Assignment 2

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I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If $f : A \rightarrow A$ and $A = R - \{\frac{8}{5}\}$, show that the function $f(x) = \frac{8x+3}{5x-8}$ is one-one onto. Hence, find f^{-1}.

II. SOLUTION

Defination: Let $f : X \rightarrow Y$ be a function, we say f is one-one or injective, if and only if $\forall$ x_1 , $x_2 \in X$. if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Suppose that x_1 and x_2 are arbitrary integers and $f(x_1) = f(x_2)$, we need to show that $x_1 = x_2$. Since $f(x_1) = f(x_2)$.

$$f(x_1) = \frac{8x_1 + 3}{5x_1 - 8} \quad and \quad f(x_2) = \frac{8x_2 + 3}{5x_2 - 8}$$

Now, equating $f(x_1) = f(x_2)$ since from the defination.

$$\implies \frac{8x_1 + 3}{5x_1 - 8} = \frac{8x_2 + 3}{5x_2 - 8} \tag{1}$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1 - x_2) = 0. \tag{2}$$

From the eqn(2) we can say that, the equation satisfies only when $x_1 - x_2 = 0$. Which implies,

$$\boxed{x_1 = x_2}$$

Hence, the given function $f(x)$ is one-one.

Defination: f is called onto if $f(X) = Y$. i.e., Range of the function $f(x)$ is equal to co-domain.

Let,

$$y = f(x) = \frac{8x + 3}{5x - 8} \tag{3}$$

On solving the eqn(3),
we get,

$$\boxed{x = \frac{8y+3}{5y-8}} \tag{4}$$

Where y is element of co-domain.
Now eqn(4) is defined $\forall$ $y \in R - \{\frac{8}{5}\}$. i.e. $y \in A$.

Hence, the given function $f(x)$ is onto.

Defination: $f: X \rightarrow Y$ is bijective function i.e., both one-one and onto then there exit a unique function called inverse function and is denoted by f^{-1}, such that,

$$f^{-1}(y) = x \iff f(x) = y$$

Now, from defination $f^{-1}(y) = x$. From eqn(4) we get the value of x

$$f^{-1}(y) = \frac{8y + 3}{5y - 8}$$

i.e., the inverse of function $f(x)$ is,

$$\boxed{f^{-1}(x) = \frac{8x+3}{5x-8}}$$

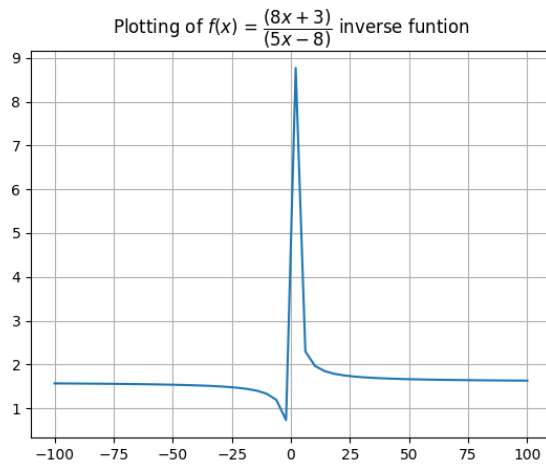Plotting of $f(x) = \dfrac{(8x+3)}{(5x-8)}$ inverse funtion

Fig. 1. graph

From the above graph we observe that, since the graph is continuous hence it is onto

NOTE: The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.