

# Assignment 2

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## I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If  $f : A \rightarrow A$  and  $A = R - \{\frac{8}{5}\}$ , show that the function  $f(x) = \frac{8x+3}{5x-8}$  is one-one onto. Hence, find  $f^{-1}$ .

## II. SOLUTION

Defination: Let  $f : X \rightarrow Y$  be a function, we say  $f$  is one-one or injective, if and only if  $\forall x_1, x_2 \in X$ . if  $f(x_1) = f(x_2)$  then  $x_1 = x_2$ .

Suppose that  $x_1$  and  $x_2$  are arbitrary integers and  $f(x_1) = f(x_2)$ , we need to show that  $x_1 = x_2$ . Since  $f(x_1) = f(x_2)$ .

$$f(x_1) = \frac{8x_1+3}{5x_1-8} \quad \text{and} \quad f(x_2) = \frac{8x_2+3}{5x_2-8}$$

Now, equating  $f(x_1) = f(x_2)$  since from the defination.

$$\implies \frac{8x_1+3}{5x_1-8} = \frac{8x_2+3}{5x_2-8} \quad (1)$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1 - x_2) = 0. \quad (2)$$

From the eqn(2) we can say that, the equation satisfies only when  $x_1 - x_2 = 0$ . Which implies,

$$x_1 = x_2$$

Hence, the given function  $f(x)$  is one-one.

Defination:  $f$  is called onto if  $f(X) = Y$ . i.e., Range of the function  $f(x)$  is equal to co-domain.

Let,

$$y = f(x) = \frac{8x+3}{5x-8} \quad (3)$$

On solving the eqn(3), we get,

$$x = \frac{8y+3}{5y-8} \quad (4)$$

Where  $y$  is element of co-domain. Now eqn(4) is defined  $\forall y \in R - \{\frac{8}{5}\}$ . i.e.  $y \in A$ .

now, from eqn(4) substitute the value of  $x$ .

$$\rightarrow f(x) = f\left(\frac{8y+3}{5y-8}\right) \quad (5)$$

$$\rightarrow f(x) = \frac{8\left(\frac{8y+3}{5y-8}\right) + 3}{5\left(\frac{8y+3}{5y-8}\right) - 8} \quad (6)$$

$$\rightarrow f(x) = \frac{8(8y+3) + 3(5y-8)}{5(8y+3) - 8(5y-8)} \quad (7)$$

$$\rightarrow f(x) = \frac{79y}{79} \quad (8)$$

$$\rightarrow f(x) = y \quad (9)$$

Hence, from eqn(9) we can say, given function  $f(x)$  is onto.

Defination:  $f: X \rightarrow Y$  is bijective function i.e., both one-one and onto then there exit a unique function called inverse function and is denoted by  $f^{-1}$ , such that,

$$f^{-1}(y) = x \iff f(x) = y$$

Now, from defination  $f^{-1}(y) = x$ . From eqn(4) we get the value of  $x$

$$f^{-1}(y) = \frac{8y + 3}{5y - 8}$$

i.e., the inverse of function  $f(x)$  is,

$$f^{-1}(x) = \frac{8x+3}{5x-8}$$

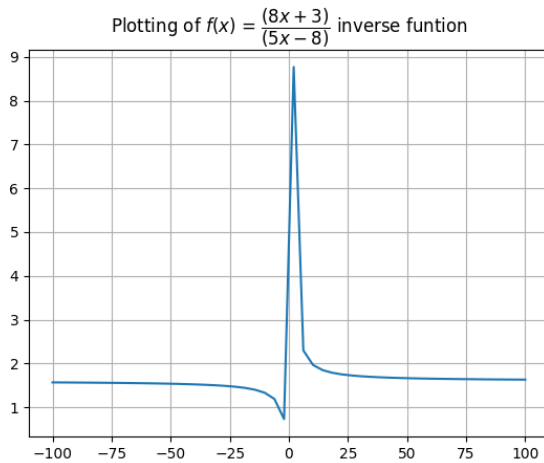


Fig. 1. graph

From the above graph we observe that, since the graph is continuous hence it is onto

NOTE: The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.