Name: Solutions

10/8/14

1. Give an LU factorization of

$$A = \begin{pmatrix} 2 & 1 & -2 \\ -4 & -3 & 5 \\ 0 & -1 & 4 \end{pmatrix}.$$

$$A \xrightarrow[\substack{1^{st} \text{ row} \times 2 \\ \text{added to} \\ 2^{nd} \text{ row}}]{} \begin{pmatrix} 2 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & 4 \end{pmatrix} \xrightarrow[\substack{2^{nd} \text{ row} \times (-1) \\ \text{added to} \\ 3^{rd} \text{ row}}]{} \begin{pmatrix} 2 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{pmatrix} = U$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\boxed{A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{pmatrix}}$$

2. Suppose the LU factorization of a matrix A is

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 3 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Use this to find all vectors in the nullspace of A. (Note: You should NOT compute A to do this problem!)

$$N(A) = N(U)$$

$$U = \begin{pmatrix} \boxed{-1} & 2 & 3 & -1 \\ 0 & 0 & \boxed{2} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} x \quad y \quad z \quad w \end{array}$$

pivot variables x, z
free variables y, w

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2y - \frac{5}{2}w \\ y \\ -\frac{1}{2}w \\ w \end{pmatrix} = \boxed{y \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + w \begin{pmatrix} -\frac{5}{2} \\ 0 \\ -\frac{1}{2} \\ 1 \end{pmatrix}}$$

$$2z + w = 0$$
$$z = -\frac{1}{2}w$$

$$-x + 2y + 3z - w = 0$$
$$x = 2y + 3\left(-\frac{1}{2}w\right) - w = 2y - \frac{5}{2}w$$