Stacked time solution method with first-order terminal condition

IrisToolbox Knowledge Base

Overview

- Creating a stacked time system
- Simulation procedure and terminal condition
- Changes in information sets

Model equations

System of n dynamic conditional-expectations equations

$$
\begin{gathered}
\mathrm{E}_t\left[\; f_1\left(x_{t-k},\ldots,x_{t+m}\right)\;\right]=0\\
\mathrm{E}_t\left[\; f_2\left(x_{t-k},\ldots,x_{t+m}\right)\;\right]=0\\
\vdots\\
\mathrm{E}_t\left[\; f_n\left(x_{t-k},\ldots,x_{t+m}\right)\;\right]=0
\end{gathered}
$$

where

- n is the number of model equations
- x_t is an $n\times 1$ vector model variables
- $\mathrm{E}_t[\cdot]$ is a conditional expectations operator
- k is the maximum lag
- m is the maximum lead

Stacked time setup

- Simulation range $t = 1, \ldots, T$
- Drop the expectations operator
- Stack the n equations for the T simulation periods
- Create a large static system of $T \times n$ equations in $T \times n$ unknowns
- Known initial conditions $x_{1-k}, \ldots, x_0$
- Unknown terminal conditions $x_{T+1}, \ldots, x_{T+m}$

Stacked time system of equations and unknowns

- A total of $n \cdot T$ equations
- A total of $n \cdot T$ unknows, $x_t,\ t = 1, \ldots, T$

$$
\begin{gathered}
f_1\left(x_{1-k}, \ldots, x_{1+m}\right) = 0 \\
f_2\left(x_{1-k}, \ldots, x_{1+m}\right) = 0 \\
\vdots \\
f_n\left(x_{1-k}, \ldots, x_{1+m}\right) = 0 \\
\vdots \\
\vdots \\
f_1\left(x_{T-k}, \ldots, x_{T+m}\right) = 0 \\
f_2\left(x_{T-k}, \ldots, x_{T+m}\right) = 0 \\
\vdots \\
f_n\left(x_{T-k}, \ldots, x_{T+m}\right) = 0
\end{gathered}
$$

Simulation setup

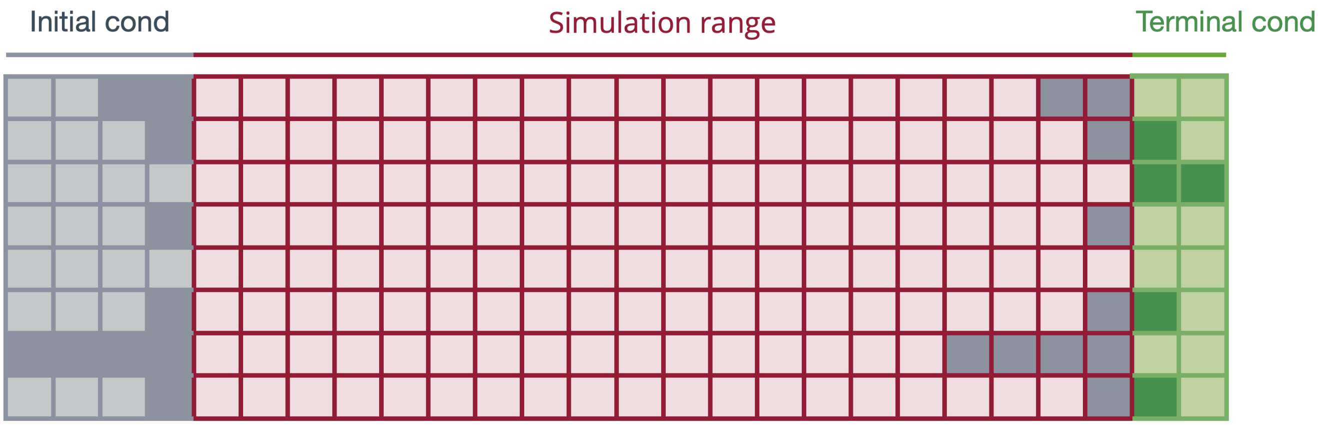
Initialize

- Create an $n \times (T + k + m)$ matrix
- Fill in initial condition in columns $1, \ldots, k$

In each iteration

- Fill in the simulation range columns
- Taking the last simulation range columns as initial condition, use the first order simulator to fill in terminal codnition
- Evaluate the LHS–RHS discrepancy for all $n \times T$ equations and send this information to the solver

Visualization of the simulation setup



Terminal condition derived from first-order solution

First-order solution of the model (model-consistent expectations of endogenous variables integrated away)

$$x_t \approx T\, x_{t-1} + K + R_0\, \varepsilon_t + \cdots + R_h\, \mathrm{E}_t[\varepsilon_{t+h}]$$

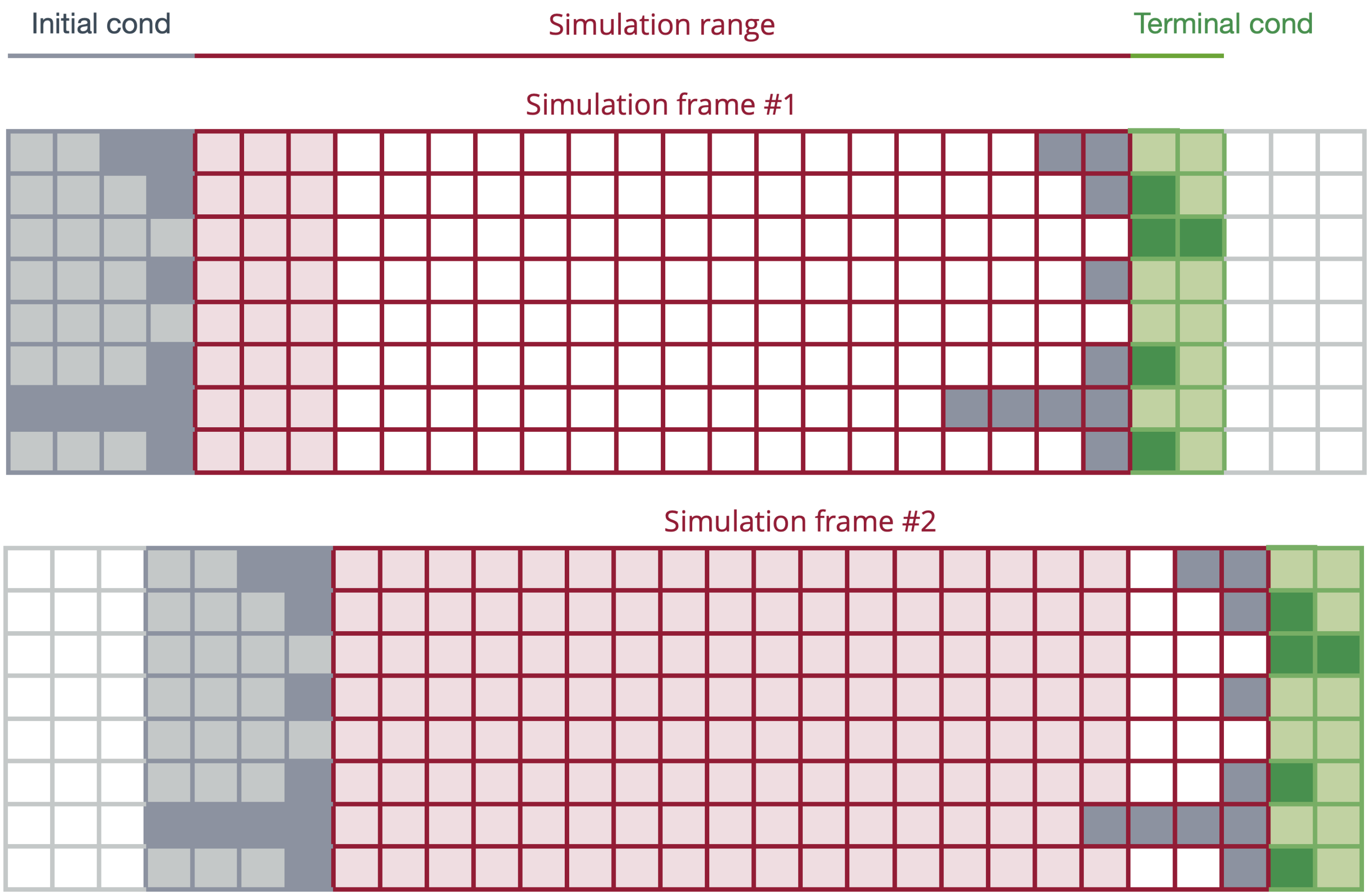
Create a stacked system to calculate the terminal condition points needed

$$\begin{bmatrix} x_{T+1} \\ \vdots \\ x_{T+m} \end{bmatrix} = T^{\text{term}} \begin{bmatrix} x_{T+1-k} \\ \vdots \\ x_{T} \end{bmatrix} + K^{\text{term}}$$

Changes in information sets within a simulation

- By design, a stacked time simulation is consistent with an assumption of all future events (shocks, swaps) are anticipated
- To simulate a sequence of unanticipated events, the simulation needs to be broken down into a sequence of sub-simulations (simulation frames)

Breakdown of simulation into frames



Implemenation in IrisT

Syntax of the `simulate` function for the stacked time method

```
[outputDb, info, frameDb] = simulate( ...
    model, inputDb, range ...
    , method="stacked" ...
);
```

Output structure `info`

Auxiliary output databank with simulation frames `frameDb`

Options to control the setup of the stacked time method

`startIter=`

- `"firstOrder"` (default)
- `"data"`

`terminal=`

- `"firstOrder"` (default)
- `"data"`