Quiz 7
Math 201 F01

Name: Key

03/23/12

Determine whether the following series converge or diverge, using any appropriate method (computation of partial sums, nth term test, integral test, direct comparison test, limit comparison test, alternating series test). State which method you are using, and show enough work to indicate that you have checked any needed conditions.

1. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

Direct comparison to $\sum \frac{1}{n^{3/2}}$ which converges since $3/2 > 1$

Check $0 \le \frac{\sqrt{n}}{n^2+1} \le \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$

The series converges

2. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

Integral test $\int_2^{\infty} \frac{1}{x \ln x}\,dx = \lim_{b\to\infty} \int_2^b \frac{1}{x \ln x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{1}{u}\,du = \lim_{b\to\infty} \ln|u| \Big|_{\ln 2}^{\ln b}$

$= \lim_{b\to\infty} \ln|\ln b| - \ln|\ln 2| = \infty$

Check $f(x) = \frac{1}{x \ln x} \ge 0$ for large x & decreasing

$f'(x) = \frac{d}{dx}(x \ln x)^{-1} = -(x \ln x)^{-2}\left(\ln x + x \cdot \frac{1}{x}\right) = -\frac{\ln x + 1}{(x \ln x)^2} \le 0$

The series diverges.