Alchemist by Juame Plensa (location: MIT W20)

Nonlinear Optimization

MIT MANAGEMENT SLOAN SCHOOL

15.060: Data, Models, and Decisions

Podimata, Ramakrishnan, Yao

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A Classification of Optimization Problems

		Decision Variables	
		All continuous	Some (or all) integer
Objective Function & Constraints	All linear	*Linear Optimization* ✅	*Integer Linear Optimization** ✅
	Some (or all) non-linear	*Nonlinear Optimization*	*Integer Nonlinear Optimization*

* also known as Discrete Optimization

Nonlinear Optimization is Very Versatile

Examples of important applications of nonlinear optimization

- Portfolio optimization
- Revenue/Price/Promotions Optimization
- Salesforce Optimization
- Marketing Mix Optimization
- Predictive Analytics
  - When we build Linear Regression and Logistic Regression models, we are actually solving Nonlinear Optimization problems!
- Numerous applications in science and engineering …

Optimizing Campaign Expenditure

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This is a hypothetical example designed for educational purposes only

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- In particular, the campaign has $20 million left to spend on State-level campaign efforts (advertising, get-out-the-vote efforts , etc.)

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- In particular, the campaign has $20 million left to spend on State-level campaign efforts (advertising, get-out-the-vote efforts , etc.)
- They want to know how to allocate the remaining funds across the states

The State of the Campaign

- At this stage, the campaign is confident their candidate has **229** electoral votes almost guaranteed

- Their opponent has **242** electoral votes almost guaranteed

- There are only three swing states left:
  - FL (29 electoral votes)
  - OH (18 electoral votes)
  - PA (20 electoral votes)

Electoral Vote Math

Electoral votes secured by our candidate	229
Electoral votes secured by the opponent	242
Electoral votes up for grabs	67 (= 29 + 18 + 20)
Total votes in the Electoral College	538
Total votes needed by a candidate to win	270
To win the election, our candidate needs	41 votes!

How best to spend the $20m campaign funds that remain?

Proposal: Allocate the remaining funds *in proportion to the number of electoral votes* in each of the remaining contested states

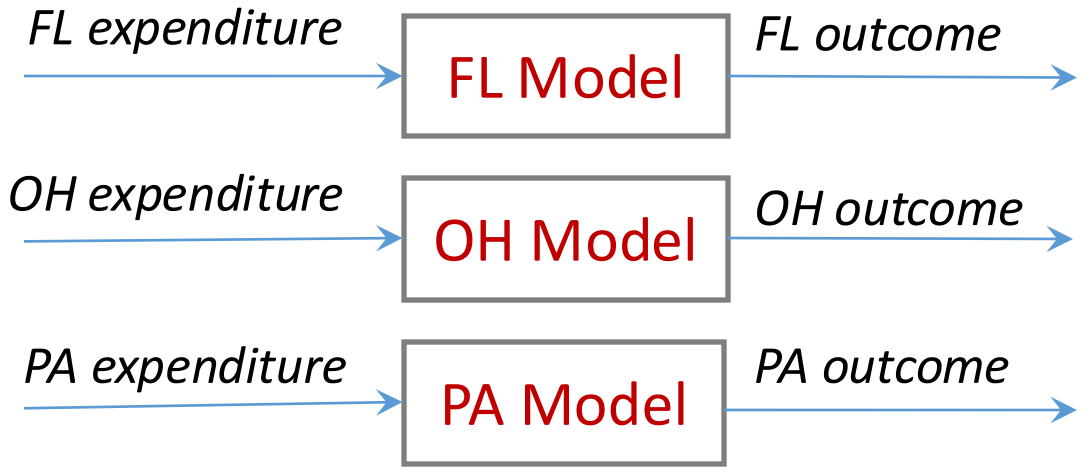
State	**FL**	**OH**	**PA**
Electoral votes	29	18	20
Expenditure [$ million]	20 * 29/67 = **8.66**	20 * 18/67 = **5.37**	20 * 20/67 = **5.97**

- **How good is this plan?**
- **Can we devise a better plan?**

Proposed Approach

<u>Step 1</u>: Predictive

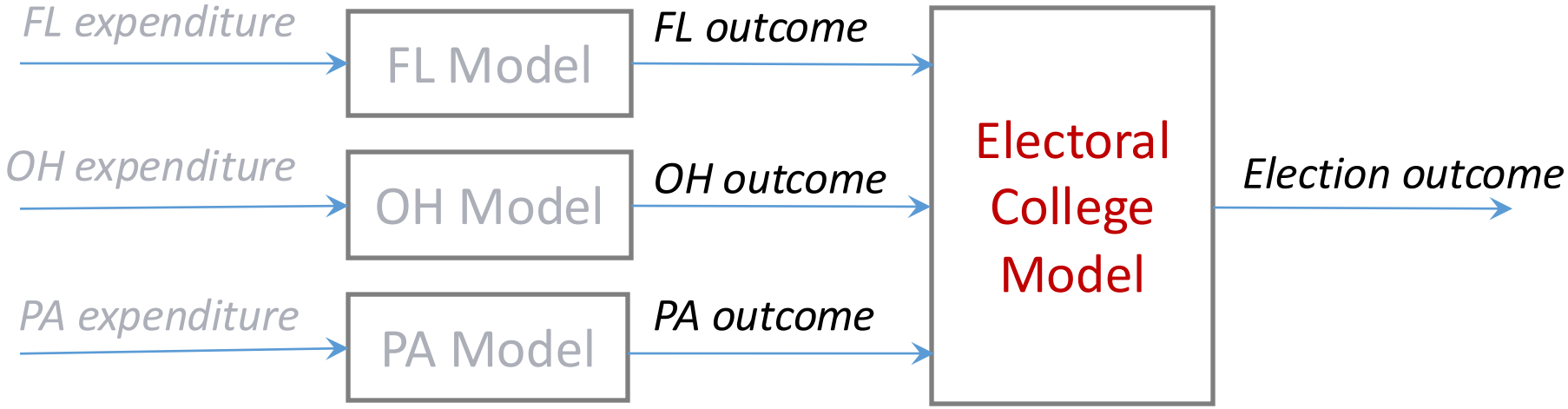
- Develop State-level models that relate expenditure in a state to the election outcome in that state.



Proposed Approach

Step 1: Predictive

- Develop State-level models that relate expenditure in a state to the election outcome in that state.
- Combine the State-level models with the logic of the Electoral College to predict the outcome of the election given an expenditure plan.



Proposed Approach

Step 1: Predictive

- Develop State-level models that relate proposed expenditure in a state to the election outcome in that state.
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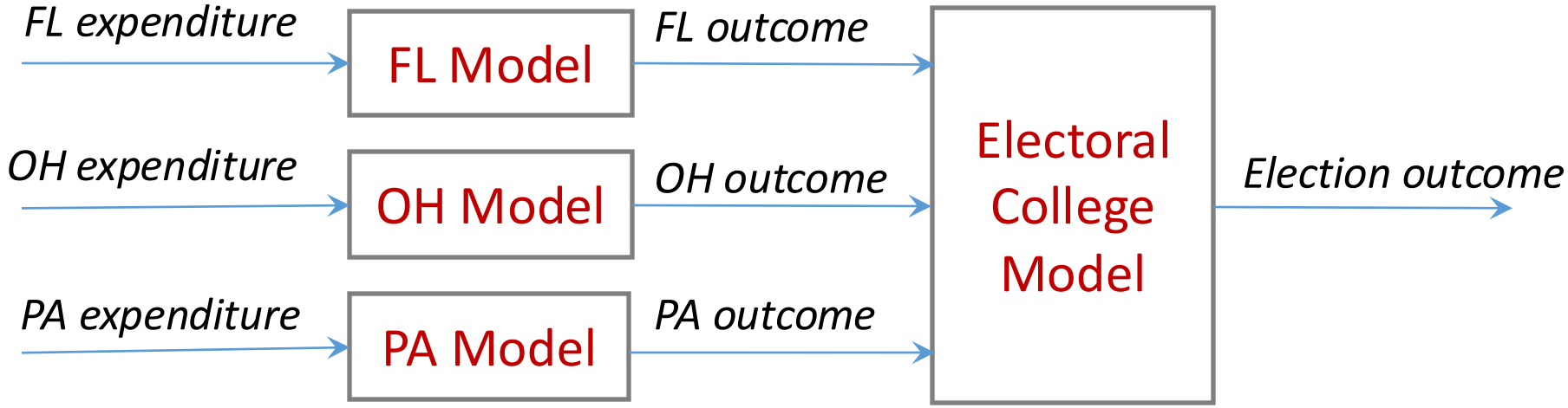

Step 2: Prescriptive

- Use the predictive models to formulate an expenditure optimization model

Building a Predictive Model

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- Data that would be nice to have:
  - Expenditure of past campaigns in this state and whether the candidate won
  - Also: Features of the candidate, opponent, political/economic/social landscape (to build a more accurate model)

Expenditure	Features/ Independent variables	Win/Loss?
$10m	…	Win
$8m	…	Win
$2m	…	Win
$1m	…	Loss
$0m	…	Loss

What can we do if there is limited data?

- The campaign has very limited reliable historical data on how campaign expenditure influences election results in swing states.

- Luckily, the campaign has **pundits (experts)** in each state, who know the state well, and who can provide (subjective) assessments of the chances of winning the state given certain levels of expenditure.

Pundit Assessments - Florida

Florida	Probability of Winning Florida for Expenditure Level				
	$4 million	$8 million	$10 million	$12 million	$16 million
FL Pundit 1	0.34	0.36	0.43	0.50	0.52
FL Pundit 2	0.41	0.43	0.52	0.60	0.63
FL Pundit 3	0.45	0.48	0.58	0.67	0.70
FL Pundit 4	0.50	0.53	0.63	0.74	0.77
FL Pundit 5	0.56	0.60	0.72	0.84	0.87

Let’s visualize the pundit assessments for Florida