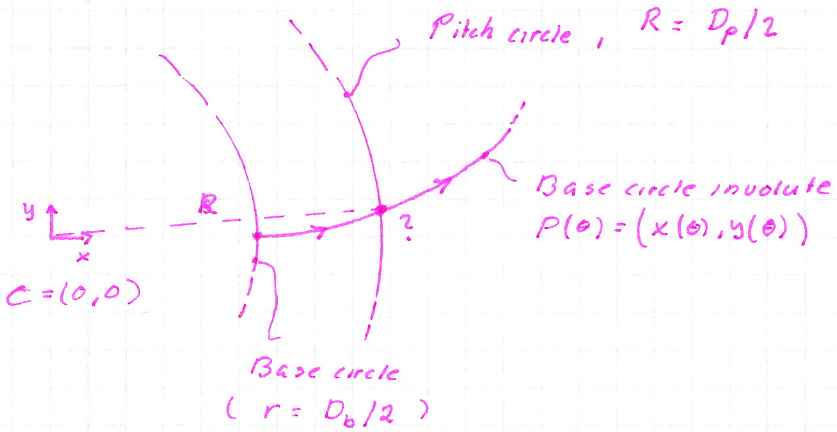
Analytical solutions to involute circle parameter values.



Base circle involut parametric eqn.

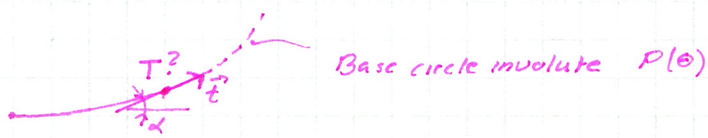
$$P(\theta): \begin{cases} x(\theta) = r \cdot (\cos\theta + \theta \cdot \sin\theta) \\ y(\theta) = r \cdot (\sin\theta - \theta \cdot \cos\theta) \end{cases}$$

Condition: $|P(\theta)| = R$

$$r^2 \cdot (\cos^2\theta + 2\theta \sin\theta \cos\theta + \theta^2 \sin^2\theta) + r^2 (\sin^2\theta - 2\theta \sin\theta \cos\theta + \theta^2 \cos^2\theta) = R^2$$

$$\underbrace{\cos^2\theta + \sin^2\theta}_{=1} + \theta^2 \underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} = \left(\frac{R}{r}\right)^2$$

$$\boxed{\theta = \pm \sqrt{\left(\frac{R}{r}\right)^2 - 1}}$$



$\vec{t} = (\cos\alpha, \sin\alpha)$, α = Pressure angle

Differentian of $P(\theta)$ w.r.t. θ

$$x'(\theta) = -r\sin\theta + r\theta\cos\theta + r \cdot 1 \cdot \sin\theta = r\theta\cos\theta$$
$$y'(\theta) = r\cos\theta + r\theta\sin\theta - r \cdot 1 \cdot \cos\theta = r\theta\sin\theta$$

$$\tan\alpha = \frac{y'(\theta)}{x'(\theta)} = \frac{r\theta\sin\theta}{r\theta\cos\theta} = \tan\theta$$

$$\boxed{\theta = \alpha}$$
