Free All The Things

Markus Hauck

codecentric

Free All The Things

- well known: free monads
- maybe known: free applicatives
- free monoids
- free <you name it>

Goal Of This Talk

- how many of you wrote a Free X
- how many of you used Free...
  - Monad
  - Applicative
  - Functor
  - Boolean Algebra
  - other?
- Goal: explain the technique behind “Free X”
- Be able to apply the “pattern” yourself

The Road Ahead

What Is Free

A free functor is left adjoint to a forgetful functor

What's the problem?

What Is Free

A free “thing” **FreeA** on a type *A* is a *A* and a function

```
def inject(x: A): FreeA
```

s. t. for any other “thing” *B* and a function

```
val f: A => B
```

there exists a unique homomorphism `g` such that

```
g.compose(inject) === f
```

What Is Free

- the good news: there is a recipe
  1. create an AST for ops + vars
  2. modify it such that laws ensured during construction*

Why Free

- having a Free X is good for a number of reasons
- use Free X as if it was X
- but the program is reified into some (data-)structure
- this structure can often be analyzed and optimized
- many interpreters of the same program

Scales of Power

- the structures we will look at, are able to capture computations that have different power abilities
- monad: depend on previous values and branching
- applicative: fixed structure with arbitrary applicative effects in between
- functor: apply a function to the content
- monoid: limited power, but very flexible and composable
- surprise

Disclaimer

- we will mostly look at the data structure version of Free X
- the alternative is to use finally tagless representations (Next Talk)

Freeing The Monad

Freeing The Monad

- what are the operations?

```
trait Monad[F[_]] {
  def pure[A](x: A): F[A]
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

Freeing The Monad

- what are the laws?
- (pseudocode)

```
// Left identity
pure(a).flatMap(f) === f(a)

// Right identity
fa.flatMap(pure) === fa

// Associativity
fa.flatMap(f).flatMap(g) ===
  fa.flatMap(a => f(a).flatMap(g))
```

Freeing The Monad

- todo: the minimal “thing” that has a *Monad* instance **satisfies** the laws
- simple idea: capture as data
- any minimal combination works

Freeing The Monad

```scala
trait Monad[F[_]] {
  def pure[A](x: A): F[A]
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

```scala
sealed abstract class Free[F[_], +A]

final case class Pure[F[_], A](a: A)
    extends Free[F, A]

final case class FlatMap[F[_], A, B](
    fa: Free[F, A],
    f: A => Free[F, B])
    extends Free[F, B]
```

Freeing The Monad

```scala
implicit def freeMonad[F[_], A]: Monad[Free[F, ?]] =
  new Monad[Free[F, ?]] {
    def pure[A](x: A): Free[F, A] = Pure(x)

    def flatMap[A, B](fa: Free[F, A])(
        f: A => Free[F, B]): Free[F, B] =
      FlatMap(fa, f)
  }
```

Interpreter

```scala
def runFree[F[_], M[_]: Monad, A](
    nat: FunctionK[F, M])(free: Free[F, A]): M[A] =
  free match {
    case Pure(x) => Monad[M].pure(x)
    case FlatMap(fa, f) =>
      Monad[M].flatMap(runFree(nat)(fa))(x =>
        runFree(nat)(f(x)))
  }
```

What about the laws?

```scala
// The associativity law
fa.flatMap(f).flatMap(g) ===
  fa.flatMap(a => f(a).flatMap(g))
```

```scala
val exp1 = FlatMap(FlatMap(fa, f), g)
val exp2 = FlatMap(fa, (a: Int) => FlatMap(f(a), g))

exp1 != exp2
```

What about the laws?

The Laws

- actually, we don’t satisfy them
- programmer: after interpretation it’s no longer visible
- mathematician: that’s not the free monad!
- use them to make it faster
- tradeoff: during construction vs during interpretation

Faster Free Monads

- common optimization: associate `flatMap`'s to the right
- avoids having to rebuild the tree repeatedly during construction
- how: during construction time

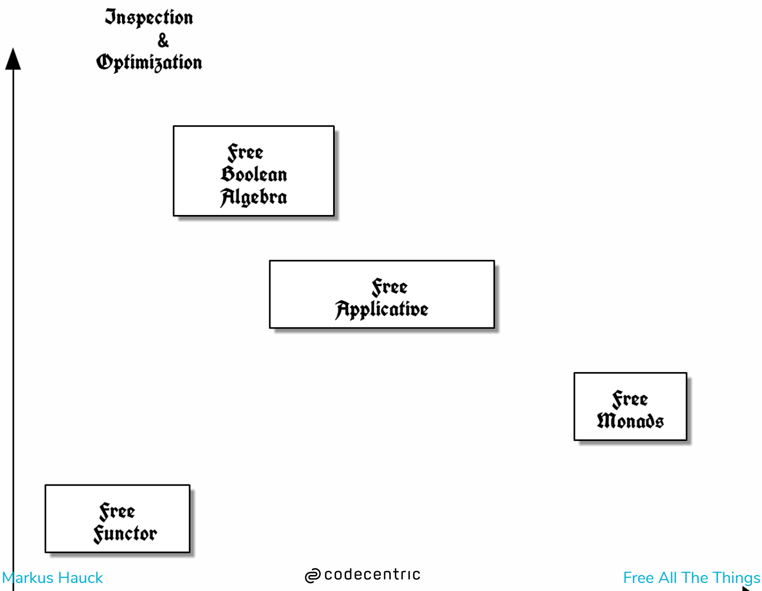
So What?

- the laws tell us what “rewriting” is possible
- here: **flatMap** has to be associative, that means we can re-associate
- why? Let’s look at what happens with normal **flatMap**s

Use Cases

- DSL with monadic expressiveness
- branching, loops, basically everything

Tradeoffs


Inspection
&
Optimization
Free
Boolean
Algebra
Free
Applicative
Free
Monads
Free
Functor

Freeing The Monad

- that’s it for the Monad
- what else?

Freeing The Applicative

- free monads are great, but also limited
- we can't analyze the programs
- how about a smaller gun?

Freeing The Applicative

- we follow the same pattern
- look at typeclass operations
- create datastructure
- “interpreter”

The Applicative Class

```scala
trait Applicative[F[_]] {
  def pure[A](x: A): F[A]
  def ap[A, B](fab: F[A => B], fa: F[A]): F[B]
}
```

Freeing The Applicative

- again the same pattern: we model it as an ADT

```scala
sealed abstract class FreeAp[F[_], A]

final case class Pure[F[_], A](a: A)
    extends FreeAp[F, A]

final case class Ap[F[_], A, B](
    fab: FreeAp[F, A => B],
    fa: FreeAp[F, A])
    extends FreeAp[F, B]
```

1

- of course we also need the interpreter

Less Power?!

- why would we consider Applicative if it's less powerful?
- less is more: we can inspect the AST

Freeing The Functor

- we are well equipped by now

Freeing The Functor

```scala
sealed abstract class FreeFunctor[F[_], A]
case class Fmap[F[_], X, A](fa: F[X])(f: X => A)
    extends FreeFunctor[F, A]
```

Freeing The Functor

- clean code alarm: only one subclass
- can we get rid of it?

Disclaimer

- Once upon a time: https://engineering.wingify.com/posts/Free-objects/
- really awesome article about free objects
- use free boolean algebra to define DSL for event predicates
- all credits to Chris Stucchio (@stucchio)

Free Boolean Algebra

- Wikipedia: boolean algebra + set of generators
- let’s go

Boolean Algebras

- seen: common fp type classes
- apply our knowledge to another example: boolean algebras

Your conclusion here