Assignment 4

JARPULA BHANU PRASAD - AI21BTECH11015

Download codes from:

Download python code from - Python
Download latex code from - Latex

I. Problem-CBSE-9th Q)example 2

Q)Two coins are tossed simultaneously 500 times, and we get

$$Two\ heads : 105\ times$$
$$One\ head : 275\ times$$
$$No\ head : 120\ times$$

Find the probability of occurrence of each of these events.

II. Solution

Let the random variable $X \in \{0,1,2\}$ denote the number of heads in the coin-tossing experiment.
Now,

$$Pr(X=i) = \frac{n(X=i)}{\sum_{i=0}^{2} n(X=i)} \tag{1}$$

Where $i \in \{0,1,2\}$ and $n(X=i)$ is the frequency of getting i heads.
Also,
Number of times 2 coins were tossed = 500

$$\implies \sum_{i=0}^{2} n(X=i) = 500 \tag{2}$$

Probability of getting zero heads

$$Pr(X=0) = \frac{120}{500} = 0.24 \tag{3}$$

Probability of getting one head

$$Pr(X=1) = \frac{275}{500} = 0.55 \tag{4}$$

Probability of getting two heads

$$Pr(X=2) = \frac{105}{500} = 0.21 \tag{5}$$

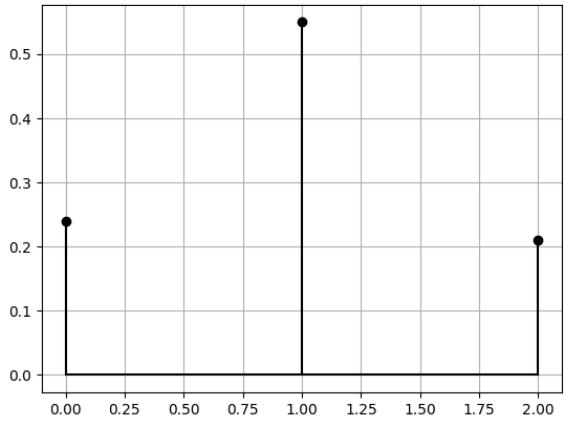

Fig. 1. Plot of PMF using above data

Now considering fair coins: Let probability of getting a head be a success and equal to p and probability of getting a tail be a failure and equal to q where $p + q$ = 1. We can express this as a binomial distribution

$$\sum_{i=0}^{n} Pr(X=i) = \sum_{i=0}^{n} {}^{n}C_i (p)^i (1-p)^{n-i} \tag{6}$$

Where n = 2 for 2 coins. Therefore,

$$Pr(X=i) = {}^{2}C_i (p)^i (q)^{2-i} \tag{7}$$

For fair coins,

$$p = \frac{1}{2} \quad and \quad q = \frac{1}{2} \tag{8}$$

Therefore,

$$Pr(X=0) = {}^2C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad (9)$$

$$Pr(X=1) = {}^2C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^1 = \frac{1}{2} \quad (10)$$

$$Pr(X=2) = {}^2C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^0 = \frac{1}{4} \quad (11)$$

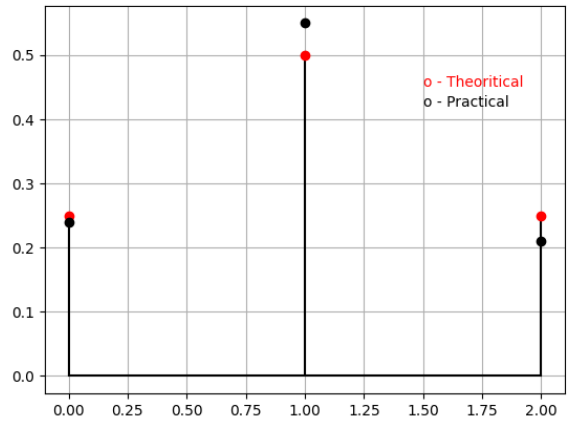

Fig. 2. Comparison of theoretical and practical PMF plots