### 1.

# **Unrestricted Algorithm**

#### Minimum Function:

This function is just performing lookups in the 2d trackback array which all happen in constant time. It also compares the currMin to a new number which is also constant time, so **Time complexity is O(1)**.

The **space complexity here is O(1)** since we are not making any new space in the array we are just overwriting the (i, j) space in the trackback\_array.

#### **CalculateSequences Function:**

The **time complexity here is O(nm)** because of the while loop can, in the worst case, go through the traceback\_array the length of the topSequence \* the length of the sideSequence. This is n \* m since it keeps looping all the way to the first cell at (0,0) since that is where the None is stored.

The space complexity is O(n + m) since we are making two new strings for the sequences and they may have "-" in them. Thus the overall space complexity would be O(d) where d is equal to which ever length is greater between n (topSequence) and m (sideSequence).

#### **Unrestricted Function:**

The overall time complexity here is O(nm) where m is the size of the sideSequence and n is the size of the topSequence. This comes from making the 2d arrays as well as the for loop that runs mn time, and the calculateSequences. Thus it equals O(4nm) but the 4 is dropped since it is a constant and it just comes out to be O(nm).

The overall space complexity here is O(nm) where m is the size of the sideSequence and n is the size of the topSequence. This comes from making the two 2d arrays so it would be O(2nm) but the 2 is dropped since it is a constant and it just comes out to be O(nm).

# Thus my unrestricted algorithm has an overall time and space complexity of O(nm).

#### **Banded Algorithm**

#### bandedMin Function:

This function is just performing lookups in the 2d array which all happen in constant time. It also compares the currMin to a new number which is also constant time, so **Time complexity is O(1)**.

The space complexity here is O(1) since we are not making any new space in the array we are just overwriting the (i, j) space in the traceback\_array.

#### BandedCalculateSequences Function:

The **time complexity here is O(kn)** because of the while loop can, in the worst case, go through the trackback\_array n times which is the amount of rows and k

which is the amount of columns. This is n \* k since it keeps looping all the way to the first cell at (0,3) since that is where the None is stored.

The space complexity is O(n + m) since we are making two new strings for the sequences and they will may have "-" in them. Thus the overall space complexity would be O(d) where d is equal to which ever length is greater between n (topSequence) and m (sideSequence).

#### bandedAlg Function:

The overall time complexity here is O(kn) where k is 7 and n is the size of the topSequence or sideSequence whichever is shorter, or if there is a tie the topSequence is chosen as n. The number of rows (n) can at most be the shorter sequence + 3 since d = 3 which comes to O((n+3)k). But the 3 is dropped in Big-O since it is a constant making it O(nk). This comes from making the 2d arrays as well as the for loop that runs kn time, and the bandedCalculateSequences. Thus it equals O(4kn) but the 4 is dropped since it is a constant and it just comes out to be O(kn). The overall space complexity here is O(kn) where k is 7 and n is the size of the topSequence or sideSequence whichever is shorter, or if there is a tie the topSequence is chosen as n. The number of rows (n) can at most be the shorter sequence + 3 since d = 3 which comes to O((n+3)k). But the 3 is dropped in Big-O since it is a constant making it O(nk). This comes from making the two 2d arrays so it would be O(2kn) but the 2 is dropped since it is a constant and it just comes out to be O(nk).

## Thus my banded algorithm has an overall time and space complexity of O(kn).

2.

How string alignment extraction works with my unrestricted algorithm: When I made my alignment cost array I also made a traceback array. This traceback array is updated along side of my alignment cost array with the correct value of where the minimum value came from. A "D" (diagonal) means that the value came from the cell up one and over to the left one from the current cell. An "A" (above) means that the value came from the cell up one from the current cell. A "L" (left) means that the value came from the cell to the left one from the current cell. Once the minimum value is found the correct letter is set in the trace array. All ties store the trace back as a "D". Once all costs are found calculateSequences is called which starts at the last row, last column cell in the traceback array. It works its way through the traceback array till it gets to the (0,0) cell where "None" is found. If the traceback array's cell has a "D" then the next location is up one and over to the left one. It adds the topSequence's letter at that location to tempSeqI as well as the sideSequence's letter at that location to tempSeq. If the traceback array's cell has a "L" then the next location is one over to the left. It adds the topSequence's letter at that location to tempSeqI. It also adds a "-" to tempSeqI since a deletion occurred. If the traceback array's cell has an "A" then the next location is one up. It adds a "-" to tempSeqI since an insertion occurred. It also adds the sideSequence's letter at that location to tempSeq. It continues through the matrix till (0,0) is reached at the top.

Then the string is reversed to be in the correct order and then cut down to 100 characters.

How string alignment extraction works with my banded algorithm: When I made my alignment cost array I also made a traceback array. This traceback array is updated along side of my alignment cost array with the correct value of

where the minimum value came from. A "D" (diagonal) means that the value came from the cell up one from the current cell. This occurs because the columns became squished in the banded traceback array so that diagonals in the normal array are now directly above a cell. An "A" (above) means that the value came from the cell up one and over to the right one from the current cell. Again this happens because the columns became squished in the banded traceback array so that aboves in the normal array are now up one and over to the right one. A "L" (left) means that the value came from the cell to the left one from the current cell. Once the minimum value is found the correct letter is set in the trace array. All ties store the trace back as a "D". Since the banded array has some areas that would be out of range in the normal array it has areas in it that have stored in their cells math.inf within the cost array. This means that the index is now out of range for the topSequence and thus the character cannot be looked up. It fills in any areas that are out of range with this math.inf value. It then finds in the last row which column does not have a math.inf stored in its cell and hands that column number to the bandedCalculateSequences. Using the last row and the column handed to it. It goes through the traceback array until it hits None which is found at the top of the array at (0,3). If the traceback array's cell has a "D" then the next location is up one. It adds the topSequence's letter at that location to tempSeqI as well as the sideSequence's letter at that location to tempSeqQ. If the traceback array's cell has a "L" then the next location is one over to the left. It adds the topSequence's letter at that location to tempSeqI. It also adds a "-" to tempSeqQ since a deletion occurred. If the traceback array's cell has an "A" then the next location is one up and to the right one. It adds a "-" to tempSeqI since an insertion occurred. It also adds the sideSequence's letter at that location to tempSeqQ. It continues through the matrix till (0,3) is reached at the top. Then the string is reversed to be in the correct order and then cut down to 100 characters.

#### Results:

3. Unrestricted k = 1000

					Gene Se	equence Aliç	gnment				
		sequence1	sequence2	sequence3	sequence4	sequence5	sequence6	sequence7	sequence8	sequence9	sequence10
	sequence1	-30	-1	4956	4956	4956	4956	4956	4956	4956	4956
	sequence2		-33	4948	4948	4948	4948	4948	4948	4948	4948
	sequence3			-3000	-2996	-2956	-2944	-1431	-1448	-1399	-1448
	sequence4				-3000	-2960	-2948	-1431	-1448	-1399	-1448
	sequence5					-3000	-2988	-1423	-1452	-1391	-1448
	sequence6						-3000	-1426	-1452	-1394	-1448
	sequence7							-3000	-2771	-2814	-2767
	sequence8								-3000	-2731	-2996
	sequence9									-3000	-2727
	sequence10										-3000
Label I: Sequence I: Sequence J:											
Label J:											
	Process Clear  Banded Align Length: 1000										

Banded k = 3000

					Gene Se	equence Alig	gnment				
		sequence1	sequence2	sequence3	sequence4	sequence5	sequence6	sequence7	sequence8	sequence9	sequence10
	sequence1	-30	-1	inf	inf	inf	inf	inf	inf	inf	inf
	sequence2		-33	inf	inf	inf	inf	inf	inf	inf	inf
	sequence3			-9000	-8984	-8888	-8848	-2735	-2743	-1429	-2735
	sequence4				-9000	-8888	-8848	-2739	-2748	-1426	-2740
	sequence5					-9000	-8960	-2711	-2739	-1426	-2727
	sequence6						-9000	-2708	-2728	-1415	-2716
	sequence7							-9000	-8103	-1256	-8099
	sequence8								-9000	-1310	-8980
	sequence9									-9000	-1315
	sequence10										-9000
abel I:											
equence I:											
Sequence J:											
Label J:											
					Proc	ess	Clear				
				✓ Ban	ded Align L	ength: 30	000				

4.

Extracted Alignment, Unrestricted, k = 1000 #3, #10 gattgcgagcgatttgcgtgcgtgcat-ccc--gcttcact-gatctcttgttagatcttttcataatctaaactttataaaaacatccactcctgt-a -a-taagagtgattggcgtccgtacgtaccctttctactctcaaactcttgttagtttaaatc-taatctaaactttat--aaac-ggcacttcctgtgt

Extracted Alignment, Banded, k = 3000 #3, #10 gattgcgagcgatttgcgtgcgtcat-ccc--gcttcact-gatctcttgttagatcttttcataatctaaactttataaaacatccactcctgt-a -a-taagagtgattggcgtccgtacgtaccctttctactctcaaactcttgttagtttaaatc-taatctaaactttat--aaac-ggcacttcctgtgt

```
#!/usr/bin/python3

from which_pyqt import PYQT_VER

if PYQT_VER == 'PYQT5':
    from PyQt5.QtCore import QLineF, QPointF
elif PYQT_VER == 'PYQT4':
    from PyQt4.QtCore import QLineF, QPointF
else:
    raise Exception('Unsupported Version of PyQt: {}'.format(PYQT_VER))
```

```
mport math
MAXINDELS = 3
MATCH = -3
INDEL = 5
SUB = 1
    self.alignment1 = None
     self.alignment2 = None
  def align(self, sequences, table, banded, align length):
     self.banded = banded
     self.MaxCharactersToAlign = align length
     for i in range(len(sequences)): # Rows O(10) so constant O(1)
          if self.banded:
            cost = self.bandedAlg(sequences[i], sequences[j]) # O(kn) time and space
            score = cost # O(1)
            alignment1 = self.alignment1.format(i + 1, len(sequences[i]), align length,
                                   ,BANDED' if banded else ")
            alignment2 = self.alignment2.format(j + 1, len(sequences[j]), align_length,
                                   ',BANDED' if banded else ")
            s = {'align cost': score, 'seqi first100': alignment1, 'seqj first100': alignment2}
            table.item(i, j).setText('{}'.format(int(score) if score != math.inf else score))
            table.repaint()
         iresults.append(s)
       results.append(jresults)
     return results
```

```
def unrestricted(self, sequence1, sequence2):
  maxAlignment = self.MaxCharactersToAlign # O(1)
  topSequence = list(sequence 1) # O(1)
  sideSequence = list(sequence2) # O(1)
  if (len(sequence1) > maxAlignment): # O(1)
    n = maxAlignment # O(1)
    topSequence = list(sequence1[:maxAlignment]) # O(1) grabs from start to maxAlignment of the list
  if (len(sequence2) > maxAlignment): # O(1)
     m = maxAlignment # O(1)
     sideSequence = list(sequence2[:maxAlignment]) # O(1) grabs from start to maxAlignment of the list
    m = len(sequence2) # O(1)
  array = [[0 for i in range(columns)] for i in
  traceback array = [["-" for i in range(columns)] for j in range(rows)] # O(nm) time and space
  traceback_array[0][0] = None # O(1)
     array[i][0] = i * INDEL # O(1)
     traceback array[i][0] = "A" # O(1) A means that the number came from the above one it was an insert
     array[0][j] = j * INDEL
     traceback array[0][j] = "L" # O(1) L means that the number came from the left one it was a delete
       char1 = sideSequence[i - 1] # O(1)
       char2 = \overline{topSequence[j-1] \# O(1)}
       sameChar = char1 == char2 \# O(1)
       array[i][j] = self.minimum(array, traceback array, i, j, sameChar) # O(1)
  self.calculateSequences(topSequence, sideSequence, traceback array) # O(nm)
  return array[rows - 1][columns - 1] # O(1)
```

```
def minimum(self, array, trackback array, i, j, sameChar):
  left = array[i][j - 1] # O(1)
  diagonal = array[i - 1][j - 1] # O(1)
  if sameChar: # O(1)
    currMin = diagonal + MATCH # O(1)
    trackback array[i][j] = "D" \# O(1) D means it came from the diagonal
    currMin = diagonal + SUB # O(1)
    trackback array[i][j] = "D" \# O(1) D means it came from the diagonal
  if currMin > (left + INDEL): # O(1)
    trackback_array[i][j] = "L" # O(1) L means it came from the left a delete
  if currMin > (above + INDEL):
    trackback_array[i][j] = "A" \# O(1) A means it came from above an insert
  return currMin
# The time complexity here is O(nm) because of the while loop which will at the worst case go through the
def calculateSequences(self, topSequence, sideSequence, traceback array):
  tempSeqI = None # O(1)
  tempSeqJ = None # O(1)
  currTopChar = len(topSequence)
  currSideChar = len(sideSequence)
  while traceback_array[currSideChar][
    currTopChar] is not None: # O(nm) cause it repeats all the way to (0,0) in traceback array
    source = traceback array[currSideChar][currTopChar] # O(1)
         tempSeqI += topSequence[currTopChar - 1] # O(1)
         tempSeqI = topSequence[currTopChar - 1] # O(1)
       if tempSeqJ is not None: # O(1)
         tempSeqJ += sideSequence[currSideChar - 1] # O(1)
         tempSeqJ = sideSequence[currSideChar - 1] # O(1)
       currTopChar = 1 \# O(1)
       currSideChar -= 1 # O(1)
    elif source is "L": # O(1) a deletion
       if tempSeqI is not None: # O(1)
         tempSeqI += topSequence[currTopChar - 1] # O(1)
         tempSeqI = topSequence[currTopChar - 1] # O(1)
         tempSeqJ += "-" # O(1)
```

```
tempSeqJ = "-" \# O(1)
         currTopChar -= 1 # O(1) move over a column only since we moved left
         if tempSeqI is not None: # O(1)
           tempSeqI += "-" \# O(1)
           tempSeqI = "-" \# O(1)
         if tempSeqJ is not None: # O(1)
           tempSeqJ += sideSequence[currSideChar - 1] # O(1)
           tempSeqJ = sideSequence[currSideChar - 1] # O(1)
         currSideChar -= 1 # O(1) move up a row since we inserted
    if tempSeqI is None:
       self.alignment1 = "No Alignment Possible"
       tempSeqI = tempSeqI[::-1] # O(n) where n is the size of tempSeqI this reverses the string
       self.alignment1 = tempSeqI[:100] # O(100) = O(1) takes first 100 chars and puts into alignment1
    if tempSeqJ is None:
       self.alignment2 = "No Alignment Possible"
       tempSeqJ = tempSeqJ[::-1] # O(m) where m is the size of tempSeqJ this reverses the string
       self.alignment2 = tempSeqJ[:100] # O(100) = O(1) takes first 100 chars and puts into alignment2
bandedCalculateSequences. Thus
  def bandedAlg(self, sequence1, sequence2):
    maxAlignment = self.MaxCharactersToAlign # O(1)
    topSequence = list(sequence1) # O(1)
    sideSequence = list(sequence2) # O(1)
    if (len(sequence1) > maxAlignment): # O(1)
      n = maxAlignment # O(1)
    if (len(sequence2) > maxAlignment): # O(1)
       m = maxAlignment # O(1)
       sideSequence = list(sequence2[:maxAlignment]) # O(1) grabs from start to maxAlignment of the list
       m = len(sequence2) # O(1)
```

```
diff = cols - rows # O(1)
    # If the difference is big then it will have much more insertions and deletes then are allowed in a row
    if diff > MAXINDELS or diff < -MAXINDELS:
       self.alignment1 = "No Alignment Possible" # O(1)
       return math.inf
    if len(topSequence) > len(sideSequence): # O(1)
       topSequenceBigger = True \# O(1)
       topSequenceBigger = False \# O(1)
the other one
    differentOfLengths = abs(len(topSequence) - len(sideSequence))
    if (topSequenceBigger):
       rows = len(sideSequence) + 1 + differentOfLengths # O(1) this is n as seen below, sideSequence is smaller
       temp = topSequence
       topSequence = sideSequence
       sideSequence = temp
       rows = len(topSequence) + 1 + differentOfLengths # O(1) this is n as seen below, topSequence is smaller
    columns = (2 * MAXINDELS) + 1 # O(1) this is k = 7
    array = [["nil" for i in range(columns)] for j in range(rows)] # O(kn) time & space makes extra row for -
    traceback array = [["-" for i in range(columns)] for j in range(rows)] # O(kn) time and space
    array[0][3] = 0 # O(1)
    array[0][4] = 5 \# O(1)
    array[0][5] = 10 # O(1)
    array[1][2] = 5 \# O(1)
    array[3][0] = 15 \# O(1)
    traceback\_array[0][3] = None # O(1)
    traceback\_array[0][4] = "L" # O(1)
    traceback_array[0][6] = "L" # O(1)
traceback_array[0][6] = "L" # O(1)
    traceback_array[1][2] = "A" # O(1)
    traceback_array[2][1] = "A" \# O(1)
    traceback_array[3][0] = "A" \# O(1)
    array[0][0] = math.inf # O(1)
    array[0][1] = math.inf # O(1)
    array[0][2] = math.inf # O(1)
    array[1][0] = math.inf # O(1)
    array[1][1] = math.inf # O(1)
    array[2][0] = math.inf # O(1)
```

```
array[rows - 1][4] = math.inf # O(1)
  array[rows - 2][6] = math.inf \# O(1)
  for i in range(0, rows): # O(n) n is length of which ever sequence is shorter
       if array[i][j] == "nil":
         array[i][j] = self.bandedMin(array, traceback_array, topSequence, sideSequence, i, j) # O(1)
  column = MAXINDELS # O(1)
    column -= 1
  self.bandedCalculateSequences(topSequence, sideSequence, traceback array, rows, column) # O(kn)
  return array[rows - 1][column]
def bandedMin(self, array, traceback_array, topSequence, sideSequence, i, j):
  currMin = math.inf # O(1)
    above = math.inf \# O(1)
    left = array[i][j-1] # O(1)
    diagonal = array[i - 1][j] \# O(1)
    left = array[i][j - 1] # O(1)
     if i - (4 - j) > len(topSequence) - 1:
       return math.inf
       if topSequence[i - (4 - j)] == sideSequence[i - 1]: # O(1)
         currMin = diagonal + MATCH # O(1)
         traceback array[i][j] = "D" \# O(1)
         currMin = diagonal + SUB # O(1)
         traceback array[i][j] = "D" \# O(1)
  if left is not math.inf: # O(1)
    if currMin > (left + INDEL): # O(1)
       traceback_array[i][j] = "L" # O(1) L means it came from the left a delete
    if currMin > (above + INDEL): # O(1)
       currMin = above + INDEL # O(1)
       traceback array[i][j] = "A" # O(1) A means it came from above an insert
  return currMin
# trackback array the n times which is the amount of rows and k which is the amount of columns
```

```
def bandedCalculateSequences(self, topSequence, sideSequence, traceback array, rows, column):
    tempSeqI = None # O(1)
    tempSeqQ = None # O(1)
    while traceback_array[rows - 1][column] is not None: #O(nk) cause it repeats all the way to (0,3) in traceback
array
       source = traceback array[rows - 1][column] # O(1)
         if tempSeqI is not None: # O(1)
           tempSeqI += topSequence[rows - (4 - column) - 1] # O(1)
         if tempSeqQ is not None: # O(1)
           tempSeqQ += sideSequence[rows - 2] # O(1)
           tempSeqQ = sideSequence[rows - 2] \# O(1)
         if tempSeqI is not None: # O(1)
           tempSeqI += topSequence[rows - (4 - column) - 1] # O(1)
           tempSeqI = topSequence[rows - (4 - column) - 1] # O(1)
         if tempSeqQ is not None: # O(1)
           tempSeqQ += "-" # O(1)
           tempSeqQ = "-" \# O(1)
         if tempSeqI is not None: # O(1)
           tempSeqI += "-" \# O(1)
           tempSeqI = "-" # O(1)
         if tempSeqQ is not None: # O(1)
           tempSeqQ += sideSequence[rows - 2] # O(1)
           tempSeqQ = sideSequence[rows - 2] # O(1)
         print("Error: Table didn't contain an A, L or D!")
    if tempSeqI is None:
       self.alignment1 = "No Alignment Possible"
       tempSeqI = tempSeqI[::-1] # O(n) where n is the size of tempSeqI this reverses the string
       self.alignment1 = tempSeqI[:100] # O(100) = O(1) takes first 100 chars and puts into alignment1
    if tempSeqQ is None:
```

self.alignment2 = "No Alignment Possible"
else:

tempSeqQ = tempSeqQ[::-1] # O(m) where m is the size of tempSeqJ this reverses the string self.alignment2 = tempSeqQ[:100] # O(100) = O(1) takes first 100 chars and puts into alignment2