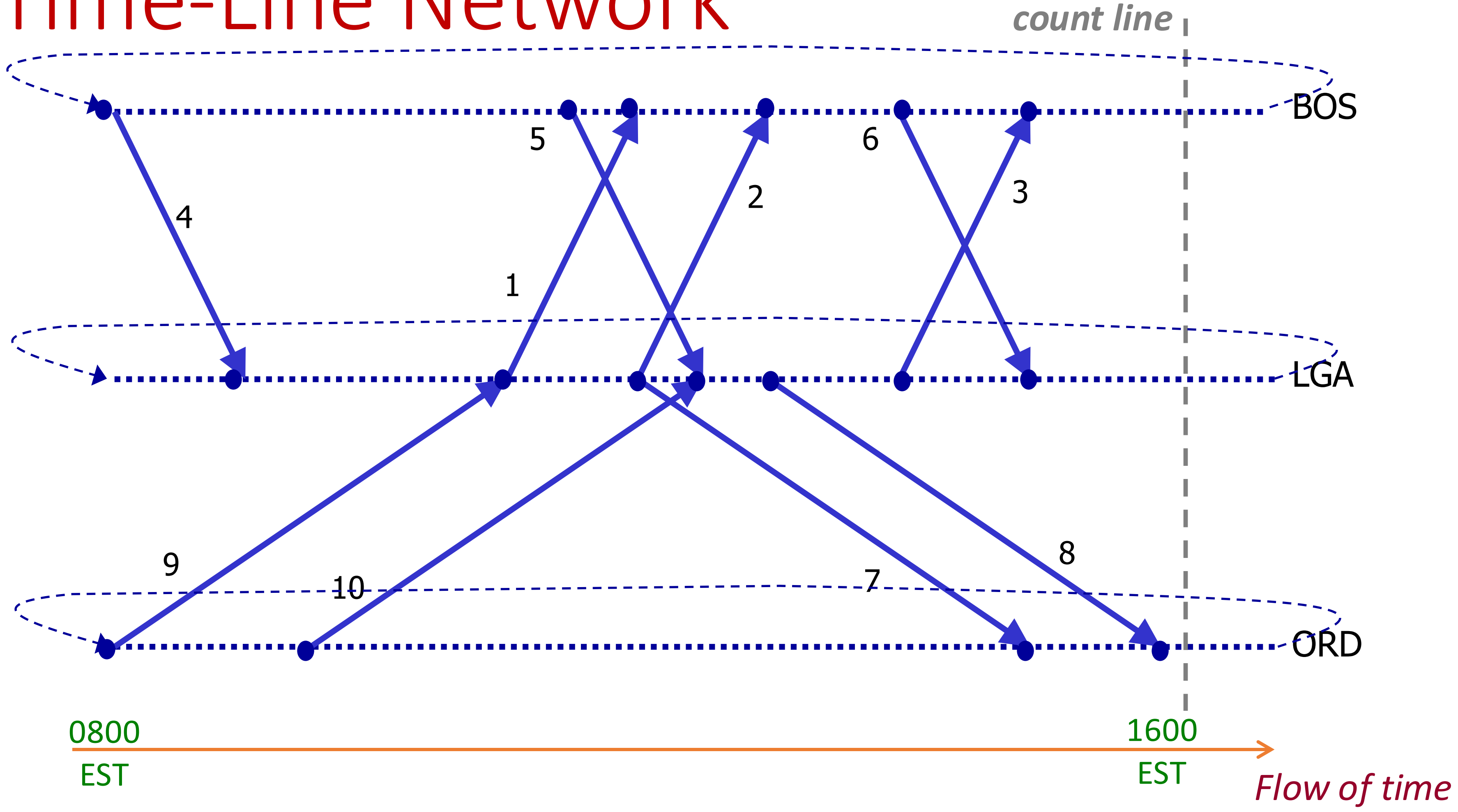
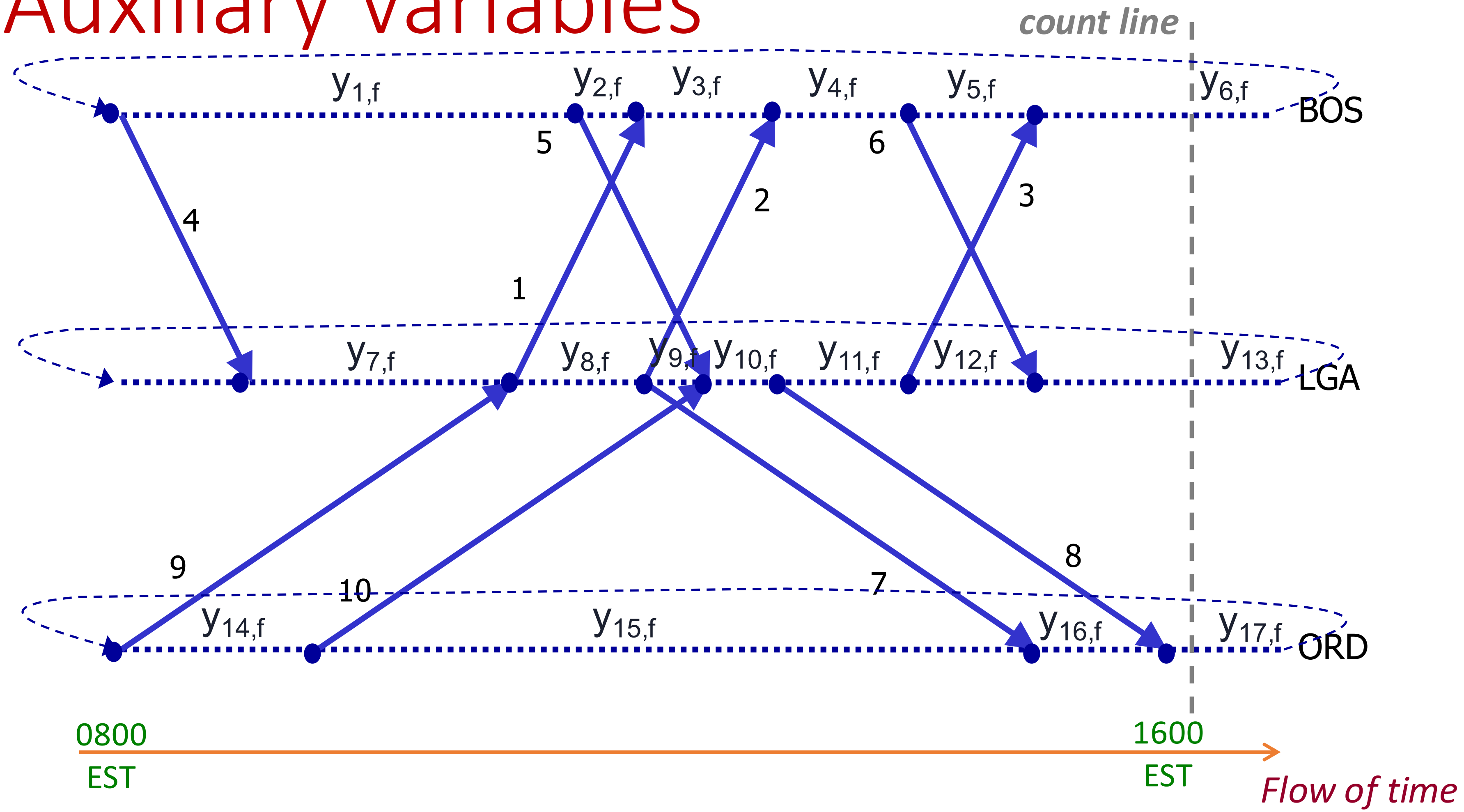
Time-Line Network



Each ● is a node representing an airport at a specific time of day.

Reference: "The Fleet Assignment Problem: Solving a Large-Scale Integer Program" (1995) by C. A. Hane, C. Barnhart, E. L. Johnson, R. E. Marsten, G. L. Nemhauser, G. Sigismondi, *Mathematical Programming*, **70**, 211-232.

Auxiliary Variables



Each ● is a node representing an airport at a specific time of day.

Formulation Constraints – (2/3)

Constraints (cont.):

- Balance
  make sure the number of inbound airplanes to a node equals the number of outbound planes for each fleet type.

  For the first node at BOS & fleet type A: $y_{6,A} - x_{4,A} - y_{1,A} = 0$

  For the first node at BOS & fleet type B: $y_{6,B} - x_{4,B} - y_{1,B} = 0$

  and so on for each node and fleet type...

- Non-negativity (on the auxiliary y variables)
  make sure the number of aircraft on the ground of each fleet type is not negative at any time.

Formulation Constraints – (3/3)

Constraints (cont.):

- Count
  the number of airplanes on the ground and in the air at the "countline" of any given fleet type must not exceed fleet availability.

$$y_{6,A} + y_{13,A} + y_{17,A} \leq 1$$

$$y_{6,B} + y_{13,B} + y_{17,B} \leq 2$$

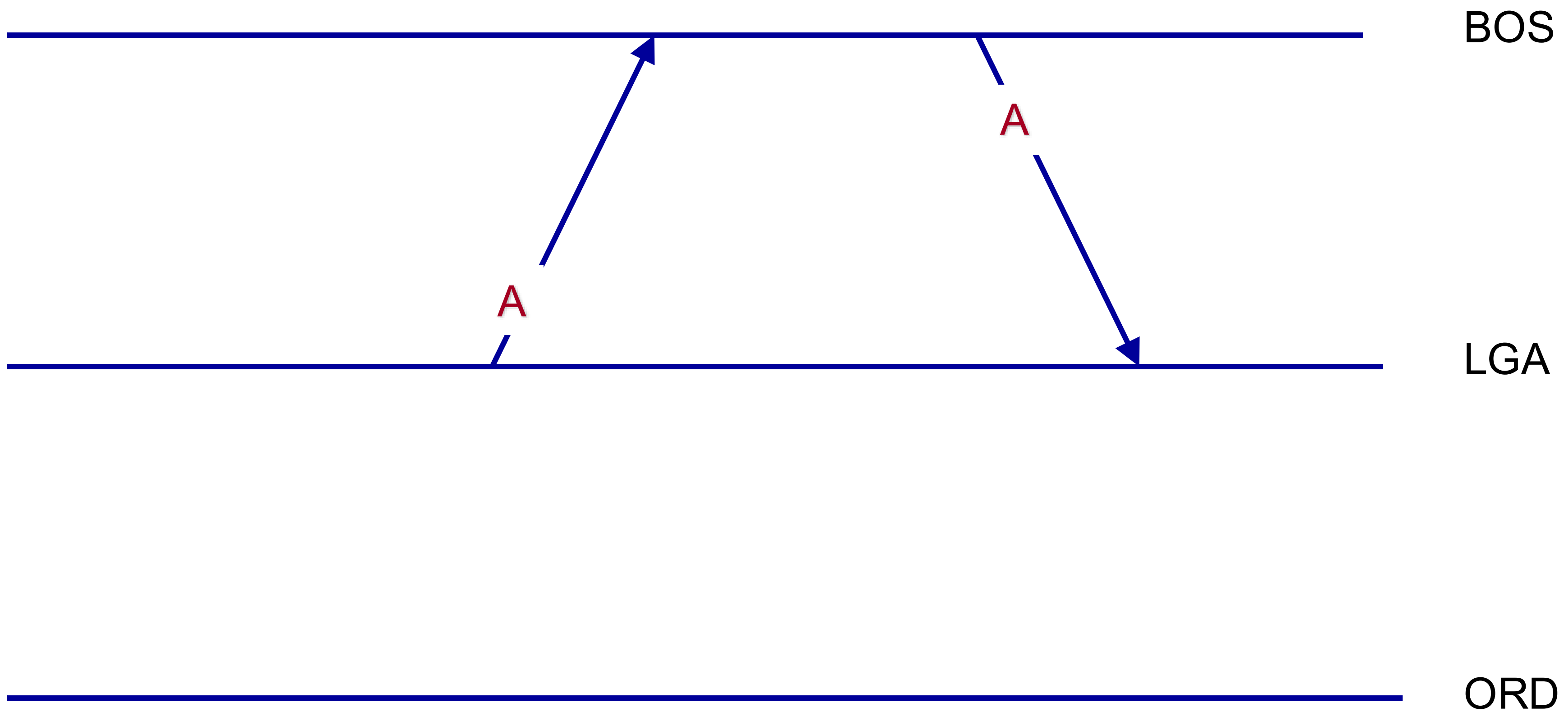
$$y_{6,C} + y_{13,C} + y_{17,C} \leq 2$$

Formulation Constraints

<u>Constraints</u>:

- Binary
  Each assignment decision variable must be either 0 or 1
- Cover
  make sure each flight is assigned an airplane.
- Balance
  make sure the number of inbound airplanes to a node equals the number of outbound planes for each fleet type.
- Non-negativity (on the auxiliary y variables)
  make sure the number of aircraft on the ground of each fleet type is not negative at any time.
- Count
  the number of airplanes on the ground and in the air at the "countline" of any given fleet type must not exceed fleet availability.

Example Solution – Type A Aircraft

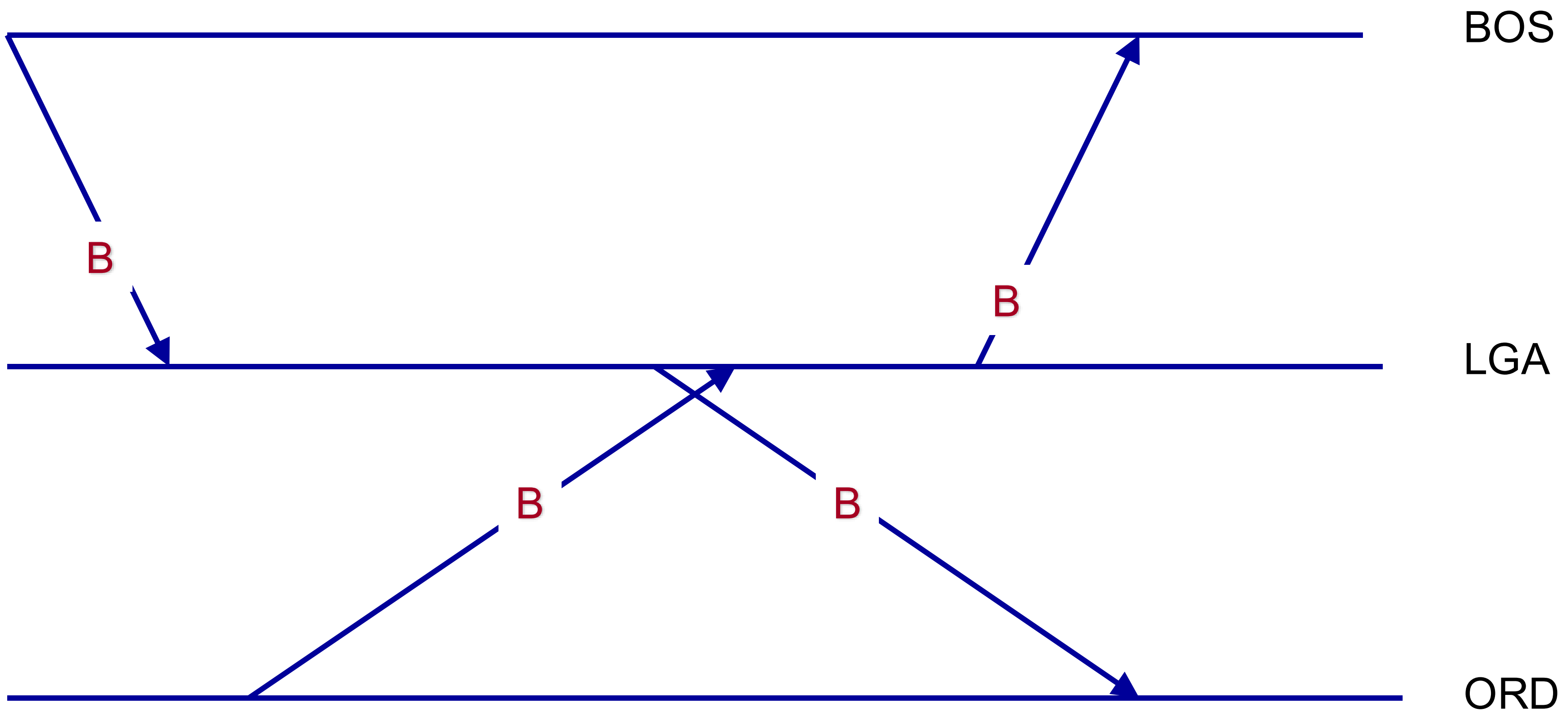


Revenue = $428,500

Cost = $148,000

Profit = $280,500

Example Solution - Type B Aircraft

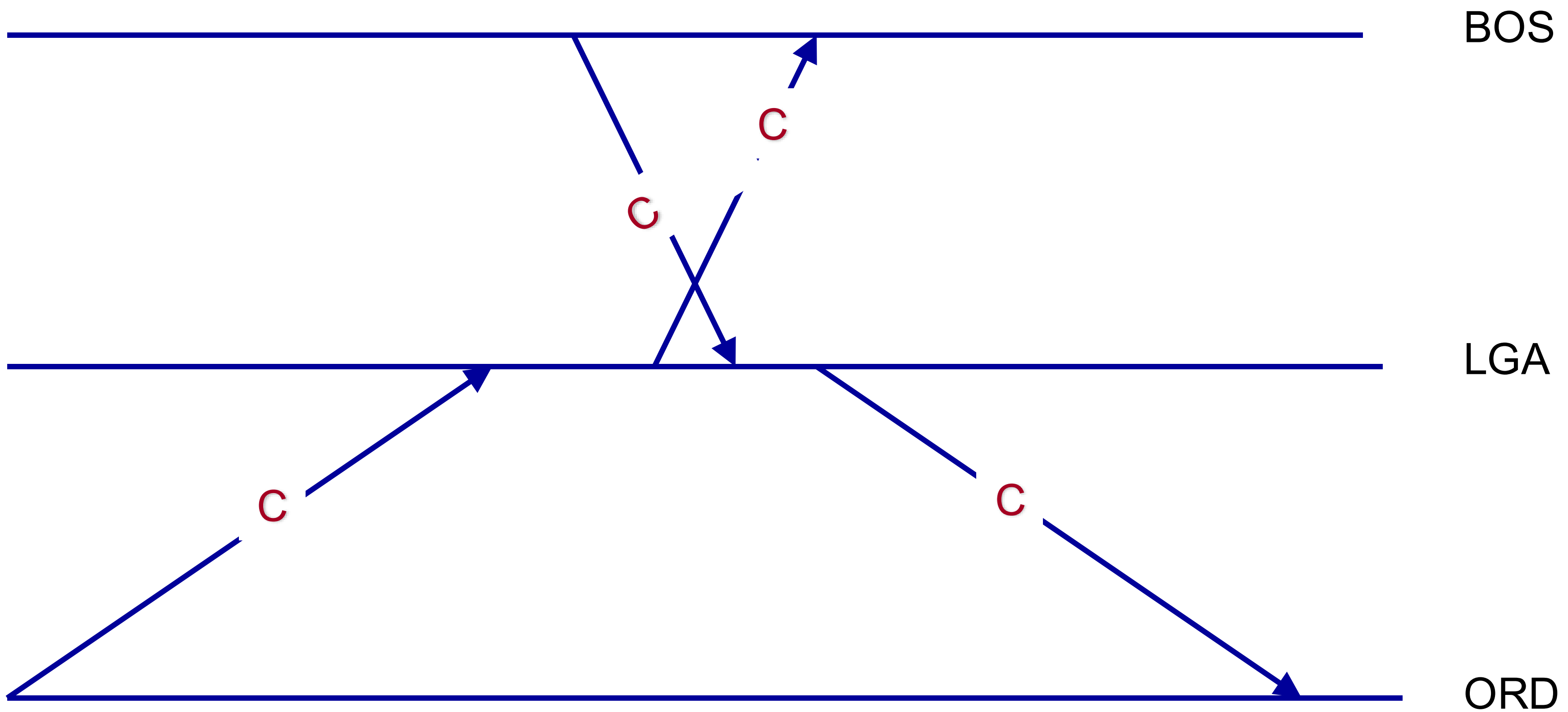


Revenue = $428,500

Cost = $148,000

Profit = $280,500

Example Solution - Type C Aircraft

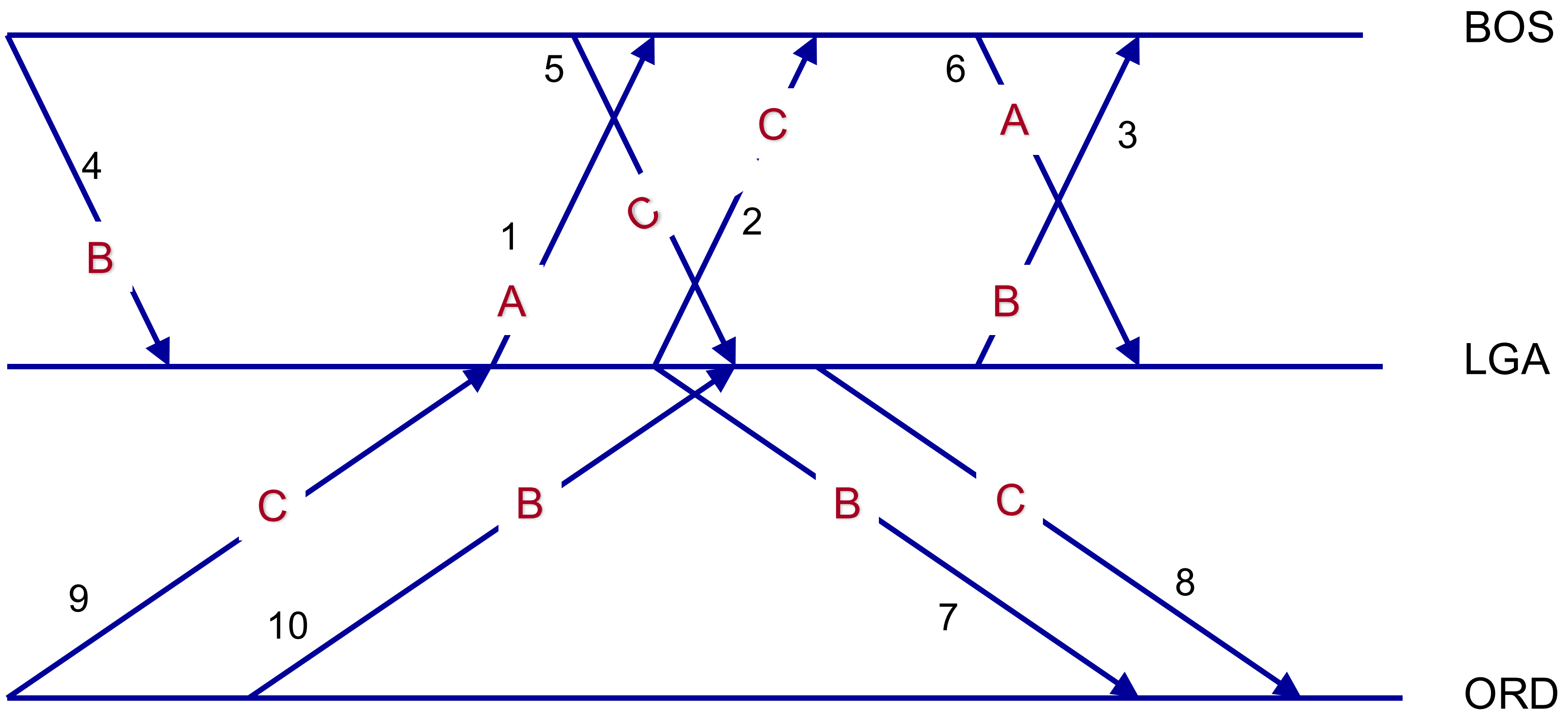


Revenue = $428,500

Cost = $148,000

Profit = $280,500

Example Solution – All Aircraft Types



Revenue = $428,500

Cost = $148,000

Profit = $280,500

Computational Experience with Actual Problems

- The "F" problem consists of 2,044 flights serving 76,741 itineraries covering 112 airports using a fleet of 9 aircraft types.
- The "A" problem consists of 1,888 flights serving 75,484 itineraries covering 106 airports using a fleet of 9 aircraft types.

Table 5. Comparison of models.

Run	FAM	SFAM0	SFAM4	IFAM (1 hr.)	IFAM (5 hrs.)
F[0.8]					
Profit change	0	43.234	45.834	45.834	45.834
Solution time	989	1,371	1,077	1,570	1,574
F[1.0]					
Profit change	0	14.55	32.922	29.333	32.993
Solution time	762	925	704	3,813	14,109
F[1.2]					
Profit change	0	32.596	41.421	28.86	28.86
Solution time	1,331	1,004	3,838	3,821	18,237
F[1.5]					
Profit change	0	71.477	73.585	no	8.979
Solution time	1,829	1,361	3,914	solution	18,267
A[0.8]					
Profit change	0	4.922	5.032	(2.350)	2.814
Solution time	517	884	1,005	3,762	18,171
A[1.0]					
Profit change	0	10.764	11.335	(18.266)	(4.945)
Solution time	776	930	871	3,806	18,203
A[1.2]					
Profit change	0	17.207	25.999	no	(41.606)
Solution time	877	1,007	1,676	solution	18,189
A[1.5]					
Profit change	0	(11.729)	1.157	no	(63.785)
Solution time	534	1,449	2,314	solution	18,307

Notes. Profit changes are reported in [$million/365 days] relative to FAM. Negative numbers are reported in parentheses. Solution times are reported in CPU seconds.

Reference: "Airline Fleet Assignment with Enhanced Revenue Modeling" (2009) by C. Barnhart, A. Farahat, M. Lohatepanont, *Operations Research*, **57**(1), 231-244.