Nonlinear Systems: Homework Assignment #4

Due by midnight on Tuesday March 3rd.

Question 1: (NC 3.5 (1) and (3)):

For each of the following systems, determine whether the origin is stable, asymptotically stable or unstable.

(a) $\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \dot{x}_3 = -2\sin(x_1) - 2x_2 - 2x_3$
(b) $\dot{x}_1 = x_2 + x_3, \quad \dot{x}_2 = -\sin(x_1) - x_3, \quad \dot{x}_3 = -\sin(x_1) + x_2$

Note: Easton and Seth both asked questions in class about when LaSalle's theorem will not work. I believe (b) is an example that shows this case.

Question 2 (NC 3.8):

Consider the system,

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -\tanh(x_1 + x_2)$$

(a) Show that

$$V(x) = \int_0^{x_1} \tanh(\sigma)\, d\sigma + \int_0^{x_1+x_2} \tanh(\sigma)\, d\sigma + x_2^2$$

is positive definite for all x and radially unbounded.

(b) Show that the origin is globally asymptotically stable.

Question 3 (NC 3.13):

Show that the origin of the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + \mu x_2 - x_2^3$$

Is globally asymptotically stable for $\mu \leq 0$ and unstable for $\mu > 0$. Is it exponentially stable when $\mu = 0$?

Question 4 (NC 3.16 (1)):

Consider the system

$$\dot{x}_1 = -(x_1 + x_1^3) + 2x_2, \quad \dot{x}_2 = 2x_1 - (x_2 + x_2^3)$$

and do the following:

(a) Find all equilibrium points and study their stability using linearization.
(b) Using quadratic Lyapunov functions, estimate the regions of the attraction of each asymptotically stable equilibrium point. Try to make the estimates as large as you can.
(c) Draw the phase portrait of the system to find the exact regions of attraction.

Question 5 (NC 3.18):

Consider the system:

$$\dot{x}_1 = x_1^3 - x_2, \quad \dot{x}_2 = x_1 - x_2,$$

Show that the origin is asymptotically stable. Is it exponentially stable? Is it globally asymptotically stable? If not, estimate the region of attraction.