Assignment 2

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I. PROBLEM-ICSE-2019-12 Q)4-B

Q) If f : A $\rightarrow$ A and A = R - $\{\frac{8}{5}\}$, show that the function $f(x)$ = $\frac{8x+3}{5x-8}$ is one-one onto. Hence, find f^{-1}.

II. SOLUTION

Defination: Let $f : X \rightarrow Y$ be a function, we say f is one-one or injective, if and only if $\forall$ x_1 , $x_2 \in X$. if $f(x_1)$ = $f(x_2)$ then x_1 = x_2.

Suppose that x_1 and x_2 are arbitrary integers and $f(x_1)$ = $f(x_2)$, we need to show that x_1 = x_2. Since $f(x_1)$ = $f(x_2)$.

$$f(x_1) = \frac{8x_1+3}{5x_1-8} \quad and \quad f(x_2) = \frac{8x_2+3}{5x_2-8}$$

Now, equating $f(x_1)$ = $f(x_2)$ since from the defination.

$$\implies \frac{8x_1+3}{5x_1-8} = \frac{8x_2+3}{5x_2-8} \tag{1}$$

On cross multiplying and simplifying the eqn(1), we get,

$$\implies 49(x_1-x_2) = 0. \tag{2}$$

From the eqn(2) we can say that, the equation satisfies only when $x_1 - x_2$= 0. Which implies,

$$\boxed{x_1 = x_2}$$

Hence, the given function $f(x)$ is one-one.

Defination: f is called onto if $f(X) = Y$. i.e., Range of the function $f(x)$ is equal to co-domain.

Let,

$$y = f(x) = \frac{8x+3}{5x-8} \tag{3}$$

On solving the eqn(3), we get,

$$\boxed{x = \frac{8y+3}{5y-8}} \tag{4}$$

Where y is element of co-domain. Now eqn(4) is defined $\forall$ $y \in R$ - $\{\frac{8}{5}\}$. i.e.$y \in$ A.

now,

$$\rightarrow f(x) = f(\frac{8y+3}{5y-8}) \tag{5}$$

$$\rightarrow f(x) = \frac{8(\frac{8y+3}{5y-8})+3}{5(\frac{8y+3}{5y-8})-8} \tag{6}$$

$$\rightarrow f(x) = \frac{8(8y+3)+3(5y-8)}{5(8y+3)-8(5y-8)} \tag{7}$$

$$\rightarrow f(x) = \frac{79y}{79} \tag{8}$$

$$\rightarrow f(x) = y \tag{9}$$

Hence, from eqn(9)we can say, given function $f(x)$ is onto.

Defination: $f: X \to Y$ is bijective function i.e., both one-one and onto then there exit a unique function called inverse function and is denoted by f^{-1}, such that,

$$f^{-1}(y) = x \iff f(x) = y$$

Now, from defination $f^{-1}(y) = x$. From eqn(4) we get the value of x

$$f^{-1}(y) = \frac{8y+3}{5y-8}$$

i.e., the inverse of function $f(x)$ is,

$$f^{-1}(x) = \frac{8x+3}{5x-8}$$

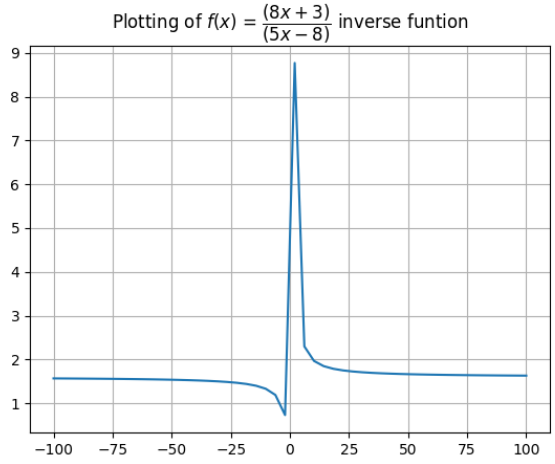

Fig. 1. graph

From the above graph we observe that, since the graph is continuous hence it is onto

NOTE: The above shown graph is graph of inverse function. In general for inverse function that is not one-one may have multiple images.