Intro to estimation and calibration using system priors

IrisToolbox Knowledge Base

Priors in bayesian estimation

- Traditionally, priors on individual parameters
- However, more often than not we simply wish to control some properties of the model as a whole system: **system properties**
- And not so much the individual parameters

Examples of system properties

- Model-implied correlation between output and inflation
- Model-implied sacrifice ratio
- Frequency response function from output to potential output (band of periodicities ascribed to potential output)
- Suppress secondary cycles in shock responses (e.g. more than 90% of a shock response has to occur within the first 10 quarter)
- Make sure a type 2 policy error is costly (delays in the policy response to inflationary shocks calls for a larger reaction later)
- Anything...
- Even **qualitative** properties (e.g. sign restrictions) can be expressed as system priors

System priors formally

Posterior density

$$\underbrace{p\left(\theta \mid Y, m\right)}_{\text{Posterior}} \propto \underbrace{p\left(Y \mid \theta, m\right)}_{\text{Data likelihood}} \times \underbrace{p\left(\theta \mid m\right)}_{\text{Prior}}$$

Prior density typically consists of independent marginal priors

$$p\left(\theta \mid m\right) = p_1\left(\theta_1 \mid m\right) \times p_2\left(\theta_2 \mid m\right) \times \cdots \times p_n\left(\theta_n \mid m\right)$$

Complement or replace with density involving a property of the model as a whole, $h\left(\theta\right)$

$$p\left(\theta \mid m\right) = p_1\left(\theta_1 \mid m\right) \times \cdots \times p_n\left(\theta_n \mid m\right) \times q_1\big(h(\theta) \mid m\big) \times \cdots \times q_k\big(h(\theta) \mid m\big)$$

Benefits of system priors in estimation

- A relatively low number of system priors can push parameter estimates into a region where the properties of the model as a whole make sense and are well-behaved…
- …without enforcing a tighter prior structure on individual parameters

Non-bayesian interpretation of priors: Penalty/shrinkage

- Shrinkage (or penalty) function
- Keep the parameters close to our “preferred” values
- ”Close” is defined by the shape/curvature of the shrinkage/penalty function
- Example: Normal priors are equivalent to quadratic shrinkage/penalty

Priors in calibration: Maximize prior mode

- Exclude/disregard data likelihood
- Only maximize prior mode
- Case 1: only independent priors on individual parameters
  $\Rightarrow$ modes of marginals

$$p\left(\theta \mid m\right) = p_1\left(\theta_1 \mid m\right) \times \cdots \times p_n\left(\theta_n \mid m\right) \times \cdots$$

- Case 2: only a small number of system priors
  $\Rightarrow$ very likely underdetermined (singular)

$$p\left(\theta \mid m\right) = q_1\big(h(\theta) \mid m\big) \times \cdots \times q_k\big(h(\theta) \mid m\big)$$

- Case 3: Combination of priors on individual parameters and system priors
  $\Rightarrow$ deviate as little as possible from the "preferred" values of parameters while delivering sensible system properties

$$p\left(\theta \mid m\right) = p_1\left(\theta_1 \mid m\right) \times \cdots \times p_n\left(\theta_n \mid m\right) \times q_1\big(h(\theta) \mid m\big) \times \cdots \times q_k\big(h(\theta) \mid m\big)$$

Implemenation in IrisT

The following `@Model` class functions (methods) can be used to construct a `@SystemProperty` object for efficient evaluation of system properties

Function	Description
`simulate`	Any kind of simulation, including complex simulation design
`acf`	Autocovariance and autocorrelation functions
`xsf`	Power spectrum and spectral density functions
`ffrf`	Filter frequency response function