

1. Find the tangent line to the parametrized curve

$$x = \sqrt{t} - 1, \quad y = 5t$$

at the point where  $t = 4$ .

$$(x(4), y(4)) = (1, 20)$$

$$\left. \frac{dy}{dx} \right|_{x=4} = \left. \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right|_{t=4} = \left. \frac{5}{\frac{1}{2} t^{-1/2}} \right|_{t=4} = 10\sqrt{t} \Big|_{t=4} = 20$$

$$(y - 20) = 20(x - 1)$$

or

$$y = 20x$$

2. Compute the arclength of the parameterized curve

$$x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1.$$

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2} dt$$

$$= \int_0^1 6t \sqrt{1 + t^2} dt$$

$$u = 1 + t^2$$

$$du = 2t dt$$

$$= 3 \int_1^2 \sqrt{u} du = 2 u^{3/2} \Big|_1^2$$

$$= 2 \cdot 2^{3/2} - 2 = 2^{5/2} - 2$$