Math 400 Fall 2022 Final Project 1 due: 5PM on Fri November 18

Project Directions

- Include a report on every group member's contribution.
- Submit the group's well commented code used for the project with instructions on how to compile and run.
- Make a **10** to **20** minute video presentation of your results.

The project consists of four problems. One from each section.

Section 1

The problem in this section is the same for all groups.

a. The purpose of this exercise is to understand when Newton's method works and fails. To this end, solve

$$\tanh x = 0,$$

by Newton's method and study the intermediate details of the algorithm. Start with $x_0 = 1.08$. Plot the the tangent in each iteration of Newton's method. Then repeat the calculations and the plotting when $x_0 = 1.09$. Explain what you observe.

b. Does the secant method behave better than Newton's method in the problem described in part **a**? Try the initial guesses

(i) $x_0 = 1.08$ and $x_1 = 1.09$

(ii) $x_0 = 1.09$ and $x_1 = 1.1$

(iii) $x_0 = 1$ and $x_1 = 2.3$

(iv) $x_0 = 1$ and $x_1 = 2.4$.

c. Solve the same problem as in part **a**, using the bisection method, but let the initial interval be $[-5, 3]$.

d. An attractive idea is to combine the reliability of the bisection method with the speed of Newton's method. Such a combination is implemented by running the bisection method until we have a narrow interval, and then switch to Newton's method for speed.

Write a program that implements this idea. Start with an interval $[a, b]$ and switch to Newton's method when the current interval in the bisection method is a fraction s of the initial interval (i.e., when the interval has length $s(b - a)$). Potential divergence of Newton's method is still an issue, so if the approximate root jumps out of the narrowed interval (where the solution is known to lie), one can switch back to the bisection method. The value of s must be given as argument to the program, but it may have a default value of 0.1.

Try the new method on $\tanh x = 0$ with the initial interval $[-10, 15]$.

Use numerical differentiation to compute the derivative in each Newton step.

Section 2

Solve the nonlinear system using Newton's method with the given initial vector. Terminate the process when the maximum norm of the difference between successive iterates is less than 5×10^{-6}.

1. $$\begin{aligned} f(x,y,z) &= x^2+y^2+z^2-1 &= 0 \\ g(x,y,z) &= x^2+z^2-0.25 &= 0 \\ h(x,y,z) &= x^2+y^2-4z &= 0 \end{aligned} \qquad \mathbf{x}^{(0)} = [1 \ \ 1 \ \ 1]^T.$$

2. $$\begin{aligned} f(x,y,z) &= x^2+y^2+z^2-10 &= 0 \\ g(x,y,z) &= x+2y-2 &= 0 \\ h(x,y,z) &= x+3z-9 &= 0 \end{aligned} \qquad \mathbf{x}^{(0)} = [2 \ \ 0 \ \ 2]^T.$$

3. $$\begin{aligned} f(x,y,z) &= x^2+50x+y^2+z^2-200 &= 0 \\ g(x,y,z) &= x^2+20y+z^2-50 &= 0 \\ h(x,y,z) &= -x^2-y^2+40z+75 &= 0 \end{aligned} \qquad \mathbf{x}^{(0)} = [2 \ \ 2 \ \ 2]^T.$$

Implement Gaussian elimination with partial pivoting to solve the linear system in each step. Use numerical differentiation to compute the Jacobian matrix at each step.

Section 3

The problem in this section is the same for all groups.

Riemann sums can be used to approximate integrals and they do so by using piecewise constant functions to approximate the function. The trapezoidal rule uses piecewise linear functions to approximate the function and then the area of a trapezoid to approximate the area. Simpson's rule uses piecewise parabolas to approximate the function. The process used to build Simpson's rule (refer to section 3.3-3.4 of our online text, Numerical Methods: An Inquiry Based Approach) can be extended to any higher-order polynomial. Develop your own integration algorithm that uses piecewise cubic functions. You need to show all of the mathematics necessary to derive the algorithm, provide several test cases to show that the algorithm works, and produce a numerical experiment that shows the order of accuracy of the algorithm.

Section 4

The problem in this section is the same for all groups. Go to the USGS water data repository: https://maps.waterdata.usgs.gov/mapper/index.html.

Here you'll find a map with information about water resources around the country.

- Zoom in to a dam of your choice (make sure that it is a dam).
- Click on the map tag then click "`Access Data`"
- From the drop down menu at the top select either "`Daily Data`" or "`Current/Historical Data`". If these options don't appear then choose a different dam.

- Change the dates so you have the past year's worth of information.
- Select `Tab-separated` under "`Output format`" and press `Go`. Be sure that the data you got has a flow rate (ft^3/sec).
- At this point you should have access to the entire data set. Copy it into a `csv` file and save it to your computer.

For the data that you downloaded, perform the following tasks:

1. Plot the data in a reasonable way giving appropriate units
2. Find the total amount of water that has been discharged from the dam during the past calendar year
3. Report any margin of error in your calculation based on the numerical method that you used in part (2)