Visualizing Neurons: A Comparative Analysis of PDE-Based Image Reconstruction

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February 15, 2020

Project Proposal

Image segmentation is the process of partitioning raw digital data (an image) into image objects, e.g. pixels, line segments, closed contours/surfaces. The purpose is to generate meaningful data e.g. a 2D closed contour or 3D triangulation that then can be utilized as a computational domain to analyze/solve PDE model equations on [4]. The process itself will require the use of non-linear equations PDEs. In particular one methodology utilizes active contours (or snakes) [3], that is an evolving curve which is constrained by a given raw image $u_0(x)$, where $x \in \Omega \subset \mathbb{R}^2$ in 2D as an example. The snake models that are used [3] are based on

$$F_1(C) = \alpha \int_0^1 |C'(s)|^2 \, ds + \beta \int_0^1 |C''(s)| \, ds - \lambda \int_0^1 |\nabla u_0(C(s))|^2 \, ds$$

where the contour C that minimizes the above functional is the contour that realizes the image and this is analogous to a non-linear PDE. A more compact version [3] is given by

$$F_2(C) = \int_0^1 g(\nabla(u_0(s))) \, ds$$

which is analogous to

$$\phi_t = |\nabla \phi| g(\nabla u_0)\kappa + \nabla g(\nabla u_0) \cdot \nabla \phi$$

where ϕ are the isocontours, level sets, of our image u_0, this formulation is given by [5]. Another methodology involves gradient vector flows [2]

$$E_{\mathrm{GVF}} = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |\mathbf{v} - \nabla f|^2 \, dx \, dy$$

where μ is a controllable smoothing term, which is solved by the Euler equations

$$\mu \nabla^2 u - \left(u - \frac{\partial}{\partial x} F_{\mathrm{ext}}\right)\left(\frac{\partial}{\partial x} F_{\mathrm{ext}}(x,y)^2 + \frac{\partial}{\partial y} F_{\mathrm{ext}}(x,y)^2\right) = 0$$
$$\mu \nabla^2 v - \left(v - \frac{\partial}{\partial y} F_{\mathrm{ext}}\right)\left(\frac{\partial}{\partial x} F_{\mathrm{ext}}(x,y)^2 + \frac{\partial}{\partial y} F_{\mathrm{ext}}(x,y)^2\right) = 0$$

A third methodology is to use the Mumford-Shah functional [1]

$$E[J,B] = C \int_D (I(\vec{x}) - J(\vec{x}))^2 \, \mathrm{d}\vec{x} + A \int_{D/B} \vec{\nabla} J(\vec{x}) \cdot \vec{\nabla} J(\vec{x}) \, \mathrm{d}\vec{x} + B \int_B ds$$

optimizing this functional leads to a criteria for segmenting an image into sub-image regions and Ambrosio-Tortorelli give an algorithm for acheiving the minimum and also show the minimum is well-defined. For this project we will implement the aforementioned methodologies to realize computational domains from 2D and then 3D raw neuronal cell data. From these implementations we will perform a comparative analysis of the reproduced image data and determine if there is a particular methodology that is more efficient given the unstructured nature of neuron cells.

References

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[5] Hong-Kai Zhao, T. Chan, B. Merriman, and S. Osher. A variational level set approach to multiphase motion. *Journal of Computational Physics*, 127(1), 8 1996.