8.3 $\boxed{\#101}$: $\int \cos^{m}(x)\sin^{n}(x)\,dx$.

$u = \sin^{n-1}(x)$ $\qquad$ $dv = \cos^{m}(x)\sin(x)\,dx$

$du = (n-1)\sin^{n-2}(x)\cos(x)\,dx$ $\qquad$ $V = \dfrac{-\cos^{m+1}(x)}{m+1}$

$$\int \cos^{m}(x)\sin^{n}(x)\,dx = -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m+2}(x)\,dx$$

$$= -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)\cos^{2}(x)\,dx$$

$$= -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)(1-\sin^{2}(x))\,dx$$

$$= -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)\,dx - \frac{n-1}{m+1}\int \sin^{n}(x)\cos^{m}(x)\,dx$$

Group the term $\int \sin^{n}(x)\cos^{m}(x)\,dx$ together to the left :

$$\left(1+\frac{n-1}{m+1}\right)\int \cos^{m}(x)\sin^{n}(x)\,dx = -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)\,dx$$

$$\frac{m+n}{m+1}\int \cos^{m}(x)\sin^{n}(x)\,dx = -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)\,dx$$

Divide the equation by $\dfrac{m+n}{m+1}$:

$$\int \cos^{m}(x)\sin^{n}(x)\,dx = \frac{m+1}{m+n}\cdot\frac{-\sin^{n-1}(x)\cos^{m+1}(x)}{m+1} + \frac{m+1}{m+n}\cdot\frac{n-1}{m+1}\int \sin^{n-2}(x)\cos^{m}(x)\,dx$$

$$= -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+n} + \frac{n-1}{m+n}\int \sin^{n-2}(x)\cos^{m}(x)\,dx$$