Mathematics 265: Introduction to Mathematical Proofs
Spring 2020

Instructor: John Rhodes

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Office Hours: M 9:30-10:30, W 1:00-2:00, and by appointment

Web page: https://jarhodesuaf.github.io/M265.html

Prerequisites: Math 252 Calculus II

Credit Hours: 3.0

Text: Book of Proof, 3rd ed., by R. Hammack, free pdf at
https://www.people.vcu.edu/~rhammack/BookOfProof/
or available on Amazon, Barnes & Noble, etc.

Class Meetings: MWF 11:45-12:45, in 409 Gruening

Midterm Exams: February 26, April 1, in class

Final Exam: 10:15 a.m. – 12:15 p.m., Wednesday, April 29

Course overview and learning outcomes:

This course is intended to give you an understanding of the role of rigorous, logical arguments in mathematics, and develop your abilities to construct and present your own.

The concept of a 'proof' is unique to mathematics, and is what distinguishes the field from the sciences, in which (unprovable) mathematical descriptions may be given of empirical phenomena. At least in principal, a proof is simply a logical argument that certain assumptions must imply some conclusion. Once a statement has been proved mathematically, a reasonable person will view it as 'true.'

In practice, though, things are more complicated, and knowing the rules of logic isn't enough. The rough idea for a proof often comes from some vague intuition of why something should be true. Gradually, the intuition is fleshed out into a more formal argument. Along the way, gaps may be discovered in the reasoning, and changes may need to be made to fill them. Even what is considered to constitute a complete proof can vary, depending on the background of the reader. Although in principal we should be able to write proofs whose validity could be checked by a computer, human proofs often leave out many of the smallest details. A proof written for an expert in a field may leave out so many details that a non-expert cannot even follow the argument.

A very good way to view a mathematical proof, which will be emphasized in this course, is that it is a form of communication. It is intended not only to establishes that some statement is true, but also, and equally importantly, to explain why it is true to another human. Thus the form and level of detail needed for a proof must take into account the intended audience. Over many years, the mathematical community has developed approaches and styles of writing that work well for conveying arguments; you will be learning to use these.

During the course of the semester, you will learn both logical and conceptual tools for constructing your own mathematical proofs. This includes foundational material (basic set theory, logic) and more elaborate concepts that arise in many mathematical settings (equivalence relations, functions, cardinality), as well as a number of standard argument structures (direct proof, proof by contrapositive, proof by contradiction, proof by induction). All of these will appear again in more advanced mathematics courses, and are essential to a mathematics major.

You will also gain experience in evaluating proofs for correctness and quality. To facilitate this, you and your classmates will present your arguments to each other, sometimes by speaking in class, and sometimes through anonymized copies of homework. You should be willing to give and accept constructive criticism in a respectful and mature way. While it isn't always pleasant, there is no better way to learn how to communicate mathematical ideas than hearing from a peer how you have failed to get a point across.

Mechanics of the course:

Class meetings will be a mixture of the instructor lecturing, students presenting their own work, and interactive group work of various forms. You should bring your textbook regularly, as it will be needed for some of the group activities.

When I am presenting material at the board, I recommend that you take notes. I will also ask for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers, but I do expect you to be actively following and participating — that makes the class more interesting for us all.

When students present work, or you are working in small groups, you are expected to participate in discussions. If you are presenting your own work, you should not worry about being evaluated. While clear and correct arguments are the goal, the point of the discussion is to determine what is clear and correct and what is not. An interesting 'mistake' may be more valuable to the class than a perfect proof.

Class attendance and participation. While I will not formally take roll, missing class can have a severe impact on your grade through the participation component. If you must miss class due to sickness, or other unavoidable reasons, you should be sure to inform me.

Homework assignments will be given in class, but also posted on the course web page soon after class is over.

Homework problems from the text will usually be assigned daily, posted on the class website, and collected each Monday. **Assignments are due in class** but will be accepted up until the time I leave the office that day. Beyond that, late homework will only be accepted if there are serious extenuating circumstances.

I encourage you to work with others on the homework, but you must *write up solutions independently*. You will learn nothing from simply copying someone's solution. The best approach is 1) make a first attempt at all problems alone, 2) meet with a classmate to work out difficult issues, 3) write up complete solutions alone.

The entire homework assignment will be checked to be sure you have attempted everything. Selected problems will be graded completely.

Some of the homework will be required to be typed, using the mathematical typesetting system LaTeX. There are many free implementations of this for all types of computers, so that you may use any computer you have access to, including ones in the DMS computer lab. You will receive instruction on LaTeX's use in the course of the semester. If you wish, you can use it for all your homework, but only designated problems must be submitted typeset. I will also create a Dropbox folder where you will be asked to submit your LaTeX files.

Missed examinations or assignments that are not approved in advance will result in a zero grade on that exam or assignment. No make-ups will be allowed except in extreme circumstances (e.g., family death, documented illness, etc.). Notifying me by email or a note that you will miss an exam or due date is not sufficient for advance approval; you must speak with me in person if you believe you have a valid excuse.

Calculators will be useless in this course, except as paperweights.

Auditing of this course will only be allowed for those who agree to participate fully, as evidenced by completion of homework, midterm exam, and class participation.

Grades:

Your performance will be evaluated based on 25% daily homework, 5% class participation, 20% each midterm exam, 30% final exam.

Course grades will be determined according to the following cutoffs:

$$A \geq 90\%,$$
$$B \geq 80\%,$$
$$C \geq 70\%,$$
$$D \geq 60\%.$$

The top 3 points of each grade range will receive a '+', and the bottom 3 points a '–'.

I reserve the right to move the cutoff points downward if particular assignments or exams turn out to be unexpectedly difficult. Note that you are not in competition with your peers – everyone in the class may get an $A+$, or everyone may get an F.

University and Department Policies:

Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams which can be found at

`http://www.dms.uaf.edu/dms/Policies.html`.

Disabilities:

If you have any disabilities that I should know about, you should bring them to my attention soon so that we can work with the Office of Disability Services to set up any necessary accommodations.

Tentative Schedule

Week 1–3	Jan. 13 – Jan. 31 NO CLASS, Jan. 20 (AK Civil Rights Day)	Chapters 1, 2
Week 4–6	Feb. 3 – Feb 21	Chapters 4, 5, 6
Week 7–8	Feb 24 – March 6 MIDTERM EXAM (Chap 1-6), Feb 26	Chapter 7, 8, 9
SPRING BREAK	March 9 – 13	
Week 9–10	March 16 – March 27	Chapter 10, 11
Week 11–12	March 30 – April 10 MIDTERM EXAM (Chap 7-11), April 1	Chapter 12, 14
Week 13–14	April 13 – April 27 FINAL EXAM, April 29 10:15 a.m. – 12:15 p.m.	Chapter 13, Additional topics (if time permits)