DELIVERABLE 8

Due by Wednesday, December 3, 11:59 p.m.

Work with your assigned Core Team members only. Each team needs to upload its solutions to Canvas as a single pdf file and an excel file (any team member can do so through their Canvas account). Make sure that only team members who contributed to the submission are listed on the cover page.

Problem 1 (25 points)

In planning the DMD final exam in a pre-Covid year, we needed to determine how many students from each section are allocated to each exam classroom. In true DMD spirit, let's view this as an optimization problem.

Suppose we have 6 sections with the following number of students enrolled in each section:

Section	A	B	C	D	E	F
Ocean	Atlantic	Baltic	Caribbean	Indian	Mediterranean	Pacific
Enrollment	63	62	63	61	62	60

We have 6 exam classrooms with the following seat capacities:

Classroom	1	2	3	4	5	6
Location	E51-315	E51-325	E51-335	E51-345	E51-376	E51-395
Capacity	86	86	76	128	54	70

In this problem we will assume that the objective function is to minimize the density of the highest density classroom. The density of a classroom is defined to be the number of students allocated to that classroom divided by the classroom's capacity.

We require that each classroom holds at most 70 students. This would allow the TA assigned to the classroom to distribute and collect exam papers quickly at the start and end of the exam.

a) Formulate the problem as a **linear optimization (also known as linear program, or LP)** problem to find out how many students from each session are allocated to each exam classroom. Define your decision variables clearly (your decision variables should be continuous, i.e. you can ignore integrality constraints on the number of students assigned to classrooms) and write down the objective function and constraints.
Hint: Include constraints of the form: "density of classroom i ≤ z" where z is a decision variable representing an upper limit on classroom density, and "density of classroom i" is expressed in terms of your remaining decision variables. Then write the objective function in terms of z.

b) Implement and solve your LP model in Excel Solver. **Submit an Excel Sheet**. Report the optimal assignment and the optimal objective value.

c) Suppose you solve the optimization problem formulated in part (a) and find that the optimal value (density) equals 81%. The shadow price of the constraint that limits the number of students in classroom E51-345 to 70 is equal to -0.00269. Its allowable increase is equal to 25 and its allowable decrease is equal to 2. The TA assigned to E51-345 finds someone to assist her/him with paper distribution and is confident that, together, they'll be able to handle up to 95 students. What is the new optimal value (density) of the problem formulated in part (a) if we get the additional assistance to classroom E51-345?

d) Now, formulate the LP in part (a) as an **integer linear optimization**. Implement and solve the formulation in the Excel Solver. **Submit an Excel Sheet.** Report the optimal solution and the optimal objective value.

You should observe that the optimal objective value of the integer model is no less than that of the LP model. Is this a coincidence or must be the case? Explain. Can you still do sensitivity analysis as in part c) for the integer model?

Problem 2 (30 points)

A call center management company has approached your consulting firm to explore developing a work shift optimization model for scheduling call center operators.

As a prototype application, consider a small call center with 10 operators labelled A-J. The call center operates 12 hours each weekday, from 7AM – 7PM. Prior call volume analysis has concluded that the minimum number of operators that need to be on duty during each hour is as given in the following table:

Hour	7AM – 8AM	8AM – 9AM	9AM – 10AM	10AM – 11AM	11AM – 12PM	12PM – 1PM	1PM – 2PM	2PM – 3PM	3PM – 4PM	4PM – 5PM	5PM – 6PM	6PM – 7PM
Minimum required number of operators	2	3	5	8	7	5	6	7	5	5	4	4

An operator is compensated at the rate of $18 per hour for each hour worked between 9AM and 5PM (including lunch break) and at the rate of $25 per hour for each hour worked earlier than 9AM or later than 5 PM. The call center management company would like the optimization tool to indicate which specific hours of the day will be worked by each operator to meet the minimum required number of operators. Its current contractual agreement imposes the following constraints:

- Each operator must work exactly 7 hours per day, not counting lunch break. The hours need not be contiguous (Note: the operators work remotely from home).
- Each operator must be provided with a one-hour lunch break sometime between 11AM and 2PM
- The hours assigned to an operator cannot span more than 10 hours. In particular, if an operator is asked to work the 7AM – 8AM hour then they cannot work either the 5PM – 6PM or the 6PM – 7PM hours. Similarly, an operator assigned to work the 8AM – 9AM hour cannot work the 6PM – 7PM.

The objective of the operator is to minimize cost while meeting the above service and contractual requirements.

a) Provide an **integer linear optimization model** of the problem clearly stating the decision variables, objective function, and constraints;
b) Implement and solve the model using Microsoft Excel Solver. **Submit your spreadsheet model.**
c) Provide a table of operator schedule and report the associated daily cost.

d) Operators in general prefer to have fewer switches from an on hour to an off hour i.e. they work in one hour and then stop working in the next hour. You want to make sure that each operator has at most 3 such switches during his/her daily schedule (including lunch break and the end of day). That is, for each operator, there would be at most three switches from on (working) to off (non-working) during a day. **Formulate new linear constraints to enforce this requirement**. You do not need to solve this in Excel. Hint: You may need to introduce new binary variables.

Problem 3 (20 points)

You're organizing a winter food festival in Kendall Square's open plaza near MIT. After reviewing proposals, you've narrowed the field to 8 excellent catering vendors, each of which provide some utility to the success of your event. However, due to space constraints in the plaza, you can accommodate **5 vendor stalls**. Some information about the vendors is provided below.

Vendor Number	Utility	Serves Drinks?
1	13	No
2	12	No
3	11	No
4	10	No
5	8	No
6	7	Yes
7	6	Yes
8	5	Yes

While each vendor's utility was estimated in isolation, the combination of vendors creates interesting synergies that add additional utility or potentially negative utility. As a result, you turn to optimization to help you find the combination of vendors that has the highest utility.

Your start with the following optimization formulation that doesn't consider any vendor synergies.

Maximize: $13\,y_1 + 12\,y_2 + \ldots + 5\,y_8$

Subject to: $y_1 + y_2 + \ldots + y_8 \le 5$

$y_1, \ldots, y_8$ are binary.

Here, y_i is a binary variable for whether you choose vendor i-th for the food festival. After doing your research, you have observed the following synergies between vendors.

a) Vendors 1, 2, and 3 fit into a cohesive theme that will attract many more customers. If all 3 vendors are chosen, the utility increases by an extra 10, in addition to the individual utilities.
b) Vendors 4, 5 and 6 also fit into a nice theme. Choosing at least two of these three vendors will net an additional 5 utility.
c) Historic data shows that drink options are very important at a food festival. If you choose no vendors with drinks, then your utility will take a -15 hit.

d) Vendors 7 and 8 are run by the same parent company. They can share a stall at the food festival, meaning that together they only take up 1 stall. Utilities are unaffected by the two vendors operating from the same stall.

For each of the synergies above, please add a single variable named z_a, z_b, z_c or z_d to the formulation. State how you would incorporate the variable into the existing objective / constraints. Then, write down the additional constraint(s) that connect z_a, z_b, z_c and z_d to y_1, y_2, …, y_8. Your constraints should enforce that z_a, z_b, z_c and z_d always take on the correct binary value (i.e.., you should have both a ≤ and ≥ constraint).

Hint: Part (b) is tricky. You'll need to use $2z_b \geq \ldots$ and $2z_b \leq \ldots$ in your constraints instead of just z_b

Problem 4 (25 points)

A contractor is working with a venue to set up the stage for the long awaited Dire Straits reunion concert. Setting up the stage requires 20 hours of structure building and 15 hours of electrical wiring (for lighting and sound). Four workers are available to work on these two tasks. The hourly rate and available number of hours for each worker are provided in the table below.

Worker	Hours available to work	Hourly rate
1	20	\$20
2	18	\$18
3	15	\$15
4	10	\$12

Task	Description	Hours of work required
1	Structure building	20
2	Electrical wiring	15

a) Formulate an **integer linear optimization** that determines the number of hours each worker should spend on each task so as to minimize total cost while ensuring the stage setup is completed and each worker's available hours are not exceeded. Define your variables clearly and write down the mathematical expressions for the objective function and constraints in terms of these variables. The number of hours each worker should spend must be an integer.

b) Implement and solve the above IP model in Excel Solver. Report the optimal solution and the optimal objective value. **Submit the Excel sheet.**

c) Extend the formulation of part (a) to account for the following additional safety and union rule requirements. Clearly specify what additional (binary) variables are required and write down the mathematical expressions for the additional linear inequalities.
The requirements are:
- R1: At least three workers must work on structure building;
- R2: Worker 3 can work on structure building or electrical wiring, but not both;
- R3: If Worker 4 works on electrical wiring, then Worker 4 must work at least 5 hours on electrical wiring.

For each new constraint, please indicate the requirement (e.g., R1, etc.) it represents.

d) Implement and solve the above IP model part (c) in Excel Solver. Report the optimal solution and the optimal objective value. **Submit the Excel sheet.**