

Recall the guidelines:

- A. domain
- B. intercepts
- C. symmetry
- D. asymptotes
- E. increase/decrease (and critical numbers)
- F. local maxima/minima
- G. concavity (and inflection points)
- H. sketch the graph

1. Sketch the graph by applying the guidelines:

$$y = \frac{\sin x}{2 + \cos x}, \quad 0 \leq x \leq 2\pi$$

A.

B. $x=0 \Rightarrow y=0$

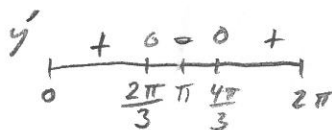
$$y=0 \Rightarrow x=0, \pi, 2\pi$$

C. none (period is 2π)D. none (continuous since $2 + \cos x \neq 0$)

$$E, F. y' = \frac{(\cos x)(2 + \cos x) - (\sin x)(-\sin x)}{(2 + \cos x)^2}$$

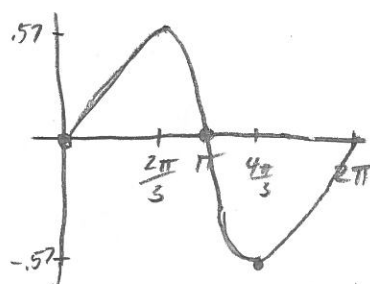
$$= \frac{2\cos x + \cos^2 x + \sin^2 x}{(2 + \cos x)^2} = \frac{2\cos x + 1}{(2 + \cos x)^2}$$

$$y' = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$$



$$G. y'' = \frac{(-2\sin x)(2 + \cos x)^2 - (2\cos x + 1)(2)(2 + \cos x)(-\sin x)}{(2 + \cos x)^4}$$

$$= \frac{(2 + \cos x)(-2\sin x)[-2 - \cos x + 2\cos x + 1]}{(2 + \cos x)^4} = \frac{(2 + \cos x)(-2\sin x)(\cos x - 1)}{(2 + \cos x)^4}$$



$$f\left(\frac{2\pi}{3}\right) = \frac{\sin\left(\frac{2\pi}{3}\right)}{2 + \cos\left(\frac{2\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{2 + (-\frac{1}{2})}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3} \approx .57$$

$$f\left(\frac{4\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$

concavity determined by sign of $(2\sin x)(\cos x - 1)$

2. Sketch the graph by applying the guidelines:

$$y = \frac{1}{x^2 - 4} = (x^2 - 4)^{-1}$$

A. domain $x \neq \pm 2$

B. $y = 0$ never

$$x = 0 \quad y = -\frac{1}{4}$$

C. even $f(x) = f(-x)$

$$D. \lim_{x \rightarrow \infty} \frac{1}{x^2 - 4} = 0$$

$$\lim_{x \rightarrow 2^+} \frac{1}{x^2 - 4} = \infty$$

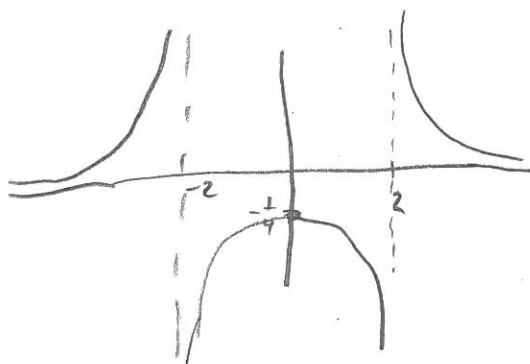
$$\lim_{x \rightarrow 2^-} \frac{1}{x^2 - 4} = -\infty$$

E, F. $y' = \frac{-1}{(x^2 - 4)^2} < 0$ always where defined

$$G. y'' = \frac{2}{(x^2 - 4)^3} \text{ not defined at } \pm 2$$

$$y'' \quad + \quad \text{DNE} \quad - \quad \text{DNE} \quad +$$

$$\quad \quad \quad -2 \quad \quad \quad 2$$



3. Sketch the graph by applying the guidelines:

$$y = \frac{x}{\sqrt{x^2 + 1}}$$

A. domain $(-\infty, \infty)$

$$B. y = 0 \Rightarrow x = 0$$

$$x = 0 \Rightarrow y = 0$$

C. odd $f(x) = -f(-x)$

$$D. \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{x}{x\sqrt{1 + \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x^2}}} = 1$$

$$E, F. y' = \frac{\sqrt{x^2 + 1} - x \cdot \frac{1}{2}(x^2 + 1)^{-1/2} \cdot 2x}{x^2 + 1}$$

$$= \frac{(x^2 + 1) - x^2}{(x^2 + 1)\sqrt{x^2 + 1}} = \frac{1}{(x^2 + 1)^{3/2}} > 0 \text{ always}$$

$$G. y'' = -\frac{3}{2}(x^2 + 1)^{-5/2}(2x)$$

$$= \frac{-3x}{(x^2 + 1)^{5/2}} \quad y'' \quad + \quad 0 \quad -$$

$$\quad \quad \quad 0$$

