

1. Determine the radius of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{3n-1}$$

$$\left| \frac{\frac{(-2)^{n+1} x^{n+1}}{3(n+1)-1}}{\frac{(-2)^n x^n}{3n-1}} \right| = \frac{3n-1}{3n+2} 2|x| \xrightarrow[n \rightarrow \infty]{\text{as}} 2|x| < 1 \text{ for convergence}$$

$$|x| < \frac{1}{2}$$

$$\underline{R = \frac{1}{2}}$$

2. Determine the interval of convergence of the series above.

We know the series converges for $|x| < \frac{1}{2}$ & diverges for $|x| > \frac{1}{2}$.

If $x = \frac{1}{2}$, the series is $\sum \frac{(-1)^n}{3n-1}$. Since $\frac{1}{3n-1} \rightarrow 0$
 $\frac{1}{3n-1} > \frac{1}{3(n+1)-1}$, this

converges by the alternating series test.

If $x = -\frac{1}{2}$, the series is $\sum \frac{1}{3n-1}$. We perform a limit comparison test to $\sum \frac{1}{n}$. Since $\frac{\frac{1}{3n-1}}{\frac{1}{n}} = \frac{n}{3n-1} \rightarrow \frac{1}{3} \neq 0$
the series diverges.

The interval of convergence is $\underline{\left(-\frac{1}{2}, \frac{1}{2}\right]}$.