3 Self-organized criticality: the Oslo model

This section provides the tasks and questions related to implementing and analyzing the Oslo model of self-organized criticality. **The tasks should be finished till 21.11.2024**.

The Oslo rice-pile model is a theoretical model of self-organized criticality (SOC) used to study the behavior of granular materials and avalanche-like phenomena in a simple, discrete system. It was introduced as a variation of the sandpile model, and it's named after the city of Oslo, Norway, where it was developed.

1. Implement the Oslo model using the following algorithm focusing on slopes z_i:
   (a) Initialize the system in arbitrary stable configuration $z_i \leqslant z_i^T$, where z_i^T is i-th slope threshold $\in \{1, 2\}$;
   (b) Drive the system by adding a grain at left-most site;
   (c) If $z_i > z_i^T$, relax the site i,

   $$\begin{aligned} &\text{for} \quad i = 1 : z_1 \to z_1 - 2, z_2 \to z_2 + 1; \\ &\text{for} \quad i = 2 \ldots L-1 : z_i = z_i - 2, z_{i\pm1} \to z_{i\pm1} + 1; \\ &\text{for} \quad i = L : z_L \to z_L - 1, z_{L-1} \to z_{L-1} + 1. \end{aligned}$$

   During relaxation do not forget about choosing randomly new threshold $z_i^T \in [1, 2]$ for the relaxed site. Continue relaxation until $z_i \leqslant z_i^T$ for all i;
   (d) Return to point (b).
2. Plot scaled avalanche size s/s_{max} in function of time t (measured in terms of grain additions). Does it makes sense to analyze data for small t? Which condition should be satisfied in avalanche size statistical analysis?
3. Plot in log-log scale avalanche size probability $P(s, L)$ with respect to avalanche size s for several lengths of the system (L should be at least 64). Do you observe power law behavior? Why this power law breaks for large s?