Alchemist by Juame Plensa (location: MIT W20)

Discrete Optimization II

15.060: Data, Models, and Decisions

Podimata, Ramakrishnan, Yao

Class 17 (Nov 19)

A Classification of Optimization Problems

		Decision Variables	
		All continuous	Some (or all) integer
Objective Function & Constraints	All linear	*Linear Optimization*	*Integer Linear Optimization**
	Some (or all) non-linear	*Nonlinear Optimization*	*Integer Nonlinear Optimization*

* also known as Discrete Optimization

Recap: The Power of Binary Variables

- Binary variables:
  - Are very useful to model business decisions that involve doing something or not doing something (as opposed to how much of something to do) …
  - … but make solving an optimization model more challenging

- Binary variables allow us to model IF-THEN relationships and non-linearities using “tricks” so that the formulation remains linear.

- A vast array of real-world decision problems can be modeled with binary variables

Today’s Class

- Today, we’ll tackle another discrete optimization problem, this time in **supply-chain management**.

- This problem falls under the umbrella of **matching** problems (more on this later), a broad category of real-world problems that discrete optimization is particularly well suited to solve.

- We’ll start with a base formulation. Then, as an in-class exercise, you’ll be responsible for **modifying the formulation** to account for some real-world changes

Optimization in Supply Chains

Supply Chain Optimization

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    - the location of facilities
    - the sizing of facilities
    - the sourcing and distribution flows

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  - the structure of the supply chain
  - the location of facilities
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- Examples where supply chain network optimization is performed:
  - Expansion into a new market
  - Launching a new product
  - Evaluating mergers and acquisitions
  - Responding to supply, demand, or distribution disruptions
  - Responding to cost, demand, or other economic realities
  - Responding to new regulations
  - Simply evolving to adapt to changes in the business environment

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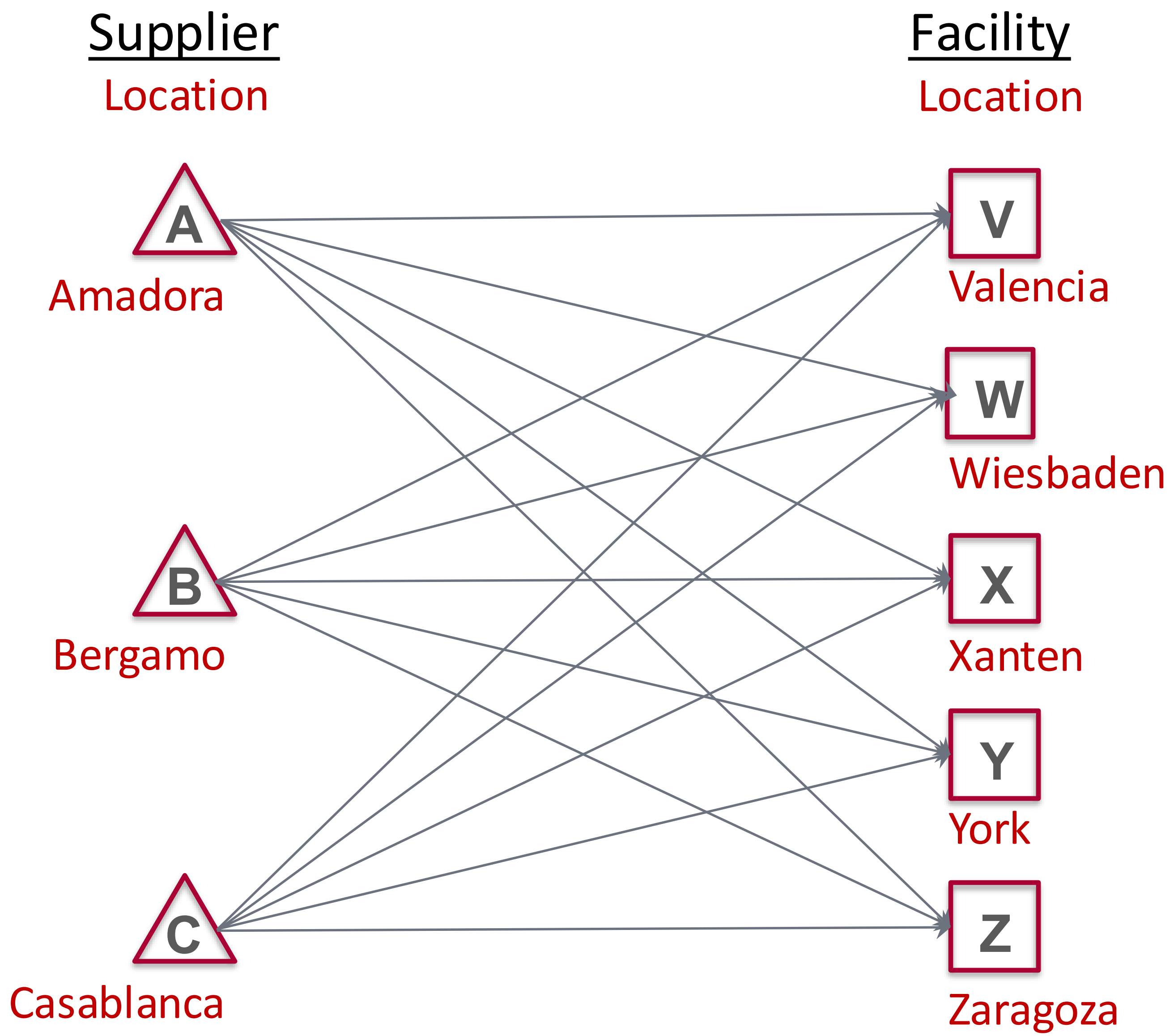
  - Responding to new regulations
  - Simply evolving to adapt to changes in the business environment
- **Optimization is a critical success factor in supply chain design and operation worldwide**

Any Supply Chain experience?

Motivating Example

- Consider a large apparel retailer in Europe with 5 production facilities and 3 fabric suppliers.
  - The suppliers are located in **A**madora (Portugal), **B**ergamo (Italy), and **C**asablanca (Morocco)
  - The production facilities are located in **V**alencia (Spain), **W**iesbaden (Germany), **X**anten (Germany), **Y**ork (England), and **Z**aragoza (Spain)

Supply Network



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- Can we use optimization to minimize supply costs while meeting demand?

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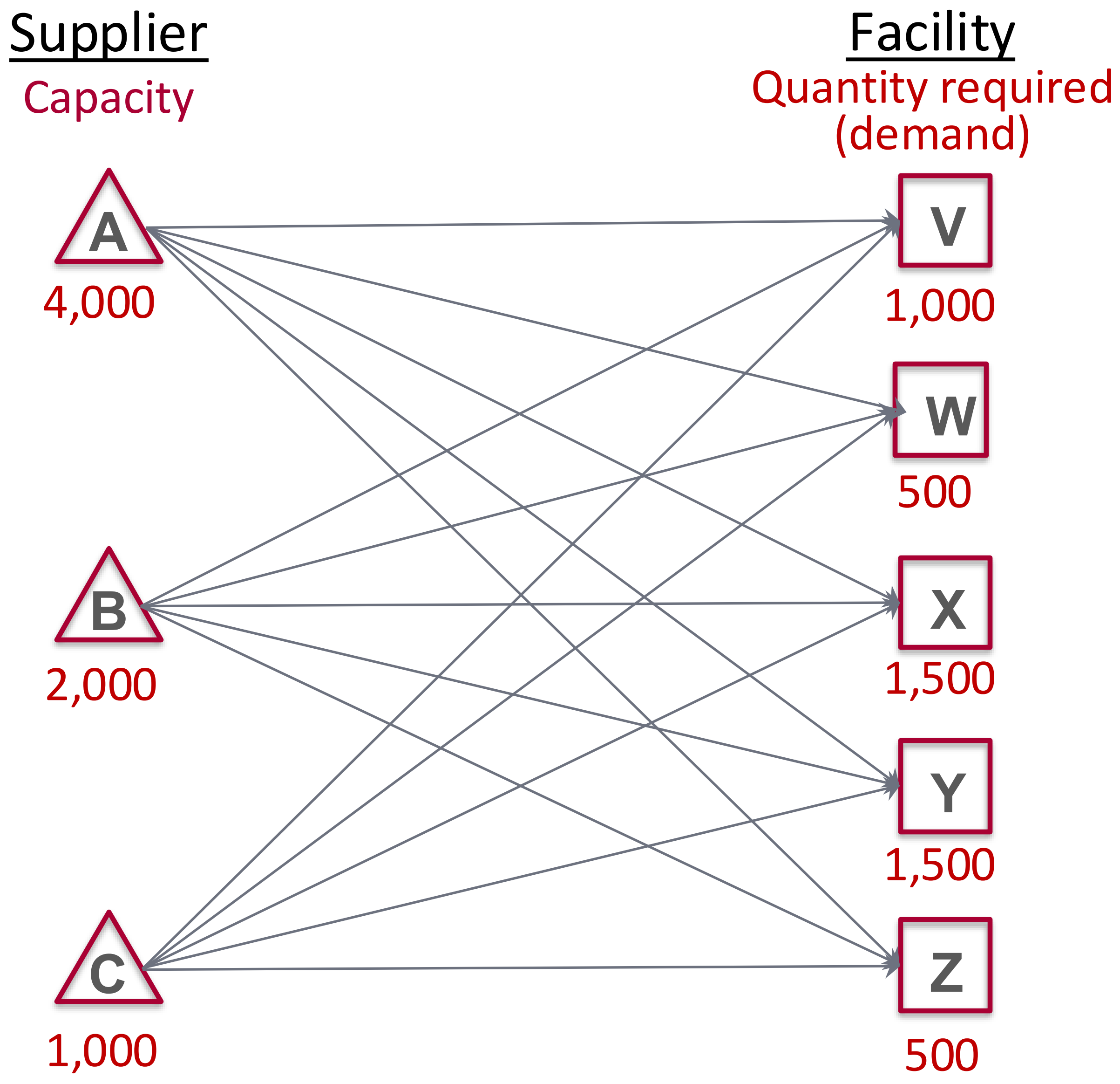
This example is highly simplified. Supply chains are much more complex (multiple stages, multiple products, multiple time periods, many design decisions at each stage, and complex operational constraints). However, it provides an illustration of how optimization models can capture key design decisions, and practice in modeling using binary variables

What sort of input data we do need to formulate the problem?

Facility Demands and Supply Capacities



Capacity and quantity required figures are in tons of fabric per month

Supply Costs

- Supply Costs in €1,000 per ton of fabric:

Supplier	Cost of supplying 1 ton to Facility				
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**
Amadora	1.78	2.26	2.22	2.30	1.45
Bergamo	1.64	2.70	2.00	2.44	2.30
Casablanca	1.70	2.15	2.58	1.28	1.95

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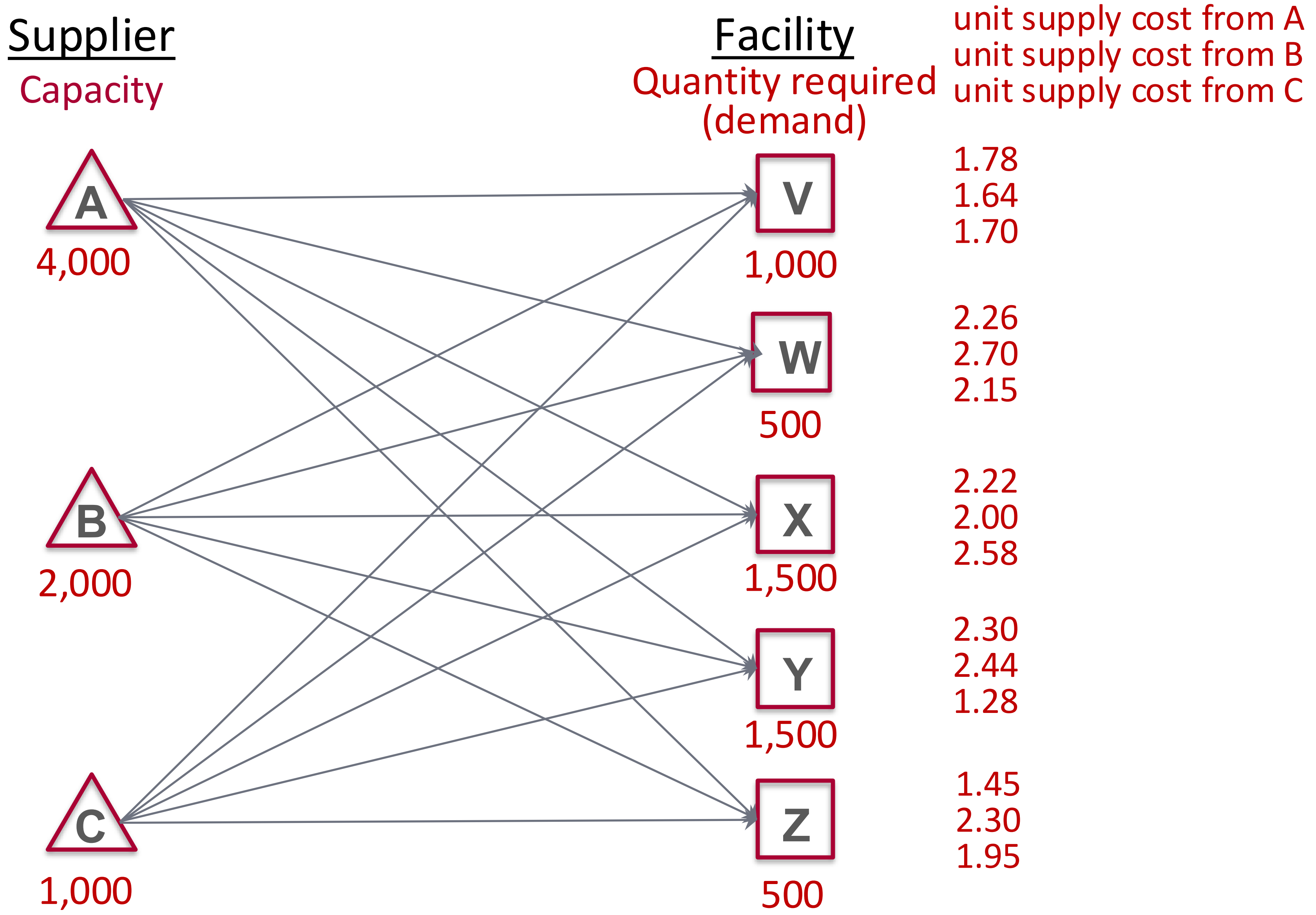
- Supply costs include purchase, transportation, and all other variable costs
    - Example: It costs €1,780 to supply 1 ton of fabric from the supplier in Amadora to the production facility in Valencia

- Ignore fixed supply costs (for now)

Summary



Supplier
Capacity
Facility
Quantity required (demand)
unit supply cost from A
unit supply cost from B
unit supply cost from C
A
4,000
B
2,000
C
1,000
V
1,000
1.78
1.64
1.70
W
500
2.26
2.70
2.15
X
1,500
2.22
2.00
2.58
Y
1,500
2.30
2.44
1.28
Z
500
1.45
2.30
1.95

This is clearly a simplification. We ignored …

- Fixed costs
- Economies of scale (e.g., bulk discount)
- Multiple time periods
- Multiple products
- Transportation mode options + lead time
- Capacity constraints on the routes
- …

Yet, the formulation can be easily modified to incorporate any of the above real-world factors. *Isn't that incredible?*

Let’s formulate an optimization model!

- Decision variables

- Objective function

- Constraints

Decision Variables

$$\mathbf{s}_{A,V}, \mathbf{s}_{A,W}, \ldots, \mathbf{s}_{C,Z}$$

Objective Function

minimize

(total supply cost) $1.78\ \mathbf{s}_{A,V} + 2.26\ \mathbf{s}_{A,W} + \ldots + 1.95\ \mathbf{s}_{C,Z}$

Supplier	Cost of supplying 1 ton to Facility				
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**
Amadora	1.78	2.26	2.22	2.30	1.45
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$\mathbf{s}_{A,V}, \mathbf{s}_{A,W}, \ldots, \mathbf{s}_{C,Z}$

Supply side constraints

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A) $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$

(capacity at B) $s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$

(capacity at C) $s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$

Supply side constraints

$s_{A,V}, s_{A,W}, \ldots, s_{C,Z}$

Demand-side Constraints

minimize

$$\begin{array}{ll} \text{(total supply cost)} & 1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z} \end{array}$$

subject to

$$\left.\begin{array}{ll} \text{(capacity at A)} & s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000 \\ \text{(capacity at B)} & s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le 2{,}000 \\ \text{(capacity at C)} & s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000 \end{array}\right\} \text{Supply side constraints}$$

$$\left.\begin{array}{ll} \text{(demand at V)} & s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000 \\ \text{(demand at W)} & s_{A,W} + s_{B,W} + s_{C,W} \ge 500 \\ \text{(demand at X)} & s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500 \\ \text{(demand at Y)} & s_{A,Y} + s_{B,Y} + s_{C,Y} \ge 1{,}500 \\ \text{(demand at Z)} & s_{A,Z} + s_{B,Z} + s_{C,Z} \ge 500 \end{array}\right\} \text{Demand side constraints}$$

$$s_{A,V},\ s_{A,W},\ \ldots,\ s_{C,Z}$$

Don’t forget non-negativity constraints

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A) $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$

(capacity at B) $s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$

(capacity at C) $s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$

Supply side constraints

(demand at V) $s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$

(demand at W) $s_{A,W} + s_{B,W} + s_{C,W} \geq 500$

(demand at X) $s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$

(demand at Y) $s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$

(demand at Z) $s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$

Demand side constraints

(nonnegativity) $s_{A,V} \geq 0,\ s_{A,W} \geq 0,\ \ldots,\ s_{C,Z} \geq 0$

Base Formulation

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

$$\left.\begin{array}{ll}
\text{(capacity at A)} & s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000 \\
\text{(capacity at B)} & s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000 \\
\text{(capacity at C)} & s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000
\end{array}\right\} \text{Supply side constraints}$$

$$\left.\begin{array}{ll}
\text{(demand at V)} & s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000 \\
\text{(demand at W)} & s_{A,W} + s_{B,W} + s_{C,W} \geq 500 \\
\text{(demand at X)} & s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500 \\
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\text{(demand at Z)} & s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500
\end{array}\right\} \text{Demand side constraints}$$

(nonnegativity) $s_{A,V} \geq 0,\ s_{A,W} \geq 0,\ \ldots,\ s_{C,Z} \geq 0$

Base Formulation Spreadsheet Model

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**	
Amadora	1.78	2.26	2.22	2.30	1.45	4,000
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000
Quantity required	1,000	500	1,500	1,500	500	

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**	
Amadora	**500**	**500**	**0**	**500**	**500**	2,000
Bergamo	**500**	**0**	**1,500**	**0**	**0**	2,000
Casablanca	**0**	**0**	**0**	**1,000**	**0**	1,000
Quantity received	1,000	500	1,500	1,500	500	

OBJECTIVE

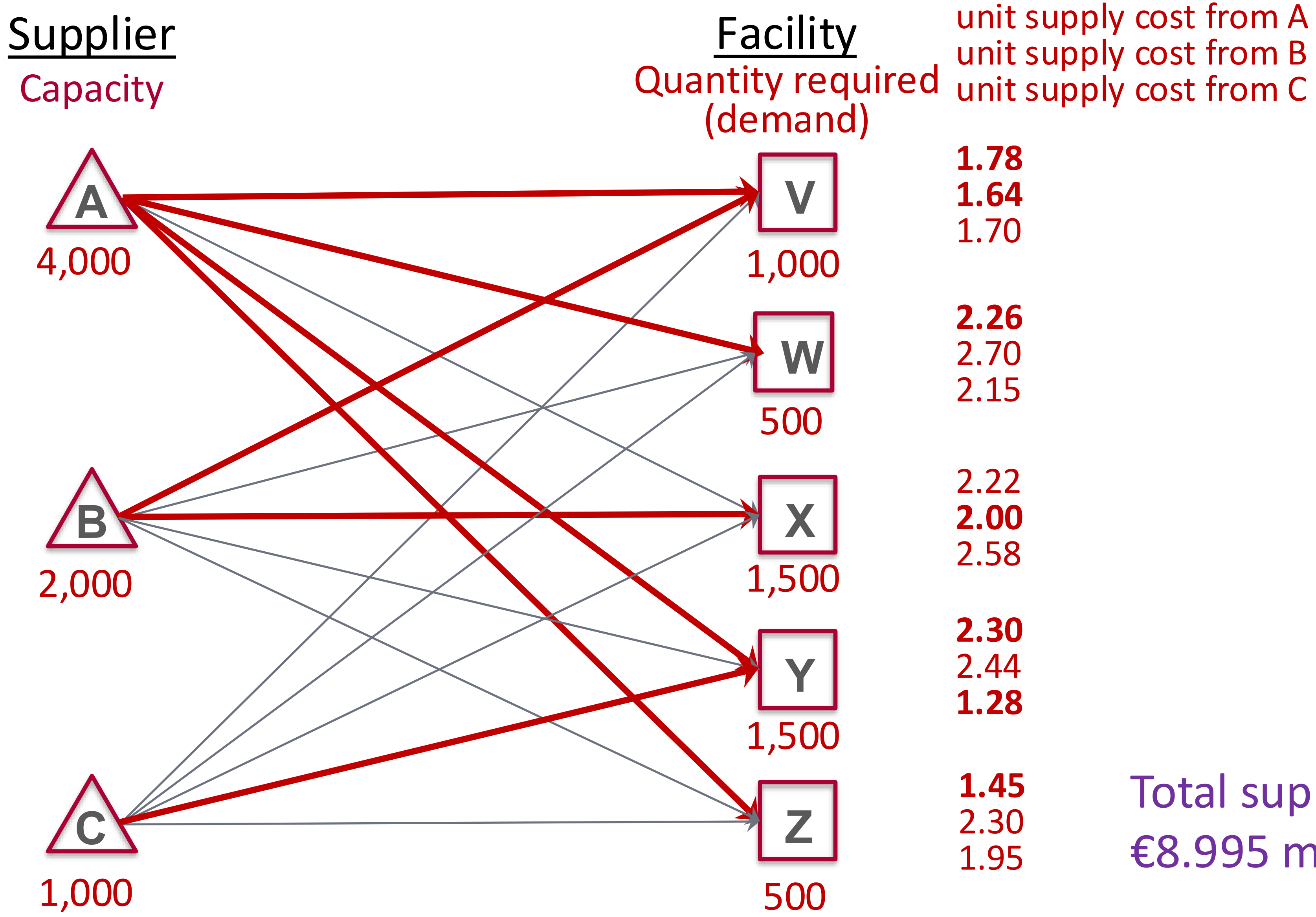
Total Cost	**8,995**

minimize

CONSTRAINTS

Supply does not exceed capacity
Quantity delivered equals quantity required
Nonegativity

Solution to Base Formulation

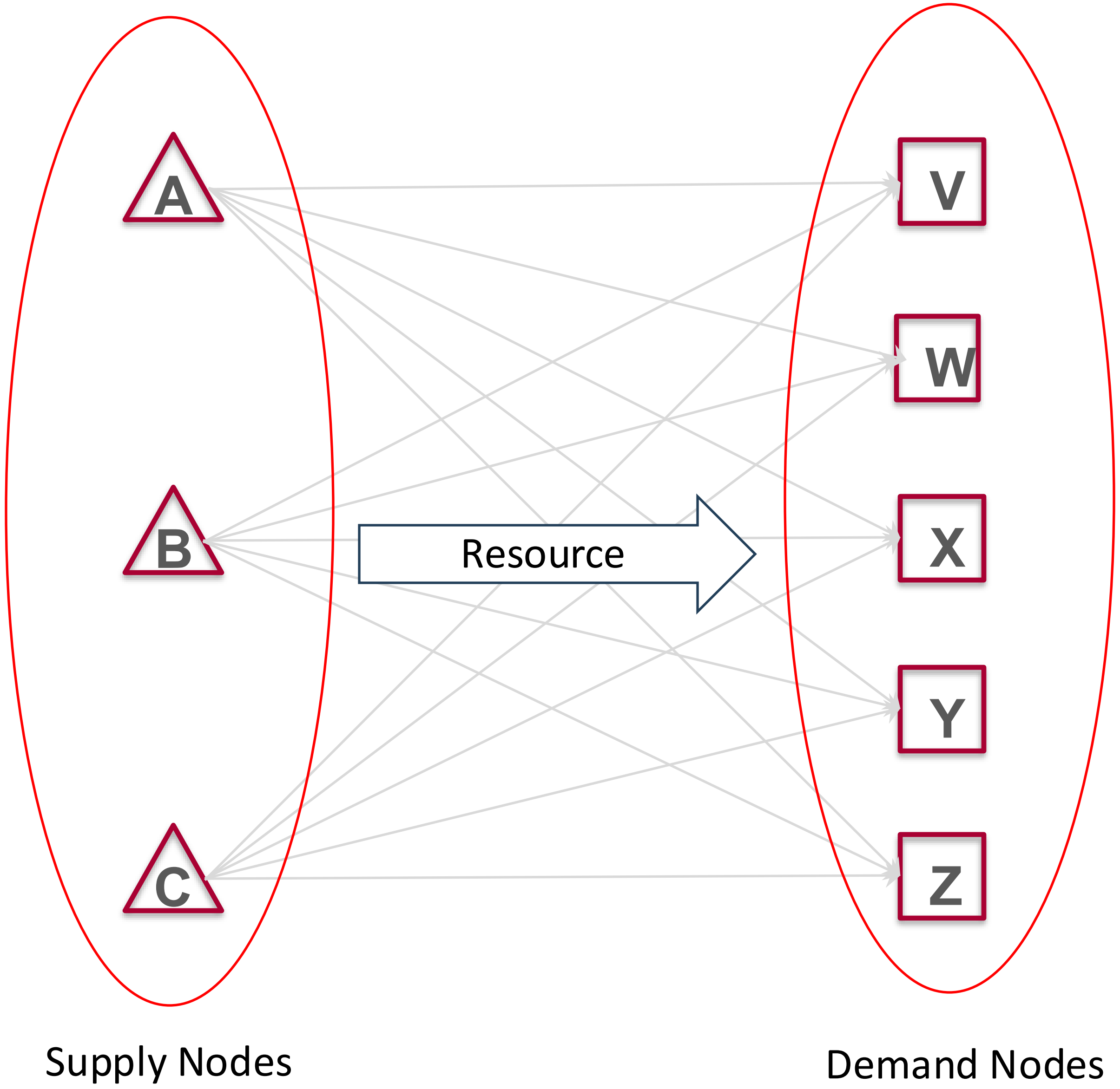


Total supply cost = €8.995 million

More generally, discrete optimization is often used for matching problems

- **Matching** problems generally involve some **resource** that must be allocated from a set of **supply nodes** to **demand nodes**
  - The resource may be physical goods or abstract concepts such as time.

- **Goal**: Find the best way to match supply and demand to minimize cost or maximize utility



Matching appears everywhere!

Application Domain	Supply Nodes	Demand Nodes	Resource	Objective
Supply Chain	Suppliers	Production Facilities	Raw materials / goods	Minimize transportation and procurement cost
Healthcare Staffing	Available nurses	Hospital shifts	Labor hours	Maximize nurse satisfaction and satisfy staffing requirements
School Assignment	Students (with preferences)	Schools	Students seats	Maximize student satisfaction
Ride Sharing	Available drivers	Passengers requesting rides	Rides	Minimize passenger wait time and driver idle time

Optimization is really well suited for these applications!

Class Exercise 1

Some advice before you start

- In Linear Optimization, we allowed the decision variables to be fractional
- In Monday's class on Discrete Optimization, we required the decision variables to be binary
- In today's exercise, you need to use both fractional and binary decision variables
  - Tip: Fractional variables should be used for "how much" decisions and binary variables should be used for "should we do this? yes or no?" decisions
  - The tricky part will be to "connect" the two using the right constraints
  - But whatever you do, the objective and the constraints MUST be linear!

Evaluating Operational Proposals

The retailer's senior management is considering two separate supply strategy proposals aimed at better streamlining its supply chain operations:

- Proposal 1: **Supplier Consolidation** (using exactly 2 instead of 3 suppliers)
- Proposal 2: **Single Sourcing** (serve **V**alencia from a single supplier only)

In-Class Exercise

- You will have 15 minutes to evaluate the two proposals. Work on it by yourself for a few minutes, then work on it with your neighbors
- For <u>each</u> proposal:
  - Write down on your handout precisely what additional binary decision variables and what modifications to the objective function and/or constraints are required
  - Your objective function and constraints must be linear
- We will stop by to answer any questions you may have. Good luck!

Class Exercise - Part 1 Solutions

(Full solutions will be posted to Canvas after class)

Base Formulation

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$	Supply side constraints
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$	
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$	
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$	Demand side constraints
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$	
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$	
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(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$	
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$	

Proposal 1 Formulation: *Will this work*?

(Answer)

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le 2{,}000$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000$
(two suppliers)	$\boxed{y_A + y_B + y_C = 2}$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \ge 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500$
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(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \ge 0$
(binary)	$\boxed{y_A, y_B, y_C \text{ are binary}}$

Idea:
y_i = 1 if we use Supplier i, and 0 if we don't...

Proposal 1 Formulation

(Answer)

Idea:
y_i = 1 if we use Supplier i, and 0 if we don't...

minimize

(total supply cost)	$1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000\, y_A$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000\, y_B$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000\, y_C$
(two suppliers)	$\boxed{y_A + y_B + y_C = 2}$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$
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(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$
(binary)	$\boxed{y_A, y_B, y_C \text{ are binary}}$

Let's see how this constraint works

$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000\ y_A$

If $y_A = 1$, the constraint becomes $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$

Total shipments from Amadora cannot exceed capacity

If $y_A = 0$, the constraint becomes $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 0$

$s_{A,V} = 0,\ s_{A,W} = 0,\ s_{A,X} = 0,\ s_{A,Y} = 0,\ s_{A,Z} = 0$

No shipments from Amadora

Proposal 1 Spreadsheet Model*

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**	
Amadora	1.78	2.26	2.22	2.30	1.45	4,000
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000
Quantity required	1,000	500	1,500	1,500	500	

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied	Select supplier?	Effective capacity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**			
Amadora	1,000	500	1,500	500	500	4,000	1	4000
Bergamo	0	0	0	0	0	0	0	0
Casablanca	0	0	0	1,000	0	1,000	1	1000
Quantity received	1,000	500	1,500	1,500	500		2	

OBJECTIVE

Total Cost	9,395

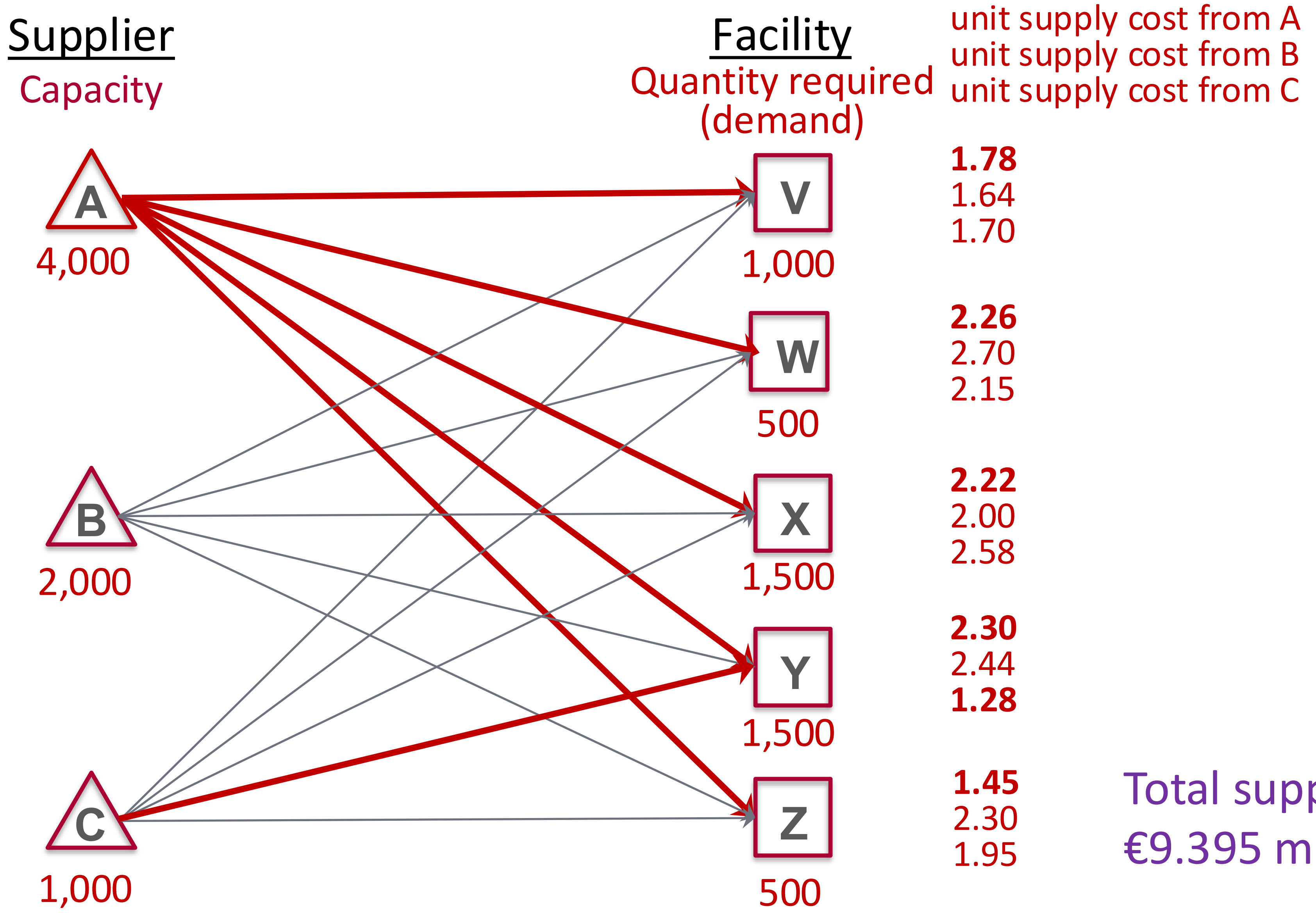
minimize

CONSTRAINTS

Supply does not exceed effective capacity
Quantity delivered equals quantity required
Select two suppliers
Nonnegativity + binary selection variables

*will be uploaded after class

Proposal 1 Supply Plan



Total supply cost = €9.395 million

Proposal 2 Formulation – *Will this work?*

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A) $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$

(capacity at B) $s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$

(capacity at C) $s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$

(single sourcing for V) $y_{A,V} + y_{B,V} + y_{C,V} = 1$

(linking constraint for A) $s_{A,V} \leq 4{,}000$

(linking constraint for B) $s_{B,V} \leq 2{,}000$

(linking constraint for C) $s_{C,V} \leq 1{,}000$

(demand at V) $s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$

(demand at W) $s_{A,W} + s_{B,W} + s_{C,W} \geq 500$

(demand at X) $s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$

(demand at Y) $s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$

(demand at Z) $s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$

(nonnegativity / binary) $s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$; $y_{A,V}, y_{B,V}, y_{C,V}$ are binary

Idea:
$y_{i,V}$ = 1 if we use Supplier i to serve Valencia, and 0 if we don't...

Proposal 2 Formulation

(Answer)

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A) $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$

(capacity at B) $s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$

(capacity at C) $s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$

(single sourcing for V) $\boxed{y_{A,V} + y_{B,V} + y_{C,V} = 1}$

(linking constraint for A) $s_{A,V} \leq 4{,}000\, y_{A,V}$

(linking constraint for B) $s_{B,V} \leq 2{,}000\, y_{B,V}$

(linking constraint for C) $s_{C,V} \leq 1{,}000\, y_{C,V}$

(demand at V) $s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$

(demand at W) $s_{A,W} + s_{B,W} + s_{C,W} \geq 500$

(demand at X) $s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$

(demand at Y) $s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$

(demand at Z) $s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$

(nonnegativity / binary) $s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$; $\boxed{y_{A,V}, y_{B,V}, y_{C,V} \text{ are binary}}$

Idea:

$y_{i,V}$ = 1 if we use Supplier i to serve Valencia, and 0 if we don't…

Proposal 2 Spreadsheet Model*

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**	
Amadora	1.78	2.26	2.22	2.30	1.45	4,000
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000
Quantity required	1,000	500	1,500	1,500	500	

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied	Used to source Valencia?	Valencia demand * binary
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**			
Amadora	**0**	**500**	**500**	**500**	**500**	2,000	**0**	0
Bergamo	**1,000**	**0**	**1,000**	**0**	**0**	2,000	**1**	1000
Casablanca	**0**	**0**	**0**	**1,000**	**0**	1,000	**0**	0
Quantity received	1,000	500	1,500	1,500	500		1	

OBJECTIVE

Total Cost	9,035

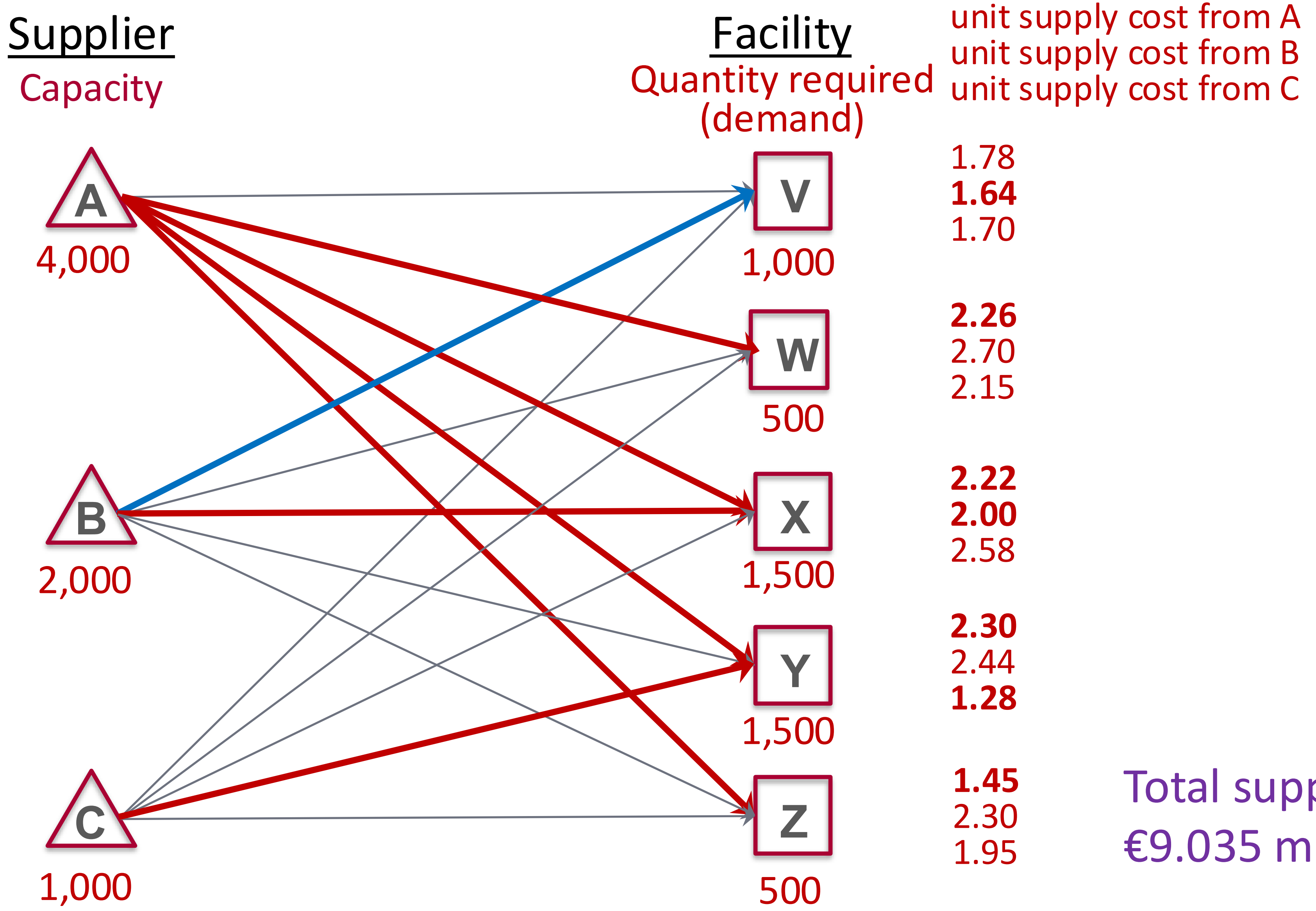
minimize

CONSTRAINTS

Supply does not exceed capacity
Quantity delivered equals quantity required
Sum of binaries equals 1 (select only one supplier for location 1)
Cells C12:14 cannot exceed Cells J12:J14
Nonegativity
Binary

*will be uploaded after class

Proposal 2 Supply Plan



Total supply cost = €9.035 million

Operational Proposals: Cost Implications Summary

	Without supplier consolidation	With supplier consolidation
Without single sourcing	€8.995 million $\xrightarrow{+4.44\%}$	€9.395 million
	$\downarrow$ +0.44%	
With single sourcing	€9.035 million	(optional)

Class Exercise 2

Assessing Impact of New Supply Conditions

- Bergamo's Condition: The supplier at **B**ergamo has indicated that a renewal of their supply contract would require a **fixed cost** of €1 million for ordering any amount from the supplier. This is on top of the variable cost.

- Amadora's Condition: The supplier at **A**madora has indicated that a renewal of their supply contract would require a **minimum order quantity** of 3,000 tons.

The retailer's senior management wants to understand the cost implications of each of these two new contract conditions (separate and jointly) in order to inform its contract negotiations

In-Class Exercise

- You will have 15 minutes to evaluate the two proposals. Work on it by yourself for a few minutes, then work on it with your neighbors

- For <u>each</u> proposal:
  - Write down on your handout precisely what additional binary decision variables and what modifications to the objective function and/or constraints are required
  - Your objective function and constraints must be linear

- We will stop by to answer any questions you may have. Good luck!

Class Exercise - Part 2 Solutions

(Full solutions will be posted to Canvas after class)

Base Formulation

minimize

(total supply cost)	$1.78\ s_{A,V} + 2.26\ s_{A,W} + \ldots + 1.95\ s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$
(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$

Bergamo's Condition Formulation

(Answer)

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z} + \boxed{1{,}000\, y_B}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le \boxed{2{,}000\ y_B}$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \ge 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \ge 1{,}500$
(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \ge 500$
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \ge 0$
(binary)	$\boxed{y_B \text{ is binary}}$

Idea:
$y_B = 1$ if we continue to supply from Bergamo, and 0 otherwise ...

Bergamo's Condition Spreadsheet Model*

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity	Supplier fixed cost
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**		
Amadora	1.78	2.26	2.22	2.30	1.45	4,000	0
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000	1,000
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000	0
Quantity required	1,000	500	1,500	1,500	500		

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied	Supply from Bergamo?	Effective capacity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**			
Amadora	1,000	500	1,500	500	500	4,000	1	4000
Bergamo	0	0	0	0	0	0	0	0
Casablanca	0	0	0	1,000	0	1,000	1	1000
Quantity received	1,000	500	1,500	1,500	500			

OBJECTIVE

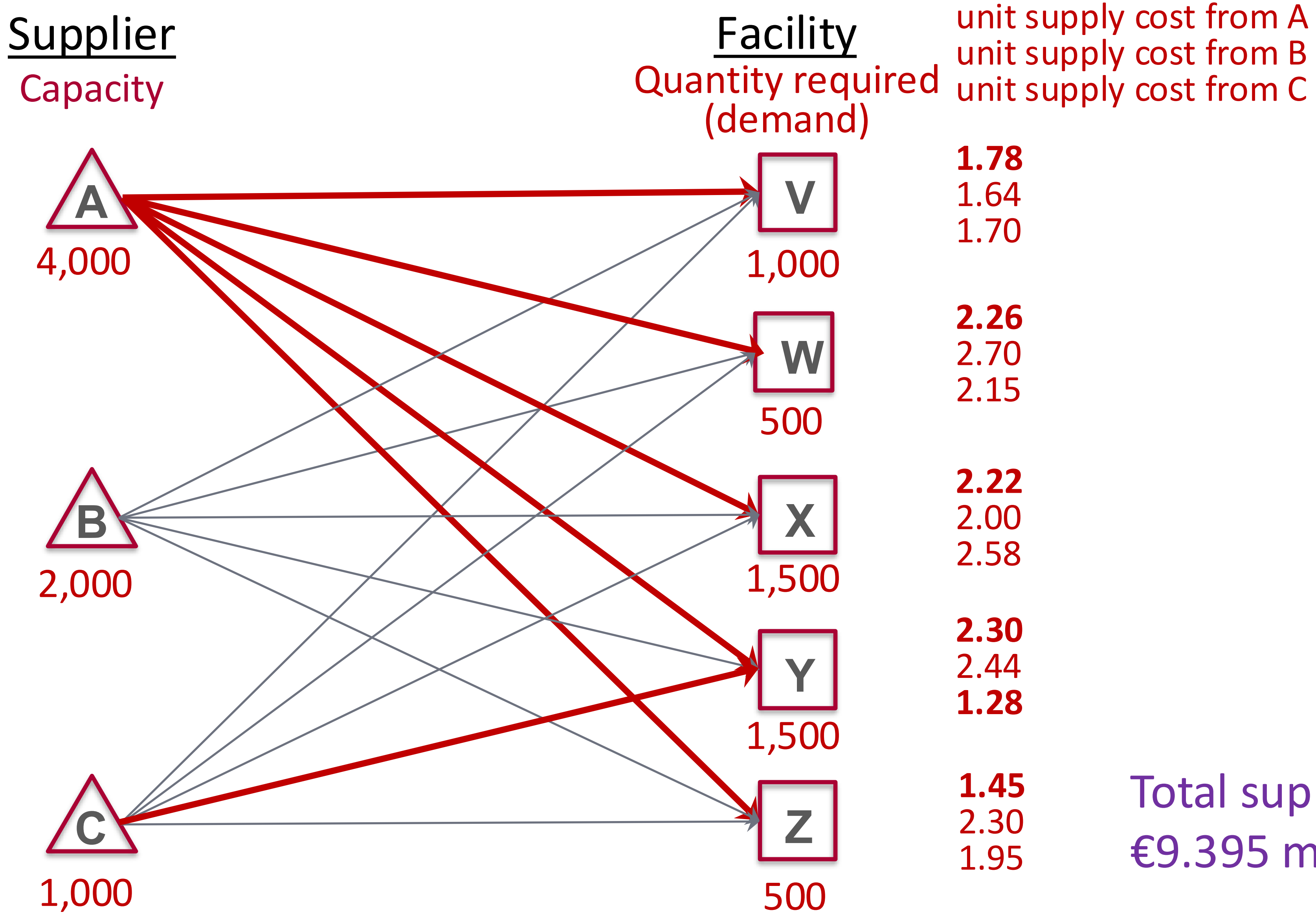
Total Cost	**9,395**	*minimize*

CONSTRAINTS

Supply does not exceed effective capacity
Quantity delivered equals quantity required
Nonegativity
Binary Bergamo supply variable

*will be uploaded after class

Supply Plan under Bergamo's Condition



Total supply cost = €9.395 million

Amadora's Condition Formulation

(Answer)

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000\, y_A$
(min order quantity from A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \ge 3{,}000\, y_A$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le 2{,}000$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \ge 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \ge 1{,}500$
(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \ge 500$
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \ge 0$
(binary)	y_A is binary

Idea:

y_A = 1 if we continue to supply from Amadora, and 0 otherwise.

Notice that the modified constraints ensure that if y_A = 1 then the supply from Amadora has to be at least 3000. But if y_A = 0 then the supply equals 0.

Amadora's Condition Spreadsheet Model*

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity	Minimum quantity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**		
Amadora	1.78	2.26	2.22	2.30	1.45	4,000	3,000
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000	
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000	
Quantity required	1,000	500	1,500	1,500	500		

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied	Supply from Amadora?	Effective capacity	Effective min quantity
	Valencia	**Wiesbaden**	**Xanten**	**York**	**Zaragoza**				
Amadora	**1,000**	**500**	**500**	**500**	**500**	3,000	**1**	4,000	3,000
Bergamo	**0**	**0**	**1,000**	**0**	**0**	1,000			
Casablanca	**0**	**0**	**0**	**1,000**	**0**	1,000			
Quantity received	1,000	500	1,500	1,500	500				

OBJECTIVE

Total Cost	**9,175**

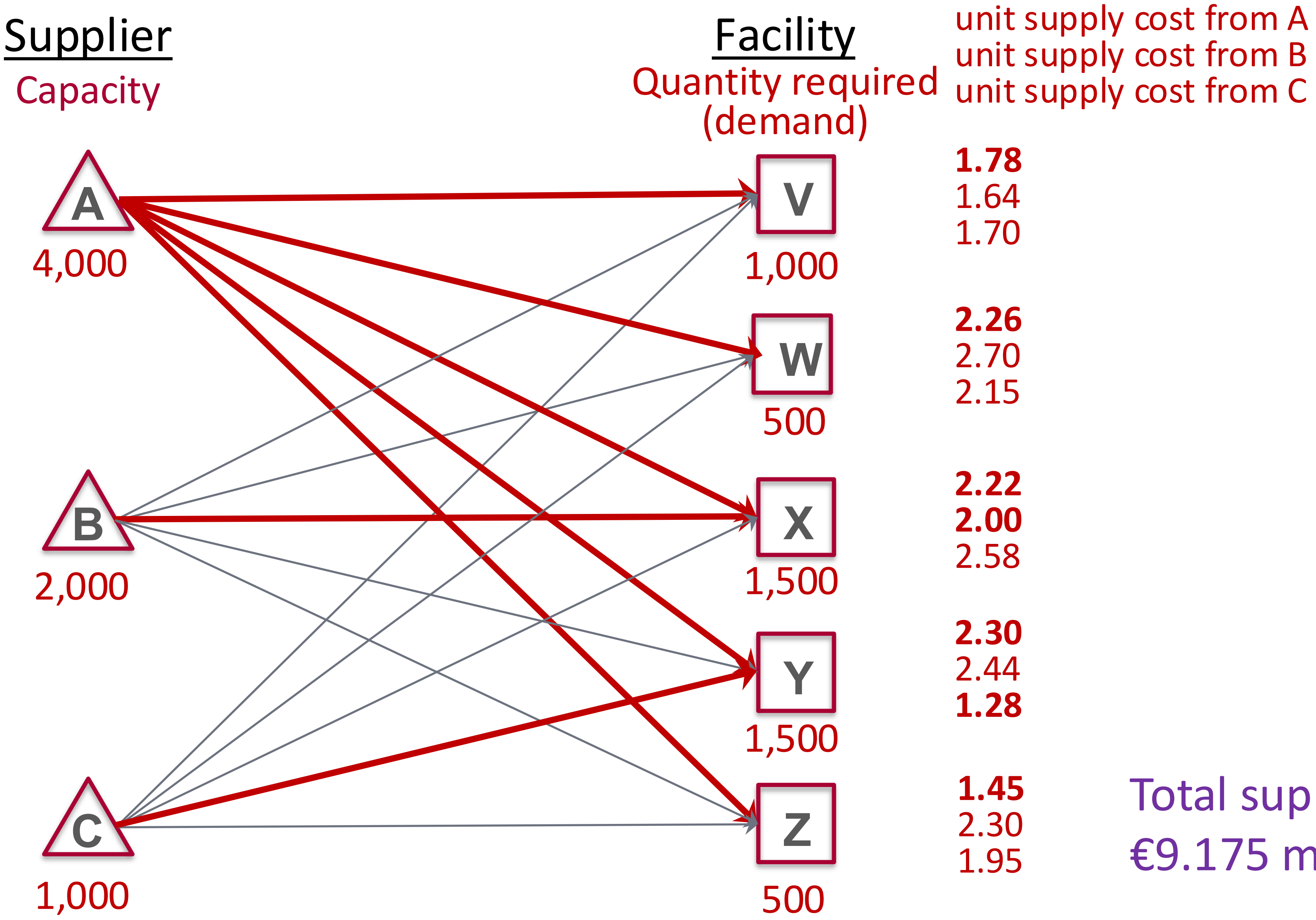
minimize

CONSTRAINTS

Supply from Amadora does not exceed its effective capacity
Supply from Amadora at least equal to its effective minimum order quantity
Supply from Bergamo and Casablanca does not exceed supplier capacities
Quantity delivered equals quantity required
Nonegativity
Binary Amadora supply variable

*will be uploaded after class

Supply Plan under Amadora's Condition



Total supply cost = €9.175 million

New Contract Conditions: Cost Implications Summary

	Without Bergamo's condition		With Bergamo's condition
Without Amadora's condition	€8.995 million	+4.44% →	€9.395 million (drop Bergamo)
	↓ +2.00%		
With Amadora's condition	€9.175 million (continue to source from Amadora)		(optional)

Analytics in Practice: Quick Vignette

Supply Chain Optimization

- The purpose of a well-designed supply chain is to deliver the right products to customers in a cost-effective and timely fashion.
- *Supply Chain Network Optimization* is a strategic planning process whose purpose is to determine or improve:
  - the structure of the supply chain
  - the location of facilities
  - the sizing of facilities
  - the sourcing and distribution flows
- Examples where supply chain network design is performed:
  - Expansion into a new market
  - Launching a new product
  - Evaluating mergers and acquisitions
  - Responding to supply, demand, or distribution disruptions
  - Responding to cost, demand, or other economic realities
  - Responding to new regulations
  - Simply evolving to adapt to changes in the business environment
- **Optimization is a critical success factor in supply chain design and operation.**

Newspaper Distribution*

- Newspapers are typically distributed through a two-level supply chain

Printing plants → Distribution Depots → Delivery Routes

- A larger newspaper/media company had the opportunity to buy distribution assets (a set of distribution depots and associated paper delivery routes) and needed to determine how much to bid for the assets

- To determine the maximum bid price, the newspaper wanted to understand the cost savings from combining the new depots/routes with its existing supply chain

*Simplified version of a problem from my professional experience

Newspaper Distribution

- This is a supply chain network optimization problem!

- Decision variables: which depots and routes to use

- Constraints
  - Each newspaper subscriber has to be on at least one route
  - A route can be used only if its associated depot is operational

- Objective function: Minimize Cost of Newspaper Delivery*
  - Each depot (current or new) has a fixed monthly cost of ongoing operation if we use it
  - Each paper delivery route has a variable monthly cost depending on the length of the route

*simplified version

Newspaper Distribution

Integer Optimization Approach

- Decision Variables: A binary decision variable x_D for each depot D and binary decision variable y_R for each route R

- Constraints
  - Each newspaper subscriber has to be on at least one route
    - For each subscriber, identify the routes that include that subscriber i.e. let's say routes 1, 4, 7 and 8 include that subscriber's address. Then, the constraint is:
      - $y_1 + y_4 + y_7 + y_8 \geq 1$
  - A route can be used only if its associated depot is operational
    - For each route-depot combination: route binary variable <= depot binary variable
      - $y_R \leq x_D$

- Objective function: Minimize Cost of Newspaper Delivery
  - Each depot (current or new) has a fixed monthly cost C_D of ongoing operation if we use it
  - Each delivery route has a variable monthly cost L_R depending on the length of the route

Total Delivery Cost = $\sum_{all\ depots} C_D x_D$ + + $\sum_{all\ routes} L_R y_R$

Newspaper Distribution

- By solving this problem and comparing the cost of the optimal solution with the current delivery cost, we estimated the incremental cost savings IF we owned the new depots and routes.

- These savings helped inform the maximum price we would want to pay for the distribution assets

Key Themes

- Optimization models, especially those utilizing **binary variables**, are **extraordinarily applicable**.
- We have seen several applications where optimization has made significant **business impact**
- But equally important, **optimization** is:
  - A framework for **structuring** one’s thinking about decisions
  - A **bridge** from data and predictive models to decisions
  - A must-have **fluency** for **analytics bilinguals** (like you!)

What’s Next

- *Today:* Deliverable #8 (the last one – yay!) will be posted today, due only on December 3rd
- *Thursday*: Deliverable #7 due
- *Friday*: Recitation on Discrete Optimization
- 1-on-1 Meetings
  - *Please book via https://calendly.com/ramamit*
  - *I have added more Calendly slots. If the Calendly times don’t work, please email my assistant Laura (brentrup@mit.edu) to find a time.*