Mathematics/Computer Science 661: Optimization
Fall 2013

Instructor: John Rhodes, j.rhodes@alaska.edu; 102 Chapman, 474-5445

Office Hours: M 9:00 –10:00, W 2:00-3:00, F 10:00-11:00, and by appointment

Web page: `http://www.dms.uaf.edu/~jrhodes/M661.html`

Prerequisites: The main mathematical prerequisites are multivariable calculus and linear algebra. Basic computer programming skills will be needed, but can be acquired along the way. Strength and experience in either mathematics or computer science, along with a willingness to fill in gaps, can offset a weaker background in the other. All computer programming can be done in MATLAB, or in a general purpose language like C++.

Credit Hours: 3.0

Texts: Numerical Optimization, 2nd ed., by Nocedal and Wright, Springer; Linear programming with MATLAB, by Ferris, Mangasarian, and Wright, MPS-SIAM

Class Meetings: MWF 11:45-12:45 in Duckering 406;
Due to the instructor's travel commitments, at least 3 class meetings (the week of Oct 21-25) will be rescheduled and held some M 4:00-5:00. Several additional meetingsmay be held to free up time for student presentations at the end of the semester.

Exams: Take-home Midterm: week of Oct. 21-25;
Take-home Final: due Wed., Dec. 18, 12:15pm

Course overview and goals: Optimization problems occur throughout applied mathematics and the sciences. A simple textbook example that you've probably seen before provides a good orientation:

> A farmer has 40 feet of fencing with which to form 3 sides of a rectangular pen, the fourth side being the wall of a barn. What should the dimensions of the pen be to maximize its area?

The standard Calculus I approach to this problem is to express the area of the pen as a function of the length of one of its sides, and then use calculus to find the maximum by setting a derivative equal to 0 and solving an equation. For this problem, it is simple to calculate the needed derivative, and simple to solve the resulting equation.

In more realistic problems, the functions to be maximized might have hundreds of variables, so many partial derivatives would need to computed and set equal to 0. The resulting system of equations is likely to be horrendous, and impossible to solve by naive methods. (If we are lucky, though, these might be linear equations, and linear algebra could be utilized.) Clearly, we need a more sophisticated approach.

Returning to the simple problem, notice also that there were implicit *constraints*, that all sides have non-negative lengths, $s_i \geq 0$, and the sum of lengths of the sides satisfy $2s_1 + s_2 = 40$. In fact, many optimization problems naturally have constraints on the variables, expressed by equalities, inequalities, or both. From multivariable calculus, you may already be familiar with the method of Lagrange multipliers for optimization with equality constraints, but as we'll see other theory is needed as well.

The goal of this course is to introduce you, as mathematicians, scientists, or engineers, to some of the main ideas and approaches to solving both constrained and unconstrained optimization problems. This introduction will necessarily be incomplete, as optimization remains an area of active research. Specific optimization problems arising in applications often have special features that can be exploited in clever ways, so that general approaches may not be the very best to use for any particular application you may work on. However, this course will give you a solid background, providing you with a basic framework and tools.

Optimization is very much an applied field — its problems arise from concrete applications, and without those applications it would not remain vibrant. However, for most of this course we will not be focusing on the applications directly. We will also not be attempting to produce professional-level computer code. Rather we will understand the theory, and implement it in simple computer programs to understand performance on more artificial problems.

We will begin our survey with the general theory of Non-linear Optimization, which makes few special assumptions about the function being optimized, or the constraints. The later half of the course turns to Linear Programming, which deals with optimizing linear functions with linear constraints. Although this is really just a special case of the general theory, the special features of the problem allow a much more thorough development. Linear Programming is both heavily used in practice, and has provided motivation for the development of many algorithms in theoretical computer science.

Mechanics of the course:

Lectures The class will be run as an interactive lecture. That means that while I will be presenting material at the board, and you will be taking notes, I will also be asking for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers, but I do expect you to be actively following and participating — that makes the class more interesting for us all.

Although I will not formally take attendance, it will factor into your evaluation in class participation. If you miss class, you should get notes from another student.

Homework You are expected to read and assimilate assigned sections of the textbooks. Readings will sometimes include material dealt with only cursorily, or not at all, in lectures.

Homework problems will usually be assigned daily, but only collected each Monday (due in class, but accepted until 5pm at my office or mailbox). Homework problems will be posted on the course web page as they are assigned.

I will not accept any late homework that has not been cleared ahead of time. *There will be no exceptions to this* other than for a genuine emergency (e.g., a death in the family, documented illness, etc.).

The entire homework assignment will be checked to be sure you have attempted everything. Selected problems will be graded more completely. Even though you may find you can't do every problem, you must make a reasonable attempt on them all.

I encourage you to work with others on the homework, *but to write up the solutions independently.* In writing up your work, you should present your arguments in such a way that an intelligent, but ignorant, person can understand them. In particular complete sentences and a logical presentation are expected.

Project Each student will complete an individual project in this course, which can be tailored to particular interests. Possibilities include producing a piece of high-quality optimization software, delving more deeply into mathematical theory, exploring an optimization approach beyond what we are able to touch on in the lectures, or investigating a particular application. Midway through the course, you will submit for approval a brief description of the project you would like to undertake.

Grading Your performance will be evaluated based on 15% homework, 5% class participation, 25% midterm exam , 30% final exam, 25% project. Course grades will be determined according to the following cutoffs:

$$A \geq 90\%, \quad B \geq 80\%, \quad C \geq 70\%, \quad D \geq 60\%.$$

I reserve the right to move the cutoff points downward if particular exams turn out to be unexpectedly difficult. Note that you are not in competition with your peers – everyone in the class may get an A, or everyone may get an F.

For missed examinations that are not approved in advance, no make-up exams will be given, except in case of emergencies.

Any form of cheating will be dealt with harshly. At a minimum, the examination or assignment will receive a score of zero, so a passing course grade will require strong performance on all other work. I may also request a University Disciplinary and Honor Code Committee hearing which could result in suspension or expulsion.

University and Department Policies Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams which can be found at

`http://www.dms.uaf.edu/dms/Policies.html`.

If you have any disabilities that I should know about, you should bring them to my attention soon so that we can work with the Office of Disability Services to set up any necessary accommodations.

Tentative Schedule The following schedule is only approximate. The instructor will be out of town on Oct. 21-25 (and possibly other days), so classes will not be held then. To make up class time, and to allow for student presentations of projects, we will schedule 4+ extra class meetings on some Monday afternoons.

Week	Starting date	Material to be covered
0-1	Sept. 6	N&W, Chapters 1,2, LABOR DAY
2	Sept. 16	N&W, Chapters 3
3	Sept. 23	N&W, Chapter 3,5
4	Sept. 30	N&W, Chapters 5
5	Oct. 7	N&W, Chapters 6,7
6	Oct. 14	N&W, Chapter 12
7	Oct. 21	MIDTERM TAKE-HOME EXAM (no meetings this week)
8	Oct. 28	F,M&W, Chapter 1
9	Nov. 4	F,M&W, Chapter 2,3
10	Nov. 11	F,M&W, Chapters 3
11	Nov. 18	F,M&W, Chapter 4
12	Nov. 25	F,M&W, Chapter 5, THANKSGIVING (no class Nov. 28)
13	Dec. 2	F,M&W, Chapter 5
14	Dec. 9	Catch up; Project presentations(?)
		FINAL TAKE-HOME EXAM