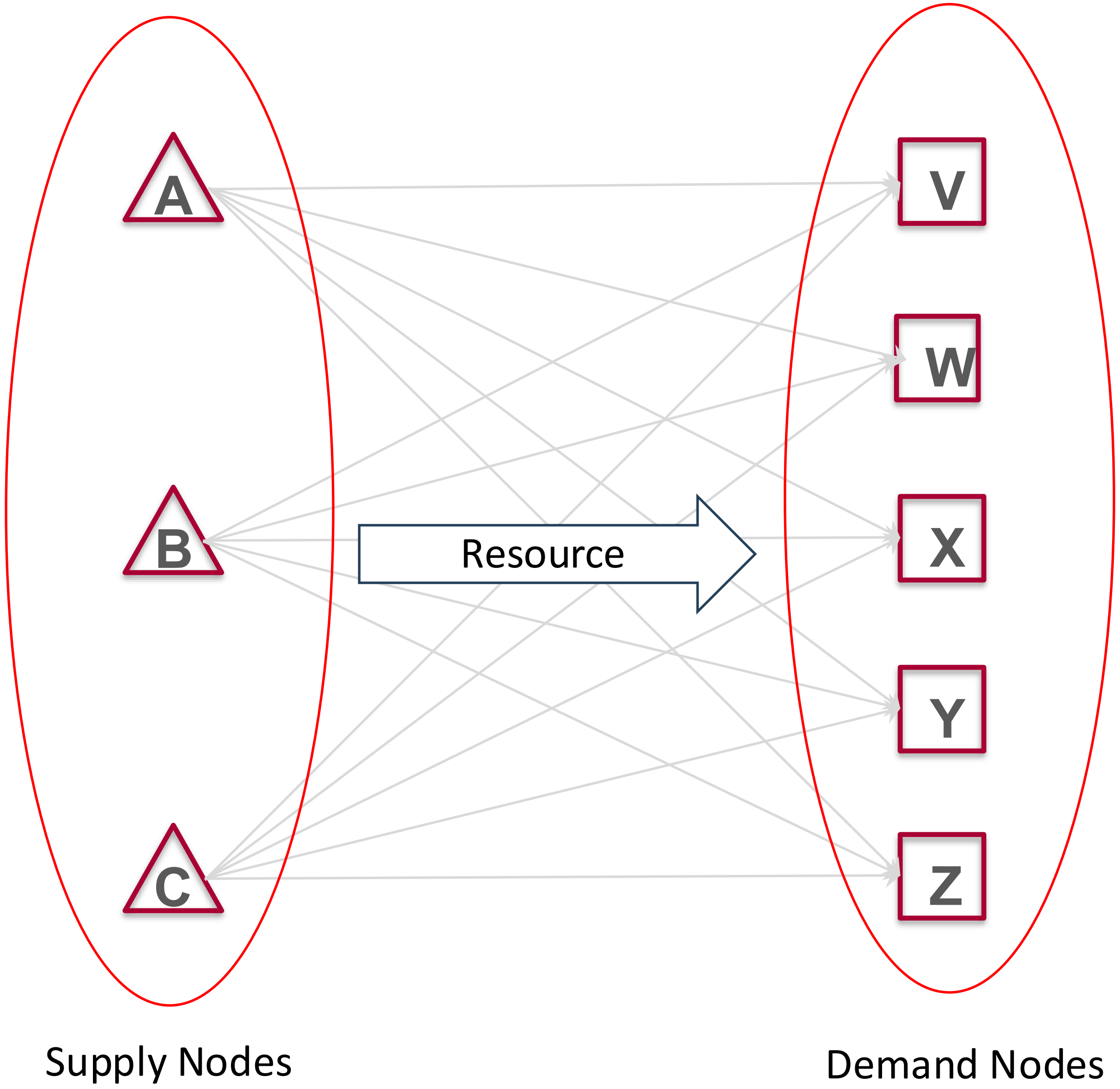
More generally, discrete optimization is often used for matching problems

- **Matching** problems generally involve some **resource** that must be allocated from a set of **supply nodes** to **demand nodes**
  - The resource may be physical goods or abstract concepts such as time.
- **Goal**: Find the best way to match supply and demand to minimize cost or maximize utility



Matching appears everywhere!

Application Domain	Supply Nodes	Demand Nodes	Resource	Objective
Supply Chain	Suppliers	Production Facilities	Raw materials / goods	Minimize transportation and procurement cost
Healthcare Staffing	Available nurses	Hospital shifts	Labor hours	Maximize nurse satisfaction and satisfy staffing requirements
School Assignment	Students (with preferences)	Schools	Students seats	Maximize student satisfaction
Ride Sharing	Available drivers	Passengers requesting rides	Rides	Minimize passenger wait time and driver idle time

Optimization is really well suited for these applications!

Class Exercise 1

Some advice before you start

- In Linear Optimization, we allowed the decision variables to be fractional
- In Monday's class on Discrete Optimization, we required the decision variables to be binary
- In today's exercise, you need to use both fractional and binary decision variables
    - Tip: Fractional variables should be used for "how much" decisions and binary variables should be used for "should we do this? yes or no?" decisions
    - The tricky part will be to "connect" the two using the right constraints
    - But whatever you do, the objective and the constraints MUST be linear!

Evaluating Operational Proposals

The retailer's senior management is considering two separate supply strategy proposals aimed at better streamlining its supply chain operations:

- Proposal 1: **Supplier Consolidation** (using exactly 2 instead of 3 suppliers)
- Proposal 2: **Single Sourcing** (serve **V**alencia from a single supplier only)

In-Class Exercise

- You will have 15 minutes to evaluate the two proposals. Work on it by yourself for a few minutes, then work on it with your neighbors

- For <u>each</u> proposal:
  - Write down on your handout precisely what additional binary decision variables and what modifications to the objective function and/or constraints are required
  - Your objective function and constraints must be linear

- We will stop by to answer any questions you may have. Good luck!

Class Exercise - Part 1 Solutions

(Full solutions will be posted to Canvas after class)

Base Formulation

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

$$
\left.\begin{array}{lr}
\text{(capacity at A)} & s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000 \\
\text{(capacity at B)} & s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le 2{,}000 \\
\text{(capacity at C)} & s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000
\end{array}\right\} \text{Supply side constraints}
$$

$$
\left.\begin{array}{lr}
\text{(demand at V)} & s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000 \\
\text{(demand at W)} & s_{A,W} + s_{B,W} + s_{C,W} \ge 500 \\
\text{(demand at X)} & s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500 \\
\text{(demand at Y)} & s_{A,Y} + s_{B,Y} + s_{C,Y} \ge 1{,}500 \\
\text{(demand at Z)} & s_{A,Z} + s_{B,Z} + s_{C,Z} \ge 500
\end{array}\right\} \text{Demand side constraints}
$$

(nonnegativity) $s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \ge 0$

Proposal 1 Formulation: *Will this work*?

(Answer)

minimize	
(total supply cost)	$1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$
subject to	
(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4{,}000$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2{,}000$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1{,}000$
(two suppliers)	$\boxed{y_A + y_B + y_C = 2}$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \geq 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \geq 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1{,}500$
(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \geq 0$
(binary)	$\boxed{y_A, y_B, y_C \text{ are binary}}$

<u>Idea</u>:
y_i = 1 if we use Supplier i, and 0 if we don't...

Proposal 1 Formulation

(Answer)

minimize

(total supply cost) $1.78\, s_{A,V} + 2.26\, s_{A,W} + \ldots + 1.95\, s_{C,Z}$

subject to

(capacity at A)	$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \le 4{,}000\, y_A$
(capacity at B)	$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \le 2{,}000\, y_B$
(capacity at C)	$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \le 1{,}000\, y_C$
(two suppliers)	$y_A + y_B + y_C = 2$
(demand at V)	$s_{A,V} + s_{B,V} + s_{C,V} \ge 1{,}000$
(demand at W)	$s_{A,W} + s_{B,W} + s_{C,W} \ge 500$
(demand at X)	$s_{A,X} + s_{B,X} + s_{C,X} \ge 1{,}500$
(demand at Y)	$s_{A,Y} + s_{B,Y} + s_{C,Y} \ge 1{,}500$
(demand at Z)	$s_{A,Z} + s_{B,Z} + s_{C,Z} \ge 500$
(nonnegativity)	$s_{A,V}, s_{A,W}, \ldots, s_{C,Z} \ge 0$
(binary)	y_A, y_B, y_C are binary

Idea:
$y_i = 1$ if we use Supplier i, and 0 if we don't...