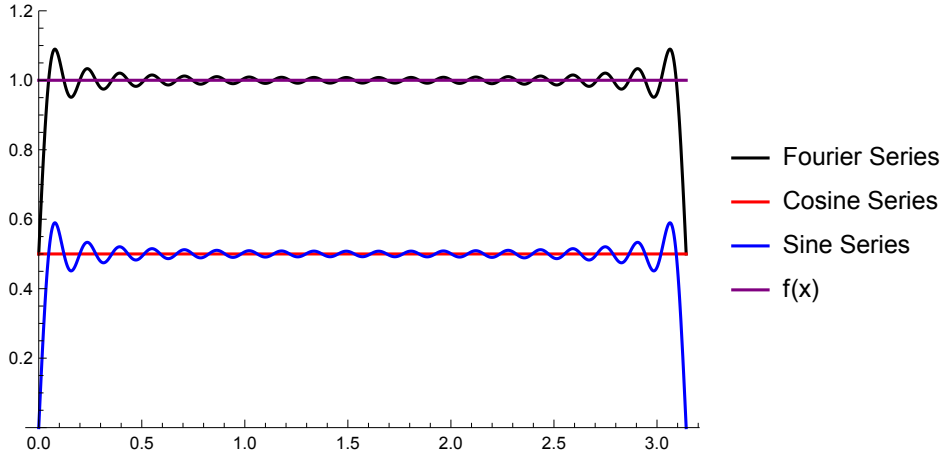
In[1133]:=
```
(*#8 For the following functions,
sketch the Fourier series of f(x) (on the interval -L < x <\.11)4. Compare
f(x) to its Fourier series:*)
```

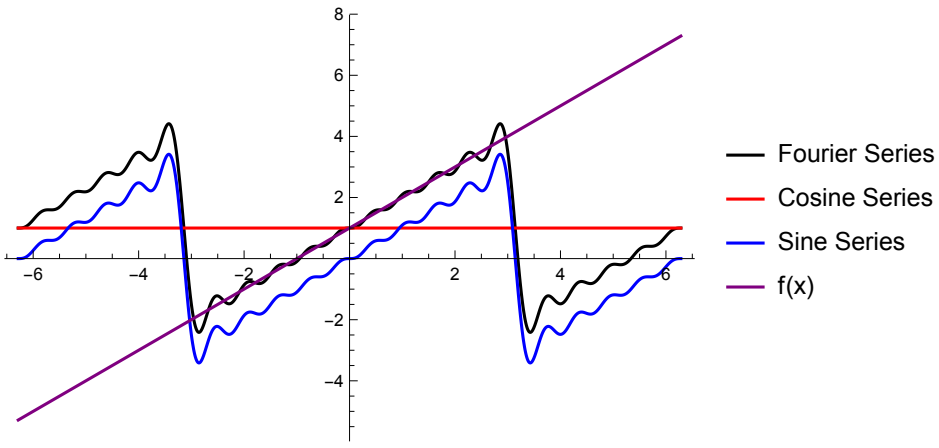
In[1134]:=
```
(*Problem #8 part a*)
(*Plot the Fourier,Cosine,Sine Series f[x] = 1 on the interval [-L,L]*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, 0, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, 0, L}] * (1 / L)
f[x_] = 1;
L = Pi;

a[n];
b[n];

Table[a[n], {n, 0, 10}];
Table[b[n], {n, 0, 10}];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2

Plot[{Evaluate[myFourier[x, 40]], Evaluate[myFCos[x, 40]],
  Evaluate[myFSin[x, 40]], f[x]}, {x, 0, L}, PlotStyle → {Black, Red, Blue, Purple},
 PlotLegends → {"Fourier Series", "Cosine Series", "Sine Series", "f(x)"},
 PlotRange → {0, 1.2}]
(*Graph of part a f(x) = 1*)
```

Out[1146]=



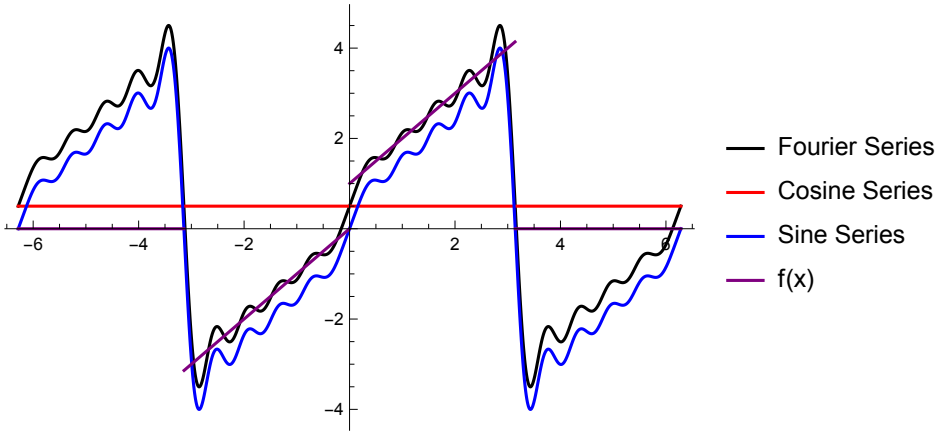
In[1147]:=
```
(*Problem #8 part b f(x) =1+x*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] = 1 + x;
L = Pi;

a[n];
b[n];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2

Plot[{Evaluate[myFourier[x, 10]], Evaluate[myFCos[x, 10]], Evaluate[myFSin[x, 10]],
   f[x]}, {x, -2 L, 2 L}, PlotStyle → {Black, Red, Blue, Purple},
 PlotLegends → {"Fourier Series", "Cosine Series", "Sine Series", "f(x)"}]
```

Out[1157]=



In[1246]:=
```
(*Problem #8 part c f(x) is a piecewise defined function*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] := Piecewise[{{x, x >= -L && x < 0}, {1 + x, x > 0 && x <= L}}]
L = Pi;

a[n];
b[n];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2


Plot[{Evaluate[myFourier[x, 10]], Evaluate[myFCos[x, 10]], Evaluate[myFSin[x, 10]],
   f[x]}, {x, -2 L, 2 L}, PlotStyle → {Black, Red, Blue, Purple},
 PlotLegends → {"Fourier Series", "Cosine Series", "Sine Series", "f(x)"}]
(*Graph of part c f(x) = piecewise*)
(*Notice the change in the 'slopes' of the 'x' and 'x+1' part*)
```

Out[1256]=



In[1257]:=
```
(*Problem #8 part d f(x) is a piecewise defined function*)
Clear[a, b, M, x, n, L, f, myFCos, myFSin, myFourier]
a[n_] := Integrate[f[x] * Cos[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
b[n_] := Integrate[f[x] * Sin[(n * Pi / L) * x], {x, -L, L}] * (1 / L)
f[x_] := Piecewise[{{0, x >= -L && x < 0}, {1, x > 0 && x <= L}}]
L = Pi;

a[n];
b[n];

(*Below are the fourier cosine & sine series definitions*)
myFCos[x_, M_] := Sum[a[n] * Cos[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2
myFSin[x_, M_] := Sum[b[n] * Sin[(n * Pi / L) * x], {n, 1, M}]

(*Here is the fourier series definition*)
myFourier[x_, M_] :=
 Sum[a[n] * Cos[(n * Pi / L) * x] + b[n] * Sin[(n * Pi / L) * x], {n, 1, M}] + a[0] / 2


Plot[{Evaluate[myFourier[x, 30]], Evaluate[myFCos[x, 30]],
  Evaluate[myFSin[x, 30]], f[x]}, {x, -L, L}, PlotStyle → {Black, Red, Blue, Purple},
 PlotLegends → {"Fourier Series", "Cosine Series", "Sine Series", "f(x)"}]
(*Graph of part d piecewise*)
(*Notice that the graph switches form 0 and 1 at the origin*)
```

Out[1267]=