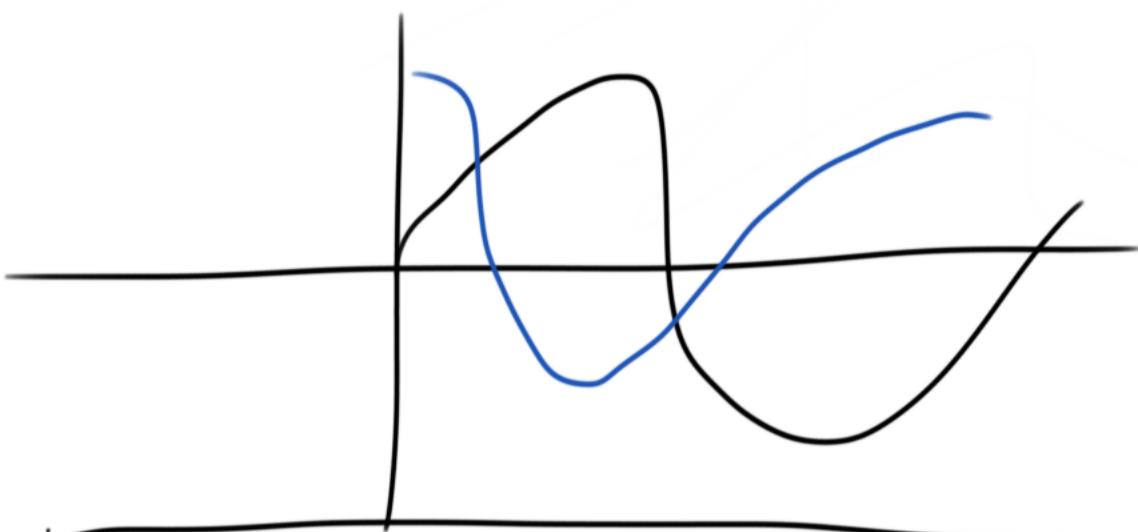


## 6. Goniometrie

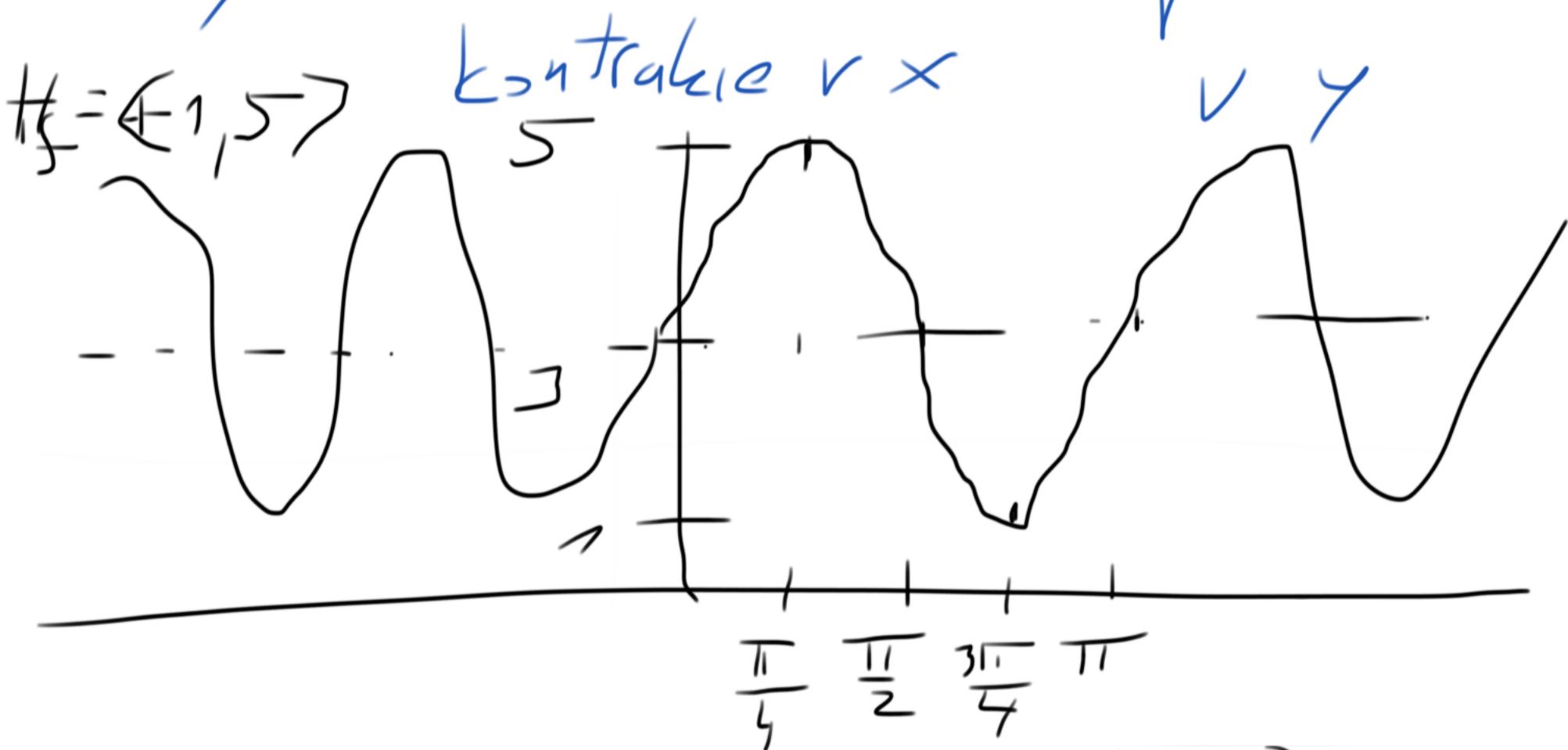


$$y = 2 \sin [3x + 3] \quad \text{graf}$$

dilatace  
 $v_y$

$a \cdot x$   
 $a > 0$

posunutí  
 $v_y$



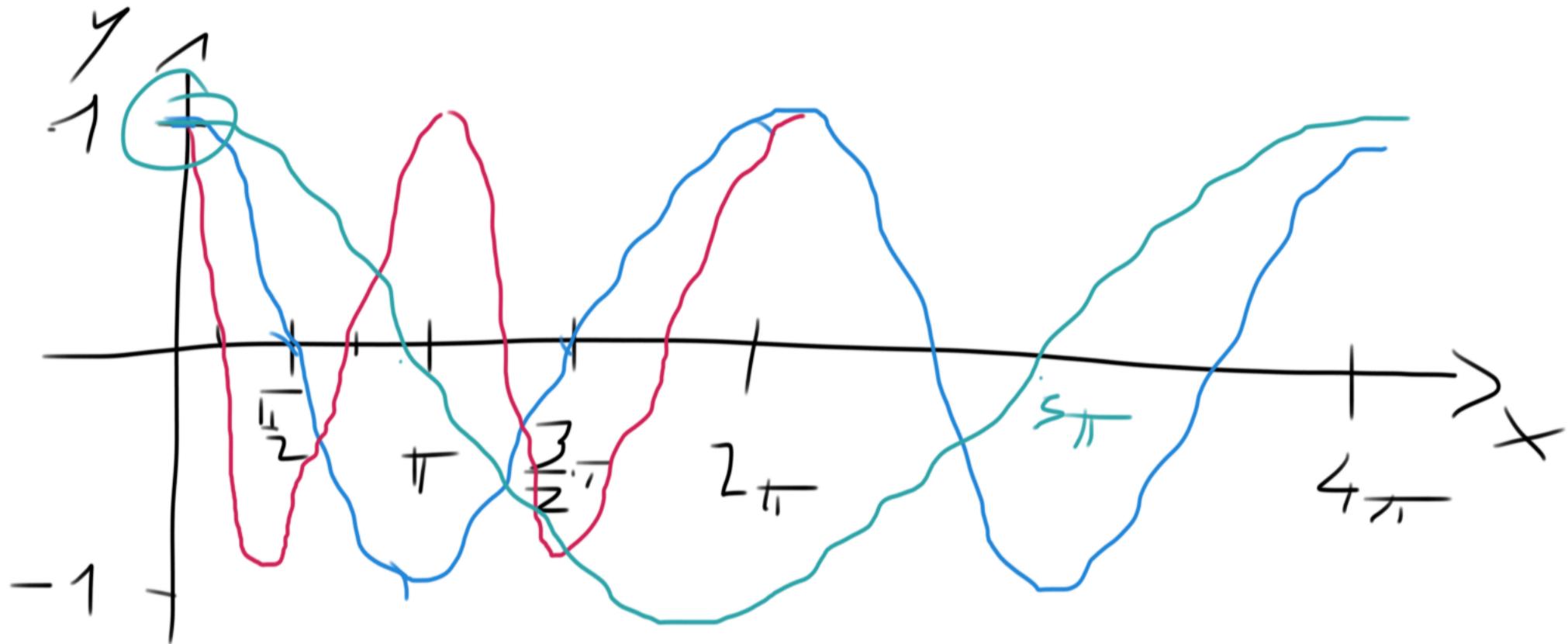
$$\sin(x \cdot f) \rightarrow \sin(2x)$$

f - frekvence

$$f = 2 \rightarrow T = \pi$$

$$f = 1 \sim T = L$$

$\cos 2x$	$T = \pi$	
$\cos x$	$T = 2\pi$	
$\cos \frac{x}{2}$	$T = 4\pi$	



Null:  $\pi + k \cdot 2\pi = \pi + k \cdot \frac{\pi}{2}$

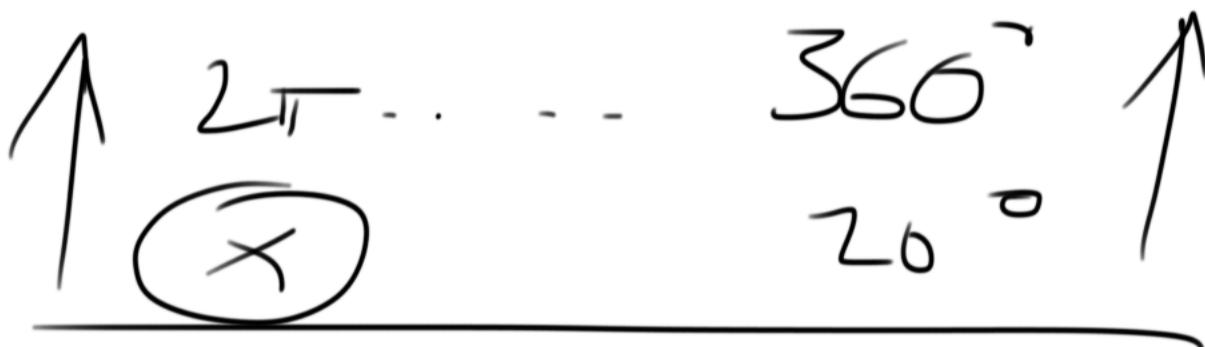
maxima:  $k \cdot 4\pi = k \cdot \pi$

minima:  $2\pi + k \cdot 4\pi = 2\pi + k \cdot \pi$

Práctický úloží  $\alpha$  a  $\beta$

Vyjádřit  $\alpha$  v oblékové mřížce

$$\alpha = 20^\circ$$



$$\frac{\alpha}{2\pi} = \frac{20}{360}$$

$$\alpha = \frac{1}{18} \cdot 2\pi = \frac{\pi}{9}$$

✓

$$\alpha = 20^\circ = \frac{60^\circ}{3} \quad 60 \approx \frac{\pi}{3}$$

$$20^\circ \approx \frac{\frac{\pi}{3}}{3} = \frac{\pi}{9}$$

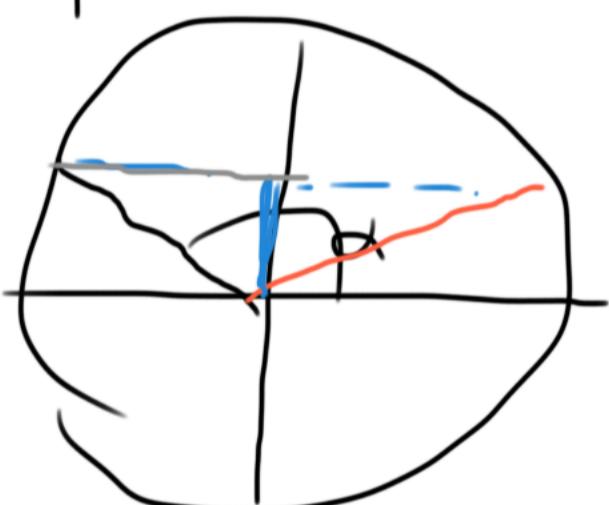
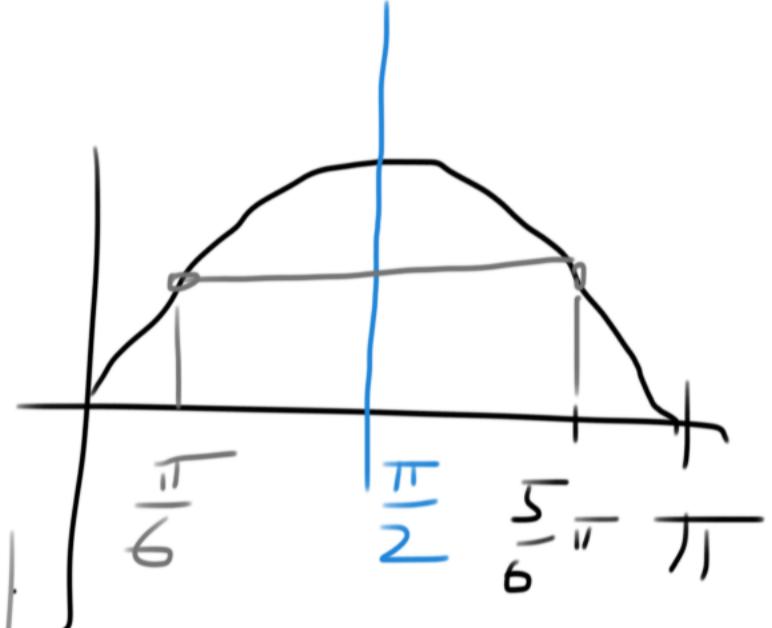
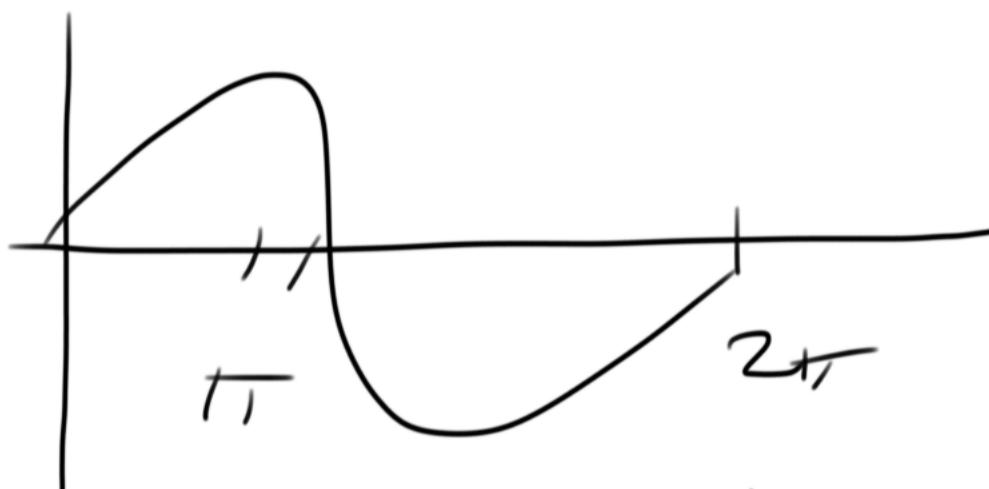
$$\beta = \frac{15}{6}\pi \quad \frac{\pi}{6} \approx 30^\circ \quad \beta = 15 \cdot 30^\circ = 450^\circ$$

$$\frac{\pi}{6} - 180^\circ \quad x = 180 \cdot \frac{15}{6} \\ x = 30 \cdot 15 \\ x = 450^\circ$$

Hodnota funkce

Vypočítejte (bez kalkulačky)

$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$



$$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$$

$$\cos\left(\frac{3}{4}\pi\right)$$



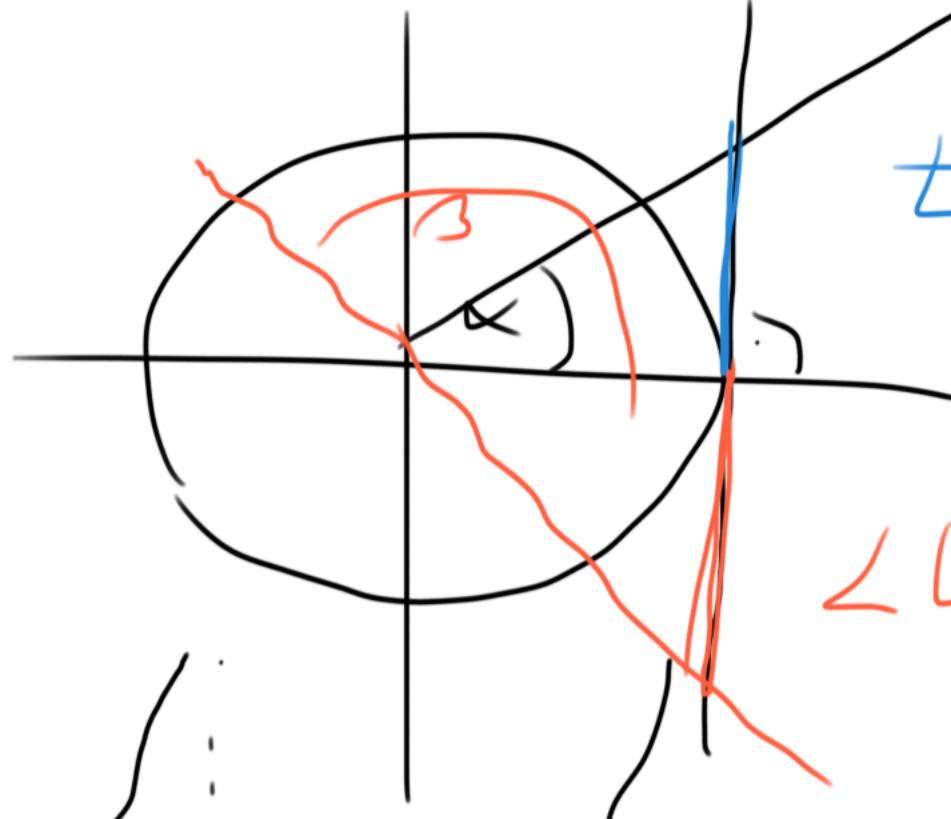
$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{4} \approx 45^\circ$$

$$\frac{3\pi}{4} \approx 135^\circ$$

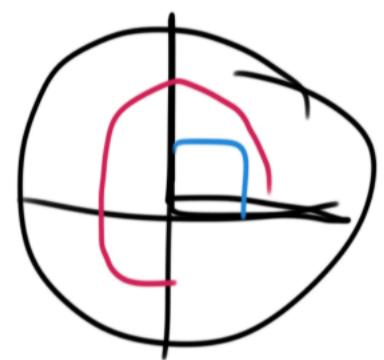
$$\cos\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2}$$

$$\operatorname{tg} \frac{7}{6}\pi$$

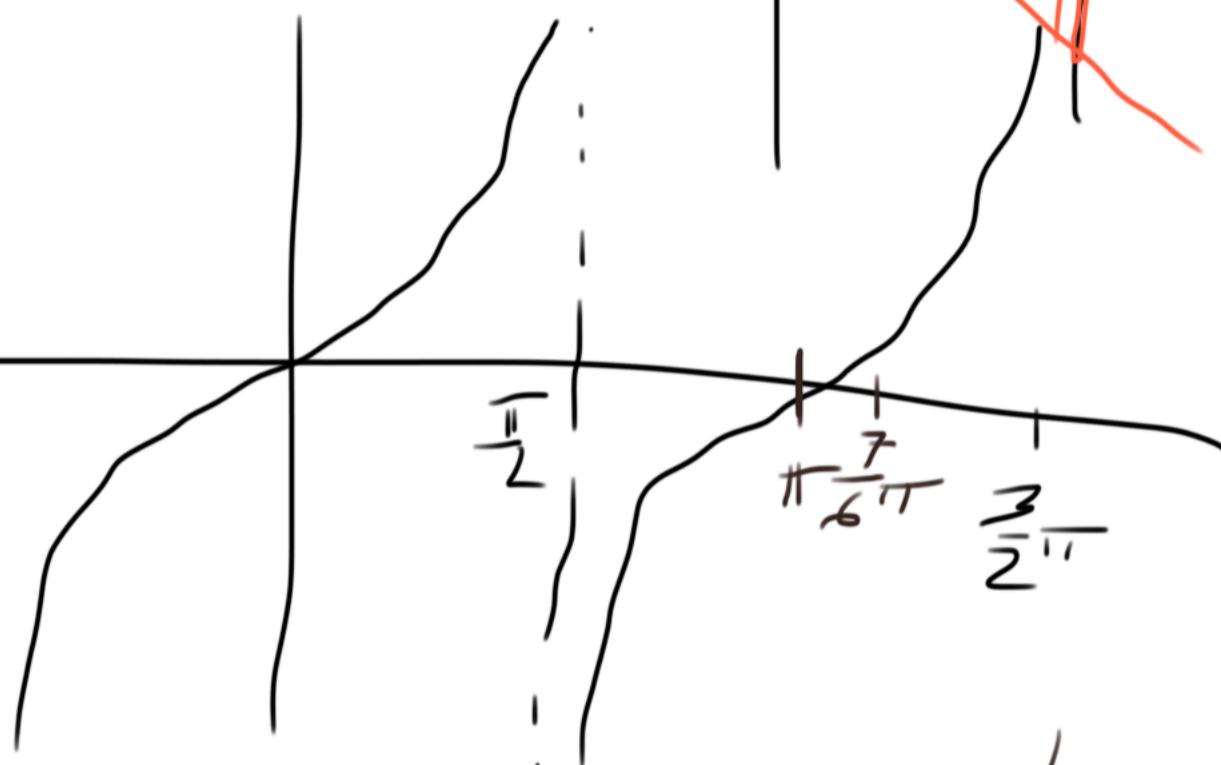


$$\operatorname{tg} \alpha$$

$\angle D$



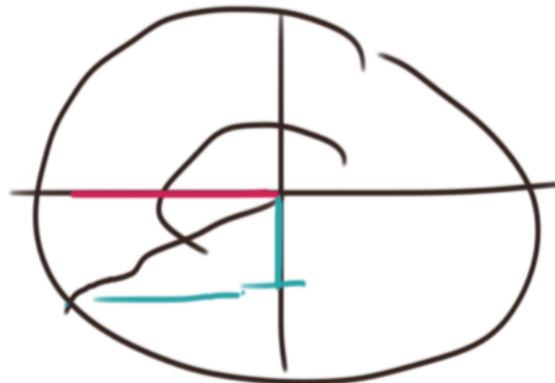
$$\frac{\pi}{6} \quad \frac{1}{2}\pi$$



$$\frac{7}{6}\pi = \pi + \frac{1}{6}\pi$$

$$\left( \operatorname{tg}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \right)$$

$$\operatorname{tg}\left(\frac{7}{6}\pi\right) = \frac{\sqrt{3}}{3}$$



$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\sin x$$

$$\cos x$$

$$\sin\left(\frac{7}{6}\pi\right) = -\frac{1}{2}$$

$$\operatorname{tg}\left(\frac{7}{6}\pi\right) = -\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$\cos\left(\frac{7}{6}\pi\right) = -\frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\begin{array}{l} \text{tg } x \quad T = \pi \\ \sin x \quad T = 2\pi \end{array}$$

$$\sin\left(\frac{13}{6}\pi\right) = \sin\left(\frac{12}{6}\pi + \frac{1}{6}\pi\right) =$$

$$\frac{13}{6}\pi > 2\pi$$

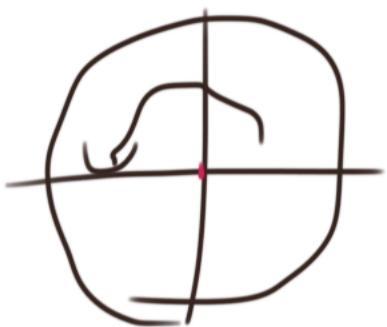
$$= \sin\left(\frac{\pi}{6} + 2\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\operatorname{tg}\left(\frac{7}{6}\pi\right) = \operatorname{tg}\left(\frac{\pi}{6}\right) = \underline{\frac{\sqrt{3}}{3}}$$

$$\cotg\left(\frac{7}{6}\pi\right) = \cotg\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$\cotg x = \frac{1}{\operatorname{tg} x} \Rightarrow \operatorname{ctg}\left(\frac{7}{6}\pi\right) = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

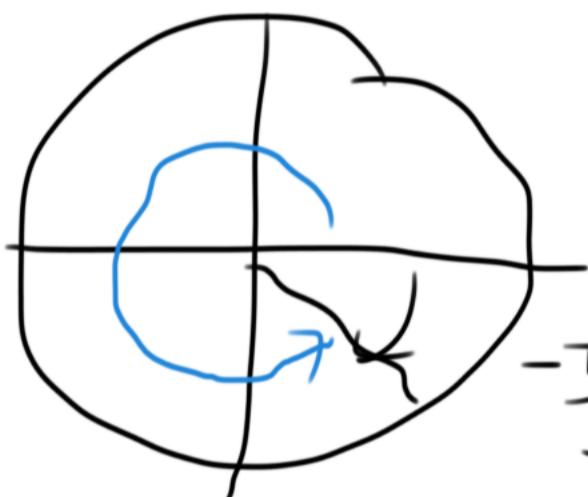
$$\sin\left(\frac{10}{3}\pi\right) = \sin(5\pi) = \sin(\pi) = 0$$



$$5\pi = \pi + 2 \cdot 2\pi$$

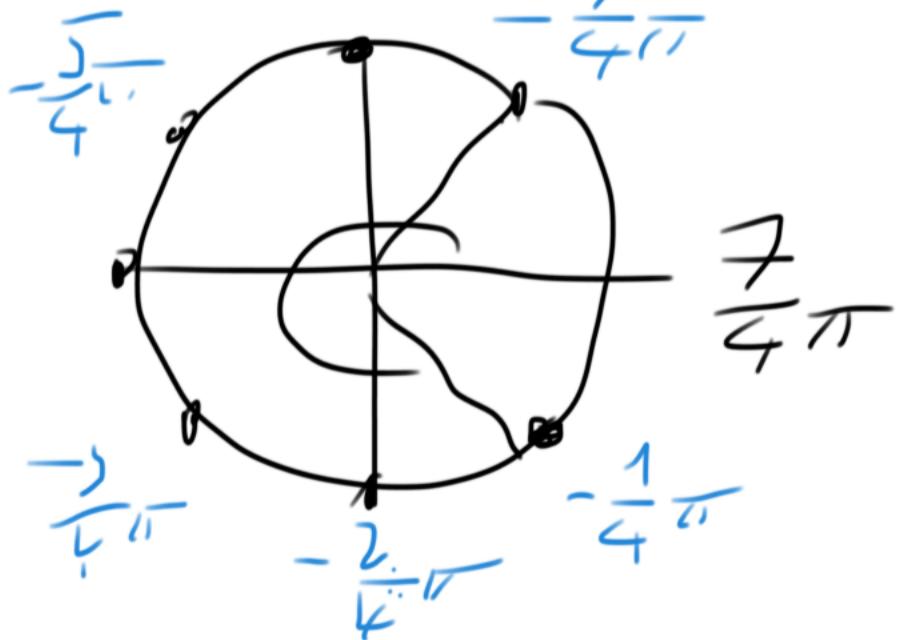
$b=2$

$$\sin\left(-\frac{7}{4}\pi\right) =$$



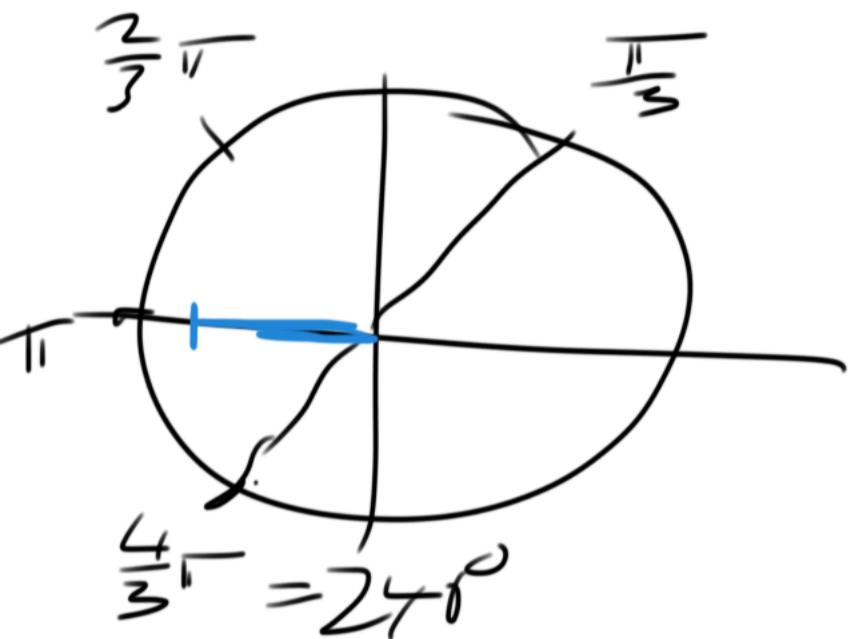
$$-\frac{\pi}{3} \approx 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$$

$$\sin\left(-\frac{7}{4}\pi\right) = \sin\left(-\frac{7}{4}\pi + 2\pi\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



$$\cos(240^\circ)$$

$$240^\circ = 4 \cdot 60^\circ \approx 4 \cdot \frac{\pi}{3} = \frac{4\pi}{3}$$



$$\begin{aligned}\cos(240^\circ) &= \cos\left(\frac{4\pi}{3}\right) \\ &= -\frac{1}{2}\end{aligned}$$

$$\underline{\cotg \frac{\pi}{2} = 0}$$



$$\cot \frac{\pi}{2} = \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}}$$

$$\rightarrow \frac{0}{1} = 0$$

$$\cotg(210^\circ) = \cotg\left(\frac{7}{6}\pi\right)$$

$\neq 30^\circ$

$$= \cotg\left(\frac{\pi}{6}\right)$$

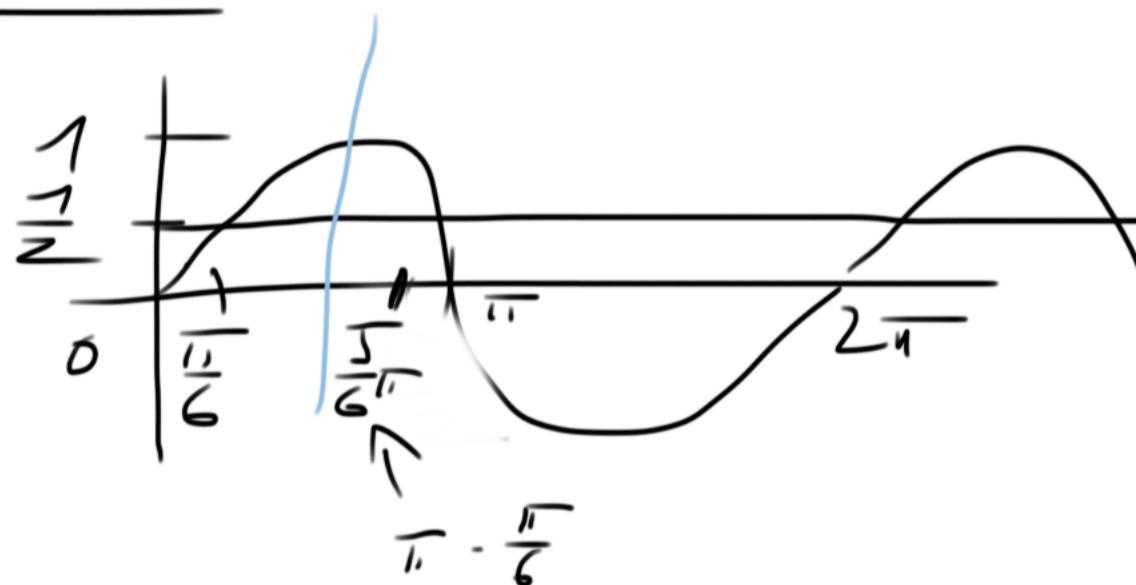
$$= \sqrt{3}$$

$T = \pi$  ✓

---

Reite  $\sim R$

$$\sin x = \frac{1}{2}$$



$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\sin \frac{5}{6}\pi = \frac{1}{2}$$

$$x_1 = \frac{\pi}{6} + k \cdot 2\pi \quad x_2 = \frac{5}{6}\pi + k \cdot 2\pi$$


---

$$\operatorname{tg} x = \sqrt{3}$$

$$x = \frac{\pi}{3} + k \cdot \pi$$

	$\frac{\pi}{3}$	$\frac{\pi}{6}$
$\sin x$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\cos x$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\operatorname{tg} x$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$

$$T = \pi$$


---

Feste  $\vee R$

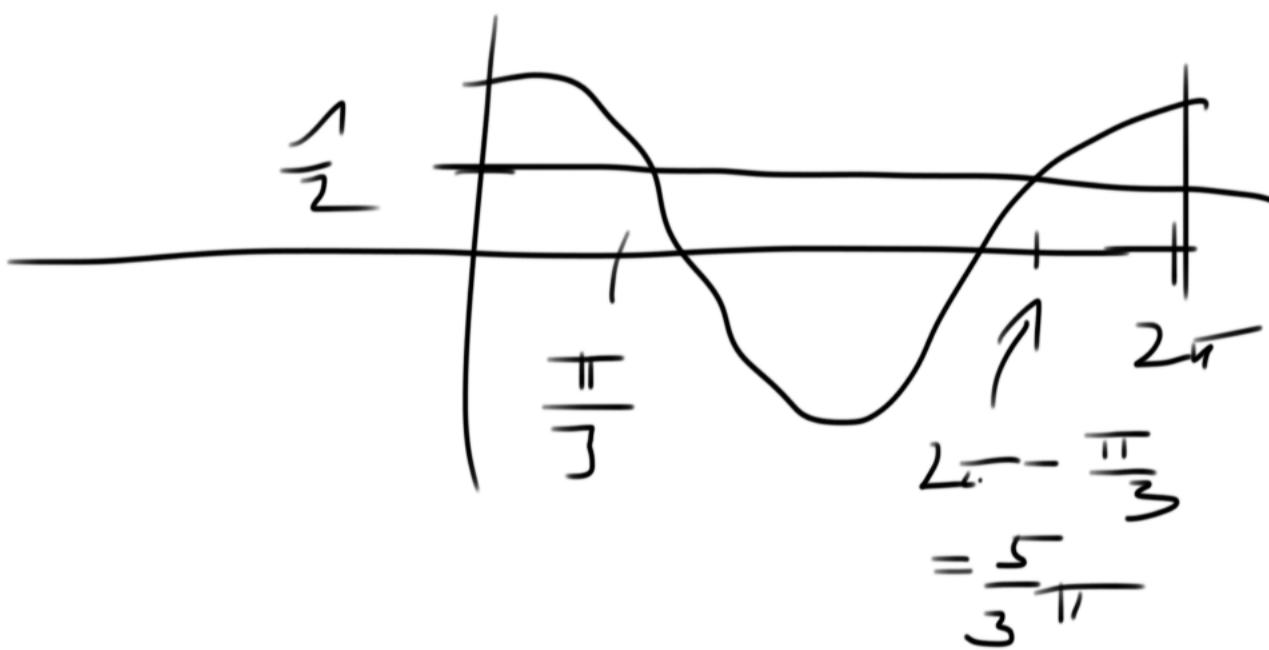
$$2 \cdot \frac{\cos x + 1}{3} - \frac{4 \cdot \cos x - 1}{2} = 1 - \cos x$$

$$4 \cos x + 4 - 12 \cos x + 3 = 6 - 6 \cos x$$

$$-8 \cos x + 7 = 6 - 6 \cos x$$

$$1 = 2 \cos x$$

$$\cos x = \frac{1}{2}$$



$$\left. \begin{array}{l} x_1 = \frac{\pi}{3} + k \cdot 2\pi \\ x_2 = \frac{5}{3}\pi + k \cdot 2\pi \end{array} \right\}$$

Reste v R

$$\frac{5 \sin x + 4}{10 \sin x + 4} = 1 \quad |(10 \sin x + 4)$$

$$10 \sin x + 4 \neq 0$$

$$\sin x \neq -\frac{4}{10}$$

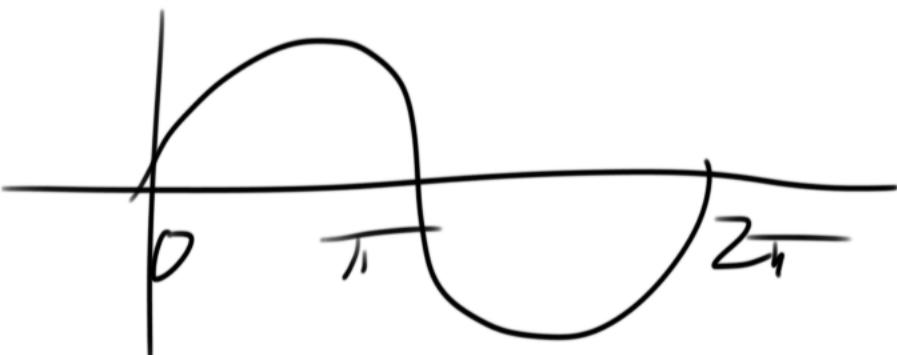
$$\sin x \neq -\frac{2}{5}$$

$$x \neq ?$$

$$5 \sin x + 4 = 10 \sin x + 4$$

$$0 = 5 \sin x$$

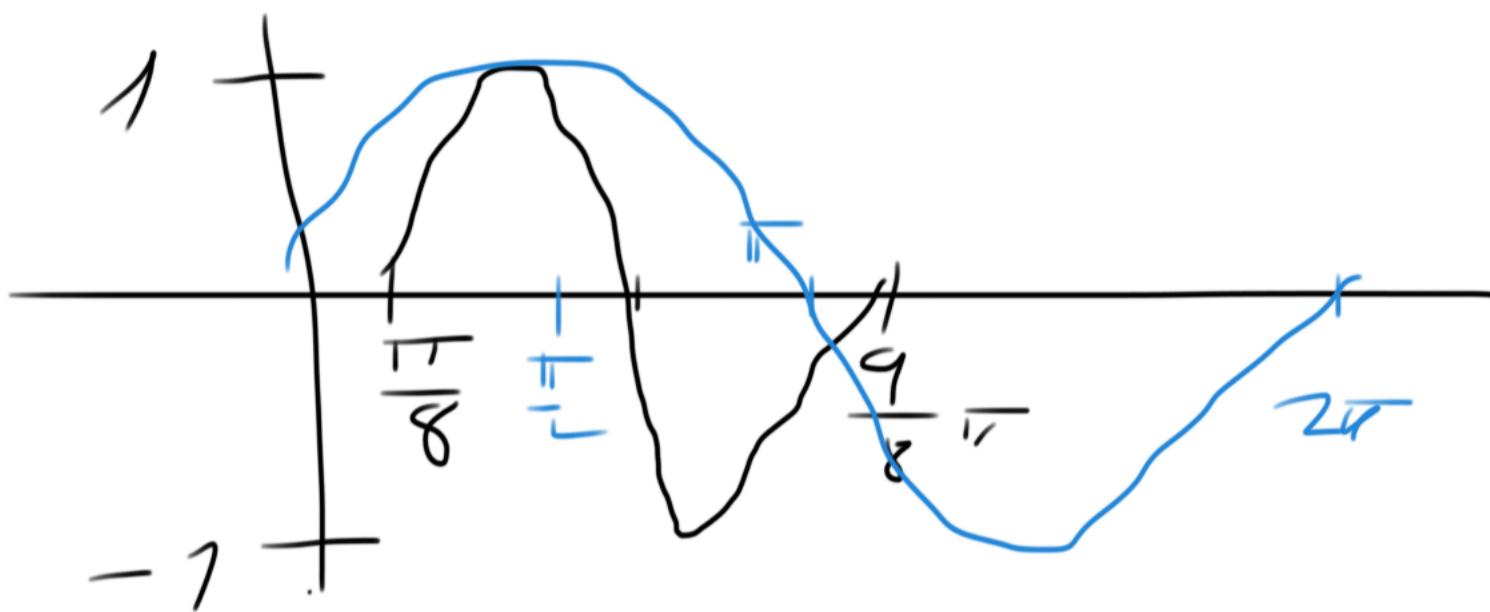
$$\sin x = 0$$



$$\left\{ \begin{array}{l} x = k \cdot \pi \\ k \in \mathbb{Z} \end{array} \right.$$

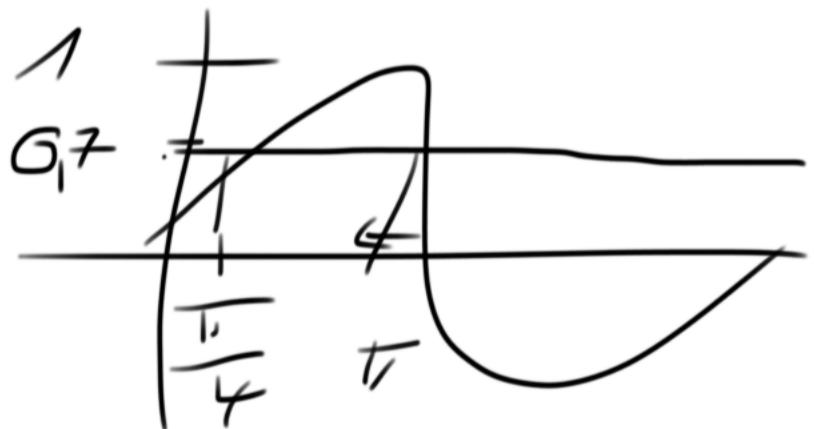
Rechte  $\sqrt{R}$

$$\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



Substiface:  $z = 2x - \frac{\pi}{4}$

$$\sin z = \frac{\sqrt{2}}{2} = 0, \pi, \dots$$



$$\begin{cases} z_1 = \frac{\pi}{4} + k \cdot 2\pi \\ z_2 = \frac{3}{4}\pi + k \cdot 2\pi \end{cases}$$

$$2x_1 - \frac{\pi}{4} = \frac{\pi}{4} + k \cdot 2\pi \quad / + \frac{\pi}{4}$$

$$2x_1 = \frac{\pi}{2} + k \cdot 2\pi \quad / : 2$$

$$\begin{cases} x_1 = \frac{\pi}{4} + k \cdot \pi \\ x_2 = \frac{\pi}{2} + k \cdot \pi \end{cases}$$

Reite r R

$$2 \cos^2 x - \cos x - 1 = 0$$

$$y = \cos x$$

$$2y^2 - y - 1 = 0$$

$$y_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

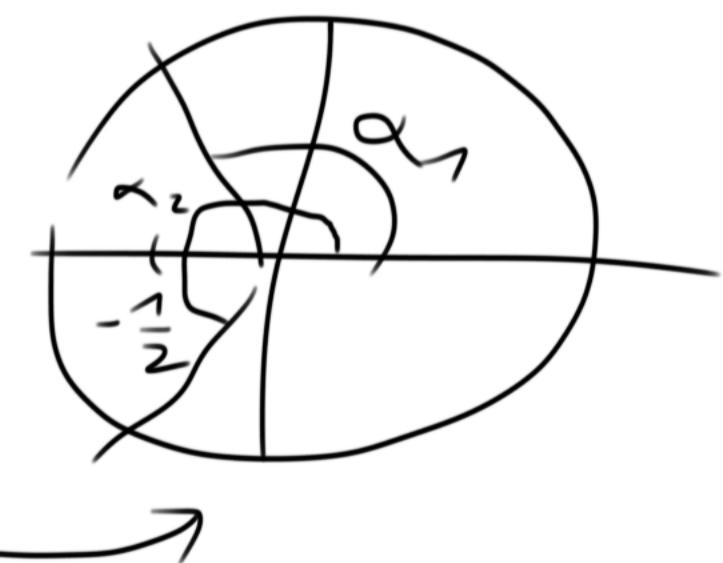
$$\cos x_1 = 1$$

$$x_1 = 2\pi \cdot k \quad k \in \mathbb{Z}$$

$$\cos x_2 = -\frac{1}{2}$$

$$\alpha_1 = \frac{2}{3}\pi$$

$$\alpha_2 = \frac{4}{3}\pi$$



$$x_2 = \frac{2}{3}\pi + 2\pi \cdot k$$

$$x_3 = \frac{4}{3}\pi + 2\pi \cdot k \quad k \in \mathbb{Z}$$

$$\sin^2 x - 3 \sin x + 2 = 0$$

$$y = \sin x$$

$$y^2 - 3y + 2 = 0$$

$$y_{1,2} = \frac{3 \pm \sqrt{9 - 8}}{2} = \begin{cases} 1 \\ 2 \end{cases}$$

$$\sin x_1 = 2 \quad \leftarrow \text{NR}^-$$

$$\sin x_2 = 1$$

$$x_2 = \frac{\pi}{2} + k \cdot 2\pi$$

$$f_{\sin, \cos} = \langle -1, 1 \rangle$$

$$\forall x \in R : |\sin x| \leq 1$$

$$\sin(x) = \frac{3}{5} \quad \cancel{x}$$


---


$$\sin(2x) = ?$$

$$\sin(2x) = 2 \cdot \underline{\sin x} \cdot \underline{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$= \sqrt{1 - \frac{9}{25}} \\ = \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}}$$

$$\sin(2x) = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{4\sqrt{21}}{25} \quad \checkmark$$

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grafy, rovnice, výčíslení,

$$\sin \frac{\pi}{6} =$$

$$\sin(60^\circ)$$