HOMEWORK 2 PROBLEM #3:

A) Show that $u(x,t) = (x+t)^3$ satisfies the wave equation $u_{tt} = u_{xx}$

We will find the first and second derivatives with respect to *x* and *t*,

1. $u_t = 3(x+t)^2$
2. $u_{tt} = 6(x+t)$
3. $u_x = 3(x+t)^2$
4. $u_{xx} = 6(x+t)$

If we substitute into the wave equation we get:

$$u_{tt} = u_{xx} \implies 6(x+t) = 6(x+t)$$

Since both sides of the equal symbol are the same, equality is held and therefore our function $u(x,t)$ does satisfy the wave equation.

B) What initial conditions does $u(x,t) = (x+t)^3$?

Recall initial conditions for the wave equation are given by:

$$u(x,0) = f(x) \text{ and } u_t(x,t) = g(x)$$

Therefore, we will substitute 0 in for *t* in our solution,

$$u(x,0) = x^3 = f(x)$$

Then we will substitute 0 in for *t* in equation (1) to get,

$$u_t(x,0) = 3x^2 = g(x)$$

Notice that g(x) = f'(x), so we could think of the initial conditions as,

$u(x,0) = x^3 = f(x)$

$u_t(x,0) = 3x^2 = f'(x)$

C) Plot Solution Surface (See mathematica file for code) Below is the plot

D) Using the plot of the solution surface discuss the difference between u(x,0) and u(0,t). From the plot u(x,0) represents the initial position of the wave at time zero this is represented by the function x^3. No at u(0,t) this represents the behavior of the wave at one end of our 1-dimensional rod in this case at the end of the rod u(0,t) = t^3.