Assignment 10

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Problem

Q)x and y are jointly normal with parameters $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho_{xy})$. Find

- a) $E\{\mathbf{y}|\mathbf{x} = x\}$
- b) $E\{\mathbf{x}^2|\mathbf{y} = y\}$.

Solution

a)We shall show that, if the random varaible x and y are jointly normal with zero mean, then

$$f(y|x) = \frac{1}{\sigma_2\sqrt{2\pi(1-r^2)}} exp\left(-\frac{(y - r\sigma_2 x/\sigma_1)^2}{2\sigma_2^2(1-r^2)}\right) \tag{1}$$

The same reason leads to comclusion that if x and y are jointly normal with $E(x) = \eta_1$ and $E(y) = \eta_2$, then $f(y|x)$ is given by eqn(1) if y and x are raplaced by $y - \eta_2$ and $x - \eta_@$respectively.

In other words for a given x, $f(y|x)$ is a normal density with mean $\eta_2 + r\sigma_2(x-\eta_1)/\sigma_1$ and varaince $\sigma_2^2(1-r^2)$.
From above we get

$$E(Y|X=x) = \mu_y + \frac{\rho_{xy}\sigma_y(x-\mu_x)}{\sigma_x} \tag{2}$$

b) Similarly

$$f_{x|y}\{\mathbf{X}|\mathbf{Y}=y\} \sim N(\mu, \sigma^2) \tag{3}$$

Where

$$\mu = \mu_x + \frac{\rho_{xy}\sigma_x(y-\mu_y)}{\sigma_y} \tag{4}$$

and

$$\sigma^2 = \sigma_x^2(1-\rho_{xy}^2) \tag{5}$$

since

$$E(\mathbf{X}^2|\mathbf{Y}=y) = Var(\mathbf{X}|\mathbf{Y}=y) + (E|\mathbf{X}|\mathbf{Y}=y|)^2 \tag{6}$$

We obtain

$$E(\mathbf{X}^2|\mathbf{Y}=y) = \sigma^2 + \mu^2 \tag{7}$$

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