Neural Networks

10. Recurrent Networks

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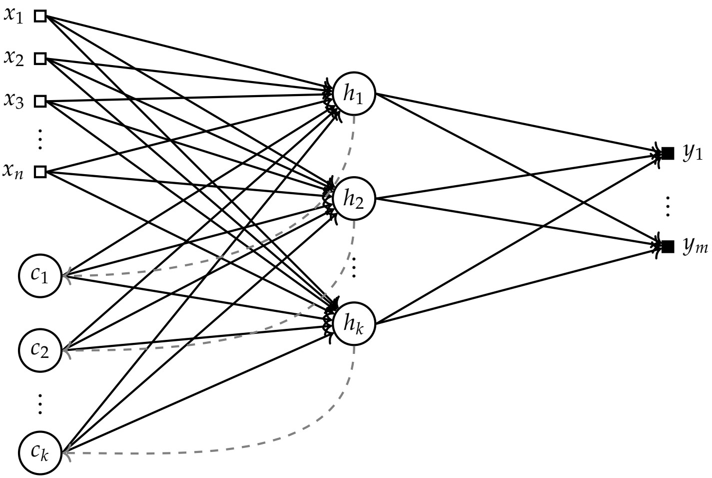
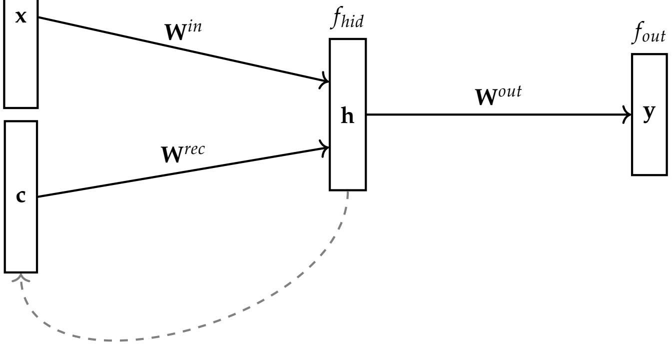
Recurrent Networks

Simple Recurrent Network (nodes)



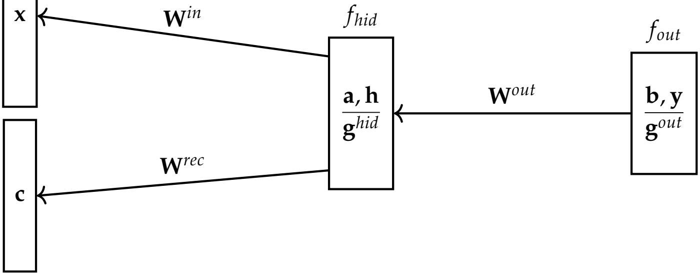
(Elman, 1990)

Simple Recurrent Network (blocks)



$$\mathbf{c}(t) = \mathbf{h}(t\text{–}1) \quad \mathbf{h} = f_{hid}(\mathbf{W}^{in}\mathbf{x} + \mathbf{W}^{rec}\mathbf{c}) \quad \mathbf{y} = f_{out}(\mathbf{W}^{out}\mathbf{h})$$

Simple Backpropagation (Elman)



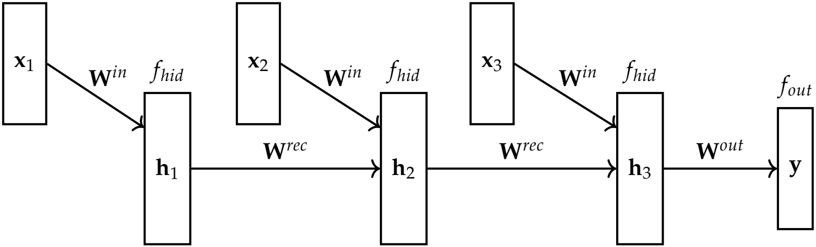
$$\begin{aligned}\mathbf{g}^{out} &= f'_{out}(\mathbf{b}) \odot (\mathbf{d} - \mathbf{y}) \\ \mathbf{g}^{hid} &= f'_{hid}(\mathbf{a}) \odot \mathbf{W}^{out}\mathbf{g}^{out}\end{aligned}$$

$$\begin{aligned}\Delta\mathbf{W}^{in} &= \mathbf{g}^{hid}\mathbf{x}^T \\ \Delta\mathbf{W}^{rec} &= \mathbf{g}^{hid}\mathbf{c}^T \\ \Delta\mathbf{W}^{out} &= \mathbf{g}^{out}\mathbf{h}^T\end{aligned}$$

Back-Propagation Through Time

Unfolding in time

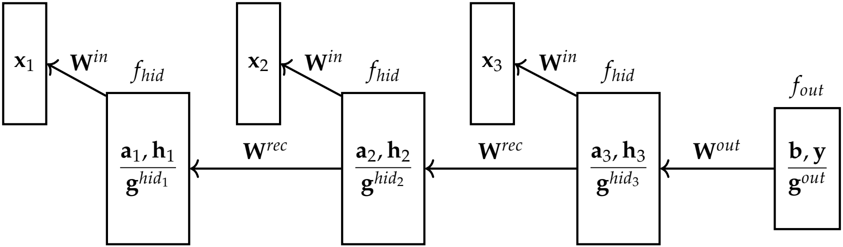
- input sequence $\langle \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \rangle \mapsto \mathbf{y}$:



$$\begin{aligned}
\mathbf{h}_1 &= f_{hid}\left(\mathbf{W}^{in}\mathbf{x}_1\right) \\
\mathbf{h}_2 &= f_{hid}\left(\mathbf{W}^{in}\mathbf{x}_2 + \mathbf{W}^{rec}\mathbf{h}_1\right) \\
\mathbf{h}_3 &= f_{hid}\left(\mathbf{W}^{in}\mathbf{x}_3 + \mathbf{W}^{rec}\mathbf{h}_2\right) \\
\mathbf{y} &= f_{out}\left(\mathbf{W}^{out}\mathbf{h}_3\right)
\end{aligned}$$

Gradients and Weight Update

- input sequence $\langle \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \rangle \mapsto \mathbf{y}$, correct output $\mathbf{d}$:



$$
\begin{aligned}
\mathbf{g}^{out} &= f'_{out}(\mathbf{b}) \odot (\mathbf{d} - \mathbf{y}) \\
\mathbf{g}^{hid_3} &= f'_{hid}(\mathbf{a}_3) \odot \mathbf{W}^{out} \mathbf{g}^{out} \\
\mathbf{g}^{hid_2} &= f'_{hid}(\mathbf{a}_2) \odot \mathbf{W}^{rec} \mathbf{g}^{hid_3} \\
\mathbf{g}^{hid_1} &= f'_{hid}(\mathbf{a}_1) \odot \mathbf{W}^{rec} \mathbf{g}^{hid_2}
\end{aligned}
$$

$$
\begin{aligned}
\Delta \mathbf{W}^{out} &= \mathbf{g}^{out} \mathbf{h}_3^T \\
\Delta \mathbf{W}^{rec} &= \mathbf{g}^{hid_2} \mathbf{h}_1^T + \mathbf{g}^{hid_3} \mathbf{h}_2^T \\
\Delta \mathbf{W}^{in} &= \mathbf{g}^{hid_1} \mathbf{x}_1^T + \mathbf{g}^{hid_2} \mathbf{x}_2^T + \mathbf{g}^{hid_3} \mathbf{x}_3^T
\end{aligned}
$$