Assignment 2

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I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If f : A $\rightarrow$ A and A = R - $\{\frac{8}{5}\}$, show that the function $f(x)$ = $\frac{8x+3}{5x-8}$ is one-one onto. Hence, find f^{-1}.

II. SOLUTION

Definition: Let $f : X \rightarrow Y$ be a function,
we say f is one-one or injective,
if and only if $\forall$ x_1 , $x_2 \in X$.
if $f(x_1)$ = $f(x_2)$ then x_1 = x_2.

Suppose that x_1 and x_2 are arbitrary integers and $f(x_1)$ = $f(x_2)$, we need to show that x_1 = x_2. Since $f(x_1)$ = $f(x_2)$.

$$f(x_1) = \frac{8x_1+3}{5x_1-8} \quad and \quad f(x_2) = \frac{8x_2+3}{5x_2-8}$$

Now, equating $f(x_1)$ = $f(x_2)$ since from the defination.

$$\implies \frac{8x_1+3}{5x_1-8} = \frac{8x_2+3}{5x_2-8} \tag{1}$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1 - x_2) = 0. \tag{2}$$

From the eqn(2) we can say that,
the equation satisfies only when $x_1 - x_2$= 0.
Which implies,

$$\boxed{x_1 = x_2}$$

Hence, the given function $f(x)$ is one-one.

Definition: f is called onto if $f(X) = Y$.
i.e., Range of the function $f(x)$ is equal to co-domain.

Let,

$$y = f(x) = \frac{8x+3}{5x-8} \tag{3}$$

On solving the eqn(3),
we get,

$$\boxed{x = \frac{8y+3}{5y-8}} \tag{4}$$

Where y is element of co-domain.
Now eqn(4) is defined $\forall$ $y \in R$ - $\{\frac{8}{5}\}$.
i.e.$y \in$ A.

From eqn(4) we can say x = $\frac{8y+3}{5y-8}$ is continuous $\forall$ $y \in R$ - $\{\frac{8}{5}\}$.
Now for continuous function there are no discontinuous points in domain
Hence, we can say for every element in domain there must exist an image.
Which impels that given function eqn(4) (continuous function) is onto. Since co-domain = range (from definition).

now, from eqn(4) substitute the value of x.

$$\implies f(x) = f(\frac{8y+3}{5y-8}) \tag{5}$$

$$\implies f(x) = \frac{8(\frac{8y+3}{5y-8})+3}{5(\frac{8y+3}{5y-8})-8} \tag{6}$$

$$\implies f(x) = \frac{8(8y+3)+3(5y-8)}{5(8y+3)-8(5y-8)} \tag{7}$$

$$\implies f(x) = \frac{79y}{79} \tag{8}$$

$$\implies f(x) = y \tag{9}$$

Hence, from eqn(9) we can say, given function $f(x)$ is onto.

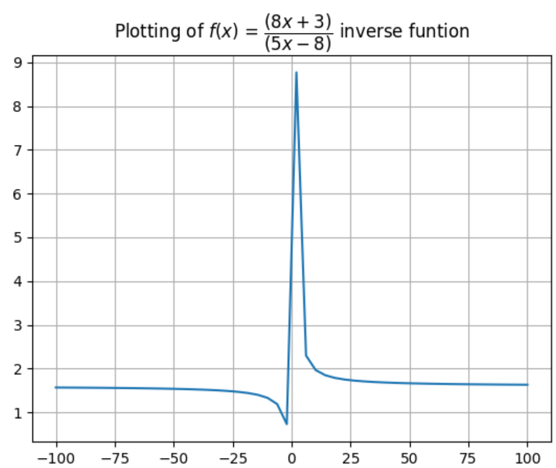

Fig. 1. graph

From the graph 1 we observe that,
since the graph is continuous hence it is onto

NOTE: The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.

Definition: $f\colon X \to Y$ is bijective function i.e., both one-one and onto then there exit a unique function called inverse function and is denoted by f^{-1}, such that,

$$f^{-1}(y) = x \iff f(x) = y$$

Now, from defination $f^{-1}(y) = x$. From eqn(4) we get the value of x

$$f^{-1}(y) = \frac{8y+3}{5y-8}$$

i.e., the inverse of function $f(x)$ is,

$$\boxed{f^{-1}(x) = \frac{8x+3}{5x-8}}$$