

# This is clearly a simplification. We ignored ...

- Fixed costs
- Economies of scale (e.g., bulk discount)
- Multiple time periods
- Multiple products
- Transportation mode options + lead time
- Capacity constraints on the routes
- ...

Yet, the formulation can be easily modified to incorporate any of the above real-world factors. *Isn't that incredible?*

# Let's formulate an optimization model!

- Decision variables
- Objective function
- Constraints

# Decision Variables

$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$

# Objective Function

minimize

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

Supplier	Cost of supplying 1 ton to Facility				
	Valencia	Wiesbaden	Xanten	York	Zaragoza
Amadora	1.78	2.26	2.22	2.30	1.45
Bergamo	1.64	2.70	2.00	2.44	2.30
Casablanca	1.70	2.15	2.58	1.28	1.95

$$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$$

# Supply side constraints

**minimize**

(total supply cost)  $1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$

**subject to**

(capacity at A)  $s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4,000$

(capacity at B)  $s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2,000$

(capacity at C)  $s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1,000$

Supply side  
constraints

$$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$$

# Demand-side Constraints

**minimize**

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

**subject to**

(capacity at A)

$$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4,000$$

(capacity at B)

$$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2,000$$

(capacity at C)

$$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1,000$$

(demand at V)

$$s_{A,V} + s_{B,V} + s_{C,V} \geq 1,000$$

(demand at W)

$$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$$

(demand at X)

$$s_{A,X} + s_{B,X} + s_{C,X} \geq 1,500$$

(demand at Y)

$$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1,500$$

(demand at Z)

$$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$$

$$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$$

Supply side  
constraints

Demand side  
constraints

# Don't forget non-negativity constraints

**minimize**

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

**subject to**

(capacity at A)

$$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4,000$$

(capacity at B)

$$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2,000$$

(capacity at C)

$$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1,000$$

(demand at V)

$$s_{A,V} + s_{B,V} + s_{C,V} \geq 1,000$$

(demand at W)

$$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$$

(demand at X)

$$s_{A,X} + s_{B,X} + s_{C,X} \geq 1,500$$

(demand at Y)

$$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1,500$$

(demand at Z)

$$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$$

(nonnegativity)

$$s_{A,V} \geq 0, s_{A,W} \geq 0, \dots, s_{C,Z} \geq 0$$

Supply side  
constraints

Demand side  
constraints

# Base Formulation

**minimize**

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

**subject to**

(capacity at A)

$$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4,000$$

(capacity at B)

$$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2,000$$

(capacity at C)

$$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1,000$$

(demand at V)

$$s_{A,V} + s_{B,V} + s_{C,V} \geq 1,000$$

(demand at W)

$$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$$

(demand at X)

$$s_{A,X} + s_{B,X} + s_{C,X} \geq 1,500$$

(demand at Y)

$$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1,500$$

(demand at Z)

$$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$$

(nonnegativity)

$$s_{A,V} \geq 0, s_{A,W} \geq 0, \dots, s_{C,Z} \geq 0$$

Supply side  
constraints

Demand side  
constraints

# Base Formulation Spreadsheet Model

## PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity
	Valencia	Wiesbaden	Xanten	York	Zaragoza	
<b>Amadora</b>	1.78	2.26	2.22	2.30	1.45	4,000
<b>Bergamo</b>	1.64	2.70	2.00	2.44	2.30	2,000
<b>Casablanca</b>	1.70	2.15	2.58	1.28	1.95	1,000
Quantity required	1,000	500	1,500	1,500	500	

## DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied
	Valencia	Wiesbaden	Xanten	York	Zaragoza	
<b>Amadora</b>	500	500	0	500	500	2,000
<b>Bergamo</b>	500	0	1,500	0	0	2,000
<b>Casablanca</b>	0	0	0	1,000	0	1,000
Quantity received	1,000	500	1,500	1,500	500	

## OBJECTIVE

Total Cost **8,995** minimize

## CONSTRAINTS

Supply does not exceed capacity

Quantity delivered equals quantity required

Nonnegativity

# Solution to Base Formulation

