Cap. 4, Vol. 2 - Oscilações Amortecidas e Forçadas

01) i) Amortecimento crítico: $\begin{cases} \omega_0 = \gamma/2 \\ x(t) = t \cdot e^{-\frac{\gamma}{2}t} \end{cases} \rightarrow x(t) = t \cdot e^{-\omega_0 t}$

$\rightarrow \dot{x}(t) = e^{-\omega_0 t} - \omega_0 t \cdot e^{-\omega_0 t} = e^{-\omega_0 t} - \omega_0 x$

$\rightarrow \ddot{x}(t) = -\omega_0 e^{-\omega_0 t} - \omega_0 \dot{x}$

Logo, $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = -\omega_0 e^{-\omega_0 t} - \omega_0 \dot{x} + 2\omega_0 e^{-\omega_0 t} - 2\omega_0^2 x + \omega_0^2 \cancel{x}$

$= \omega_0 (e^{-\omega_0 t} - \dot{x} - \omega_0 x)$

$= \omega_0 (e^{-\omega_0 t} - e^{-\omega_0 t} + \omega_0 x - \omega_0 x) = 0$

ii) Para oscilações forçadas e $\omega = \omega_0$: $x(t) = \frac{F_0}{2m\omega_0} t \operatorname{sen}(\omega_0 t)$

$\rightarrow \dot{x}(t) = \frac{F_0}{2m\omega_0} \operatorname{sen}(\omega_0 t) + \frac{F}{2m\omega_0} t \cdot \omega_0 \cdot \cos(\omega_0 t) = \frac{F_0}{2m\omega_0} \operatorname{sen}(\omega_0 t) + \frac{F}{2m} t \cdot \cos(\omega_0 t)$

$\rightarrow \ddot{x}(t) = \frac{F_0}{2m\omega_0} \cdot \omega_0 \cdot \cos(\omega_0 t) + \frac{F}{2m} \cos(\omega_0 t) - \frac{F}{2m} t \cdot \omega_0 \operatorname{sen}(\omega_0 t)$

$= \frac{F}{m} \cos(\omega_0 t) - \frac{F\omega_0}{2m} t \operatorname{sen}(\omega_0 t)$

Logo: $\ddot{x} + \omega_0^2 x = \frac{F}{m} \cos(\omega_0 t) - \frac{F\omega_0}{2m} t \operatorname{sen}(\omega_0 t) + \omega_0^2 \frac{F_0 t \operatorname{sen}(\omega_0 t)}{2m\omega_0} =$

$= \frac{F}{m} \cos(\omega_0 t) - \frac{F\omega_0}{2m} t \operatorname{sen}(\omega_0 t) + \frac{F\omega_0}{2m} t \operatorname{sen}(\omega_0 t) = \frac{F}{m} \cos(\omega_0 t)$

02) i) $Q = 10 = \frac{\omega_0}{\gamma} \rightarrow \omega_0 = 10\gamma$, onde $\gamma = b/m \rightarrow b = m\gamma$

ii) $E_m(t) = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \Rightarrow \frac{dE_m}{dt} = m\dot{x}\ddot{x} + kx\dot{x} = \dot{x}(m\ddot{x} + kx)$

Mas: $m\ddot{x} = -kx - b\dot{x} \Rightarrow \frac{dE_m}{dt} = -b\dot{x}^2 = -m\gamma\dot{x}^2$

Mas: $\frac{dE_m}{dt} = -4E_c \Rightarrow -m\gamma\dot{x}^2 = -4 \cdot \frac{1}{2} m \dot{x}^2 \Rightarrow \gamma = 2 \rightarrow \omega_0 = 20\,\text{rad/s}$

iii) $\omega = \sqrt{\omega_0^2 - (\frac{\gamma}{2})^2} = \sqrt{400 - 1} \cong 20\,\text{rad/s}$

v) $x(0)=0 \therefore x(t) = A e^{-\frac{\gamma}{2}t} \cdot \cos(\omega t+\varphi)$

$\Rightarrow 0 = A \cdot \cos(\varphi) \Rightarrow \cos\varphi = 0$

$\sin\varphi = \pm 1$

$\dot{x}(t) = -\frac{A\gamma}{2} e^{-\frac{\gamma}{2}t} \cdot \cos(\omega t+\varphi) - \omega A e^{-\frac{\gamma}{2}t} \cdot \sin(\omega t+\varphi)$

$\dot{x}(0) = 5 \Rightarrow 5 = -A \cdot \cos\varphi - 20A \sin(\varphi)$

$\Rightarrow 5 = -20A \cdot \sin\varphi \Rightarrow A\sin\varphi = -\frac{1}{4} \Rightarrow \begin{cases} A = \frac{1}{4} \\ \sin\varphi = -1 \\ \varphi = -\pi/2 \end{cases}$

Logo, $\boxed{x(t) = \frac{1}{4} \cdot e^{-t} \cdot \cos(20t - \frac{\pi}{2})}$

03 Oscilador livre amortecido:

a) $x(t) = A \cdot e^{-\frac{\gamma}{2}t} \cdot \cos(\omega t+\varphi)$, $\omega = \sqrt{\omega_0^2 - (\frac{\gamma}{2})^2} \cong \omega_0$, $\gamma << \omega_0$

$$r = \frac{x(t+\tau)}{x(t)} \overset{t=0}{=} \frac{x(\tau)}{x(t)} = \frac{A \cdot e^{-\frac{\gamma}{2}\tau} \cdot \cos(\frac{2\pi}{\tau}\cdot\tau+\varphi)}{A \cdot e^{-\frac{\gamma}{2}\cdot 0} \cdot \cos(\varphi)} = \frac{e^{-\frac{\gamma}{2}\cdot\tau} \cdot \cos(2\pi+\varphi)}{\cos\varphi} = e^{-\frac{\gamma}{2}\tau}$$

Logo: $\delta = |\ln r| = |-\frac{\gamma}{2}\tau| = +\frac{\tau \cdot \gamma}{2}$

b) $A(t) = A \cdot e^{-\frac{\gamma}{2}t} \Rightarrow \frac{A}{2} = A \cdot e^{-\frac{\gamma}{2}t} \Rightarrow 2^{-1} = e^{-\frac{\gamma}{2}t}$

$\Rightarrow -\ln 2 = -\frac{\gamma}{2}t \Rightarrow t = \frac{2\ln 2}{\gamma}$

Logo, $n = \frac{t}{\tau} = \frac{2\ln 2}{\tau \cdot \gamma} = \frac{2\ln 2}{2\delta} \rightarrow n = \frac{\ln 2}{\delta}$

04 a) Oscilador criticamente amortecido: $\gamma/2 = \omega_0 \rightarrow \gamma = 2\omega_0$

$x(t) = e^{-\frac{\gamma}{2}t}[C + Dt]$

Equilíbrio: $x(0) = 0 \Rightarrow e^{-\frac{\gamma}{2}\cdot 0}[C + D\cdot 0] = 0 \Rightarrow C = 0$

$\Rightarrow x(t) = D \cdot t \cdot e^{-\frac{\gamma}{2}t} = Dt e^{-\omega_0 t}$

$\Rightarrow \dot{x}(t) = De^{-\omega_0 t} - \omega_0 D t e^{-\omega_0 t} = De^{-\omega_0 t}(1 - \omega_0 t)$

$x(t=1) = 3{,}68 \Rightarrow 3{,}68 = D \cdot 1 \cdot e^{-\omega_0} \Rightarrow D = 3{,}68 \cdot e^{\omega_0}$

Deslocamento máximo em $t = 1s \Rightarrow \dot{x}(1) = 0 \Rightarrow D \cdot e^{-\omega_0 \cdot 1}(1 - \omega_0) = 0$

$E: D = 3,68 \cdot e^{1} \cong 10$

Logo, $V_0 = \dot{x}(t=0) = D \cdot e^{-w_0 \cdot 0}(1 - w_0 \cdot 0) = D = 10\, m/s$

b) $x_0 = 2m \Rightarrow x(0) = 2$

$$\Rightarrow e^{-w_0 \cdot 0}[C + D \cdot 0] = 2 \Rightarrow C = 2$$

$$\Rightarrow x(t) = 2e^{-w_0 t} + D.t.e^{-w_0 t}$$

Velocidade: $\dot{x}(t) = -2w_0 e^{-w_0 t} + De^{-w_0 t} - Dw_0 t e^{-w_0 t}$

$$V_0 = \dot{x}(0) = 10 \Rightarrow -2w_0 + D - w_0 D 0 = 10$$

$$-2.1 + D = 10 \Rightarrow D = 12$$

Logo: $x(t) = e^{-w_0 t}[C + Dt] \Rightarrow x(t) = e^{-t}(2 + 12t)\ m$

05 $m\ddot{z} = -\rho\dot{z} \Rightarrow \ddot{z} + \frac{\rho}{m}\dot{z} = 0$

Possível solução: $z = z_0 e^{wt} \rightarrow \begin{cases} \dot{z} = z_0 w e^{wt} \\ \ddot{z} = z_0 w^2 e^{wt} \end{cases}$

$$\Rightarrow z_0 w^2 e^{wt} + \frac{\rho}{m} z_0 w e^{wt} = 0$$

$$\Rightarrow z_0 w e^{wt}\left(w + \frac{\rho}{m}\right) = 0 \Rightarrow \begin{cases} w_1 = 0 \\ w_2 = -\frac{\rho}{m} \end{cases}$$

Solução geral: $z(t) = Ae^{w_1 t} + Be^{w_2 t} = A + Be^{w_2 t}$

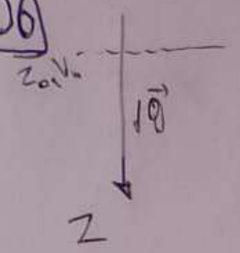
Condições iniciais: $z_0 = z(0) = A + B$

Velocidade: $\dot{z}(t) = Bw_2 e^{w_2 t} \rightarrow V_0 = \dot{z}(0) = B.\left(-\frac{\rho}{m}\right) \Rightarrow B = -\frac{mV_0}{\rho}$

$$A = z_0 + \frac{mV_0}{\rho}$$

$$\Rightarrow z(t) = z_0 + \frac{mV_0}{\rho}\left(1 - e^{-\frac{\rho}{m}t}\right)$$

06



$m\ddot{z} = mg - \rho\dot{z} \Rightarrow \ddot{z} + \frac{\rho}{m}\dot{z} = g$ ← não-homogênea

Solução: $z(t) = z_h(t) + z_p(t)$

Onde ~~: $z_h(t) = z_0 + \frac{mv_0}{\rho}(1 - e^{-\frac{\rho}{m}t})$, da questão anterior~~

$z_h(t) = A + B e^{-\frac{\rho}{m}t}$, da questão anterior

Como $f(x) = g$ tem grau 0 → $z_p(t) = Ct + D$ → $\begin{cases} \dot{z}_p(t) = C \\ \ddot{z}_p(t) = 0 \end{cases}$

Na EDO: $0 + \frac{\rho}{m} \cdot C = g \to C = \frac{mg}{\rho}$

Logo: $z(t) = z_h(t) + z_p(t) = A + B e^{-\frac{\rho}{m}t} + \frac{mg}{\rho}t + D$, seja: $E = A + D$ = cte

Velocidade: $\dot{z}(t) = -\frac{B\rho}{m} \cdot e^{-\frac{\rho}{m}t} + \frac{mg}{\rho}$

Condições iniciais: $z(0) = z_0 \Rightarrow E + B = z_0$

$\dot{z}(0) = v_0 \Rightarrow -\frac{B\rho}{m} + \frac{mg}{\rho} = v_0 \to B = \frac{m^2 g}{\rho^2} - \frac{mv_0}{\rho}$

Então: $z(t) = z_0 + \frac{mv_0}{\rho} - \frac{m^2 g}{\rho^2} + \frac{m^2 g}{\rho^2} e^{-\frac{\rho}{m}t} - \frac{mv_0}{\rho} e^{-\frac{\rho}{m}t} + \frac{mg}{\rho}t$ $\quad E = z_0 + \frac{mv_0}{\rho} - \frac{m^2 g}{\rho^2}$

$\Rightarrow z(t) = z_0 + \frac{mv_0}{\rho}(1 - e^{-\frac{\rho}{m}t}) - \frac{m^2 g}{\rho^2}(1 - e^{-\frac{\rho}{m}t}) + \frac{mg}{\rho}t$

$\Rightarrow z(t) = z_0 + \frac{mg}{\rho}t + \left(\frac{mv_0}{\rho} - \frac{m^2 g}{\rho^2}\right)(1 - e^{-\frac{\rho}{m}t})$

Para $\dot{z}(t \to \infty) = \frac{mg}{\rho}$, velocidade terminal, que é quando a força de resistência do ar se torna igual a força peso e a partícula passa a cair com velocidade constante:

$v_T = \frac{mg}{\rho}$

$\boxed{07}$ $m\ddot{x} = -Kx + F \Rightarrow m\ddot{x} + Kx = F_0 sen(\omega t)$

$$\Rightarrow \ddot{x} + \frac{K}{m}x = \frac{F_0}{m} sen(\omega t)$$

$$\Rightarrow \ddot{x} + \omega_0^2 x = \frac{F_0}{m} sen(\omega t)$$

- Solução homogênea $x_h(t)$: $\ddot{x} + \omega_0^2 x = 0$

Do MHS, sabemos que $x_h(t)$ é da forma: $A sen(\omega_0 t + \varphi)$
(pode ser cosseno, mas com seno facilitará
as contas, pois $F = F_0 sen(\omega t)$)

$$\rightarrow x_h(t) = A sen(\omega_0 t + \varphi)$$

- Solução particular: $x_p(t) = B sen(\omega t) \Rightarrow \begin{cases} \dot{x}_p(t) = \omega B cos(\omega t) \\ \ddot{x}_p(t) = -\omega^2 B sen(\omega t) \end{cases}$

$\rightarrow -\omega^2 B sen(\omega t) + \omega_0^2 B sen(\omega t) = \frac{F_0}{m} sen(\omega t)$

Para $sen(\omega t) \neq 0$: $\Rightarrow B(\omega_0^2 - \omega^2) = \frac{F_0}{m} \rightarrow B = \frac{F_0}{m(\omega_0^2 - \omega^2)}$

Assim:

$x(t) = x_h(t) + x_p(t) \Rightarrow x(t) = A sen(\omega_0 t + \varphi) + B sen(\omega t)$

$\rightarrow \dot{x}(t) = A\omega_0 cos(\omega_0 t + \varphi) + B\omega cos(\omega t)$

Condições iniciais: $x(0) = 0 \rightarrow 0 = A sen\varphi \rightarrow sen\varphi = 0 \rightarrow \begin{cases} \varphi = 0 \\ cos\varphi = 1 \end{cases}$

$\dot{x}(0) = 0 \rightarrow 0 = A\omega_0 + B\omega$

$$\Rightarrow A = -\frac{\omega}{\omega_0} \cdot \frac{F_0}{m(\omega_0^2 - \omega^2)}$$

Solução: $x(t) = x_h(t) + x_p(t)$

$$x(t) = -\frac{\omega}{\omega_0} \frac{F}{m(\omega_0^2 - \omega^2)} \cdot sen(\omega_0 t) + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cdot sen(\omega t)$$

$$\Rightarrow x(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)} \left[sen(\omega t) - \frac{\omega}{\omega_0} sen(\omega_0 t) \right]$$

08 Oscilador não amortecido forçado:

$$m\ddot{x} = -Kx + F \Rightarrow \ddot{x} + \underbrace{\frac{k}{m}}_{\omega_0^2} x = \frac{F_0}{m} e^{-\beta t}$$

Solução homogênea: $x_h(t) = \ddot{x} + \frac{k}{m}x = 0 \rightarrow x_h(t) = A\cos(\omega_0 t + \varphi)$

Possível solução particular: $x_p(t) = C . e^{-\beta t}$

$$\Rightarrow \begin{cases} \dot{x}_p(t) = -\beta C e^{-\beta t} \\ \ddot{x}_p(t) = \beta^2 C e^{-\beta t} \end{cases}$$

Na EDO: $\beta^2 C e^{-\beta t} + \omega_0^2 . C . e^{-\beta t} = \frac{F_0}{m} e^{-\beta t} \rightarrow \boxed{C = \frac{F_0}{m(\omega_0^2 + \beta^2)}}$

Logo: $\begin{cases} x(t) = x_h(t) + x_p(t) = A\cos(\omega_0 t + \varphi) + C . e^{-\beta t} \\ \dot{x}(t) = -\omega_0 A \,\text{sen}(\omega_0 t + \varphi) - \beta C e^{-\beta t} \end{cases}$

Condições iniciais:

$x(0) = 0 \Rightarrow A . \cos\varphi + C = 0 \rightarrow \cos\varphi = -C/A$

$\dot{x}(0) = 0 \Rightarrow -\omega_0 A \,\text{sen}\,\varphi - \beta C = 0 \rightarrow \text{sen}\,\varphi = \frac{-\beta C}{\omega_0 A}$

$\rightarrow \frac{C^2}{A^2} + \frac{\beta^2 C^2}{\omega_0^2 A^2} = 1$

$A = \frac{C\sqrt{\omega_0^2 + \beta^2}}{\omega_0}$

Solução geral: $x(t) = x_h(t) + x_p(t)$

$$\Rightarrow x(t) = A . [\cos(\omega_0 t) . \cos\varphi - \text{sen}(\omega_0 t) . \text{sen}\,\varphi] + C e^{-\beta t}$$

$$\Rightarrow x(t) = A\left[\cos(\omega_0 t) . \left(\frac{-C}{A}\right) - \text{sen}(\omega_0 t) . \left(\frac{-\beta}{\omega_0}\right) . \frac{C}{A}\right] + C e^{-\beta t}$$

$$\Rightarrow x(t) = C\left[e^{-\beta t} + \frac{\beta}{\omega_0}\text{sen}(\omega_0 t) - \cos\omega_0 t\right]$$

$$x(t) = \frac{F_0}{m(\omega_0^2 + \beta^2)}\left[e^{-\beta t} - \cos(\omega_0 t) + \frac{\beta}{\omega_0}\text{sen}(\omega_0 t)\right]$$

09 $m = dV = 8 . 10^3 = 8000\,g = 8\,kg$

No equilíbrio, a mola se desloca x_0:

$Kx_0 + E = P \Rightarrow Kx_0 = mg - d_A . V . g = 8 . 9{,}8 - 1{,}25 . 10^3 . 10^{-3} . 9{,}8$

$\Rightarrow x_0 = \frac{66{,}15}{40} = 1{,}65\,m$

Equação do movimento em relação ao ponto de equilíbrio x_0:

$m\ddot{z} = -kz - \rho . \dot{z} + (P - E)$

$\Rightarrow \ddot{z} + \frac{b}{m} \cdot \dot{z} + \frac{k}{m} z = \frac{66,15}{8} = 8,27$ $\begin{cases} \gamma = b/m = 1/4 \\ \omega_0 = \sqrt{\frac{k}{m}} = 2,23 \end{cases}$

i) Solução particular: $x_p(t) = C \begin{cases} \dot{x}_p = 0 \\ \ddot{x}_p = 0 \end{cases}$

$\rightarrow 0 + \gamma \cdot 0 + \omega_0^2 \cdot C = 8,27 \rightarrow C = 1,65 = x_p(t)$

ii) Solução da equação homogênea: $x_h(t)$

Como $\omega_0^2 > \left(\frac{\gamma}{2}\right)^2$, o sistema é subamortecido:

$x_h(t) = A e^{-\frac{\gamma}{2}t} \cos(\omega t + \varphi)$, com: $\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2} = \sqrt{5 - \left(\frac{1}{8}\right)^2} \cong 2,23$

Velocidade: $\dot{z}(t) = \dot{x}_p(t) + \dot{x}_h(t)$

$\rightarrow \dot{z}(t) = -\frac{A\gamma}{2} \cdot e^{-\frac{\gamma}{2}t} \cdot \cos(\omega t + \varphi) - \omega A e^{-\frac{\gamma}{2}t} \operatorname{sen}(\omega t + \varphi)$

Condições iniciais:

$z(0) = x_0 + 1\text{cm} = 1,65 + A \cdot e^{-\frac{\gamma}{2} \cdot 0} \cdot \cos(\varphi) \Rightarrow A \cdot \cos\varphi = 0,01$

$\dot{z}(0) = 0 \Rightarrow -\frac{A}{2} \cdot \frac{1}{4} \cdot \cos\varphi - 2,23 \cdot A \cdot \operatorname{sen}\varphi = 0$

$\Rightarrow \operatorname{sen}\varphi = -0,0559 \cos\varphi \rightarrow \varphi = -3,2 \rightarrow \begin{cases} \operatorname{sen}\varphi = -0,0558 \\ \cos\varphi = 0,9984 \end{cases}$

$\rightarrow A = 0,01\,\text{m}$

Logo: $z(t) = 1,65 + 0,01 \cdot e^{-\frac{t}{8}} \cos(2,23t - 3,2)$, em relação a x_0.

Em relação a $l_0 = 0,5$ e lembrando que:

$\cos(2,23t - 3,2) = \cos(2,23t) \cdot \cos(-3,2) - \operatorname{sen}(\omega t) \cdot \operatorname{sen}(-3,2)$

$\cong \cos(2,23t) + 0,056 \operatorname{sen}(2,23t)$

$\rightarrow z'(t) = 0,5 + 1,65 + A \cdot e^{-\frac{\gamma}{2}t} \cdot \cos(\omega t + \varphi)$

$\rightarrow z'(t) = 2,15 + 0,01 \cdot e^{-\frac{t}{8}} \cdot \left[\cos(2,23t) + 0,056 \operatorname{sen}(2,23t)\right] \text{m}$

1 08 Equação 4.4.6 : $A(\omega) = \dfrac{F_0}{m\sqrt{[(\omega_0^2-\omega^2)]^2+\gamma^2\omega^2}}$

a)

Para $A(\omega)$ máximo: $(\omega_0^2-\omega^2)^2+\gamma^2\omega^2 = f(\omega)$ deve ser mínimo

$$\frac{df}{d\omega} = 0 \rightarrow 2(\omega_0^2-\omega^2)(-2\omega)+2\gamma^2\omega = 0$$
$$\rightarrow 2\gamma^2\omega = 4\omega(\omega_0^2-\omega^2) \Rightarrow \omega_0^2-\omega^2 = \frac{\gamma^2}{2} \rightarrow \omega = \sqrt{\omega_0^2-\frac{\gamma^2}{2}}$$

Em $A(\omega)$: $$A(\omega) = \frac{F_0}{m\sqrt{\left(\frac{\gamma^2}{2}\right)^2+\gamma^2\left(\omega_0^2-\frac{\gamma^2}{2}\right)}} = \frac{F_0}{m\sqrt{\frac{\gamma^4}{4}+\gamma^2\omega_0^2-\frac{\gamma^4}{2}}}$$

$$\Rightarrow A(\omega) = \frac{F_0}{m\sqrt{\gamma^2\omega_0^2-\frac{\gamma^4}{4}}} = \frac{F_0}{m\gamma\sqrt{\omega_0^2-\frac{\gamma^2}{2}}}$$

b) $$A\omega = \frac{F_0\,\omega}{m\sqrt{[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]}} = \frac{F_0}{m}\sqrt{\frac{\omega^2}{[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]}}$$

Seja: $$g(\omega) = \frac{\omega^2}{[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]}$$

$$\frac{dg}{d\omega} = \frac{2\omega[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]-\omega^2[-4\omega(\omega_0^2-\omega^2)+2\gamma^2\omega]}{[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]^2} = 0$$

$$\rightarrow 2\omega[\omega_0^4-2\omega_0^2\omega^2+\omega^4+\gamma^2\omega^2] = 2\omega^2[-2\omega(\omega_0^2-\omega^2)+\gamma^2\omega]$$
$$\rightarrow \omega_0^4\cancel{-2\omega_0^2\omega^2}+\omega^4+\cancel{\gamma^2\omega^2} = \cancel{-2\omega_0^2\omega^2}+2\omega^4+\cancel{\gamma^2\omega^2}$$
$$\rightarrow \omega^4 = \omega_0^4 \rightarrow \boxed{\omega = \omega_0}$$

Logo: $$A\omega_{máx} = \frac{F_0}{m}\sqrt{\frac{\omega^2}{(\omega_0^2-\omega^2)+\gamma^2\omega^2}} = \frac{F_0}{m\gamma}$$

$\boxed{11}$ A mão se move em MHS em torno de A:

$$Z_{mão} = S\,sen(\omega t) \quad (Z(0)=0)$$

Quando o ~~a~~ mola se distende Z, pela extremidade inferior, ligado ao bloco, a mola se distende:

$$\Delta x = Z - S\,sen(\omega t)$$

Força da mola: $F_k = k\Delta x = k\,[Z - A\,sen(\omega t)]$

Força na mão: $F(t) = mg + k\,[Z - A\,sen(\omega t)]$

No bloco:

$$m\ddot{z} = -k[Z - A\,sen(\omega t)] + mg$$

$$\Rightarrow m\ddot{z} = -kz + kA\,sen(\omega t) + mg$$

$$\Rightarrow \ddot{z} + \frac{k}{m}z = g + \frac{k}{m}A\,sen(\omega t)$$

$$\Rightarrow \ddot{z} + \omega_0^2 z = A\omega_0^2\,sen(\omega t)$$

Solução homogênea: $z_h(t) = A'\cos(\omega_0 t + \varphi)$ (MHS)

Solução particular: $z_p(t) = B\,sen(\omega t)$ $\begin{cases} \dot{z}_p = B\omega\cos(\omega t) \\ \ddot{z}_p = -B\omega^2 sen(\omega t) \end{cases}$

$$\Rightarrow -B\omega^2\,\cancel{sen(\omega t)} + \omega_0^2 B\,\cancel{sen(\omega t)} = A\omega_0^2\,\cancel{sen(\omega t)}$$

$$\Rightarrow B(\omega_0^2 - \omega^2) = A\omega_0^2 \Rightarrow B = \frac{\omega_0^2}{\omega_0^2 - \omega^2}A$$

Para $z(0)=0$: $A'\cdot\cos\varphi = 0 \rightarrow \begin{cases} \cos\varphi = 0 \\ sen\,\varphi = \pm 1 \end{cases}$

Velocidade: $\dot{z}(t) = -\omega_0 A'\,sen(\omega_0 t + \varphi) + \omega B\cos(\omega t)$

$\dot{z}(0) = 0$: $-\omega_0 A'\,sen\varphi + \omega B = 0 \rightarrow A' = \frac{\omega\omega_0}{\omega_0^2 - \omega^2}A$

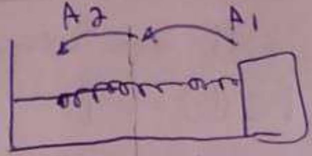
Logo: $z(t) = z_h(t) + z_p(t)$

$$z(t) = A'[\cos(\omega_0 t)\cancel{\cos\varphi}^{\,0} - sen(\omega_0 t)\cdot\cancel{sen\varphi}^{\,1}] + B\,sen(\omega t)$$

$$z(t) = B\,sen(\omega t) - A'\,sen(\omega_0 t) = \frac{\omega_0^2}{\omega_0^2 - \omega^2}\cdot A\cdot sen(\omega t) - \frac{\omega\omega_0}{\omega_0^2 - \omega^2}A\,sen(\omega_0 t)$$

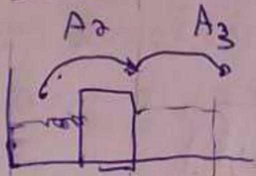
$$\Rightarrow z(t) = A\cdot\frac{\omega_0^2}{\omega_0^2 - \omega^2}\left[sen(\omega t) - \frac{\omega}{\omega_0}sen(\omega_0 t)\right]$$

12 1º Semi-período:



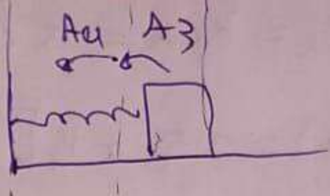
$$\frac{kA_1^2}{2} = \frac{kA_2^2}{2} + \mu m g (A_1 + A_2)$$

2º Semi-período:



$$\frac{kA_2^2}{2} = \frac{kA_3^2}{2} + \mu m g (A_2 + A_3)$$

3º Semi-período:



$$\frac{kA_3^2}{2} = \frac{kA_4^2}{2} + \mu m g (A_3 + A_4)$$

⋮

$$\rightarrow \frac{kA_n^2}{2} = \frac{kA_{n+1}^2}{2} + \mu m g (A_{n+1} + A_n)$$

$$\rightarrow \frac{k}{2}(A_n^2 - A_{n+1}^2) = \mu m g (A_{n+1} + A_n)$$

$$\rightarrow (A_n + A_{n+1})\cancel{(A_{n+1} + A_n)} = \frac{2\mu m g}{k}\cancel{(A_{n+1} + A_n)}$$

$$\rightarrow \Delta A = A_n - A_{n+1} = \frac{2\mu m g}{k} = \frac{2 \cdot 0{,}25 \cdot 1 \cdot 9{,}8}{100} = 4{,}9 \text{ cm}$$

A amplitude cai 4,9 cm em cada semi-período. Isso acontece:

$$n = \frac{24{,}5}{4{,}9} = 5 \text{ vezes}$$

E dura: $5 \cdot \frac{T}{2} = \frac{5}{2} \cdot \frac{2\pi}{\omega} = 5\pi\sqrt{\frac{m}{k}} = \frac{\pi}{2}$ s

13 $P = F \cdot v \rightarrow P(t) = F(t) \cdot \dot{x}(t)$

$$\Rightarrow P(t) = F_0 \cos(\omega t)\left[-a\omega \operatorname{sen}(\omega t) + b\omega \cos(\omega t)\right]$$

$$\Rightarrow P(t) = -F_0 a\omega \cdot \operatorname{sen}(\omega t)\cos(\omega t) + F_0 b\omega \cos^2(\omega t)$$

$$\Rightarrow P(t) = -\frac{F_0 a\omega}{2}\operatorname{sen}(2\omega t) + F_0 b\omega \cos^2(\omega t)$$

$$\Rightarrow \overline{P(t)} = -\frac{F_0 a\omega}{2}\overset{\rightarrow 0}{\overline{\operatorname{sen}(2\omega t)}} + F_0 b\omega \overline{\cos^2(\omega t)}$$

$$\Rightarrow \overline{P(t)} = F_0 b\omega \cdot \frac{1}{2}$$

14 De 4.6.13: $x_1(t) = A_1\cos(\omega_0 t+\varphi_1) + A_2\cos(\omega_2 t+\varphi_2)$

$$x_2(t) = A_1\cos(\omega_0 t+\varphi_1) - A_2\cos(\omega_2 t+\varphi_2)$$

Velocidades: $\dot{x}_1(t) = -\omega_0 A_1 \operatorname{sen}(\omega_0 t+\varphi_1) - \omega_2 A_2 \operatorname{sen}(\omega_2 t+\varphi_2)$

$$\dot{x}_2(t) = -\omega_0 A_1 \operatorname{sen}(\omega_0 t+\varphi_1) + \omega_2 A_2 \operatorname{sen}(\omega_2 t+\varphi_2)$$

Condições iniciais

$$\begin{cases} x_1(0)=0 \Rightarrow A_1\cos\varphi_1 + A_2\cos\varphi_2 = 0 \\ x_2(0)=0 \Rightarrow A_1\cos\varphi_1 - A_2\cos\varphi_2 = 0 \end{cases} \rightarrow \cos\varphi_2 = 0 \rightarrow \cos\varphi_1 = 0 \Rightarrow \varphi_1=\varphi_2=\frac{\pi}{2}$$

$$\begin{matrix} \dot{x}_1(0)=0 \Rightarrow \\ \dot{x}_2(t=0)=V \Rightarrow \end{matrix} \begin{cases} -\omega_0 A_1 - \omega_2 A_2 = 0 \\ -\omega_0 A_1 + \omega_2 A_2 = V \end{cases} \quad A_2 = \frac{V}{2\omega_2} \quad A_1 = -\frac{V}{2\omega_0}$$

Onde: $\omega_0 = \sqrt{\frac{g}{l}} = 4{,}43 \text{ rad/s}$

$$\omega_2 = \sqrt{\omega_0^2 + \frac{2k}{m}} = \sqrt{4{,}43^2 + \frac{2\cdot 25}{0{,}25}} = 14{,}8 \text{ rad/s}$$

$$A_1 = -\frac{0{,}1}{2\cdot 4{,}43} = -1{,}13 \text{ cm} \quad e \quad A_2 = \frac{0{,}1}{2\cdot 14{,}8} = 0{,}34 \text{ cm}$$

$$\rightarrow \begin{cases} x_1(t) = -1{,}13\cdot\cos(4{,}43t+\frac{\pi}{2}) + 0{,}34\cdot\cos(14{,}8t+\frac{\pi}{2}) \\ x_2(t) = -1{,}13\cdot\cos(4{,}43t+\frac{\pi}{2}) - 0{,}34\cos(14{,}8t+\frac{\pi}{2}) \end{cases}$$

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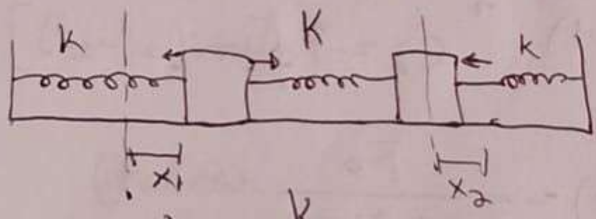

$$m\ddot{x}_1 = -kx_1 + K(x_1-x_2)$$

$$m\ddot{x}_2 = -kx_2 - K(x_1 x_2)$$

$$\rightarrow \begin{cases} \ddot{x}_1 + \omega_1^2 x_1 = \frac{K}{m}(x_1-x_2) \\ \ddot{x}_2 + \omega_1^2 x_2 = -\frac{K}{m}(x_1-x_2) \end{cases} \xrightarrow{\text{Soma}} (\ddot{x}_1+\ddot{x}_2) + \omega_1^2(x_1+x_2) = 0$$

Para $q_1 = \frac{x_1+x_2}{2} \rightarrow \ddot{q}_1 + \omega_1^2 q_1 = 0$

$$q_2 = \frac{x_2-x_1}{2} \rightarrow \ddot{q}_2 + (\omega_1^2+k)q_2 = 0$$

$$\rightarrow \begin{cases} x_1(t) = A_1(\cos\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2) \\ x_2(t) = A_1(\cos\omega_1 t + \varphi_1) - A_2\cos(\omega_2 t + \varphi_2) \end{cases}$$

Condições Iniciais:

$$\begin{cases} x_1(0) = 0 & \Rightarrow \\ x_2(0) = 0 & \Rightarrow \end{cases} \begin{cases} A_1\cos\varphi_1 + A_2\cos\varphi_2 = 0 \\ A_1\cos\varphi_1 - A_2\cos\varphi_2 = 0 \end{cases} \rightarrow \varphi_1 = \varphi_2 = \pi/2$$

$$\begin{cases} \dot{x}_1(0) = 0 & \Rightarrow \omega_1 A_1 + \omega_2 A_2 = 0 \\ \dot{x}_2(0) = 0 & \Rightarrow \omega_1 A_1 - \omega_2 A_2 = V \end{cases} \rightarrow \begin{matrix} A_1 = V/2\omega_1 \\ A_2 = -V/2\omega_2 \end{matrix}$$

Logo
$$x_1(t) = \frac{V}{2\omega_1}\operatorname{sen}(\omega_1 t) - \frac{V}{2\omega_2}\operatorname{sen}(\omega_2 t) \qquad \omega_2 = \sqrt{\omega_1^2 + 2\frac{k}{m}}$$

$$x_2(t) = \frac{V}{2\omega_1}\operatorname{sen}(\omega_1 t) + \frac{V}{2\omega_2}\operatorname{sen}(\omega_2 t) \qquad \omega_1 = \sqrt{\frac{k}{m}}$$

$\boxed{16}$
$$\begin{cases} \ddot{x}_1 + \omega_0^2 x_1 = k(x_2 - x_1) \\ \ddot{x}_2 + \omega_0^2 x_2 = -k(x_2 - x_1) + \frac{F_0}{m}\cos(\omega_0 t) \end{cases}$$

Das questões anteriores:
$$\ddot{q}_1 + \omega_0^2 q_1 = \frac{F_0}{2m}\cos(\omega t)$$
$$\ddot{q}_2 + \omega_1^2 q_2 = \frac{-F_0}{2m}\cos(\omega t)$$

Solução particular:
$$\begin{cases} q_1 = c_1\cos(\omega t) \\ q_2 = c_2\cos(\omega t) \end{cases} \rightarrow \begin{matrix} c_1 = F_0/[2m(\omega_0^2 - \omega^2)] \\ c_2 = -F_0/[2m(\omega_1^2 - \omega^2)] \end{matrix}$$

$$\rightarrow q_1(t) = \frac{F_0}{2m(\omega_0^2 - \omega^2)}\cdot\cos(\omega t) \quad \text{e} \quad q_2(t) = -\frac{F_0}{2m(\omega_1^2 - \omega^2)}\cdot\cos(\omega t)$$

Logo:
$$x_1(t) = \frac{F_0}{2m}\left(\frac{1}{\omega_0^2 - \omega^2} - \frac{1}{\omega_1^2 - \omega^2}\right)\cos(\omega t) = \frac{F_0}{m}\frac{k}{(\omega_0^2 - \omega^2)(\omega_1^2 - \omega^2)}\cdot\cos(\omega t)$$

$$x_2(t) = \frac{F_0}{2m}\left(\frac{1}{\omega_0^2 - \omega^2} + \frac{1}{\omega_1^2 - \omega^2}\right)\cos(\omega t) = \frac{F_0}{2m}\frac{(\omega_0^2 + \omega_1^2 - 2\omega^2)}{(\omega_0^2 - \omega^2)(\omega_1^2 - \omega^2)}\cdot\cos(\omega t)$$

17 a)

$$\begin{cases} M\ddot{x}_1 = -K(x_1 - x_2) \\ m\ddot{x}_2 = -K(x_2 - x_3) + K(x_1 - x_2) \\ M\ddot{x}_3 = K(x_2 - x_3) = -K(x_3 - x_2) \end{cases}$$

Esquerda: $x_1 - x_2$

Segunda mola: $x_2 - x_3$

$$\rightarrow x_{cm} = \frac{Mx_1 + mx_2 + Mx_3}{M + m + M} \rightarrow \ddot{x}_{cm} = \frac{M\ddot{x}_1 + m\ddot{x}_2 + M\ddot{x}_3}{2M + m}$$

$$= \frac{-K(x_1 - x_2) - K(x_2 - x_3) + K(x_1 - x_2) + K(x_2 - x_3)}{2M + m} = 0$$

18 $m\ddot{z}_1 = -Kz_1 + K(z_2 - z_1) \Rightarrow \ddot{z}_1 = \frac{K}{m}(z_2 - 2z_1)$

$$\ddot{z}_2 = \frac{K}{m}(z_1 - z_2)$$

$$\ddot{q} = \alpha\ddot{z}_1 + \beta\ddot{z}_2 = \alpha\frac{K}{m}(z_2 - 2z_1) + \beta\frac{K}{m}(z_1 - z_2) = z_1 K(\beta - 2\alpha) + z_2(\alpha - \beta)$$