Nonlinear Systems: Homework Assignment #5

Due by midnight on Tuesday March 24th.

Question 1 (NC 4.1):

Consider the linear time-varying system $\dot{x} = [A + B(t)]x$ where A is Hurwitz and $B(t)$ is piecewise continuous. Let P be the solution of the Lyapunov equation $PA + A^T P = -I$. Show that the origin is globally exponentially stable if $2\|PB(t)\| \leq a < 1, \forall t \geq 0$.

Question 2 (NC 4.2 (1) and (2)):

For the following systems, determine whether the origin is uniformly stable, uniformly asymptotically stable, exponentially stable, or none of the above. $g(t)$ is piecewise continuous and bounded.

(1) $\dot{x}_1 = -x_1^3 + g(t)x_2,\ \dot{x}_2 = -g(t)x_1 - x_2$
(2) $\dot{x} = -g(t)h(x)$, where $xh(x) \geq ax^2\ \forall x$ with $a > 0$ and $g(t) \geq k > 0\ \forall t \geq 0$.

Question 3 (NC 4.11):

For the following system, find a compact set Ω such that for all $x(0) \in \Omega$, the solution $x(t)$ is ultimately bounded and estimate the ultimate bound in terms of d. Hint: Apply Theorem 4.5 using $V(x)$ from Example 3.14.

$$\dot{x}_1 = -x_2, \qquad \dot{x}_2 = x_1 + (x_1^2 - 1)x_2 + d$$

Note: for those of you without NC book I will post the relevant theorems and an example problem. Example 3.14 is already contained in the region of attraction section posted for last homework.

Second Note: If you are able to derive all the inequalities used in this problem (and the example problem) then you will receive 5pts extra credit. However, to get the extra credit you must derive them independently.

Third Note: The problem in the NC book has a typo with a missing negative sign on the first dynamics equation. It has been corrected here.

Question 4 (NC 4.12 (1)):

For the following system, show that the origin is globally asymptotically stable when $a = 0$ and the solution is globally ultimately bounded by a class K function of $|a|$ when $a \neq 0$.

$$\dot{x}_1 = -x_1^3 + x_2 \qquad \dot{x}_2 = -x_2 + a$$

Hint: Treat the system as a cascade connection then use Lemma 4.6 NC (Lemma 4.7 NS) to show it is ISS, i.e. show that the first equation is ISS w.r.t. x_2 as the input. Then show that the second system is ISS.

Question 5 (NC 4.14 (2) and (6)):

For each of the following systems, investigate input-to-state stability. The function h is locally Lipschitz, $h(0) = 0$, and $yh(y) \geq ay^2 \;\; \forall y$, with $a > 0$.

(1) $\dot{x} = h(x) + u$

(2) $\dot{x}_1 = -x_1 - x_1^3 + x_2, \quad \dot{x}_2 = -x_1 - x_2 + u$

Question 6 (NC 9.4):

Consider the inverted pendulum on a cart (A.41)-(A.44) in NC or the equations in problem 1.15 in NS, where it is required to stabilize the pendulum at $\theta = 0$ and the cart at $y = 0$.

(a) Show that the task cannot be achieved by open-loop control
(b) Using linearization, design a stabilizing state feedback controller.
(c) Using simulation with the (A.45) data, find
    a. The range of$x_1(0)$, when $x(0) = [x_1(0), 0, 0, 0]^T$, for which the pendulum is stabilized;
    b. The effect of $\pm 30\%$ perturbation in m and J.

Note: rather than using the equations provided in Kahlil, use the equations (and parameters) derived for the inverted pendulum from the control book: http://controlbook.byu.edu/doku.php. The purpose of this question is to remind you of the work you did in the undergrad control class (or have you learn if you did not take that class) using state feedback to stabilize a system. In the next homework we will be doing more simulation problems so this will set you up for those problems as well.