company is particularly anxious that the impurity level in a consignment not exceed 5% and want to purchase from the supplier more likely to meet that specification. Which supplier should be chosen?

	Mean	Standard Deviation
Supplier A	4.4	0.4
Supplier B	4.2	0.6

10 One vs Multiple Projects

Using the simulation tool in Excel, simulate the potential outcomes associated with taking the 23 projects as described in Section 1.1. Do your numbers agree (approximately) with the calculations in the notes (mean, variance, probability of losing money)? Also compare to the normal approximation numbers we got in the slides in Section 1.1. Close enough?

11 Retirement Simulation

Assume my current retirement account has $100,000 invested in the the S&P500. You are my financial advisor and I need you to provide me with a plausible range for my investment by the end of my working life (say 30 years from now).

What is the chance I will outperform a fixed income investment with a yield of 2% per year?

12 Portfolios

I am interested in building a portfolio of stocks and bonds... a very convenient way is to invest in two ETFs (Exchange Traded Funds). Let's choose VTI and VGLT as the ETFs. What allocation between the two assets gives you the best Sharpe Ratio? If you decide on a 50-50 allocation, what is the probability you will get a return larger than 1% next month? (You will need to get data on this... use monthly returns in the last 5 years. Yahoo Finance is a great resource!)

13 Option Pricing

1. Revisit the Nvidia option pricing example in the Section 1. What is the "fair price of the option" if the we change the potential stock price upwards move to $1,100 and the potential downside to $700? Does that make sense?

2. In the same Nvidia example, what happens to the option price if the probability of a upwards stock price move is 60%? Does that make sense?

3. (Optional) Revisit the option pricing simulation exercise we ran in class... How does the option price changes if we increase the horizon of the option? How does the price change if we increase the variance of each step? How does the price change if we assume that at each step the probability of an upward move is higher in each step?