Moysés Nussenzveig, vol. 2, cap 9. Propriedade dos Gases
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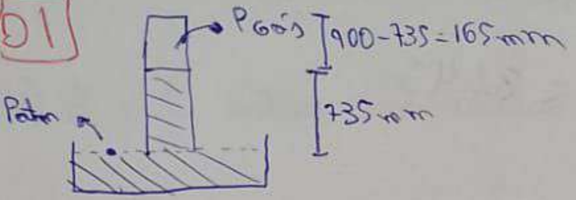
Física / Bacharelado - UFC 2024.2

01



Lei de Stevin:

$P_{atm} = P_{coluna} + P_{Gás}$

$750 = 735 + P_{Gás} \Rightarrow P_{Gás} = 15 \text{ mmHg}$

$\Rightarrow P_{Gás} = \frac{15}{760} \text{ atm}$

Gás ideal: $P_{Gás} \cdot V = nRT$

$\Rightarrow \frac{15}{760} \cdot \frac{1 \cdot 16,5 \cdot 10^{-4} m^3}{1 m^3 / 10^3 l} = n \cdot 0,082 \cdot 293 \rightarrow \frac{15}{760} \cdot 1,65 = n \cdot 0,082 \cdot 293$

$\rightarrow n = 1,35 \cdot 10^{-3}$ mols

02 a) $PV = nRT \rightarrow 1 \cdot 2 = n \cdot 0,082 \cdot 298 \rightarrow n = 0,082$ mols

$m = nM = 0,082 \cdot 32 = 2,62 g$

b) No equilíbrio, a pressão é a mesma em ambos recipientes (P), mas teremos n_1 mols de oxigênio no recipiente 1 e n_2 no 2.

$\rightarrow \begin{cases} P_1 \cdot V_1 = n_1 R T_1 \\ P_2 \cdot V_2 = n_2 R T_2 \end{cases} \rightarrow \begin{matrix} P \cdot 1 = n_1 \cdot R \cdot 373 \\ P \cdot 1 = n_2 \cdot R \cdot 298 \end{matrix} \rightarrow n_1 = 0,8 n_2$

A quantidade de gás se preserva: $n = n_1 + n_2 \Rightarrow n = 1,8 n_2$

$\rightarrow n_2 = 0,045$ mols

Logo: $P \cdot 1 = 0,045 \cdot 0,082 \cdot 298 \rightarrow \boxed{P = 1,11 \text{ atm}}$

c) $\begin{matrix} n_1 = 0,0364 \text{ mols} \\ n_2 = 0,0456 \text{ mols} \end{matrix} \rightarrow \begin{matrix} m_1 = 1,46 g \\ m_2 = 1,16 g \end{matrix} \rightarrow \begin{matrix} \Delta m_1 = 1,46 - 1,31 = 0,15 g \\ \Delta m_2 = 1,16 - 1,31 = -0,15 g \end{matrix} \rightarrow \boxed{\Delta m = 0,15 g}$

03 a) Pressão ocasionada pelo peso: $\frac{mg}{A} = \frac{10 \cdot 9,81}{200 \cdot 10^{-4}} = 4905 Pa \cong 0,049$ atm

$\rightarrow P_{Gás} = 1 + 0,049 = 1,049$ atm (constante e igual à externa)

Gás ideal: $P_{Gás} \cdot V_1 = nRT_1 \rightarrow P_{Gás} \cdot V_1 = \frac{m}{M} R T_1$

$\Rightarrow \rho = \frac{m}{V_1} = \frac{P_{Gás} \cdot M}{RT_1} = \frac{1,049 \cdot 4}{0,082 \cdot 293} = 0,174 \text{ kg/m}^3$

b) $\begin{cases} P_{Gás} \cdot V_1 = nRT_1 \\ P_{Gás} \cdot V_2 = nRT_2 \end{cases} \rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \rightarrow V_2 = V_1 \cdot \frac{T_2}{T_1} = 3 \cdot \frac{343}{293} = 3,51 \; l$

c) $W = P_{Gás} \cdot \Delta V = 1{,}049 \cdot 101325 \cdot (3{,}51 - 3) \cdot 10^{-3} = 54{,}4\,J$

Pascal

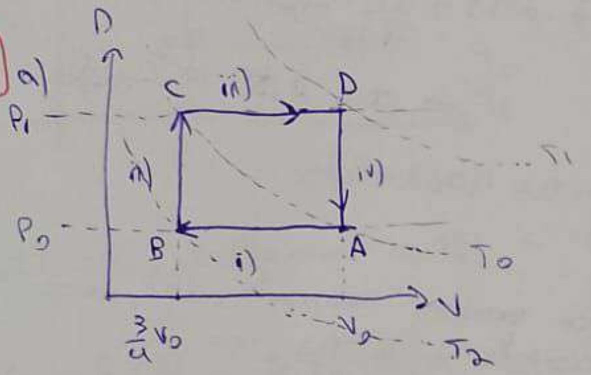
d) $d = \frac{m}{V} = \frac{nM}{V} \rightarrow n = \frac{dV}{M} = \frac{0{,}174 \cdot 3}{4} = 0{,}1305$ mols

$\Delta U = n C_V \Delta T = 0{,}1305 \cdot \frac{3}{2} \cdot 8{,}314 \cdot 50 = 81{,}4\,J$

e) 1ª Lei da Termodinâmica:

$\Delta U = Q - \Delta W \Rightarrow 81{,}4 = Q - 54{,}4 \rightarrow Q = 135{,}8\,J$

Q4 a)



b) Em A: $P_0 V_0 = nRT_0 \rightarrow 1 \cdot V_0 = 1 \cdot 0{,}082 \cdot 300 \rightarrow V_0 = 24{,}6\,\ell$

Em C: $P_1 \cdot \frac{3}{4}V_0 = nRT_0 \rightarrow P_1 \cdot \frac{3}{4} \cdot 24{,}6 = 1 \cdot 0{,}082 \cdot 24{,}6 \rightarrow P_1 = \frac{4}{3}$ atm

$W = \oint p\,dV \stackrel{A}{=} (V_0 - \frac{3}{4}V_0) \cdot (\frac{4}{3} - 1) = \frac{1}{4} \cdot 24{,}6 \cdot 10^{-3} \cdot \frac{1}{3} \, 101325 = 207{,}7\,J$

c) $\frac{C_P}{C_V} = \frac{7}{5} \quad \therefore \quad C_P - C_V = R \rightarrow \frac{7}{5}C_V - C_V = R \rightarrow \begin{cases} C_V = \frac{5}{2}R \\ C_P = \frac{7}{2}R \end{cases}$

Em B: $P_0 \cdot \frac{3}{4}V_0 = nR \cdot T_2 \rightarrow 1 \cdot \frac{3}{4} \cdot 24{,}6 = 1 \cdot 0{,}082 \cdot T_2$

$\rightarrow T_2 = 225\,K$

Logo: $Q_i = nC_P \Delta T = 1 \cdot \frac{7}{2} \cdot 8{,}314 \cdot (225 - 300) = -2182{,}425\,J$

$Q_{ii} = nC_V \Delta T = 1 \cdot \frac{5}{2} \cdot 8{,}314 \cdot (300 - 225) = 1558{,}875\,J$

+

$-623{,}55\,J$

Fornecido: $623{,}55\,J$

d) Em D : $P_1 V_0 = nR.T_1 \rightarrow \frac{4}{3} \cdot 24,6 = 1 \cdot 0,082 \cdot T_1 \rightarrow T_1 = 400K$

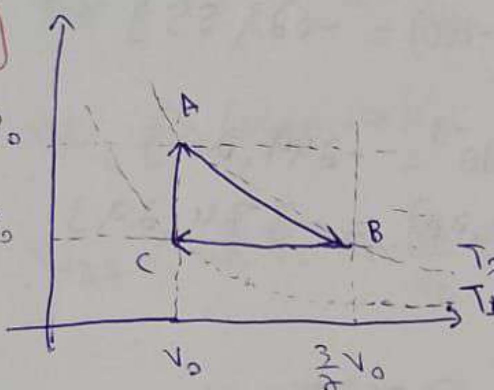
Mínimo : $T_2 = 225K$

Máximo : $T_1 = 400K$

e) A e C estão em uma mesma isoterma : $U_A = U_C$

$\Rightarrow \Delta U_{AC} = 0$

05



Isoterma : $P_A V_A = P_B V_B$

$P_0 . V_0 = P_B \, 1,5 V_0 \rightarrow P_B = \frac{2}{3} P_0$

b) $\Delta U_i = 0$ (isoterma) $\Rightarrow W_i = nRT \ln \frac{V_F}{V_i} = 1 . 8,314 . 293 \ln 1,5$

$= 987,7 J$

$W_{ii} = P \Delta V = \frac{2}{3} P_0 . (V_0 - 1,5 V_0) = -\frac{P_0 V_0}{3}$

$= -\frac{nRT_2}{3} = -\frac{1 . 8,314 . 293}{3} = -812 J$

$W_{iii} = 0$ (ΔV)

$\rightarrow W_T = W_i + W_{ii} + W_{iii} = 987,7 - 812 + 0 = 175,7 J$

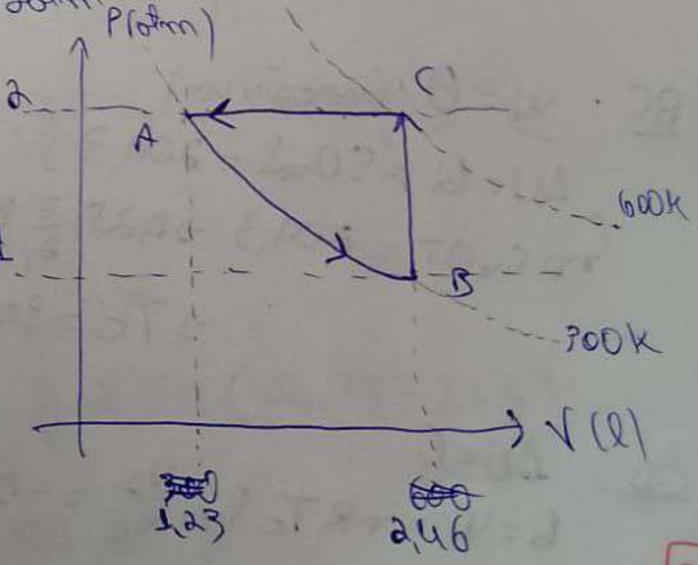
06 a) O, B e C estão sob uma mesma linha : $\Delta O.300.B$ semelhante ao triângulo $\Delta BAC \Rightarrow P_1 = 2 . 1 = 2 atm$

$P_A . V_A = nRT_A \rightarrow V_A = 1,23 \, l$

$P_B . V_B = nRT_B \rightarrow V_B = 2,46 \, l$

$P_C . V_C = nRT_C \rightarrow V_C = 2,46 \, l$

$C_P = C_V + R = \frac{5}{2} R$



b) AB: $\Delta U = 0$ (mesma temperatura)

$Q = W = nRT \ln \frac{V_f}{V_i} = 0{,}1 . 8{,}314 . 300 . \ln 2 = 172{,}88\ J$

BC: $W = 0$ (mesmo volume)

$Q = \Delta U = n . C_v . \Delta T = 0{,}1 . \frac{3}{2} . 8{,}314 . (600-300) = 374{,}13\ J$

CA: $Q = nC_p\Delta T = 0{,}1 . \frac{5}{2} . 8{,}314\ (300-600) = -623{,}55\ J$

$W = P\Delta V = 2 . 101325 . (1{,}23 - 2{,}46) . 10^{-3} = -249{,}26\ J$

$\Delta U = Q - \Delta W = -623{,}55 - (-249{,}26) = -374{,}29\ J$

Ciclo: $\Delta U = 0 + 374 - 374 = 0$

$Q = 172{,}88 + 374{,}13 - 623{,}55 = -76{,}54\ J$

$W = 172{,}88 + 0 - 249{,}26 = -76{,}38\ J$

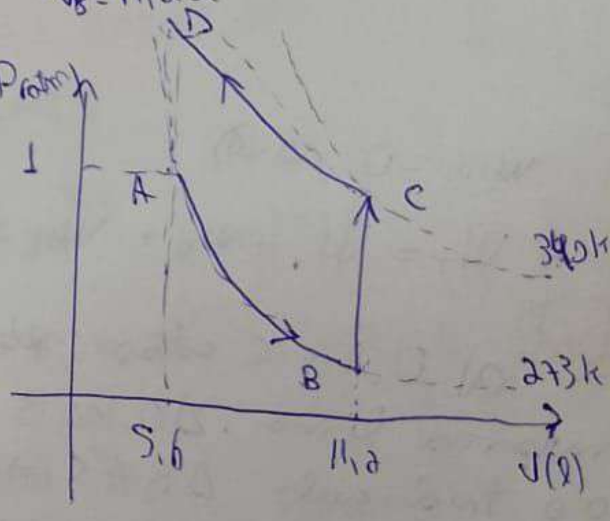
07 $n = \frac{1}{4} = 0{,}25$ mols ; 1 atm ; 273 K $\rightarrow \begin{cases} V_A = 5{,}6\ l \\ V_B = 11{,}2\ l \end{cases}$

i) AB: $\Delta U = 0$ (isotérmica)

$Q = W = nRT \ln \frac{V_B}{V_A}$

$= 0{,}25 . 8{,}314 . 273 . \ln 2$

$= 393{,}3\ J$



ii) BC: $W = 0$ (isocórico)

$\Delta U = Q = 50\ cal = 209{,}3\ J$

$(\rightarrow n C_v \Delta T = 209{,}3 \rightarrow 0{,}25 . \frac{3}{2} . 8{,}314 . (T_C - T_B) = 209{,}3$

$\rightarrow T_C = 340\ K$

iii) CD: $\Delta U = 0$

$Q = W = nRT_C \ln \frac{V_D}{V_C} = -0{,}25 . 8{,}314 . 340 . \ln 2 = -489{,}4\ J$

08 AB: $W_{AB} = P_0(V_B - V_A)$

Mas: $P_0 . V_A = nRT_1 \rightarrow V_A = \frac{RT_1}{P_0}$ $\rightarrow W_{AB} = P_0\left(\frac{RT_2 - RT_1}{P_0}\right) = R(T_2 - T_1)$

$P_0 . V_B = nRT_2 \rightarrow V_B = \frac{RT_2}{P_0}$

BC: $W_{BC} = nRT_2 . \ln \frac{V_0}{V_B} = RT_2 \ln \frac{P_0 V_0}{RT_2}$

CD: $W_{CD} = 0$ (isocórico)

DA: $W_{DA} = nRT_1 \ln \frac{V_A}{V_D} = RT_1 . \ln \frac{RT_1}{P_0 V_0} = - RT_1 \ln \frac{P_0 V_0}{RT_1}$

$\rightarrow W_{Tot} = R(T_2 - T_1) + RT_2 . \ln\left(\frac{P_0 V_0}{RT_2}\right) - RT_1 . \ln\left(\frac{P_0 V_0}{RT_1}\right)$

09 a) $C_p = C_V + R \rightarrow C_p = \frac{5}{2} R \rightarrow \gamma = \frac{C_p}{C_V} = \frac{5}{3}$

Inicialmente: $P_0 V_0 = nRT_0 \rightarrow 10 . V_0 = 1 . 0{,}082 . 273$

$\rightarrow V_0 = 2{,}24\ l$

Processo adiabático: $P_0 V_0^{\gamma} = P_F . V_F^{\gamma} \rightarrow 10 . 2{,}24^{5/3} = 1 . V_F^{5/3}$

$\rightarrow V_F = 2{,}24 . 10^{3/5} = 8{,}92\ l$

Final: $P_F . V_F = nRT_F \rightarrow 1 . 8{,}92 = 1 . 0{,}082 . T_F$

$\rightarrow T_F = 108{,}75\ K = -164{,}25\ °C$

b) $Q = 0 \rightarrow \Delta U = -W \rightarrow W = -\Delta U$

$= -n C_V \Delta T$

$= -1 . \frac{3}{2} . 8{,}314 (108{,}75 - 273)$

$= 2048{,}35\ J$

(10) $P_0V_0 = nRT_0 \rightarrow 1 \cdot 1 = n \cdot 0{,}082 \cdot 300 \rightarrow n = 0{,}0406$ mols

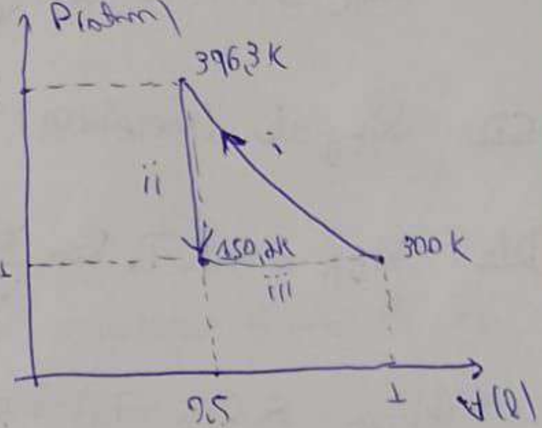
i) $Q = 0 \rightarrow \Delta U = -W$

Adiabático: $P_0V_0^{\gamma} = P_1 V_1^{\gamma} \rightarrow 1 \cdot 1^{\gamma} = P_1 \cdot 0{,}5^{7/5} \rightarrow P_1 = 2{,}639$ atm

$P_1 V_1 = nRT_1 \rightarrow 2{,}6 \cdot 0{,}5 = 0{,}04 \cdot 0{,}082 \cdot T_1 \rightarrow T_1 = 396{,}3$ K

$\Delta U = n C_v \Delta T = 0{,}0406 \cdot \frac{5}{2} \cdot 8{,}314 \cdot (396{,}3 - 300) = 81{,}3$ J

$W = -81{,}3$ J



ii) $V = 0 \rightarrow W = 0$

$P_2V_2 = nRT_2 \rightarrow 1 \cdot 0{,}5 = 0{,}0406 \cdot 0{,}082 \cdot T_2$

$\rightarrow T_2 = 150{,}19$ K

$\Delta U = Q = n C_v \Delta T = 0{,}0406 \cdot \frac{5}{2} \cdot 8{,}314 \cdot (150{,}19 - \cancel{360{,}3}\ 396{,}3)$

$= -207{,}69$ J

iii) $W = P\Delta V = 101325 \cdot (1 - 0{,}5) \cdot 10^{-3} = 50{,}66$ J

$Q = n C_p \Delta T = 0{,}0406 \cdot \frac{7}{2} \cdot 8{,}314 \cdot (300 - 150{,}19) = 176{,}99$ J

$\Delta U = Q - W = 126{,}32$ J

$W_{Tot} = -81{,}3 + 0 + 50{,}66 = -30{,}64$ J

(11) i) $V_i = V_i \rightarrow \Delta W = 0$

$\frac{P_0 \cdot V_i}{T_0} = \frac{P_i \cdot V_i}{T_i} \rightarrow \frac{P_0 V_i}{290} = \frac{P_0/2 \cdot V_i}{T_i} \rightarrow T_i = 145$ K

$\Delta U = n C_v \Delta T = 1 \cdot \frac{3}{2} \cdot 8{,}314 \cdot (145 - 290) = -1808$ J

ii) $\Delta U = 0$ (isotérmico)

$\frac{P_0 V_i}{T_0} = \frac{P_{ii} \cdot V_{ii}}{T_{ii}} \Rightarrow \frac{P_0 V_i}{T_0} = \frac{\frac{P_0}{2} \cdot V_{ii}}{T_0} \rightarrow V_{ii} = 2 \cdot V_i$

$W = Q = nRT \cdot \ln \frac{V_{ii}}{V_i} = 1 \cdot 8{,}314 \cdot 290 \cdot \ln 2$

$= 1671$ J

(iii) $C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R \to \gamma = \frac{C_p}{C_V} = \frac{5}{3}$

$$P_0 V_i^{\gamma} = P_{iii} \cdot V_{iii}^{\gamma} \Rightarrow P_0 \cdot V_i^{\gamma} = \frac{P_0}{2} \cdot V_{iii}^{\gamma} \to V_{iii} = 1{,}51 \cdot V_i$$

$$\frac{P_0 \cdot V_i}{T_0} = \frac{P_{iii} \cdot V_{iii}}{T_{iii}} \to \frac{P_0 V_i}{290} = \frac{P_0/2 \cdot 1{,}51 V_i}{T_{iii}} \to T_{iii}' = 218{,}95\,K$$

(iv) $V_{iv} = 2V_i$

Expansão livre: $\Delta W = 0$

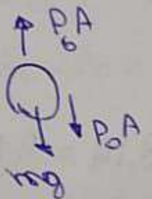
Isotérmico $\to \Delta U = 0$

$\to T_{iv} = 290\,K$

$\boxed{12}$ a) A pressão externa e interna atuam numa área A da bolinha, que corresponde a secção reta dela:

$$A = \pi a^2$$

Equilíbrio inicial:



$$\to P_G A = mg + P_0 A$$

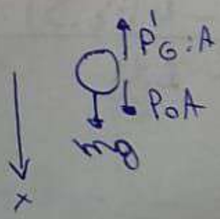
$$\to P_G = P_0 + \frac{mg}{A}$$, que é a pressão do gás.

Quando deslocamos a bola de um valor x bem pequeno o volume de gás diminui para: $V' = V - Ax$, e a pressão sobe para: P_G'

Processo adiabático: $P_G \cdot V^{\gamma} = P_G' \cdot (V - Ax)^{\gamma}$

$$\to P_G' = P_G \cdot \left(\frac{V}{V - Ax}\right)^{\gamma}$$

Resultante:



$$\to F_R = mg + P_0 A - A P_G \left(\frac{V}{V - Ax}\right)^{\gamma}$$

$$\to F_R = A\left(P_0 + \frac{mg}{A}\right) - A \cdot \left(P_0 + \frac{mg}{A}\right) \cdot \frac{V^{\gamma}}{(V - Ax)^{\gamma}}$$

$$\to F_R = A\left(P_0 + \frac{mg}{A}\right)\left[1 - \frac{V^{\gamma}}{(V - Ax)^{\gamma}}\right]$$

$$\to F_R = A \cdot P_G \left[\frac{(V - Ax)^{\gamma} - V^{\gamma}}{(V - Ax)^{\gamma}}\right]$$

$$= A.P_0\left[\frac{V^\gamma(1-\frac{A}{V}x)^\gamma - V^\gamma}{V^\gamma(1-\frac{Ax}{V})^\gamma}\right] \simeq A.P_0\left[\frac{1-\frac{A\gamma}{V}x - 1}{1}\right] = -\frac{A^2 P_0.\gamma}{V}.x$$

$$= m.a$$

$$\rightarrow a = -\underbrace{\frac{A^2.P_0\gamma}{mV}}_{\omega^2}.x \quad , \text{M.H.S}$$

$$\rightarrow \frac{2\pi}{T} = A.\sqrt{\frac{P_0\gamma}{mV}} \rightarrow \boxed{T = \frac{2}{a^2}\sqrt{\frac{mV}{P_0.\gamma}}}$$

b) $1{,}5 = \frac{2}{25.10^{-6}}\sqrt{\frac{10^{-2}.5.10^{-3}}{101325.\gamma}} \Rightarrow 1{,}875.10^{-5} = \sqrt{\frac{4{,}9.10^{-10}}{\gamma}} \rightarrow \gamma = \frac{4{,}9.10^{-10}}{3{,}5.10^{-10}} = 1{,}4$

B a) i) $P_0V_0 = P_i.V_i \rightarrow 1.22{,}4 = P_i.5 \rightarrow P_i = 4{,}48$ atm

ii) $P_{ii}.V_{ii} = nRT_{ii} \rightarrow T_{ii} = \frac{0{,}55.22{,}4}{1.0{,}082} = 150{,}24$ K

b) Em ii): $P_i.V_i^\gamma = P_{ii}.V_{ii}^\gamma$

$\rightarrow 4{,}48.5^\gamma = 0{,}55.22{,}4^\gamma \rightarrow 8{,}14 = 4{,}48^\gamma \rightarrow \ln 8{,}14 = \gamma.\ln 4{,}48$

$\rightarrow \gamma = 1{,}4$

$$\rightarrow \begin{cases} C_p = 1{,}4C_V \\ C_p - C_V = R \end{cases} \rightarrow 0{,}4C_V = R \rightarrow \begin{cases} C_V = \frac{5}{2}R \\ C_p = \frac{7}{2}R \end{cases}$$

c) $\Delta U = n.C_V.\Delta T = 1.\frac{5}{2}.8{,}314.(150{,}24 - 273{,}15) = -2554{,}685$

b) $W = W_i + W_{ii} = nRT.\ln\frac{V_{ii}}{V_i} + \frac{P_F.V_F - P_iV_i}{1-\gamma}$

$= 1.8{,}314.273.\ln\frac{5}{22{,}4} + \frac{(0{,}55.22{,}4 - 4{,}48.5).101325.10^{-3}}{1 - 1{,}4}$

$= -3403{,}73 + \frac{(-1021{,}356)}{(-0{,}4)}$

$= -850{,}34$ J