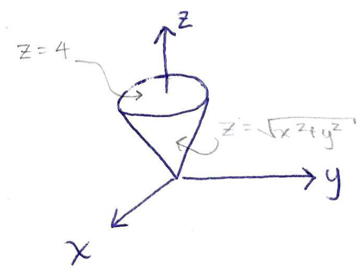
MATH 202 Name :________________________________

Midterm 3 April 25, 2012

Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized.

1. (12 pts.) Set up, but **do not integrate**, a triple integral **in cylindrical coordinates** that computes the volume of the solid that lies below $z = 4$ and above $z = \sqrt{x^2 + y^2}$. See figure.



2. (12 pts.) Compute the iterated integral:

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\, dx dy$$

3. (14 pts.) A solid sphere B of radius 2 centered at the origin has charge density

$$\rho(x, y, z) = e^{(x^2+y^2+z^2)^{\frac{3}{2}}} \text{ coulombs/cm}^3$$

at any point (x, y, z) in the sphere in cm. Compute the total electrical charge of the solid B. Include units.

4. (14 pts.) Use Green's theorem to evaluate the integral

$$\oint_C (e^{\cos(x)} - \frac{1}{3}y^3)\, dx + (\ln y + \frac{1}{3}x^3)\, dy$$

where C is the circle of radius 3 centered at the origin oriented in the counterclockwise direction.

5. (20 pts.) Consider the vector field with continuous partial derivatives defined on all of $\mathbb{R}^2$,

$$\mathbf{F}(x, y, z) = \left\langle\, ye^x + \sin\left(\frac{\pi}{2}y\right),\, e^x + \frac{\pi}{2}x\cos\left(\frac{\pi}{2}y\right) + 2y \,\right\rangle.$$

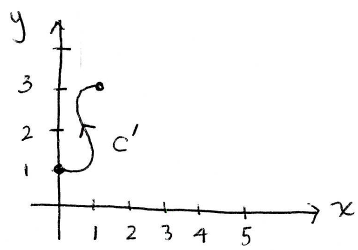
(a) (8 pts.) By finding a potential function f, prove that $\mathbf{F}$ is conservative.

(b) (7 pts.) Supposing $\mathbf{F}$ represents a force field, compute the amount of work done by $\mathbf{F}$ on a particle moving along the path

$$\mathbf{r}(t) = \langle\, t,\, 2t + 1 \,\rangle, \quad 0 \le t \le 1.$$

(Assume that $\mathbf{F}$ is measured in Newtons and distances are measured in meters.)

(c) (5 pts.) Now compute the work done by $\mathbf{F}$ on a particle moving along the path C' pictured. Explain your answer.



6. (18 pts.) Consider the vector field

$$\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - x^2\,\mathbf{k}$$

(a) (7 pts.) Compute $\operatorname{curl}\mathbf{F}$.

(b) (7 pts.) Compute $\operatorname{div}\mathbf{F}$.

(c) (4 pts.) Suppose the vector field $\mathbf{F}$ represents the velocity field for some fluid. Compute the divergence of $\mathbf{F}$ at the point $(1, 1, 1)$ and indicate what $\operatorname{div}\mathbf{F}(1, 1, 1)$ tells you about the net fluid flow at $(1, 1, 1)$.

7. (10 pts.)

True or False? If the following statements are correct, mark them 'True.' If they are false, give corrected versions.

(a) If $-C$ denotes 'C backwards,' then $\int_C F \cdot d\mathbf{r} = \int_{-C} F \cdot d\mathbf{r}$.

(b) If $-C$ denotes 'C backwards,' then $\int_C f\,ds = \int_{-C} f\,ds$.