Math 314 – MATLAB Exercise 1 Name:____________________________
Due: Friday, February 22, 2013

Fitting Polynomials to Points

This exercise is intended to familiarize you with the use of MATLAB, while also getting you to think about systems of linear equations. You are encouraged to work with another student on this, but should each write and turn in your own solutions on this handout.

1. Consider the two points $(.3, .7)$ and $(.8, .5)$. To plot them in MATLAB, first set $\mathbf{x} = \begin{pmatrix} .3 \\ .8 \end{pmatrix}$ to be the vector of x-values using the command:

   $$\texttt{x} = [.3; .8]$$

   Similarly set $\mathbf{y} = \begin{pmatrix} .7 \\ .5 \end{pmatrix}$ to be the vector of y-values. Then enter the command:

   $$\texttt{plot}(\texttt{x}, \texttt{y}, '*')$$

2. Replot the points as green circles. (Type `help plot` to learn how.)

3. We want to find an equation of the form $y = mx + b$ that the two points satisfy. By plugging in the two points, find two equations in two unknowns that must be solved. Record them here:

4. Write these equations as $A\mathbf{z} = \mathbf{d}$, where A is a 2×2 matrix, $\mathbf{z}$ is a vector of unknowns, and $\mathbf{d}$ is a vector of numbers. Record your matrix equation here:

5. Enter A and $\mathbf{d}$ into MATLAB (with commands like `A=[1, 2; 3, 4]`, `d=[5; 6]`) and create the augmented matrix with the command `B=[A d]`. Then use the `rref` command to perform Gauss-Jordan elimination, and find m and b. Record the equation $y = mx + b$ that you found:

6. Find m and b two different ways using the commands `inv(A)*d`, and then `A \ d`. Do all three commands give you the same solution?

   (This last command, with the backslash, is MATLAB's way of saying "Perform elimination with back substitution to solve $A\mathbf{x} = \mathbf{d}$. For large matrices, it's the preferred command to use, but it doesn't make much difference for a 2×2 system. We'll also learn later that the backslash does something different if the system has infinitely-many solutions, to somehow pick out a 'best' one.)

7. To add a plot of the line you've found to your earlier plot, first enter the MATLAB command `hold on` so that your earlier plot will not be wiped out. Then enter the command `ezplot('m*x +b',[0 1])` to plot the line for x between 0 and 1. (In this command, you'll need to substitute in the values you've found for `m` and `b`.)

8. Do similar work to find a polynomial of the form $y = ax^2 + bx + c$ that goes through the points $(-1, 1.5)$, $(3, 32.2)$, and $(5, -42.6)$. (Enter `hold off` before beginning so that your old graph will be replaced.)

   As you work this problem record the following:

   (a) The system of equations to be solved:

   (b) The matrix equation expressing the system:

   (c) The values you found for a, b, and c:

   (d) A rough sketch (copied from the computer screen) of the graph of the parabola you found and the three points on it:

9. Explain, in terms of intersecting planes in 3-space, why if only two points were given in step (7) you would expect there to be infinitely many parabolas through them:

10. Explain, in terms of intersecting planes in 3-space, why if four points were given in step (7) you would expect there to be no parabolas through them:

11. If you had seven points in the x-y plane, what degree polynomial equation of the form $y = p(x)$ should you look for so that its graph will pass through the points? In a few sentences, explain your reasoning in terms of "hyperplanes" in a large dimensional space.

12. Find an equation of the form in step (10) through the points $(1, 4)$, $(2, -1)$, $(4, 7)$, $(5, -3)$, $(8, 1)$, $(9, -10)$, $(11, 3)$. (To do this easily it helps to know the following: If $\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$ has been entered into MATLAB as `c`, then the command `vander(c)` will return the matrix $\begin{pmatrix} c_1^{n-1} & c_1^{n-2} & \dots & c_1 & 1 \\ c_2^{n-1} & c_2^{n-2} & \dots & c_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_n^{n-1} & c_n^{n-2} & \dots & c_n & 1 \end{pmatrix}$. This is called the *Vandermonde matrix*.)

    Record the equation $y = p(x)$ you found here:

13. In the last question, you solved what appeared to be a non-linear problem in 2-d (fit a 7th degree polynomial through some points in the plane). But you did this by solving a linear problem in 8-d (find the intersection of a collection of hyperplanes in an 8-dimensional space). In a sentence or two, explain how a 2-d non-linear problem became a 8-d linear one.