Name: _Solutions_

09/26/14

1. What is the entry a_{23} of the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & -1 \end{pmatrix}?$$

$$a_{23} = \begin{pmatrix} 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 2\cdot 0 + 1\cdot 1 + 0\cdot(-1) = \boxed{1}$$

2. Solve the following, using elimination on the augmented matrix, and back substitution:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -2 \\ 0 & -6 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}.$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -1 & 1 & -2 & 1 \\ 0 & -6 & 1 & -7 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 3 \\ 0 & -6 & 1 & -7 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & -2 & 2 \end{array}\right)$$

$$x + y + z = 2$$
$$2y - z = 3$$
$$-2z = 2$$

$$z = -1; \quad 2y - (-1) = 3 \Rightarrow y = 1 \quad; \quad x + 1 + (-1) = 2 \Rightarrow x = 2$$

$$\boxed{\vec{x} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}}$$

3. Find A^{-1}, or show it doesn't exist, for

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ -1 & 2 & -1 \end{pmatrix}.$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right)$$

So $\boxed{A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 2 & -1 & 0 \end{pmatrix}}$