

ICCS342 - Lecture 14

# Decision Tree and Random Forest

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Slides adapted from lecture slides by Michael I. Jordan, UC – Berkeley,  
and lecture slides by Tom Mitchell, CMU



Mahidol University  
International College



# Machine Learning



what society thinks I do



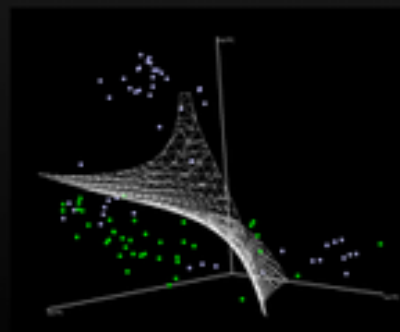
what my friends think I do



what my parents think I do

$$\begin{aligned} \ell_p &= \frac{1}{2} \|\mathbf{w}\|^2 = \frac{1}{2} \sum_{i=1}^n \alpha_i y_i (\mathbf{x}_i \cdot \mathbf{w} + b) + \frac{1}{2} \sum_{i=1}^n \alpha_i \\ \alpha_i &\geq 0, \forall i \\ \mathbf{w} &= \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i, \sum_{i=1}^n \alpha_i y_i = 0 \\ \nabla_{\theta} \ell(\theta_t) &= \frac{1}{n} \sum_{i=1}^n \nabla \ell(x_i, y_i; \theta_t) + \nabla r(\theta_t) \\ \theta_{t+1} &= \theta_t - \eta \nabla \ell(x_{(t)}, y_{(t)}; \theta_t) - \eta \cdot \nabla r(\theta_t) \\ \mathbb{E}_{(t)}[\ell(x_{(t)}, y_{(t)}; \theta_t)] &= \frac{1}{n} \sum_{i=1}^n \ell(x_i, y_i; \theta_t). \end{aligned}$$

what other programmers think I do



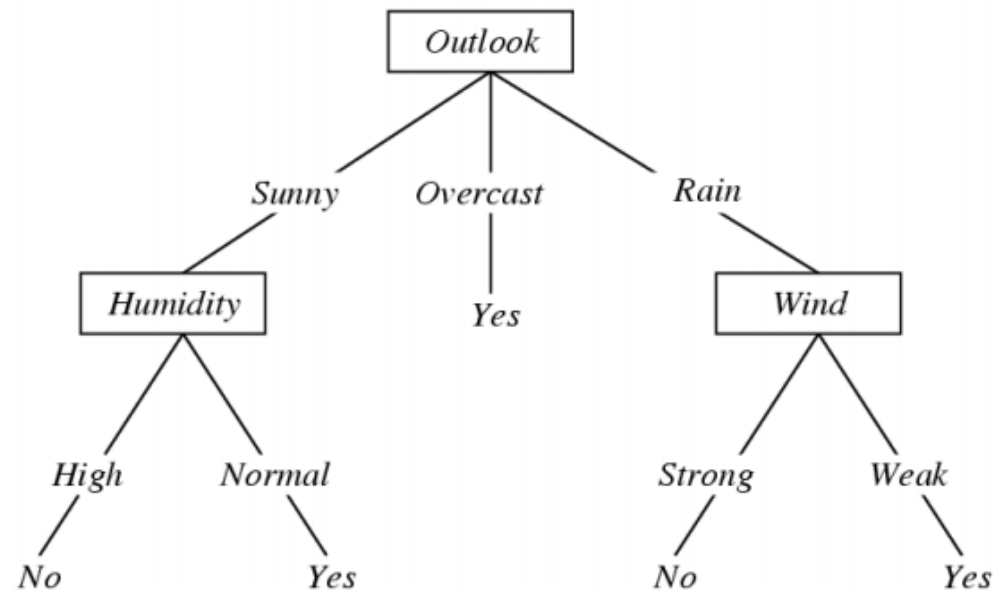
what I think I do

```
>>> from scipy import SVM
```

what I really do

# Decision Tree

- Example: Play tennis?



- Each node tests an attribute  $X_i$
- Each branch from a node selects a value for  $X_i$
- Each **leaf node** predicts  $y$

# Top-Down Induction of Decision Trees

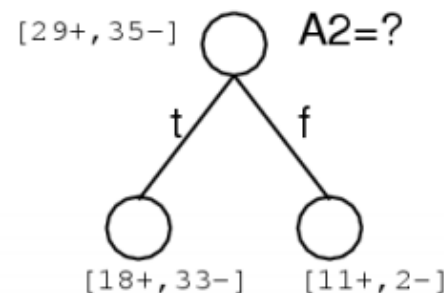
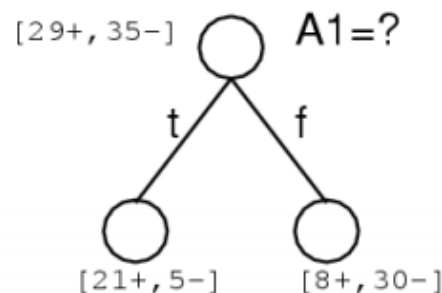
[ID3, C4.5, Quinlan]

*node* = Root

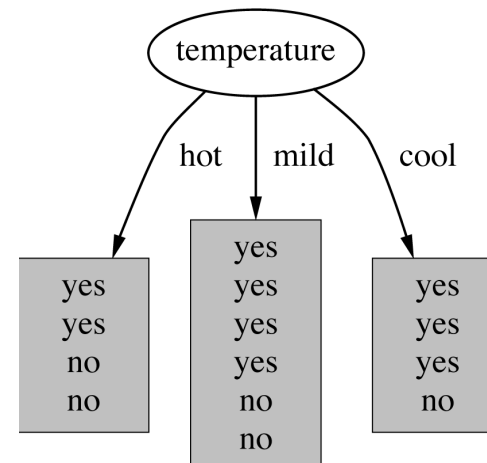
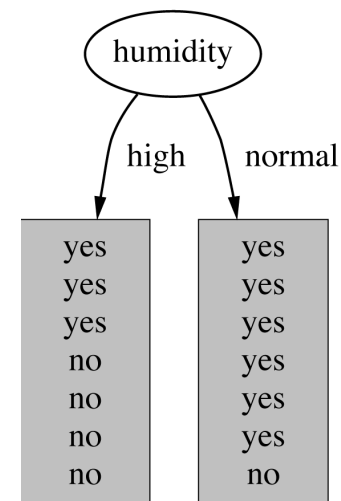
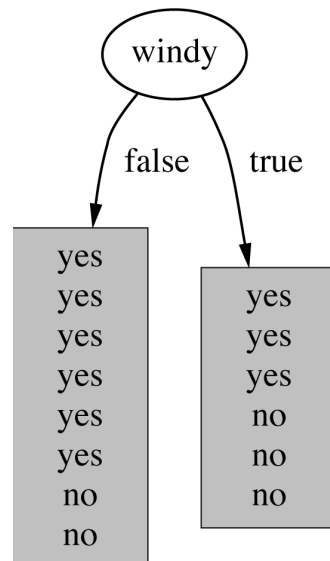
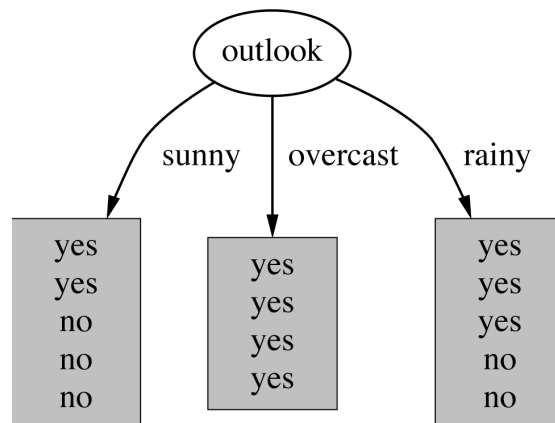
Main loop:

1.  $A \leftarrow$  the “best” decision attribute for next *node*
2. Assign  $A$  as decision attribute for *node*
3. For each value of  $A$ , create new descendant of *node*
4. Sort training examples to leaf nodes
5. If training examples perfectly classified, Then STOP, Else iterate over new leaf nodes

Which attribute is best?



# Which attribute to select?



# Gini Impurity

- Given a set of items with  $J$  classes, suppose  $i \in \{1, 2, 3, \dots, J\}$ , and let  $p_i$  be the fraction of items labeled with class  $i$  in the set:

$$Gini(p) = \sum_{i=1}^J p_i(1 - p_i)$$

# Gini Impurity

Example:  $S = \{1, 1, 2, 2, 3\}$

$$p_1 = 2/5, p_2 = 2/5, p_3 = 1/5$$

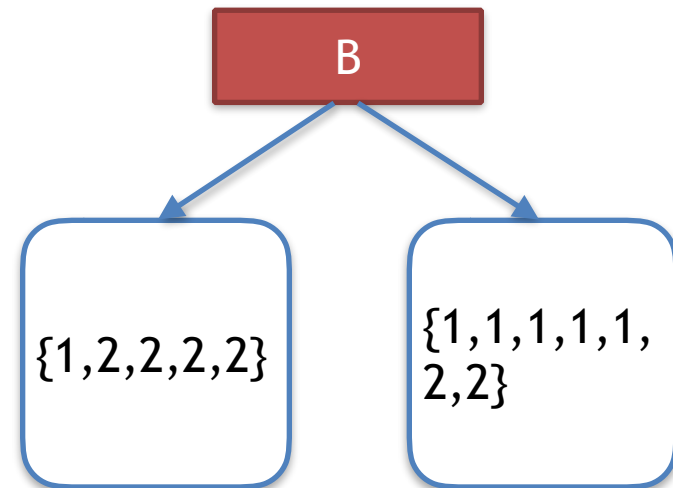
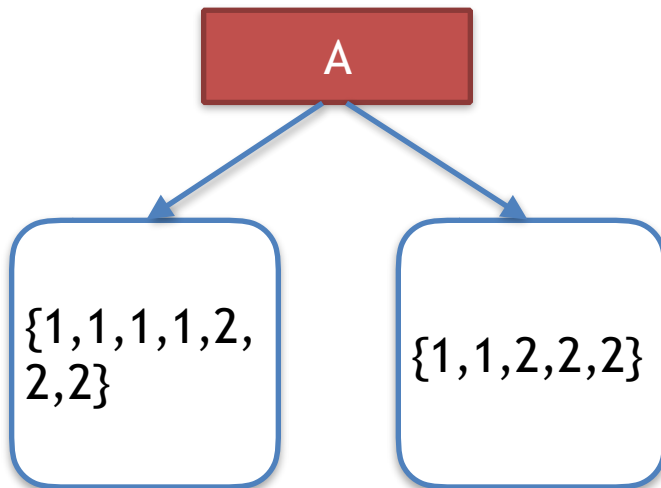
Gini Impurity of  $S$

$$= 2/5(1-2/5) + 2/5(1-2/5) + 1/5(1-1/5)$$

$$= 6/25 + 6/25 + 4/25$$

$$= 16/25 = 0.64$$

# Which split is better?





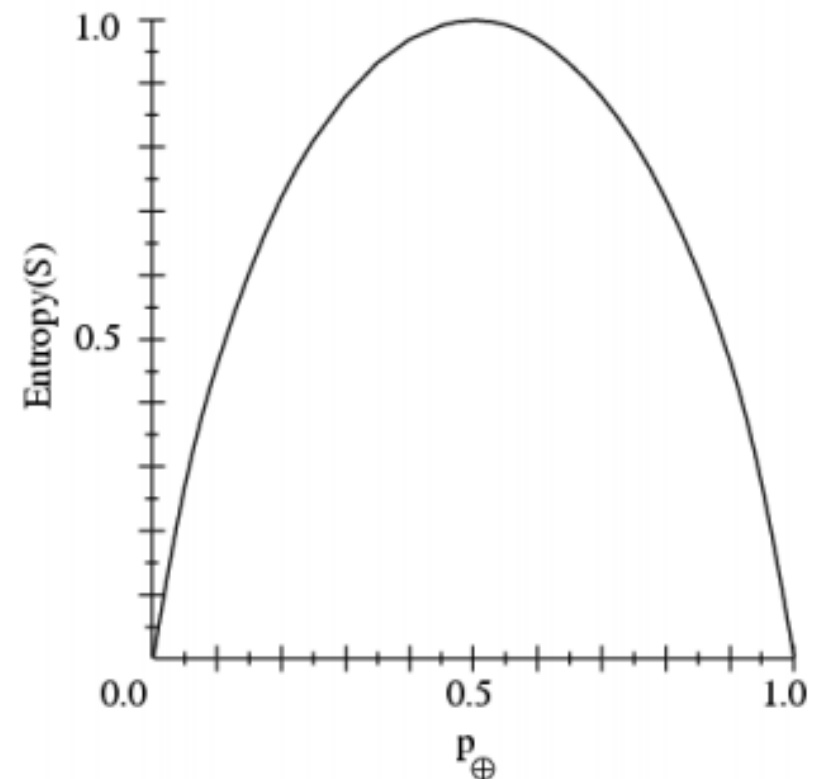
# Information Entropy

$$H(X) = - \sum_{i=1}^n p(x_i) \log_b p(x_i)$$

- Quantifies “randomness”
  - High entropy  $\longrightarrow$  more random
  - Low entropy  $\longrightarrow$  less random
  - When  $b=2$ ,  $H(X)$  is the expected number of bits to encode the random variable  $X$

# Sample Entropy

- $S$  is a sample of training examples.
- $p_{\oplus}$  is the proportion of positive examples in  $S$ .
- $p_{\ominus}$  is the proportion of negative examples in  $S$ .
- Entropy measures the impurity of  $S$ :
- $H(S) = -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$



# Information Gain

- Based on Shannon Entropy
- IG calculates effective change in entropy after making a decision based on the value of an attribute.
- For decision trees, it's ideal to base decisions on the attribute that provides the largest change in entropy, the attribute with the highest gain.

# Information Gain

- Also known as Mutual Information:

$$I(X, A) = H(X) - H(X|A)$$

where

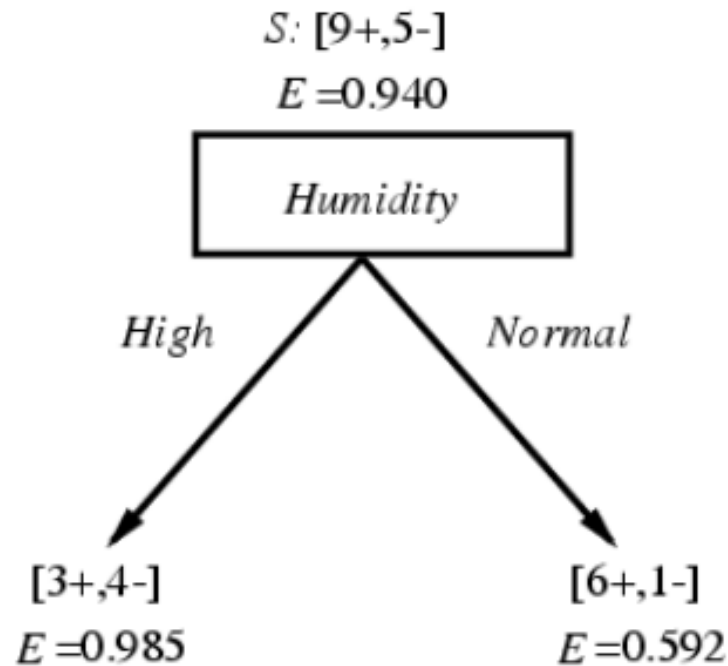
$$H(X|A) = \sum_a P(A = a) H(X|A = a)$$

- IG is the expected reduction in entropy of the target variable  $X$ , due to sorting on variable  $A$

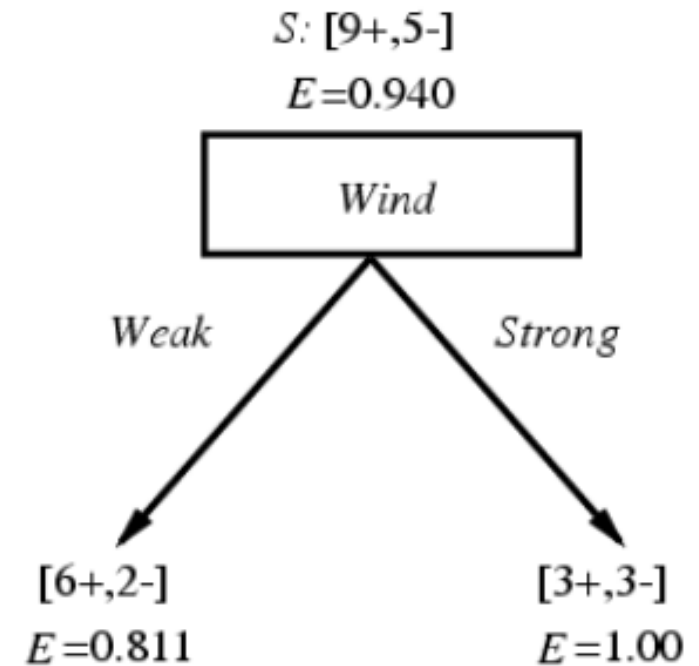
# Exercise: Play tennis?

<i>Outlook</i>	<i>Tem</i>	<i>Humid</i>	<i>Windy</i>	<i>Play</i>	<i>Outlook</i>	<i>Tem</i>	<i>Humid</i>	<i>Windy</i>	<i>Play</i>
<i>Sunny</i>	<i>Hot</i>	<i>High</i>	<i>FALSE</i>	<i>NO</i>	<i>Sunny</i>	<i>Mild</i>	<i>High</i>	<i>FALSE</i>	<i>NO</i>
<i>Sunny</i>	<i>Hot</i>	<i>High</i>	<i>TRUE</i>	<i>NO</i>	<i>Sunny</i>	<i>Cool</i>	<i>Norm</i>	<i>FALSE</i>	<i>YES</i>
<i>Overcast</i>	<i>Hot</i>	<i>High</i>	<i>FALSE</i>	<i>YES</i>	<i>Rainy</i>	<i>Mild</i>	<i>Norm</i>	<i>FALSE</i>	<i>YES</i>
<i>Rainy</i>	<i>Mild</i>	<i>High</i>	<i>FALSE</i>	<i>YES</i>	<i>Sunny</i>	<i>Mild</i>	<i>Norm</i>	<i>TRUE</i>	<i>YES</i>
<i>Rainy</i>	<i>Cool</i>	<i>Norm</i>	<i>FALSE</i>	<i>YES</i>	<i>Overcast</i>	<i>Mild</i>	<i>High</i>	<i>TRUE</i>	<i>YES</i>
<i>Rainy</i>	<i>Cool</i>	<i>Norm</i>	<i>TRUE</i>	<i>NO</i>	<i>Overcast</i>	<i>Hot</i>	<i>Norm</i>	<i>FALSE</i>	<i>YES</i>
<i>Overcast</i>	<i>Cool</i>	<i>Norm</i>	<i>TRUE</i>	<i>YES</i>	<i>Rainy</i>	<i>Mild</i>	<i>High</i>	<i>TRUE</i>	<i>NO</i>

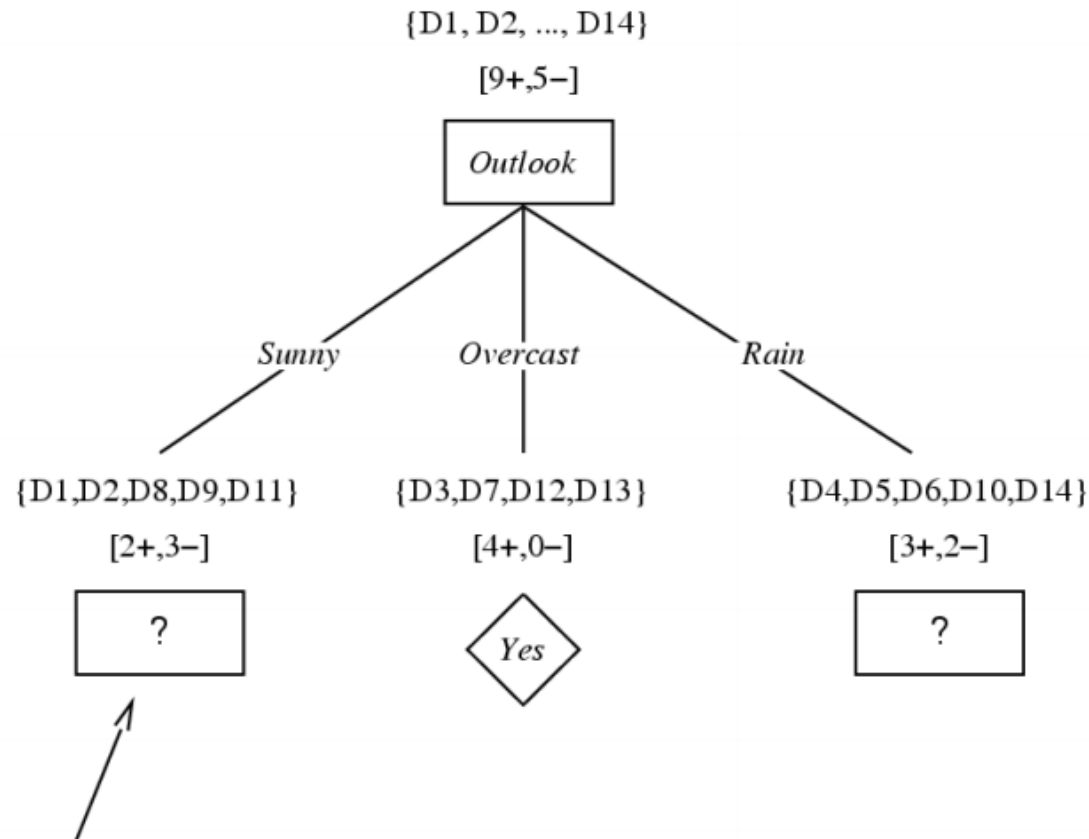
## Which attribute is the best classifier?



$$\begin{aligned}
 \text{Gain}(S, \text{Humidity}) &= .940 - (7/14).985 - (7/14).592 \\
 &= .151
 \end{aligned}$$



$$\begin{aligned}
 \text{Gain}(S, \text{Wind}) &= .940 - (8/14).811 - (6/14)1.0 \\
 &= .048
 \end{aligned}$$



Which attribute should be tested here?

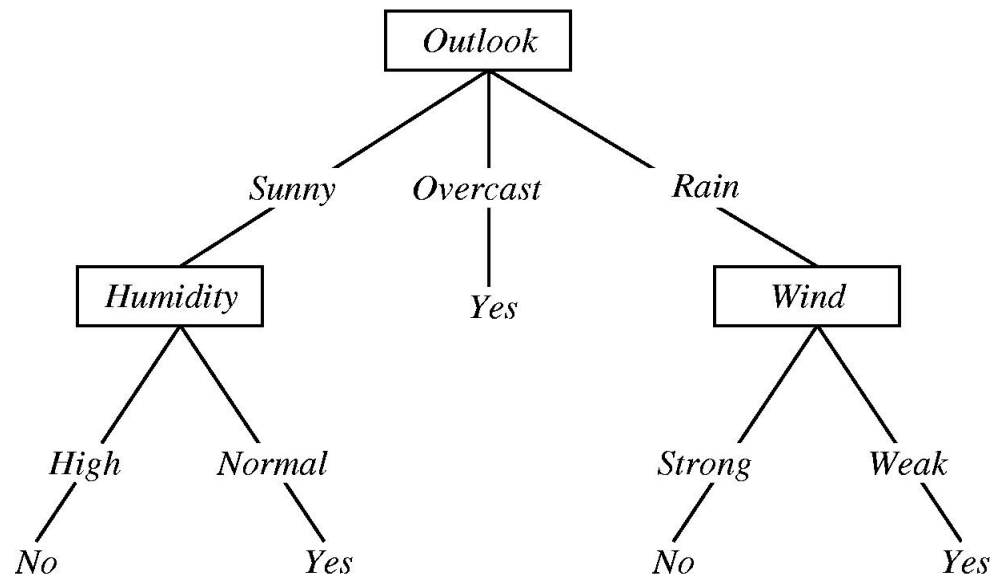
$$S_{\text{sunny}} = \{D1,D2,D8,D9,D11\}$$

$$\text{Gain}(S_{\text{sunny}}, \text{Humidity}) = .970 - (3/5) 0.0 - (2/5) 0.0 = .970$$

$$\text{Gain}(S_{\text{sunny}}, \text{Temperature}) = .970 - (2/5) 0.0 - (2/5) 1.0 - (1/5) 0.0 = .570$$

$$\text{Gain}(S_{\text{sunny}}, \text{Wind}) = .970 - (2/5) 1.0 - (3/5) .918 = .019$$

## Overfitting in Decision Trees



Consider adding a noisy training example:

*Sunny, Hot, Normal, Strong, PlayTennis=No*

What effect on tree?



# Overfitting

Consider error of hypothesis  $h$  over

- training data:  $error_{train}(h)$
- entire distribution  $\mathcal{D}$  of data:  $error_{\mathcal{D}}(h)$

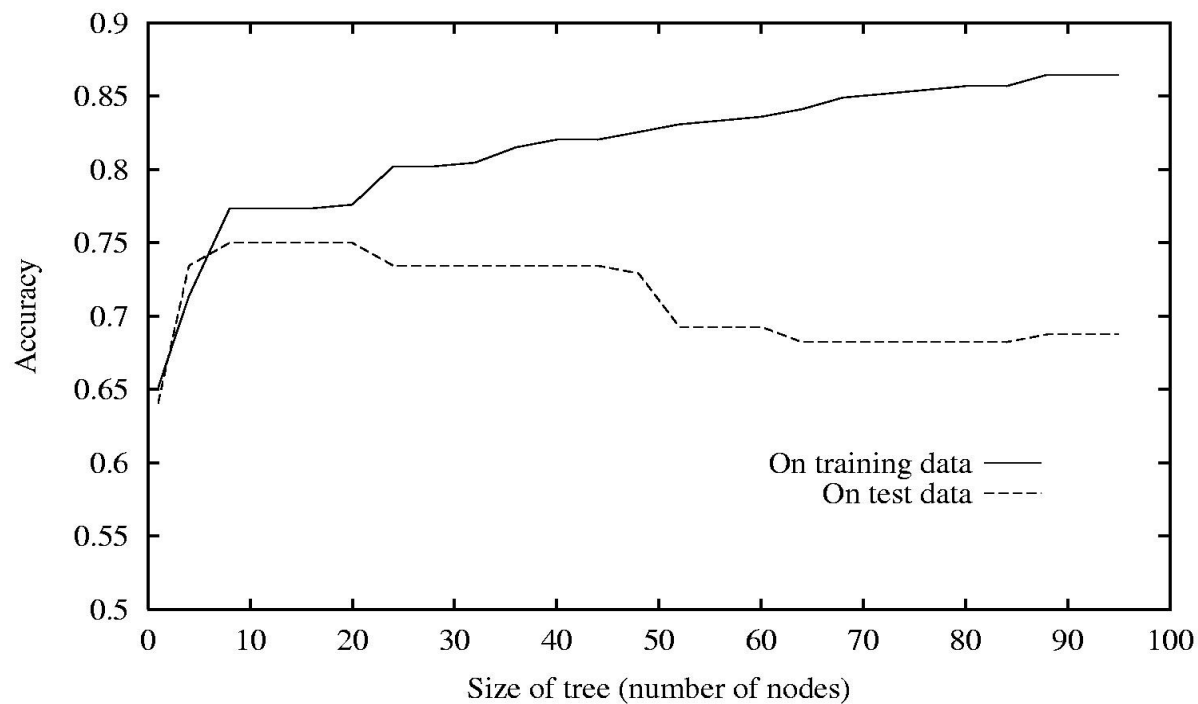
Hypothesis  $h \in H$  **overfits** training data if there is an alternative hypothesis  $h' \in H$  such that

$$error_{train}(h) < error_{train}(h')$$

and

$$error_{\mathcal{D}}(h) > error_{\mathcal{D}}(h')$$

# Overfitting in Decision Tree Learning



# Avoiding Overfitting

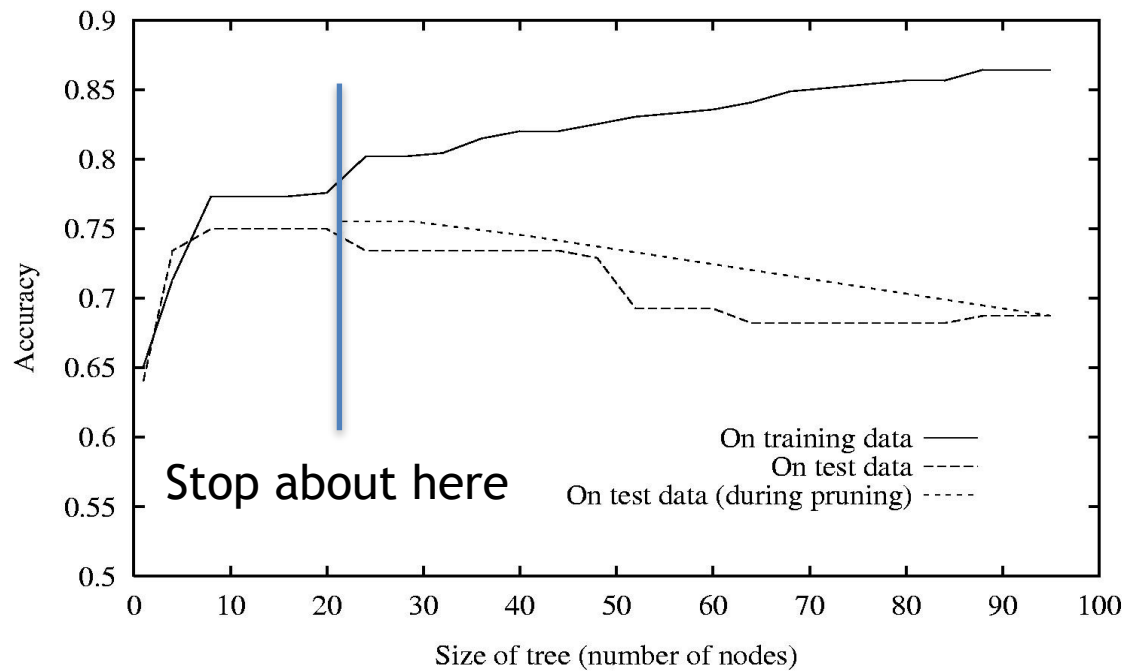
How can we avoid overfitting?

- Stop growing when data split not statistically significant
- Grow full tree, then post-prune

How to select “best” tree:

- Measure performance over training data
- Measure performance over separate validation data set
- Add complexity penalty to performance measure

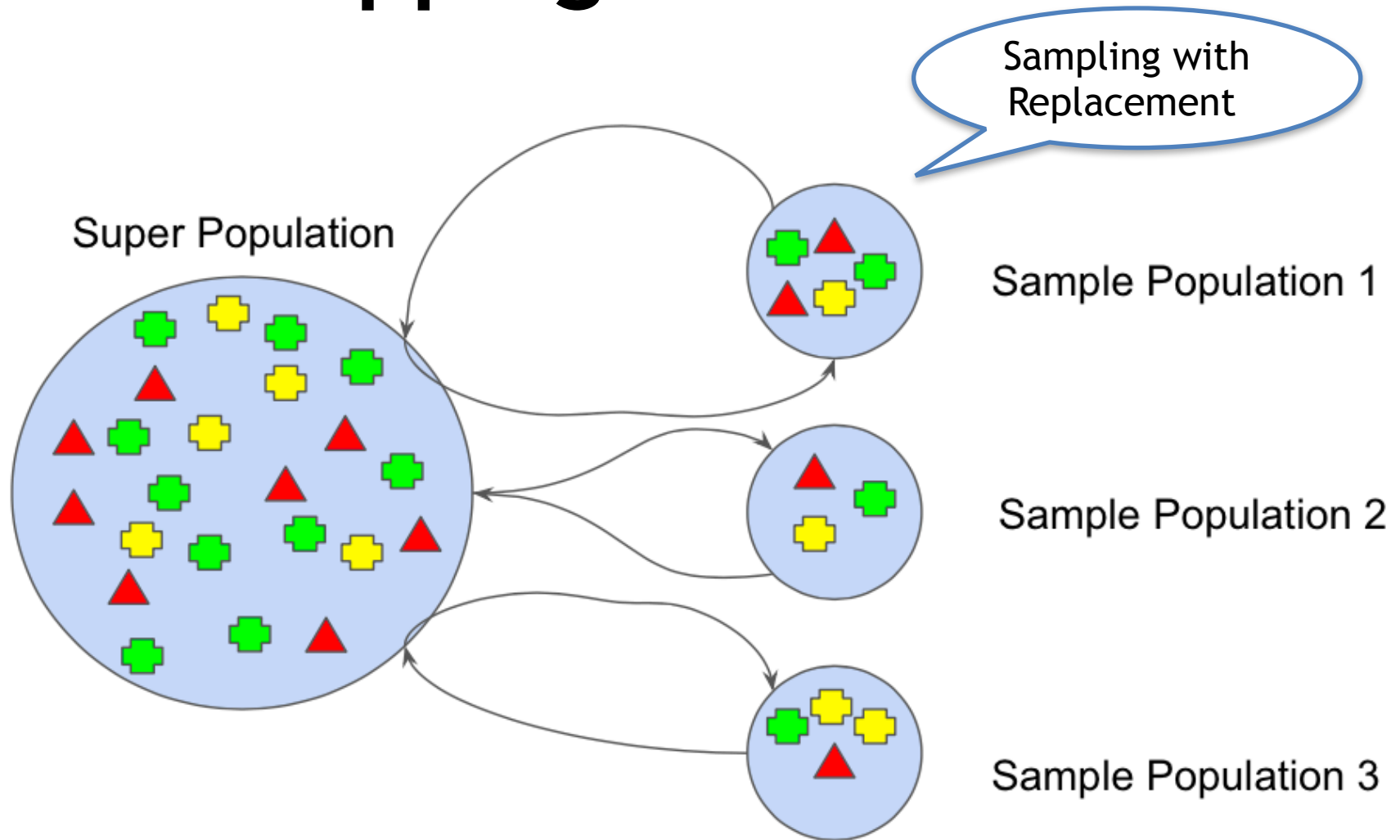
## Effect of Reduced-Error Pruning



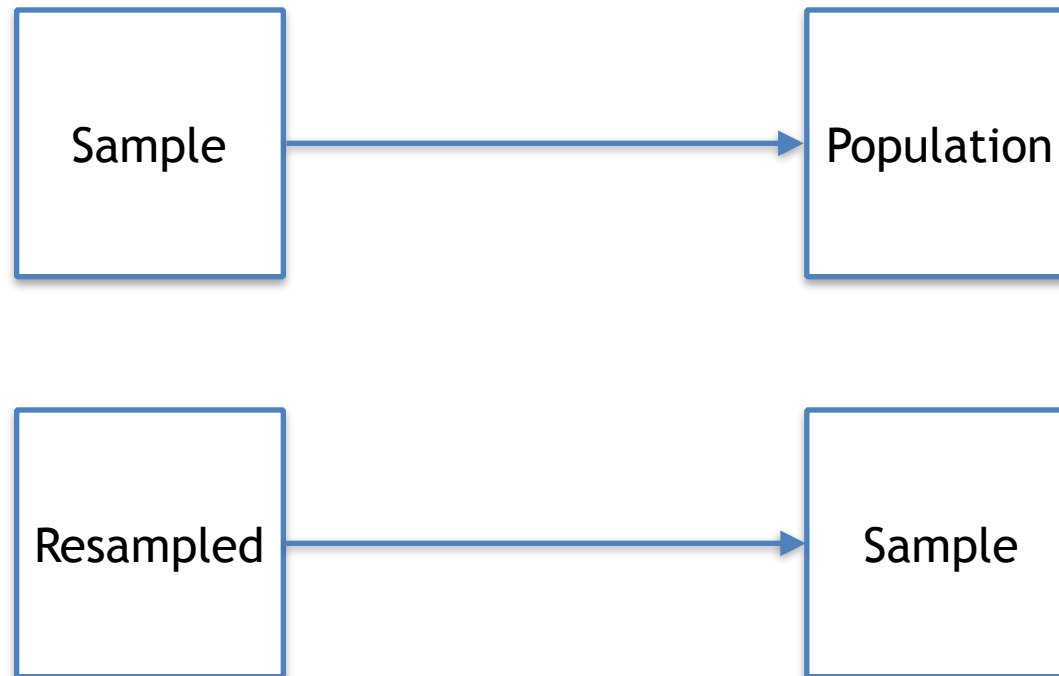
# Ensemble Learning

- Combines a set of weak hypotheses (classifiers) to create a strong classifier that obtains better performance than a single one.

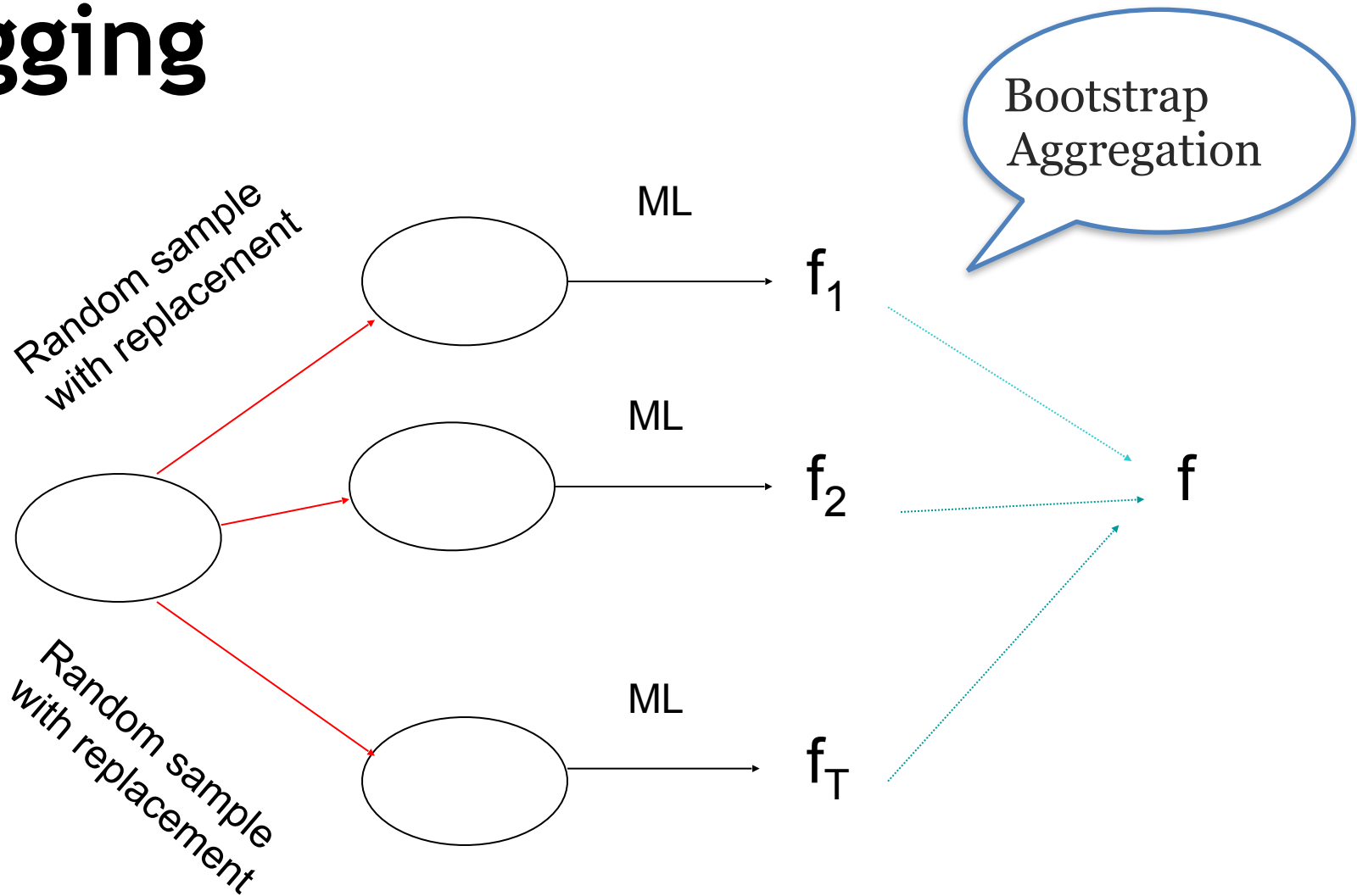
# Bootstrapping



# Learning about population from sample

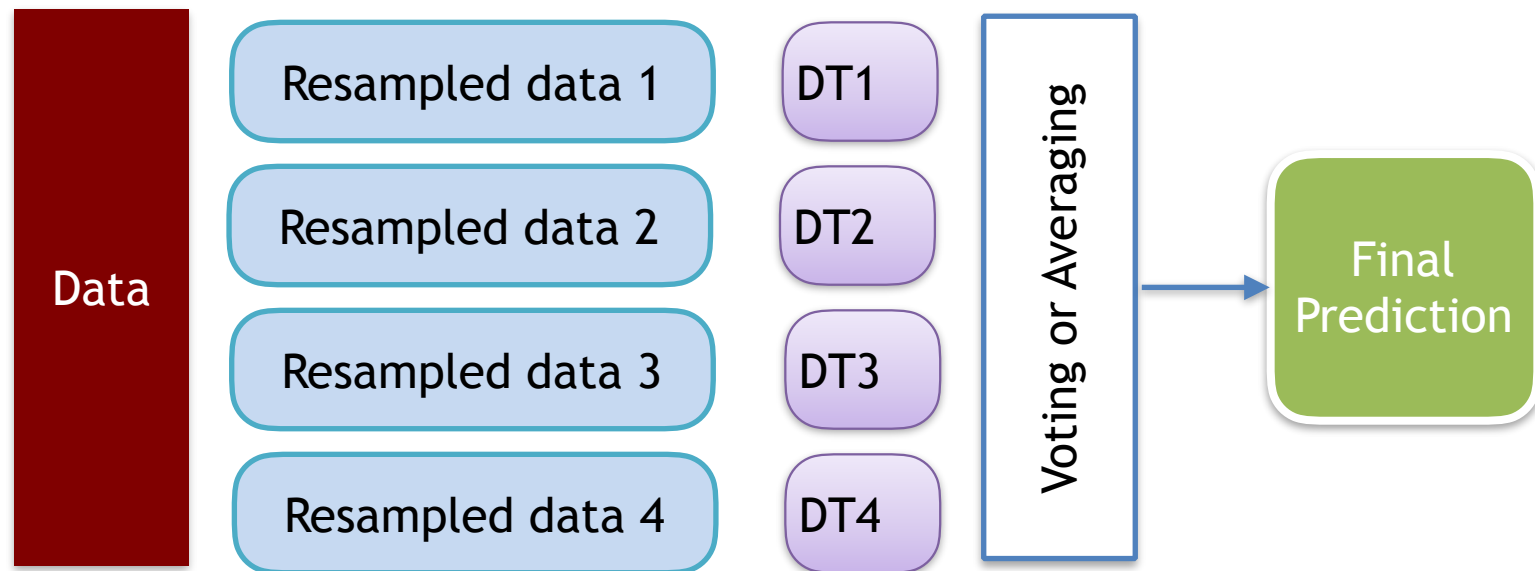


# Bagging





# Random Forest



More detailed in Python notebook