Wake Model Description for Case Study 3

IEA Task 37 on System Engineering in Wind Energy

This is an explanatory enclosure to accompany `iea37-cs3-announcement.pdf`. For case study 3 (cs3), we will use the enclosed Python file `iea37-aepcalc.py` to evaluate your reported optimal turbine locations in `.yaml` format. If you desire to implement the AEP calculations in a language other than Python, the algorithm's description and wake model equations are provided below. Please insure your implementation computes the same AEP value given in the baseline layout (`iea37-ex-opt3.yaml`) also enclosed.

Wake Model Equations

The wake model for cs3 is a simplified version of Bastankhah's Gaussian wake model [1]. The governing equations for the velocity deficit in a waked region are:

$$\frac{\Delta V}{V_\infty} = \begin{cases} \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y_i - y_g}{\sigma_y}\right)^2\right), & \text{if } (x_i - x_g) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\sigma_y = k_y \cdot (x_i - x_g) + \frac{D}{\sqrt{8}} \tag{2}$$

Variable	Value	Definition
$\Delta V/V_\infty$	Eq. (1)	Normalized wake velocity deficit
C_T	8/9	Thrust coefficient
$x_i - x_g$	-	Dist. from hub generating wake (x_g) to hub of interest (x_i), along freestream
$y_i - y_g$	-	Dist. from hub generating wake (y_g) to hub of interest (y_i), $\perp$ to freestream
σ_y	Eq. (2)	Standard deviation of the wake deficit
k_y	0.0324555	Variable based on a turbulence intensity of 0.075 [1, 2]
D	198 m	Turbine diameter [3]

The two cases in the wake velocity equation are needed because wakes are assumed to only affect points downstream. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the total velocity deficit is calculated using the square root of the sum of the squares:

$$\left(\frac{\Delta V}{V_\infty}\right)_{total} = \sqrt{\left(\frac{\Delta V}{V_\infty}\right)_1^2 + \left(\frac{\Delta V}{V_\infty}\right)_2^2 + \left(\frac{\Delta V}{V_\infty}\right)_3^2 + \ldots} \tag{3}$$

AEP Algorithm

1. Read the following input from `.yaml` files:
   - Turbine (x,y) locations.
   - Turbine attributes (cut-in\cut-out\rated wind speed\rated power).
   - Number of wind directional bins, n ($n = 60$ for cs3).
   - Wind frequency at each binned direction, $f(\theta)$.
   - Number of speed bins to compute, m ($m = 60$ for cs3).
   - Wind speed at each binned direction, $w(\theta)$.
2. Calculate the power produced in the farm for one wind direction:

(a) For each binned direction θ, rotate the turbine locations (x, y) into the wind frame of reference (x_w, y_w):

$$\begin{aligned}\Psi &= -\left(\frac{\pi}{2} + \theta\right) \\ x_w &= x\cos(\Psi) + y\sin(\Psi) \\ y_w &= -x\sin(\Psi) + y\cos(\Psi)\end{aligned}$$

(b) For each wind speed bin as V_∞, iterate through each turbine in the field to compute its power:

- Compute the wake deficit between each turbine pair Eq. (1) (there is no wake effect of a turbine on itself).
- Use Eq. (3) to calculate the total wake loss.
- Compute effective wind speed (V_e) at each turbine:

$$V_e = V_\infty \left[1 - \left(\frac{\Delta V}{V_\infty}\right)_{total}\right]$$

- Use V_e and the IEA37 10MW power curve to calculate each turbine's power:

$$P_{turb}(V_e) = \begin{cases} 0 & V_e < V_{cut\text{-}in} \\ P_{rated} \cdot \left(\frac{V_e - V_{cut\text{-}in}}{V_{rated} - V_{cut\text{-}in}}\right)^3 & V_{cut\text{-}in} \leq V_e < V_{rated} \\ P_{rated} & V_{rated} \leq V_e < V_{cut\text{-}out} \\ 0 & V_e \geq V_{cut\text{-}out} \end{cases} \tag{4}$$

(c) Sum powers from all t turbines for that wind speed

$$P_{speed} = \sum_{k=1}^{t} P_{turb,k} \tag{5}$$

(d) Sum powers for all m wind speed bins for that wind direction, multiplied by the probability that each speed will occur

$$P_{dir} = \sum_{j=1}^{m} w_j P_{speed,j} \tag{6}$$

3. Compute AEP using farm power for all n directions where P is the wind farm power for direction i and f is the corresponding frequency for direction i. The factor of 8760 is to multiply by hours in a year:

$$AEP = \left(\sum_{i=1}^{n} f_i P_{dir,i}\right) 8760 \frac{\text{hrs}}{\text{yr}} \tag{7}$$

References

[1] Thomas, J. J. and Ning, A., "A method for reducing multi-modality in the wind farm layout optimization problem," *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.

[2] Niayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.

[3] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., "IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines," Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.