Assignment 9

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Problem

Q)The distribution of x^2.

Solution

Let $y = x^2$

- If $y \geq 0$, then $x^2 \leq y$ for $-\sqrt{y} \leq x \leq \sqrt{y}$ (see fig(1)) . Hence $F_y(y) = P(-\sqrt{y} \leq x \leq \sqrt{y}) = F_x(\sqrt{y}) - F_x(-\sqrt{y})$, $y > 0$
- If $y < 0$, then there are no values of x such that $x^2 < y$.Hence $F_y(y) = P(\phi) = 0$,$y < 0$
- By direct differentiation of $F_y(y)$, we get

$$f_y(y) = \begin{cases} \frac{f_x(\sqrt{y})+f_x(-\sqrt{y})}{2\sqrt{y}} & y > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

- If $f_x(x)$ represents an even function, then eqn(1) reduces to

$$f_y(y) = \frac{f_x(\sqrt{y})U(y)}{\sqrt{y}} \tag{2}$$

- In particular if $x \sim N(0, 1)$, so that

$$f_x(x) = \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} \tag{3}$$

and substituting this into eqn(2), we obtain the p.d.f of $y = x^2$ to be

$$f_y(y) = \frac{e^{\frac{-y}{2}}U(y)}{\sqrt{2\pi y}} \tag{4}$$

On comparing this with **CHI-SQUARE DISTRIBUTION**,
we notice that eqn(4) represent a chi-square random varaible with n=1, since $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. Thus, if x is a Guassian random varaible with $\mu = 0$, then $y = x^2$ represents a chi-square random varaible with one degree of freedom.

$y = x^2$

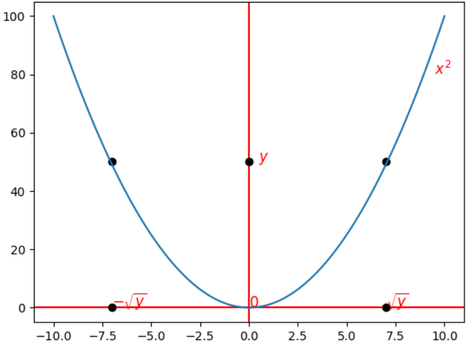

Figure: $y = x^2$

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