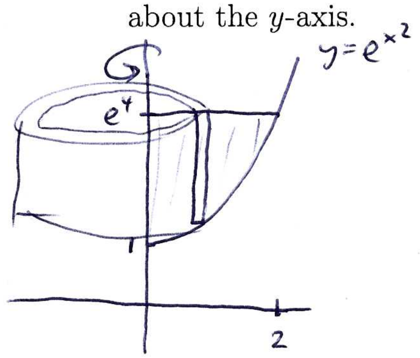
Quiz 2 Name: Key

Math 201 F01 01/31/12

1. Using the method of cylindrical shells, set-up and evaluate an integral to find the volume obtained by rotating the region bounded by the graphs of

$$y = e^{x^2}, \quad x = 0, \quad y = e^4$$

about the y-axis.



$$V \approx \sum_{\text{shells}} \Delta V = \sum (2\pi x)(e^4 - e^{x^2})\Delta x$$

$$V = \int_0^2 2\pi x(e^4 - e^{x^2})\,dx = \int_0^2 2\pi(e^4 x - x e^{x^2})\,dx$$

$$= 2\pi\left(e^4\frac{x^2}{2} - \frac{e^{x^2}}{2}\right)\Big|_0^2 = \pi\left(4e^4 - e^4 - (0-1)\right)$$

$$= \pi(3e^4 + 1)$$

2. Compute the arclength of the curve given by

$$y = \frac{x^3}{6} + \frac{1}{2x}$$

for $1 \leq x \leq 2$.

$y' = \frac{x^2}{2} - \frac{1}{2x^2}$ so $ds = \sqrt{1+(y')^2}\,dx = \sqrt{1+\left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\,dx = \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}\,dx$

$$= \sqrt{\frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}}\,dx = \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\,dx = \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)dx$$

$$\text{Arclength} = \int_1^2 ds = \int_1^2 \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)dx = \left(\frac{x^3}{6} - \frac{1}{2x}\right)\Big|_1^2 = \frac{8}{6} - \frac{1}{4} - \left(\frac{1}{6} - \frac{1}{2}\right)$$

$$= \frac{7}{6} + \frac{1}{4} = \frac{17}{12}$$