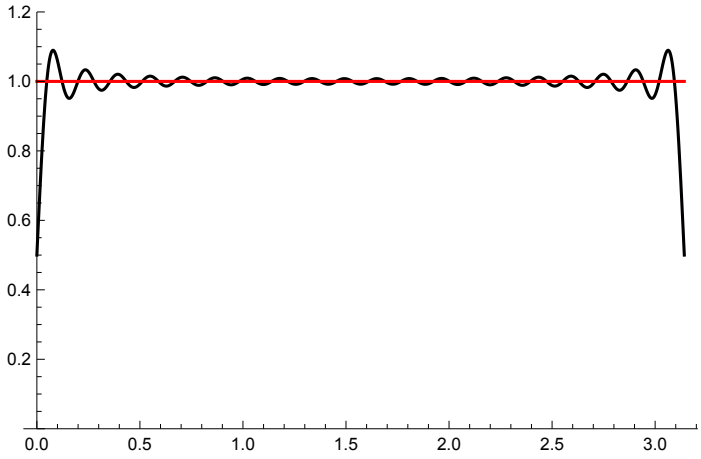
In[990]:=
```
(*#7 For the following functions,
sketch the Fourier series of f(x) (on the interval -L < x <\.11)4. Compare
f(x) to its Fourier series:*)
```

In[845]:=
```
(*Problem #7 part a*)
(*Plot the Fourier Series f[x] = 1 on the interval [-L,L]*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, 0, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, 0, L}] * (1 / L)
f[x_] = 1;
L = Pi;

a[n];
b[n];

Table[a[n], {n, 0, 10}];
Table[b[n], {n, 0, 10}];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2

Plot[{Evaluate[myFourier[x, 40]], f[x]},
 {x, 0, L}, PlotStyle → {Black, Red}, PlotRange → {0, 1.2}]
(*Graph of part a f(x) = 1*)
```

Out[857]=

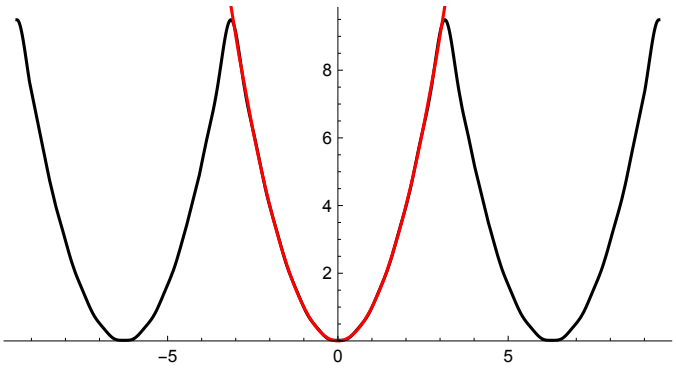


In[509]:=
```
(*Problem #7 part b f(x) = x^2*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] = x^2;
L = Pi;
a[n]
b[n]

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2


Plot[{Evaluate[myFourier[x, 10]], f[x]}, {x, -3 L, 3 L},
 PlotStyle → {Black, Red}, PlotRange → {-0.01, f[L]}, AspectRatio → 1 / 2]
```

Out[514]=
$$\frac{1}{n^3\,\pi}\left(4\,n\,\pi\,\text{Cos}\left[n\,\pi\right]+2\,\left(-2+n^2\,\pi^2\right)\,\text{Sin}\left[n\,\pi\right]\right)$$

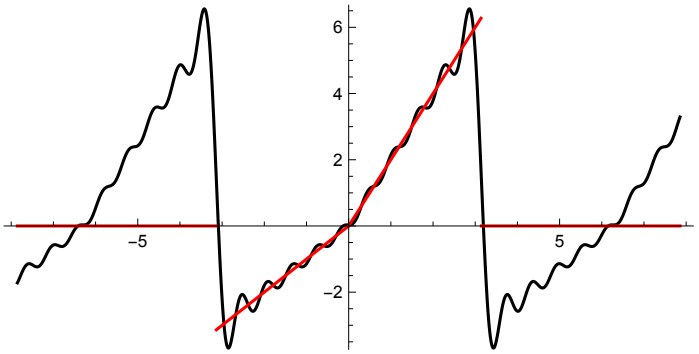
Out[515]= 0

Out[519]=



In[979]:=
```
(*Problem #7 part c f(x) is a piecewise defined function*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] := Piecewise[{{x, x >= -L && x < 0}, {2 x, x > 0 && x <= L}}]
L = Pi;

a[n];
b[n];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2


Plot[{Evaluate[myFourier[x, 10]], f[x]}, {x, -2.5 L, 2.5 L},
 PlotStyle → {Black, Red}, PlotRange → {f[-L] - .6, f[L] + .4}, AspectRatio → 1 / 2]
(*Graph of part c f(x) = piecewise*)
(*Notice that the periodicity of the 'x' part and '2x' are
 not the same*)
```

Out[989]=



In[924]:=
```
(*Problem #7 part d f(x) is a piecewise defined function*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] := Piecewise[{{0, x >= -L && x < 0}, {1, x > 0 && x <= L}}]
L = Pi;

a[n];
b[n];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2


Plot[{Evaluate[myFourier[x, 40]], f[x]}, {x, -L, L},
 PlotStyle → {Black, Red}, PlotRange → {f[-L] - .4, f[L] + .4}, AspectRatio → 1 / 2]
(*Graph of part d piecewise*)
(*Notice that the graph switches form 0 and 1 at the origin*)
```

Out[934]=