When we add a new constraint…

Does the original optimal solution satisfy the new constraint?

- **Yes** => The original solution is still optimal. We don't need to resolve.
- **No** => We need to explicitly add the additional constraint and re-solve. The optimal objective value may stay the same or get worse (it cannot improve).

Model	Optimal Utility	Optimal Course Selection
Basic	46	Bid on A, E, F, G, H, I, J
Additional Constraints	44	**Bid on A, C, E, F, G, H, J**

Additional Constraints

- ✅ (A B conflict) $y_A + y_B \leq 1$
- ✅ (B or C required) $y_B + y_C \geq 1$
- ✅ (E pre-req to H) $y_H \leq y_E$

More challenging: what if H has two pre-requisites?

- If you take H, you must also take **both E and F** (E and F are both pre-requisite to H)

???

- If you take H, you must also take either E or F (Either E or F can be a pre-requisite to H).

????

More challenging: what if H has two pre-requisites?

- If you take H, you must also take **both E and F** (E and F are both pre-requisite to H)

$$y_H \leq y_E$$

$$y_H \leq y_F$$

- If you take H, you must also take either E or F (Either E or F can be a pre-requisite to H).

More challenging: what if H has two pre-requisites?

- If you take H, you must also take **both E and F** (E and F are both pre-requisite to H)

$$y_H \leq y_E$$

$$y_H \leq y_F$$

- If you take H, you must also take either E or F (Either E or F can be a pre-requisite to H).

$$y_H \leq y_E + y_F$$

Other examples of IF-THEN constraints

Let's say we have binary variables x, y and z. How would you encode each of the following if-then statements as a *linear* constraint?

- If x = 1 and y = 1, then z must be 1.

$$z \geq x + y - 1$$

- If either of x = 0 or y = 0, then z must be 0.

$$z \leq x$$

$$z \leq y$$

- If x and y are different, then z must be 1. Otherwise, no restrictions on z.

$$z >= x - y$$

$$z >= y - x$$

Synergy

Applying IF-THEN constraints to model relationships between variables

Course Bidding Formulation - “Synergy Effects”

- Recall the objective function:

  maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

- Suppose the utility of taking courses I and J *together* is 20 (greater than their sum of 12) because they allow you to complete the Analytics Certificate.
- How can you incorporate this into your model?

Course Bidding Formulation - "Synergy Effects"

- Recall the objective function:

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

- Suppose the utility of taking courses I and J *together* is 20 (greater than their sum of 12) because they allow you to complete the Analytics Certification.
- How can you incorporate this into your model?

$8 = (20 - (6+6))$

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J + 8\ y_I\ y_J$

Course Bidding Formulation - "Synergy Effects"

- Recall the objective function:

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

- Suppose the utility of taking courses I and J *together* is 20 (greater than their sum of 12) because they allow you to complete the Analytics Certification.
- How can you incorporate this into your model?

$8 = (20 - (6+6))$

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J + 8\ y_I\ y_J$

Will this work?

Course Bidding Formulation - "Synergy Effects"

- Recall the objective function:

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

- Suppose the utility of taking courses I and J *together* is 20 (greater than their sum of 12) because they allow you to complete the Analytics Certification.
- How can you incorporate this into your model?

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J + 8\ y_I\ y_J$

This is not linear ☹