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Math 405 Homework

p. 53, #2

Theorem. *If G is a group, its identity is unique.*

Proof. Suppose e_1 and e_2 are both identities in G. Then

$$e_1 = e_1 e_2 = e_2$$

where the first equality follows from e_2 being an identity, and the second from e_1 being an identity. $\square$

p. 54 #5 Let

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}.$$

Note rank $A \leq \min(2, 4) = 2$, since A is 2×4. Since the rows of A are non-zero, and not multiples of one another, $\text{rank}(A) \neq 0, 1$, so $\text{rank}(A) = 2$.

p. 63, #1 I don't know how to do this, and foolishly did not realize that until it was too late to seek help. It will never happen again.