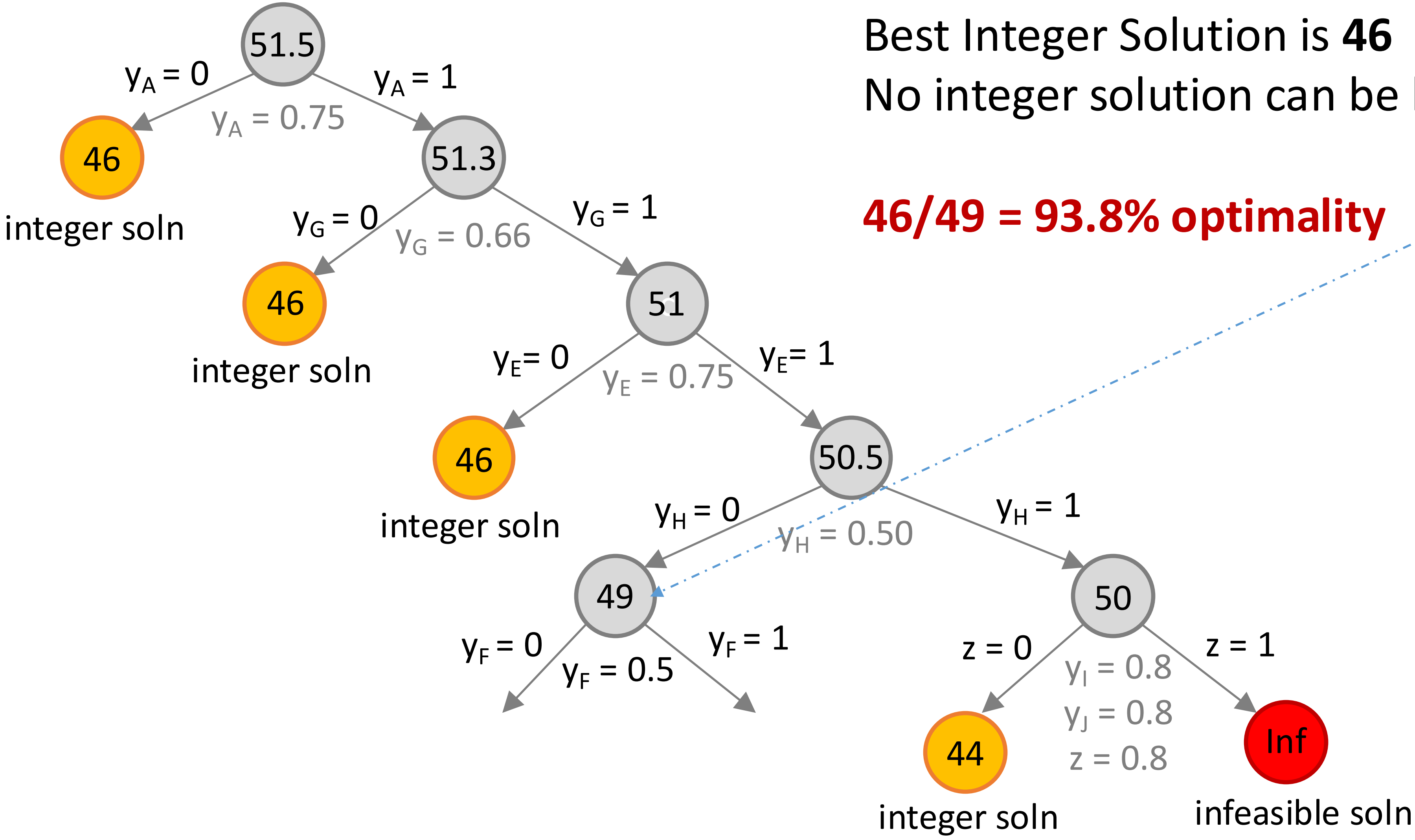
Branch and Bound

- Branch and bound can solve a binary optimization problems with N variables by **solving way fewer than 2^N** linear optimization problems.

- Practitioners typically set a time limit (e.g., 1 hour).
  - Branch-and-bound will find the **best solution** it can within that time limit.
  - Branch-and-bound will give tell you **how far you** are to the optimal solution.

Time's up!



Best Integer Solution is **46**

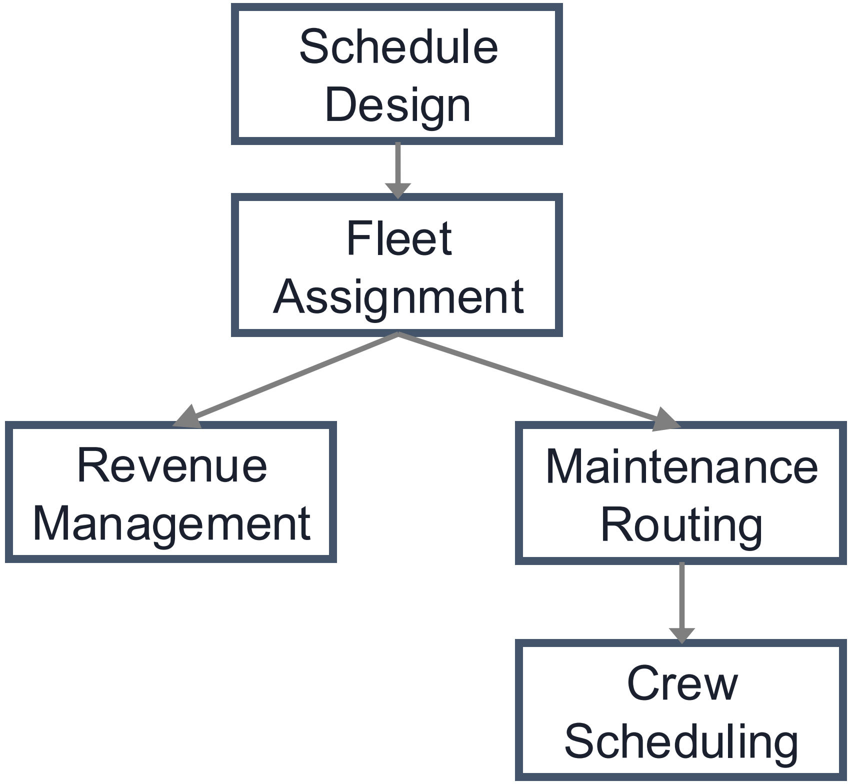
No integer solution can be better than **49**

46/49 = 93.8% optimality

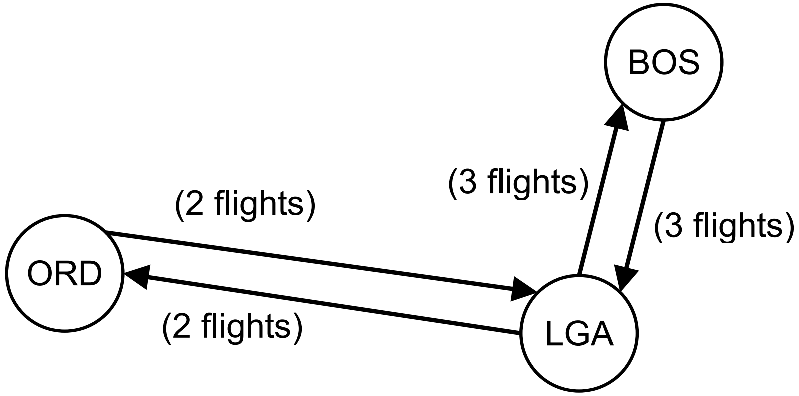
APPENDIX
<u>Optional</u>
Real-world application of 0-1 integer optimization: AIRLINE FLEET ASSIGNMENT

Applications of Optimization in Airline Operations

- Airlines use optimization models to plan many aspects of their operations such as assigning aircraft and crew to flights.
- Airlines also use optimization (especially linear programming) to manage their revenue.
- The scale of operations is huge. A major US airline would typically fly 1,500-2,500 domestic flights daily and offer over 100,000 different itinerary/fare combinations.



Airline Fleet Assignment – Simplified Example (1/2)



Flight #	From	To	Dept Time (EST)	Arr Time (EST)	Fare [$]	Demand [passengers]
1	LGA	BOS	1000	1100	150	250
2	LGA	BOS	1100	1200	150	250
3	LGA	BOS	1800	1900	150	100
4	BOS	LGA	0700	0800	150	150
5	BOS	LGA	1030	1130	150	300
6	BOS	LGA	1800	1900	150	150
7	LGA	ORD	1100	1400	400	150
8	LGA	ORD	1500	1800	400	200
9	ORD	LGA	0700	1000	400	200
10	ORD	LGA	0830	1130	400	150

Airline Fleet Assignment – Simplified Example (2/2)

Fleet type	Number of aircraft owned	Capacity [seats]	Per flight operating cost [$000]	
			LGA - BOS	LGA – ORD
A	1	120	10	15
B	2	150	12	17
C	2	250	15	20

The Airline Fleet Assignment Problem

Given:

- Flight schedule: (set of daily flight legs)
- Estimated passenger demand
- Aircraft fleet characteristics
- Revenue and operating cost data

Find:

- A feasible fleet assignment (an allocation of fleet types to flight legs) that maximizes:

  Profit contribution = Revenue – Operating Costs

Formulation – Decision Variables

Decision variables:

For each flight leg – fleet type combination, we need a binary variable that indicates whether that fleet type is assigned to that flight leg.

For example:

$x_{1,A}$ = 1 if fleet type A is assigned to flight leg 1

= 0 otherwise

Flight #	Fleet Type		
	A	B	C
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Formulation – Objective Function

- What is the (estimated) profit contribution of flight 1 when assigned fleet type A?
  $\approx 150 \cdot \min(250,120) - 10{,}000 = 8{,}000$

- What is the (estimated) profit contribution of flight 2 when assigned fleet type B?

  $\approx 150 \cdot \min(250,150) - 12{,}000 = 10{,}500$

<u>Objective Function</u>:

maximize $8{,}000\, x_{1,A} + 10{,}500\, x_{1,B} + 22{,}500\, x_{1,C}$
$+ \ldots$

Flight #	Fleet Type A	Fleet Type B	Fleet Type C
1	8,000	10,500	22,500
2	8,000	10,500	22,500
3	5,000	3,000	0
4	8,000	10,500	7,500
5	8,000	10,500	22,500
6	33,000	10,500	7,500
7	33,000	43,000	40,000
8	33,000	43,000	60,000
9	33,000	43,000	60,000
10	33,000	43,000	10,000

Formulation Constraints – (1/3)

Constraints:

- Binary
  Each assignment decision variable must be either 0 or 1

  $x_{1,A}$, $x_{1,B}$, $x_{1,C}$, … , $x_{10,A}$, $x_{10,B}$, $x_{10,C}$ are binary variables

- *Make sure each flight is assigned an airplane.*

  For flight 1: $x_{1,A} + x_{1,B} + x_{1,C} = 1$ and so on for each flight