

# Assignment 2

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### I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If  $f : A \rightarrow A$  and  $A = R - \{\frac{8}{5}\}$ , show that the function  $f(x) = \frac{8x+3}{5x-8}$  is one-one onto. Hence, find  $f^{-1}$ .

Hence, the given function  $f(x)$  is one-one.

Definition:  $f$  is called onto if  $f(X) = Y$ . i.e., Range of the function  $f(x)$  is equal to co-domain.

### II. SOLUTION

Definition: Let  $f : X \rightarrow Y$  be a function, we say  $f$  is one-one or injective, if and only if  $\forall x_1, x_2 \in X$ . if  $f(x_1) = f(x_2)$  then  $x_1 = x_2$ .

Suppose that  $x_1$  and  $x_2$  are arbitrary integers and  $f(x_1) = f(x_2)$ , we need to show that  $x_1 = x_2$ . Since  $f(x_1) = f(x_2)$ .

$$f(x_1) = \frac{8x_1 + 3}{5x_1 - 8} \quad \text{and} \quad f(x_2) = \frac{8x_2 + 3}{5x_2 - 8}$$

Now, equating  $f(x_1) = f(x_2)$  since from the definition.

$$\implies \frac{8x_1 + 3}{5x_1 - 8} = \frac{8x_2 + 3}{5x_2 - 8} \quad (1)$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1 - x_2) = 0. \quad (2)$$

From the eqn(2) we can say that, the equation satisfies only when  $x_1 - x_2 = 0$ . Which implies,

$$x_1 = x_2$$

$$y = f(x) = \frac{8x + 3}{5x - 8} \quad (3)$$

On solving the eqn(3), we get,

$$x = \frac{8y+3}{5y-8} \quad (4)$$

Where  $y$  is element of co-domain. Now eqn(4) is defined  $\forall y \in R - \{\frac{8}{5}\}$ . i.e.  $y \in A$ .

From eqn(4) we can say  $x = \frac{8y+3}{5y-8}$  is continuous  $\forall y \in R - \{\frac{8}{5}\}$ .

Now for continuous function there are no discontinuous points in domain

Hence, we can say for every element in domain there must exist an image.

Which impels that given function eqn(4) (continuous function) is onto. Since co-domain = range (from definition).

now, from eqn(4) substitute the value of  $x$ .

$$\Rightarrow f(x) = f\left\{\frac{8y+3}{5y-8}\right\} \quad (5)$$

$$\Rightarrow f(x) = \frac{8\left(\frac{8y+3}{5y-8}\right) + 3}{5\left(\frac{8y+3}{5y-8}\right) - 8} \quad (6)$$

$$\Rightarrow f(x) = \frac{8(8y+3) + 3(5y-8)}{5(8y+3) - 8(5y-8)} \quad (7)$$

$$\Rightarrow f(x) = \frac{79y}{79} \quad (8)$$

$$\Rightarrow f(x) = y \quad (9)$$

Hence, from eqn(9) we can say, given function  $f(x)$  is onto.

Now, from definition  $f^{-1}(y) = x$ . From eqn(4) we get the value of  $x$

$$f^{-1}(y) = \frac{8y+3}{5y-8}$$

i.e., the inverse of function  $f(x)$  is,

$$f^{-1}(x) = \frac{8x+3}{5x-8}$$

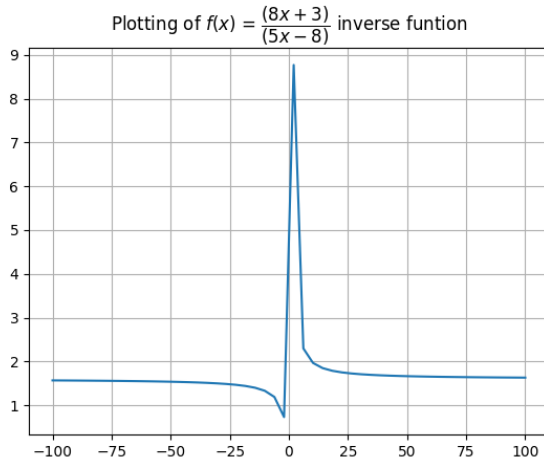


Fig. 1. graph

From the graph 1 we observe that, since the graph is continuous hence it is onto

**NOTE:** The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.

**Definition:**  $f: X \rightarrow Y$  is bijective function i.e., both one-one and onto then there exist a unique function called inverse function and is denoted by  $f^{-1}$ , such that,

$$f^{-1}(y) = x \iff f(x) = y$$