Transport in "sparse" networks: From linear response to effective range hopping.

Yaron de Leeuw, Doron Cohen

Ben-Gurion University of the Negev

The model

Motivated by [1, 2, 3], we consider 1D, quasi-1D and 2D networks with:

$$\frac{dp_n}{dt} = \sum_m w_{nm} p_m$$

The random site model ($1D$ / $2D$) :

$$w_{nm} = w_0 e^{-\epsilon_{nm}} e^{-|x_n - x_m|/\xi}$$

$$x_n = \text{random with avg. distance } r_0$$

$$s \equiv \xi / r_0$$

$$\epsilon_{nm} = \text{random activation potential} \in [0, \sigma]$$

The banded lattice model (quasi-$1D$) :

$$w_{nm} = w_0 e^{-\epsilon_{nm}} B(n-m)$$

$$b = \text{"bandwidth"} = \text{width of } B(r)$$

Diffusion

The long time dynamics are characterized by the spreading $S(t)$, the survival probability $\mathcal{P}(t)$ and the spectral counting function $\mathcal{N}(\lambda)$. In the case of diffusion these are:

$$S(t) = \langle r^2(t) \rangle \sim (2d)Dt$$

$$\mathcal{P}(t) \sim \frac{1}{(Dt)^{d/2}}$$

$$\mathcal{N}(\lambda) \sim \left[\frac{\lambda}{D}\right]^{d/2}$$

The $1D$ chain model

For $s > 1$ we get:

$$D = \left(\frac{1}{N}\sum_n \frac{1}{w_n}\right)^{-1} = \frac{s-1}{s} w_0$$

For $0<s<1$, we get subdiffusion (D=0) with:

$$S(t) \sim t^{2s/(1+s)}$$

$$\mathcal{P}(t) \sim t^{-s/(1+s)}$$

$$\mathcal{N}(\lambda) \sim \lambda^{s/(1+s)}$$

These are analytic results (see [1]), and they are reflected in the spectral analysis, figure 2.

Effective Range Hopping

According to linear response:

$$D_{\text{LRT}} = \frac{1}{2d} \iint w(r,\epsilon) r^2 \rho(r,\epsilon) d\epsilon dr$$

However, this will not work for "sparse" systems, because only percolating paths contribute to the transport. We define a percolation threshold by :

$$\iint_{w(r,\epsilon)>w*} \rho(r,\epsilon) dr d\epsilon = p_c$$

with p_c of order unity.
Then we calculate D as follows:

$$D_{\text{ERH}} = \frac{1}{2d} \iint \min\{w(r,\epsilon), w^*\} r^2 \rho(r,\epsilon) d\epsilon dr$$

Diffusion as a function of sparsity

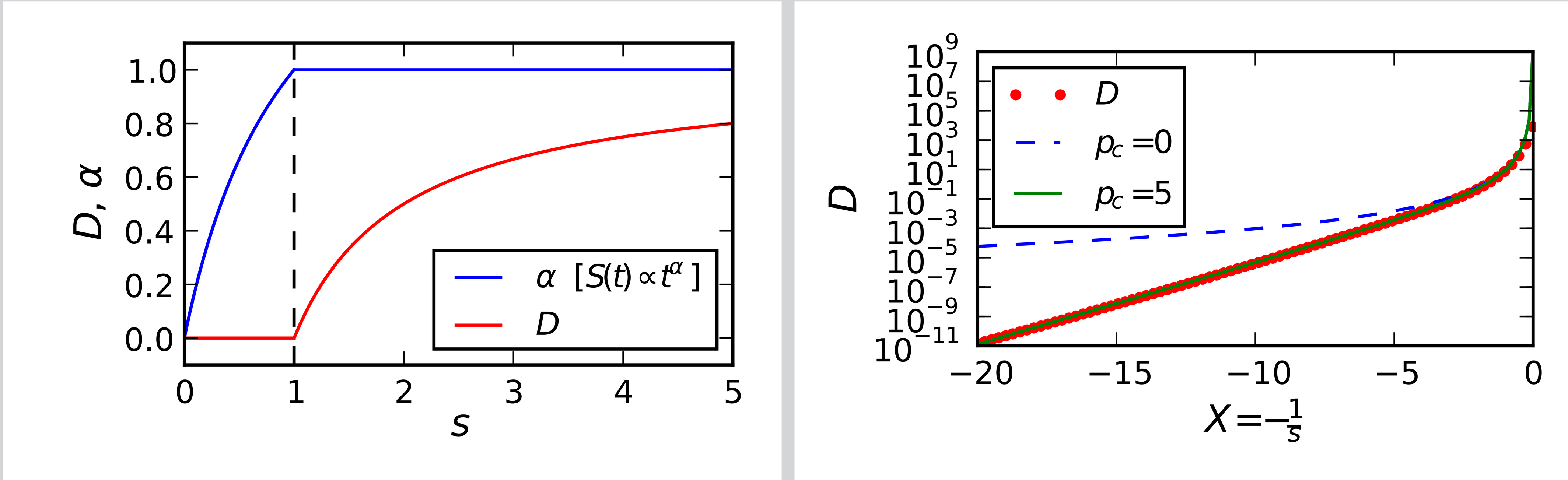

Figure 1: The zero activation energy model: (left) The dependence of D and α on s as predicted by the theory for a $1D$ network. (right) Numerical resuls for D in the case of a $2D$ network, based on spectral analysis of Fig2 compared with our ERH prediction. The dashed line is LRT.

Eigenvalue distributions

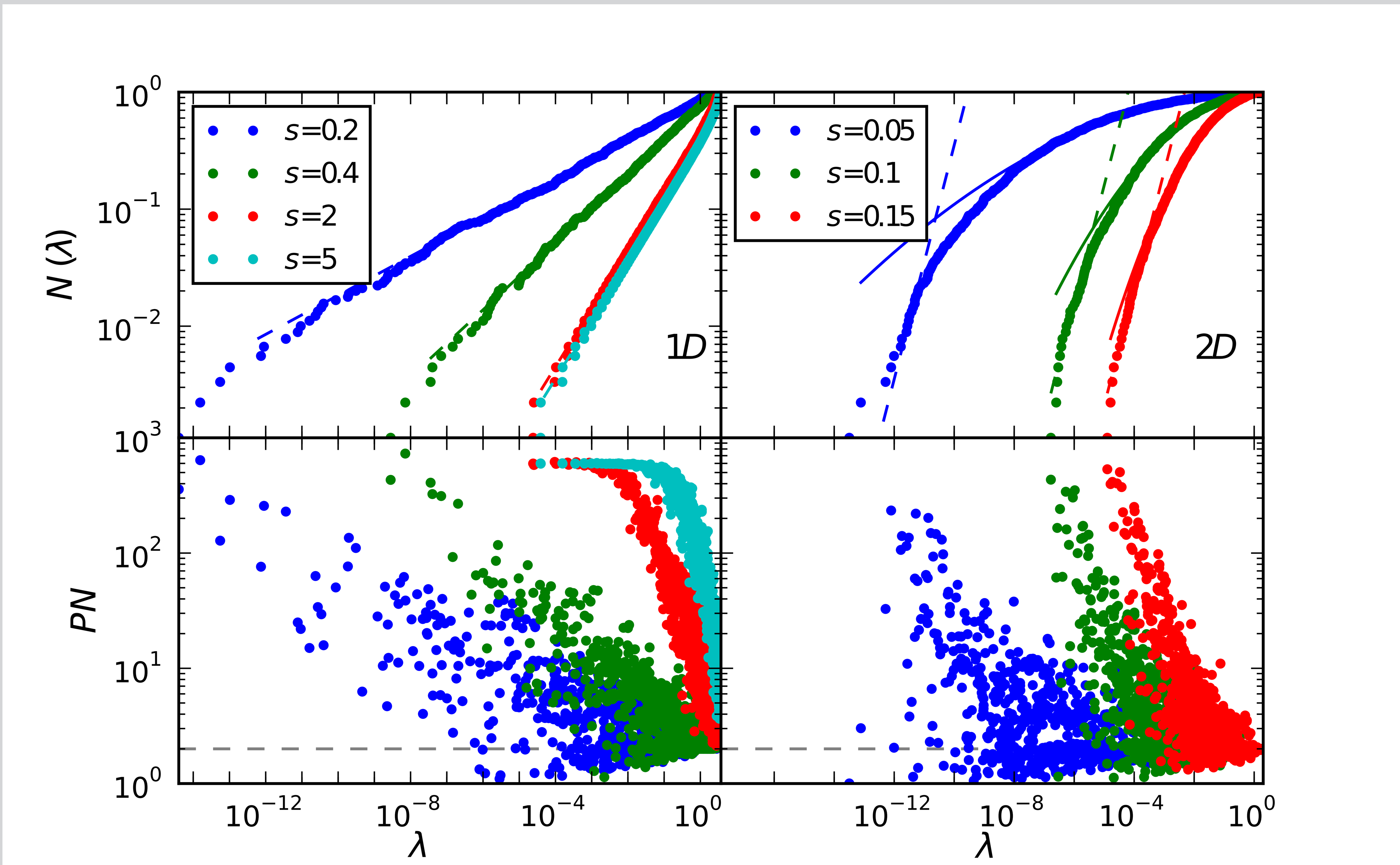

Figure 2: Spectral properties for $1D$ and $2D$. The upper panels show the cumulative count of the eigenvalues, while the lower panels present the participation number of the corresponding eigenmodes. In the $2D$ case the RG analysis of the localized modes [2] predicts the solid lines. The numerics show clearly a departure towards a diffusive behavior, as implied by ERH.

Banded Lattice Model

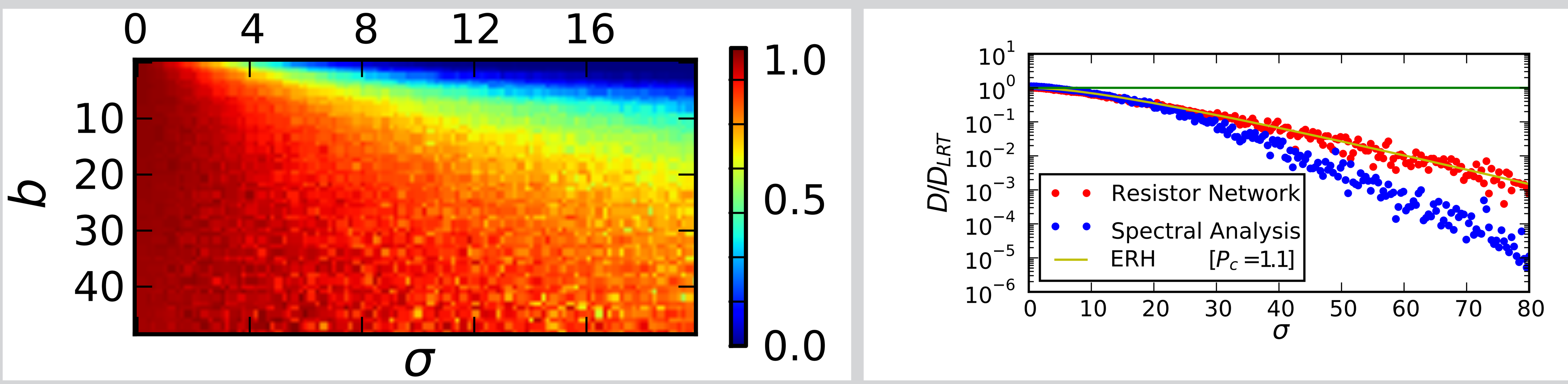

Figure 3: (left) The numerical result for D/D_{LRT} imaged as a function of σ and b. (right) Plot of the results for a $b = 10$ matrix. The blue points are based on spectral analysis (involving an undeterminded pre-factor). The curve is our ERH prediction.

References

[1] S. Alexander, J. Bernasconi, W.R. Schneider, R. Orbach, Rev. Mod. Phys. 53, 175 (1981).

[2] A. Amir, Y. Oreg, Y. Imry, Phys. Rev. Lett. 105, 070601 (2010); Phys. Rev. B 77, 165207 (2008).

[3] A. Stotland, T. Kottos, D. Cohen, Phys. Rev. B 81, 115464 (2010).