Assignment 2

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I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If $f : A \to A$ and $A = R$ - $\{\frac{8}{5}\}$, show that the function $f(x) = \frac{8x+3}{5x-8}$ is one-one onto. Hence, find f^{-1}.

II. SOLUTION

Defination: Let $f : X \to Y$ be a function, we say f is one-one or injective, if and only if $\forall\ x_1\ ,\ x_2 \in X$. if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Suppose that x_1 and x_2 are arbitrary integers and $f(x_1) = f(x_2)$, we need to show that $x_1 = x_2$. Since $f(x_1) = f(x_2)$.

$$f(x_1) = \frac{8x_1 + 3}{5x_1 - 8} \quad and \quad f(x_2) = \frac{8x_2 + 3}{5x_2 - 8}$$

Now, equating $f(x_1) = f(x_2)$ since from the defination.

$$\implies \frac{8x_1 + 3}{5x_1 - 8} = \frac{8x_2 + 3}{5x_2 - 8} \qquad (1)$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1 - x_2) = 0. \qquad (2)$$

From the eqn(2) we can say that, the equation satisfies only when $x_1 - x_2 = 0$. Which implies,

$$x_1 = x_2$$

Hence, the given function $f(x)$ is one-one.

Defination: f is called onto if $f(X) = Y$. i.e., Range of the function $f(x)$ is equal to co-domain.

Let,

$$y = f(x) = \frac{8x + 3}{5x - 8} \qquad (3)$$

On solving the eqn(3), we get,

$$x = \frac{8y+3}{5y-8} \qquad (4)$$

Where y is element of co-domain. Now eqn(4) is defined $\forall\ y \in R$ - $\{\frac{8}{5}\}$. i.e. $y \in A$.

now, from eqn(4) substitute the value of x.

$$\implies f(x) = f(\frac{8y + 3}{5y - 8}) \qquad (5)$$

$$\implies f(x) = \frac{8(\frac{8y+3}{5y-8}) + 3}{5(\frac{8y+3}{5y-8}) - 8} \qquad (6)$$

$$\implies f(x) = \frac{8(8y + 3) + 3(5y - 8)}{5(8y + 3) - 8(5y - 8)} \qquad (7)$$

$$\implies f(x) = \frac{79y}{79} \qquad (8)$$

$$\implies f(x) = y \qquad (9)$$

Hence, from eqn(9) we can say, given function $f(x)$ is onto.

Defination: $f\colon X \to Y$ is bijective function i.e., both one-one and onto then there exit a unique function called inverse function and is denoted by f^{-1}, such that,

$$f^{-1}(y) = x \iff f(x) = y$$

Now, from defination $f^{-1}(y) = x$. From eqn(4) we get the value of x

$$f^{-1}(y) = \frac{8y + 3}{5y - 8}$$

i.e., the inverse of function $f(x)$ is,
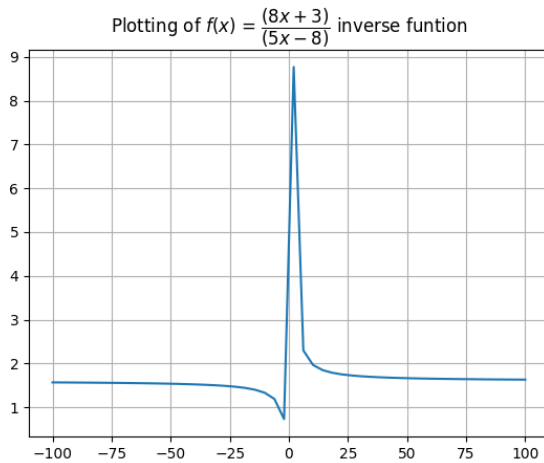
$$f^{-1}(x) = \frac{8x+3}{5x-8}$$

Fig. 1. graph

From the above graph we observe that, since the graph is continuous hence it is onto

NOTE: The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.