Objective Function in Terms of Expenditure Allocations

$$\text{Pr(Winning the Election)} = \text{Pr(Win FL)}\,[\,1 - \text{Pr(Lose OH)} \times \text{Pr(Lose PA)}\,]$$

Objective Function in Terms of Expenditure Allocations

Pr(Winning the Election) = Pr(Win FL) [1 - Pr(Lose OH) x Pr(Lose PA)]

$$\underbrace{\left(0.45 + \frac{(0.70 - 0.45)}{1 + e^{-(x_{FL} - 10)}}\right)}_{\text{Pr(Win FL)}} [1 - \qquad\qquad\qquad\qquad]$$

Objective Function in Terms of Expenditure Allocations

Pr(Winning the Election) = Pr(Win FL) [1 - Pr(Lose OH) x Pr(Lose PA)]

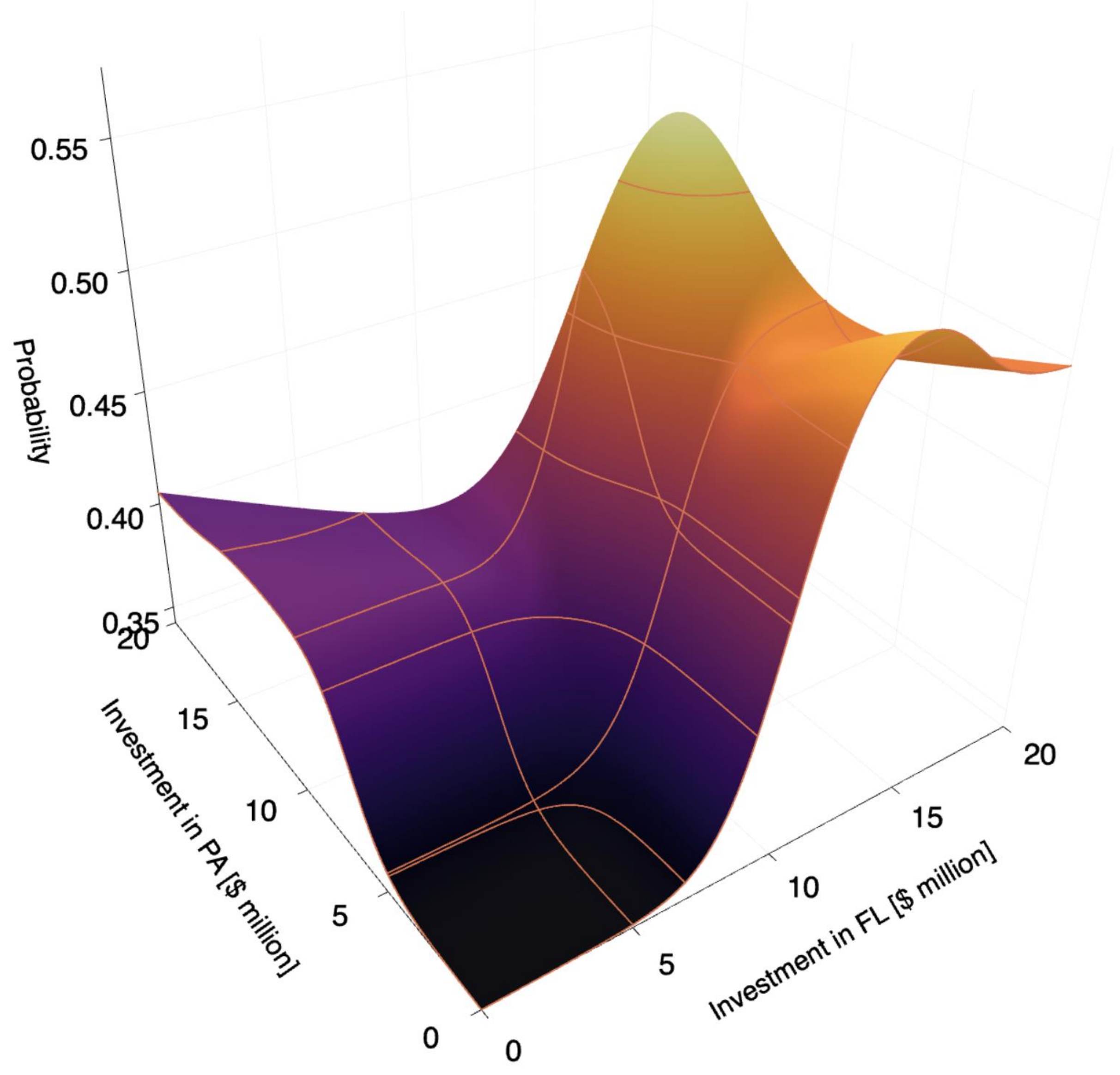
$$\underbrace{\left(0.45+\frac{(0.70-0.45)}{1+e^{-(x_{FL}-10)}}\right)}_{\text{Pr(Win FL)}}\left[1-\underbrace{\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(x_{OH}-5)}}\right)}_{\text{Pr(Lose OH) = 1 - Pr(Win OH)}}\qquad\qquad\qquad\right]$$

Objective Function in Terms of Expenditure Allocations

Pr(Winning the Election) = Pr(Win FL) [1 - Pr(Lose OH) x Pr(Lose PA)]

$$\underbrace{\left(0.45+\frac{(0.70-0.45)}{1+e^{-(x_{FL}-10)}}\right)}_{\text{Pr(Win FL)}}\left[1-\underbrace{\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(x_{OH}-5)}}\right)}_{\text{Pr(Lose OH) = 1 - Pr(Win OH)}}\underbrace{\left(1\ -\ 0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(x_{PA}-7)}}\right)}_{\text{Pr(Lose PA) = 1 – Pr(Win PA)}}\right]$$

Let’s visualize the Objective Function!



We set OH expenditure to be
= 20 – FL expenditure
– PA expenditure

We can use this objective function to predict the probability of a win for any expenditure plan

Plan	Expenditure [$ million]			Predicted Outcome
	FL	OH	PA	
No expenditure	0	0	0	31.53%

$$\left(0.45+\frac{(0.70-0.45)}{1+e^{-(0-10)}}\right)\left[1-\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(0-5)}}\right)\left(1\ -\ 0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(0-7)}}\right)\right]$$

We can use this objective function to predict the probability of a win for any expenditure plan

Plan	Expenditure [$ million]			Predicted Outcome
	FL	OH	PA	
No expenditure	0	0	0	31.53%
Current Plan	8.66	5.37	5.97	39.23%

$$\left(0.45+\frac{(0.70-0.45)}{1+e^{-(8.66-10)}}\right)\left[1-\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(5.37-5)}}\right)\left(1\ -\ 0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(5.97-7)}}\right)\right]$$

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But we want to find the best plan, so let's return to the optimization formulation

Objective Function

Decision variables

x_{FL} : \$millions to spend in Florida
x_{OH} : \$millions to spend in Ohio
x_{PA} : \$millions to spend in Pennsylvania

Objective function

Maximize

$$\left(0.45+\frac{(0.70-0.45)}{1+e^{-(x_{FL}-10)}}\right)\left[1-\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(x_{OH}-5)}}\right)\left(1\ -\ 0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(x_{PA}-7)}}\right)\right]$$

Constraints

Constraints

Decision variables

x_{FL} : \$millions to spend in Florida
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Objective function

Maximize

$$\left(0.45 + \frac{(0.70 - 0.45)}{1 + e^{-(x_{FL}-10)}}\right)\left[1 - \left(1 - 0.50 - \frac{(0.60 - 0.50)}{1 + \mathrm{e}^{-(x_{OH}-5)}}\right)\left(1 - 0.40 - \frac{(0.80 - 0.40)}{1 + \mathrm{e}^{-(x_{PA}-7)}}\right)\right]$$

Constraints

What constraints do we need?

The complete formulation

Decision variables

x_{FL} : \$millions to spend in Florida
x_{OH} : \$millions to spend in Ohio
x_{PA} : \$millions to spend in Pennsylvania

Objective function

Maximize

$$\left(0.45+\frac{(0.70-0.45)}{1+e^{-(x_{FL}-10)}}\right)\left[1-\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(x_{OH}-5)}}\right)\left(1\ -\ 0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(x_{PA}-7)}}\right)\right]$$

Constraints

subject to $x_{FL} + x_{OH} + x_{PA} \leq 20$

$x_{FL}\,,\, x_{OH}\,,\, x_{PA} \geq 0$

This is a nonlinear optimization model

Decision variables

x_{FL} : \$millions to spend in Florida
x_{OH} : \$millions to spend in Ohio
x_{PA} : \$millions to spend in Pennsylvania

Objective function

Non-linear objective function

Maximize

$$\left(0.45+\frac{(0.70-0.45)}{1+e^{-(x_{FL}-10)}}\right)\left[1-\left(1-0.50-\frac{(0.60-0.50)}{1+\mathrm{e}^{-(x_{OH}-5)}}\right)\left(1\;-\;0.40-\frac{(0.80-0.40)}{1+\mathrm{e}^{-(x_{PA}-7)}}\right)\right]$$

Constraints

subject to $x_{FL} + x_{OH} + x_{PA} \leq 20$ Linear constraints

$x_{FL}, x_{OH}, x_{PA} \geq 0$

Solving the NLO Formulation

Switch to Excel

Recommendations

Plan	Expenditure [$ million]			Predicted Outcome
	FL	OH	PA	
No expenditure	0	0	0	31.53%
Current Plan	8.66	5.37	5.97	39.23%
Solver Solution (starting from no expenditure)	0	7.91	12.09	41.31%

Recommendations

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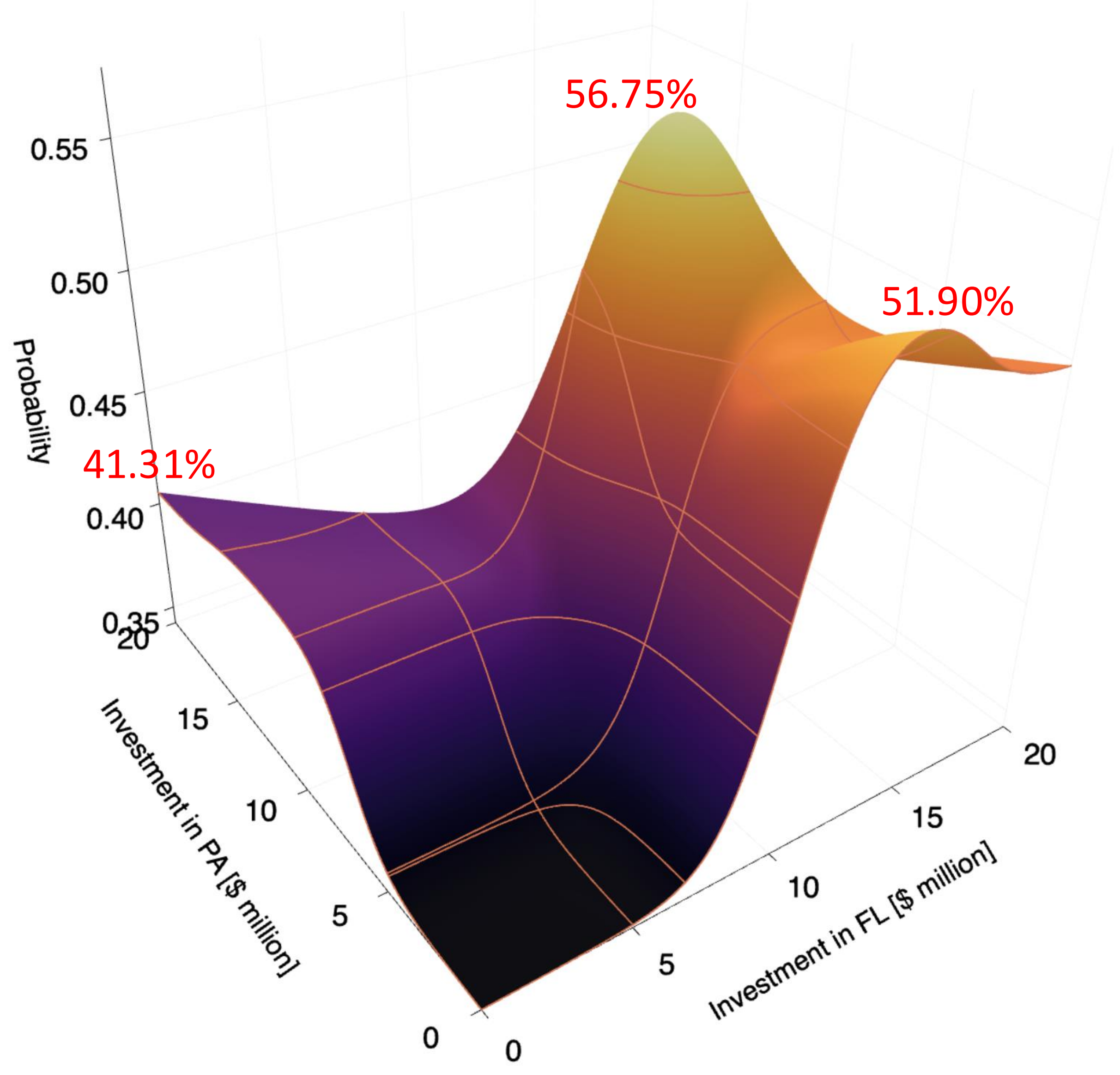
Solver Solution (starting from "Current Plan")	11.87	0	8.13	56.75%

How the different solutions look visually



What is the best solution?

Recommendations

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This is in fact the best solution!!

Optimal Expenditures for FL, OH and PA