Free All The Things

Markus Hauck



Free All The Things

- well known: free monads
- maybe known: free applicatives
- free monoids
- free <you name it>



Goal Of This Talk

- how many of you wrote a Free X
- how many of you used Free...
 - Monad
 - Applicative
 - Functor
 - other?
- Goal: explain the technique behind "Free X" + example

The Road Ahead

- Demonstrate a recipe to "free" things
- Using: Free Monads, Applicatives, Functors
- New thing: Free Boolean Algebra + Example



What's The Problem

A free functor is left adjoint to a forgetful functor what's the problem?



What Is Free

A free "thing" **FreeA** on a type(class) A is a A and a function

```
def inject(x: A): FreeA
```

such that for any other "thing" B and a function

val $f: A \Rightarrow B$

there exists a unique homomorphism g such that

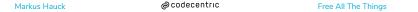
What Is Free

- still sounds complicated?
- there is a recipe
 - AST
 - inject
 - interpreter
 - check laws



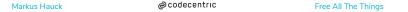
Why Free

- nice API using typeclass
- use Free X as if it was X
- program reified into datastructure
- structure can be analyzed/optimized
- one program many interpretations

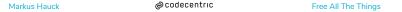


Disclaimer Before We Start

- deep embeddings / initial encoding / data structure representation
- not: finally tagless, optimization



Freeing The Monad



The Monad Typeclass

```
trait Monad[F[_]] {
    def pure[A](x: A): F[A]

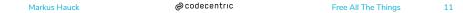
def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

Give Me The Laws

```
// Left identity
pure(a).flatMap(f) === f(a)

// Right identity
fa.flatMap(pure) === fa

// Associativity
fa.flatMap(f).flatMap(g) ===
fa.flatMap(a => f(a).flatMap(g))
```



Applying The Recipe

```
trait Monad[F[_]] {
   def pure[A](x: A): F[A]

def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

- now comes our recipe
 - AST
 - inject
 - interpreter
 - check laws

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Freeing The Monad

```
sealed abstract class Free[F[ ], A]
1
2
      final case class Pure[F[ ], A](a: A)
3
          extends Free[F, A]
4
5
      final case class FlatMap[F[], A, B](
6
          fa: Free[F, A],
7
          f: A \Rightarrow Free[F, B]
8
          extends Free[F, B]
9
10
      final case class Inject[F[_], A](fa: F[A])
11
          extends Free[F, A]
12
```

Freeing The Monad

```
implicit def freeMonad[F[_], A]: Monad[Free[F, ?]] =
new Monad[Free[F, ?]] {
    def pure[A](x: A): Free[F, A] = Pure(x)

def flatMap[A, B](fa: Free[F, A])(
    f: A => Free[F, B]): Free[F, B] =
FlatMap(fa, f)
}
```

Interpreter

```
def runFree[F[_], M[_]: Monad, A](nat: F ~> M)(
    free: Free[F, A]): M[A] = free match {
    case Pure(x) => Monad[M].pure(x)
    case Inject(fa) => nat(fa)
    case FlatMap(fa, f) =>
        Monad[M].flatMap(runFree(nat)(fa))(x =>
        runFree(nat)(f(x)))
}
```

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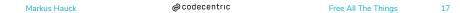
What about the laws?

```
// The associativity law
  fa.flatMap(f).flatMap(g) ===
     fa.flatMap(fa, a \Rightarrow f(a).flatMap(q))
  val exp1 = FlatMap(FlatMap(fa, f), q)
  val exp2 = FlatMap(fa, (a: Int) => FlatMap(f(a), q))
3
  exp1 != exp2
```



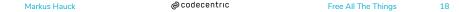
What about the laws?





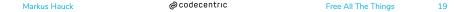
The Laws

- · actually, we don't satisfy them
- programmer: after interpretation it's no longer visible
- mathematician: that's not the free monad!
- tradeoff: during construction vs during interpretation



The Right Free Monad

- common transformation: associate flatMap's to the right
- avoids having to rebuild the tree repeatedly during construction
- how: during construction time



Transforming Free Monads

```
def flatMap[A, B](fa: Free[F, A])(
    f: A => Free[F, B]): Free[F, B] = fa match {
    case Pure(x) => f(x) // Left identity
    case Inject(fa) => FlatMap(Inject(fa), f)
    case FlatMap(ga, g) => // Associativity
    FlatMap(ga, (a: Any) => FlatMap(g(a), f))
}
```

Transforming Free Monads

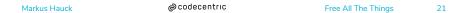
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def flatMap[A, B](fa: Free[F, A])(
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    FlatMap(ga, (a: Any) => FlatMap(g(a), f))
}
```

Transforming Free Monads

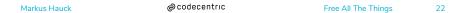
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def flatMap[A, B](fa: Free[F, A])(
    f: A => Free[F, B]): Free[F, B] = fa match {
    case Pure(x) => f(x) // Left identity
    case Inject(fa) => FlatMap(Inject(fa), f)
    case FlatMap(ga, g) => // Associativity
    FlatMap(ga, (a: Any) => FlatMap(g(a), f))
}
```

We Freed Monads

- DSL with monadic expressiveness
- context sensitive, branching, loops, fancy control flow
- familiarity with monadic style for DSL
- big drawback: interpreter has limited possibilities

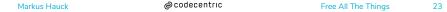


Freeing The Applicative



Freeing The Applicative

- free monads are great, but also limited
- we can't analyze the programs
- how about a smaller abstraction?



Recall

- we follow the same pattern
- AST
- inject
- interpreter
- check laws



The Applicative Typeclass

```
trait Applicative[F[_]] {
   def pure[A](x: A): F[A]

def ap[A, B](fab: F[A => B], fa: F[A]): F[B]
}
```



AST for FreeApplicative

```
sealed abstract class FreeAp[F[ ], A]
2
      final case class Pure [F], A (a: A)
3
          extends FreeAp[F. A]
4
5
      final case class Ap[F[ ], A, B](
6
          fab: FreeAp[F, A \Rightarrow B],
7
          fa: FreeAp[F, A])
8
          extends FreeAp[F, B]
9
10
      final case class Inject[F[\ ], A](fa: F[A])
11
          extends FreeAp[F, A]
12
```

Laws

```
1 // identity
   Ap(Pure(identity), v) === v
3
  // composition
   Ap(Ap(Ap(Pure(.compose), u), v), w) ===
     Ap(u, Ap(v, w))
  // homomorphism
   Ap(Pure(f), Pure(x)) === Pure(f(x))
10
   // interchange
   Ap(u, Pure(y)) === Ap(Pure((v)), u)
12
```



Don't Forget The Laws

```
def ap[A, B](fab: FreeAp[F, A \Rightarrow B],
1
                    fa: FreeAp[F, A]): FreeAp[F, B] =
2
        (fab, fa) match {
3
          case (Pure(f), Pure(x)) =>
4
            Pure(f(x)) // homomorphism
5
          case (u. Pure(v)) =>
6
            Ap(Pure((f: A \Rightarrow B) \Rightarrow f(y)), u) // interchange)
7
          case ( , ) => Ap(fab, fa)
8
```

Don't Forget The Laws

```
def ap[A, B](fab: FreeAp[F, A \Rightarrow B],
1
                    fa: FreeAp[F, A]): FreeAp[F, B] =
2
        (fab, fa) match {
3
          case (Pure(f), Pure(x)) =>
4
            Pure(f(x)) // homomorphism
5
          case (u. Pure(v)) =>
6
            Ap(Pure((f: A \Rightarrow B) \Rightarrow f(y)), u) // interchange)
7
          case ( , ) => Ap(fab, fa)
8
```

Running FreeApplicatives

```
def runFreeAp[F[\_], M[\_]: Applicative, A](
1
         nat: F \sim M)(free: FreeAp[F, A]): M[A] =
2
       free match {
3
         case Pure(x) => Applicative[M].pure(x)
4
         case Inject(fa) => nat(fa)
5
         case Ap(fab, fa) =>
6
           Applicative[M]
7
             .ap(runFreeAp(nat)(fab), runFreeAp(nat)(fa))
8
```

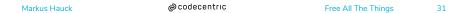
We Freed Applicatives

- DSL with applicative expressiveness
- context insensitive
- pure computation over effectful arguments
- more freedom during interpretation



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Freeing The Functor



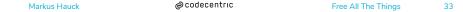
And Once Again

- AST
- inject
- interpreter
- check laws



The Functor Typeclass

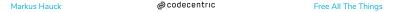
```
trait Functor[F[_]] {
    def map[A, B](fa: F[A])(f: A => B): F[B]
}
```



```
sealed abstract class FreeFunctor[F[_], A]

case class Fmap[F[_], X, A](fa: F[X])(f: X => A)
    extends FreeFunctor[F, A]

case class Inject[F[_], A](fa: F[A])
    extends FreeFunctor[F, A]
```



```
sealed abstract class FreeFunctor[F[_], A]

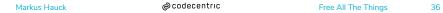
case class Fmap[F[_], X, A](fa: F[X])(f: X => A)
    extends FreeFunctor[F, A]

def inject[F[_], A](value: F[A]) =
    Fmap(value)(identity)
```

Clean Code Police



• only one subclass?



```
sealed abstract class Fmap[F[_], A] {
1
       type X
2
       def fa: F[X]
       def f: X => A
4
6
     def inject[F[], A](v: F[A]) = new Fmap[F, A] {
       type X = A
8
       def fa = v
9
       def f = identity
10
11
```

```
sealed abstract class Coyoneda [F[], A]
1
       type X
2
       def fa: F[X]
       def f: X => A
6
     def inject[F[\_], A](v: F[A]) = new Coyoneda[F, A] {
       type X = A
8
       def fa = v
9
       def f = identity
10
11
```



Free Functor Instance

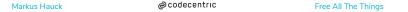
```
implicit def covoFun[F[ ]]: Functor[Covoneda[F, ?]] =
1
        new Functor[Covoneda[F, ?]] {
2
          def map[A, B](coyo: Coyoneda[F, A])(
3
              q: A \Rightarrow B: Covoneda[F, B] =
4
            new Coyoneda[F, B] {
5
              type X = coyo.X
6
              def fa = coyo.fa
7
              def f = q.compose(coyo.f)
8
9
10
```

Free Functor Instance

```
implicit def coyoFun[F[_]]: Functor[Coyoneda[F, ?]] =
        new Functor[Coyoneda[F, ?]] {
2
          def map[A, B](coyo: Coyoneda[F, A])(
3
              q: A \Rightarrow B: Covoneda[F, B] =
4
            new Coyoneda[F, B] {
5
              type X = coyo.X
6
              def fa = coyo.fa
7
              def f = q.compose(coyo.f)
8
9
10
```

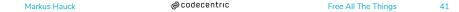
Free Functor Interpreter

```
def runCoyo[F[]: Functor, A](
coyo: Coyoneda[F, A]): F[A] =
Functor[F].map(coyo.fa)(coyo.f)
```



We Freed Functors

- DSL with hmm functorial expressiveness?
- map fusion! (functor law)
- boring interpreter, though



We Freed Functors

- DSL with hmm functorial expressiveness?
- map fusion! (functor law)
- boring interpreter, though
- still fun!

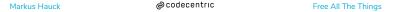


Now That We Can Free Anything



What should we free?

Monads Applicatives Functors

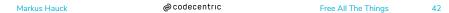


Now That We Can Free Anything



What should we free?

Monads Applicatives Functors Monoids

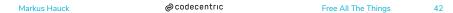


Now That We Can Free Anything



What should we free?

Monads Applicatives Functors Monoids Semigroups



Now That We Can Free Anything



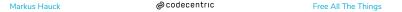
What should we free?

Monads Applicatives Functors Monoids Semigroups Groups

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Credit Where It's Due

- Once upon a time: https://engineering.wingify.com/posts/Free-objects/
- use free boolean algebra to define DSL for event predicates
- credits to Chris Stucchio (@stucchio)



Let's Free A Boolean Algebra

- DSL: and, or, not, true, false
- we know what to do, so let's go!
- AST
- inject
- interpreter
- check laws



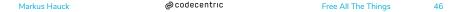
Boolean Algebras

```
trait BoolAlgebra[A] {
  def tru: A
  def fls: A

def not(value: A): A

def and(lhs: A, rhs: A): A
  def or(lhs: A, rhs: A): A
}
```

```
sealed abstract class FreeBool[+A]
1
2
     case object Tru extends FreeBool[Nothing]
3
     case object Fls extends FreeBool[Nothing]
4
5
     case class Not[A](value: FreeBool[A])
6
          extends FreeBool[A]
7
     case class And[A](lhs: FreeBool[A], rhs: FreeBool[A])
8
          extends FreeBool[A]
9
     case class Or[A](lhs: FreeBool[A], rhs: FreeBool[A])
10
          extends FreeBool[A]
11
     case class Inject[A](value: A) extends FreeBool[A]
12
```



```
def runFreeBool[A, B](fb: FreeBool[A])(f: A \Rightarrow B)(
1
          implicit B: BoolAlgebra[B]): B = {
2
        fb match {
3
          case Tru
                       => B.tru
4
          case Fls
                       => B.fls
5
          case Inject(v) \Rightarrow f(v)
6
          case Not(v) => B.not(runFreeBool(v)(f))
7
          case Or(lhs, rhs) =>
8
            B.or(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
9
          case And(lhs, rhs) =>
10
            B.and(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
11
12
13
```

```
def runFreeBool[A, B](fb: FreeBool[A])(f: A \Rightarrow B)(
1
          implicit B: BoolAlgebra[B]): B = {
2
        fb match {
3
          case Tru
                      => B.tru
4
          case Fls
                       => B.fls
5
          case Inject(v) \Rightarrow f(v)
6
          case Not(v) => B.not(runFreeBool(v)(f))
7
          case Or(lhs, rhs) =>
8
            B.or(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
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            B.and(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
11
12
13
```

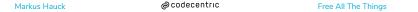
```
def runFreeBool[A, B](fb: FreeBool[A])(f: A \Rightarrow B)(
1
          implicit B: BoolAlgebra[B]): B = {
2
       fb match {
3
         case Tru
                      => B.tru
4
         case Fls
                       => B.fls
5
         case Inject(v) => f(v)
6
         case Not(v) => B.not(runFreeBool(v)(f))
7
          case Or(lhs, rhs) =>
8
            B.or(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
9
          case And(lhs, rhs) =>
10
            B.and(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
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12
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```

```
def runFreeBool[A, B](fb: FreeBool[A])(f: A \Rightarrow B)(
1
          implicit B: BoolAlgebra[B]): B = {
2
        fb match {
3
          case Tru
                       => B.tru
4
          case Fls
                       => B.fls
5
          case Inject(v) \Rightarrow f(v)
6
          case Not(v) => B.not(runFreeBool(v)(f))
7
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8
            B.or(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
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12
13
```

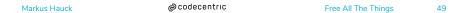
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1
          implicit B: BoolAlgebra[B]): B = {
2
        fb match {
3
          case Tru
                       => B.tru
4
          case Fls
                       => B.fls
5
          case Inject(v) \Rightarrow f(v)
6
          case Not(v) => B.not(runFreeBool(v)(f))
7
          case Or(lhs, rhs) =>
8
            B.or(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
9
          case And(lhs, rhs) =>
10
            B.and(runFreeBool(lhs)(f), runFreeBool(rhs)(f))
11
12
13
```

Using Free Bool

- that was simple (though boilerplate-y)
- what can we do with our new discovered structure
- reminder: boolean operators
 - true, false
 - and, or
 - · xor, implies, nand, nor, nxor

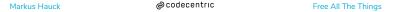


Free Bool Example: Search



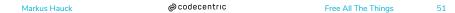
Partial Evaluation

- assume: implemented short circuiting
- what if you don't have all the information?
- partially evaluate predicates
- if evaluates successfully, done
- else, send it on



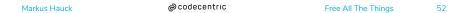
We Freed Boolean Algebras

- good example of underused free structure
- partial evaluation
- serialize the AST (JSON, Protobuf, Avro, ...)
- exercise: minimize AST representation



Resources

- Free Boolean Algebra by Chris Stucchio https://engineering.wingify.com/posts/Free-objects/
- Source Code: https://github.com/markus1189/ free-all-the-things/tree/flatmap



Go And Free All The Things!



Introduction

Free Monad

Free Applicative

Free Functor

Free Boolean Algebra

Conclusion



Bonus

- remove Inject cases from Monad and Applicative
- apply recipe to Monoid and get List (hint: laws)
- free Magmas
- define free X using alternative minimal set of ops of the typeclass