In[1365]:=
```
(*Problem #2a) Plot the wave equation solution with L=1, c=1, f(x)= 180x^2(1-x)
   with I.C. u(x,0)=f(x) and u_t(x,0)=g(x) *)
Clear[a, b, f, x, t, n, L, g, myFSin, myFCos, c]

(*Here I define the coefficients for the terms of the series expansions*)
a[n_] := Integrate[f[x] * Sin[n * Pi * x / L] * (2 / L), {x, 0, L}]
b[n_] := Integrate[g[x] * Cos[n * Pi * x / L] * (2 / (n * c * Pi)), {x, 0, L}]
L = 1;
c = 1;

(*Below are our initial conditions when t = 0*)
f[x_] := 180 * x^2 * (1 - x)
g[x_] := 0

Print["Coefficients"]
Simplify[a[n], Assumptions → {n > 0 && n ∈ Integers}]
b[n]

(*This is the definition of the solution surface*)
u[x_, t_, M_] := Sum[a[n] * Cos[n * c * Pi * t / L] * Sin[n * Pi * x / L], {n, 1, M}];

(*This definies a new function which is just first 10 terms
 of the series that equals our solution surface*)
myFunction[x_, t_] := Evaluate[u[x, t, 10]]

Print["u[x,t] to 10 terms:"]
myFunction[x, t]
```

Coefficients

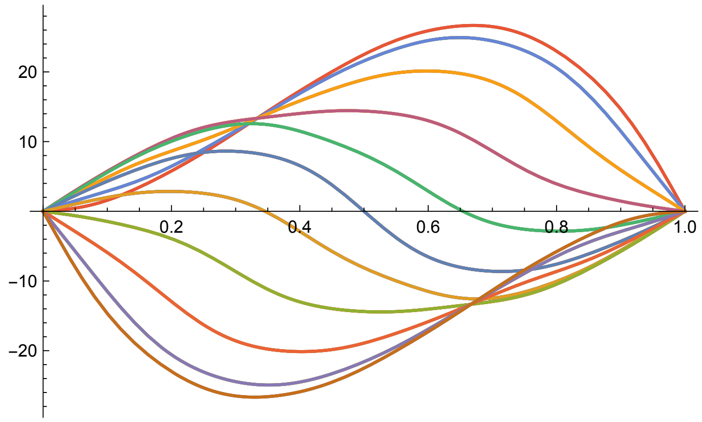
Out[1373]= $-\frac{720\,(1+2\,(-1)^n)}{n^3\,\pi^3}$

Out[1374]= 0

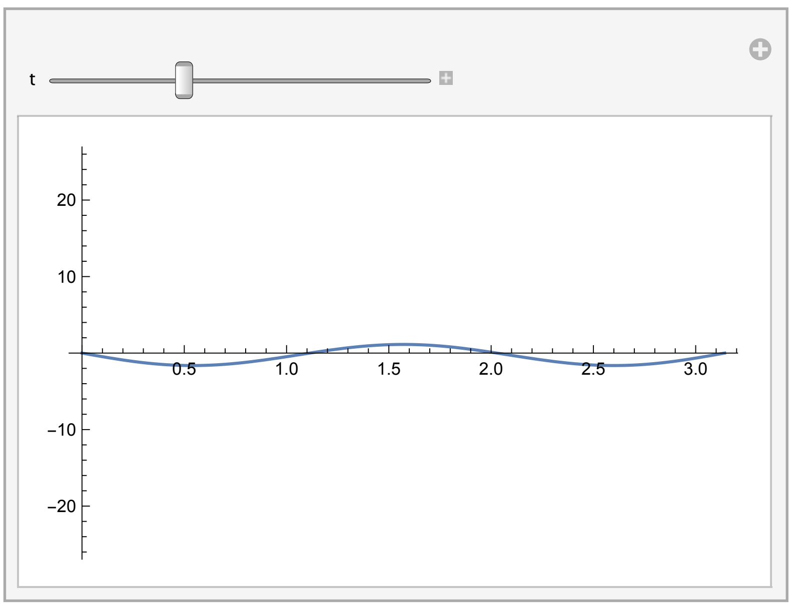
u[x,t] to 10 terms:

Out[1378]= $\frac{720\,\mathrm{Cos}[\pi\,t]\,\mathrm{Sin}[\pi\,x]}{\pi^3} - \frac{270\,\mathrm{Cos}[2\,\pi\,t]\,\mathrm{Sin}[2\,\pi\,x]}{\pi^3} + \frac{80\,\mathrm{Cos}[3\,\pi\,t]\,\mathrm{Sin}[3\,\pi\,x]}{3\,\pi^3} - \frac{135\,\mathrm{Cos}[4\,\pi\,t]\,\mathrm{Sin}[4\,\pi\,x]}{4\,\pi^3} + \frac{144\,\mathrm{Cos}[5\,\pi\,t]\,\mathrm{Sin}[5\,\pi\,x]}{25\,\pi^3} - \frac{10\,\mathrm{Cos}[6\,\pi\,t]\,\mathrm{Sin}[6\,\pi\,x]}{\pi^3} + \frac{720\,\mathrm{Cos}[7\,\pi\,t]\,\mathrm{Sin}[7\,\pi\,x]}{343\,\pi^3} - \frac{135\,\mathrm{Cos}[8\,\pi\,t]\,\mathrm{Sin}[8\,\pi\,x]}{32\,\pi^3} + \frac{80\,\mathrm{Cos}[9\,\pi\,t]\,\mathrm{Sin}[9\,\pi\,x]}{81\,\pi^3} - \frac{54\,\mathrm{Cos}[10\,\pi\,t]\,\mathrm{Sin}[10\,\pi\,x]}{25\,\pi^3}$

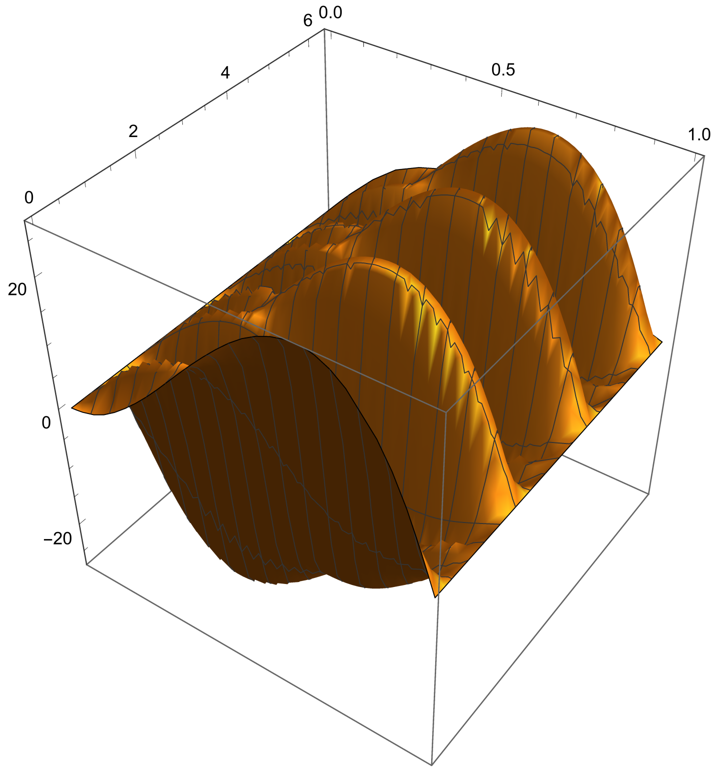
```
(*Below are the snapshots of the vibrating string at different times*)
myTable := Table[myFunction[x, t], {t, 0, 5, 0.1}]
Plot[myTable, {x, 0, L}]
```




```
(*Here you can manipulate the plot*)
Manipulate[Plot[myFunction[x, t], {x, 0, L}, PlotRange → {-27, 27}], {t, 0, 10, .05}]
```



```
Clear[t]
(*Here is the behavior over t = 0 to 2Pi of the vibrating string*)
(*From the to graphs I can conclude that that string wave is bouncing back and
 forth along the entirety of the string since. This
 demonstrates the oscillatory nature of the vibrating string,
this also shows the behavior of the vibration being UNDAMPED,
the wave will continue to translate back and forth as t goes to infinity*)
Plot3D[Evaluate[myFunction[x, t]], {x, 0, L}, {t, 0, 2 * Pi}, BoxRatios → {1, 1, 1}]
```



In[1379]:=
```
(*Problem #2b) Plot the wave equation solution with L=1, c=1, f(x)= 180x^2(1-x)
   with I.C. u(x,0)=f(x) and u_t(x,0)=g(x) *)
Clear[a, b, f, x, t, n, L, g, myFSin, myFunction, myFCos, c]

(*Here I define the coefficients for the terms of the series expansions*)
a[n_] := Integrate[f[x] * Sin[n * Pi * x / L] * (2 / L), {x, 0, L}]
b[n_] := Integrate[g[x] * Cos[n * Pi * x / L] * (2 / (n * c * Pi)), {x, 0, L}]
L = Pi;
c = 1;

(*Below are our initial conditions when t = 0*)
f[x_] := 2 * Sin[3 x]
g[x_] := 1 - x

Print["Coefficients"]
Simplify[a[n], Assumptions → {n > 0 && n ∈ Integers}]
Simplify[b[n], Assumptions → {n > 0 && n ∈ Integers}]

(*This is the definition of the solution surface*)
u[x_, t_, M_] := Sum[
   (a[n] * Cos[n * c * Pi * t / L] + b[n] Sin[n * c * Pi * t / L]) * Sin[n * Pi * x / L], {n, 1, M}];

(*This definies a new function which is just first 10 terms
 of the series that equals our solution surface*)
myFunction[x_, t_] := Evaluate[u[x, t, 10]]

Print["u[x,t] to 10 terms:"]
myFunction[x, t]
```

Coefficients

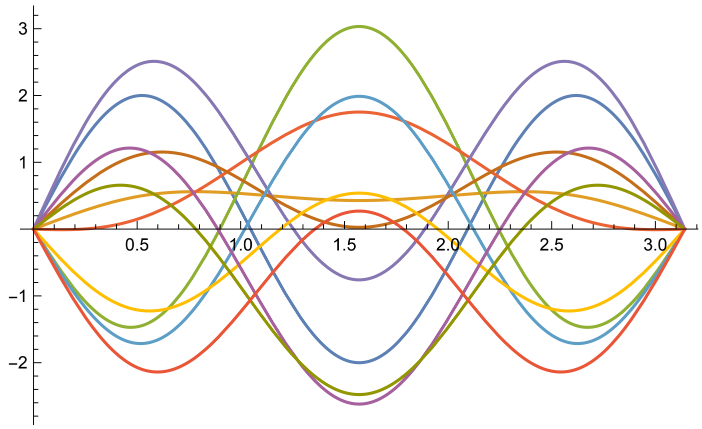
Out[1387]= 0

Out[1388]= $-\frac{2\,(-1+(-1)^n)}{n^3\,\pi}$

u[x,t] to 10 terms:

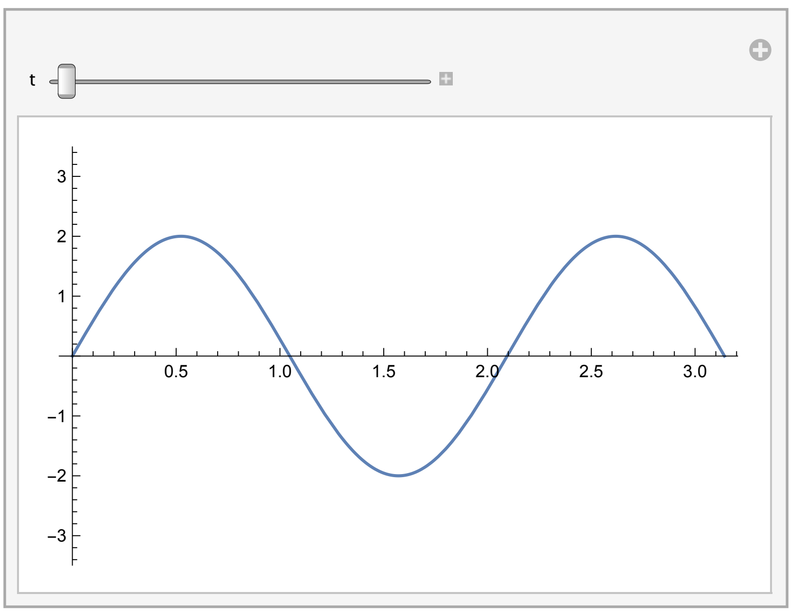
Out[1392]= $\frac{4\,\mathrm{Sin}[t]\,\mathrm{Sin}[x]}{\pi} + \left(2\,\mathrm{Cos}[3\,t] + \frac{4\,\mathrm{Sin}[3\,t]}{27\,\pi}\right)\mathrm{Sin}[3\,x] + \frac{4\,\mathrm{Sin}[5\,t]\,\mathrm{Sin}[5\,x]}{125\,\pi} + \frac{4\,\mathrm{Sin}[7\,t]\,\mathrm{Sin}[7\,x]}{343\,\pi} + \frac{4\,\mathrm{Sin}[9\,t]\,\mathrm{Sin}[9\,x]}{729\,\pi}$

```
(*Below is a plot of the solution curves for
 values of t form 0 to 5 sample at 0.5 intervals*)
myTable := Table[myFunction[x, t], {t, 0, 5, 0.5}]
Plot[myTable, {x, 0, L}]
```




```
(*Here you can manipulate the plot*)

Manipulate[
 Plot[myFunction[x, t], {x, 0, L}, PlotRange → {-3.5, 3.5}], {t, 0, 10, 0.05}]
```



```
Clear[t]
(*Here is the behavior over t = 0 to 2Pi of the vibrating string*)
(*From the to graphs I can conclude that that string wave is bouncing back and
 forth along the entirety of the string since. This
 demonstrates the oscillatory nature of the vibrating string,
this also shows the behavior of the vibration being UNDAMPED,
the wave will continue to translate back and forth as t goes to infinity*)
Plot3D[Evaluate[myFunction[x, t]], {x, 0, L}, {t, 0, 2 * Pi}, BoxRatios → {1, 1, 1}]
```