

Decision Tree :- Decision tree is a supervised learning technique that can be used for both classification and regression problems but mostly it is preferred for solving classification problems. It is a tree structured classifier, where internal nodes represent the features of dataset, branches represent the decision rule and each leaf node represents the outcome.

Why use decision tree?

Decision trees usually mimic human thinking ability while making a decision so it is easy to understand.

②

the logic behind the decision tree can be easily understood because it shows a tree like structure.

Outlook	Temperature	Routine	Wear Jacket
Sunny	Cold	Indoor	No
Sunny	Warm	Outdoor	No
Cloudy	Warm	Indoor	No
Sunny	Warm	Indoor	No
Cloudy	Cold	Indoor	Yes
Cloudy	Cold	Outdoor	Yes
Sunny	Cold	Outdoor	Yes

$$E(X) = \sum_{i=1}^n p_i \log_2 p_i$$

$$IG(Y/X) = E(Y) - E(Y|X)$$

$$\text{Gini index} = 1 - \sum_{i=1}^n p_i^2$$

- ⇒ Entropy
- ⇒ Information gain
- ⇒ integer numbers

$$\log_2 4$$

$$\Rightarrow \log_2 2^2$$

$$\Rightarrow 2 \log_2 2$$

$$\Rightarrow 2 \text{ Ans}$$

or,

$$\log_2 4$$

$$\Rightarrow \frac{\log 4}{\log 2}$$

$$= 2 \text{ Ans}$$

$$\log_2 2^2 = 2$$

base change
rule:-

$$\log_a b = \frac{\log b}{\log a}$$

4

(ii) Fraction number -

$$\log_2\left(\frac{1}{4}\right) \Rightarrow \frac{\log_2(1)}{\log_2(4)}$$

$$\log\left(\frac{a}{b}\right) = \frac{\log a}{\log b}$$

$$\Rightarrow \frac{\log 1 - \log 4}{\log 2}$$

$$\Rightarrow \frac{0 - \log 2^2}{\log 2}$$

$$\Rightarrow \frac{-2 \log 2^2}{\log 2}$$

$$= -2 \frac{\log 2^2}{\log 2}$$

	wear	Jacket
1	Yes	m times
2	No	y times

Find Entropy :- $E(x) = \sum_{i=1}^n p_i \log_2 p_i$

$E(x)$ = Entropy before Partition

$E(x|y)$ = Entropy after Partition

Target $E(x) \rightarrow E(x|y)$

⇒ find Entropy 2
(y, m)

$$\Rightarrow -(-p_i \log_2 p_i) + (-p_i \log_2 p_i)$$

$$\Rightarrow \left(-\frac{4}{7} \log_2 \frac{4}{7}\right) + \left(-\frac{3}{7} \log_2 \frac{3}{7}\right)$$

$$\Rightarrow \cancel{0.70 + 0.59}$$

$$\Rightarrow (-0.57 \log_2 0.57) + (-0.42 \log_2 0.42)$$

$$\Rightarrow 0.98 = E(x)$$

	wear	Jacket
1	Yes	m times
2	No	y times

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6

Outlook

Sunny	4
cloudy	3

$E(\text{Outlook}, \text{Sunny})$

$$\Rightarrow - \left(\frac{1}{4} \log_2 \frac{1}{4} + \frac{3}{4} \log_2 \frac{3}{4} \right)$$

$$\Rightarrow 0.812$$

$E(\text{Outlook}, \text{cloudy})$

$$\Rightarrow - \left(\frac{2}{5} \log_2 \frac{2}{5} + \frac{1}{5} \log_2 \frac{1}{5} \right)$$

$$\Rightarrow 0.918$$

$$\text{info gain} \Rightarrow E(S) - \left(\frac{4}{7} * 0.812 \right) - \left(\frac{3}{7} * 0.918 \right)$$

$$= 0.127$$

$$\text{Outlook} = 0.127$$

7

Temperature

cold	4
warm	3

E (Temperature, cold)

$$\Rightarrow - \left(\frac{m}{4} \log_2 \frac{m}{4} + \frac{1}{4} \log_2 \frac{1}{4} \right)$$

$$\Rightarrow - (0.75 \log_2 0.75 + 0.25 \log_2 0.25)$$

$$= 0.811$$

E (Temperature, warm)

$$\Rightarrow - \left(\frac{m}{m} \log_2 \frac{m}{m} + \frac{0}{m} \log_2 \frac{0}{m} \right)$$

$$= 0.00$$

$$\text{Information gain} \Rightarrow H(S) - \left(\frac{4}{7} \times 0.811 \right)$$

$$= \left(\frac{13}{7} \times 0.50 \right)$$

$$H(S) = 0.98$$

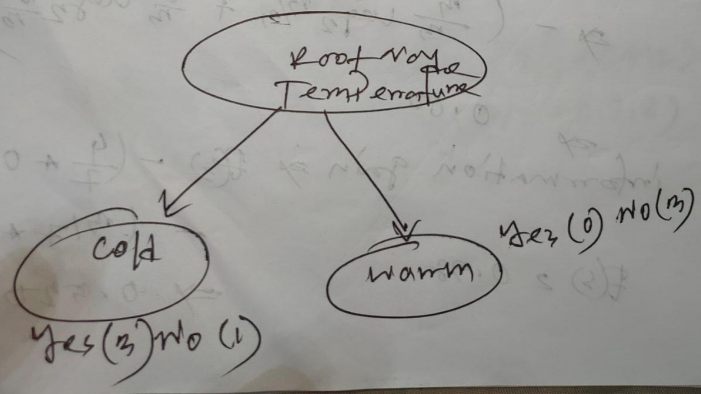
$$\Rightarrow 0.520 \text{ Ans}$$

8

Routine	
indoor	4
outdoor	3

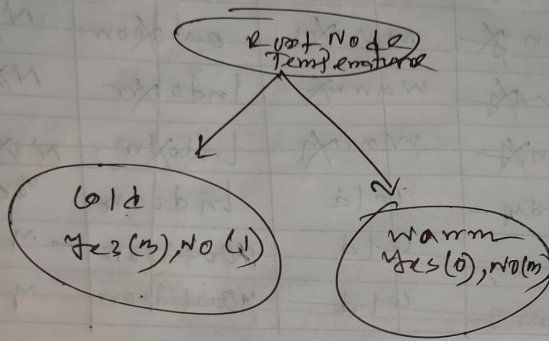
$\Rightarrow E(\text{routine, indoor})$
 $\Rightarrow - \left(\frac{1}{4} \log_2 \frac{1}{4} + \frac{3}{4} \log_2 \frac{3}{4} \right)$
 $\Rightarrow 0.812$

$\Rightarrow E(\text{routine, outdoor})$
 $\Rightarrow - \left(\frac{2}{3} \log_2 \frac{2}{3} + \frac{1}{3} \log_2 \frac{1}{3} \right)$
 $\Rightarrow 0.918$



9

$$\text{Gini index} = 1 - \sum_{i=1}^n p_i^2$$



$$\text{gini index} = 1 - \left(\frac{0}{3}\right)^2 + \left(\frac{3}{3}\right)^2$$

$$\Rightarrow 1 - 0 + 1$$

For warm

$$\Rightarrow 0$$

$$1 - 0 = 1$$

$$= 0$$

For cold

$$\Rightarrow 1 - \left\{ \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \right\}$$

$$1 - \left\{ (0.75)^2 + (0.25)^2 \right\}$$

$$1 - (0.5625 + 0.0625)$$

$$= 0.375$$

10

Outlook	Temperature	Routine	Wear Jacket
sunny	cold	indoor	no
sunny	warm	outdoor	no
cloudy	warm	indoor	no
sunny	warm	indoor	no
cloudy	cold	indoor	yes
cloudy	cold	outdoor	yes
sunny	cold	outdoor	yes

New Entropy

yes

no

11

New Entropy S_2

$$\begin{aligned}
 S_2 &= (-P_1' \log_2 P_1') + (-P_1' \log_2 P_1') \\
 &= \left(-\frac{3}{4} \log_2 \frac{3}{4}\right) + \left(-\frac{1}{4} \log_2 \frac{1}{4}\right) \\
 &= (-0.75 \log_2 \frac{3}{4}) + (-0.25 \log_2 \frac{1}{4}) \\
 &= 0.311 + 0.5 \\
 &= 0.811
 \end{aligned}$$

$E(\text{Routine, Indoor})$

$$\begin{aligned}
 &= -\left(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2}\right) \\
 &= 1
 \end{aligned}$$

$E(\text{Routine, Outdoor})$

$$\begin{aligned}
 &= -\left(\frac{2}{2} \log_2 \frac{2}{2} + \frac{0}{2} \log_2 \frac{0}{2}\right) \\
 &= 0
 \end{aligned}$$

Information gain:-

$$E(S_2) - \frac{2}{4} * 1 - \frac{2}{4} * 0$$

$$\begin{aligned}
 &= 0.811 - \frac{2}{4} * 1 - \frac{2}{4} * 0 \\
 &= 0.312
 \end{aligned}$$

12

$$E(\text{outlook, sunny})$$

$$\Rightarrow -\left(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2}\right)$$

$$E(\text{outlook, cloudy}) \Rightarrow 1$$

$$\Rightarrow -\left(\frac{2}{2} \log_2 \frac{2}{2} + \frac{0}{2} \log_2 \frac{0}{2}\right)$$

$$\Rightarrow 0$$

Information gain

$$E(32) = \frac{2}{4} \times 1 + \frac{2}{4} \times 0$$

$$\Rightarrow 0.5$$

$$\Rightarrow 0.312$$

13

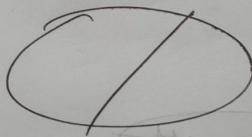
Routine gini index

$$1 - \sum_{i=1}^n p_i^2$$

Routine

Indoor
No (1) Yes (1)

Outdoor
Yes (2) No (2)



14

