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Cap. 3

01 Velocidade v_0 do sistema $(m+M)$ após o impacto:

$$\vec{P}_1 = \vec{P}_2 \Rightarrow mV = (m+M).V_0 \Rightarrow V_0 = \frac{m}{m+M}.V$$

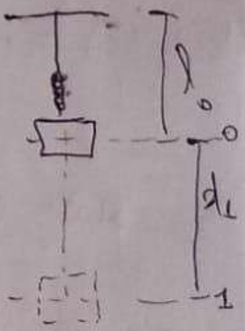
$$\text{MHS}: \omega = \sqrt{\frac{K}{m_T}} \rightarrow \omega = \sqrt{\frac{K}{m+M}}$$

$$x(t) = A.\cos(\omega t + \frac{\pi}{2}) = A\,\text{sen}(\omega t)$$

Em $t=0$ $\begin{cases} x(t=0) = 0 \\ v(t=0) = \pm\omega A \end{cases} \rightarrow |A| = \frac{V_0}{\omega} = \frac{mV}{m+M}\sqrt{\frac{m+M}{K}}$

$$\Rightarrow \boxed{x = \left(\frac{mV}{\sqrt{K(m+M)}}\right)\cdot \text{sen}\left(\sqrt{\frac{K}{m+M}}\cdot t\right)}$$

02 Seja d_1 a distância máxima da distensão da mola, a partir de l_0. Inicialmente, $V(0)=0$ e a mola está relaxada. Conservação da energia mecânica, em relação ao ponto mais baixo $U_G=0$

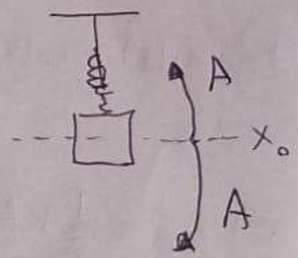


$$K_0 + U_{K_0} + U_{G_0} = K_1 + U_{K_1} + U_{G_1}$$

$$\Rightarrow 0 + 0 + mgd_1 = 0 + \frac{1}{2}Kd_1^2 + 0 \rightarrow d_1 = \frac{2mg}{K}$$

No equilíbrio, a mola se distende x_0:

$$mg = Kx_0 \Rightarrow x_0 = \frac{mg}{K}$$



A massa oscila com amplitude A, acima e abaixo de x_0, logo:

$$d_1 = x_0 + A \rightarrow \frac{2mg}{K} = \frac{mg}{K} + A \Rightarrow A = \frac{mg}{K}$$

Em $t=0$: $\begin{cases} Z(0) = l_0 + x_0 = l_0 + \frac{mg}{K} \\ V(0) = -\omega A\,\text{sen}(\alpha) \rightarrow \alpha = 0^\circ \end{cases}$, $\omega = \sqrt{\frac{K}{m}}$

$$Z_{máx} = l_0 + x_0 + A = l_0 + \frac{2mg}{K}$$

$$Z_{mín} = l_0 + x_0 - A = l_0$$

$$\rightarrow Z(t) = l_0 + \frac{mg}{K} - \frac{mg}{K}\cos\left(\sqrt{\frac{K}{m}}\,t\right)$$

03 a) $\begin{cases} x_1(t) = A_1 \cdot \cos(\omega t + \varphi_1) \\ v_1(t) = -\omega A_1 \operatorname{sen}(\omega t + \varphi_1) \end{cases}$ $\quad \therefore \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{100}{10^{-2}}} = 10^2$ rad/s

(mesma para ambos)

Condições iniciais:

$x_1(0) = -1cm \Rightarrow -10^{-2} = A_1 \cdot \cos\varphi_1$ $\rightarrow \frac{\omega \operatorname{sen}\varphi_1}{\cos\varphi_1} = \frac{-\sqrt{3}}{10^{-2}}$

$v_1(0) = -\sqrt{3}$ m/s $\Rightarrow -\sqrt{3} = -\omega A_1 \operatorname{sen}\varphi_1$

$\rightarrow tg\varphi_1 = -\sqrt{3} \Rightarrow \varphi_1 = -\frac{\pi}{3}$

Em $x_1(0) = -1cm = A_1 \cdot \cos\left(-\frac{\pi}{3}\right) \Rightarrow A_1 = -2cm$

$\begin{cases} x_2(t) = A_2 \cdot \cos(\omega t + \varphi_2) \\ v_2(t) = -\omega A_2 \operatorname{sen}(\omega t + \varphi_2) \end{cases}$

Condições iniciais:

$x_2(0) = +1cm \Rightarrow 10^{-2} = A_2 \cdot \cos\varphi_2$ $\rightarrow \frac{-\omega \operatorname{sen}\varphi_2}{\cos\varphi_2} = \frac{\sqrt{3}}{10^{-2}}$

$V_2(0) = +\sqrt{3}$ m/s $\Rightarrow \sqrt{3} = -\omega A_2 \operatorname{sen}\varphi_2$

$\rightarrow tg\varphi_2 = -\sqrt{3} \rightarrow \varphi_2 = -\frac{\pi}{3}$

E $x_2(0) = 1cm \Rightarrow 1cm = A_2 \cdot \cos\left(-\frac{\pi}{3}\right) \rightarrow A_2 = 2cm$

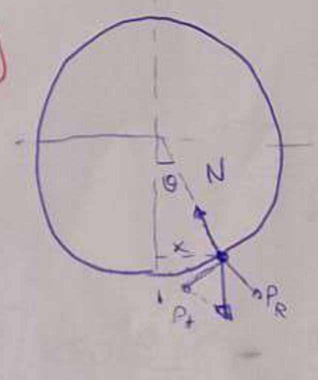
Logo $\begin{cases} x_1(t) = -2 \cdot \cos\left(100t - \frac{\pi}{3}\right), \text{ cm} \\ x_2(t) = 2 \cdot \cos\left(100t - \frac{\pi}{3}\right), \text{ cm} \end{cases}$

b) $x_1(t) = x_2(t) \Leftrightarrow \cos\left(100t - \frac{\pi}{3}\right) = 0 \Rightarrow 100t - \frac{\pi}{3} = \frac{\pi}{2}$

$\Rightarrow 100t = \frac{5\pi}{6} \rightarrow t = \frac{\pi}{120}$ s

c) $E_M = \frac{1}{2}KA_1^2 + \frac{1}{2}KA_2^2 = \frac{K}{2}(A_1^2 + A_2^2) = \frac{100}{2}(0{,}02^2 + 0{,}02^2) = 0{,}04$ J

04



Força restauradora: $F_R = -mg \operatorname{sen}\theta$

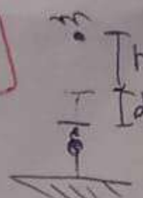
Para θ muito pequeno: arco $\cong x$

$\rightarrow F_R = -mg \cdot \frac{x}{R} = -\underbrace{\left(\frac{mg}{R}\right)}_{K} \cdot x$ → proporcional a x

→ M.H.S

$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{R}{g}}}$

05



A máxima compressão da mola é d.
Conservação da energia entre esse ponto e o inicial:

a) $mg(h+d) = \frac{K}{2}d^2 \rightarrow Kd^2 = 2mgh + 2mgd$

$$\rightarrow d^2 + \left(-\frac{2mg}{K}\right)\cdot d + \left(-\frac{2mgh}{K}\right) = 0$$

$$\Delta = \frac{4m^2g^2}{K^2} + \frac{8mgh}{K} = \frac{4m^2g^2}{K^2}\left(1 + \frac{2Kh}{mg}\right)$$

$$\text{Logo: } d = \frac{\frac{2mg}{K} \pm \frac{2mg}{K}\sqrt{1+\frac{2Kh}{mg}}}{2} = \underbrace{\frac{mg}{K}}_{x_0} \pm \underbrace{\frac{mg}{K}\sqrt{1+\frac{2Kh}{mg}}}_{A}$$

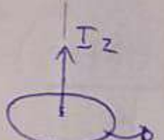
Deslocamento da mola devido ao peso de m:

$$mg = Kx_0 \rightarrow x_0 = \frac{mg}{K}$$

Temos um MHS em torno desse ponto: $x_0 \pm A \rightarrow \boxed{A = \frac{mg}{K}\sqrt{1+\frac{2Kh}{mg}}}$

b) $E_m = \frac{1}{2}KA^2 = \frac{K}{2}\cdot\frac{m^2g^2}{K^2}\left(1+\frac{2Kh}{mg}\right) = \frac{m^2g^2}{2K} + mgh$

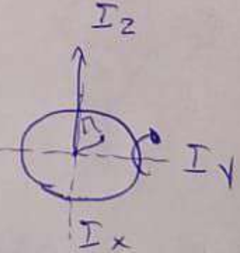
06 a) $\tau = -K.\theta \Rightarrow I\alpha = -K.\theta \rightarrow \ddot{\theta} = -\frac{K}{I}.\theta \rightarrow \omega^2 = \frac{K}{I}$



$I = \frac{MR^2}{2}$ $\rightarrow \frac{2\pi}{T_a} = \sqrt{\frac{K}{I_z}} \rightarrow T_a = 2\pi\sqrt{\frac{I_z}{K}}$

$$\rightarrow T_a = 2\pi\sqrt{\frac{MR^2}{2K}} = \pi\sqrt{\frac{4MR^2}{2K}} \rightarrow T_a = \pi R\sqrt{\frac{2M}{K}}$$

b)

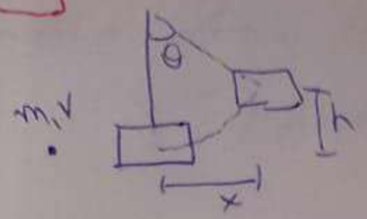


$$I_z = \int r^2 dm = \int (x^2+y^2)\,dm = \int x^2 dm + \int y^2 dm = I_x + I_z$$

Simetria: $I_x = I_Y = \frac{I_z}{2} = \frac{MR^2}{4}$

$$T_b = 2\pi\sqrt{\frac{I_Y}{K}} = 2\pi\sqrt{\frac{MR^2}{4K}} = \frac{2\pi R}{2}\sqrt{\frac{M}{K}} \rightarrow T_b = \pi R\sqrt{\frac{M}{K}}$$

07



$m.V = (M+m).V_0 \rightarrow V_0 = \dfrac{10^{-2}}{10+10^{-2}} \cdot 300 = 0{,}299\,m/s$

$\dfrac{M_T V_0^2}{2} = M_T g.h \rightarrow V_0^2 = 2g\,l(1-\cos\theta)$

$\rightarrow \cos\theta = 0{,}9954 \rightarrow \theta_{máx} = 5{,}487°$

$\rightarrow \text{sen}\,\theta = 0{,}0956$

$\theta_{máx}$ é pequeno: $x \cong l.\theta \rightarrow \theta(t) = \dfrac{x(t)}{l} = x(t)$ (MHS)

$A = l.\text{sen}\,\theta_{máx} = 1.\,0{,}0956 = 9{,}56\,cm$

$V_{máx} = V_0 = |\omega A| \rightarrow \omega = \dfrac{V_0}{A} = 3{,}12\,rad/s$ $\rightarrow \theta(t) = 9{,}56.\cos(3{,}12t + \frac{\pi}{2})$

$x(t=0) = 0 = A.\cos\varphi \rightarrow \varphi = \frac{\pi}{2}$

08 $\omega = \sqrt{\dfrac{K}{M_T}} = \sqrt{\dfrac{K}{M+m}}$

Somente a força de atrito atua na horizontal, em m. Logo, ela é a responsável pelo MHS de m.

$F_R = F_e \leq \mu_e N \Rightarrow m.a(t) \leq \mu_e mg \rightarrow a(t) \leq \mu_e . g$

Mas: $a(t) = -\omega^2 A\cos(\omega t + \varphi) \rightarrow a_{máx} = \omega^2.A$

$\rightarrow \omega^2.A \leq \mu_e g \Rightarrow \dfrac{K}{M+m}.A \leq \mu_e g \rightarrow A_{máx} = \dfrac{\mu_e.g\,(M+m)}{K}$

09 Massa do densímetro:

$P = E_0 \rightarrow mg = \rho V_0 g \rightarrow m = \rho V_0$

Afundando x:

$F_R = P - E_x = mg - \rho V g = \rho V_0 g - \rho(V_0 + Ax)g$

$= \rho V_0 g - \rho V_0 g - \rho A g.x$

$= -\rho A g.x = -Kx \rightarrow$ MHS: $k = \rho A g$

$\omega = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{\rho A g}{\rho V_0}} \rightarrow \omega = \sqrt{\dfrac{Ag}{V_0}}$

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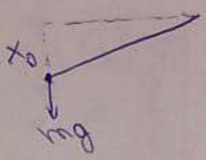


$mg = Kx_0 \to \frac{k}{m} = \frac{g}{x_0}$

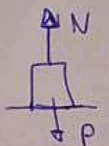
Força restauradora: $-Kx \Rightarrow \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{x_0}} = \sqrt{\frac{9,8}{0,05}} = 14 \, rad/s$

11 a) $x(t) = A\cos(\omega t + \varphi)$ ∴ $x(0) = 0 \to A\cos\varphi = 0$

$v(t) = -\omega A \sin(\omega t + \varphi)$ $\quad v(0) = +0,8 \to \sin\varphi = \frac{0,8}{-0,8} = -1$

$a(t) = -\omega^2 A \cdot \cos(\omega t + \varphi)$

$\to \varphi = -\frac{\pi}{2}$



$F_R = N - P = m \cdot a(t)$

Perda de contato: $N = 0 \to 0 - mg = m \cdot a(t) \to a(t) = -g$

$\Rightarrow -g = -\omega^2 \cdot x \to x = \frac{g}{\omega^2} = \frac{9,8}{20^2} = 0,0245 \, m = 2,45 \, cm$

b) Em $x = 2,45 \, cm \to 2,45 = 4 \cdot \cos(20t - \frac{\pi}{2}) \to t = 0,124 \, s$

$\to V(t = 0,124 \, s) = -20 \cdot 0,04 \sin(20 \cdot 0,124 - \frac{\pi}{2}) \to |V| = 0,63 \, m/s$

Conservação da energia: $\frac{mV^2}{2} = mgh \to h = \frac{V^2}{2g} = \frac{0,63^2}{2 \cdot 9,8} = 2,03 \, cm$

12 Sistema conservativo $\to E$ constante $\to \frac{dE}{dt} = 0$

$\to \frac{d(\dot{q})^2}{dt} = \frac{d(\dot{q})^2}{d\dot{q}} \cdot \frac{d\dot{q}}{dt} = 2\ddot{q} \cdot \dot{q}$

$\frac{dq^2}{dt} = \frac{dq^2}{dq} \cdot \frac{dq}{dt} = 2q \cdot \dot{q}$

$\to \frac{dE}{dt} = 2a\ddot{q}\dot{q} + 2a\omega^2 q \cdot \dot{q} = 0$

$\to \dot{q}(\ddot{q} + \omega^2 q) = 0$

$\hookrightarrow \ddot{q} + \omega^2 q = 0$

$\hookrightarrow$ MHS, ω

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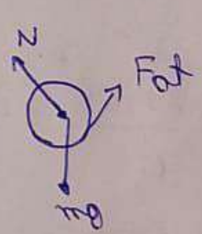


Sem deslizar: Fat gera torque

$\tau = -F_{at} \cdot r = I \cdot \alpha$

$\Rightarrow -F_{at} \cdot r = \frac{2}{5} m r^2 \cdot \left(\frac{a}{r}\right) \to F_{at} = -\frac{2}{5} m a$

Força restauradora: $F_R = -mg\sin\theta + F_{at} = m \cdot a$

$\Rightarrow -mg \cdot \frac{x}{R} - \frac{2ma}{5} = ma \Rightarrow -\frac{mg}{R} \cdot x = \frac{7ma}{5} \Rightarrow a = -\left(\frac{5g}{7R}\right) \cdot x \to$ MHS

$\omega^2 = \frac{5g}{7R} \to \omega = \sqrt{\frac{5g}{7R}}$

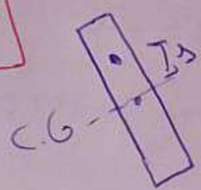
14 a) $I_a = I_{cm_a} + Md^2 = MR^2 + MR^2 = 2MR^2$

$$\to \tau_a = 2\pi\sqrt{\frac{I_a}{MgD_a}} = 2\pi\sqrt{\frac{2MR^2}{Mg.R}} = 2\pi\sqrt{\frac{2R}{g}} = 2\pi\sqrt{\frac{l}{g}} = \tau$$

b) $I_b = I_{cm_b} + Md^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

$$\tau_b = 2\pi\sqrt{\frac{I_b}{MgD_b}} = 2\pi\sqrt{\frac{3MR^2}{2Mg R}} = 2\pi\sqrt{\frac{2R}{g}}\cdot\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}\cdot\tau$$

15



$$I_o = I_{cm} + md^2 = \frac{Ml^2}{12} + Ms^2 = \frac{M}{12}(l^2 + 12s^2)$$

$$T = 2\pi\sqrt{\frac{I_o}{MgD}} = 2\pi\sqrt{\frac{M}{12gM}\frac{(l^2+12s^2)}{s}} = 2\pi\sqrt{\frac{1}{12g}\left(\frac{l^2}{s}+12s\right)}$$

$$f(s) = l^2 s^{-1} + 12s \to f'(s) = 0 \Rightarrow -l^2 s^{-2} + 12 = 0 \Rightarrow \frac{l^2}{s^2} = 12 \to s = \frac{l}{2\sqrt{3}}$$

Logo: $f\left(\frac{l}{2\sqrt{3}}\right) = l^2\cdot\frac{2\sqrt{3}}{l} + 12\cdot\frac{l}{2\sqrt{3}} = 2\sqrt{3}\,l + 2\sqrt{3}\,l = 4\sqrt{3}\,l$

$$T_{min} = 2\pi\sqrt{\frac{1}{12g}\cdot 4\sqrt{3}\,l} = \cancel{2\pi\sqrt{\frac{l}{g}}\sqrt{\frac{\sqrt{3}}{3}}} = 2\pi\sqrt{\frac{l}{g}\cdot\frac{\sqrt{3}}{3}}$$

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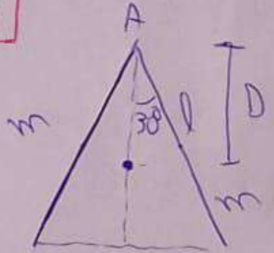


$$D = \frac{h}{2} = \frac{1}{2}\,l\,\frac{\sqrt{3}}{2} = \frac{l\sqrt{3}}{4}$$

$$I_A = 2(I_{cm} + Md^2) = 2\left(\frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2\right) = \frac{ml^2}{6} + \frac{ml^2}{2} = \frac{ml^2 + 3ml^2}{6} = \frac{2ml^2}{3}$$

$$\to T = 2\pi\sqrt{\frac{I_A}{(2m)gD}} = 2\pi\sqrt{\frac{2ml^2}{3.2mg.\frac{l\sqrt{3}}{4}}} = 2\pi\sqrt{\frac{4l}{3\sqrt{3}g}} = 4\pi\sqrt{\frac{l}{g}\frac{\sqrt{3}}{3.3}}$$

$$= \frac{4\pi}{3}\sqrt{\frac{l}{g}.\sqrt{3}}$$

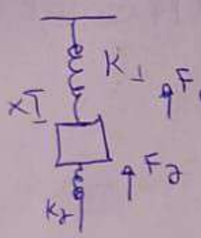
17 $\begin{cases} x(t) = A\cos(\omega t + \varphi) \\ v(t) = -\omega A \operatorname{sen}(\omega t + \varphi) \end{cases}$ $\therefore$ $\begin{cases} \omega = \sqrt{\frac{K}{m}} \to K = m\omega^2 \\ \omega t = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2} \end{cases}$

$E_c = 3E_p \to \frac{1}{2}mv^2 = 3\cdot\frac{1}{2}Kx^2$

$$\frac{1}{2}\cancel{m\omega^2A^2}\operatorname{sen}^2\left(\frac{\pi}{2}+\varphi\right) = \frac{3}{2}\cancel{m\omega^2A^2}\cos^2\left(\frac{\pi}{2}+\varphi\right)$$

$\to tg\left(\frac{\pi}{2}+\varphi\right) = \pm\sqrt{3}$ $\begin{cases} \frac{\pi}{2}+\varphi = \frac{\pi}{3} \to \varphi = -\frac{\pi}{6} \\ \frac{\pi}{2}+\varphi = \pi-\frac{\pi}{3} \to \varphi = \frac{\pi}{6} \end{cases}$ $\to \boxed{\varphi = \pm\frac{\pi}{6} + K\pi}$

18 a) Deslocando x além do ponto de equilíbrio



$F_R = -K_1x - K_2x = -(K_1+K_2)x = -K.x \to K = K_1+K_2$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{K_1+K_2}{m}}$$

b) Deslocando o bloco em x, os molas deslocam x_1 e x_2.

$x = x_1+x_2 \to \frac{F}{K_R} = \frac{F}{K_1} + \frac{F}{K_2} \Rightarrow \frac{1}{K_R} = \frac{K_1+K_2}{K_1K_2} \to K_R = \frac{K_1K_2}{K_1+K_2}$

$$\omega = \sqrt{\frac{K_R}{m}} = \sqrt{\frac{K_1K_2}{m(K_1+K_2)}}$$

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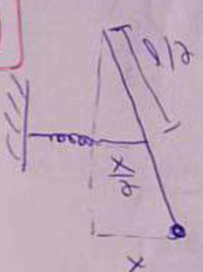

Se afastarmos a massa m de x, a mola estica $\frac{x}{2}$

A mola produz torque, vamos usar o MHS com torque:

$\sum \tau = -K\left(\frac{x}{2}\right)\cdot\frac{l}{2} - mg\operatorname{sen}\theta\cdot l$, mas $\begin{cases} \operatorname{sen}\theta \cong \theta \\ x \cong l.\theta \end{cases}$

$$\Rightarrow \sum \tau = -\frac{Kl^2}{4}\cdot\theta - mgl\,\theta = -\left(\frac{Kl^2}{4}+mgl\right)\cdot\theta = -K_T\cdot\theta$$

$$\omega = \sqrt{\frac{K_T}{I}} = \sqrt{\frac{\frac{Kl^2}{4}+mgl}{ml^2}} = \sqrt{\frac{K}{4m}+\frac{g}{l}}$$

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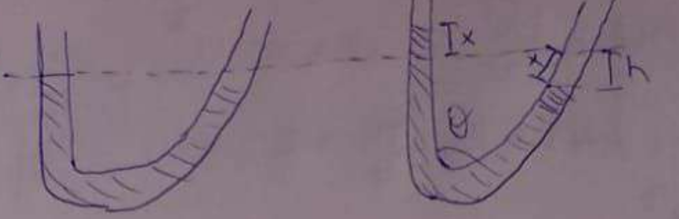

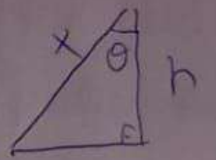

$\cos\theta = \frac{h}{x}$

$\rightarrow h = x\cos\theta$

Quando o fluido sobe x na esquerda, desce $x\cos\theta$ na direita.
Assim, na esquerda, temos uma coluna de altura H acima do nível do líquido na direita, que é a força restauradora:

$$H = x + x\cos\theta = (1+\cos\theta)x$$

$$F_R = -P_{col} = -mg = -\rho V g = -\rho A H g = -\rho A g(1+\cos\theta).x = -kx$$

$$\Rightarrow k = \rho A g(1+\cos\theta)$$

Logo, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\rho A g(1+\cos\theta)}{m}}$

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$k = \frac{d^2U}{dr^2}$ (3.3.51, pág. 55)

a) $\frac{dU}{dr} = 0$ (mínimo potencial) $\rightarrow ke^2.r^{-2} - 10.B.r^{-11} = 0 \rightarrow B = \frac{ke^2r^9}{10}$

$$k = \frac{d^2U}{dr^2} = -2ke^2r^{-3} + 110Br^{-12} = -\frac{2ke^2}{r^3} + 11\cancel{0}.\frac{ke^2r^9}{1\cancel{0}}.\frac{1}{r^{12}} = \frac{-2ke^2}{r^3} + 11\frac{ke^2}{r^3}$$

$$= \frac{9ke^2}{r^3}$$

$$\Rightarrow k(\text{MHS}) = \frac{9.9.10^9.1,66^2.10^{-38}}{1,28^3.10^{-6}} = 989 \text{ N/m}$$

b) Massa reduzida: $\mu = \frac{m_1m_2}{m_1+m_2} \Rightarrow \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k(m_1+m_2)}{m_1m_2}}$

$$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k(m_1+m_2)}{m_1m_2}} = 4{,}2.10^{14}\text{ Hz}$$

22) a) $\begin{cases} e^{ix} = \cos x + i\,\text{sen}\,x \\ e^{-ix} = \cos x - i\,\text{sen}\,x \end{cases} \rightarrow \oplus\ \cos x = \dfrac{e^{ix}+e^{-ix}}{2}$

$\ominus\ \text{sen}\,x = \dfrac{e^{ix}-e^{-ix}}{2i}$

$\rightarrow \cos(a+b) = \dfrac{e^{i(a+b)}+e^{-i(a+b)}}{2} = \dfrac{e^{ia}\cdot e^{ib} - e^{-ia}\cdot e^{-ib}}{2} = \Big[(\cos a + i\,\text{sen}\,a)(\cos b + i\,\text{sen}\,b) - (\cos(-a)+i\,\text{sen}(-a))(\cos(-b)+i\,\text{sen}(-b))\Big]/2$

$= \underbrace{(\cos a\cos b - \text{sen}\,a\,\text{sen}\,b)}_{\cos(a+b)} + i\underbrace{(-\text{sen}\,a\cos b - \cos a\,\text{sen}\,b)}_{-\text{sen}(a+b)}$

b) $\cos 3a = \dfrac{e^{3ai}-e^{-3ai}}{2} = \dfrac{(e^{ai})^3-(e^{-ai})^3}{2} = \dfrac{(\cos a + i\,\text{sen}\,a)^3 + [\cos(-a)+i\,\text{sen}(-a)]^3}{2}$

$= \dfrac{\cos^3 a - i(3\,\text{sen}\,a\cos^2 a - \text{sen}^3 a) - 3\,\text{sen}^2 a\cos a}{2} \overset{\mathbb{R}e}{=} \cos^3 a - 3\,\text{sen}^2 a\cos a$

$\text{sen}\,3a = \dfrac{(e^{ai})^3 - [e^{(-ai)}]^3}{2i} = \dfrac{(\cos a + i\,\text{sen}\,a)^3 - [\cos(-a)+i\,\text{sen}(-a)]^3}{2i}$

$= \dfrac{\cos^3 a - i(3\,\text{sen}\,a\cos^2 a - \text{sen}^3 a) - 3\,\text{sen}^2 a\cos a}{2i} \overset{\mathbb{R}e}{=} 3\cos^2 a\,\text{sen}\,a - \text{sen}^3 a$

23) a) $\cos(ix) = \dfrac{e^{iix}+e^{-iix}}{2} = \dfrac{e^{x}+e^{-x}}{2} = \cosh x$

$\text{sen}(ix) = \dfrac{e^{iix}-e^{-iix}}{2i} = \dfrac{e^{x}-e^{-x}}{2i} = i\left(\dfrac{e^{x}-e^{-x}}{2}\right) = i\,\text{senh}\,x$

b) $\cos^2 hx - \text{sen}^2 hx = \dfrac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{4} = \dfrac{4}{4} = 1$

c) $\text{senh}(2x) = \dfrac{(e^{x})^2+(e^{-x})^2}{2} = \dfrac{(e^{x}+e^{-x})(e^{x}-e^{-x})}{2} = 2\,\text{senh}\,x\cdot\cosh x$

24 $x_1(t) = \cos\left(\omega t - \frac{\pi}{6}\right) \rightarrow \begin{cases} A_1 = 1 \\ \varphi_1 = -\frac{\pi}{6} \end{cases}$

$x_2(t) = \sin(\omega t) = \cos\left(\omega t - \frac{\pi}{2}\right) \begin{cases} A_2 = 1 \\ \varphi_2 = -\frac{\pi}{2} \end{cases}$

$\rightarrow A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1 - \varphi_2) = 1+1+2\cos\frac{\pi}{3} = 1+1+2\cdot\frac{1}{2} = 3 \Rightarrow A = \sqrt{3}$

Lei dos senos nos fasores: $\frac{\sin\beta}{A_2} = \frac{\sin(\varphi_2 - \varphi_1)}{A_1}$

$\Rightarrow \sin\beta = \frac{1}{\sqrt{3}}\sin\left(-\frac{\pi}{3}\right) = -\frac{1}{2} \Rightarrow \beta = -\frac{\pi}{6} \rightarrow \varphi = \beta + \varphi_1 = -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{\pi}{3}$

$\Rightarrow x = \sqrt{3}\cdot\cos\left(\omega t - \frac{\pi}{3}\right)$