Deliverable 5

1: Models and entailment in propositional logic

Assignment 1

Knowledge base:

- $B_{4,1}$, $B_{4,3}$ and not $B_{4,2}$
- $B_{4,1} \rightarrow P_{3,1}$ or $P_{4,2}$ (I)
- $B_{4,3} \rightarrow P_{3,3}$ or $P_{4,4}$ or $P_{4,2}$ (II)
- Not $B_{4,2} \rightarrow$ not $P_{4,1}$ and not $P_{3,2}$ and not $P_{4,3}$ (III)

Truth table:

$P_{3,1}$	$P_{3,2}$	$P_{3,3}$	$P_{4,4}$	$B_{4,1}$	$B_{4,3}$	$B_{4,2}$	Marks:	(I)	(II)	(III)
0	0	0	0	1	1	0	α_1, α_3,	0	0	1
0	0	0	1	1	1	0	α_1, α_2, α_4	0	1	1
0	0	1	0	1	1	0	α_1, α_3, α_4	0	1	1
0	0	1	1	1	1	0	α_1, α_2, α_4	0	1	1
0	1	0	0	1	1	0	α_3	0	0	0
0	1	0	1	1	1	0	α_2, α_4	0	1	0
0	1	1	0	1	1	0	α_3, α_4	0	1	0
0	1	1	1	1	1	0	α_2, α_4	0	1	0
1	0	0	0	1	1	0	α_1, α_3	1	0	1
1	0	0	1	1	1	0	α_1, α_2, α_4	1	1	1
1	0	1	0	1	1	0	α_1, α_3, α_4	1	1	1
1	0	1	1	1	1	0	α_1, α_2, α_4	1	1	1
1	1	0	0	1	1	0	α_3	1	0	0
1	1	0	1	1	1	0	α_2, α_4	1	1	0
1	1	1	0	1	1	0	α_3, α_4	1	1	0
1	1	1	1	1	1	0	α_2, α_4	1	1	0

In our table we've only considered the rows where our knowledge base can be true, i.e. the rows in which $B_{4,1}$ and $B_{4,3}$ are true, and $B_{4,2}$ is false. The rows where the entire knowledge base is true has been marked with green. The sentences that appear in all the rows in which the knowledge base is true are α_1 and α_4. This means that KB $|= \alpha_1$ and KB $|= \alpha_4$.

Assignment 2

a) True
b) False
c) True
d) False
e) False
f) True
g) False

Assignment 3

a) This is valid because the only way to make an implication invalid is to have the right side be false and the left side be true, but since both sides are equal, this is impossible.
b) This is neither, as it is possible to have clouds without rain, making the implication invalid but satisfiable.
c) This is neither, as it is possible for the implication not to be true, but also for it to be true.
d) This is neither as it is logically the same as "Clouds", and it is possible for there to not be clouds.
e) This is valid as there either are clouds or not, and both of these cases are covered by the expression.
f) This is valid. Same argument as a)
g) This is neither, as it is not valid but it is satisfiable. This is because if all variables are true then both sides are true, but if only GI and GF are true, but not GC, then the right side is true but the left side is false.

Assignment 4

a) $1*1*2^{98}$
b) 2^{97}
c) $3*2^{97}$
d) $3*2^{98}$
e) $3*2^{97}$
f) 2^{95}

2: Resolution in propositional logic

Assignment 1

a) (A v B) ^ (A v C) ^ (A v not D)
b) Not (not A v not B) ^ not (not C v not D)
   A ^ B ^ C ^ D
c) Not ((not A v B) ^ (NOT C v D))
   (A ^ NOT B) V (C ^ NOT D)
   ((A ^ NOT B) V C) ^ ((A ^ NO B) V NOT D)
   (A V C) ^(NOT B V C) ^ (A V NOT D) ^(NOT B V NOT D)
d) (A^B) V (NOT C V D)
   ((A^B) V NOT C) V ((A^B) V D)
   (A V NOT C) ^ (B V NOT C) ^(A V D)^(B V D)
e) (B V A) ^ (C V A) ^ (NOT A V NOT B V NOT C)

NOR: NOT A ^ NOT B
NAND: NOT A v NOT B
XOR: (A ^ NOT B) v (NOT A ^ B) ⇔ (A v B) ^ (NOT A v NOT B)

NAND (NAND(A,B), B) <=> NAND (NOT A v NOT B, B) ⇔ NOT (NOT A v NOT B) v NOT B
                    ⇔ (A ^ B) v NOT B ⇔ (A v NOT B) ^ (B v NOT B) ⇔ A v NOT B

Assignment 2

(c) and (a) gives us that D is false. With this in mind, (d) gives us that B is true. Finally, with this in mind, (b) gives us that A has to be true. So with this KB, A has to be true. That means that our KB entails A.

Assignment 3

X = this animal

1. hasHorn(X) ^ isHorse(X) -> isMythical (X) ⇔ not hasHorn(X) v not isHorse(X) v isMythical(X)
2. hasHorn(X) ^ isHorse(X) -> isUnicorn(X) ⇔ not hasHorn(X) v not isHorse(X) v isUnicorn(X)
3. isMythical(X) ^ onLand(X) -> isUnicorn(X) ⇔ not isMythical(X) v not onLand(X) v isUnicorn(X)
4. isUnicorn(X) -> isImmortal(X) ⇔ not isUnicorn v isImmortal(X)
5. hasHorn(X) ^ isSeal(X) -> isNarwhal(X) ⇔ not hasHorn(X) v not isSeal(X) v isNarwhal(X)
6. inSea(X) -> isSeal(X) ⇔ not inSea(X) v isSeal(X)
7. onLand(X) -> isUnicorn(X) ⇔ not onLand(X) v isUnicorn(X)

3: Representations in First-Order Logic
Assignment 1

a) (Occupation(Eliza, scientist) v Occupation(Eliza, investor)) ^ (not Occupation(Eliza, scientist) v not Occupation(Eliza, investor))
b) Occupation(Cyrus, o1) ^ Occupation(Cyrus, o2) ^ (o1 != o2)
c) ((For all) p, Occupation(p, Scientist) -> Occupation(p, teacher)) ^ ((there exists) p, Occupation(p, Teacher) ^ not Occupation(p, Scientist))
d) (Customer(Eliza, p) ^ Occupation(p, Cook)) v Occupation(Eliza, Cook)
e) Occupation(Cyrus, Scientist) -> (Boss(p, Cyrus) ^ Occupation(p, Scientist))
f) Occupation(Cyrus, Teacher) -> (Boss(p, Cyrus) ^ Occupation(p, Scientist))
g) (Boss(p1, p2) ^ Boss(p2, p3)) -> Boss (p1, p3)
h) (there exists) p1 -> Occupation(p1, Cook) -> (Customer(p2, p1) -> (Occupation(p2, investor) v Is(p2, Eliza))
i) (For all) p -> (((Occupation(p, Cook) ^ not Is(p, Cyrus)) -> (Customer(p, p2) ^ Occupation(p2, Teacher))

Assignment 2

"Everyone's DNA is unique and is derived from their parents' DNA"

(for all) p1, p2, (not isEqualDNA(p1, p2) v Is(p1, p2)) ^ (for all) p, ((isParent(parent, p) -> isDerivedDNA(p, parent))

Assignment 3

a) (there exists) child, Parent(Joan, child) ^ Female(child)
b) (there exists exactly one) child, Parent(Joan, child) ^ Female(child)
c) ((there exists exactly one) child, Parent(Joan, child)) ^ (Parent(Joan, child) -> Female(child))
d) ((there exists exactly one) child, Parent(Joan, child) ^ Parent(Kevin, child))
e) ((there exists) child, Parent(Joan, child) ^ Parent(Kevin, child)) ^
   (not (there exists) child, Parent(Joan, child) ^ Parent(person2, child) ^ person2 != Kevin)

Assignment 4

a) PlayedCharacter(Tom Holland, Spider-Man) ^ PlayedCharacter(Tobey Maguire, Spider-Man)
   ^ PlayedCharacter(Andrew Garfield, Spider-Man)
b) PlayedCharacter(p, Spider-Man) -> Male(p)
c) PlayedCharacter(p, Spider-Woman) -> Female(p)
d) Not (there exists) c: PlayedCharacter(Tom Holland, c) ^ PlayedCharacter(Willem Dafoe, c)
e) For all m: CharacterInMovie(Spider-Man, m), Directed(Joe Watts, m) -> PlayedCharacter(TH, Spider-Man)
f) (there exists) m: CharacterInMovie(Spider-Man, m) ^ CharacterInMovie(Green Goblin, m)
g) (there exists) m: Directed(WA, m) ^ PlayedInMove(WA, m)
h) (Not (there exists) m: PlayedInMovie(George Clooney, m) ^ PlayedInMovie(Quentin Tarantino, m)) ^ not (there exists) m: Directed(QT, m) ^ PlayedInMovie(GC, m)
i) Female(Uma Thurman) ^ (there exists) m: Directed (QT, m) ^ PlayedInMovie(UT, m)

4: Resolution in First-Order Logic

Assignment 2

Assume: not Hunts(Isaac, Fred) (5)

(2) ^ (3) -> TreasureHunter(Isaac) ^ Has(Fred, Goblet) (Conclusion 1)

(Conclusion 1) ^ (4) -> TreasureHunter(Isaac) ^ Has(Fred, Goblet) ^ Treasure(Goblet) (Conclusion 2)

(Conclusion 2) ^ (1) -> Hunts(Isaac, Fred) (Conclusion 3)

(Conclusion 3) ^ (5) -> Hunts(Isaac, Fred) ^ not Hunts(Isaac, Fred)

Contradiction -> Hunts(Isaac, Fred) ☐

4: Resolution in First-Order Logic

Assignment 2

Assume: not Hunts(Isaac, Fred) (5)