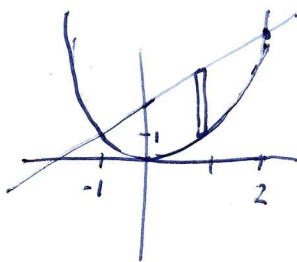1. A cable that weighs 2 lbs/ft is used to lift 800 lbs of coal up a mineshaft 500 ft deep. Find the work done in lifting both the cable and the coal.
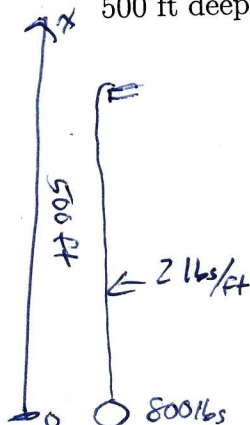
$$W \approx \sum \Delta W = \overbrace{800 \cdot 500}^{\text{coal}} + \sum 2 \cdot \Delta x \cdot (500 - x)$$

$$W = 400,000 + \int_0^{500} 2(500 - x)\, dx$$

$$= 4 \times 10^5 + \left(-(500-x)^2 \Big|_0^{500} \right) = 4 \times 10^5 + (500)^2$$

$$\boxed{= 6.5 \times 10^5 \ \text{ft-lbs.}}$$

2. Find the x-coordinate of the centroid of the region in the plane bounded by

$$y = x + 2, \quad y = x^2.$$

$$\bar{x} = \frac{M_y}{m} = \frac{\displaystyle\int_{-1}^{2} x((x+2) - x^2)\, dx}{\displaystyle\int_{-1}^{2} ((x+2) - x^2)\, dx}$$

$$\text{Numerator} = \int_{-1}^{2} -x^3 + x^2 + 2x\, dx = -\frac{x^4}{4} + \frac{x^3}{3} + x^2 \Big|_{-1}^{2} = \left(-4 + \frac{8}{3} + 4\right) - \left(-\frac{1}{4} - \frac{1}{3} + 1\right)$$

$$= 2 + \frac{1}{4} = \frac{9}{4}$$

$$\text{Denominator} = \int_{-1}^{2} -x^2 + x + 2\, dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2} = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= 5 - \frac{1}{2} = \frac{9}{2}$$

$$\bar{x} = \frac{\frac{9}{4}}{\frac{9}{2}} = \frac{2}{4} = \frac{1}{2}$$