

Free All The Things

Markus Hauck



Free All The Things

- well known: free monads
- maybe known: free applicatives
- free monoids
- free <you name it>

Goal Of This Talk

- how many of you wrote a Free X
- how many of you used Free...
 - Monad
 - Applicative
 - Functor
 - Boolean Algebra
 - other?
- Goal: explain the technique behind “Free X”
- Be able to apply the “pattern” yourself

The Road Ahead

What Is Free

A free functor is left adjoint to a forgetful functor

What's the problem?

What Is Free

A free “thing” **FreeA** on a type A is a A and a function

```
def inject(x: A): FreeA
```

s. t. for any other “thing” B and a function

```
val f: A => B
```

there exists a unique homomorphism g such that

```
g.compose(inject) === f
```

What Is Free

given a "signature" which consists of sorts, operations, and equations, you build a term algebra (i.e. an AST) of the operations and then quotient it by the equivalence relation generated by the equations

Why Free

- having a Free X is good for a number of reasons
- use Free X as if it was X
- but the program is reified into some (data-)structure
- this structure can often be analyzed and optimized
- the killer: interpreters of the program can vary

Scales of Power

- the structures we will look at, are able to capture computations that have different power abilities
- monad: depend on previous values and branching
- applicative: fixed structure with arbitrary applicative effects in between
- functor: well...
- monoid: limited power, but very flexible and composable
- surprise

Free Vs Tagless

- we will mostly look at the data structure version of Free X
- the alternative is to use finally tagless representations (Next Talk)

Freeing The Monad

- what are the operations?

```
1 trait Monad[F[_]] {  
2   def pure[A](x: A): F[A]  
3   def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]  
4 }
```

Freeing The Monad

- what are the laws?
- (pseudocode)

```
1  // Left identity
2  pure(a).flatMap(f) === f(a)
3
4  // Right identity
5  fa.flatMap(pure) === fa
6
7  // Associativity
8  fa.flatMap(f).flatMap(g) ===
9    fa.flatMap(a -> f(a).flatMap(g))
```

Freeing The Monad

- todo: the minimal “thing” that has a *Monad* instance **satisfies** the laws
- simple idea: capture as data
- btw: any minimal combination works!

Freeing The Monad

```
1 trait Monad[F[_]] {  
2   def pure[A](x: A): F[A]  
3   def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]  
4 }
```

```
1 sealed abstract class Free[F[_], A]  
2  
3 final case class Pure[F[_], A](a: A) extends Free[F, A]  
4  
5 final case class FlatMap[F[_], A, B](fa: Free[F, A],  
6                                     f: A => Free[F, B])  
7   extends Free[F, B]
```

Freeing The Monad

```
1  implicit def freeMonad[F[_], A]: Monad[Free[F, ?]] =  
2    new Monad[Free[F, ?]] {  
3      def pure[A](x: A): Free[F, A] = Pure(x)  
4  
5      def flatMap[A, B](fa: Free[F, A])(  
6        f: A => Free[F, B]): Free[F, B] = FlatMap(fa, f)  
7    }
```

The Interpreter

- but what about the laws?!
- clearly we are violating all of them!
- we need one more thing: the interpreter

```
1 def runFree[F[_], M[_]:Monad, A](  
2   nat: FunctionK[F, M]): Free[F, A] => M[A] = ???
```


The Laws

- together with the interpreter, we have to fulfill the laws

```
1  runFree(nat)(pure(a).flatMap(f)) ===  
2    runFree(nat)(f(a))
```

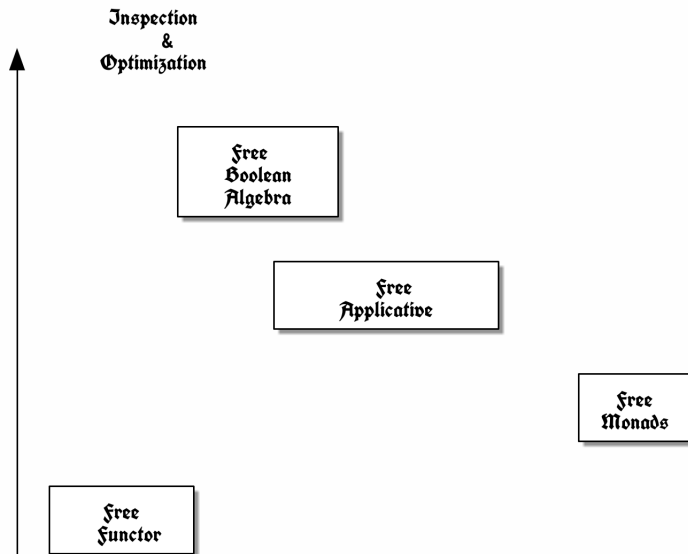
So What?

- the laws tell us what “rewriting” is possible
- here: **flatMap** has to be associative, that means we can re-associate
- why? Let's look at what happens with normal **flatMap**s

Use Cases

- DSL with monadic expressiveness
- branching, loops, basically everything

Axis of Power



Freeing The Monad

- that's it for the Monad
- what else?

Freeing The Applicative

- free monads are great, but also limited
- we can't analyze the programs
- how about a smaller gun?

Freeing The Applicative

- we follow the same pattern
- look at typeclass operations
- create datastructure
- “interpreter”

The Applicative Class

```
1 trait Applicative[F[_]] {  
2   def pure[A](x: A): F[A]  
3   def ap[A, B](fab: F[A => B], fa: F[A]): F[B]  
4 }
```


Freeing The Applicative

- again the same pattern: we model it as an ADT

```
1 sealed abstract class FreeAp[F[_], A]
2
3 final case class Pure[F[_], A](a: A) extends FreeAp[F, A]
4
5 final case class Ap[F[_], A, B](fab: FreeAp[F, A => B],
6                                fa: FreeAp[F, A])
7     extends FreeAp[F, B]
8
9
```

- of course we also need the interpreter

Less Power?!

- why would we consider Applicative if it's less powerful?
- less is more: we can inspect the AST

Freeing The Functor

- we are well equipped by now

Freeing The Functor

```
1 sealed abstract class FreeFunctor[F[_], A]
2 case class Fmap[F[_], X, A](fa: F[X])(f: X => A)
3     extends FreeFunctor[F, A]
```

Freeing The Functor

- clean code alarm: only one subclass
- can we get rid of it?

Disclaimer

- Once upon a time:
<https://engineering.wingify.com/posts/Free-objects/>
- really awesome article about free objects
- use free boolean algebra to define DSL for event predicates
- all credits to Chris Stucchio (@stucchio)

Free Boolean Algebra

- Wikipedia: boolean algebra + set of generators
- let's go

Boolean Algebras

- seen: common fp type classes
- apply our knowledge to another example: boolean algebras

Your conclusion here