```
In[1300]:= (*Homework #2 Problem 1*)
(*Solve the 1-d heat equation with conditions *)
(*u(0,t)=u(L,t)=0 and u(x,0)=f(x) and 0<x<L*)
(*f(x) = 180x(1-x)^2*)
Clear[a, b, f, L, k, t, myFSin, myFCos, mTplot, frames]

(*Here are our constants and function f[x]*)
f[x_] := 180 x * (1 - x) ^2
L = 1;
k = 0.2;

(*Here is where we calculate the coefficients in the series expansion*)
b[n_] := Integrate[f[x] * Sin[n * Pi * x / L], {x, 0, L}] * (2 / L)

Print["b[n]="]
Simplify[b[n], Assumptions → {n > 0 && n ∈ Integers}]

(*Here is the solution series to the heat equation*)
u[x_, t_, M_] := Sum[b[n] * Sin[n * Pi * x / L] * E^(-k * (n * Pi / L) ^2 * t), {n, 1, M}]

(*This is the solution to 10 terms in the series*)
myU[x_, t_] := Evaluate[u[x, t, 10]]

Print["u[x,t] to 10 terms:"]
myU[x, t]

(*Below is a plot that you can manipulate to see how the temperature
 distribution decreases*)
Manipulate[Plot[myU[x, t], {x, 0, L}, PlotRange → {0, 28}], {t, 0, 10}]

(*From the observing the graphs I can
 conclude that as t goes to infinity the heat
 will be considerably dissipated*)
```

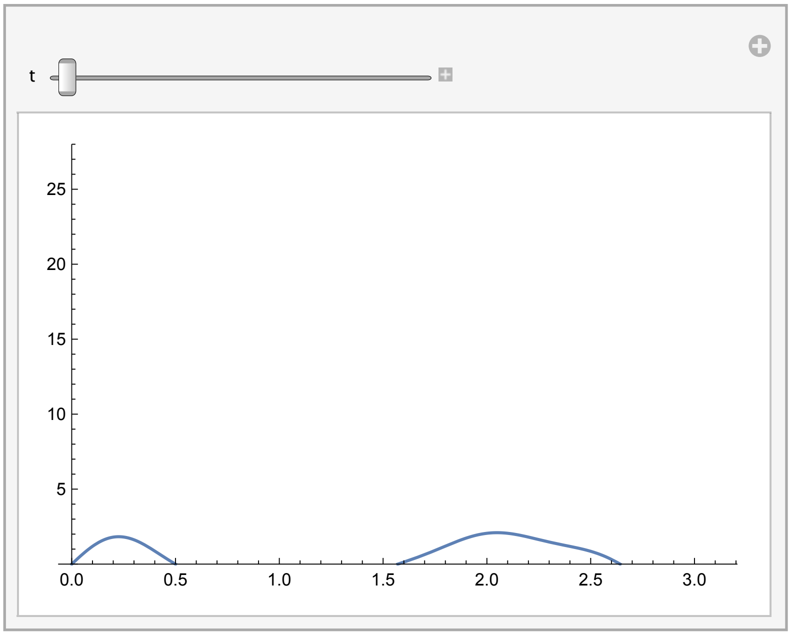
b[n]=

Out[1306]= $$\frac{720\,(2+(-1)^n)}{n^3\,\pi^3}$$

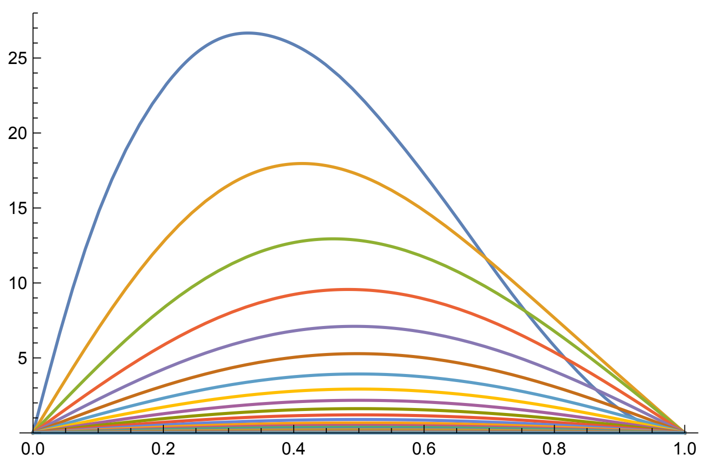
u[x,t] to 10 terms:

Out[1310]= $$\frac{720\,e^{-1.97392\,t}\,\mathrm{Sin}[\pi\,x]}{\pi^3}+\frac{270\,e^{-7.89568\,t}\,\mathrm{Sin}[2\,\pi\,x]}{\pi^3}+\frac{80\,e^{-17.7653\,t}\,\mathrm{Sin}[3\,\pi\,x]}{3\,\pi^3}+\frac{135\,e^{-31.5827\,t}\,\mathrm{Sin}[4\,\pi\,x]}{4\,\pi^3}+\frac{144\,e^{-49.348\,t}\,\mathrm{Sin}[5\,\pi\,x]}{25\,\pi^3}+\frac{10\,e^{-71.0612\,t}\,\mathrm{Sin}[6\,\pi\,x]}{\pi^3}+\frac{720\,e^{-96.7221\,t}\,\mathrm{Sin}[7\,\pi\,x]}{343\,\pi^3}+\frac{135\,e^{-126.331\,t}\,\mathrm{Sin}[8\,\pi\,x]}{32\,\pi^3}+\frac{80\,e^{-159.888\,t}\,\mathrm{Sin}[9\,\pi\,x]}{81\,\pi^3}+\frac{54\,e^{-197.392\,t}\,\mathrm{Sin}[10\,\pi\,x]}{25\,\pi^3}$$

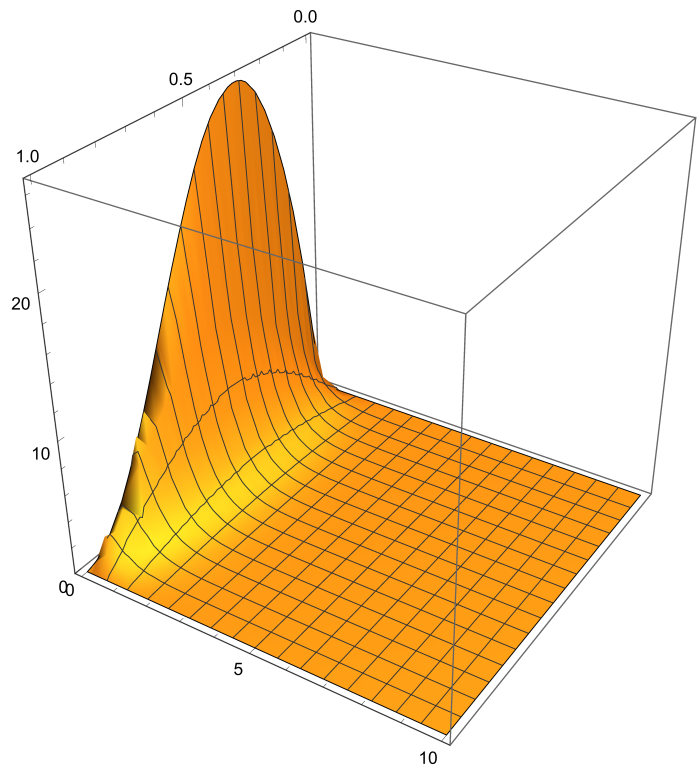
Out[1311]=




```
myPlotTable = Table[myU[x, t], {t, 0, 10, 0.15}];
Plot[myPlotTable, {x, 0, L}, PlotRange → {0, 28}]
(*Here you can see the individual time snapshots*)
(*Problem 1b:
   From the time snapshots t = 1,2,3,4,5.. the solution goes to zero,
infact from the 3D plots as t→∞ ⟹ u(x,t)=0. This makes sense because
  the B.C. conditions suggest that at both ends the temperature is 0,
therefore, the initial temperature will be forced temperature f(x)=
 u(x,0) that is initially given and that will slowly dissipate at t→∞*)
```



```
Clear[t]
(*Below is a 3d plot of the solution surface over time, notice the surface
  flattens as time continues just as I concluded with the 2-d plots above*)
function = Evaluate[u[x, t, 10]];
Plot3D[function, {x, 0, L}, {t, 0, 10},
 PlotRange → {0, 27}, ClippingStyle → None, BoxRatios → {1, 1, 1}]
```



In[1312]:=
```
(*Problem 1c let f(x) = 2cos(3x) & L = Pi & k = 1*)
(*Repeat parts b) and c) again*)

Clear[a, b, f, L, k, t, myU, x]

(*Here are our constants and function f[x]*)
f[x_] := 2 * Cos[3 x]
L = Pi;
k = 0.1;

(*Here is where we calculate the coefficients in the series expansion*)
b[n_] := Integrate[f[x] * Sin[n * Pi * x / L], {x, 0, L}] * (2 / L)

Print["b[n]="]
Simplify[b[n], Assumptions → {n > 0 && n ∈ Integers}]

(*Here is the solution series to the heat equation*)
u[x_, t_, M_] := Sum[b[n] * Sin[n * Pi * x / L] * E^(-k * (n * Pi / L)^2 * t), {n, 1, M}]

(*This is the solution to 10 terms in the series*)
myU[x_, t_] := Evaluate[u[x, t, 10]]

Print["u[x,t] to 10 terms:"]
myU[x, t]

(*Below is a plot that you can manipulate to see how the temperature
 distribution decreases*)
Manipulate[Plot[myU[x, t], {x, 0, L}, PlotRange → {-2.2, 2.2}], {t, 0, 10}]

(*From the observing the graphs I can
 conclude that as t goes to infinity the heat
 will be considerably dissipated*)
```

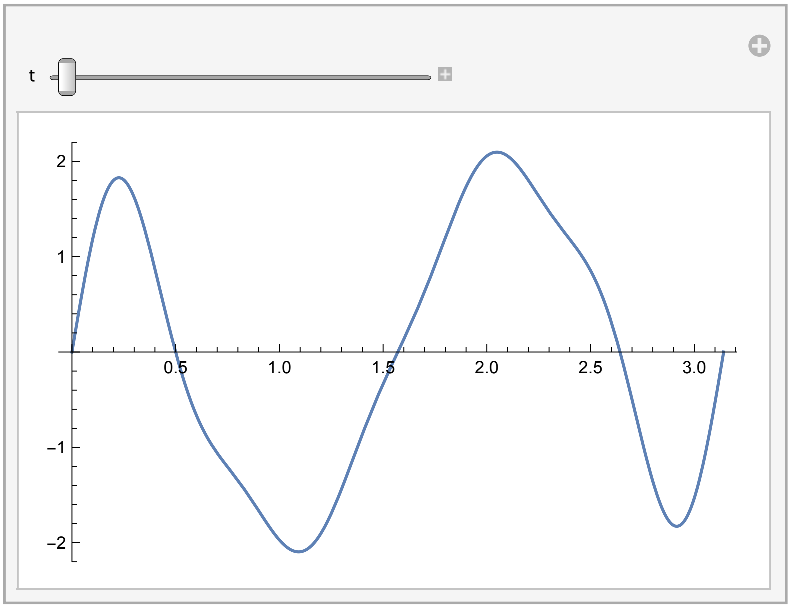
b[n]=

Out[1318]= $$\frac{4\,(1+(-1)^n)\,n}{\left(-9+n^2\right)\pi}$$

u[x,t] to 10 terms:

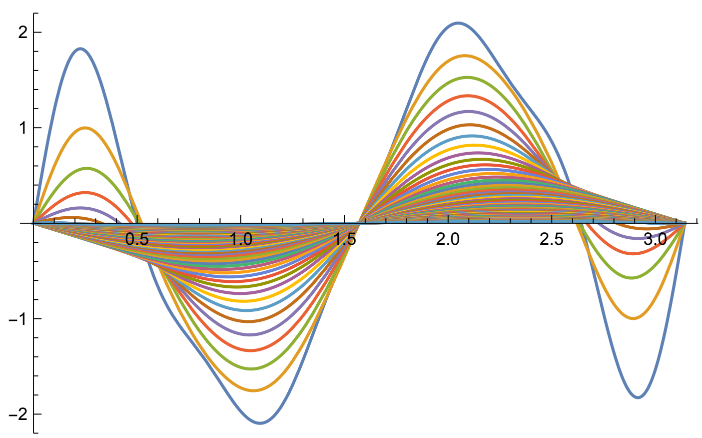
Out[1322]= $$-\frac{16\,e^{-0.4\,t}\,\text{Sin}[2\,x]}{5\,\pi}+\frac{32\,e^{-1.6\,t}\,\text{Sin}[4\,x]}{7\,\pi}+\frac{16\,e^{-3.6\,t}\,\text{Sin}[6\,x]}{9\,\pi}+\frac{64\,e^{-6.4\,t}\,\text{Sin}[8\,x]}{55\,\pi}+\frac{80\,e^{-10.\,t}\,\text{Sin}[10\,x]}{91\,\pi}$$

Out[1323]=




```
myPlotTable = Table[myU[x, t], {t, 0, 10, 0.15}];
Plot[myPlotTable, {x, 0, L}, PlotRange → {-2.2, 2.2}]

(*From the time snapshots t = 1,2,3,4,5.. the solution goes to zero,
infact from the 3D plots as t→∞ ⟹ u(x,t)=0. This makes sense because
  the B.C. conditions suggest that at both ends the temperature is 0,
therefore, the initial temperature will be forced temperature f(x)=
 u(x,0) that is initially given and that will slowly dissipate at t→∞*)
```



```
Clear[t]
(*Below is a 3d plot of the solution surface over time, notice the surface
  flattens as time continues just as I concluded with the 2-d plots above*)
Plot3D[myU[x, t], {x, 0, L}, {t, 0, 10},
 PlotRange → {-2.2, 2.2}, ClippingStyle → None, BoxRatios → {1, 1, 1}]
```