CHAPTER 6 AND 8 REVIEW

6.1 Areas Between Curves

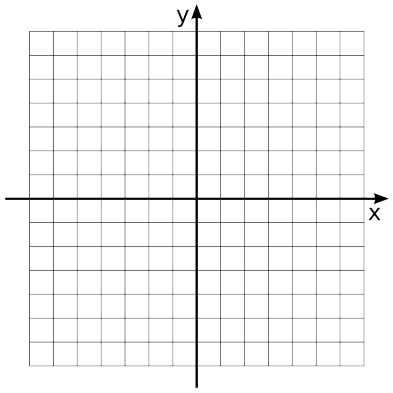
Example 1: Set up, but do not solve, an integral (or integrals) that finds the area between $y = \sqrt{x}$, $y = x^3$, $x = 0$ and $x = 2$. Begin by graphing the region bounded by these curves.

6.2 The Disc Method & 6.3 The Shell Method

- When we use the **disc method** of Section 6.2 we slice _____ to the axis of rotation. Here, we need to find the area of a general slice, $A(x)$ or $A(y)$ and sum over all of our slices.
- When you use the **shell method** of Section 6.3 we slice _____ to the axis of rotation. Here you need to find the _____ and the _____ of a slice.

Example 2: Sketch the region bounded by the curves $y = 6 - 2x - x^2$ and $y = x + 6$. Then, **SET UP BUT DO NOT SOLVE** an integral that would find the volume when this region is rotated about the following axes. Specify the method you use.

(a) x-axis



(b) y-axis

(c) $y = -2$

(d) $x = 5$

Example 3: Consider the region bounded by the curves $y = 6 - 2x - x^2$ and $y = x + 6$ from Example 2. Suppose that this region is the base of a solid and the cross sections of your solid perpendicular to the x-axis are rectangles with height 3 times the base. Set up, but do not solve, an integral that gives the volume of this solid.

6.4 Work

Example 4: A force of 100 N is required to hold a spring stretched from its natural length of 10 cm to a length of 20 cm. How much work is done in stretching the spring from 20 cm to 30 cm?

Example 5: A heavy rope, 50 feet long, weighs 100 lbs (or 2 a lb per foot) and is lying on the ground. How much work is done in pulling the rope so that it is hanging completely stretched out?

6.5 Average Value

Example 6: n a certain city, the temperature (in $^\circ$ F) t hours after 9 AM is modeled by the function $T(t) = 50 + 20\sin\left(\frac{\pi t}{12}\right)$.

(a) Find the average temperature during the period from 9 AM to 9 PM. Give an exact answer and an answer rounded to the nearest thousandth.

(b) Explain why the Mean Value for integrals applies to the equation for $T(t)$ on any interval $[a, b]$ and find the time $t = c$, such that $T(c)$ equals the average value from part (a). Give an exact and approximate answer with proper units.

8.1 Arc Length

Example 7: Find the arc length of the curve $y = 1 + \ln(\cos x)$, $0 \leq x \leq \pi/6$.

8.2 Surface Area

Example 8: The curve $y = x^2$ is rotated about the following axes. Set up an integral that finds the area of the resulting surface for $0 \leq x \leq 2$.

(a) x-axis.

(b) y-axis.

8-3 Moments, Centers of Mass and Centroids

Example 9: Find the centroid of the region bounded by $y = 5\sin x$ and $y = 0$ for $0 \leq x \leq \pi$.