MATH 202 Name :_____

Midterm 2 March 23, 2012

Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized.

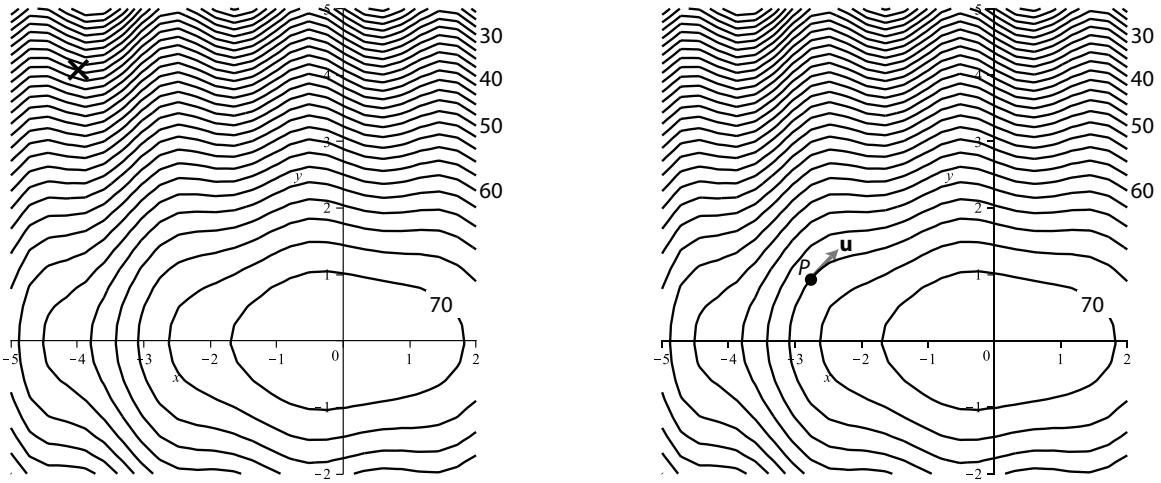
1. (15 pts.)

   (a) (7 pts.) Prove that $\lim_{(x,y)\to(0,0)} \dfrac{3xy}{x^2+y^2}$ does not exist.

   (b) (8 pts.) A particle moves along a curve in 3-space given by $\mathbf{r}(t) = \langle 4\sin(3t), 3t, 4\cos(3t)\rangle$ meters, where t is measured in seconds. Give and evaluate a definite integral that computes the distance traveled by the particle between time $t = 0$ seconds and $t = 2\pi$ seconds.

2. (8 pts.) Suppose that $w = xy^2 - x^2$ and that at time $t = 0$, $(x, y) = (1, 2)$, $\frac{dx}{dt}\big|_{t=0} = -5$, and $\frac{dy}{dt} = 3$. Use the Multivariable Chain Rule to compute $\dfrac{dw}{dt}$ when $t = 0$.

3. (12 pts. – 6 pts. each) It is well-known that mosquitoes are drawn to CO_2. Below is a contour plot for the concentration $C(x, y)$ of CO_2 at positions (x, y) in feet on the grid shown.



(a) (b)

(a) In Figure (a) on the left, sketch the path of a mosquito placed on the X at $(-4, 4)$ as it continuously moves to maximize its exposure to CO_2. Briefly justify the reasoning for drawing the path you did.

(b) In Figure (b) on the right, give the value of the directional derivative of $C(x, y)$ in the direction indicated by the vector $\mathbf{u}$ at the point P indicated with a dot. Explain your answer briefly.

4. (10 pts.) Find the equation of the tangent plane to the ellipsoid $x^2 + 2y^2 + 9z^2 = 31$ at the point $(2, 3, 1)$.

5. (10 pts.) Electrical power P in watts is given by

$$P = \frac{V^2}{R},$$

where V is voltage and R is resistance in ohms.

(a) Give a formula for the total differential dP for power.

(b) If $V = 120$ volts is applied to a 2000-ohm resistor, compute the total differential dP for power.

$dP =$ _____

6. (17 pts.) Suppose the elevation above sea level in tens of meters is given by the function

$$h(x, y) = \frac{y^2}{5 - x} \text{ tens of } m,$$

and a hiker is located at the position $(x, y) = (4, 1)$.

(a) (6 pts.) In what direction from $(4, 1)$ should the hiker move to increase his/her elevation the most?

(b) (6 pts.) If the hiker moves in the direction indicated by the vector $\mathbf{v} = \langle 1, -1 \rangle$, what is the rate of change of the hiker's elevation?

(c) (5 pts.) Using your answer to part (b), do you expect the hiker's elevation to rise or fall as the hiker moves in the direction given by $\mathbf{v}$?

7. (16 pts.) Consider the function $f(x, y) = -x^3 + 6xy - 3y^2 + 1$.

   (a) (8 pts.) Find all critical points of $f(x, y)$.

   (b) (8 pts.) Use the second derivative test to determine if the critical points are local maxima, local minima, saddle points or if there is not enough information to tell.

8. (12 pts.) **Use Lagrange multipliers** to find the maximum value of $f(x, y) = xy$ where $x > 0$ and $y > 0$, subject to the constraint $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$.

The maximum value is _____, and occurs at $(x, y) =$ _____.