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1 -

a)

$$\begin{vmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{vmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 16 \\ 38 \\ 38 \end{Bmatrix}$$

$$\frac{-7}{20} \cdot L_1 + L_2 \quad e \quad -\frac{9}{20} \cdot L_1 + L_3$$

$$\left|\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & 551/20 & 97/20 & 648/20 \\ 0 & \cancel{111/20}\ 97/20 & 519/20 & 616/20 \end{array}\right|$$

$$\frac{-97}{551} \cdot L_2 + L_3$$

$$\left|\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & 551/20 & 97/20 & 648/20 \\ 0 & 0 & \frac{276560}{11020} & \frac{276560}{11020} \end{array}\right.$$

$x_3 = 1$
$x_2 = 1$
$x_1 = 0$

b) $\left|\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 7 & 30 & 8 & 38 \\ 9 & 8 & 30 & 38 \end{array}\right.$

$A_{11} = \text{pivô} > A_{21} > A_{31}$

$\left|\begin{array}{ccc|c|} 20 & 7 & 9 & 16 \\ 0 & 551/20 & 97/20 & 648/20 \\ 0 & 97/20 & 519/20 & 616/20 \end{array}\right\} \frac{-7}{20} \cdot L_1 + L_2 \text{ e } \frac{-9}{20} \cdot L_1 + L_3$

$A_{22} = \text{pivô} > A_{32}$

$\left|\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & 551/20 & 97/20 & 648/20 \\ 0 & 0 & \frac{276560}{11020} & \frac{276560}{11020} \end{array}\right\} \frac{-97}{551} \cdot L_2 + L_3$

$x_3 = 1$
$x_2 = 1$
$x_1 = 0$

2 - $Ax = b \rightarrow LUx = b \rightarrow Ux = y$

$Ly = b$

a)

$$\left|\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 7 & 30 & 8 & 38 \\ 9 & 8 & 30 & 38 \end{array}\right| \begin{array}{l} -\frac{7}{20} \cdot \frac{L_1}{\cancel{20}} + L_2 \\ -9/20 \cdot L_1 + L_3 \end{array}$$

$$\overset{M}{\begin{bmatrix} 1 & 0 & 0 \\ -7/20 & 1 & 0 \\ -9/20 & 0 & 1 \end{bmatrix}} \cdot \overset{A}{\begin{bmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix}} = \begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 97/20 & 519/20 \end{bmatrix}$$

$$M \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -97/551 & 1 \end{bmatrix} \cdot A \begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 97/20 & 519/20 \end{bmatrix} = \begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 0 & \frac{276560}{11020} \end{bmatrix}$$

$$L \begin{bmatrix} 1 & 0 & 0 \\ 7/20 & 1 & 0 \\ 9/20 & 97/551 & 1 \end{bmatrix} \overset{U}{\begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 0 & \frac{276560}{11020} \end{bmatrix}} = A$$

$Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 7/20 & 1 & 0 \\ 9/20 & 97/551 & 1 \end{bmatrix} \cdot \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{bmatrix} 16 \\ 38 \\ 38 \end{bmatrix} \quad \frac{112}{20} + y_2 = 38$$

$$y_2 = \frac{648}{20}$$

$$y_1 = 16 \quad y_2 = \frac{648}{20} \quad y_3 = \frac{276560}{11020}$$

FORONI

$Ux = y$

$$\begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 0 & \frac{276560}{11020} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 648/20 \\ \frac{276560}{11020} \end{bmatrix}$$

$x_3 = 1$
$x_2 = 1$
$x_3 = 0$

b) Mesma maneira do item anterior já que os pivôs já estão em suas posições A_{11} e A_{22} sem precisar uma troca de linhas.

3 - a) $\begin{bmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix}$ $\frac{-7}{20} \cdot L_1 + L_2$
$\frac{-9}{20} \cdot L_1 + L_3$

$\begin{bmatrix} 20 & 7 & 9 \\ 0 & 551/20 & 97/20 \\ 0 & 97/20 & 519/20 \end{bmatrix}$ $\frac{-97}{551} \cdot L_2 + L_3$

$\begin{bmatrix} 20 & & \\ 0 & 551/20 & \\ 0 & & \end{bmatrix}$

3-a) e b)

$$\overset{A}{\begin{bmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix}} \left| \overset{I}{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} \right] \div 20 \quad L_1/20$$

$$\overset{A}{\begin{bmatrix} 1 & 7/20 & 9/20 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix}} \left| \overset{I}{\begin{matrix} 1/20 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} \right] \begin{matrix} -7L_1+L_2 \\ \\ -9L_1+L_3 \end{matrix}$$

$$\overset{A}{\begin{bmatrix} 1 & 7/20 & 9/20 \\ 0 & 551/20 & 97/20 \\ 0 & 97/20 & 519/20 \end{bmatrix}} \left| \overset{I}{\begin{matrix} 1/20 & 0 & 0 \\ -7/20 & 1 & 0 \\ -9/20 & 0 & 1 \end{matrix}} \right] \cdot 20/551$$

$-97/20 L_2 + L_3$

$$\overset{A}{\begin{bmatrix} 1 & 7/20 & 9/20 \\ 0 & 1 & \cancel{1940/11020}\ 97/551 \\ 0 & 97/20 & 519/20 \end{bmatrix}} \left| \begin{matrix} 1/20 & 0 & 0 \\ \cancel{-140/11020}\ -7/551 & 20/551 & 0 \\ -9/20 & 0 & 1 \end{matrix} \right]$$

$$A \begin{bmatrix} 1 & 7/20 & 9/20 \\ 0 & 1 & 97/551 \\ 0 & 0 & \cancel{13828}/551 \end{bmatrix} \left| \begin{matrix} 1/20 & 0 & 0 \\ -7/551 & 20/551 & 0 \\ & -97/551 & 1 \end{matrix} \right] \cdot \frac{11020}{276560}$$

$$A \begin{bmatrix} 1 & 7/20 & 9/20 \\ 0 & 1 & 97/551 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{matrix} 1/20 & 0 & 0 \\ -7/551 & 20/551 & 0 \\ \dfrac{-4280}{276560} & \dfrac{1940}{276560} & \dfrac{11020}{276560} \end{matrix} \right]$$

$$A\left[\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 97/551 & -7/551 & 20/551 & 0 \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right] \frac{-97}{551} L_3 + L_2$$

$$A\left[\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 0 & -69/6914 & 519/13828 & -97/13828 \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right]$$

$$\frac{-9}{20} \cdot L_3 + L_1 \quad e \quad \frac{-7}{20} \cdot L_2 + L_1$$

$$A\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 209/3457 & -69/6914 & -107/6914 \\ 0 & 1 & 0 & -69/6914 & 519/13828 & -97/13828 \\ 0 & 0 & 1 & -107/6914 & -97/13828 & 551/13828 \end{array}\right] A^{-1}$$

$$Det(A) = 20 \cdot \frac{551}{20} \cdot \frac{13828}{551} = 13828$$

Utilizando a matriz de Gauss

$$\begin{bmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & \frac{97}{20} \\ 0 & & \frac{13828}{551} \end{bmatrix}$$

4- a) $\left|\begin{matrix} 10 & 2 & 2 \\ 1 & 10 & 2 \\ 2 & -7 & -10 \end{matrix}\right| \begin{matrix} 28 \\ 7 \\ -17 \end{matrix}$

$10 > 2+2$ ✓

$10 > 1+2$ ✓

$|-10| > |-7| + 2$ ✓

5) $\beta_1 = \dfrac{2+2}{10} = 0.4$

$\beta_2 = \dfrac{0.4+2}{10} = 0.24$

$\beta_3 = \dfrac{2 \cdot 0,4 + 7 \cdot 0,24}{10} = 0.248$

$\beta = \max\{\beta_1, \beta_2, \beta_3\} = 0.4 < 1$ ✓

c)

4. c) $\varepsilon = 5 \cdot 10^{-1}$ $\quad x^{(0)} = \{0,0,0\}^T$

$$10d_1 + 2d_2 + 2d_3 = 28$$
$$d_1 + 10d_2 + 2d_3 = 7$$
$$2d_1 - 7d_2 + 10d_3 = -17$$

$$\{x^{(1)}\} = \begin{Bmatrix} 2.8 \\ 0.7 \\ 1.7 \end{Bmatrix} \quad \{d^{(1)}\} = \begin{Bmatrix} 2.8 \\ 0.7 \\ 1.7 \end{Bmatrix} \quad \{d_r^{(1)}\} = \frac{2.8}{2.8} = 1$$

$\{d_r^{(1)} \geq \varepsilon\}$

$$\{x^{(2)}\} = \begin{Bmatrix} 2.32 \\ 0.08 \\ 1.77 \end{Bmatrix} \quad \{d^{(2)}\} = \begin{Bmatrix} 0.48 \\ 0.62 \\ 0.07 \end{Bmatrix} \quad \{d_r^{(2)}\} = \frac{0.62}{2.32} = 0.27$$

$\{d_r^{(2)}\} \leq \varepsilon$

$$x_1^{(2)} = \frac{28 - 2x_2^{(1)} - x_3^{(1)}}{10} = \frac{28 - 2\cdot 0.7 - 2\cdot 1.7}{10} = 2.32$$

$$x_2^{(2)} = \frac{7 - x_1^{(1)} - 2x_2^{(1)}}{10} = \frac{7 - 2.8 - 3.4}{10} = 0.08$$

$$x_3^{(2)} = \frac{-17 - 2\cdot x_1^{(1)} + 7\cdot x_2^{(2)}}{-10} = \frac{-17 - 2\cdot 2.8 + 7\cdot 0.7}{-10} = 1.77$$

d)

$$d^{(1)} = \begin{Bmatrix} 2.8-0 \\ 0.42-0 \\ 1.97-0 \end{Bmatrix}$$

$$\{x^{(1)}\} = \begin{Bmatrix} 2.8 \\ 0.42 \\ 1.97 \end{Bmatrix} \qquad \{d_1^{(1)}\} = \{2.8/2.8\} \geq \varepsilon$$

$$x_1^{(1)} = 28 - 2\cdot x_2^{(0)} - 2\cdot x_3^{(0)} = \frac{28-0-0}{10} = 2.8$$

$$x_2^{(1)} = \frac{7 - 2.8 - 2\cdot x_3^{(0)}}{10} = \frac{4.2}{10} = 0.42$$

$$x_3^{(1)} = \frac{-17 - 2\cdot 2.8 + 7\cdot 0.42}{-10} = 1.97$$

$$\{x^{(2)}\} = \begin{Bmatrix} 2.32 \\ 0.074 \\ 2.11 \end{Bmatrix} \qquad d^{(2)} = \begin{Bmatrix} 0.48 \\ 0.35 \\ 0.14 \end{Bmatrix}$$

$$x_1^{(2)} = 28 - 2\cdot x_2^{(1)} - 2\cdot x_3^{(1)} = \frac{28-0.84-3.94}{10} = 2.32$$

$$x_2^{(2)} = 7 - 2.32 - 2\cdot x_3^{(1)} = \frac{7-2.32-3.94}{10} = 0.074$$

$$x_3^{(2)} = \frac{-17 - 2\cdot 2.32 + 7\cdot 0.074}{-10} = 2.11$$

$$\{d_1^{(2)}\} = \{0.48/2.32\} = 0.21 \leq \varepsilon$$