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ITCS 3153: Introduction to Artificial Intelligence

In-class assignment

Probabilistic Reasoning Over Time

1. Label each equation (filtering, smoothing, or prediction). Then label each term in right hand side of the equations.

Filtering

$$P(X_{t+1}|e_{1:t+1}) = \alpha \underbrace{P(e_{t+1}|X_{t+1})}_{\text{sensor model}} \sum_{x_t} \overbrace{P(X_{t+1}|x_t)}^{\text{transition}} \overbrace{P(x_t|e_{1:t})}^{\text{prev filtering}}$$

Normalization $\frac{1}{e_{1:t+1}}$

latest evidence

latest state

previous state

all prev evidence

smoothing

$$P(X_k|e_{1:t}) = P(X_k|e_{1:k}, e_{k+1:t})$$

→ evidence after k to now

state at k (between 1 and t)

evidence from beginning to k

prediction

$$P(X_{t+k+1}|e_{1:t}) = \sum_{x_{t+k}} \overbrace{P(X_{t+k+1}|x_{t+k})}^{\text{transition model}} \overbrace{P(x_{t+k}|e_{1:t})}^{\text{prev prediction}}$$

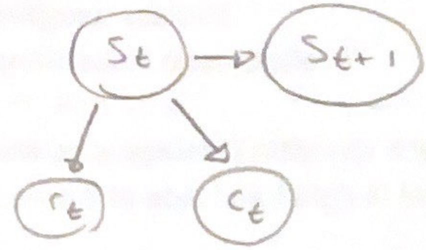
state after k time

prev state after k time from time t

all prev evidence

2. Construct a Bayesian network and show the CPTs to represent the following probabilities:

$$
\begin{aligned}
P(s_0) &= 0.7 \\
P(s_{t+1}|s_t) &= 0.8 \\
P(s_{t+1}|\neg s_t) &= 0.3 \\
P(r_t|s_t) &= 0.2 \\
P(r_t|\neg s_t) &= 0.7 \\
P(c_t|s_t) &= 0.1 \\
P(c_t|\neg s_t) &= 0.3
\end{aligned}
$$



Variable meanings:

s - whether or not a student is getting enough sleep

r - whether or not a student's eyes are red

c - whether or not a student sleeps in class

$P(s_t)$
0.7

s_t	$P(s_{t+1})$
t	.8
f	.3

s_t	$P(r_t)$
t	.2
f	.7

s_t	$P(c_t)$
t	.1
f	.3

3. Given the following evidence values:

e$_1$ = not red eyes, not sleeping in class

e$_2$ = red eyes, not sleeping in class

e$_3$ = red eyes, sleeping in class

Compute state estimation for t=1,2,3. Show the equations and exact values. State estimation: $\mathbf{P}(S_t|e_{1:t})$

Process for t=i:

1) Show $\mathbf{e}_i$
2) Write filtering equation for S_i
3) Do prediction step for t+1.
   a) Write equation, if i==1 then no $\mathbf{e}_i$
   b) Write full sum (remove summation sign)
   c) Plug in values
   d) Make vector at end
4) Do update
   a) Write equation. Plug in term from prediction step.
   b) Compute sensor model term.
   c) Multiply sensor model vector with prediction vector.
   d) Normalize

t = 1

< 0.8643, 0.1357 >

t = 2

$e_2 = \langle r_2, \neg c_2 \rangle$

$P(S_2 \mid e_{1:2}) = \alpha P(e_2 \mid S_2) \sum_{s_1} P(S_2 \mid s_1) P(s_1 \mid e_1)$

$P(S_2 \mid e_1) = \sum_{s_1} P(S_2 \mid s_1) P(s_1 \mid e_1)$

$= P(s_2 \mid s_1) P(s_1 \mid e_1) + P(s_2 \mid \neg s_1) P(\neg s_1 \mid e_1)$

$= (0.8)(0.8643) + (0.3)(0.1357)$

$P(S_2 \mid e_1) = \langle 0.73215, 0.26785 \rangle$

$P(S_2 \mid e_{1:2}) = \alpha P(e_2 \mid S_2) P(S_2 \mid e_1)$

$P(e_2 \mid s_2) = P(r_2, \neg c_2 \mid s_2) = P(r_2 \mid s_2) P(\neg c_2 \mid s_2)$

$(0.2)(0.9) = 0.18$

$P(e_2|S_2) = \langle 0.18, P(e_2|\neg S_2) \rangle = \langle 0.72, (0.7)(0.7) \rangle$

$\langle 0.18, 0.49 \rangle$

$P(S_2|e_{1:2}) = \alpha \langle .18, .49 \rangle \cdot \langle .73215, .26785 \rangle$

$= \alpha \langle .131787, .1312465 \rangle$

$\alpha = \frac{1}{.131787 + .1312465} = \frac{1}{.2630335}$

$P(S_2|e_{1:2}) = \langle \frac{.131787}{.2630335}, \frac{.1312465}{.2630335} \rangle$

$= \boxed{\langle .501, .4989 \rangle}$

$t = 3$

$e_3 = \langle r_3, c_3 \rangle$

$P(S_3|e_{1:3}) = \alpha \, P(e_3|S_3) \sum_{S_2} P(S_3|S_2) P(S_2|e_{1:2})$

$P(S_3|e_{1:2}) = \sum_{S_2} P(S_3|S_2) P(S_2|e_{1:2})$

$= P(S_3|S_2) P(S_2|e_{1:2}) + P(S_3|\neg S_2) P(\neg S_2|e_{1:}$

$= (0.8)(.501) + (0.3)(.4989)$

$P(S_3|e_{1:2}) = \langle .55047, .44953 \rangle$

$P(S_3|e_{1:3}) = \alpha \, P(e_3|S_3) \, P(S_3|e_{1:2})$

$P(e_3|S_3) = P(r, c|S_3) = P(r|S_3) P(c|S_3) = (.2)(.1)$

$= .02$

$P(e_3|S_3) = \langle .02, P(e_3|\neg S_3) \rangle$

$= \langle .02, (.7)(.3) \rangle = \langle .02, .21 \rangle$

$P(S_3|e_{1:3}) = \alpha \langle .02, .21 \rangle \langle .55047, .44953 \rangle$

$= \alpha \langle .011008, .0944013 \rangle$

$\alpha = \frac{1}{.011008 + .0944013} = \frac{1}{.1054}$

$P(S_3|e_{1:3}) = \langle \frac{.011008}{.1054}, \frac{.0944013}{.1054} \rangle = \boxed{\langle .1044, .8956 \rangle}$