Assignment-1

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Download codes from:
Python code - python.
LaTeX code - LaTeX.

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1 Problem-Oppenheim 3.6-a

1.1 Determine the inverse z-transform of

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2} \tag{1.1}$$

and determine whether the Fourier transform exist.

Solution: The Z-transfrom of $x(n)$ is defines as

$$X(z) = \mathcal{Z}[x(n)] = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{1.2}$$

Now, inverse Z-transform is defined by

$$x(n) = \mathcal{Z}^{-1}\left[\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right] \tag{1.3}$$

i.e., coefficient of z^{-1} in the expansion.
Now,

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \tag{1.4}$$

$$= \left(1+\frac{1}{2}z^{-1}\right)^{-1} \tag{1.5}$$

since, $|z| > \frac{1}{2}$ and $(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

$$= 1 - \frac{1}{2}z^{-1} + \left(\frac{1}{2}z^{-1}\right)^2 + \dots \tag{1.6}$$

$$= \sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n z^{-n} \tag{1.7}$$

$\therefore$ The inverse z-transform of $X(z)$ is $\left(-\frac{1}{2}\right)^n \forall n \geq 0$

The condition for Fourier transform to exist for function $x(n)$ is given as

$$\sum_{-\infty}^{\infty} |x(n)| < \infty \tag{1.8}$$

i.e.,

$$\sum_{-\infty}^{\infty} |x(n)| = \sum_{-\infty}^{\infty} \left|\left(-\frac{1}{2}\right)^n\right| \tag{1.9}$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} \tag{1.10}$$

$$= 1 < \infty \tag{1.11}$$

Hence we can say Fourier transform exist.