

This is clearly a simplification. We ignored ...

- Fixed costs
- Economies of scale (e.g., bulk discount)
- Multiple time periods
- Multiple products
- Transportation mode options + lead time
- Capacity constraints on the routes
- ...

Yet, the formulation can be easily modified to incorporate any of the above real-world factors. *Isn't that incredible?*

Let's formulate an optimization model!

- Decision variables
- Objective function
- Constraints

Decision Variables

$$\mathbf{s}_{A,V}, \mathbf{s}_{A,W}, \dots, \mathbf{s}_{C,Z}$$

Objective Function

minimize

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

Supplier	Cost of supplying 1 ton to Facility				
	Valencia	Wiesbaden	Xanten	York	Zaragoza
Amadora	1.78	2.26	2.22	2.30	1.45
Bergamo	1.64	2.70	2.00	2.44	2.30
Casablanca	1.70	2.15	2.58	1.28	1.95

$$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$$

Supply side constraints

minimize

(total supply cost)

$$1.78 \mathbf{s}_{A,V} + 2.26 \mathbf{s}_{A,W} + \dots + 1.95 \mathbf{s}_{C,Z}$$

subject to

(capacity at A)

$$\mathbf{s}_{A,V} + \mathbf{s}_{A,W} + \mathbf{s}_{A,X} + \mathbf{s}_{A,Y} + \mathbf{s}_{A,Z} \leq 4,000$$

(capacity at B)

$$\mathbf{s}_{B,V} + \mathbf{s}_{B,W} + \mathbf{s}_{B,X} + \mathbf{s}_{B,Y} + \mathbf{s}_{B,Z} \leq 2,000$$

(capacity at C)

$$\mathbf{s}_{C,V} + \mathbf{s}_{C,W} + \mathbf{s}_{C,X} + \mathbf{s}_{C,Y} + \mathbf{s}_{C,Z} \leq 1,000$$

} Supply side
constraints

$$\mathbf{s}_{A,V}, \mathbf{s}_{A,W}, \dots, \mathbf{s}_{C,Z}$$

Demand-side Constraints

minimize

(total supply cost)

$$1.78 s_{A,V} + 2.26 s_{A,W} + \dots + 1.95 s_{C,Z}$$

subject to

(capacity at A)

$$s_{A,V} + s_{A,W} + s_{A,X} + s_{A,Y} + s_{A,Z} \leq 4,000$$

(capacity at B)

$$s_{B,V} + s_{B,W} + s_{B,X} + s_{B,Y} + s_{B,Z} \leq 2,000$$

(capacity at C)

$$s_{C,V} + s_{C,W} + s_{C,X} + s_{C,Y} + s_{C,Z} \leq 1,000$$

Supply side
constraints

(demand at V)

$$s_{A,V} + s_{B,V} + s_{C,V} \geq 1,000$$

(demand at W)

$$s_{A,W} + s_{B,W} + s_{C,W} \geq 500$$

(demand at X)

$$s_{A,X} + s_{B,X} + s_{C,X} \geq 1,500$$

(demand at Y)

$$s_{A,Y} + s_{B,Y} + s_{C,Y} \geq 1,500$$

(demand at Z)

$$s_{A,Z} + s_{B,Z} + s_{C,Z} \geq 500$$

Demand side
constraints

$$s_{A,V}, s_{A,W}, \dots, s_{C,Z}$$

Don't forget non-negativity constraints

minimize

(total supply cost)

$$1.78 \mathbf{s}_{A,V} + 2.26 \mathbf{s}_{A,W} + \dots + 1.95 \mathbf{s}_{C,Z}$$

subject to

(capacity at A)

$$\mathbf{s}_{A,V} + \mathbf{s}_{A,W} + \mathbf{s}_{A,X} + \mathbf{s}_{A,Y} + \mathbf{s}_{A,Z} \leq 4,000$$

(capacity at B)

$$\mathbf{s}_{B,V} + \mathbf{s}_{B,W} + \mathbf{s}_{B,X} + \mathbf{s}_{B,Y} + \mathbf{s}_{B,Z} \leq 2,000$$

(capacity at C)

$$\mathbf{s}_{C,V} + \mathbf{s}_{C,W} + \mathbf{s}_{C,X} + \mathbf{s}_{C,Y} + \mathbf{s}_{C,Z} \leq 1,000$$

} Supply side
constraints

(demand at V)

$$\mathbf{s}_{A,V} + \mathbf{s}_{B,V} + \mathbf{s}_{C,V} \geq 1,000$$

(demand at W)

$$\mathbf{s}_{A,W} + \mathbf{s}_{B,W} + \mathbf{s}_{C,W} \geq 500$$

(demand at X)

$$\mathbf{s}_{A,X} + \mathbf{s}_{B,X} + \mathbf{s}_{C,X} \geq 1,500$$

(demand at Y)

$$\mathbf{s}_{A,Y} + \mathbf{s}_{B,Y} + \mathbf{s}_{C,Y} \geq 1,500$$

(demand at Z)

$$\mathbf{s}_{A,Z} + \mathbf{s}_{B,Z} + \mathbf{s}_{C,Z} \geq 500$$

} Demand side
constraints

(nonnegativity)

$$\mathbf{s}_{A,V} \geq 0, \mathbf{s}_{A,W} \geq 0, \dots, \mathbf{s}_{C,Z} \geq 0$$

Base Formulation

minimize

(total supply cost)

$$1.78 \mathbf{s}_{A,V} + 2.26 \mathbf{s}_{A,W} + \dots + 1.95 \mathbf{s}_{C,Z}$$

subject to

(capacity at A)

$$\mathbf{s}_{A,V} + \mathbf{s}_{A,W} + \mathbf{s}_{A,X} + \mathbf{s}_{A,Y} + \mathbf{s}_{A,Z} \leq 4,000$$

(capacity at B)

$$\mathbf{s}_{B,V} + \mathbf{s}_{B,W} + \mathbf{s}_{B,X} + \mathbf{s}_{B,Y} + \mathbf{s}_{B,Z} \leq 2,000$$

(capacity at C)

$$\mathbf{s}_{C,V} + \mathbf{s}_{C,W} + \mathbf{s}_{C,X} + \mathbf{s}_{C,Y} + \mathbf{s}_{C,Z} \leq 1,000$$

Supply side
constraints

(demand at V)

$$\mathbf{s}_{A,V} + \mathbf{s}_{B,V} + \mathbf{s}_{C,V} \geq 1,000$$

(demand at W)

$$\mathbf{s}_{A,W} + \mathbf{s}_{B,W} + \mathbf{s}_{C,W} \geq 500$$

(demand at X)

$$\mathbf{s}_{A,X} + \mathbf{s}_{B,X} + \mathbf{s}_{C,X} \geq 1,500$$

(demand at Y)

$$\mathbf{s}_{A,Y} + \mathbf{s}_{B,Y} + \mathbf{s}_{C,Y} \geq 1,500$$

(demand at Z)

$$\mathbf{s}_{A,Z} + \mathbf{s}_{B,Z} + \mathbf{s}_{C,Z} \geq 500$$

Demand side
constraints

(nonnegativity)

$$\mathbf{s}_{A,V} \geq 0, \mathbf{s}_{A,W} \geq 0, \dots, \mathbf{s}_{C,Z} \geq 0$$

Base Formulation Spreadsheet Model

PARAMETERS

Supplier	Cost of supplying 1 ton to Facility					Supplier capacity
	Valencia	Wiesbaden	Xanten	York	Zaragoza	
Amadora	1.78	2.26	2.22	2.30	1.45	4,000
Bergamo	1.64	2.70	2.00	2.44	2.30	2,000
Casablanca	1.70	2.15	2.58	1.28	1.95	1,000
Quantity required	1,000	500	1,500	1,500	500	

DECISIONS VARIABLES

From Supplier	To Facility					Quantity supplied
	Valencia	Wiesbaden	Xanten	York	Zaragoza	
Amadora	500	500	0	500	500	2,000
Bergamo	500	0	1,500	0	0	2,000
Casablanca	0	0	0	1,000	0	1,000
Quantity received	1,000	500	1,500	1,500	500	

OBJECTIVE

Total Cost	8,995
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minimize

CONSTRAINTS

Supply does not exceed capacity
Quantity delivered equals quantity required
Nonnegativity

Solution to Base Formulation

