Quiz 6
Math 253

Name: Key

10/09/19

1. Use the chain rule to compute $\dfrac{\partial g}{\partial u}(1,3)$ if

$$g(u,v) = f(uv, u^2 + v^2)$$

assuming g, f and the partial derivatives of f have the values given in the table:

	g	f	f_x	f_y
$(1,3)$	2	-2	2	3
$(3,10)$	-1	3	-2	-1

$g(u,v) = f(x(u,v), y(u,v))$ where $x(u,v) = uv$
$y(u,v) = u^2 + v^2$

$$\left.\frac{\partial g}{\partial u}\right|_{(1,3)} = \left.\frac{\partial f}{\partial x}\right|_{x(1,3),\,y(1,3)} \cdot \left.\frac{\partial x}{\partial u}\right|_{(1,3)} + \left.\frac{\partial f}{\partial y}\right|_{x(1,3),\,y(1,3)} \cdot \left.\frac{\partial y}{\partial u}\right|_{(1,3)}$$

$$= \left.\frac{\partial f}{\partial x}\right|_{(3,10)} \left. v\right|_{(1,3)} + \left.\frac{\partial f}{\partial y}\right|_{(3,10)} \left. 2u\right|_{(1,3)} = (-2)\cdot 3 + (-1)\cdot \quad = -8$$

2. Compute the directional derivative of the function

$$g(x,y,z) = \frac{x}{y} + xz^2 + \ln(z+x)$$

at the point $(-2,1,3)$ in the direction toward the origin.

$$\nabla g = \left< \frac{1}{y} + z^2 + \frac{1}{z+x},\ -\frac{x}{y^2},\ 2xz + \frac{1}{z+x} \right>$$

$$\nabla g(-2,1,3) = \left< 11, 2, -11 \right>$$

$$\vec{u} = \frac{-\left<-2,1,3\right>}{\sqrt{4+1+9}} = \frac{1}{\sqrt{14}}\left<2,-1,-3\right>$$

$$D_{\vec{u}}g = \left<11,2,-11\right> \cdot \frac{1}{\sqrt{14}}\left<2,-1,-3\right> = \frac{53}{\sqrt{14}}$$