# Many realistic constraints are easily represented using binary variables

- A and B cannot be taken together

$$y_A + y_B \leq 1$$

- Must take at least one of B or C

# Many realistic constraints are easily represented using binary variables

- A and B cannot be taken together

$$y_A + y_B \leq 1$$

- Must take at least one of B or C

$$y_B + y_C \geq 1$$

# If-then constraints

Writing if-then constraints using math

# Let’s add another constraint

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| H3 | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | ✓ | |
| H4 | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ |

- If you take H, then you must also take E (i.e., E is a prerequisite to H)

*Can you write down a linear constraint to capture this relationship?*

# Let’s add another constraint

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| H3 | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | | ✓ | |
| H4 | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ |

- If you take H, then you must also take E (i.e., E is a prerequisite to H)

$$y_H \leq y_E$$

(If $y_H = 1$, then $y_E \geq 1$, so $y_E$ must be equal to 1. If $y_H = 0$ then $y_E$ can be either 1 or 0)

# Formulation with more constraints

**Maximize:** $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

over variables: $y_A, y_B, \ldots, y_J$

**Subject To:**

(binary) $y_A , y_B , \ldots , y_J \geq 0, \leq 1$ and integral

(points budget) $200\ y_A + 50\ y_B + \ldots + 100\ y_J \leq 1000$

(max credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \leq 54$

(min credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \geq 36$

(MW H3 load) $y_A + y_B + y_E + y_G \leq 3$

(MW H4 load) $y_A + y_B + y_G + y_J \leq 3$

(TR H3 load) $y_C + y_D + y_I \leq 3$

(TR H4 load) $y_C + y_D + y_F + y_H + y_I \leq 3$

(A B conflict) $y_A + y_B \leq 1$

(B or C required) $y_B + y_C \geq 1$

(E pre-req to H) $y_H \leq y_E$

} Additional constraints

# Excel with additional constraints

| DECISIONS | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Course | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.0 | 1.0 |

**OBJECTIVE**

| | |
|---|---|
| Total utility | 44 |

**CONSTRAINTS**

| | LHS | □ | RHS |
|---|---|---|---|
| Points budget | 700 | <= | 1,000 |
| Course credit maximum | 54 | <= | 54 |
| Course credit minimum | 54 | >= | 36 |
| Mon, Wed H3 classes | 3 | <= | 3 |
| Tue, Thr H3 classes | 1 | <= | 3 |
| Mon, Wed H4 classes | 3 | <= | 3 |
| Tue, Thr H4 classes | 3 | <= | 3 |
| Do not take A and B | 1.0 | <= | 1 |
| Take either B or C | 1.0 | >= | 1 |
| H requires E | 1.0 | <= | 1 |
| Binary constraints | | | |

# Summary

| Model | Optimal Utility | Optimal Course Selection |
|---|---|---|
| Basic | 46 | **Bid on A, E, F, G, H, I, J** |
| Additional Constraints | 44 | **Bid on A, C, E, F, G, H, J** |

# When we add a new constraint…

*Does the original optimal solution satisfy the new constraint?*

- **Yes** => The original solution is still optimal. We don't need to resolve.

| Model | Optimal Utility | Optimal Course Selection |
|---|---|---|
| Basic | 46 | **Bid on A, E, F, G, H, I, J** |

**Additional Constraints**

✅ (A B conflict) $y_A + y_B \leq 1$

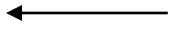
✅ (E pre-req to H) $y_H \leq y_E$

# When we add a new constraint…

*Does the original optimal solution satisfy the new constraint?*

- **Yes** => The original solution is still optimal. We don't need to resolve.
- **No** => We need to explicitly add the new constraint and re-solve.

| Model | Optimal Utility | Optimal Course Selection |
|---|---|---|
| Basic | 46 | **Bid on A, E, F, G, H, I, J** |



**Additional Constraints**

- ✅ (A B conflict) $y_A + y_B \leq 1$
- ❌ (B or C required) $y_B + y_C \geq 1$
- ✅ (E pre-req to H) $y_H \leq y_E$