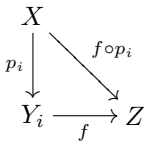
Math 651 – Homework exercises
02/15/2013

1. Let $p : X \to Y$ be a surjective continuous map. Show that p is a quotient map if and only if it takes saturated closed sets to saturated closed sets.

2. Let $p : X \to Y$ be a quotient map. Show that if A is a saturated open set or a saturated closed set, then $p|_A : A \to p(A)$ is a quotient map.

3. (The Characteristic Property of the Quotient Topology is characteristic.) Let $p : X \to Y$ be a surjective function, where X is a topological space. Consider two topologies on Y, denoting the resulting topological spaces by Y_1, Y_2, and the functions $p_i : X \to Y_i$

   Suppose both Y_1 and Y_2 have the property that for every topological space Z and every $f : Y_i \to Z$, that f is continuous if, and only if, $f \circ p_i : X \to Z$ is. Show that $Y_1 = Y_2$.



   Thus, since the quotient topology has this property, it is the unique topology on Y with this property.