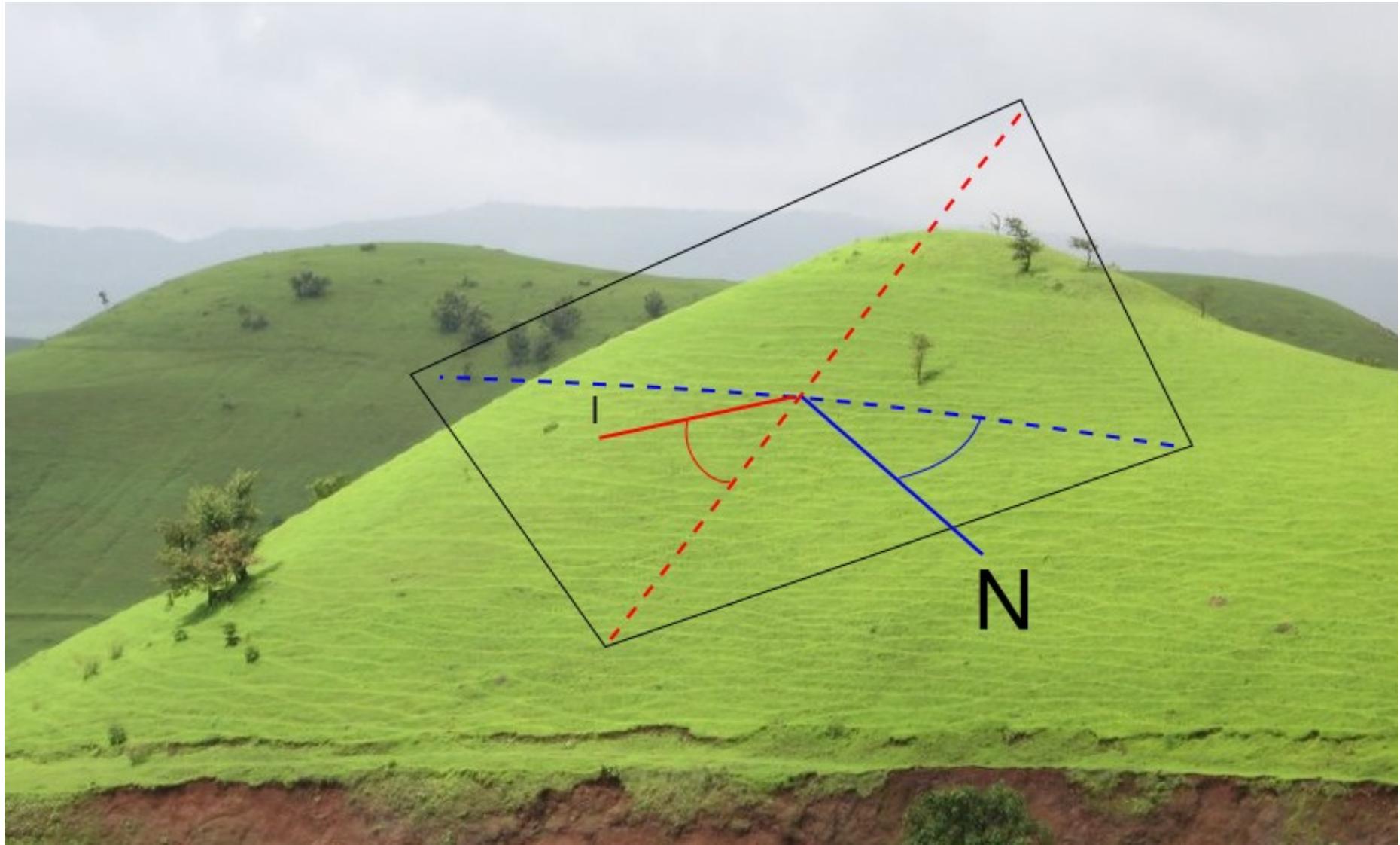


Pochodne terenu



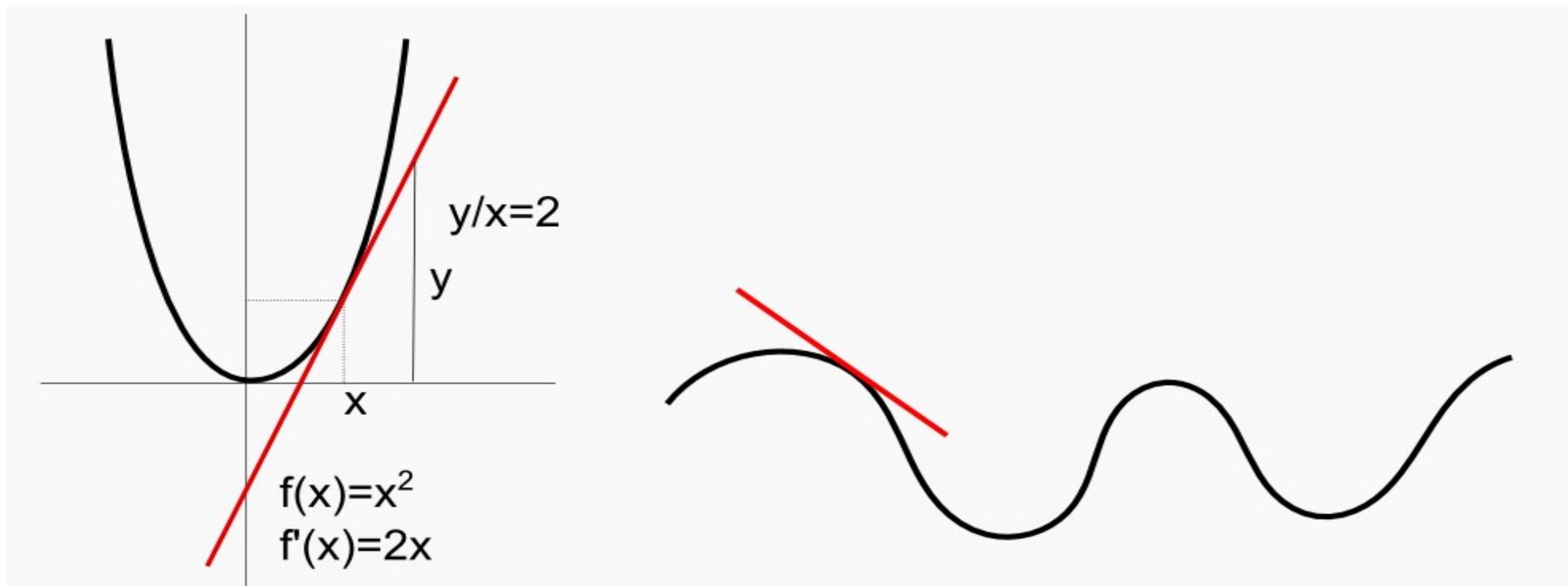
Nachylenie i ekspozycja

slope & aspect



Pojęcie pochodnej

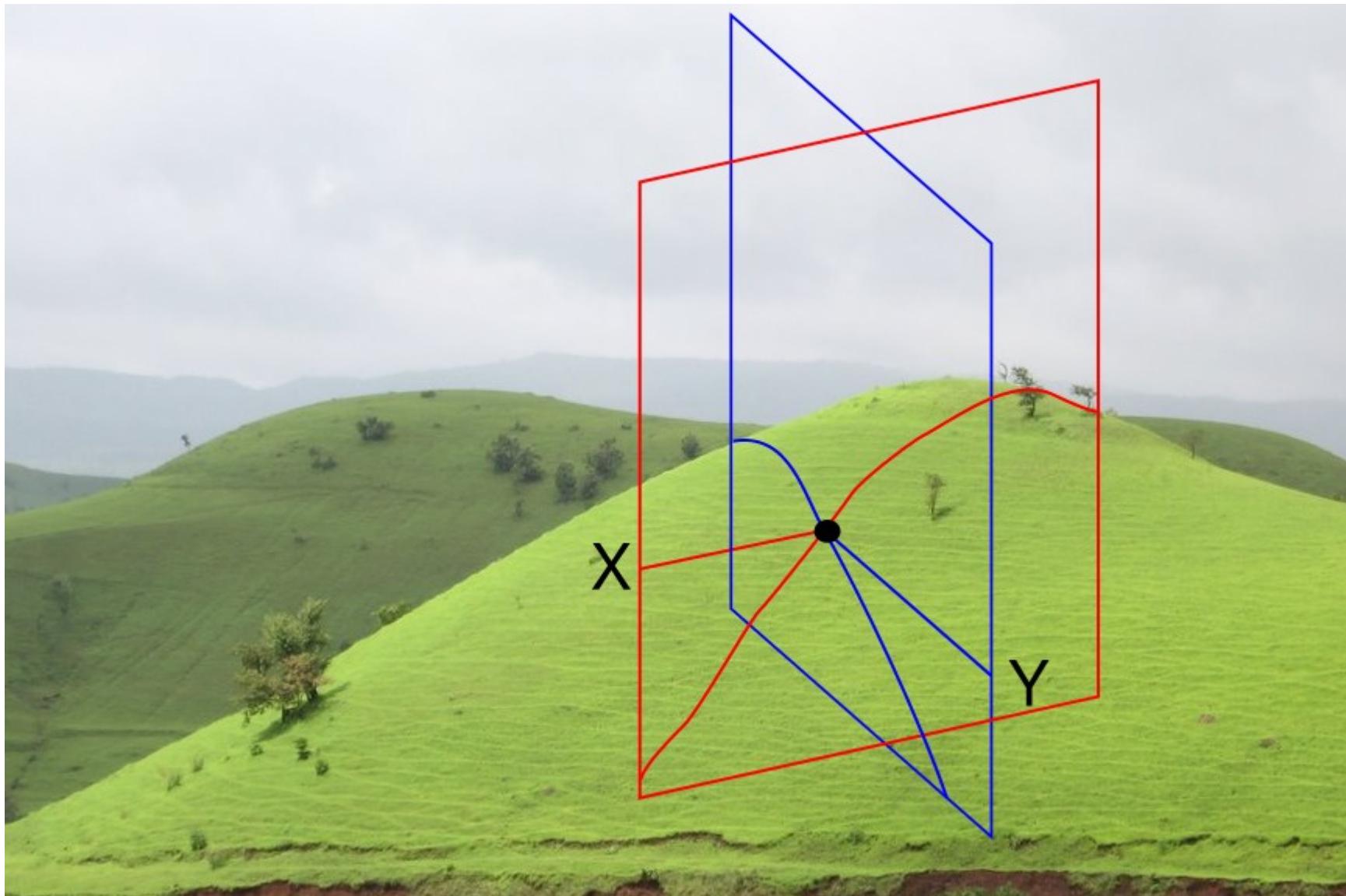
- Czym jest pochodna?
- W modelu 1d – styczna do krzywej w punkcie.
- Pochodna oznacza „wielkość zmiany wartości funkcji”
- Wartość pochodnej - nachylenie krzywej



Pochodne dla powierzchni

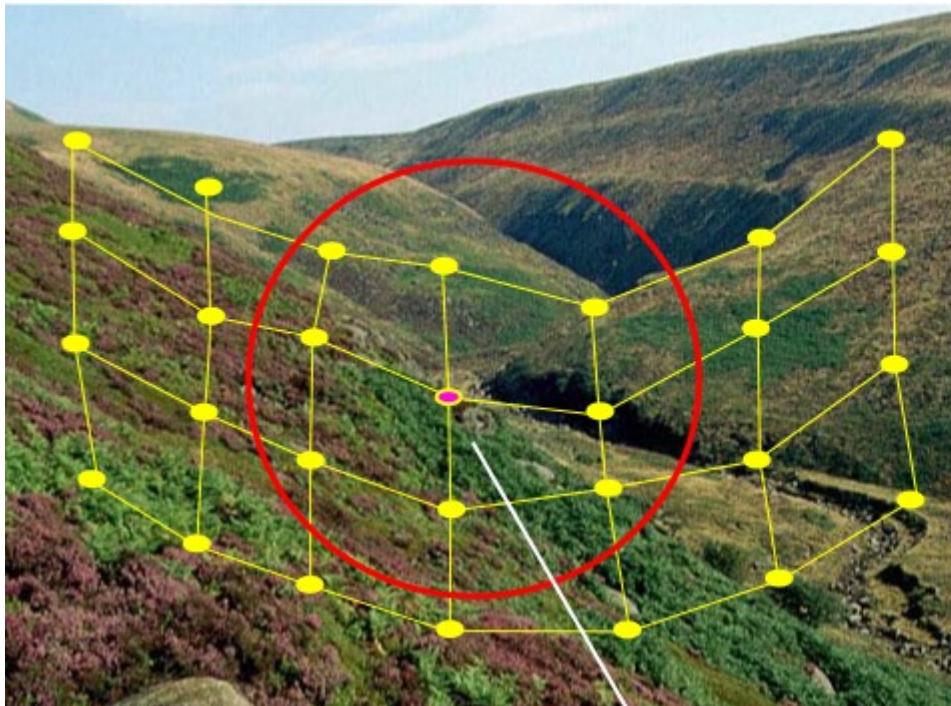


Pochodne cząstkowe w kierunkach głównych

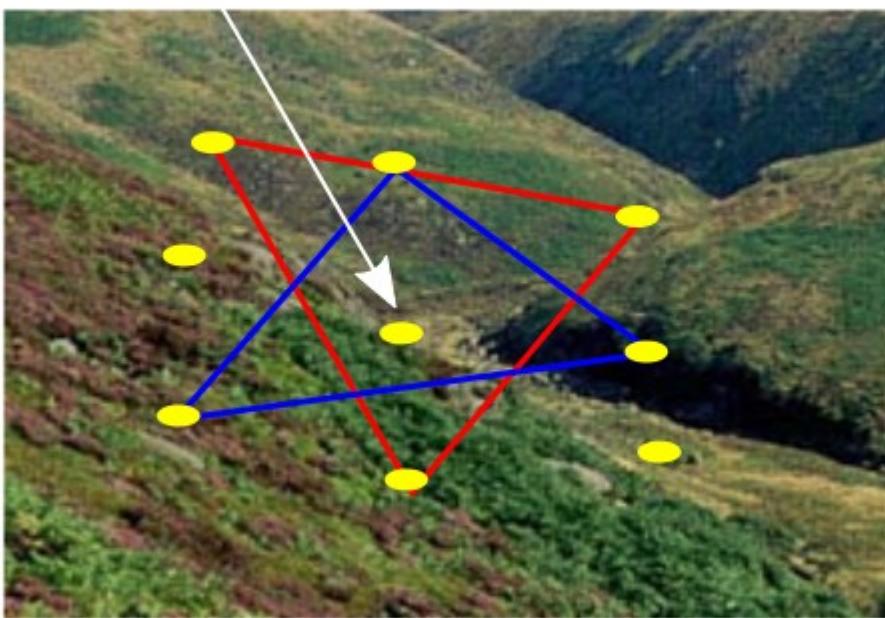
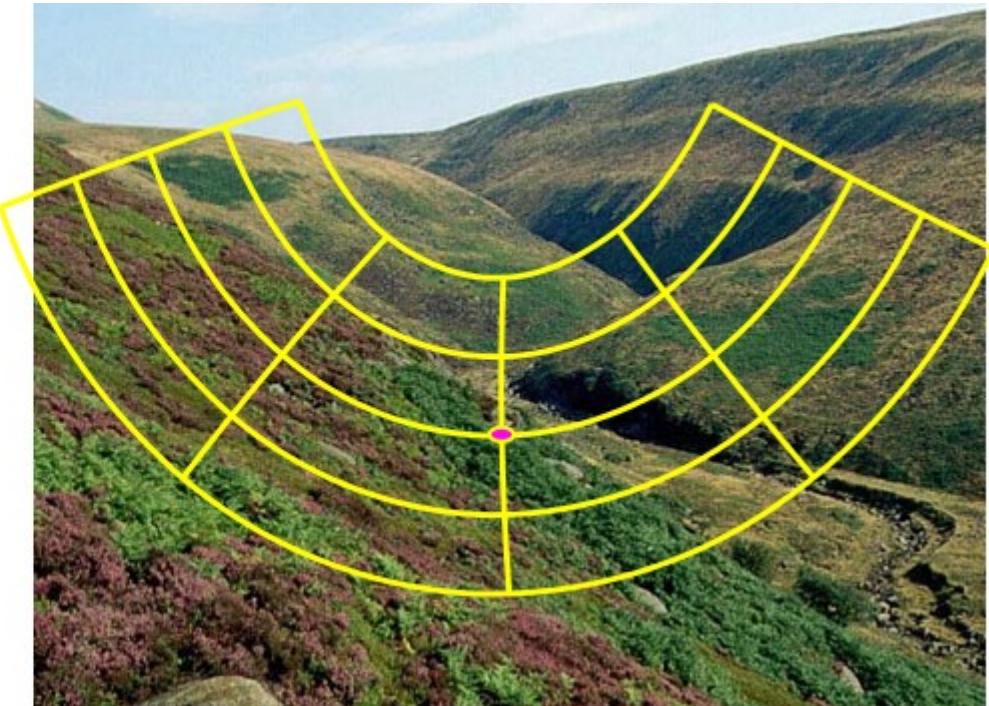


Sposoby obliczania pochodnych

Dla powierzchni wyznaczonej siatką



Dla powierzchni wyznaczonej wielomianem



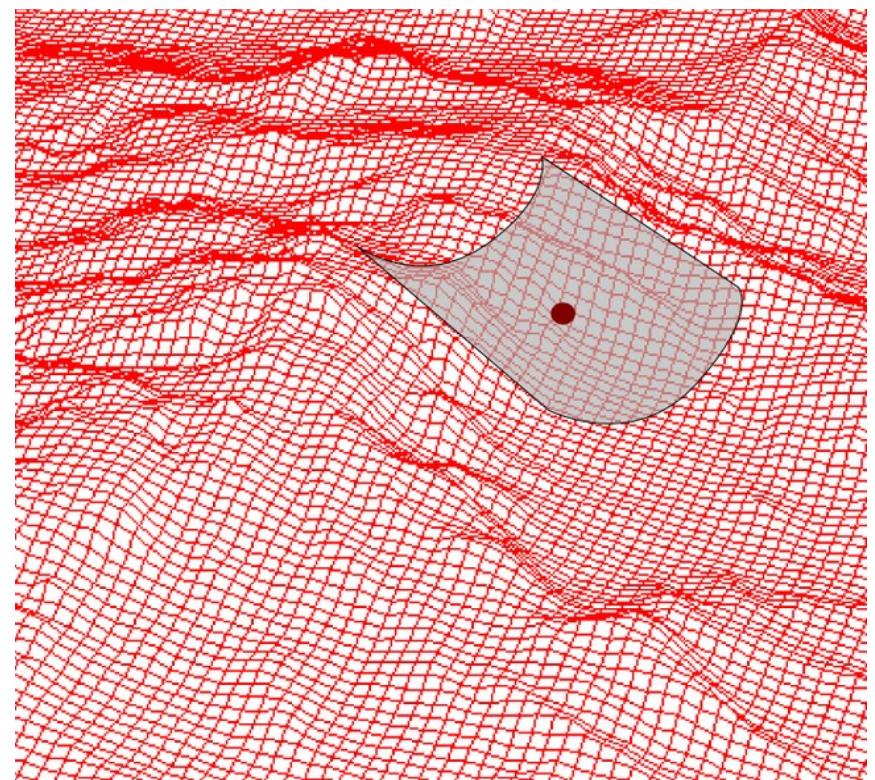
Obliczanie dwuwymiarowych pochodnych dla dopasowań ciągłych

$$ax^2+by^2+cxy+dx+ey+f=0$$

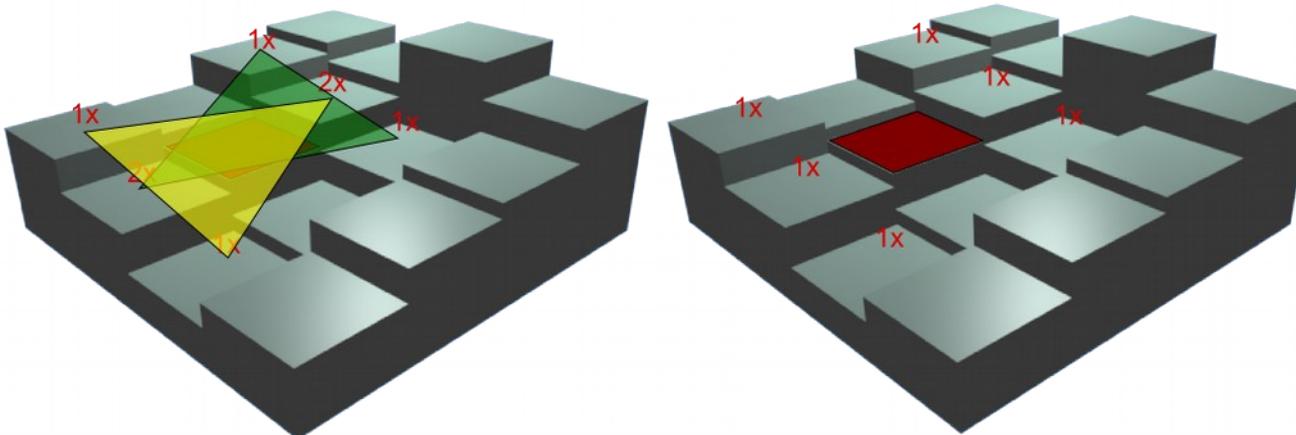
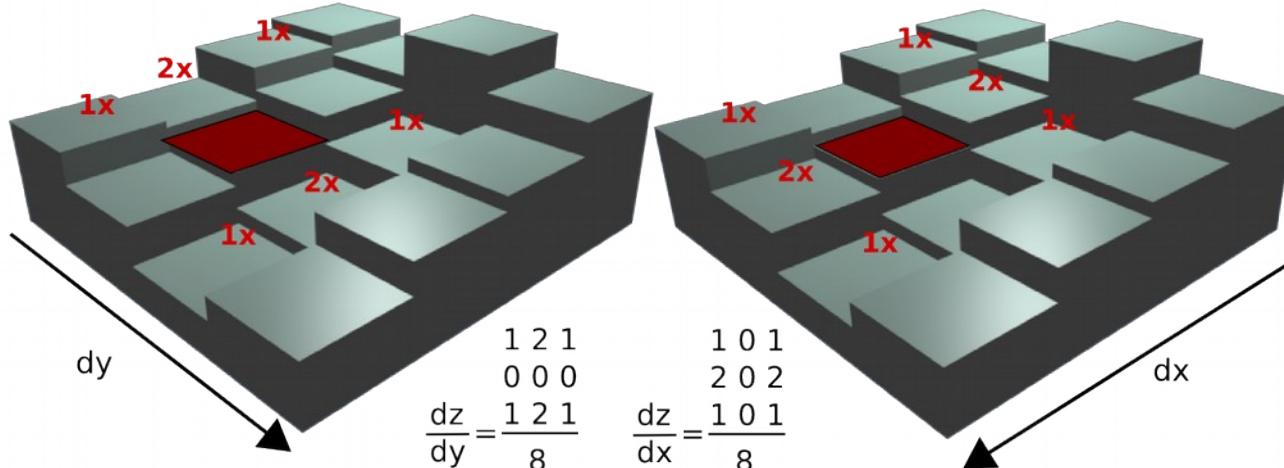
$$\nabla \bar{Z} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right)$$

$$\begin{aligned}\frac{\partial z}{\partial x} &\approx \frac{\partial(a \cdot x^2 + b \cdot y^2 + c \cdot xy + d \cdot x + e \cdot y + f)}{\partial x} \\ &= 2 \cdot a \cdot x + c \cdot y + d\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &\approx \frac{\partial(a \cdot x^2 + b \cdot y^2 + c \cdot xy + d \cdot x + e \cdot y + f)}{\partial y} \\ &= 2 \cdot b \cdot y + c \cdot x + e\end{aligned}$$



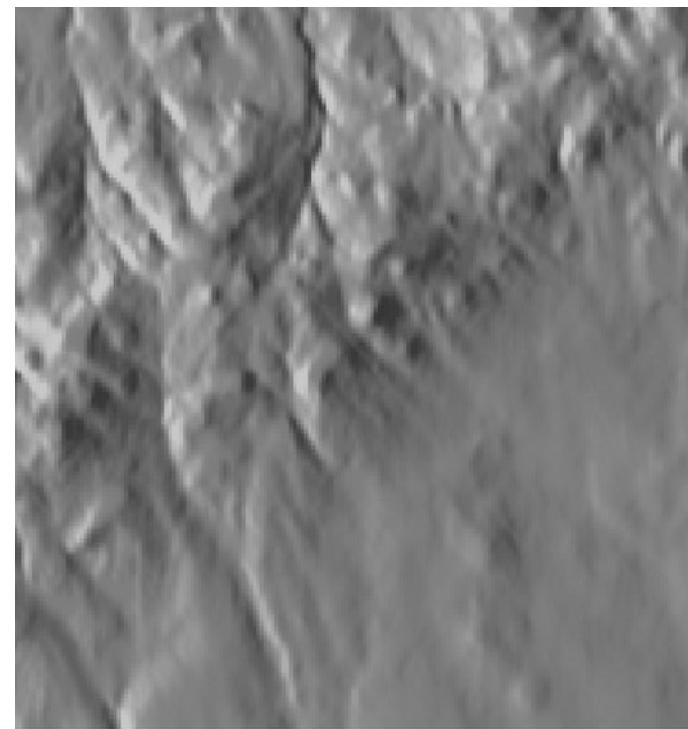
Obliczanie dwuwymiarowych pochodnych dla reprezentacji dyskretnych



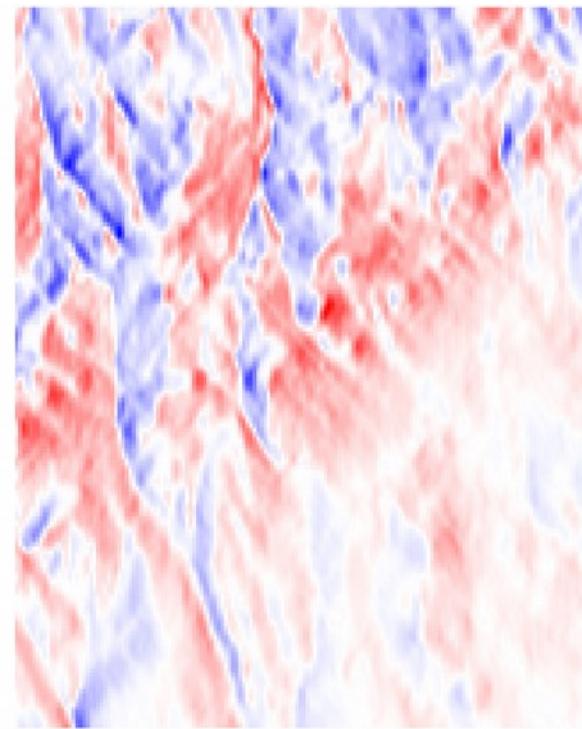
$$d = \frac{\partial z}{\partial x} = \frac{z_3 + z_6 + z_9 - z_1 - z_4 - z_7}{6 \cdot \Delta s}$$
$$e = \frac{\partial z}{\partial y} = \frac{z_1 + z_2 + z_3 - z_7 - z_8 - z_9}{6 \cdot \Delta s}$$

I pochodna terenu

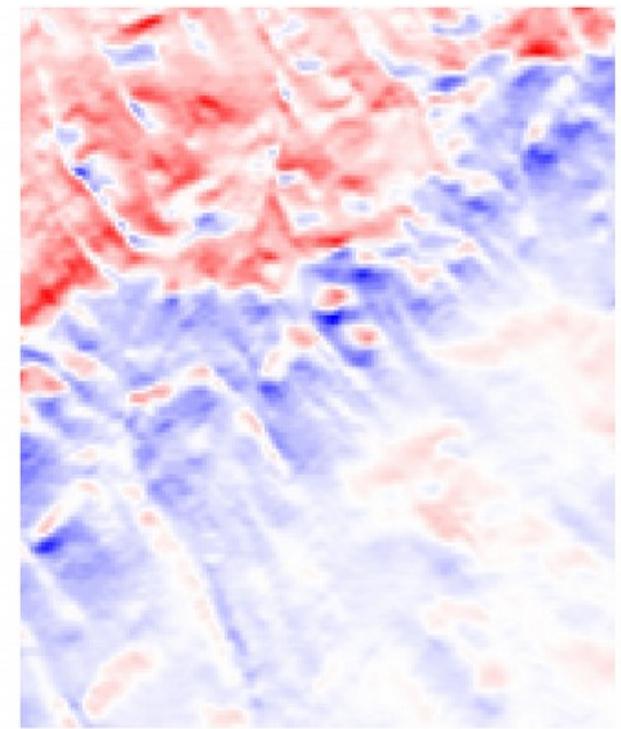
teren



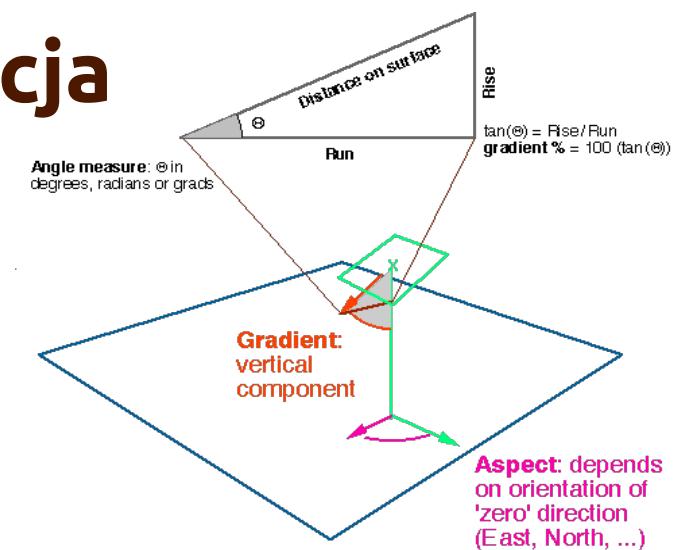
dx



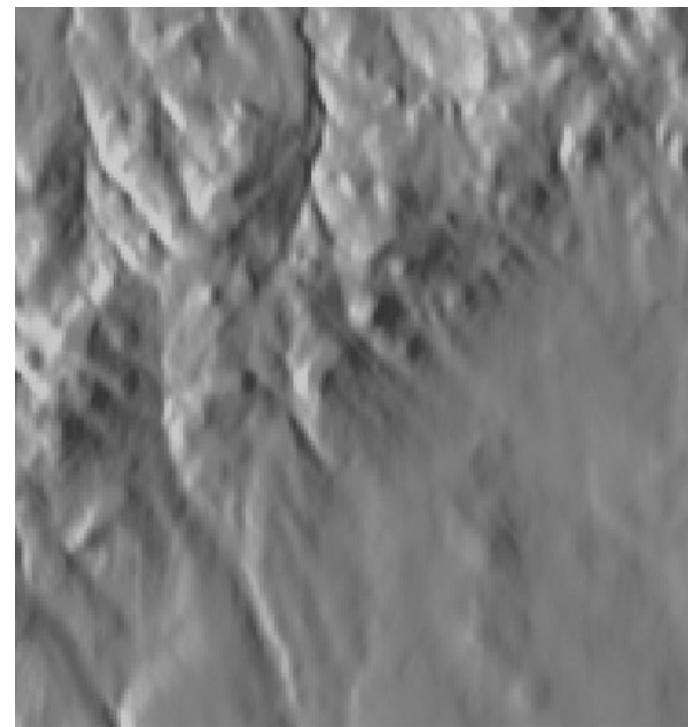
dy



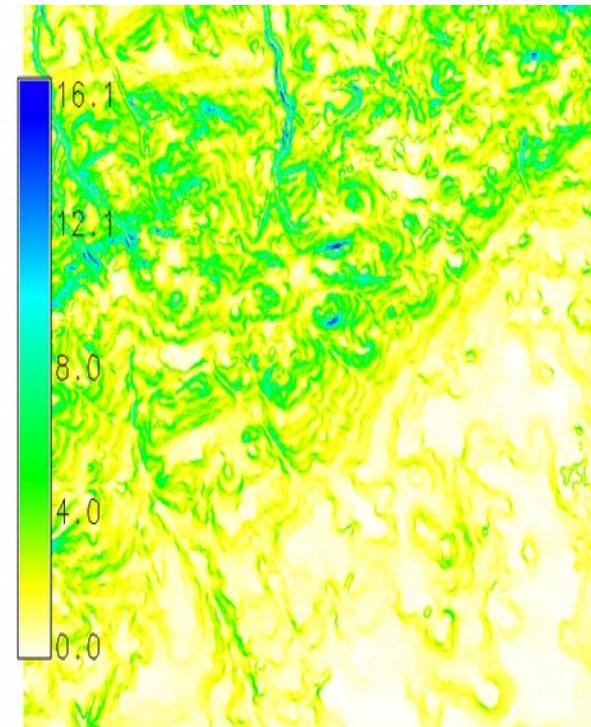
nachylenie i ekspozycja



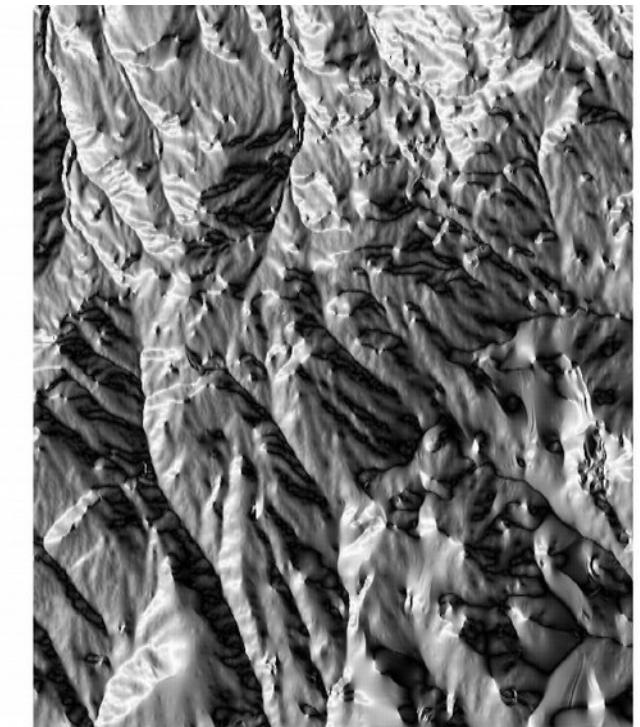
teren



slope



aspect



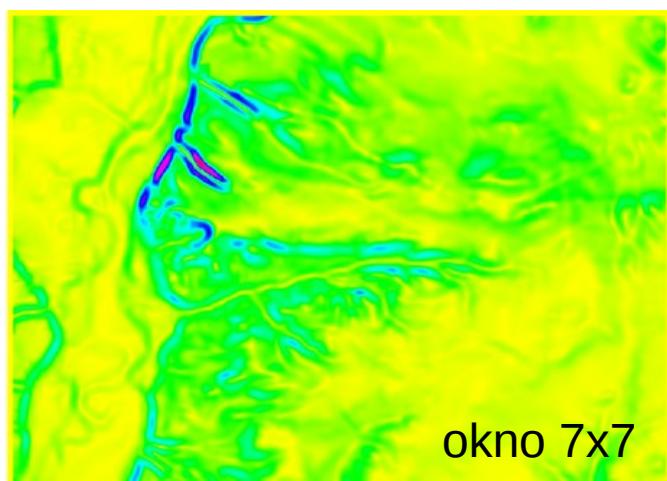
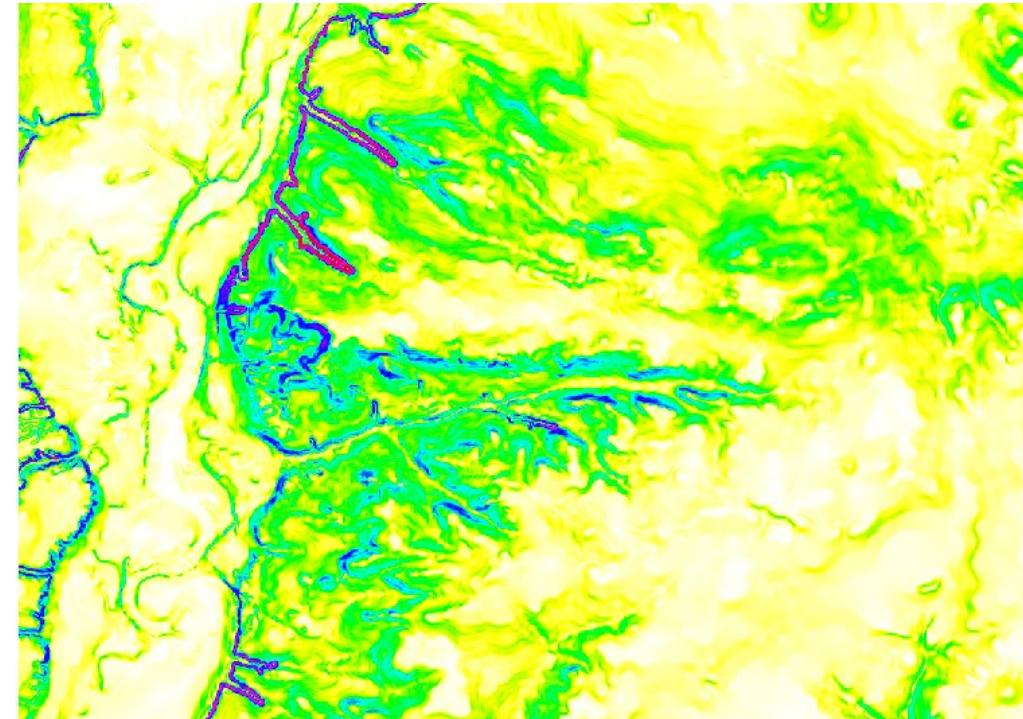
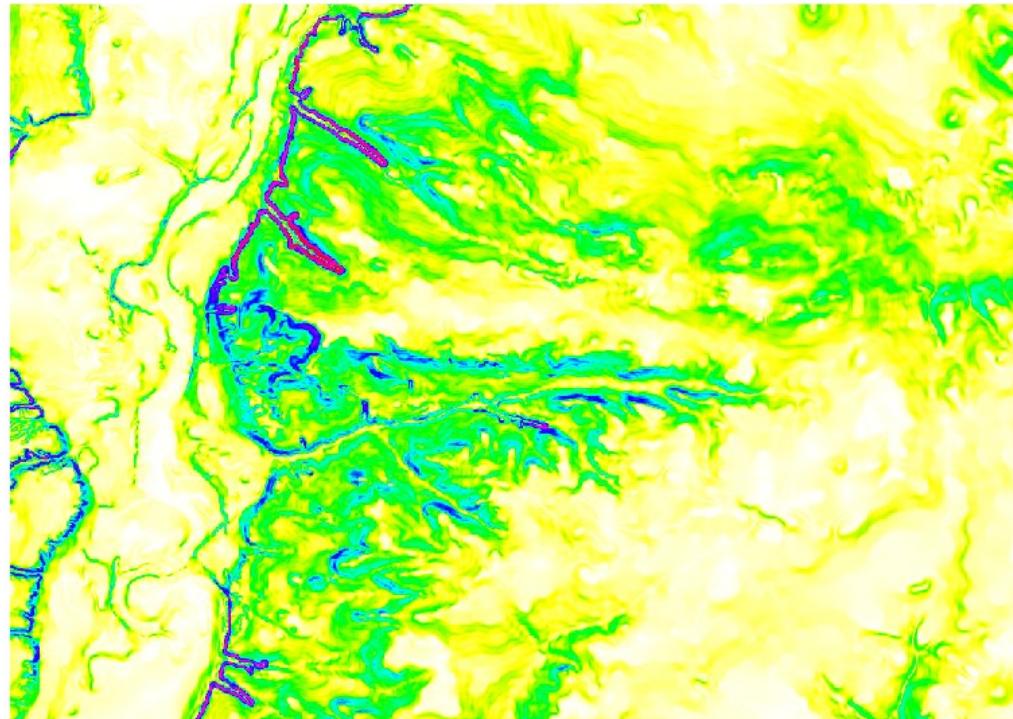
$$\text{SLOPE} = \arctan(\sqrt{d^2 + e^2})$$

$$\text{ASPECT} = 180 - \arctan\left(\frac{d}{e}\right) + 90 \cdot \frac{d}{|d|}$$

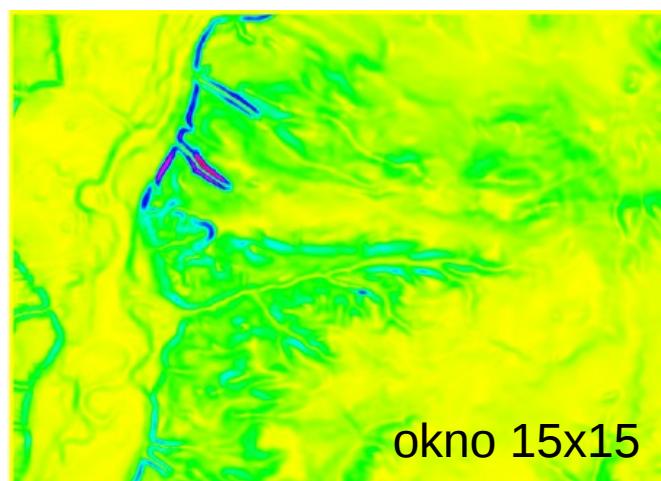
Różne metody

Dyskretna

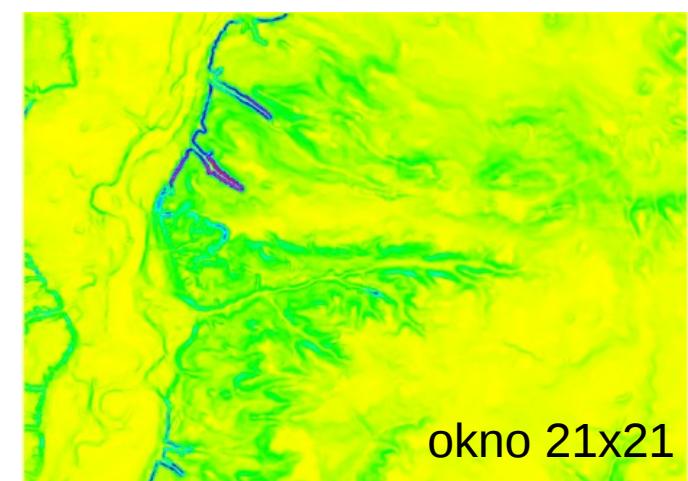
Wielomian



okno 7x7



okno 15x15

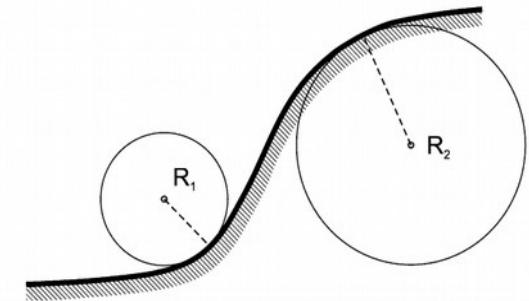
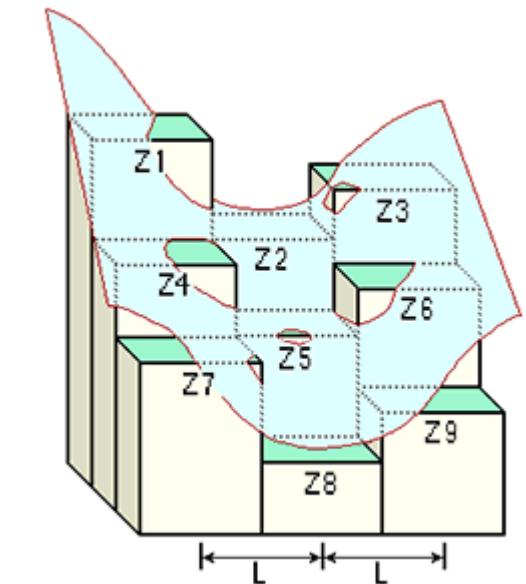
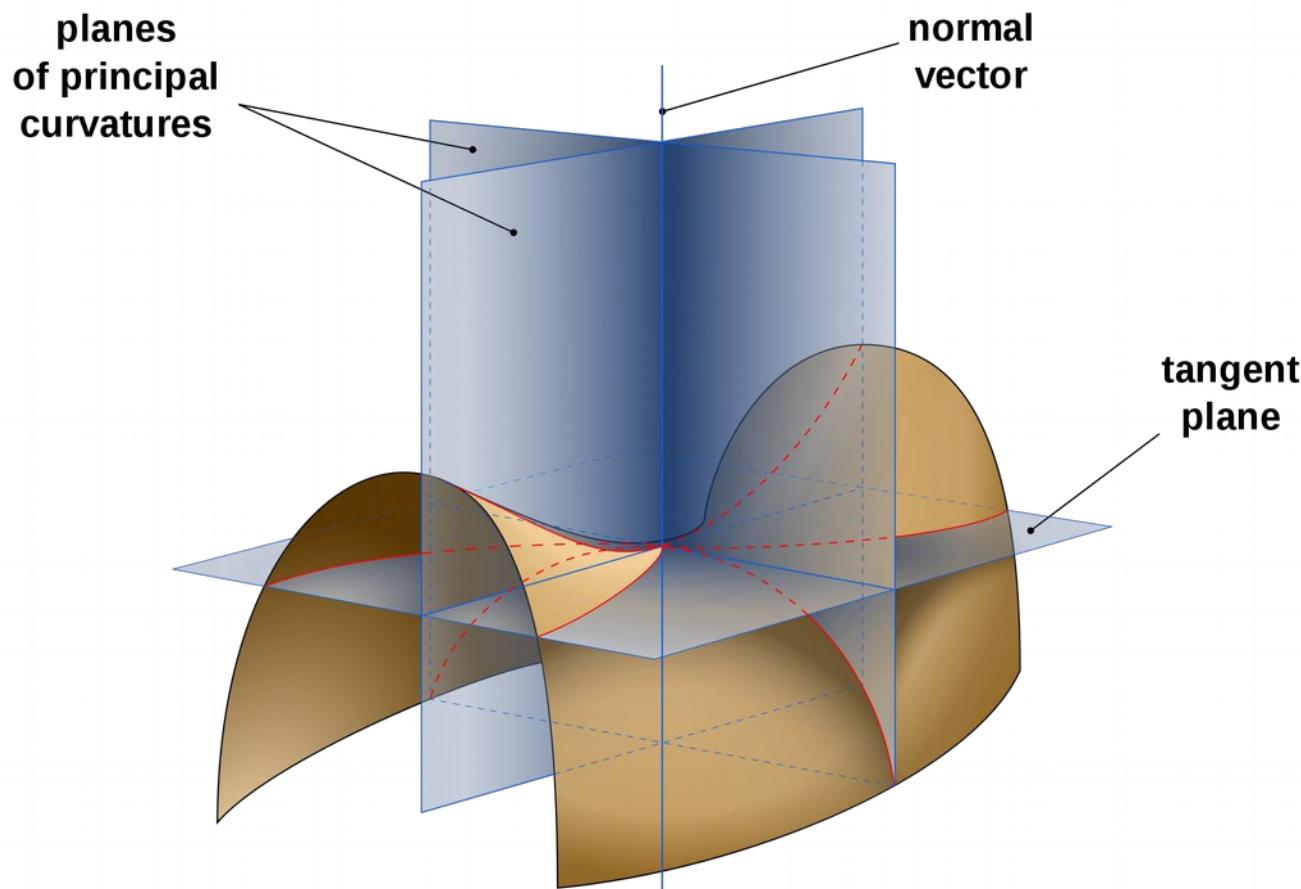


okno 21x21

Stok nie jest jednolity



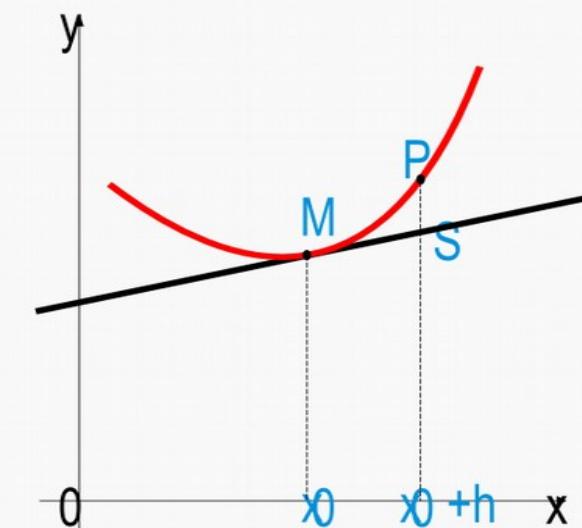
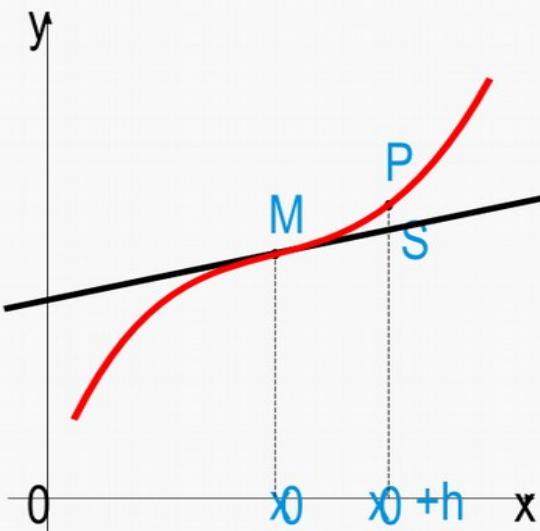
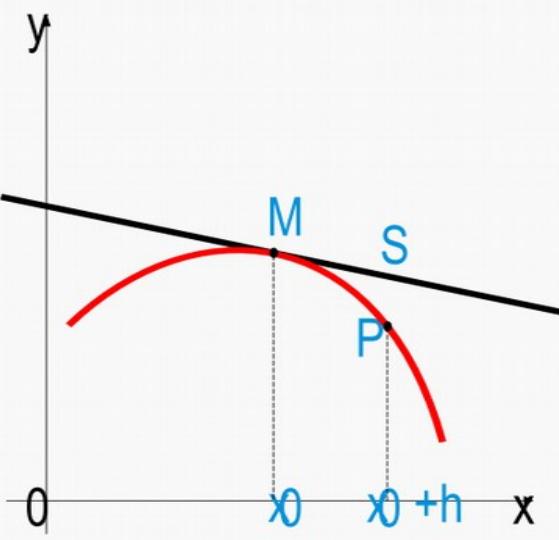
Graficzna ilustracja krzywizny w przekroju



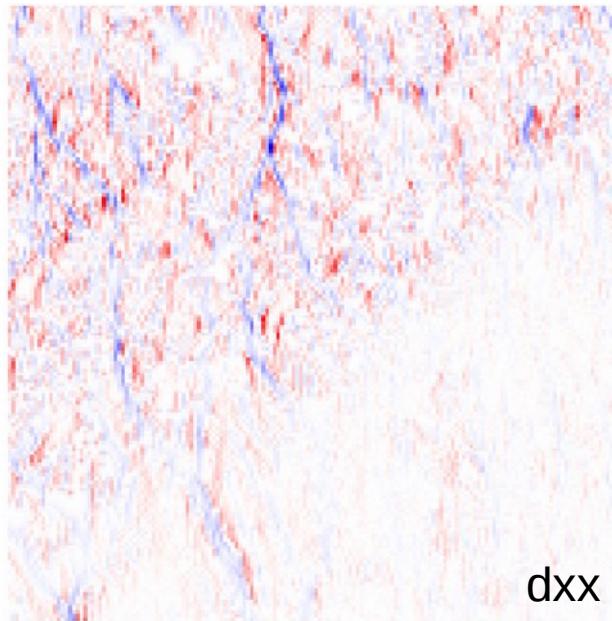
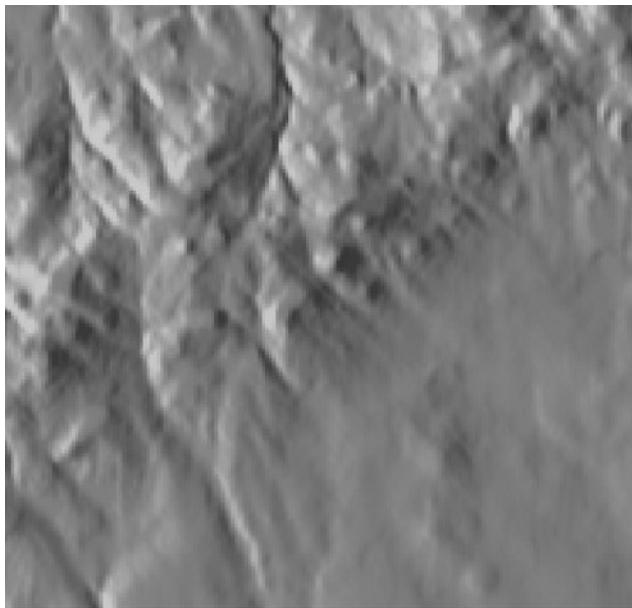
miara krzywizny

II pochodna

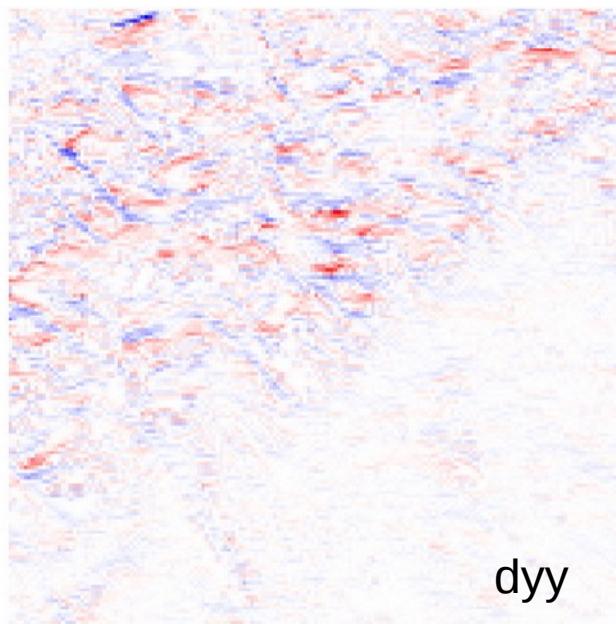
- Druga pochodna jest różniczką I pochodnej
- Druga pochodna jest definiowana jako miara wypukłości/wklesłości funkcji
- krzywizny są wyliczane z drugiej pochodnej



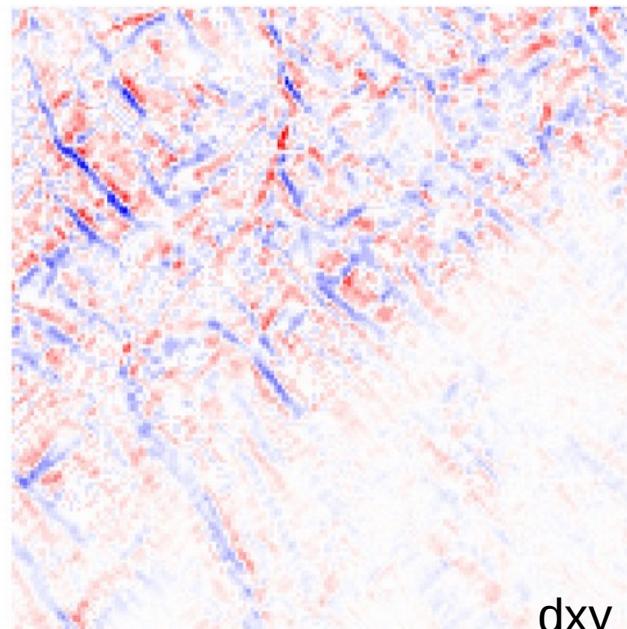
II pochodna terenu



d_{xx}

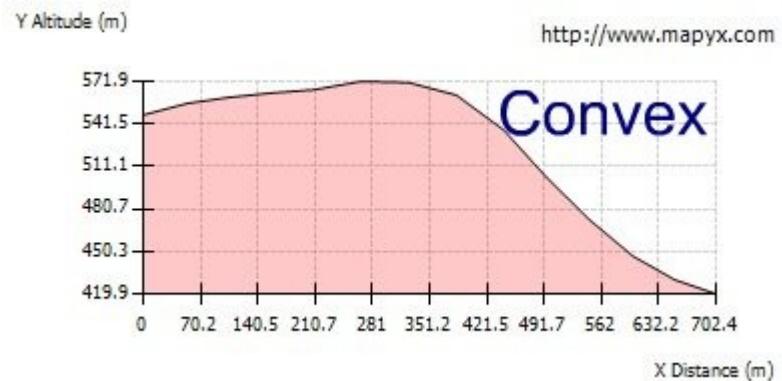
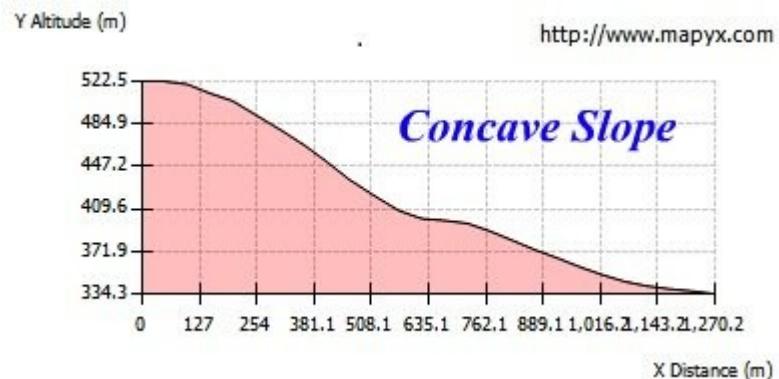
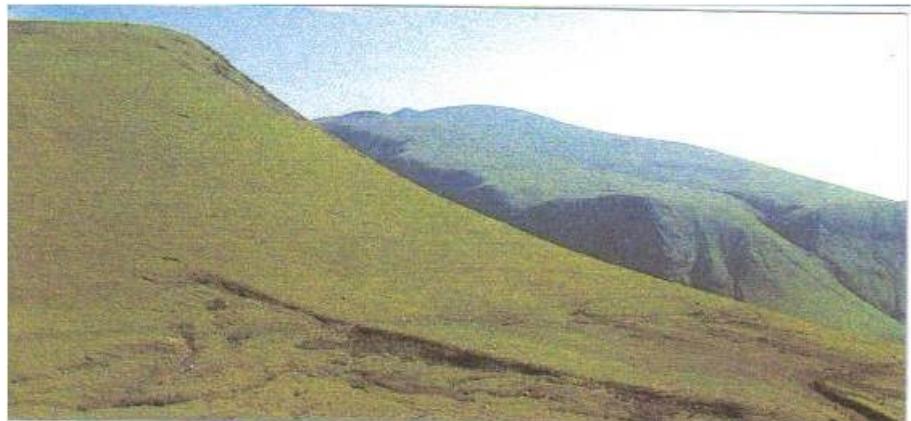


d_{yy}

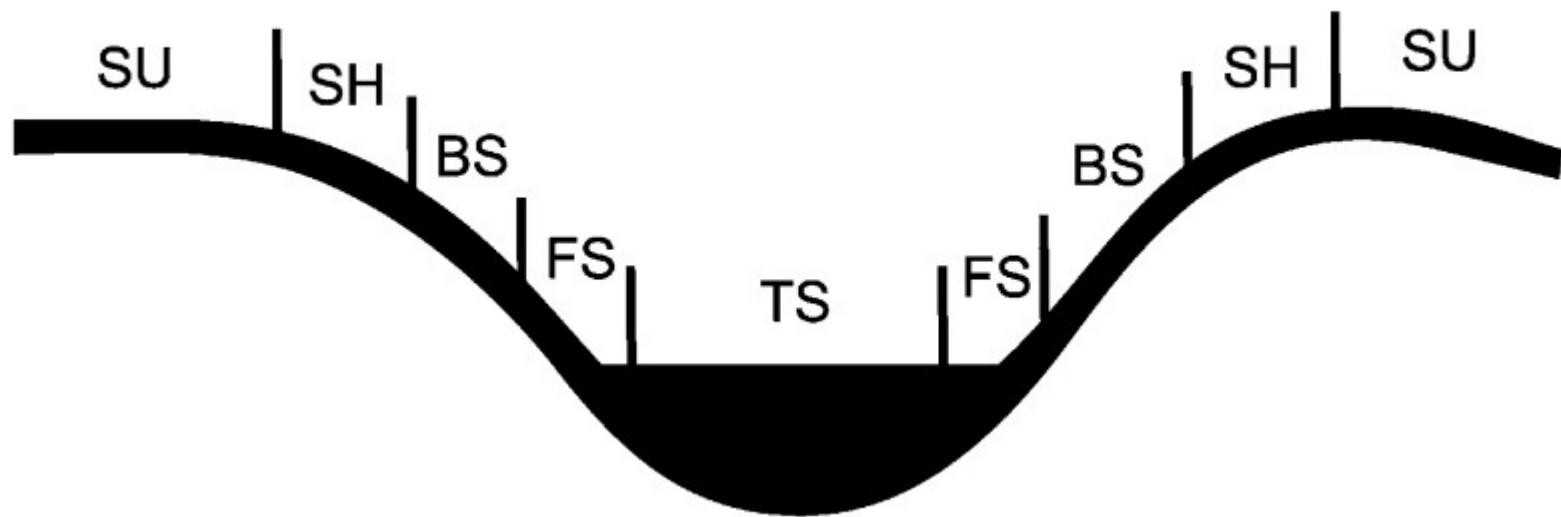


d_{xy}

Stok wklęsły stok wypukły (profil)

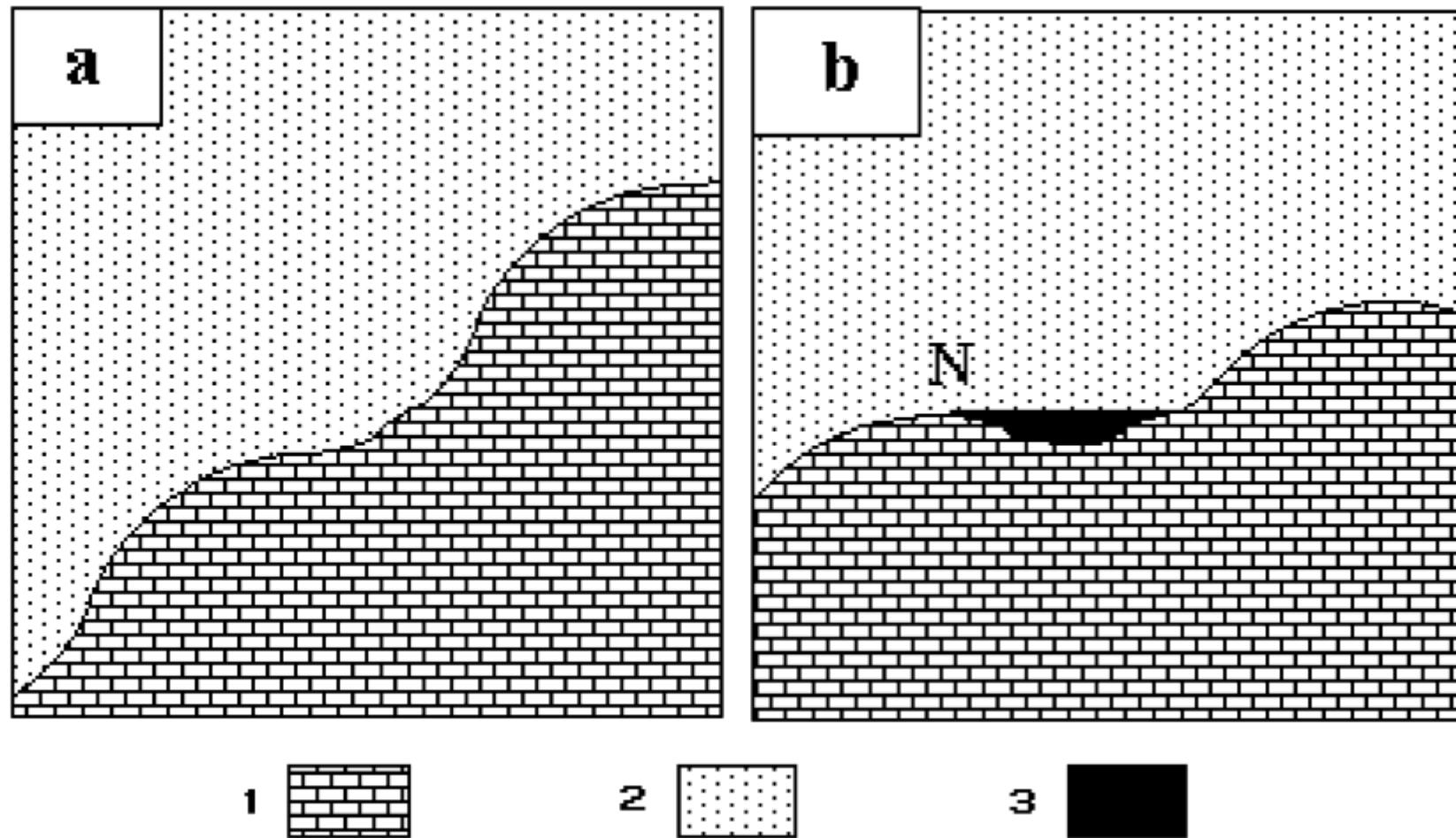


jednostki terenu a krzywizny

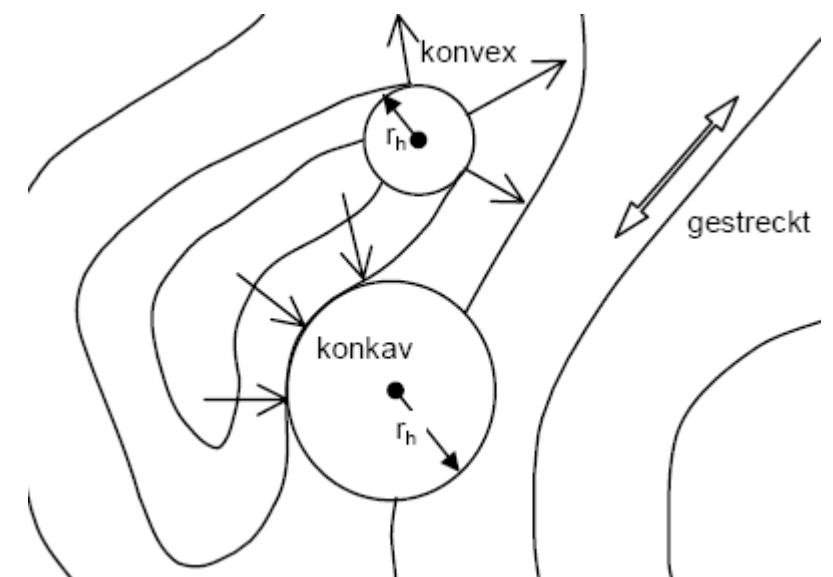
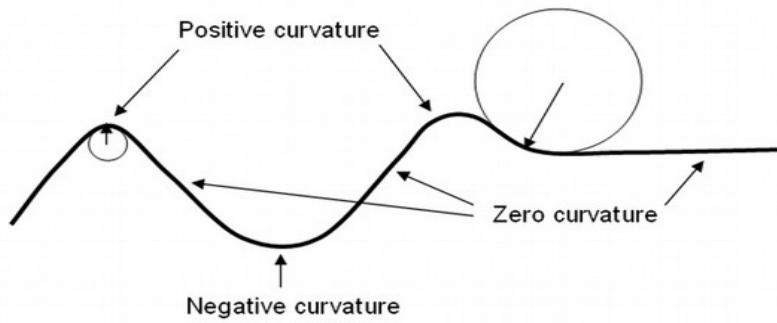
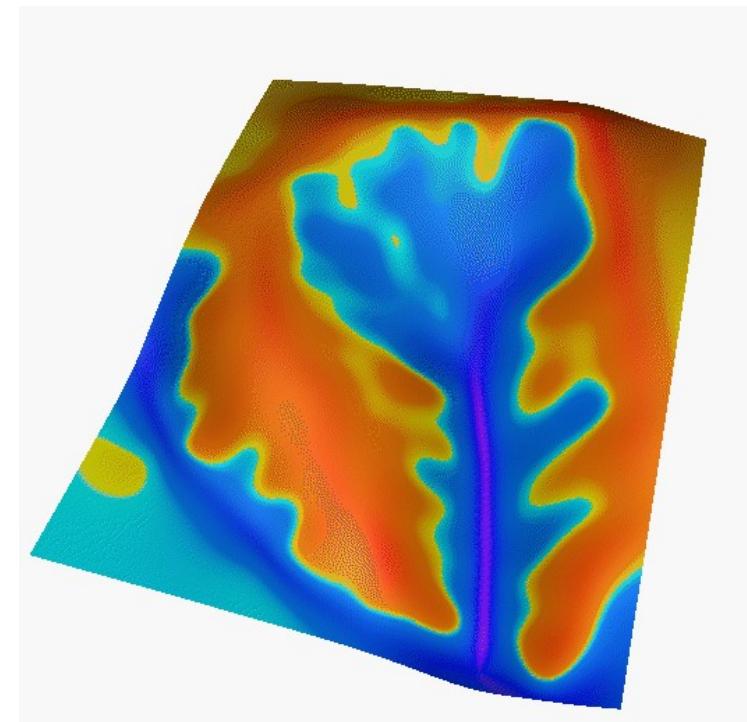


summit, shoulder, backslope, footslope

Krzywizny w polu grawitacyjnym

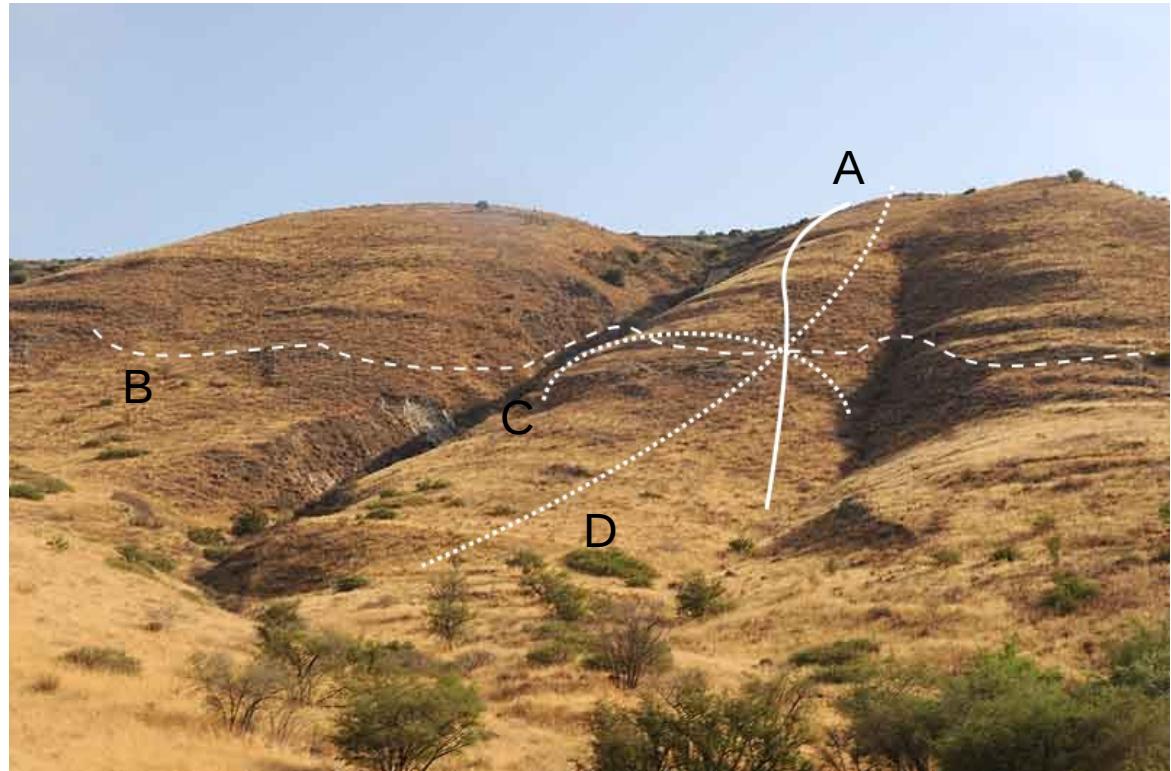
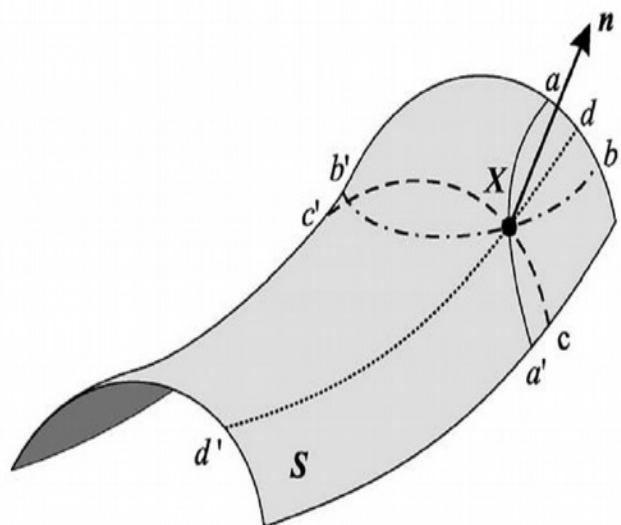


Stok wklęsty stok wypukły (styczna)



Cztery kierunki na powierzchni

- aa' - gradient
- bb' - poziomica
- cc',dd' - główne przekroje prostopadłe
- n kierunek prostopadły (normalna)



krzywizna na płaszczyźnie

- płaszczyzna krzywizny
-
- krzywizna jako funkcja 1 i 2 pochodnej:

$$f(p, q, r, s, t)$$

$$k = \frac{\frac{d^2 z}{dz^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1.5}}$$

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y} \quad r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}$$

obliczanie parametrów

$$d = \frac{\partial z}{\partial x} = \frac{z_3 + z_6 + z_9 - z_1 - z_4 - z_7}{6 \cdot \Delta s}$$

$$e = \frac{\partial z}{\partial y} = \frac{z_1 + z_2 + z_3 - z_7 - z_8 - z_9}{6 \cdot \Delta s}$$

$$r = \frac{z_1 + z_3 + z_4 + z_6 + z_7 + z_9 - 2 \cdot (z_2 + z_5 + z_8)}{3 \cdot \Delta s^2}$$

$$s = \frac{z_3 + z_7 - z_1 - z_9}{4 \cdot \Delta s^2}$$

$$t = \frac{z_1 + z_2 + z_3 + z_7 + z_8 + z_9 - 2 \cdot (z_4 + z_5 + z_6)}{3 \cdot \Delta s^2}$$

Wzory na obliczanie krzywizny

$$\text{PROFC} = -\frac{p^2 \cdot r + 2 \cdot p \cdot q \cdot r \cdot s + q^2 \cdot t}{(p^2 + q^2) \cdot \sqrt{(1 + p^2 + q^2)^3}}$$

$$\text{TANGC} = -\frac{q^2 \cdot r - 2 \cdot p \cdot q \cdot s + p^2 \cdot t}{(p^2 + q^2) \cdot \sqrt{1 + p^2 + q^2}}$$

$$\text{PLANc} = -\frac{q^2 \cdot r - 2p \cdot q \cdot s + p^2 \cdot t}{\sqrt{(1 + p^2 + q^2)^3}}$$

Średnia krzywizna, krzywizna asferyczna, krzywizna dyferencyjna

$$\text{MEANC} = \frac{\text{PROFC} + \text{TANGC}}{2}$$
$$= \frac{q^2 \cdot r - 2 \cdot p \cdot q \cdot s + p^2 \cdot t}{(p^2 + q^2) \cdot \sqrt{1 + p^2 + q^2}}$$
$$- \frac{(1 + q^2) \cdot r - 2 \cdot p \cdot q \cdot s + (1 + p^2) \cdot t}{2 \cdot (1 + p^2 + q^2)^{3/2}}$$
$$\text{UNSPHC} = \sqrt{\frac{\left(r \cdot \sqrt{\frac{1+q^2}{1+p^2}} - t \cdot \sqrt{\frac{1+p^2}{1+q^2}}\right)^2 \cdot (1 + p^2 + q^2)}{2 \cdot (1 + p^2 + q^2)^{\frac{3}{2}}} + \left(p \cdot q \cdot r \cdot \sqrt{\frac{1+q^2}{1+p^2}} - 2 \cdot s \cdot \sqrt{(1 + q^2) \cdot (1 + p^2)} + p \cdot q \cdot t \cdot \sqrt{\frac{1+p^2}{1+q^2}}\right)^2}$$

$$\text{DIFFC} = \frac{q^2 \cdot r - 2 \cdot p \cdot q \cdot s + p^2 \cdot t}{(p^2 + q^2) \cdot (1 + p^2 + q^2)^{1/2}}$$
$$- \frac{(1 + q^2) \cdot r - 2 \cdot p \cdot q \cdot s + (1 + p^2) \cdot t}{2 \cdot (1 + p^2 + q^2)^{3/2}}$$

Krzywizny pochodne

$$\text{MINC} = \text{MEANC} - \text{UNSPHC}$$

$$\text{MAXC} = \text{MEANC} + \text{UNSPHC}$$

$$\text{HEXC} = \text{UNSPHC} - \text{DIFFC}$$

$$\text{VEXC} = \text{UNSPHC} + \text{DIFFC}$$

$$\text{TOTGC} = \text{MEANC}^2 - \text{UNSPHC}^2$$

$$\text{TOTAC} = \text{MEANC}^2 - \text{DIFFC}^2$$

$$\text{TOTRC} = \text{UNSPHC}^2 - \text{DIFFC}^2$$

minimalna

maksymalna

poziome przewyższenie

pionowe przewyższenie

całkowita krzywizna Gaussa

całkowita krzywizna akumulacyjna

całkowita krzywizna pierścieniowa

$$\text{ROTOR} = \frac{(p^2 - q^2) \cdot s - p \cdot q \cdot (r - t)}{\sqrt{(p^2 + q^2)^3}}$$

Geometria różniczkowa

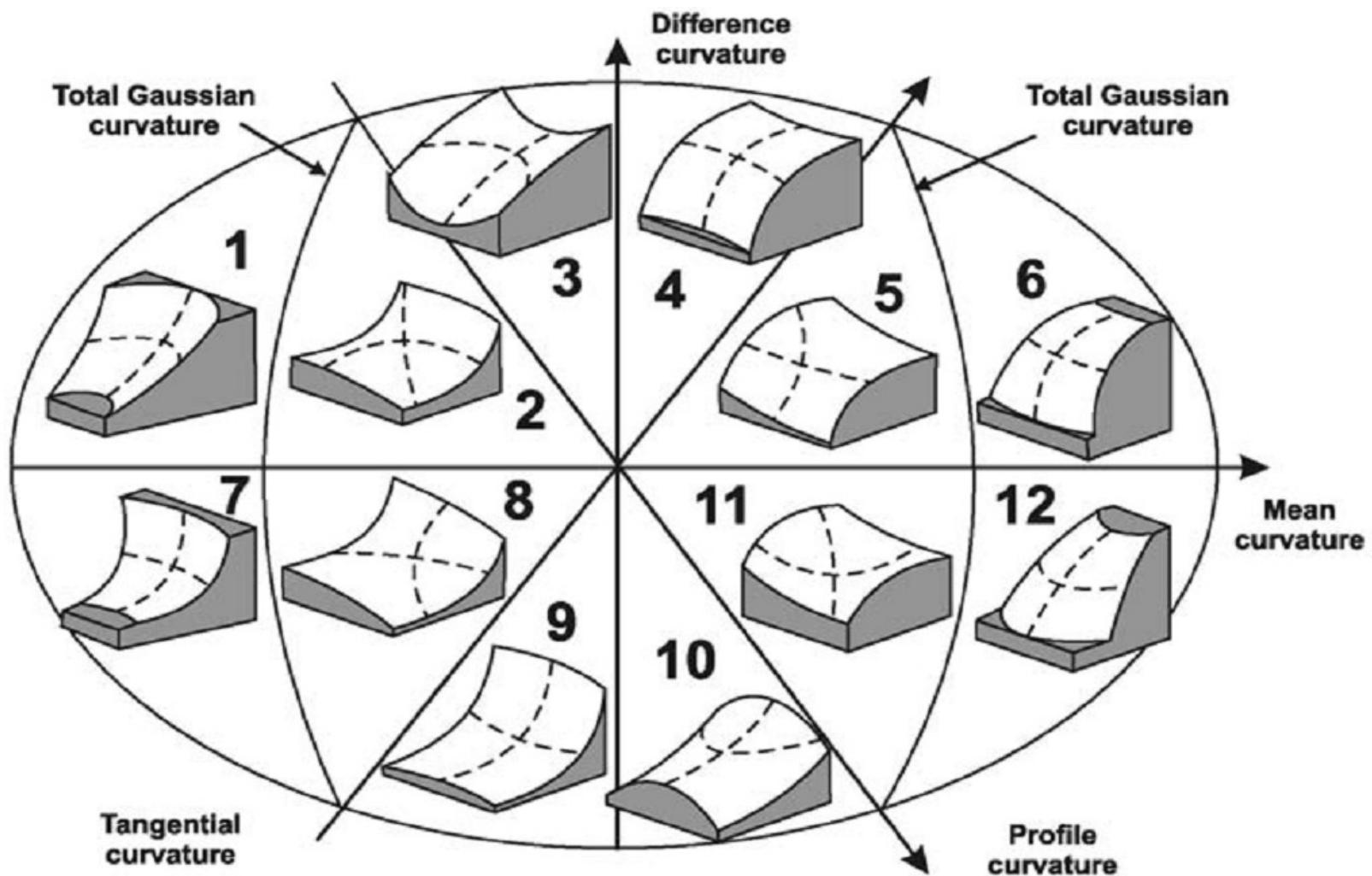
Formulae for calculation of local morphometric variables; in these formulae, $p = \partial z / \partial x$; $q = \partial z / \partial y$; $r = \partial^2 z / \partial x^2$, $s = \partial^2 z / \partial x \partial y$; $t = \partial^2 z / \partial y^2$ are designations for partial derivatives of the surface $z = z(x,y)$; the function $\text{sign}(x)$ equals 1 for $x > 0$, 0 for $x = 0$, and -1 for $x < 0$

Variable name	Class	Unit	Formula	Reference
Slope steepness	A	°	$GA = \arctan(p^2 + q^2)^{1/2}$	—
Slope gradient	A	—	$G = (p^2 + q^2)^{1/2}$	—
Gradient factor	A	—	$GF = (p^2 + q^2)^{1/2} / [1 + p^2 + q^2]^{1/2}$	—
Slope direction (or aspect)	A	°	$A_0 = -90[1 - \text{sign}(q)](1 - \text{sign}(p)) + 180[1 + \text{sign}(p)] - 180\text{sign}(p)\arccos[-q/(p^2 + q^2)^{1/2}] / \pi$	—
Mean curvature	C	1/m	$H = -[(1 + q^2)r - 2pqs + (1 + p^2)t]/[2(1 + p^2 + q^2)^{3/2}]$	(Gauss, 1827)
Unsphericity	C	1/m	$M = [(r[(1 + q^2)(1 + p^2)]^{1/2} - t[(1 + q^2)(1 + p^2)]^{1/2})^2 / (1 + p^2 + q^2) + (pqr[(1 + q^2)(1 + p^2)]^{1/2} - 2[(1 + q^2)(1 + p^2)]^{1/2}s + pqt[(1 + q^2)(1 + p^2)]^{1/2})^2 / 2(1 + p^2 + q^2)^{3/2}]$	(Shary, 1995)
Difference curvature	A	1/m	$E = (q^2r - 2pqs + p^2t)/[(p^2 + q^2)(1 + p^2 + q^2)^{1/2}] - [(1 + q^2)r - 2pqs + (1 + p^2)t]/[2(1 + p^2 + q^2)^{3/2}]$	(Shary, 1995)
Horizontal curvature	A	1/m	$kh = -(q^2r - 2pqs + p^2t)/[(p^2 + q^2)(1 + p^2 + q^2)^{1/2}]$	(Krcho, 1983)
Plan curvature ^a	A	1/m	$kp = -(q^2r - 2pqs + p^2t)/(p^2 + q^2)^{3/2}$	(Evans, 1972)
Vertical (= profile) curvature	A	1/m	$kv = -(p^2r + 2pqs + q^2t)/[(p^2 + q^2)(1 + p^2 + q^2)^{3/2}]$	(Evans, 1972)
Rotor ^a	A	1/m	$rot = [(p^2 - q^2)s - pq(r - t)]/(p^2 + q^2)^{3/2}$	(Shary, 1991)
Horizontal excess curvature	A	1/m	$khe = M - E$	(Shary, 1995)
Vertical excess curvature	A	1/m	$kve = M + E$	(Shary, 1995)
Minimal curvature	C	1/m	$kmin = H - M$	—
Maximal curvature	C	1/m	$kmax = H + M$	—
Total Gaussian curvature	C	1/m ²	$K = (rt - s^2)/(1 + p^2 + q^2)^2$	(Gauss, 1827)
Total ring curvature	A	1/m ²	$KR = [(p^2 - q^2)s - pq(r - t)]^2 / [(p^2 + q^2)^2(1 + p^2 + q^2)^2]$	(Shary, 1995)
Total accumulation curvature	A	1/m ²	$KA = H^2 - E^2$	(Shary, 1995)
Slope insolation ^b	A	%	$F = 50\{1 + \text{sign}[\cos a - \sin a * (p \sin b + q \cos b)]\}[\cos a - \sin a * (p \sin b + q \cos b)] / (1 + p^2 + q^2)$	(e.g., Onorati et al., 1992)

^a These curvatures do not belong to the complete system of 12 curvatures (Shary, 1995).

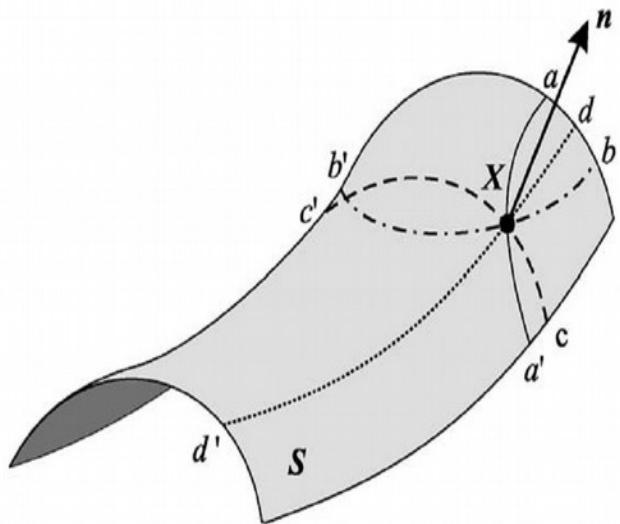
^b Here $90^\circ - a$ is Sun vertical angle from horizon, b is Sun azimuth counted from northern direction clockwise.

System 12 krzywizn

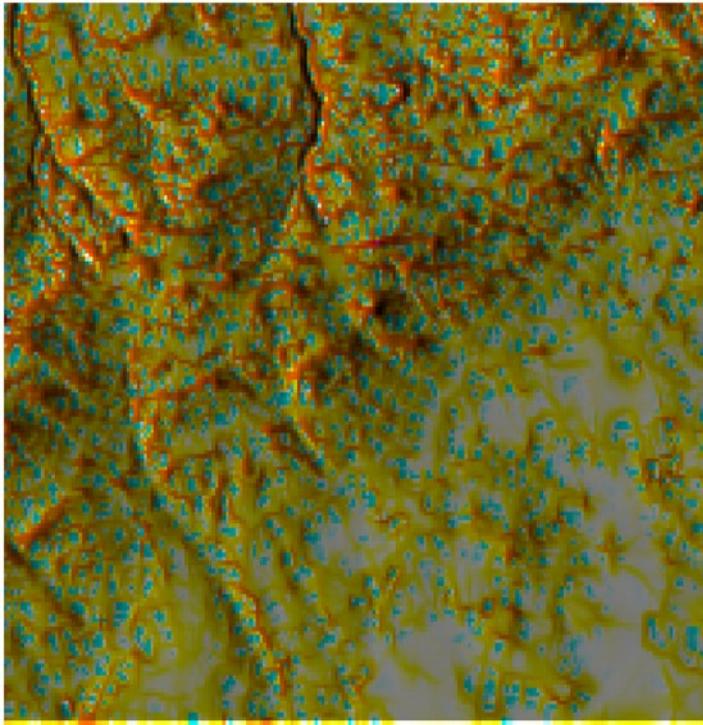


Cztery główne krzywizny

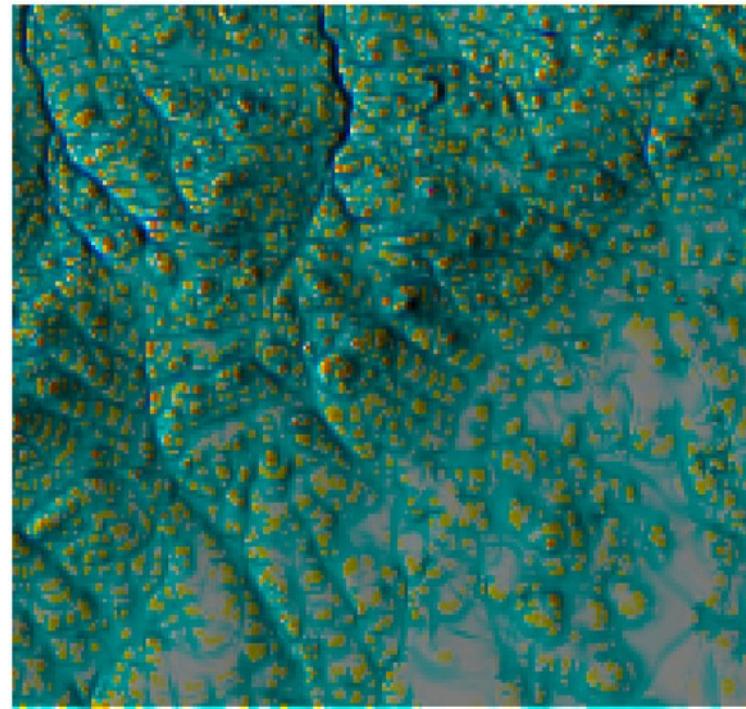
- Profilowa PLANC - aa'
- Styczna TANGC - bb'
- Maxymalna MAXC - cc'
- Minimalna MAXC - dd'



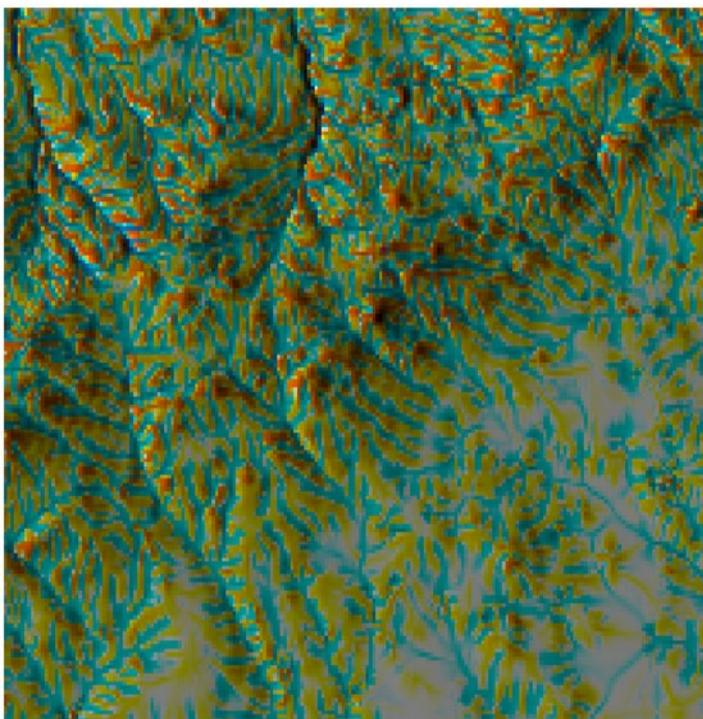
Najważniejsze krzywizny



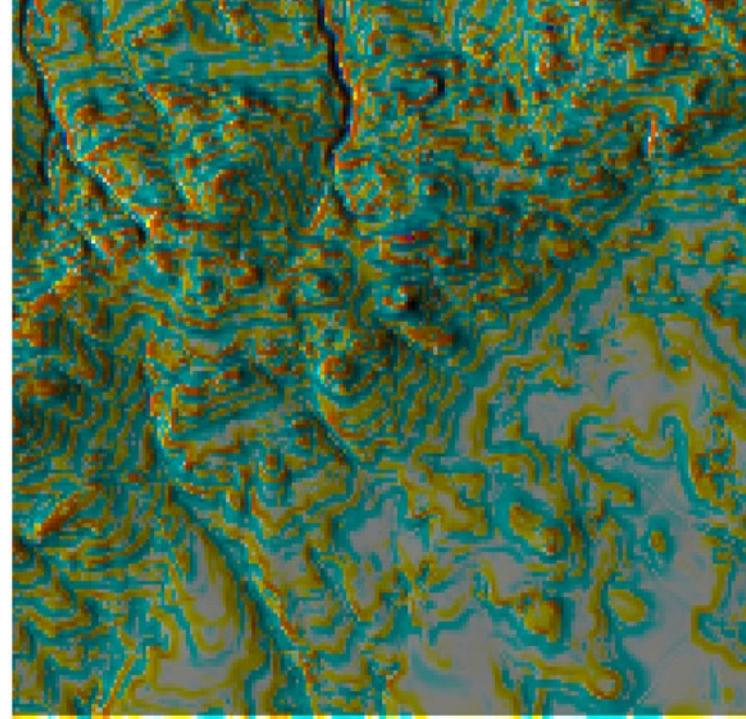
MAXC



MINC



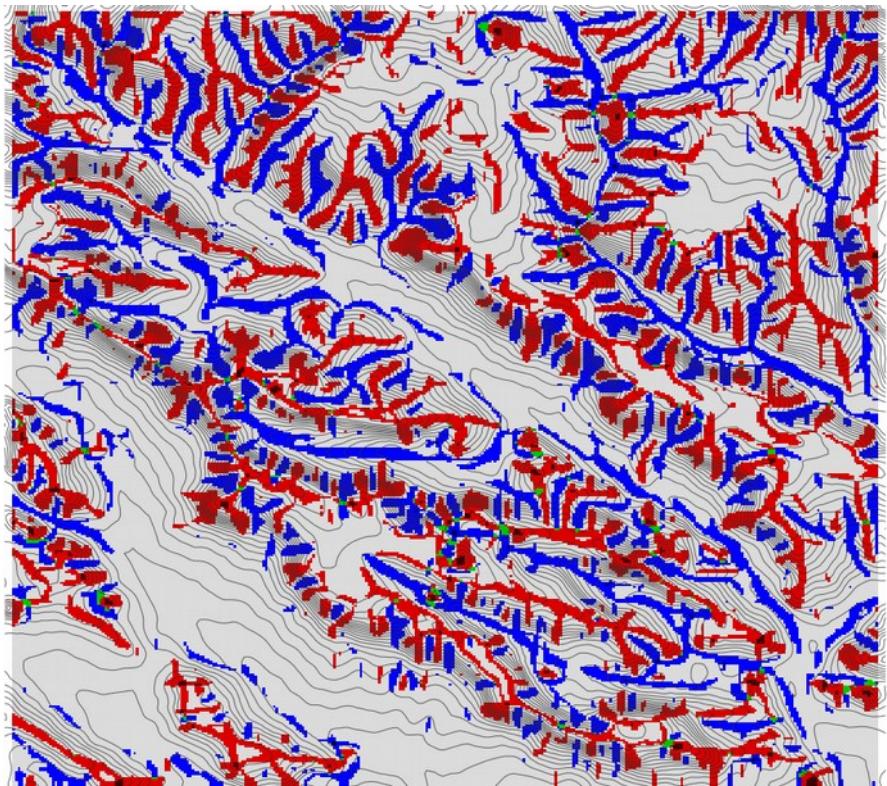
PLANC



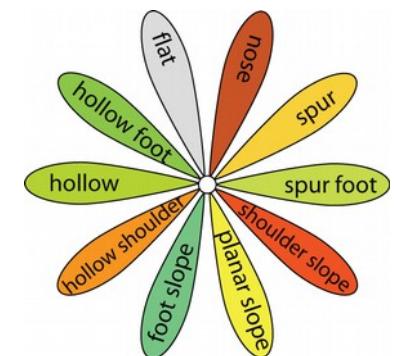
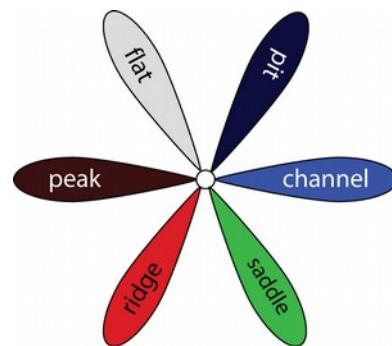
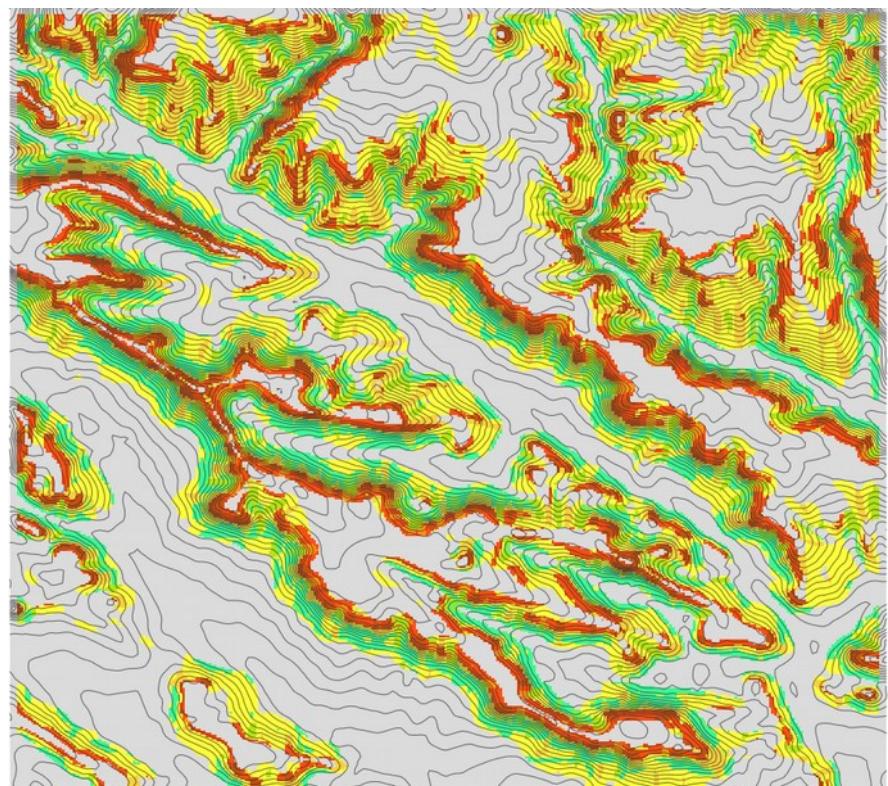
PROFC

Klasyfikacje form terenu oparte o krzywizny

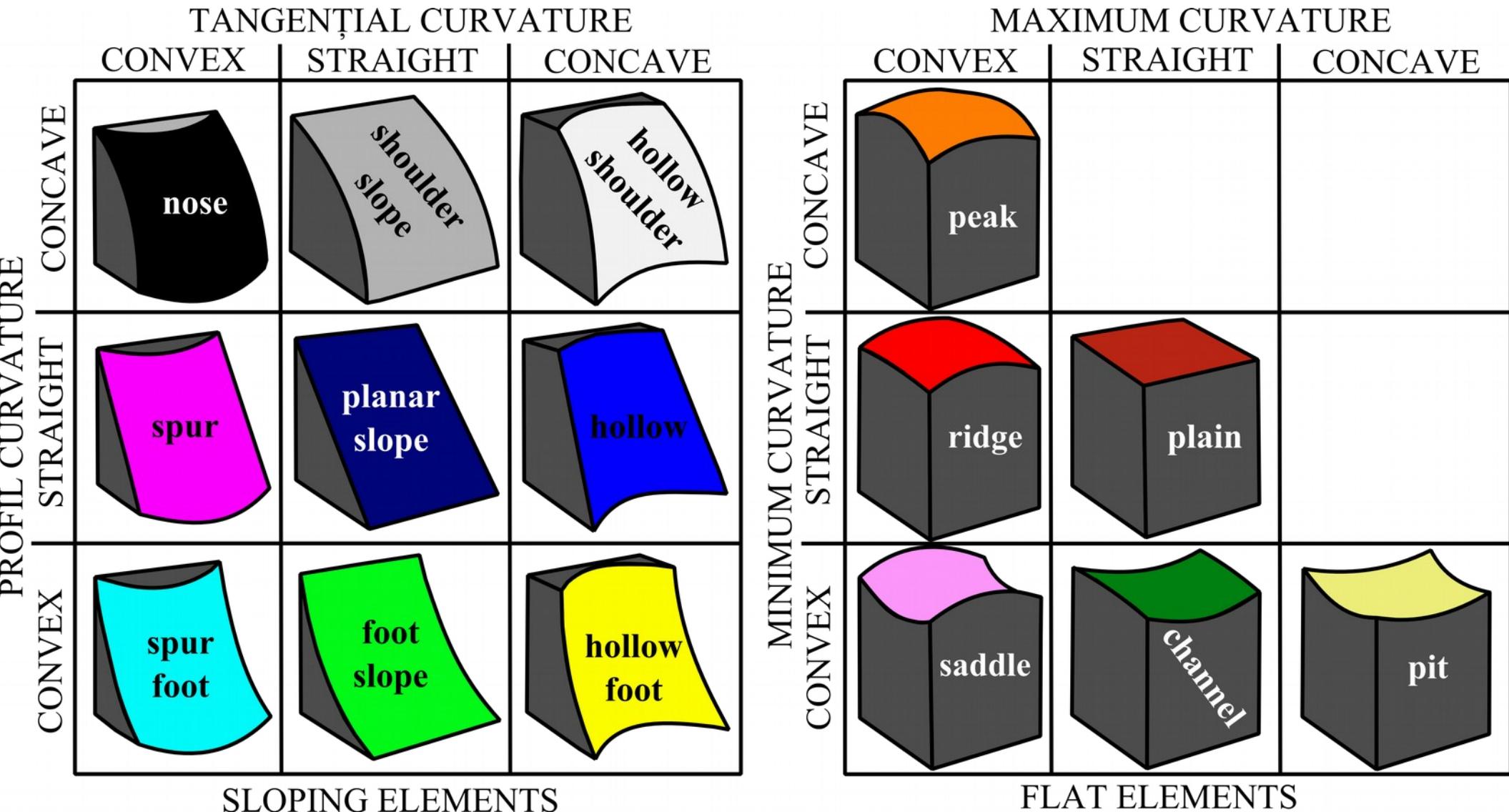
MAXC (maksymalna) + MIN (minimalna)



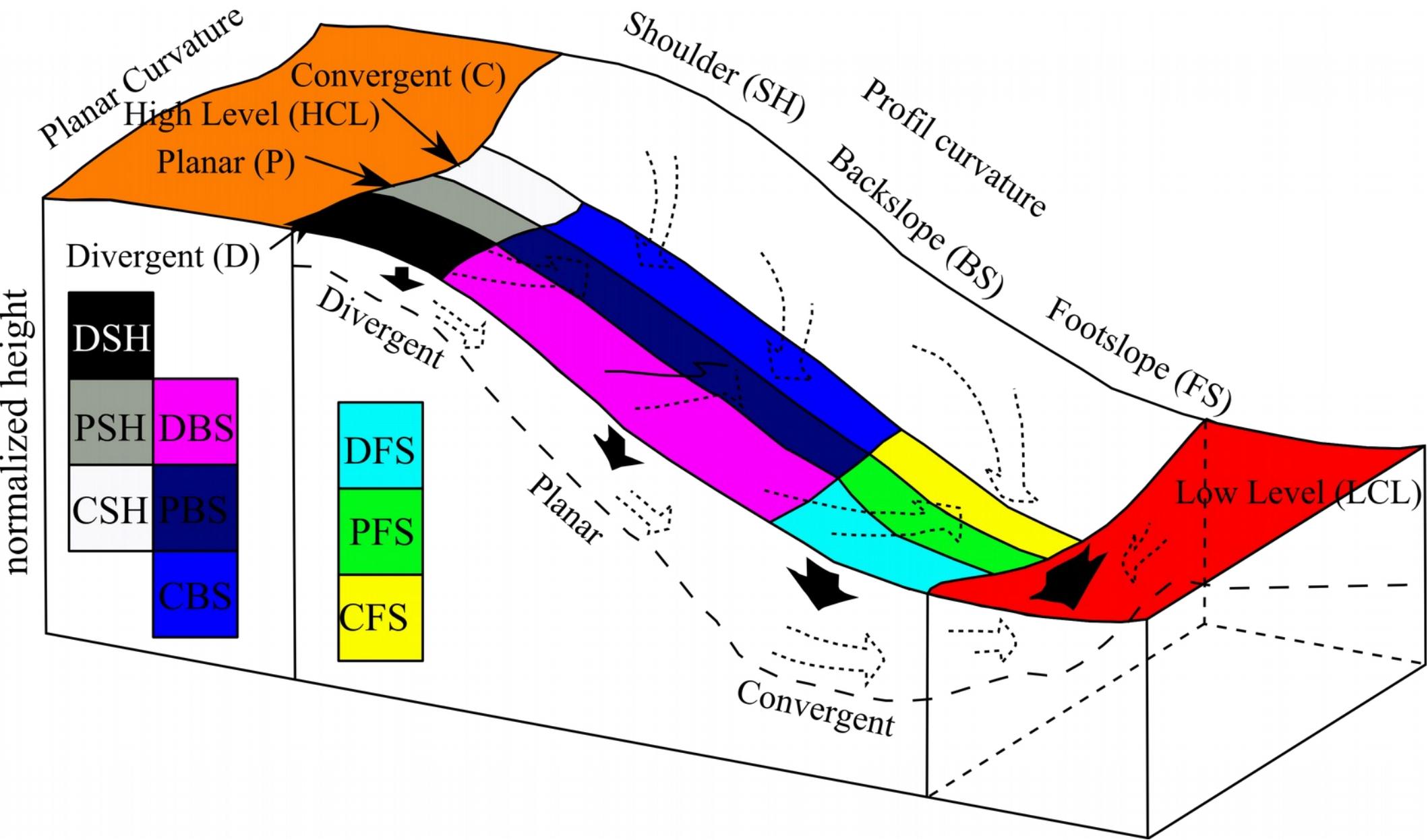
PROFC (profilowa) + PLANC (styczna)



Klasyfikacja stoków i obszarów pałskich



Fromy na stoku

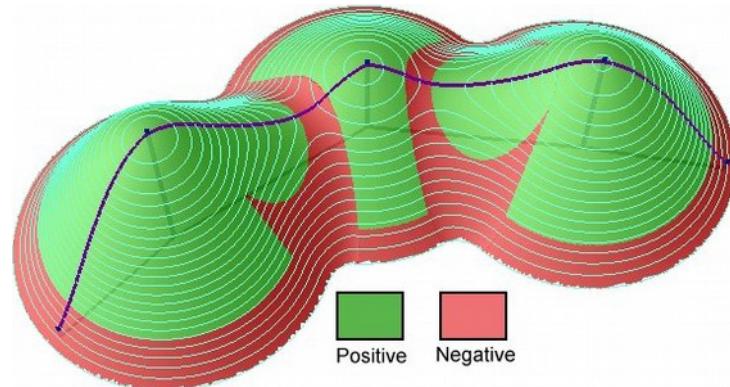
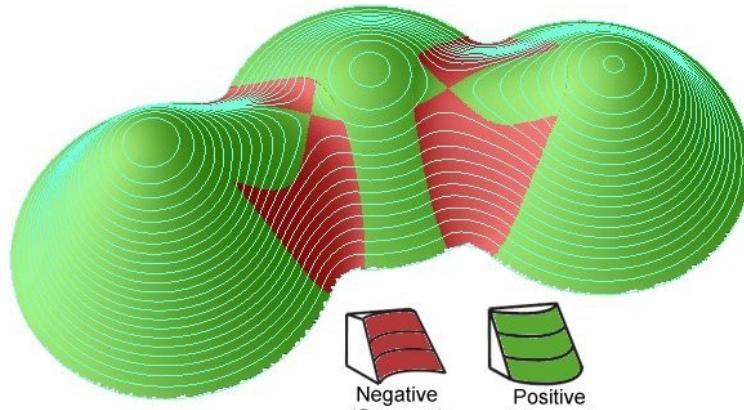
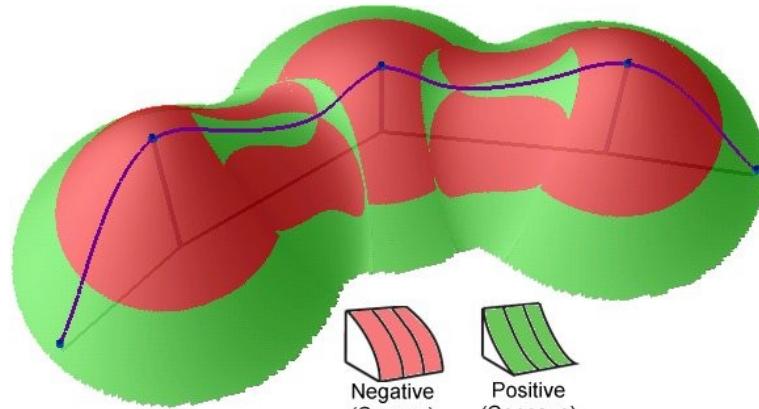


Formy na stoku

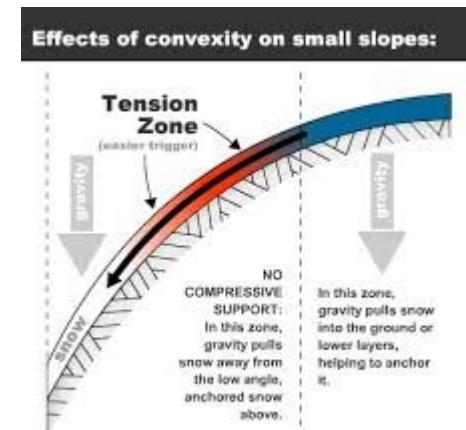
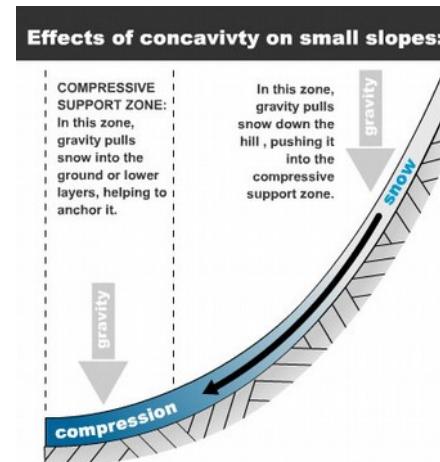
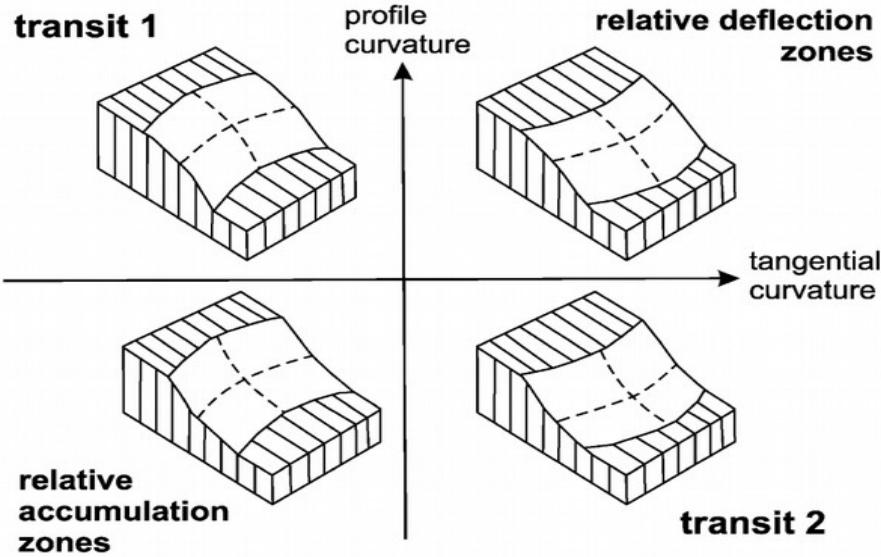
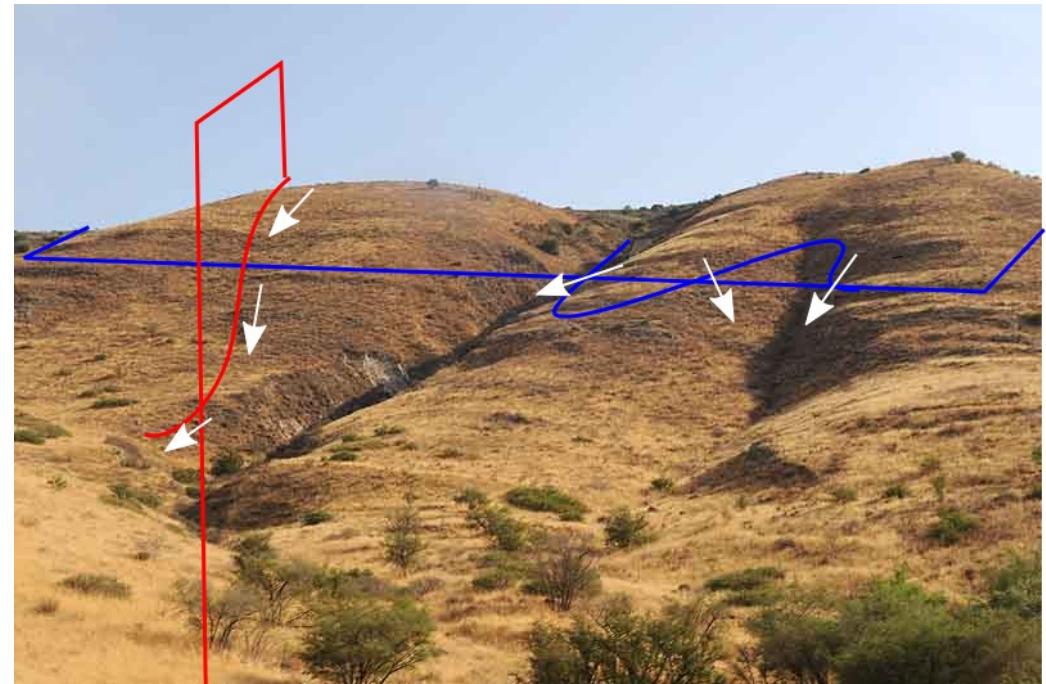
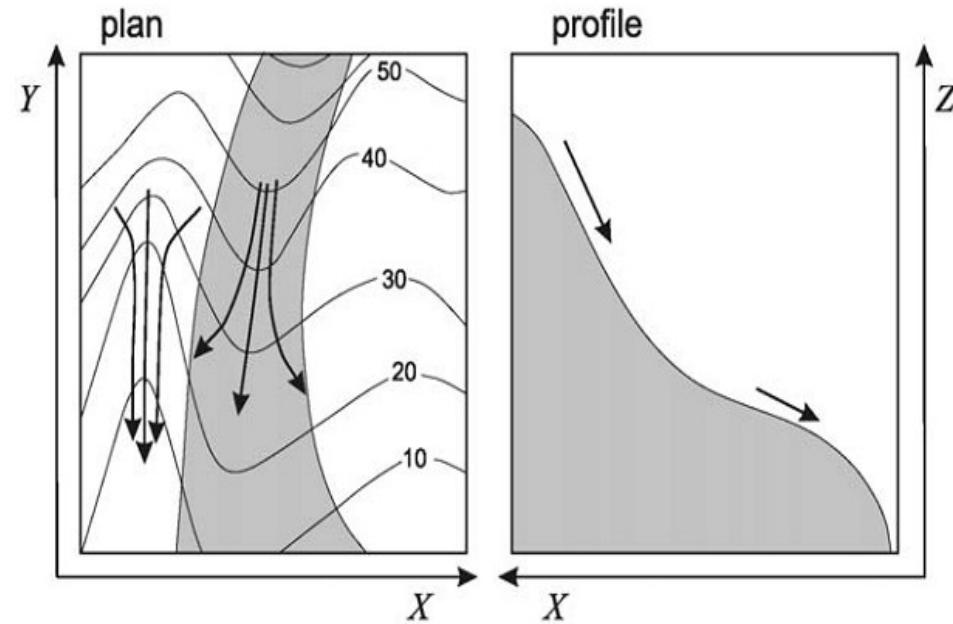


akceleracja spływu na stoku w systemie krzywizn

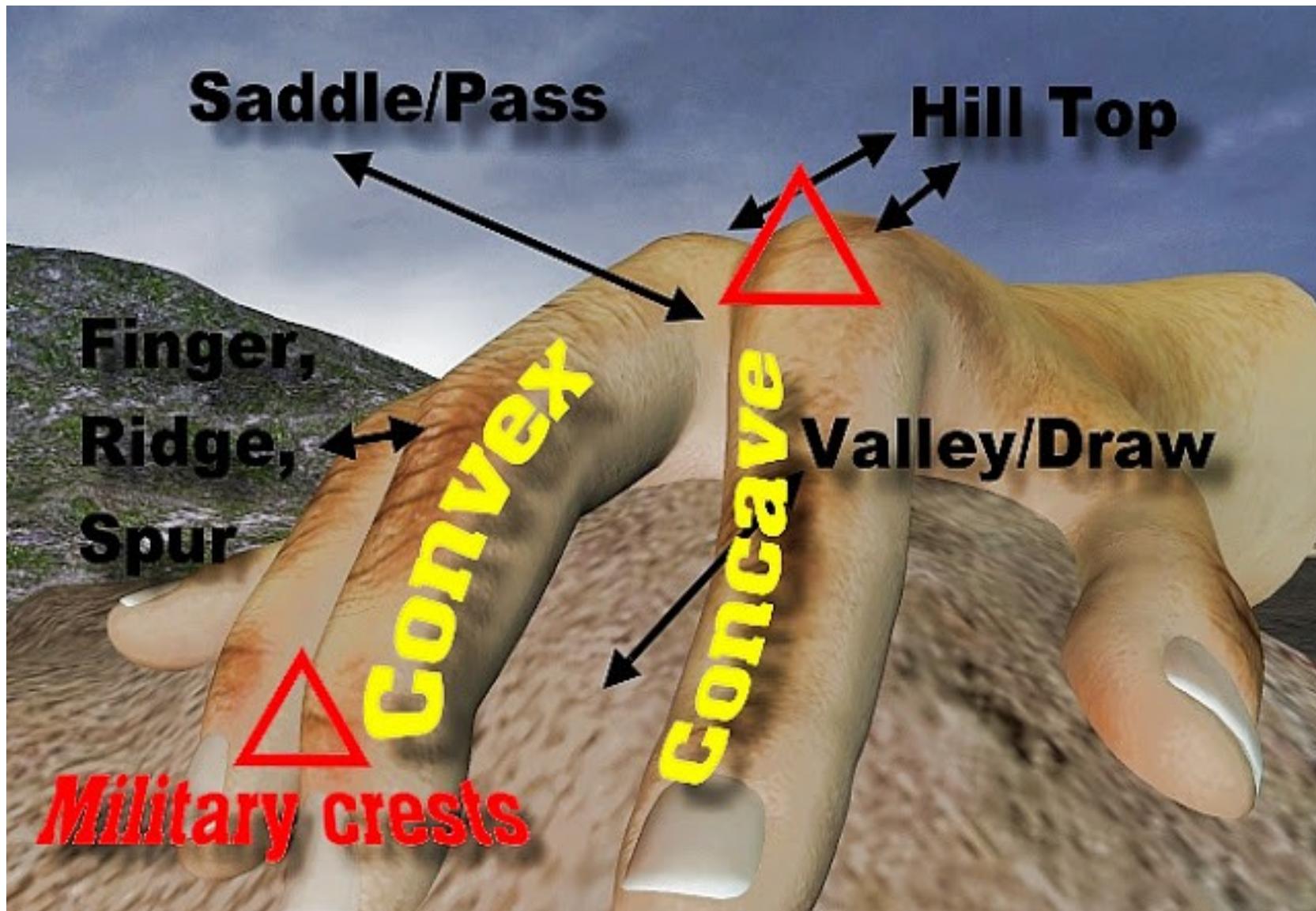
Contour	Block	
		Divergent Shoulder
		Convergent Shoulder
		Divergent Backslope
		Convergent Backslope
		Divergent Footslope
		Convergent Footslope
		Level



Krzywizna a spływy powierzchniowy



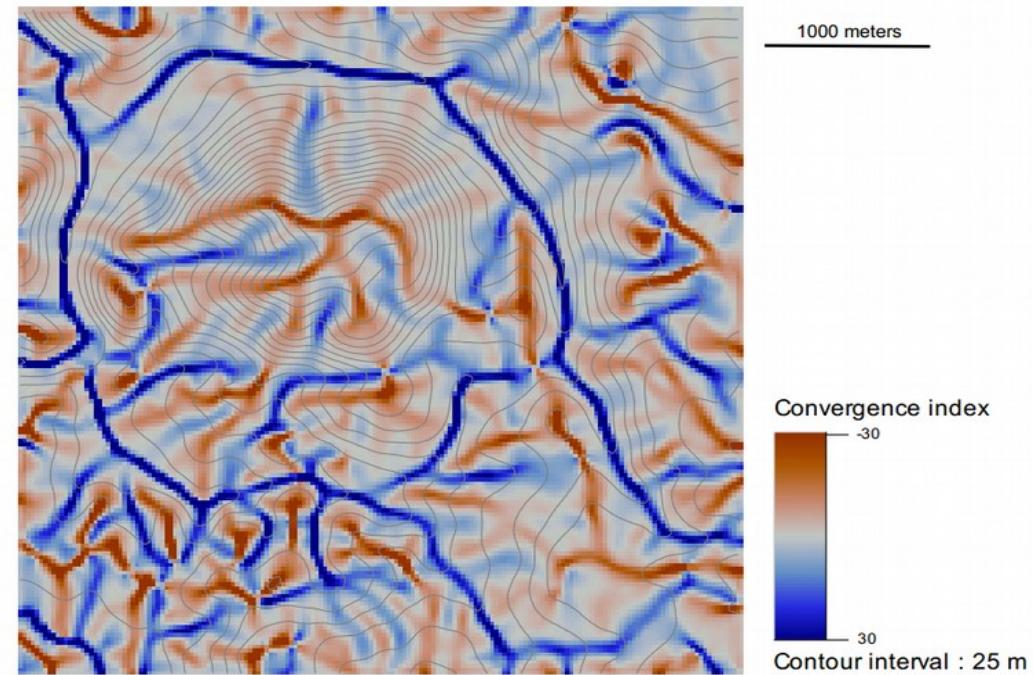
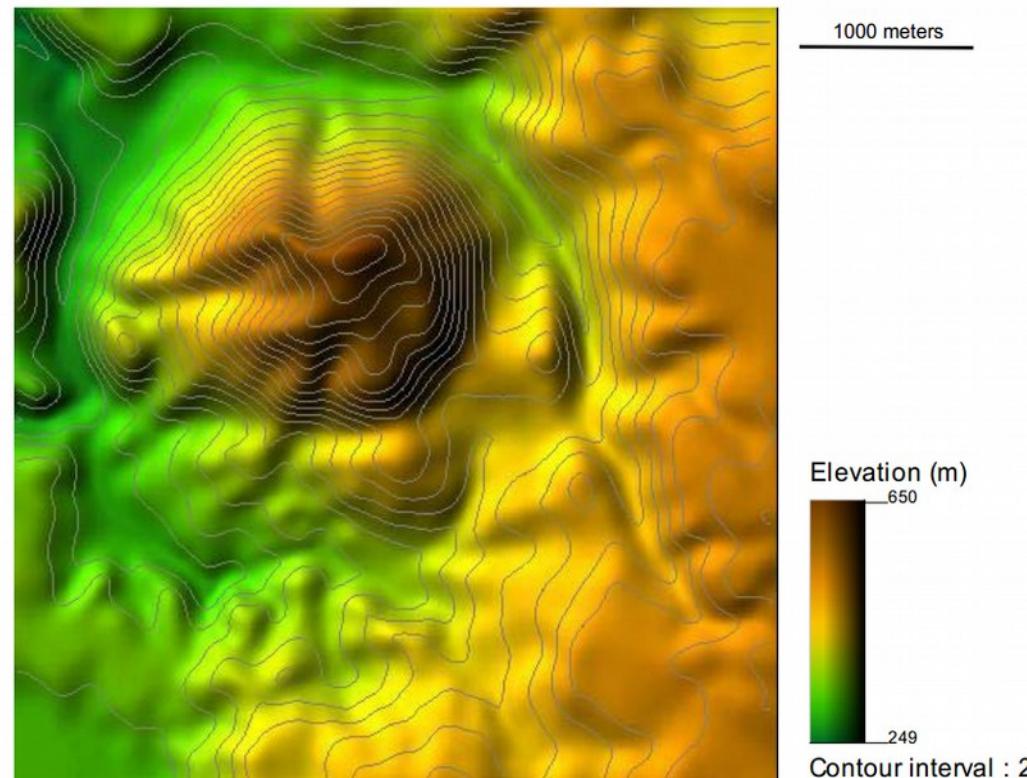
Jak zapamiętać?



Inne parametry terenu

- Konwergencja
- Prominencja i różnica wysokości (relief)
- Szorstkość
- Lokalne deniwelacje
- Otwartość
- Parametry teksturalne

Konwergencja



Zbieżność terenu/konwergencja

- Zbieżność - zgodność kierunków z idealistycznym modelem całkowitej zbieżności

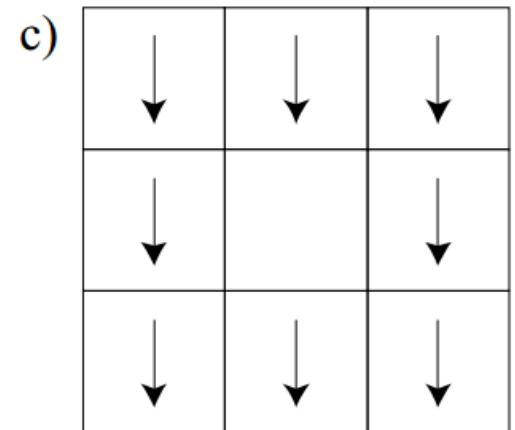
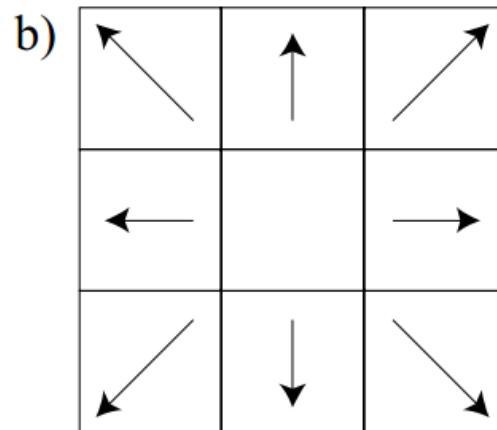
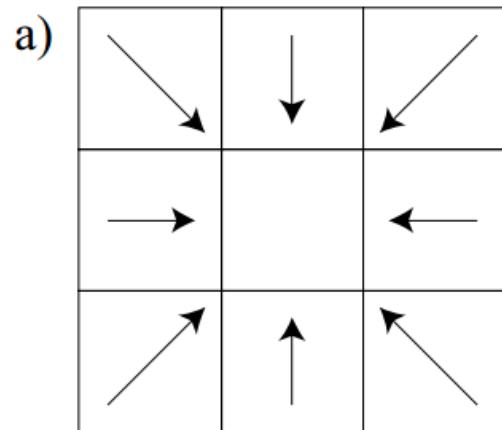
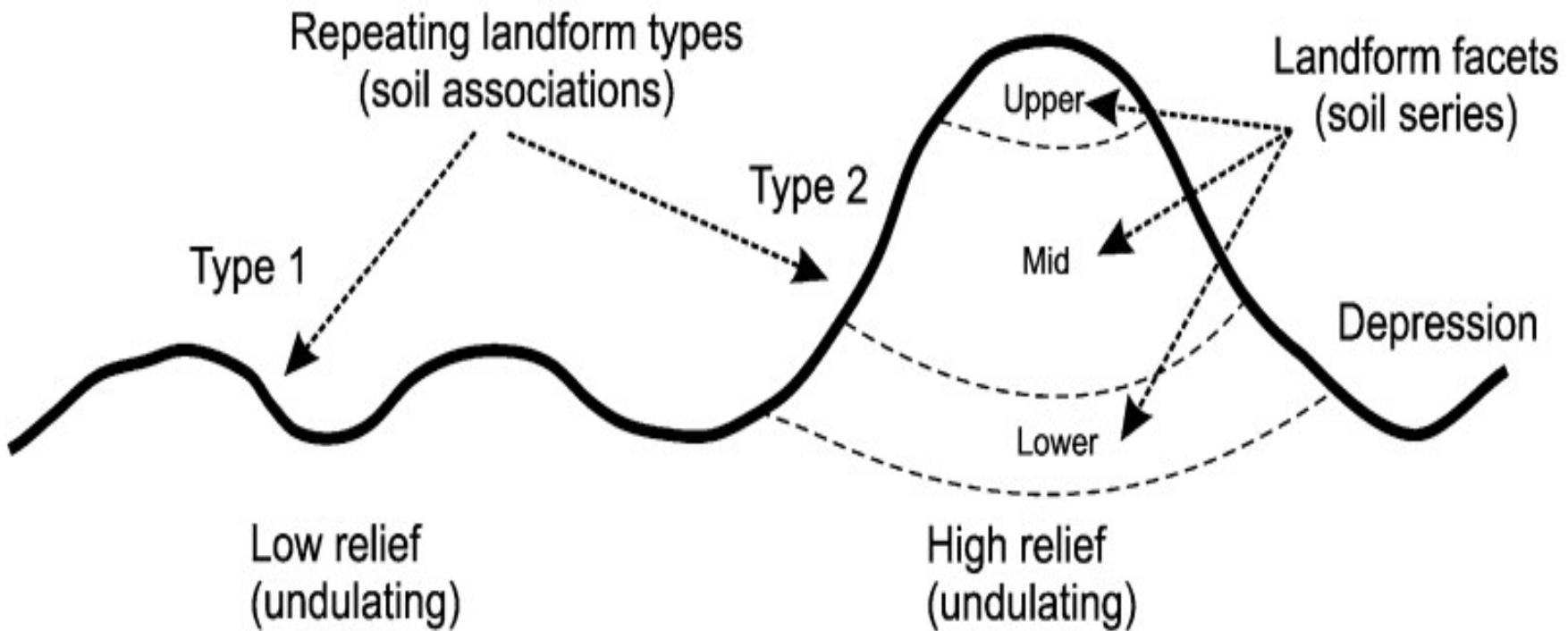


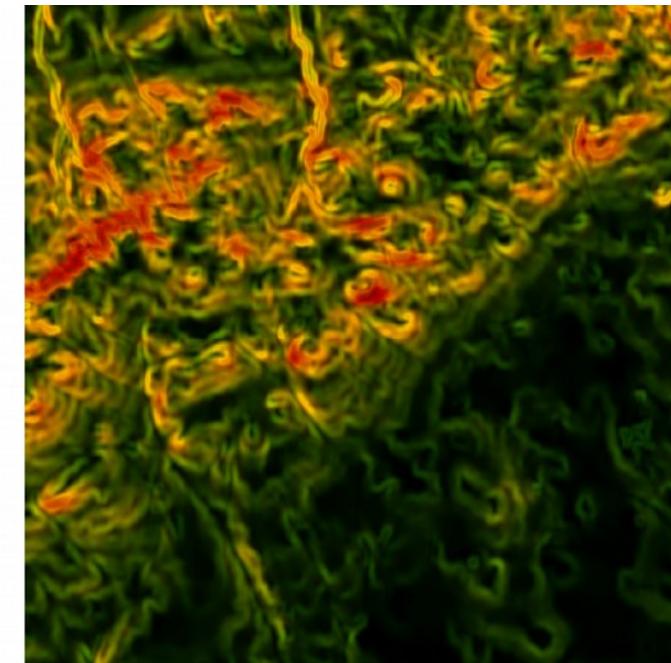
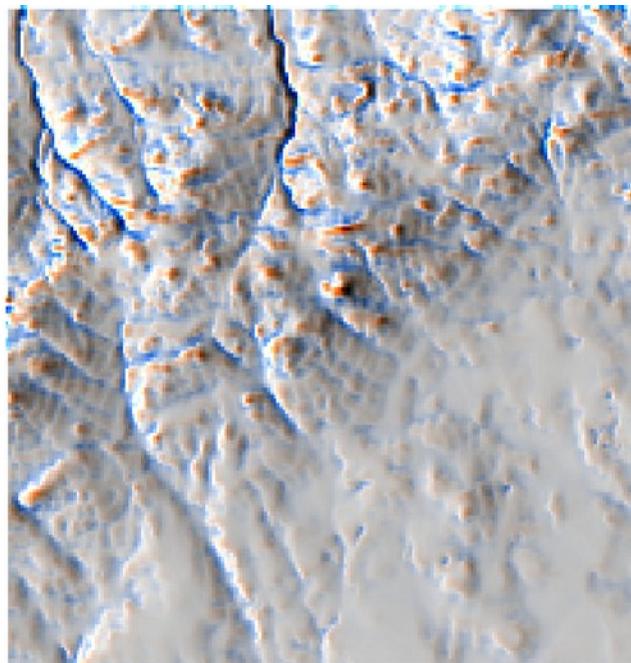
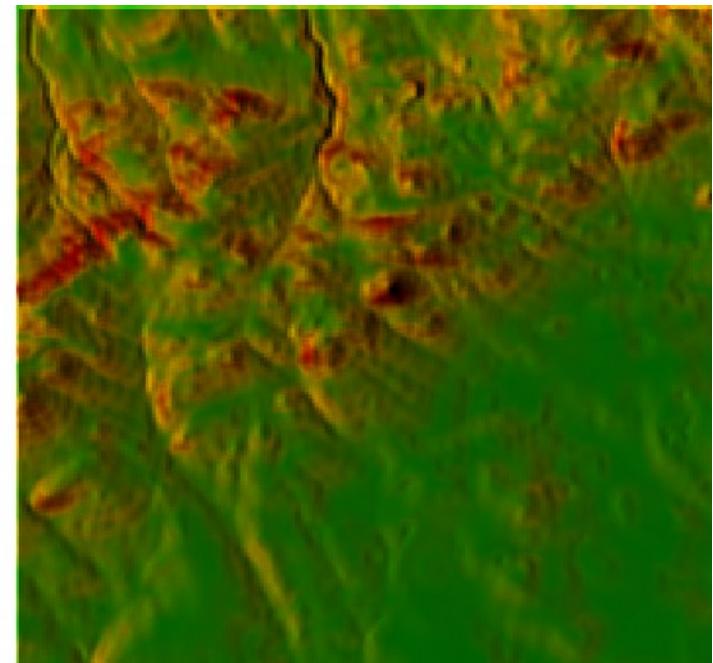
Fig. 8. Calculating the convergence index on the basis of aspect. The averages of the relative aspects are 0° (a), 180° (b), and 90° (c). The convergence indexes are -90° (a), 90° (b), 0° (c)

Relief - wysokość względna prominencja

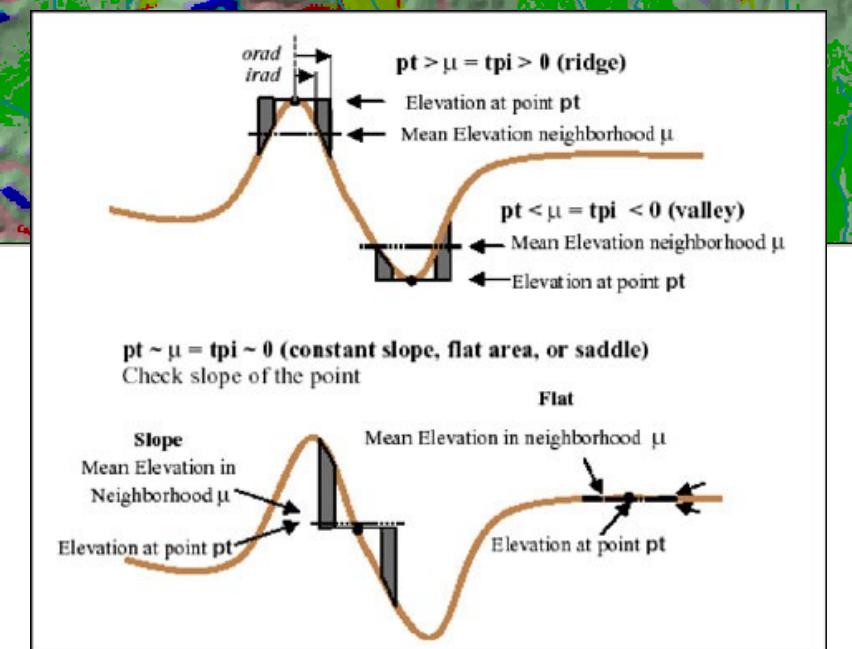
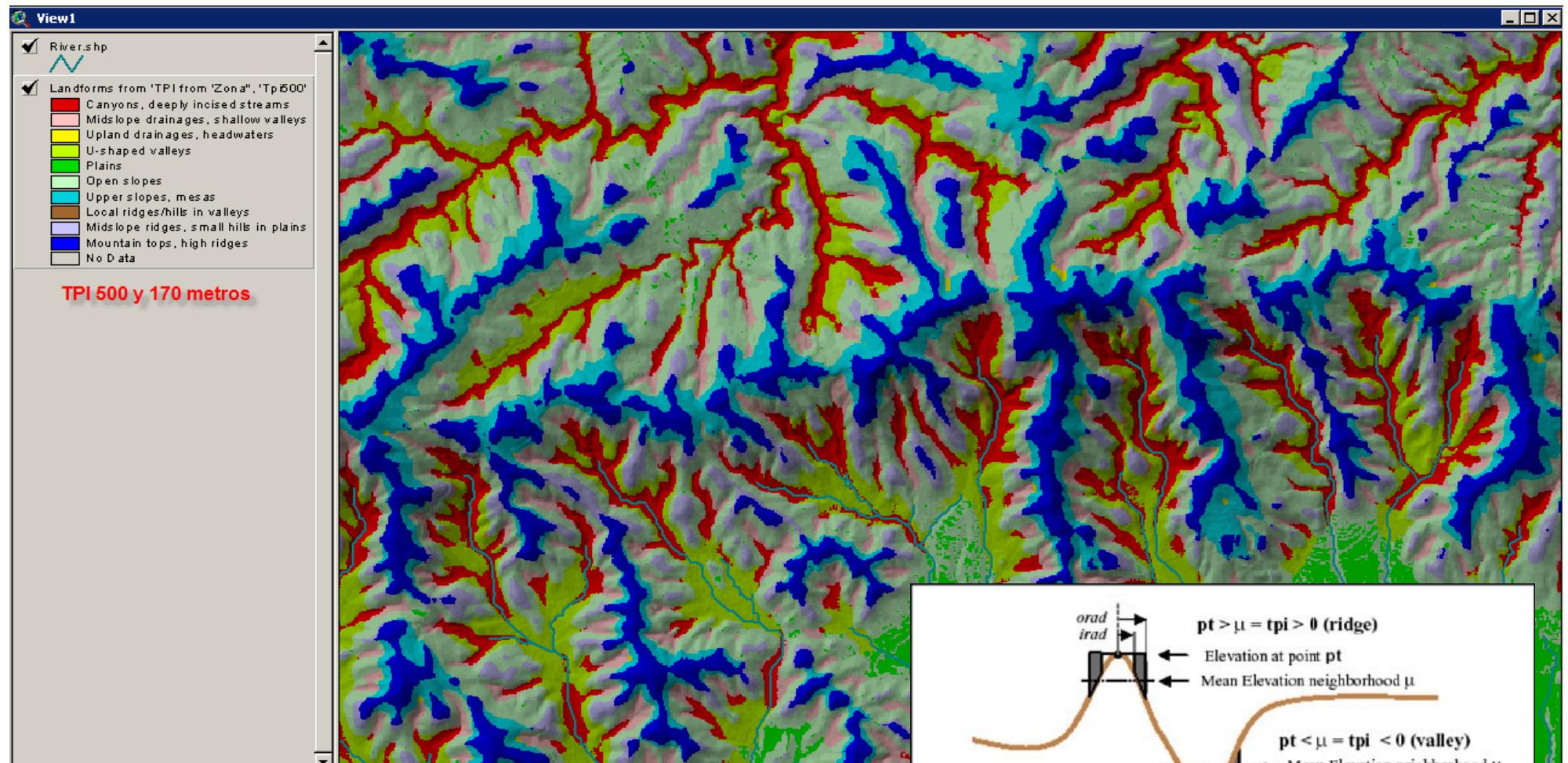


Parametry sąsiedztwa (focal)

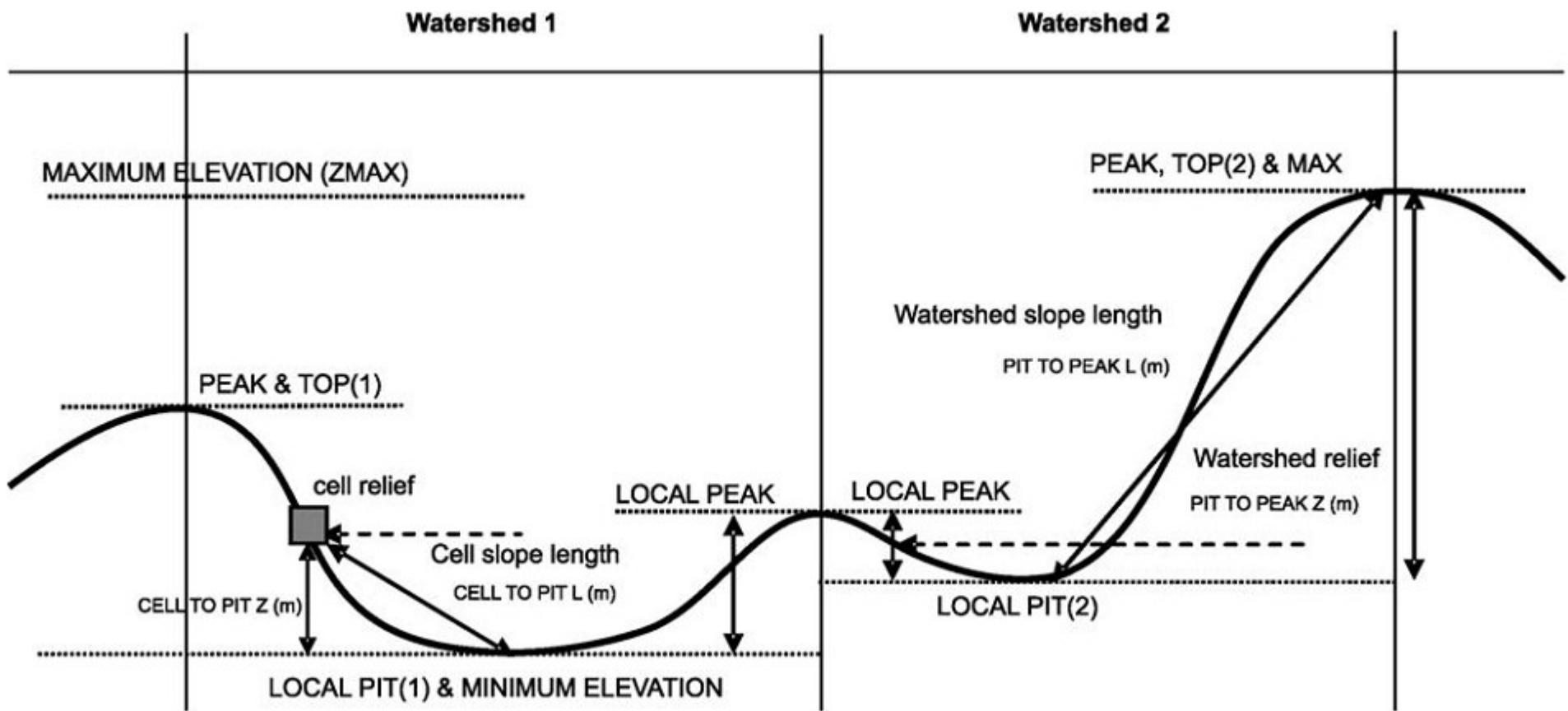
- relief lokalne różnice wysokości
- wysokość względem średniej
- odchylenie standardowe (miara szorstkości)



Wskaźnik pozycji topograficznej (TPI)



Lokalne różnice wysokości



Otwartość (Openness)

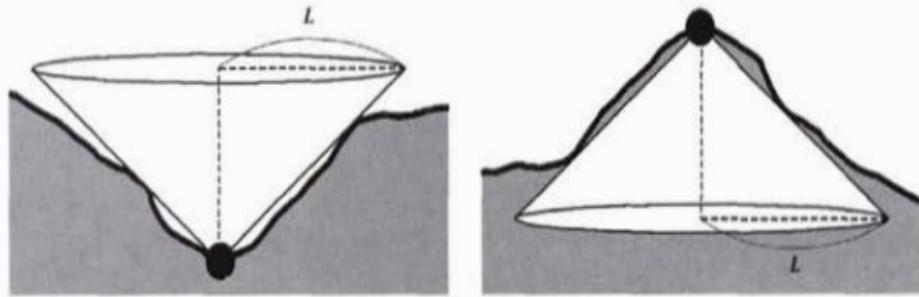
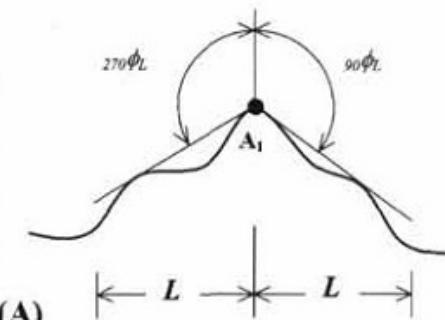


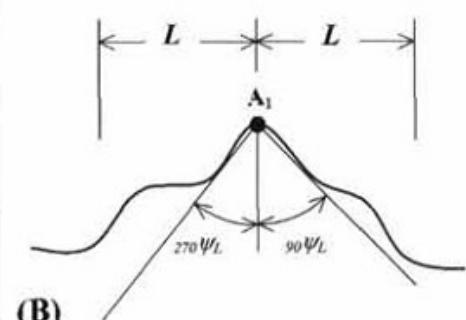
Figure 4. Positive (left) and negative (right) openness shown schematically for values $< 90^\circ$. Heavy irregular line is terrain surface; L is radial limit of calculation for chosen point (large dot) on a DEM.

A_1 has a high score of positive openness.



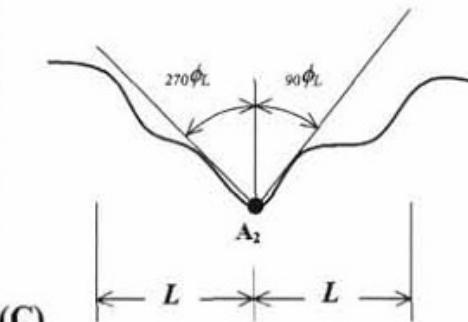
(A)

A_1 has a low score of negative openness.



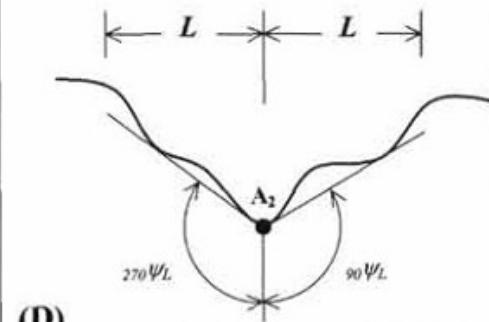
(B)

A_2 has a low score of positive openness.



(C)

A_2 has a high score of negative openness.

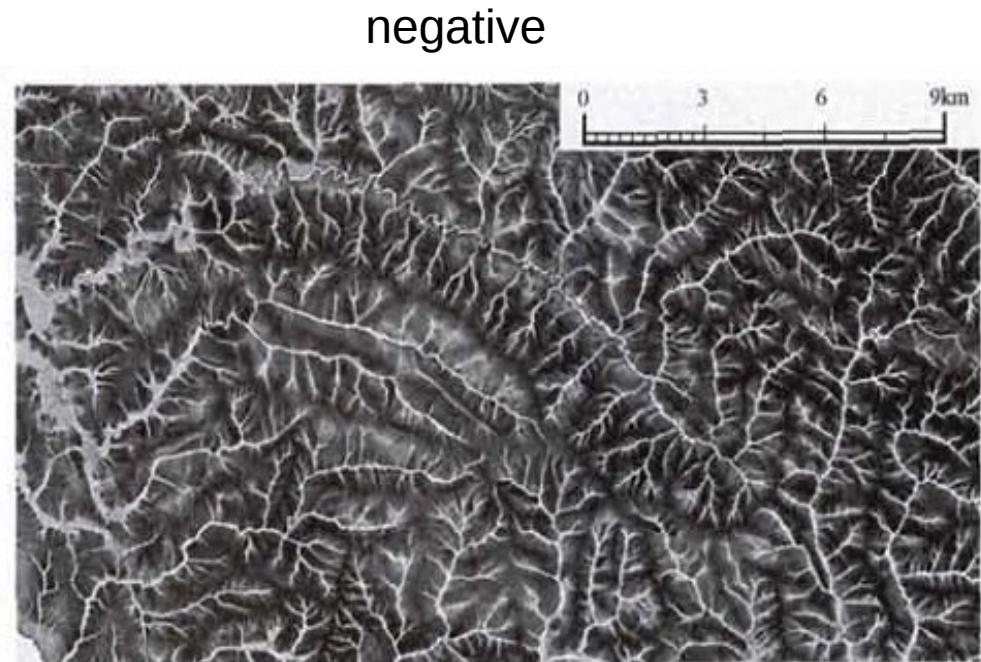


(D)

Openness przykłady



positive



negative