Exam 1 Name: Key

Math 201 F01 02/27/12

1. (50 pts. – 10 pts. each) Evaluate the following indefinite integrals, showing your work.

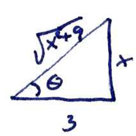
(a) $\int \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} dx = x \arctan x - \frac{1}{2}\int \frac{1}{u} du$

$u = \arctan x \quad du = \frac{1}{1+x^2} dx \qquad u = 1+x^2$

$dv = dx \quad v = x \qquad du = 2x\,dx$

$= x\arctan x - \frac{1}{2}\ln|u| + C = x \arctan x - \frac{1}{2}\ln|1+x^2| + C$

(b) $\int \frac{1}{(x^2+9)^{3/2}} dx = \int \frac{3\sec^2\theta}{(3\sec\theta)^3} d\theta = \frac{1}{9}\int \frac{1}{\sec\theta} d\theta = \frac{1}{9}\int \cos\theta \, d\theta$



$\frac{x}{3} = \tan\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta \, d\theta$

$\frac{3}{\sqrt{x^2+9}} = \cos\theta$

$\sqrt{x^2+9} = 3\sec\theta$

$= \frac{1}{9}\sin\theta + C$

$= \frac{1}{9}\frac{x}{\sqrt{x^2+9}} + C$

(c) $\int \frac{x}{\sqrt{36-x^2}} dx = -\frac{1}{2}\int u^{-1/2} du = -u^{1/2} + C$

$u = 36 - x^2$

$du = -2x\,dx$

$= -\sqrt{36-x^2} + C$

(d) $\int \sec^3 2x \tan^3 2x \, dx = \frac{1}{2}\int \sec^3 u \tan^3 u \, du = \frac{1}{2}\int \sec^2 u \tan^2 u \, (\sec u \tan u) du$

$u = 2x$

$du = 2dx$

$= \frac{1}{2}\int \sec^2 u(\sec^2 u - 1)(\sec u \tan u) du = \frac{1}{2}\int v^2(v^2-1) dv$

$v = \sec u$

$dv = \sec u \tan u$

$= \frac{1}{2}\int v^4 - v^2 \, dv = \frac{1}{2}\left(\frac{v^5}{5} - \frac{v^3}{3}\right) + C$

$= \frac{1}{10}\sec^5 u - \frac{1}{6}\sec^3 u + C$

$= \frac{1}{10}\sec^5(2x) - \frac{1}{6}\sec^3(2x) + C$

Note: This problem appeared on Quiz 5.

(e) $\int \frac{2x^2 - 2x - 2}{x^3 - x} dx$

$x^3 - x = x(x^2-1) = x(x-1)(x+1)$

so

$$\frac{2x^2-2x-2}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$2x^2-2x-2 = A(x-1)(x+1) + Bx(x+1) + C(x)(x-1)$

$2x^2-2x-2 = (A+B+C)x^2 + (B-C)x + (-A)$

so $A+B+C = 2$

$B - C = -2$

$-A = -2 \Rightarrow A = 2$

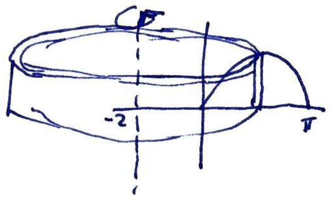
$\Rightarrow \left.\begin{array}{l} B+C=0 \\ B-C=-2 \end{array}\right\} \Rightarrow 2B = -2 \Rightarrow B = -1$

$C = 1$

$$\int \frac{2x^2-2x-2}{x^3-x} dx = \int \left(\frac{2}{x} + \frac{-1}{x-1} + \frac{1}{x+1}\right) dx = 2\ln|x| - \ln|x-1| + \ln|x+1| + C$$

$$= \ln\left|\frac{x^2(x+1)}{x-1}\right| + C$$

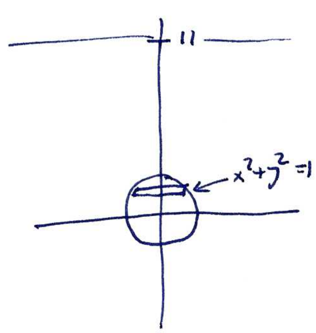
2. (12 pts.) Set up, but **do not evaluate**, an integral to compute the volume of revolution obtained by rotating the region bounded by $y = \sin x$ and $y = 0$ with $0 \le x \le \pi$ about the axis $x = -2$.



$$V \approx \underbrace{\sum \Delta V}_{\text{cylindrical shell}} = \sum 2\pi(x+2)\Delta x \sin x$$

$$V = \int_0^{\pi} 2\pi(x+2)\sin x \, dx$$

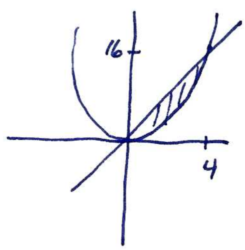
3. (13 pts.) A round window is to be located in a vertical wall of an underwater observatory. If the window has radius 1 ft and is located with its top 10 ft below the surface of the water, find the force on the window due to the water. (The weight of water is 62.4 lbs/ft^3.) Give your answer as an integral, but **do not evaluate** it.



$$F \approx \sum 62.4 \underbrace{(11-y)}_{\text{depth}} \underbrace{(2\sqrt{1-y^2})\Delta y}_{\text{area}}$$

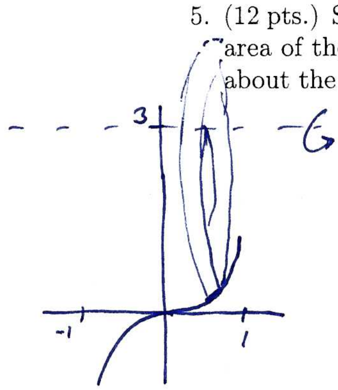
$$F = \int_{-1}^{1} 62.4(11-y)2\sqrt{1-y^2}\, dy$$

4. (13 pts.) A plate of constant density is shaped like the region between the graphs of $y = x^2$ and $y = 4x$. Give an expression involving integrals for the x-coordinate, $\bar{x}$, of the center of mass of the plate. **Do not evaluate** any integrals in your answer; do not find $\bar{y}$.



$$\bar{x} = \frac{M_y}{\text{mass}} = \frac{\rho \int_0^4 x(4x - x^2)\,dx}{\rho \int_0^4 (4x - x^2)\,dx} = \frac{\int_0^4 (4x^2 - x^3)\,dx}{\int_0^4 (4x - x^2)\,dx}$$

5. (12 pts.) Set up, but **do not evaluate**, an integral to compute the surface area of the object obtained by rotating the graph of $y = x^3$, $-1 \le x \le 1$ about the line $y = 3$.



$$S.A. \approx \sum \Delta SA = \sum 2\pi r \Delta s$$
$$= \sum 2\pi (3 - x^3) \Delta s$$

$$ds = \sqrt{1 + (f'(x))^2}\,dx$$
$$f(x) = x^3$$
$$f'(x) = 3x^2$$

$$SA = \int_{-1}^{1} 2\pi (3 - x^3)\,ds$$

$$= \int_{-1}^{1} 2\pi (3 - x^3)\sqrt{1 + 9x^4}\,dx$$