ROSADO

```
%Adaptive Integration Using Simpson
format long
clc
clear
clf
syms x
fun=@(x)(1+25.*x.^2).^(-1);
df1 = matlabFunction( diff(fun(x)) );
df2 = matlabFunction( diff(diff(fun(x))) );
df3 = matlabFunction( diff(diff(diff(fun(x)))) );
df4 = matlabFunction( diff(diff(diff(diff(fun(x))))));
a=-1;
b=1;
%tol=input('Enter a tolerance: ');
%tol=input('Enter a tolerance: ');

tol=[0.1,0.001,0.0001,0.00001,0.000001,0.000001];
frm=ceil(length(tol)/2);
for j=1:length(tol)
[myarea,plst,theError,theExactError,fevals]=adaptsimp(fun,a,b,tol(j),df4);
Iexact=integral(@(x)abs(fun(x)),a,b);
flst=fun(plst);
diff=abs(Iexact-myarea);
vec=zeros(length(plst),1);

subplot(2,frm,j)
hold on
fplot(fun,[a,b])
stem(plst,flst,'filled','r')
scatter(plst,vec,'filled','b')
ylim([0 1.25])
fprintf('For a tolerance of %f the approx area = %f.\n The exact area = %f.\n The↙
approximate error = %d.\n The exact error(comp) = %d.\n The difference in integrals is %↙
d.\n The number of function evaluations is %u.\n The number of points is %u.\n\n',tol(j),↙
myarea,Iexact,theError,theExactError,diff,fevals,length(plst))
end
```

```
function [area,ptSet,errorTotal,error,fevals] = adaptsimp(fun,xi,xf,tol,df4)
[area,error,errorTotal]=simpson(xi,xf,fun,df4);
ptSet=[xi xf];
fevals=2;
if error>tol
    m=(xi+xf)/2;
    [a1, S1,E1,Ex1,feval1]=adaptsimp(fun,xi, m, tol,df4);
    [a2, S2,E2,Ex2,feval2]=adaptsimp(fun,m,xf, tol,df4);
    area=a1+a2;
    errorTotal=E1+E2;
    error=Ex1+Ex2;
    fevals=feval1+feval2;
    ptSet=[ptSet S1 S2];
    ptSet=sort(ptSet);
    ptSet=unique(ptSet','rows').';
end
end
```

```
function [area,error,ErrorEst] = simpson(x1,x2,fun,df4)
h=abs(x2-x1)/2;
xm=(x2+x1)/2;
area=(h/3)*(fun(x1)+4*fun(xm)+fun(x2));
error=abs(integral(@(x)fun(x),x1,x2)-abs(area));
ErrorEst=abs((x2-x1)^5/90*fminbnd(@(x) -1*df4(x),x1,x2));
end
```

For a tolerance of 0.100000 the approx area = 0.530062.
The exact area = 0.549360.
The approximate error = 7.697990e-03.
The exact error(comp) = 1.929841e-02.
The difference in integrals is 1.929841e-02.
The number of function evaluations is 4.
The number of points is 3.

For a tolerance of 0.001000 the approx area = 0.549886.
The exact area = 0.549360.
The approximate error = 3.549460e-04.
The exact error(comp) = 6.367657e-04.
The difference in integrals is 5.255065e-04.
The number of function evaluations is 16.
The number of points is 9.

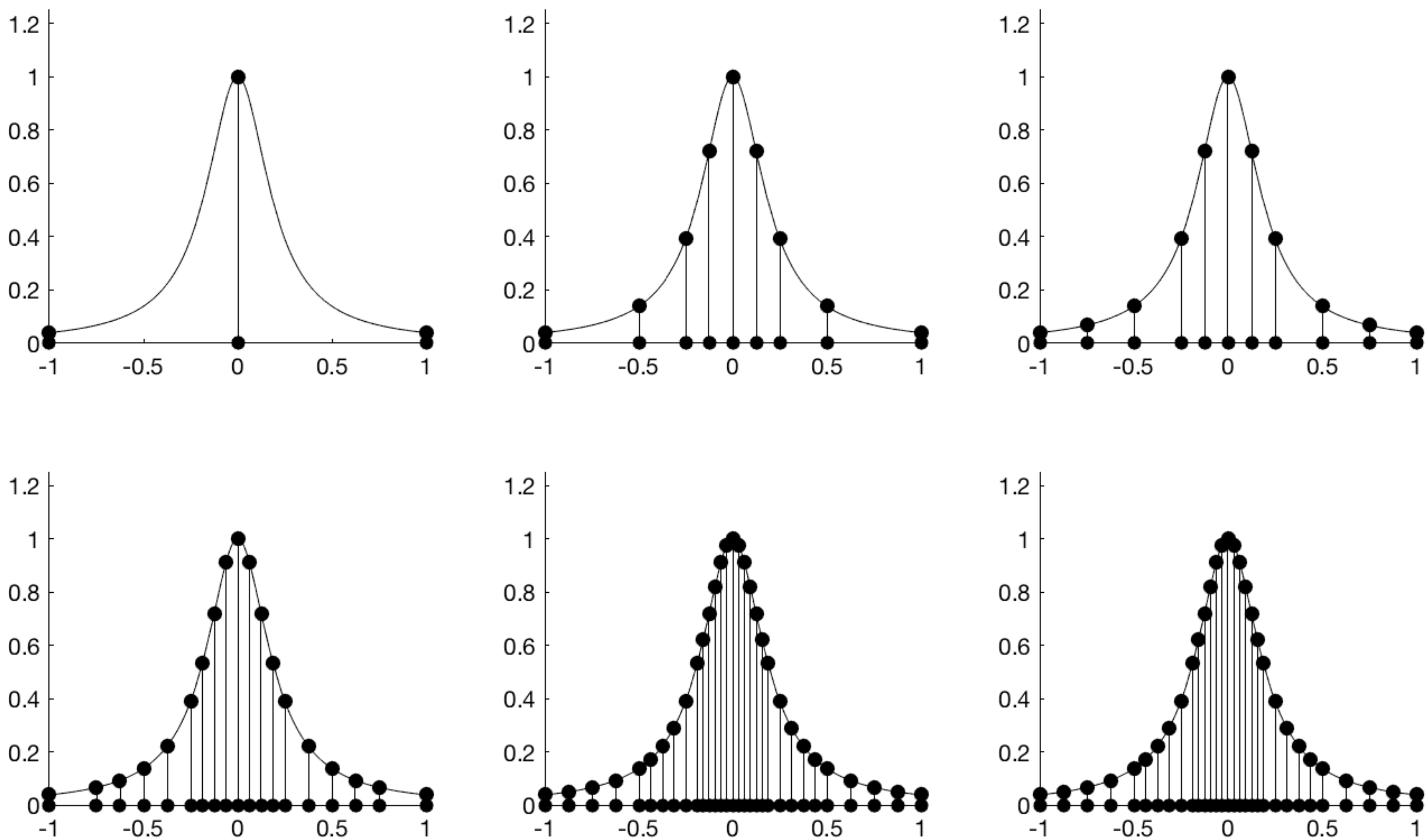
For a tolerance of 0.000100 the approx area = 0.549504.
The exact area = 0.549360.
The approximate error = 3.481520e-05.
The exact error(comp) = 2.554092e-04.
The difference in integrals is 1.441500e-04.
The number of function evaluations is 20.
The number of points is 11.

For a tolerance of 0.000010 the approx area = 0.549374.
The exact area = 0.549360.
The approximate error = 1.753973e-05.
The exact error(comp) = 2.776967e-05.
The difference in integrals is 1.362031e-05.
The number of function evaluations is 36.
The number of points is 19.

For a tolerance of 0.000001 the approx area = 0.549362.
The exact area = 0.549360.
The approximate error = 1.901950e-06.
The exact error(comp) = 4.140455e-06.
The difference in integrals is 1.872539e-06.
The number of function evaluations is 60.
The number of points is 31.

For a tolerance of 0.000001 the approx area = 0.549362.
The exact area = 0.549360.
The approximate error = 1.901950e-06.
The exact error(comp) = 4.140455e-06.
The difference in integrals is 1.872539e-06.
The number of function evaluations is 60.
The number of points is 31.

>>


1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1

PROBLEM 1

Some functions that would have problems with adaptive integration are f(x)=1/x, sin(1/x), or the dirichlet function:

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

The function 1/x has a singularity at x=0 so we will never satisfy the error tolerance.
For sin(1/x) and the Dirichelet function, both oscillate alot. For sin(1/x) the oscillation become worse as we approach x=0. The dirichelet function is nowhere differentiable and is constantly jumping from 1 to 0.
One method is to integrate "near" the jumps or discontinuities i.e. instead of [0,1] integrate from [0.0001,1] and specify a comparable tolerance.

PROBLEM 2(B)

Using the Adaptive Simpson we only need 31 points to get an error of order 10e−6 while for composite integration we need 101 sample points
With Adaptive Trapezoid we need 200 points