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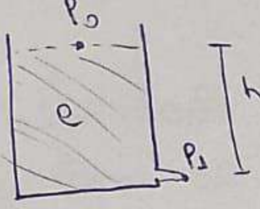
Moyses 2, cap. 02

01

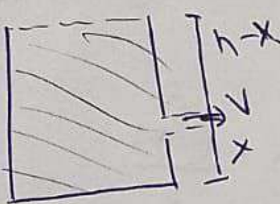


$$P_0 + \rho g h + \frac{\rho v_0^2}{2} = P_1 + \rho g \cdot 0 + \frac{\rho v_1^2}{2}$$

$$\Rightarrow v_1 = \sqrt{2gh}$$

$$\Rightarrow V = 0{,}69 \cdot A \cdot v_1 = 0{,}69 \cdot \pi \left(\frac{10^{-2}}{2}\right)^2 \sqrt{2 \cdot 9{,}8 \cdot 1} \cdot 10^3 \frac{\ell}{m^3} = 0{,}24\ \ell/s$$

02



Da questão anterior: $v = \sqrt{2g(h-x)}$

Tempo de queda: $x = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2x}{g}}$

$$\Rightarrow d = v \cdot t = \sqrt{4x(h-x)}$$

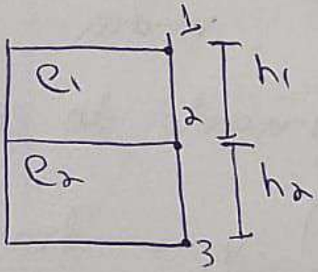
Temos que maximizar: $f(x) = x(h-x) \Rightarrow f'(x) = 0$

$$\Rightarrow h - 2x = 0 \Rightarrow h = 2x$$

a) $d = \sqrt{4 \cdot \frac{h}{2}\left(h - \frac{h}{2}\right)} = \sqrt{4 \cdot \frac{h}{2} \cdot \frac{h}{2}} = h$

b) $x = h/2$

03



i) Bernoulli entre 1 e 2:

$$P_1 + \rho_1 g h_1 + \frac{\rho_1 v_1^2}{2} = P_2 + \rho_1 g \cdot 0 + \frac{\rho_1 v_2^2}{2}$$

$$P_1 + \rho_1 g h_1 = P_2 + \frac{\rho_1 v_2^2}{2}$$

ii) Bernoulli entre 2 e 3:

$$P_2 + \rho_2 g h_2 + \frac{\rho_1 v_2^2}{2} = P_3 + \rho_2 g \cdot 0 + \frac{\rho_2 v_3^2}{2}$$

Somando i) e ii), e $P_1 = P_3 = P_0$:

$$\frac{\rho_2 v_3^2}{2} = g(\rho_1 h_1 + \rho_2 h_2) \to v_3 = \sqrt{\frac{2g(\rho_1 h_1 + \rho_2 h_2)}{\rho_2}} = \sqrt{2gh\frac{(\rho_1 + \rho_2)}{\rho_2}}$$

$$= \sqrt{2 \cdot 9{,}8 \cdot 0{,}5 \cdot \frac{(0{,}69 + 1)}{1}} = 4{,}07\ m/s$$

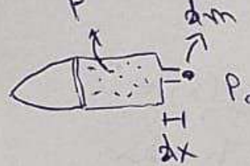
04

$$P_1 + \rho_1 g h + \cancel{\frac{\rho_1 v_1^2}{2}} = P_2 + \rho_2 g h + \frac{\rho_2 V_2^2}{2}$$

$$(1{,}25 - 1) \cdot 1{,}01 \cdot 10^5 = \frac{1{,}3}{2} V_2^2$$

$$V_2 = 197{,}4 \text{ m/s}$$

05 Seja m a massa do foguete + gás.
Uma quantidade dm de gás escapa e atinge a velocidade dada pela Eq. de Bernoulli



$$P + \rho g h + \frac{\rho V_0^2}{2} = P_0 + \rho g h + \frac{\rho V_1^2}{2}$$

$$\Rightarrow V_1 = \sqrt{2(P - P_0)/\rho}$$

Por conta da conservação do momento linear, o foguete terá velocidade V_2:

$$0 = dm \cdot V_1 + (m - dm) \cdot V_2 \Rightarrow V_2 = -\frac{dm}{m - dm} \cdot V_1$$

A variação da quantidade de movimento do foguete é Δp:

$$\Delta p = (m - dm) \cdot V_2 - m \cdot V_0 = (m - dm) \cdot \frac{(-dm)}{m - dm} \cdot V_1 = -dm \cdot V_1$$

$$\Rightarrow F_R = \frac{dp}{dt} = \frac{-dm}{dt} \cdot V_1 = -\rho \cdot A \cdot \frac{dx}{dt} \cdot V_1 = -\rho \cdot A \cdot V_1^2$$

Onde dx foi a distância linear que dm percorreu durante, dt. Ou seja: $\frac{dx}{dt} = V_1$

$$\Rightarrow |F_R| = \rho A \cdot V_1^2 = \rho \cdot A \cdot \frac{2(P - P_0)}{\rho_0} = 2A(P - P_0)$$

06 Velocidade da água ao sair do orifício:

$$P_0 + \rho g h + \frac{\rho \cdot v_1^2}{2} = P_0 + \rho g \cdot 0 + \frac{\rho v^2}{2} \Rightarrow v = \sqrt{2gh}$$

Velocidade do carrinho + tanque após uma quantidade dm de água sair:

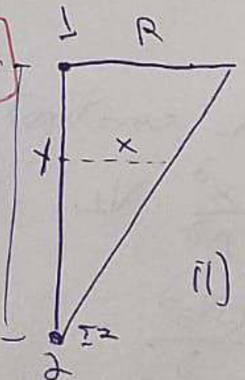
$$v_2 = \frac{dm}{M_0+m_0}\cdot v$$

$$\Rightarrow F = \frac{dp}{dt} \Rightarrow (M_0+m_0)\cdot a_0 = \frac{(M_0+m_0)\cdot\frac{dm}{M_0+m_0}\cdot v - 0}{dt}$$

$$\Rightarrow (M_0+m_0)\cdot a = \frac{dm}{dt}\cdot v \Rightarrow (M_0+m_0)\cdot a = \rho A\cdot\frac{dx}{dt}\cdot v = \rho A v^2$$

$$\Rightarrow a = \frac{2gh\rho A}{M_0+m_0}$$

07



i) Bernoulli entre 1 e 2:

$$P_1 + \rho g h + \frac{\rho v_1^2}{2} = P_2 + \rho g 0 + \frac{\rho v_2^2}{2}$$

$$\Rightarrow v_2^2 = v_1^2 + 2gh \quad (P_1 = P_2 = 0\text{, vácuo})$$

ii) Semelhança de triângulos:

$$\frac{x}{R} = \frac{y}{h} \Rightarrow y = \frac{h}{R}\cdot x \quad \text{ou} \quad x = \frac{R}{h}y$$

Quando a altura de água é y, seu volume é: $V = \frac{\pi}{3}x^2\cdot y$

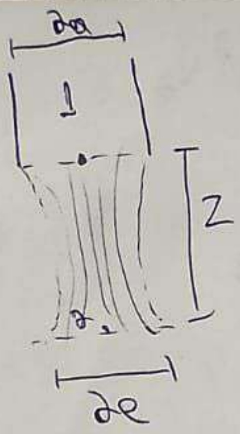
$$\Rightarrow V = \frac{\pi}{3}x^2\cdot\frac{h}{R}x = \frac{\pi h}{3R}x^3 = \frac{\pi}{3}\cdot\frac{R^2}{h^2}y^3$$

Quando a água cai um dy: $dV = -\frac{\pi R^2}{h^2}y^2\cdot dy$

No mesmo intervalo de tempo dt, o volume de água que passa pelo centro é: $dV_c = -\pi r^2\cdot dz$

$$\Rightarrow \frac{\pi R^2}{h^2}y^2 dy = \pi r^2 dz \Rightarrow \frac{R^2 y^2}{h^2}\cdot\frac{dy}{dt} = r^2\cdot\frac{dz}{dt} \Rightarrow \boxed{v_2 = \frac{R^2}{r^2 h^2}\cdot y^2 v_1}$$

08



Bernoulli entre 1 e 2:

$$P_1 + \rho g \cdot z + \frac{\rho V_1^2}{2} = P_2 + \rho g \cdot 0 + \frac{\rho V_2^2}{2}$$

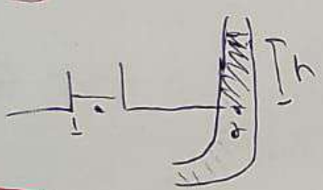
$$\Rightarrow V_2^2 = V_1^2 + 2gz \qquad (P_1 = P_2 = P_0)$$

Mas: $Q = \pi a^2 V_1 = \pi e^2 \cdot V_2 \Rightarrow \frac{e^2}{a^2} = \frac{V_1}{V_2} = \frac{V_1}{\sqrt{V_1^2 + 2gz}}$

E: $V_1 = \frac{Q}{\pi a^2}$

$$\Rightarrow \frac{e^2}{a^2} = \frac{Q/\pi a^2}{\sqrt{\left(\frac{Q}{\pi a^2}\right)^2 + 2gz}} \Rightarrow e^2 = \frac{Q \cdot a^2}{\sqrt{Q^2 + 2\pi^2 a^4 g z}}$$

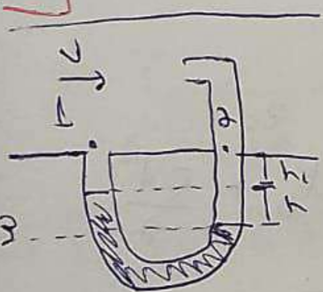
09 No cotovelo: $V = 0$. Para 1 e 2 no mesmo nível:



$$P_0 + \frac{\rho V^2}{2} = P_0 + \rho g h$$

$$\Rightarrow V = \sqrt{2gh} = 0{,}989 \text{ m/s}$$

10



Seja P_1 a pressão do fluido no escoamento. No cotovelo, $V=0$ e a pressão é P_2.

$H_1 = H_2 \rightarrow P_1 + \frac{\rho V^2}{2} = P_2$ (mesmo nível)

$$\rightarrow P_2 - P_1 = \frac{\rho V^2}{2}$$

No nível 3, aplicando Stevin:

Lado direito: $P_3 = P_2 + \rho g h + \rho g h_1$

Lado esquerdo: $P_3 = P_1 + \rho_f g h + \rho g h_1$

$$\Rightarrow P_2 + \rho g h = P_1 + \rho_f g h \Rightarrow P_2 - P_1 = gh(\rho_f - \rho)$$

$$\Rightarrow \frac{\rho V^2}{2} = gh(\rho_f - \rho) \Rightarrow V = \sqrt{\frac{2gh(\rho_f - \rho)}{\rho}}$$

a) Para $y=h \Rightarrow V_2 = \frac{R^2}{r^2}\cdot V_1$

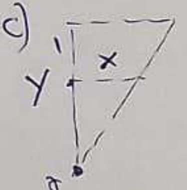
Em Bernoulli: $\left(\frac{R}{r}\right)^4 \cdot V_1^2 = V_1^2 + 2gh$

$$\Rightarrow V_1 = \sqrt{\frac{2gh}{\left(\frac{R}{r}\right)^4 - 1}} = 1{,}41\cdot 10^{-4}\ m/s$$

b) Para $y = \frac{h}{2}$;

$$V_2 = \frac{R^2}{4r^2} V_1$$

$$\Rightarrow \left(\frac{R}{2r}\right)^4 V_1^2 = V_1^2 + 2gh' \Rightarrow V_1 = \sqrt{\frac{2\cdot 9{,}8\cdot 0{,}05}{\left(\frac{10}{2\cdot 0{,}01}\right)^4 - 1}} = 3{,}96\cdot 10^{-4}\ m/s$$

c) 

Seja x o raio da base superior de água quando esta está numa altura y do centro da ampulheta.

O nível y cai com uma ~~altura~~ velocidade constante V_1.

$C = A_1\cdot V_1 = A_2\cdot V_2 \Rightarrow \pi x^2\cdot V_1 = \pi r^2\cdot V_2 \Rightarrow V_2 = \frac{x^2}{r^2}\cdot V_1$

Bernoulli: $P_1 + \rho g y + \frac{\rho V_1^2}{2} = P_2 + \rho g\cdot 0 + \frac{\rho V_2^2}{2}$

$\Rightarrow V_2^2 = V_1^2 + 2gy \Rightarrow \left(\frac{x}{r}\right)^4 V_1^2 = V_1^2 + 2gy$

$\Rightarrow y = \frac{V_1^2}{2g}\left[\left(\frac{x}{r}\right)^4 - 1\right]$

Para $x \ggg r \Rightarrow y = \frac{V_1^2}{2g}\cdot\left(\frac{x}{r}\right)^4$

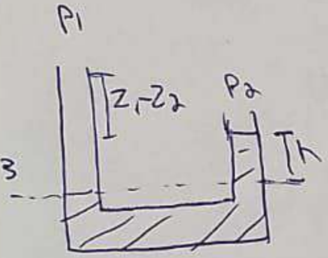
11 Continuidade: $A_1 v_1 = A_2 v_2 \Rightarrow \pi R^2 v_1 = \pi r^2 v_2$

$$\Rightarrow V_2 = \frac{R^2}{r^2} \cdot V_1$$

$$H_1 = H_2 \Rightarrow P_1 + \rho g z_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g z_2 + \frac{\rho V_2^2}{2}$$

$$\Rightarrow P_1 - P_2 = \rho g (z_2 - z_1) + \frac{\rho}{2} v_1^2 \left(\frac{R^4}{r^4} - 1\right)$$

Lei de Stevin no Manômetro no nível 3:



$$P_3 = P_2 + \rho_f g h = P_1 + \rho g h + \rho g (z_1 - z_2)$$

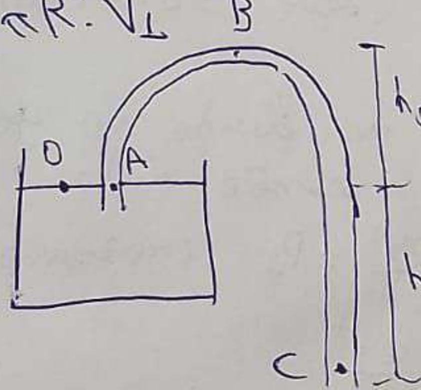
$$\Rightarrow P_1 - P_2 = gh(\rho_f - \rho) + \rho g (z_2 - z_1)$$

$$\Rightarrow gh(\rho_f - \rho) + \cancel{\rho g (z_2 - z_1)} = \cancel{\rho g (z_2 - z_1)} + \frac{\rho v_1^2}{2}\left(\frac{R^4}{r^4} - 1\right)$$

$$\Rightarrow V_1 = \sqrt{\frac{2gh(\rho_f - \rho)}{\rho\left(\frac{R^4}{r^4} - 1\right)}}$$

$$\Rightarrow Q = \pi R^2 \cdot V_1$$

12



a) $H_0 = H_A$

$$\Rightarrow P_0 = P_A + \frac{\rho V^2}{2}$$

$H_A = H_C$

$$P_A + \frac{\rho V^2}{2} + \rho g h_1 = \overset{P_C =}{P_0} + \frac{\rho V^2}{2}$$

$$\Rightarrow P_A + \cancel{\frac{\rho V^2}{2}} + \rho g h_1 = \cancel{P_A} + \cancel{\frac{\rho V^2}{2}} + \frac{\rho V^2}{2} \Rightarrow V = \sqrt{2 g h_1}$$

b) $P_A = P_0 - \frac{\rho V^2}{2} = P_0 - \frac{\rho}{2} \cdot 2 g h_1 \Rightarrow P_A = P_0 - \rho g h_1$

$H_A = H_B$: $P_A + \cancel{\frac{\rho V^2}{2}} = P_B + \rho g h_0 + \cancel{\frac{\rho V^2}{2}}$

$$\Rightarrow P_B = P_A - \rho g h_0 = P_0 - \rho g h_1 - \rho g h_0 \Rightarrow \boxed{P_B = P_0 - \rho g (h_0 + h_1)}$$

c) Em B: $H_B = P_B + \rho g h_0 + \frac{\rho v^2}{2} = cte$

Para termos h_0 máximo $\Rightarrow \begin{cases} P_B = 0 \\ V = 0 \end{cases}$

$P_B = 0 \Rightarrow P_0 - \rho g (h_0 + h_1) = 0 \Rightarrow P_0 = \rho g (h_0 + h_1)$

$\Rightarrow h_0 = \frac{P_0}{\rho g} - h_1$

$\boxed{13}$

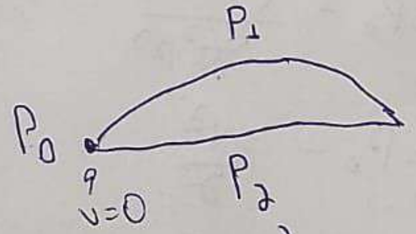
a) $V = \frac{\pi a^4}{8\eta}\left(\frac{P_1 - P_2}{L}\right)$

$\Rightarrow V = \frac{\pi (0{,}1)^4}{8 . 1}\left[\frac{(5-1) . 10^5}{50 . 10^3}\right] = 3{,}2 \, 10^{-4} \, m^3/s = 0{,}32 \, L/s$

$\Rightarrow V(24) = 3{,}2 . 10^{-4} . 24 . 60 . 60 = 2{,}75 \, 10^5 \, L/dia$

b) $V(r) = \frac{(P_1 - P_2)}{4 L \eta}(a^2 - r^2) \Rightarrow V(r=0) = \frac{(5-1) . 10^5}{4 . 50 . 10^3 . 1} . 0{,}1^2 = 0{,}2 \, m/s$

$\boxed{14}$



$H_0 = H_1 = H_2$

$\Rightarrow P_0 = P_1 + \frac{\rho V_1^2}{2} = P_2 + \frac{\rho V_2^2}{2} \Rightarrow P_2 - P_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$

$= \frac{1{,}3}{2} V_2^2 (1{,}25^2 - 1)$

$= 0{,}3656 V_2^2$

$\Delta P = \frac{F}{A} \Rightarrow 0{,}3656 V_2^2 = \frac{2000 . 9{,}8}{30}$

$\Rightarrow V_2 = 42{,}27 \, m/s \Rightarrow V_1 = 1{,}25 . V_2 = 52{,}84 \, m/s$

$\Rightarrow V_1 = 190{,}2 \, km/h$

15 $V = \frac{C}{2\pi r}$

Seja a massa: $dm = \rho \cdot dV \Rightarrow dF = a \cdot dm = a\rho \cdot dV$

Mas a é aceleração centrípeta:

$$\Rightarrow dF = \frac{V^2}{r}\rho\, dV \Rightarrow \frac{dF}{dV} = \frac{\rho V^2}{r}$$ (densidade de força)

$$\Rightarrow f = \frac{dP}{dr} = \frac{\rho V^2}{r} = \frac{\rho}{r}\cdot\frac{C^2}{4\pi^2 r^2} = \frac{\rho C^2}{4\pi^2 r^3}$$

$$\Rightarrow \int dP = \frac{\rho C^2}{4\pi^2}\int r^{-3}dr = \frac{\rho C^2}{4\pi^2}\cdot\frac{r^{-2}}{(-2)}$$

$$\Rightarrow P - P_0 = -\frac{\rho C^2}{8\pi^2 r^2} \Rightarrow P = P_0 - \frac{\rho C^2}{8\pi^2 r^2}$$

Circulação: $C = 2\pi r V \Rightarrow P = P_0 - \frac{\rho \cancel{4\pi^2} V^2 \cancel{r^2}}{8\cancel{\pi^2}\cancel{r^2}}$

$$\Rightarrow P = P_0 - \frac{\rho V^2}{2}$$