Formulating the Course Selection Problem

Constraints

Variables (binary) $y_A , y_B , \ldots , y_J \geq 0, \leq 1$ and integral

Bidding (1000 points budget) $200\, y_A + 50\, y_B + \ldots + 100\, y_J \leq 1000$

Credits (54 max credits)

(36 min credits)

	A	B	C	D	E	F	G	H	I	J
Credit Hours	12	9	9	12	6	6	9	6	9	6

Schedule (MW H3 load)

(MW H4 load)

(TR H3 load)

(TR H4 load)

	A	B	C	D	E	F	G	H	I	J
H3	✓	✓	✓	✓	✓		✓		✓	
H4	✓	✓	✓	✓		✓	✓	✓	✓	✓
Monday	✓	✓			✓		✓			✓
Tuesday			✓	✓		✓		✓	✓	
Wednesday	✓	✓			✓		✓			✓
Thursday			✓	✓		✓		✓	✓	
Friday										

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Bidding	(1000 points budget)	$200\, y_A + 50\, y_B + \dots + 100\, y_J \leq 1000$
Credits	(54 max credits)	$12\, y_A + 9\, y_B + \dots + 6\, y_J \leq 54$
	(36 min credits)	$12\, y_A + 9\, y_B + \dots + 6\, y_J \geq 36$
Schedule	(MW H3 load)	
	(MW H4 load)	
	(TR H3 load)	
	(TR H4 load)	

	A	B	C	D	E	F	G	H	I	J
H3	✓	✓	✓	✓	✓		✓		✓	
H4	✓	✓	✓	✓		✓	✓	✓	✓	✓
Monday	✓	✓			✓		✓			✓
Tuesday			✓	✓		✓		✓	✓	
Wednesday	✓	✓			✓		✓			✓
Thursday			✓	✓		✓		✓	✓	
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	(36 min credits)	$12\, y_A + 9\, y_B + \ldots + 6\, y_J \geq 36$
Schedule	(MW H3 load)	$y_A + y_B + y_E + y_G \leq 3$
	(MW H4 load)	
	(TR H3 load)	
	(TR H4 load)	

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Schedule	(MW H3 load)	$y_A + y_B + y_E + y_G \leq 3$
	(MW H4 load)	$y_A + y_B + y_G + y_J \leq 3$
	(TR H3 load)	$y_C + y_D + y_I \leq 3$
	(TR H4 load)	$y_C + y_D + y_F + y_H + y_I \leq 3$

The Complete Formulation

Maximize: $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

over variables: $y_A, y_B, \ldots, y_J$

Subject To:

(binary) $y_A , y_B , \ldots , y_J \geq 0, \leq 1$ and integral

(points budget) $200\ y_A + 50\ y_B + \ldots + 100\ y_J \leq 1000$

(54 max credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \leq 54$

(36 min credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \geq 36$

(MW H3 load) $y_A + y_B + y_E + y_G \leq 3$

(MW H4 load) $y_A + y_B + y_G + y_J \leq 3$

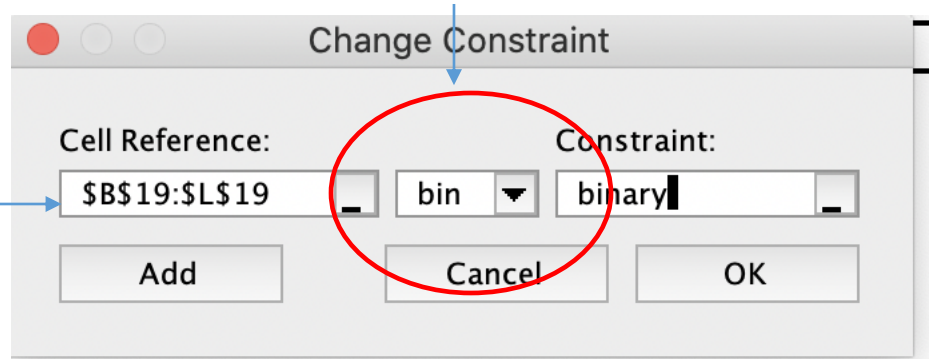
(TR H3 load) $y_C + y_D + y_I \leq 3$

(TR H4 load) $y_C + y_D + y_F + y_H + y_I \leq 3$

The only new piece here is we need to tell Solver that these decision variables are binary.

Excel's Solver makes it easy to create integer or binary variables

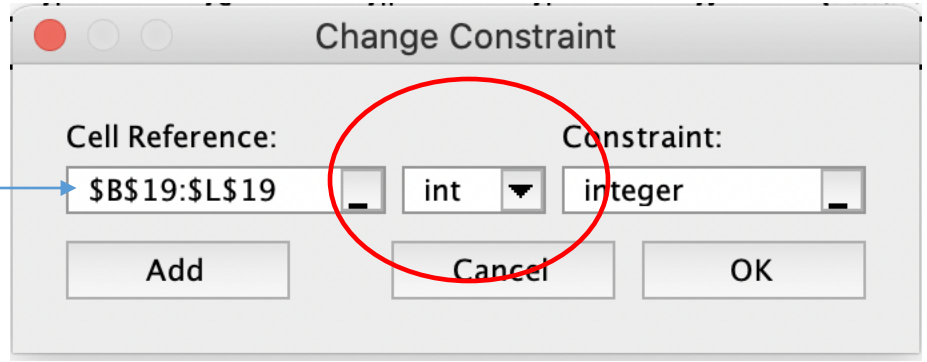
Choose the binary decision variables here



This is a shortcut for

- Integer
- >= 0
- <= 1

Choose the integer decision variables here



Solving the basic formulation in Excel

DECISIONS	A	B	C	D	E	F	G	H	I	J
Course	1.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0

OBJECTIVE

Total utility	46

CONSTRAINTS

	LHS	□	RHS
Points budget	730	<=	1,000
Course credit maximum	54	<=	54
Course credit minimum	54	>=	36
Mon, Wed H3 classes	3	<=	3
Tue, Thr H3 classes	1	<=	3
Mon, Wed H4 classes	3	<=	3
Tue, Thr H4 classes	3	<=	3
Binary constraints			

Summary

Model	Optimal Utility	Optimal Course Selection
Basic	46	**Bid on A, E, F, G, H, I, J**

Many realistic constraints are easily represented using binary variables

- A and B cannot be taken together (overlapping material)

- Must take at least one of B or C (graduation requirement)

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Can you write down linear constraints to capture these relationships?

Many realistic constraints are easily represented using binary variables

- A and B cannot be taken together

$$y_A + y_B \leq 1$$

- Must take at least one of B or C

Many realistic constraints are easily represented using binary variables

- A and B cannot be taken together

$$y_A + y_B \leq 1$$

- Must take at least one of B or C

$$y_B + y_C \geq 1$$

If-then constraints

Writing if-then constraints using math

Let’s add another constraint

	A	B	C	D	E	F	G	H	I	J
H3	✓	✓	✓	✓	✓		✓		✓	
H4	✓	✓	✓	✓		✓	✓	✓	✓	✓

- If you take H, then you must also take E (i.e., E is a prerequisite to H)

Can you write down a linear constraint to capture this relationship?

Let’s add another constraint

	A	B	C	D	E	F	G	H	I	J
H3	✓	✓	✓	✓	✓		✓		✓	
H4	✓	✓	✓	✓		✓	✓	✓	✓	✓

- If you take H, then you must also take E (i.e., E is a prerequisite to H)

$$y_H \leq y_E$$

(If $y_H = 1$, then $y_E \geq 1$, so y_E must be equal to 1. If $y_H = 0$ then y_E can be either 1 or 0)

Formulation with more constraints

Maximize: $10\ y_A + 2\ y_B + 4\ y_C + 2\ y_D + 5\ y_E + 4\ y_F + 8\ y_G + 7\ y_H + 6\ y_I + 6\ y_J$

over variables: $y_A, y_B, \ldots, y_J$

Subject To:

(binary) $y_A , y_B , \ldots , y_J \geq 0, \leq 1$ and integral

(points budget) $200\ y_A + 50\ y_B + \ldots + 100\ y_J \leq 1000$

(max credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \leq 54$

(min credits) $12\ y_A + 9\ y_B + \ldots + 6\ y_J \geq 36$

(MW H3 load) $y_A + y_B + y_E + y_G \leq 3$

(MW H4 load) $y_A + y_B + y_G + y_J \leq 3$

(TR H3 load) $y_C + y_D + y_I \leq 3$

(TR H4 load) $y_C + y_D + y_F + y_H + y_I \leq 3$

(A B conflict) $y_A + y_B \leq 1$

(B or C required) $y_B + y_C \geq 1$

(E pre-req to H) $y_H \leq y_E$

} Additional constraints

Excel with additional constraints

DECISIONS	A	B	C	D	E	F	G	H	I	J
Course	1.0	0.0	1.0	0.0	1.0	1.0	1.0	1.0	0.0	1.0

OBJECTIVE

Total utility	44

CONSTRAINTS

	LHS	□	RHS
Points budget	700	<=	1,000
Course credit maximum	54	<=	54
Course credit minimum	54	>=	36
Mon, Wed H3 classes	3	<=	3
Tue, Thr H3 classes	1	<=	3
Mon, Wed H4 classes	3	<=	3
Tue, Thr H4 classes	3	<=	3
Do not take A and B	1.0	<=	1
Take either B or C	1.0	>=	1
H requires E	1.0	<=	1
Binary constraints			

Summary

Model	Optimal Utility	Optimal Course Selection
Basic	46	**Bid on A, E, F, G, H, I, J**
Additional Constraints	44	**Bid on A, C, E, F, G, H, J**

When we add a new constraint…

Does the original optimal solution satisfy the new constraint?

- **Yes** => The original solution is still optimal. We don't need to resolve.

Model	Optimal Utility	Optimal Course Selection
Basic	46	**Bid on A, E, F, G, H, I, J**

Additional Constraints

✅ (A B conflict) $y_A + y_B \leq 1$

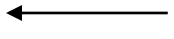
✅ (E pre-req to H) $y_H \leq y_E$

When we add a new constraint…

Does the original optimal solution satisfy the new constraint?

- **Yes** => The original solution is still optimal. We don't need to resolve.
- **No** => We need to explicitly add the new constraint and re-solve.

Model	Optimal Utility	Optimal Course Selection
Basic	46	**Bid on A, E, F, G, H, I, J**



Additional Constraints

- ✅ (A B conflict) $y_A + y_B \leq 1$
- ❌ (B or C required) $y_B + y_C \geq 1$
- ✅ (E pre-req to H) $y_H \leq y_E$