Random matrix modelling of transport in sparse systems

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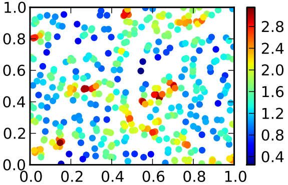
The model system

Rate Equation:

$$\frac{dp_n(t)}{dt} = \sum_m w_{nm} p_m(t)$$

$$w_{nm} = w_0 \, \mathrm{e}^{-\epsilon_{nm}} \, \mathrm{e}^{-|x_n - x_m|/\xi}$$

$$s \equiv \frac{\xi}{r_0} \qquad \leftarrow \text{ Sparsity parameter}$$



- $\epsilon_{nm} > 0$ is a random activation energy
- x_n are the locations of sites in space

$$\rho(r,\epsilon) dr d\epsilon = \frac{\Omega_d \, r^{d-1} dr}{r_0^d} \, f(\epsilon) d\epsilon, \qquad \Omega_d = 2, 2\pi, 4\pi$$

The "degenerate" model - $f(\epsilon) = \delta(\epsilon)$
The "Mott" hopping model - $f(\epsilon) = 1$

Diffusion coefficient

The long term dynamics are characterized by the spreading, the survival probability and the spectral counting function. For diffusive systems:

long time spreading - $S(t) = \left\langle r^2(t) \right\rangle \quad \sim \quad (2d)Dt$

long time survival - $\mathcal{P}(t) \sim \dfrac{1}{(Dt)^{d/2}}$

spectral density - $\mathcal{N}(\lambda) = \int^{\lambda} g(\lambda) d\lambda \quad \sim \quad \left[\dfrac{\lambda}{D}\right]^{d/2}$

D can be determined via numerical solution of a circuit equation $GV = I$ or via the small λ asymptote of $\mathcal{N}(\lambda)$.

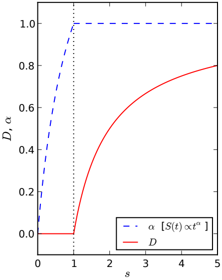
Random site model, $1d$ vs $2d$

$d = 1$, not sparse $s > 1$:

$$D = \frac{s-1}{s} w_0,$$

$d = 1$, sparse $s < 1$

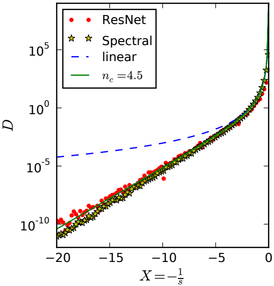
$$S(t) \sim t^{2s/(1+s)}$$



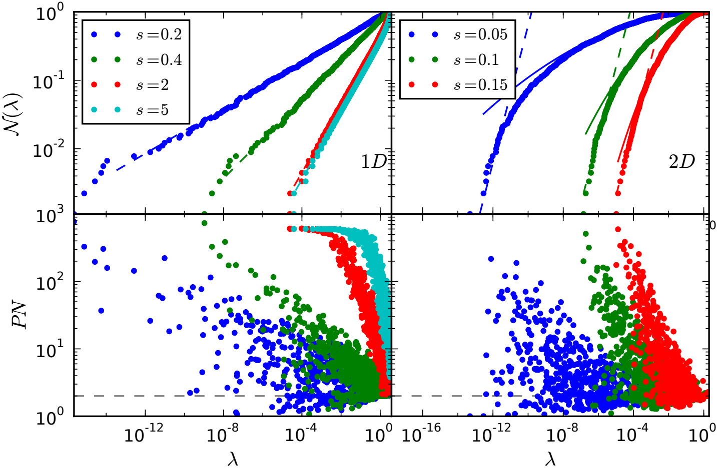
$d = 2$, always finite D.

$$D_{\mathrm{ERH}} = \mathrm{EXP}_{d+2}\left(\frac{1}{s_{\mathit{eff}}}\right) \mathrm{e}^{-1/s_{\mathit{eff}}} D_{\mathrm{linear}}$$

$$s_{\mathit{eff}} = \left(\frac{d}{\Omega_d} n_c\right)^{-1/d} \frac{\xi}{r_0}$$



Spectral comparison $1d$ vs $2d$



Solid lines - RG theory prediction [**Amir, Oreg, Imry (2010)**].
Dashed lines - Resistor network theory prediction [**YdL, DC (2012)**].

Several views over localization in this model

Localization in a disordered elastic medium near two dimensions
Sajeev John, H. Sompolinsky, M.J. Stephen, Phys. Rev. B 27, 55925603 (1983)

- All states are localized
- Localization length diverges for $\lambda \to 0$
- Debye law holds (diffusion)

Localization, Anomalous Diffusion, and Slow Relaxations
A. Amir, Y. Oreg, Y. Imry, Phys. Rev. Lett. 105, 070601 (2010)

- All states are localized
- There is sub-diffusion rather than difussion

Diffusion in sparse networks: Linear to semilinear crossover
Y. de Leeuw, D. Cohen, Phys. Rev. E 86, 051120 (2012)

- All states are localized
- There is a percolation crossover at finite λ
- Localization length diverges for $\lambda \to 0$
- Debye law holds (diffusion)

Emergent percolation length and localization in random elastic networks
A. Amir, J. J. Krich, V. Vitelli, Y. Oreg, Y. Imry, arXiv:1209.2169 (2012)

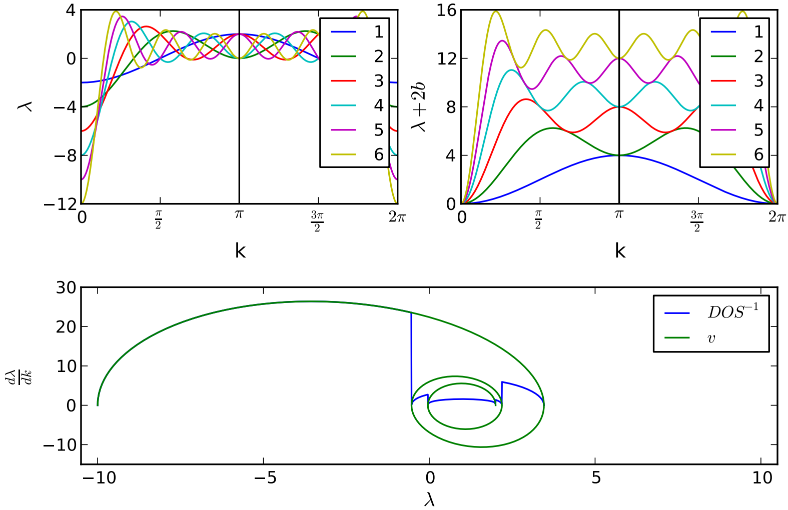
- There is an Anderson mobility edge at finite λ
- The low lying states become de-localized
- Debye law holds (diffusion)

Work plan

- Current goal: find the relation between resistor network calculation and Fourier law, or between mesoscopic transport and heat transport.
  - Study and explain the band structure of banded-sparse models.
  - Calculate heat transport and resistor network transport for these models, and look for similarities (e.g. VRH and percolation)
  - In the weak banded model, each energy λ has more than one k: check the effects on scattering and localization length calculations.
  - Check N and b scaling.
- Models with relaxation
- Sinai spectrum in $1d$
- Sinai diffusion in $2d$
- Quantum spreading
- Discrete particles (many-body physics)

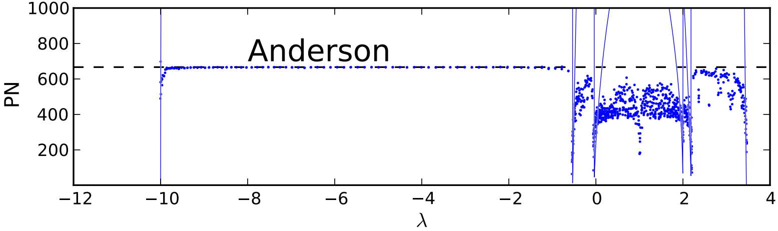
Band structure for banded weak disorder

We can analytically compute the spectrum for a banded matrix of ones (Bloch).

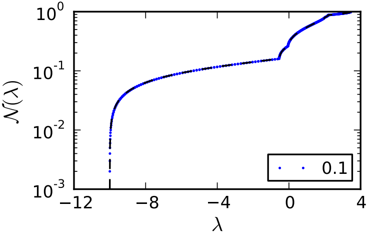


Band structure for banded weak disorder

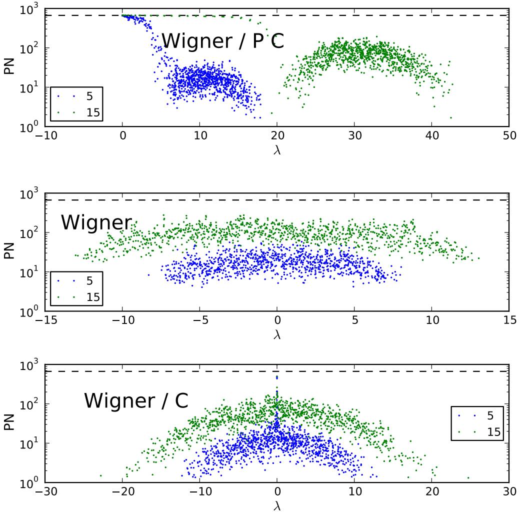
For weak disorder, PN goes like DOS^{-2} with the Bloch DOS



and $\mathcal{N}(\lambda)$ is very similar to the Bloch $\mathcal{N}(\lambda)$ (the dashed line)



PN structure for random band



PN structure for banded - sparse model

$$w_{nm} = \exp([-\eta, 0])$$