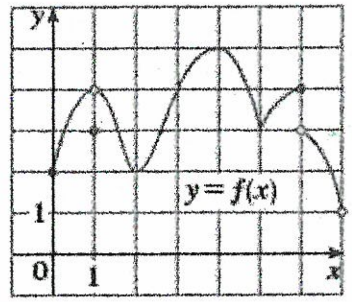
 Solutions

1. Use the graph to state the absolute and local maximum and minimum values of the function.



Absolute maximum value: $f(4)=5$

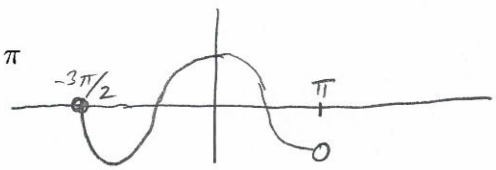
Local maximum values: $f(4)=5$
$f(6)=4$

Absolute minimum value: None

Local minimum values: $f(1)=3$
$f(2)=2$
$f(5)=3$

2. Sketch the graph f by hand and use your sketch to find the absolute and local maximum and minimum values of f.

$$f(t) = \cos(t), \quad -\frac{3\pi}{2} \le t < \pi$$



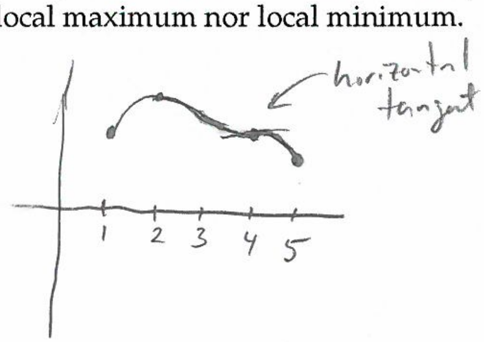
Abs. max. value: $f(0)=1$

Local max vales: $f(0)=1$

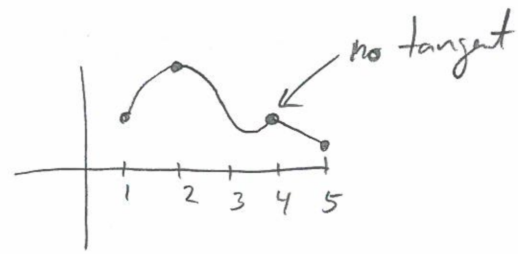
Abs. min. value: $f(-\pi)=-1$

Local min. vales: $f(-\pi)=-1$

3. Sketch a graph of a function f which is continuous on $[1,5]$, which has an absolute maximum at 2, has an absolute minimum at 5, and for which 4 is a critical number but neither a local maximum nor local minimum.



or



4. Find the absolute maximum and minimum values of f on the given interval:

$f(x) = 2x^3 - 3x^2 - 12x + 1, \qquad [-2, 3]$

$f'(x) = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

critical points $2, -1$

$f(-2) = -16 - 12 + 24 + 1 = -3$

$f(-1) = -2 - 3 + 12 + 1 = 8 \leftarrow$ abs. max value

$f(2) = 16 - 12 - 24 + 1 = -19 \leftarrow$ abs. min. value

$f(3) = 54 - 27 - 36 + 1 = -8$

5. Find the absolute maximum and minimum values of f on the given interval:

$f(x) = x^{-2} \ln x, \qquad [\frac{1}{2}, 4]$

$f'(x) = -2x^{-3} \ln x + x^{-2} \cdot x^{-1} = 0$

$\frac{-2\ln x + 1}{x^3} = 0$

$-2\ln x + 1 = 0$

$\ln x = \frac{1}{2}$

critical pt. $\quad x = e^{\frac{1}{2}} \approx 1.648$

$f(\frac{1}{2}) \approx -2.772589 \leftarrow$ abs. min value

$f(\sqrt{e}) \approx .1839397 \leftarrow$ abs. max value

$f(4) = .001341851$

6. Find the critical numbers of the function:

$h(p) = \frac{p-1}{p^2+4}$

$h'(p) = \frac{1(p^2+4) - (p-1)(2p)}{(p^2+4)^2} = \frac{p^2 + 4 - 2p^2 + 2p}{(p^2+4)^2} = \frac{-p^2 + 2p + 4}{(p^2+4)^2}$

$h'(p) = 0 \Rightarrow -p^2 + 2p + 4 = 0$

$p^2 - 2p - 4 = 0$

$p = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

$h'(p)$ DNE $\Rightarrow p^2 + 4 = 0$ never occurs

Critical pts are $1+\sqrt{5}, 1-\sqrt{5}$