Excluding a Solution

- The (previous) optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

Excluding a Solution

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- We want to exclude just this one solution but none other. We will add a constraint that will make just this solution infeasible.

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)$$

For the solution **A, C, E, G, I, J :**
Value of this expression = 6

+

For the solution **A, C, E, G, I, J :**
Value of this expression = 4

= 10 (total)

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$\underbrace{y_A + y_C + y_E + y_G + y_I + y_J}\ + \underbrace{(1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)}$$

For the solution **A, C, E, G, I, J :**
Value of this expression = 6

+

For the solution **A, C, E, G, I, J :**
Value of this expression = 4

= 10 (total)

For any other solution:
Value of this expression

+

For any other solution:
Value of this expression

= ? (total)

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$\underbrace{y_A + y_C + y_E + y_G + y_I + y_J}\ + \underbrace{(1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)}$$

For the solution **A, C, E, G, I, J :**
Value of this expression = 6 **+** For the solution **A, C, E, G, I, J :**
Value of this expression = 4 = 10 (total)

For <u>any other</u> solution:
Value of this expression **+** For <u>any other</u> solution:
Value of this expression = 9 or less (total)

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)$$

  - This expression is equal to 10 for the solution **A, C, E, G, I, J**
  - *For every other solution, the expression is guaranteed to be less than or equal to 9*

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)$$

> ➢ This expression is equal to 10 for the solution **A, C, E, G, I, J**
>
> ➢ *For every other solution, the expression is guaranteed to be less than or equal to 9*

So, we add the constraint:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H) \leq 9$$

This constraint eliminates **A, C, E, G, I, J** but no other solutions!

Excluding Solutions

- The optimal solution was **A, C, E, G, I, J** :

$$y_A^* = y_C^* = y_E^* = y_G^* = y_I^* = y_J^* = 1 \text{ and } y_B^* = y_D^* = y_F^* = y_H^* = 0$$

- Consider the following expression:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H)$$

  - This expression is equal to 10 for the solution **A, C, E, G, I, J**
  - *For every other solution, the expression is guaranteed to be less than or equal to 9*

So, we add the constraint:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H) \leq 9$$

This constraint eliminates **A, C, E, G, I, J** but no other solutions!

IN GENERAL, THE RHS WILL BE THE NUMBER OF VARIABLES - 1

Formulation with All Constraints, Synergy Effect,and Excluding Previous Optimal

maximize $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J + 8\ z$

subject to

(binary) $z, y_A, y_B, \ldots, y_J$

(points budget) $200\ y_A + 50\ y_B + \ldots + 100\ y_J \leq 1000$

(max credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \leq 54$

(min credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \geq 36$

(MW, H3 load) $y_A + y_B + y_E + y_G \leq 3$

(MW H4 load) $y_A + y_B + y_G + 6\ y_J \leq 3$

(TR H3 load) $y_C + y_D + y_I \leq 3$

(TR H4 load) $y_C + y_D + y_F + y_H \leq 3$

(A B conflict) $y_A + y_B \leq 1$

(B or C required) $y_B + y_C \geq 1$

(E pre-req to H) $y_H \leq y_E$

(Analytics Certificate Synergy)

- $z \leq y_I$
- $z \leq y_J$
- $z \geq y_I + y_J - 1$

(Exclude Previous Optimal Solution)

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H) \leq 9$$

Updated Excel Solution

Bid on B, C, E, G, H, I, J

DECISIONS	A	B	C	D	E	F	G	H	I	J	Z
Course	0.0	1.0	1.0	0.0	1.0	0.0	1.0	1.0	1.0	1.0	1.0

OBJECTIVE

Total utility	46

CONSTRAINTS

	LHS		RHS
Points budget	730	<=	1,000
Course credit maximum	54	<=	54
Course credit minimum	54	>=	36
Mon, Wed H3 classes	3	<=	3
Tue, Thr H3 classes	2	<=	3
Mon, Wed H4 classes	3	<=	3
Tue, Thr H4 classes	3	<=	3
Do not take A and B	1.0	<=	1
Take either B or C	2.0	>=	1
H requires E	1.0	<=	1
If I not chosen, Z is 0	1.0	<=	1
If J not chosen, Z is 0	1.0	<=	1
If either I or J is chosen, Z is 1	1.0	>=	1
Exclude previous optimal solution	3.0	<=	5
Binary constraints			

Summary

Model	Optimal Utility	Optimal Course Selection
Basic	46	**Bid on A, E, F, G, H, I, J**
Additional Constraints	44	**Bid on A, C, E, F, G, H, J**
Synergy Effect	47	**Bid on A, C, E, G, I, J (+ certificate)**
Exclude previous optimal solution	46	**Bid on B, C, E, G, H, I, J (+ certificate)**

Excluding Solutions

This approach can be used to exclude any number of previously-found solutions, by adding a constraint for each solution you want to exclude

Excluding Two Solutions

- We want to exclude **A, C, E, G, I, J** <u>and</u> **B, C, E, G, H, I, J**

- To eliminate **A, C, E, G, I, J** we already have this in our model:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H) \leq 9$$

- What constraint should we add to eliminate **B, C, E, G, H, I, J** ?

Excluding Two Solutions

- We want to exclude **A, C, E, G, I, J** <u>and</u> **B, C, E, G, H, I, J**

- To eliminate **A, C, E, G, I, J** we already have this in our model:

$$y_A + y_C + y_E + y_G + y_I + y_J + (1 - y_B) + (1 - y_D) + (1 - y_F) + (1 - y_H) \leq 9$$

- What constraint should we add to eliminate **B, C, E, G, H, I, J** ?

$$y_B + y_C + y_E + y_G + yH + y_I + y_J + (1 - y_A) + (1 - y_D) + (1 - y_F) \leq 9$$

Updated Excel Solution

Bid on A, C, F, G, I, J

DECISIONS	A	B	C	D	E	F	G	H	I	J	Z
Course	1.0	0.0	1.0	0.0	0.0	1.0	1.0	0.0	1.0	1.0	1.0

OBJECTIVE

Total utility	46

CONSTRAINTS

	LHS		RHS
Points budget	780	<=	1,000
Course credit maximum	51	<=	54
Course credit minimum	51	>=	36
Mon, Wed H3 classes	2	<=	3
Tue, Thr H3 classes	2	<=	3
Mon, Wed H4 classes	3	<=	3
Tue, Thr H4 classes	3	<=	3
Do not take A and B	1.0	<=	1
Take either B or C	1.0	>=	1
H requires E	0.0	<=	0
If I not chosen, Z is 0	1.0	<=	1
If J not chosen, Z is 0	1.0	<=	1
If either I or J is chosen, Z is 1	1.0	>=	1
Exclude previous optimal solution	4.0	<=	5
Exclude 2nd optimal solution	2.0	<=	6
Binary constraints			

Summary

Model	Optimal Utility	Optimal Course Selection
Basic	46	Bid on A, E, F, G, H, I, J
Additional Constraints	44	Bid on A, C, E, F, G, H, J
Synergy Effect	47	Bid on A, C, E, G, I, J (+ certificate)
Exclude previous optimal solution	46	Bid on B, C, E, G, H, I, J. (+ certificate)
Excluding two solutions	46	Bid on A, C, F, G, I, J (+ certificate)

Note that the optimal value didn't decrease when we excluded the second optimal solution

Recap: The Power of Binary Variables

- Binary variables:
  - Are very useful to model business decisions that involve doing something or not doing something (as opposed to how much of something to do) …
  - … but make solving an optimization model more challenging

- Binary variables allow us to model IF-THEN relationships and non-linearities s using “tricks” so that the formulation remains linear.

- A vast array of real-world decision problems can be modeled with binary variables

A Sample of Optimization Applications I have Worked On

- Airline/Airport Operations
    - Fleet Assignment, Crew Assignment, Gate Assignment, Staff Scheduling
- “Last Mile” Delivery and Routing
- Retail Pricing and Promotions
- Supply Chain Inventory Allocation

What's Next

- *Wednesday*: Discrete Optimization – Part 2 (Application to Supply Chain Optimization)
- *Thursday*: Deliverable #7 due
- *Friday*: Recitation on Discrete Optimization
- 1-on-1 Meetings
  - *Please book via https://calendly.com/ramamit*
  - *I have added more Calendly slots. If the Calendly times don't work, please email my assistant Laura (brentrup@mit.edu) to find a time.*

APPENDIX

Optional How Integer Optimization Problems Are Solved in Practice