Fourier Series

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Contents

***Abstract*—This manual provides a simple introduction to Fourier Series**

1 Periodic Function

Let

$$x(t) = A_0 \left|\sin\left(2\pi f_0 t\right)\right| \tag{1.1}$$

1.1 Plot $x(t)$.
Solution: The following code will plot the graph in fig (1.1)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/charger/codes/1.1.py
```

run the above code using the command

```
python3 1.1.py
```

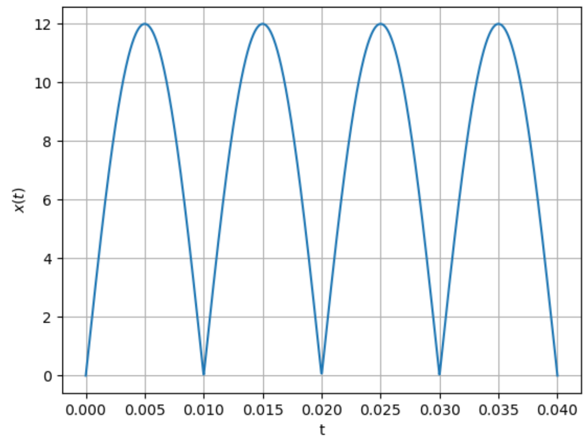

Fig. 1.1: $x(t) = A_0 \left|\sin\left(2\pi f_0 t\right)\right|$

1.2 Show that $x(t)$ is periodic and find its period.
Solution: From fig (1.1), we see that $x(t)$ is periodic. Further,

$$\text{period of } \sin(at) \quad \text{given by } \frac{2\pi}{a} \tag{1.2}$$

Now, period of $x(t)$ is

$$A_0 \left|\sin\left(2\pi f_0 t\right)\right| \implies \frac{\pi}{2\pi f_0} \tag{1.3}$$

$$\implies \frac{1}{2f_0} \tag{1.4}$$

Verification

$$x\left(t + \frac{1}{2f_0}\right) = A_0 \left|\sin\left(2\pi f_0\left(t + \frac{1}{2f_0}\right)\right)\right| \tag{1.5}$$

$$= A_0 \left|\sin\left(2\pi f_0 t + \pi\right)\right| \tag{1.6}$$

$$= A_0 \left|-\sin\left(2\pi f_0 t\right)\right| \tag{1.7}$$

$$= A_0 \left|\sin\left(2\pi f_0 t\right)\right| \tag{1.8}$$

Hence the period of $x(t)$ is $\frac{1}{2f_0}$.

2 Fourier Series

Consider $A_0 = 12$ and $f_0 = 50$ for all numerical calculations.

2.1 If

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{\jmath 2\pi k f_0 t} \tag{2.1}$$

show that

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-\jmath 2\pi k f_0 t}\, dt \tag{2.2}$$

Solution: We have for some $n \in \mathbb{Z}$,

$$x(t) e^{-\jmath 2\pi n f_0 t} = \sum_{k=-\infty}^{\infty} c_k e^{\jmath 2\pi (k-n) f_0 t} \tag{2.3}$$

But we know from the periodicity of $e^{\jmath 2\pi k f_0 t}$,

$$\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} e^{\jmath 2\pi k f_0 t}\, dt = \frac{1}{f_0}\delta(k) \tag{2.4}$$

Thus,

$$\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-\jmath 2\pi n f_0 t}\, dt = \frac{c_n}{f_0} \tag{2.5}$$

$$\implies c_n = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-\jmath 2\pi n f_0 t}\, dt \tag{2.6}$$

2.2 Find c_k for (1.1)
Solution: Using (2.2),

$$c_n = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} A_0 \left|\sin(2\pi f_0 t)\right| e^{-\jmath 2\pi n f_0 t}\, dt \tag{2.7}$$

$$= f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} A_0 \left|\sin(2\pi f_0 t)\right| \cos(2\pi n f_0 t)\ dt + \jmath f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} A_0 \left|\sin(2\pi f_0 t)\right| \sin(2\pi n f_0 t)\ dt \tag{2.8}$$

$$= 2 f_0 \int_{0}^{\frac{1}{2f_0}} A_0 \sin(2\pi f_0 t) \cos(2\pi n f_0 t)\ dt \tag{2.9}$$

$$= f_0 A_0 \int_{0}^{\frac{1}{2f_0}} \left(\sin(2\pi(n+1) f_0 t)\right) dt - f_0 A_0 \int_{0}^{\frac{1}{2f_0}} \left(\sin(2\pi(n-1) f_0 t)\right) dt \tag{2.10}$$

$$= A_0 \frac{1 + (-1)^n}{2\pi} \left(\frac{1}{n+1} - \frac{1}{n-1}\right) \tag{2.11}$$

$$= \begin{cases} \frac{2A_0}{\pi(1-n^2)} & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \tag{2.12}$$

2.3 Verify (2.1) using python.
Solution: The following code will plot the graph in fig (2.3)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/charger/codes/2.3.py
```

run the above code using the command

```
python3 2.3.py
```

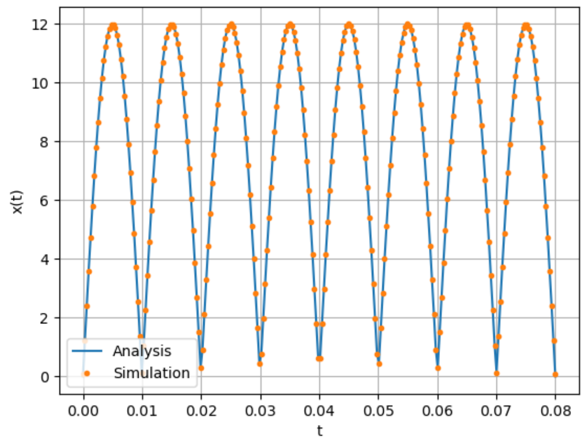

Fig. 2.3: $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{\jmath 2\pi k f_0 t}$

2.4 Show that

$$x(t) = \sum_{k=0}^{\infty} \left(a_k \cos \jmath 2\pi k f_0 t + b_k \sin \jmath 2\pi k f_0 t\right) \tag{2.13}$$

and obtain the formulae for a_k and b_k.
Solution: From (2.1),

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{\jmath 2\pi k f_0 t} \tag{2.14}$$

$$= c_0 + \sum_{k=1}^{\infty} c_k e^{\jmath 2\pi k f_0 t} + c_{-k} e^{-\jmath 2\pi k f_0 t} \tag{2.15}$$

$$= c_0 + \sum_{k=1}^{\infty} (c_k + c_{-k}) \cos(2\pi k f_0 t) + \sum_{k=0}^{\infty} (c_k - c_{-k}) \sin(2\pi k f_0 t) \tag{2.16}$$

Hence, for $k \geq 0$,

$$a_k = \begin{cases} c_0 & k = 0 \\ c_k + c_{-k} & k > 0 \end{cases} \tag{2.17}$$

$$b_k = c_k - c_{-k} \tag{2.18}$$

2.5 Find a_k and b_k for (1.1)
Solution: From (2.1), we see that since $x(t)$ is

even,

$$x(-t) = \sum_{k=-\infty}^{\infty} c_k e^{-\jmath 2\pi k f_0 t} \quad (2.19)$$

$$= \sum_{k=-\infty}^{\infty} c_{-k} e^{\jmath 2\pi k f_0 t} \quad (2.20)$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{\jmath 2\pi k f_0 t} \quad (2.21)$$

where we substitute $k \mapsto -k$ in (2.20). Hence, we see that $c_k = c_{-k}$. So, from (2.18) and for $k \geq 0$,

$$a_k = \begin{cases} \frac{2A_0}{\pi} & k = 0 \\ \frac{4A_0}{\pi(1-k^2)} & k > 0,\ k \text{ even} \\ 0 & \text{otherwise} \end{cases} \quad (2.22)$$

$$b_k = 0 \quad (2.23)$$

2.6 Verify (2.13) using python.
Solution: The following code will plot the graph in fig (2.6)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/charger/codes/2.6.py
```

run the above code using the command

```
python3 2.6.py
```

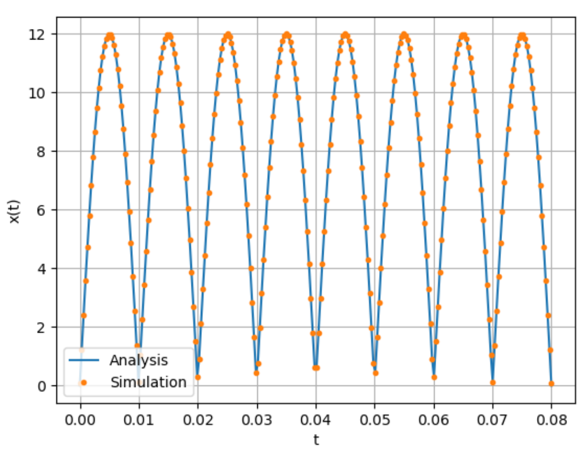

Fig. 2.6: $x(t) = \sum_{k=0}^{\infty} (a_k \cos \jmath 2\pi k f_0 t + b_k \sin \jmath 2\pi k f_0 t)$

3 Fourier Transform

3.1

$$\delta(t) = 0, \quad t \neq 0 \quad (3.1)$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad (3.2)$$

3.2 The Fourier Transform of $g(t)$ is

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt \quad (3.3)$$

3.3 Show that

$$g(t - t_0) \xleftrightarrow{\mathcal{H}} FG(f) e^{-j2\pi f t_0} \quad (3.4)$$

Solution: We write, substituting $u := t - t_0$,

$$g(t - t_0) \xleftrightarrow{\mathcal{H}} F \int_{-\infty}^{\infty} g(t - t_0) e^{-\jmath 2\pi f t}\, dt \quad (3.5)$$

$$= \int_{-\infty}^{\infty} g(u) e^{-\jmath 2\pi f (u + t_0)}\, du \quad (3.6)$$

$$= G(f) e^{-j2\pi f t_0} \quad (3.7)$$

where the last equality follows from (3.3).

3.4 Show that

$$G(t) \xleftrightarrow{\mathcal{H}} F g(-f) \quad (3.8)$$

Solution: Using the definition of the Inverse Fourier Transform,

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{\jmath 2\pi f t}\, df \quad (3.9)$$

Hence, setting $t := -f$ and $f := t$, which implies $df = dt$,

$$g(-f) = \int_{-\infty}^{\infty} G(t) e^{-\jmath 2\pi f t}\, dt \quad (3.10)$$

$$\implies G(t) \xleftrightarrow{\mathcal{H}} F g(-f) \quad (3.11)$$

3.5 $\delta(t) \xleftrightarrow{\mathcal{H}} F$?
Solution: By applying the defination fo fourier transformation for $\delta(t - t_0)$ {t_0 be time shifting}.

$$\mathcal{F}\{\delta(t - t_0)\}(f) = \mathcal{F}(t) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi f t} dt. \quad (3.12)$$

By applying the time shifting property of impulse we get

$$\mathcal{F}(f) = e^{-j2\pi f t_0} \quad (3.13)$$

$$i.e., \quad \delta(t - t_0) \xleftrightarrow{\mathcal{H}} F e^{-j2\pi f t_0} \quad (3.14)$$

Now, substitute $t_0 = 0$ we get

$$\delta(t) \overset{\mathcal{H}}{\longleftrightarrow} F1 \tag{3.15}$$

3.6 $e^{-j2\pi f_0 t} \overset{\mathcal{H}}{\longleftrightarrow} F?$
Solution: Applying the defination of inverse fourier transformation.

$$\mathcal{F}^{-1}\left\{\delta(f+f_0)\right\}(t) = f(t) = \int_{-\infty}^{\infty} \delta(f+f_0)e^{j2\pi f t}df \tag{3.16}$$

By applying the shifting property of impulse

$$f(t) = e^{-j2\pi f_0 t} \tag{3.17}$$

$$i.e., \quad e^{-j2\pi f_0 t} \overset{\mathcal{H}}{\longleftrightarrow} F\delta(f+f_0) \tag{3.18}$$

3.7 $\cos(2\pi f_0 t) \overset{\mathcal{H}}{\longleftrightarrow} F?$
Solution:

$$\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \tag{3.19}$$

$$\mathcal{F}\left[\cos(2\pi f_0 t)\right] = \int_{-\infty}^{\infty} \cos(2\pi f_0 t) e^{-j2\pi f t}dt \tag{3.20}$$

$$= \int_{-\infty}^{\infty} \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} e^{-j2\pi f t}dt \tag{3.21}$$

$$= \frac{1}{2}\Big[\int_{-\infty}^{\infty} e^{j2\pi f_0 t e^{-j2\pi f t}dt} + \int_{-\infty}^{\infty} e^{-j2\pi f_0 t}e^{-j2\pi f t}dt\Big] \tag{3.22}$$

$$= \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] \tag{3.23}$$

$$\therefore \cos(2\pi f_0 t) \overset{\mathcal{H}}{\longleftrightarrow} F\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] \tag{3.24}$$

3.8 Find the Fourier Transform of $x(t)$ and plot it. Verify using python.
Solution:

$$x(t) = A_0\left|\sin(2\pi f_0 t)\right| \tag{3.25}$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt \tag{3.26}$$

$$= \int_{-\infty}^{\infty} A_0\left|\sin(2\pi f_0 t)\right| e^{-j2\pi f t}dt \tag{3.27}$$

$$= A_0\int_{0}^{\infty} \sin(2\pi f_0 t)e^{-j2\pi f t}dt + K \tag{3.28}$$

$$= \frac{A_0}{2j}\int_{0}^{\infty} (e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})e^{-j2\pi f t}dt + K \tag{3.29}$$

$$= \frac{A_0}{2j}\int_{0}^{\infty} (e^{j2\pi(f_0-f)t} + e^{-j2\pi(f_0-f)t})dt + K \tag{3.30}$$

$$= \frac{A_0}{2j}\left[0 - \frac{1}{j2\pi(f_0-f)}\right] - \frac{A_0}{2j}\left[0 - \frac{1}{j2\pi(f_0+f)}\right] \tag{3.31}$$

$$= \frac{A_0}{2\pi(f_0^2 - f^2)} + k \tag{3.32}$$

By symmetry we get

$$k = \frac{A_0}{2\pi(f_0^2 - f^2)} \tag{3.33}$$

$$\therefore \quad X(f) = \frac{A_0}{\pi(f_0^2 - f^2)} \tag{3.34}$$

The following code will plot the graph in fig (3.8)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/charger/codes/3.8.py
```

run the above code using the command

```
python3 3.8.py
```

3.9 Show that

$$\text{rect}(t) \overset{\mathcal{H}}{\longleftrightarrow} F\text{sinc}(t) \tag{3.35}$$

Verify using python.

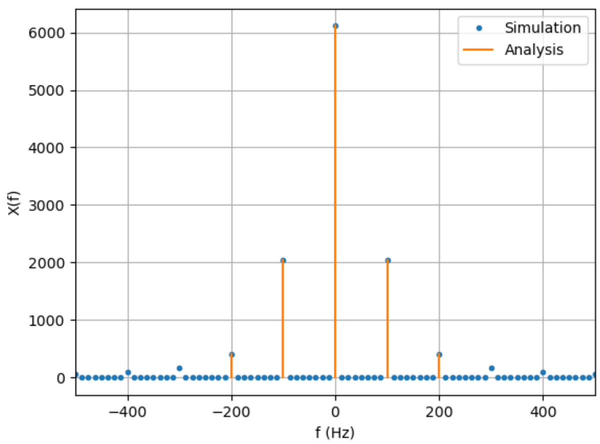

Fig. 3.8: $\mathcal{F}[x(t)]$

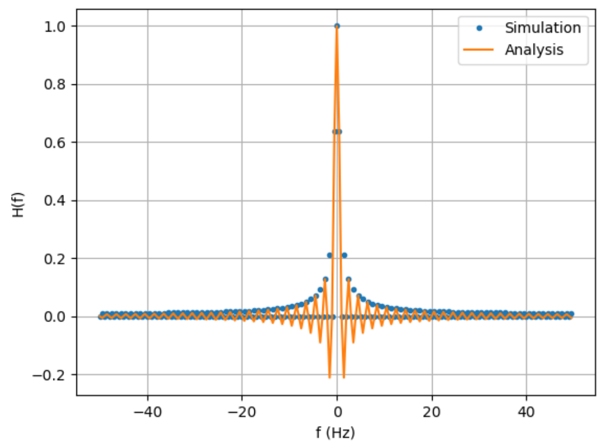

Fig. 3.9: $\mathcal{F}[\text{rect}\,(t)]$

Solution:

$$\text{rect}\,(t) = \begin{cases} 1 & |t| < T \\ 0 & \text{otherwise} \end{cases} \tag{3.36}$$

$$\mathcal{F}\,[\text{rect}\,(t)] = \int_{-\infty}^{\infty} \text{rect}\,(t)\, e^{-j2\pi f t} dt \tag{3.37}$$

$$= \int_{-T}^{T} e^{-j2\pi f t} dt \tag{3.38}$$

$$= \frac{1}{-j2\pi f}\left[e^{-j2\pi f T} - e^{j2\pi f T}\right] \tag{3.39}$$

$$= \frac{1}{\pi f}\left[\frac{e^{j2\pi f T} - e^{-j2\pi f T}}{2j}\right] \tag{3.40}$$

$$= \frac{\sin \pi f}{\pi f} \tag{3.41}$$

$$= \text{sinc}\,(f) \tag{3.42}$$

The following code will plot the graph in fig (3.9)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/charger/codes/3.9.py
```

run the above code using the command

```
python3 3.9.py
```

3.10 $\text{sinc}\,(t) \overset{\mathcal{H}}{\longleftrightarrow} F$?. Verify using python.
Solution: from (3.8) we can say

$$\text{sinc}\,(t) \overset{\mathcal{H}}{\longleftrightarrow} F\text{rect}\,(-f) \tag{3.43}$$

$$\text{rect}\,(f) \tag{3.44}$$

The following code will plot the graph in fig (3.10)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/charger/codes/3.10.py
```

run the above code using the command

```
python3 3.10.py
```

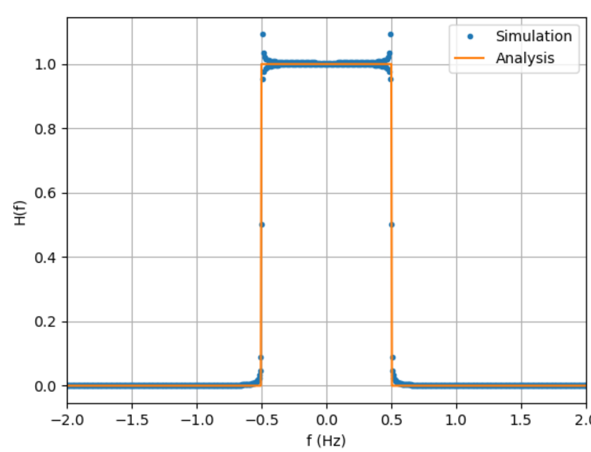

Fig. 3.10: $\mathcal{F}[\text{sinc}\,(t)]$

4 Filter

4.1 Find $H(f)$ which transforms $x(t)$ to DC 5V.
Solution: The function $H(f)$ is a low pass filter which filters out even harmonics and leaves the zero frequency component behind. The rectangular function represents an ideal

low pass filter. Suppose the cutoff frequency is $f_c = 50$ Hz, then

$$H(f) = \text{rect}\left(\frac{f}{2f_c}\right) = \begin{cases} 1 & |f| < f_c \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

Multiplying by a scaling factor to get DC 5V,

$$H(f) = \frac{\pi V_0}{2A_0}\text{rect}\left(\left(\frac{f}{2f_c}\right)\right) \tag{4.2}$$

where $V_0 = 5$ V.

4.2 Find $h(t)$.
Solution: Suppose $g(t) \xleftrightarrow{\mathcal{H}} FG(f)$. Then, for some nonzero $a \in \mathbb{R}$

$$g(at) \xleftrightarrow{\mathcal{H}} F\int_{-\infty}^{\infty} g(at)e^{-\jmath 2\pi f t}\,dt \tag{4.3}$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} g(u)e^{\left(-\jmath 2\pi \frac{f}{a} t\right)}\,dt \tag{4.4}$$

$$= \frac{1}{a}G\left(\frac{f}{a}\right) \tag{4.5}$$

where we have substituted $u := at$. Using (4.5) of the Fourier Transform in (4.1),

$$h(t) = \frac{2\pi V_0}{A_0} f_c \text{sinc}\left(2f_c t\right) \tag{4.6}$$

4.3 Verify your result using through convolution.
Solution: The following code will plot the graph in fig (4.3)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/charger/codes/4.3.py
```

run the above code using the command

```
python3 4.3.py
```

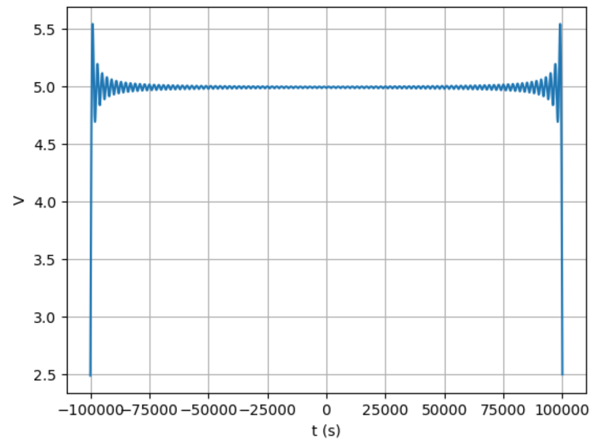

Fig. 4.3: convolution of two signals

5 Filter Design

5.1 Design a Butterworth filter for $H(f)$. **Solution:** The Butterworth filter has an amplitude response given by

$$|H(f)|^2 = \frac{1}{\left(1 + \left(\frac{f}{f_c}\right)^{2n}\right)} \tag{5.1}$$

where n is the order of the filter and f_c is the cutoff frequency. The attenuation at frequency f is given by

$$A = -10\log_{10}|H(f)|^2 \tag{5.2}$$

$$= -20\log_{10}|H(f)| \tag{5.3}$$

We consider the following design parameters for our lowpass analog Butterworth filter:

a) Passband edge, $f_p = 50$ Hz
b) Stopband edge, $f_s = 100$ Hz
c) Passband attenuation, $A_p = -1$ dB
d) Stopband attenuation, $A_s = -20$ dB

We are required to find a desriable order n and cutoff frequency f_c for the filter. From (5.3),

$$A_p = -10\log_{10}\left[1 + \left(\frac{f_p}{f_c}\right)^{2n}\right] \tag{5.4}$$

$$A_s = -10\log_{10}\left[1 + \left(\frac{f_s}{f_c}\right)^{2n}\right] \tag{5.5}$$

Thus,

$$\left(\frac{f_p}{f_c}\right)^{2n} = 10^{-\frac{A_p}{10}} - 1 \tag{5.6}$$

$$\left(\frac{f_s}{f_c}\right)^{2n} = 10^{-\frac{A_s}{10}} - 1 \tag{5.7}$$

Therefore, on dividing the above equations and solving for n,

$$n = \frac{\log\left(10^{-\frac{A_s}{10}} - 1\right) - \log\left(10^{-\frac{A_p}{10}} - 1\right)}{2\left(\log f_s - \log f_p\right)} \tag{5.8}$$

In this case, making appropriate susbstitutions gives $n = 4.29$. Hence, we take $n = 5$. Solving for f_c in (5.6) and (5.7),

$$f_{c1} = f_p \left[10^{-\frac{A_p}{10}} - 1\right]^{-\frac{1}{2n}} = 57.23\text{Hz} \tag{5.9}$$

$$f_{c2} = f_s \left[10^{-\frac{A_s}{10}} - 1\right]^{-\frac{1}{2n}} = 63.16\text{Hz} \tag{5.10}$$

Hence, we take $f_c = \sqrt{f_{c1} f_{c2}} = 60\text{Hz}$ approximately.

5.2 Design a Chebyshev filter for $H(f)$. **Solution:** The Chebyshev filter has an amplitude response given by

$$|H(f)|^2 = \frac{1}{\left(1 + \epsilon^2 C_n^2\left(\frac{f}{f_c}\right)\right)} \tag{5.11}$$

where

a) n is the order of the filter
b) ϵ is the ripple
c) f_c is the cutoff frequency
d) $C_n = \cosh^{-1}(n \cosh x)$ denotes the n$^{\text{th}}$ order Chebyshev polynomial, given by

$$c_n(x) = \begin{cases} \cos\left(n \cos^{-1} x\right) & |x| \le 1 \\ \cosh\left(n \cosh^{-1} x\right) & \text{otherwise} \end{cases} \tag{5.12}$$

We are given the following specifications:

a) Passband edge (which is equal to cutoff frequency), $f_p = f_c$
b) Stopband edge, f_s
c) Attenuation at stopband edge, A_s
d) Peak-to-peak ripple δ in the passband. It is given in dB and is related to ϵ as

$$\delta = 10 \log_{10}\left(1 + \epsilon^2\right) \tag{5.13}$$

and we must find a suitable n and ϵ. From (5.13),

$$\epsilon = \sqrt{10^{\frac{\delta}{10}} - 1} \tag{5.14}$$

At $f_s > f_p = f_c$, using (5.12), A_s is given by

$$A_s = -10 \log_{10}\left[1 + \epsilon^2 c_n^2\left(\frac{f_s}{f_p}\right)\right] \tag{5.15}$$

$$\implies c_n\left(\frac{f_s}{f_p}\right) = \frac{\sqrt{10^{-\frac{A_s}{10}} - 1}}{\epsilon} \tag{5.16}$$

$$\implies n = \frac{\cosh^{-1}\left(\frac{\sqrt{10^{-\frac{A_s}{10}} - 1}}{\epsilon}\right)}{\cosh^{-1}\left(\frac{f_s}{f_p}\right)} \tag{5.17}$$

We consider the following specifications:

a) Passband edge/cutoff frequency, $f_p = f_c = 60\text{Hz}$.
b) Stopband edge, $f_s = 100\text{Hz}$.
c) Passband ripple, $\delta = 0.5\text{dB}$
d) Stopband attenuation, $A_s = -20\text{dB}$

$\epsilon = 0.35$ and $n = 3.68$. Hence, we take $n = 4$ as the order of the Chebyshev filter.

5.3 Design a circuit for your Butterworth filter. **Solution:** Looking at the table of normalized element values L_k, C_k, of the Butterworth filter for order 5, and noting that de-normalized values L'_k and C'_k are given by

$$C'_k = \frac{C_k}{\omega_c} \qquad L'_k = \frac{L_k}{\omega_c} \tag{5.18}$$

De-normalizing these values, taking $f_c = 60$ Hz,

$$C'_1 = C'_5 = 1.64\text{mF} \tag{5.19}$$
$$L'_2 = L'_4 = 4.29\text{mH} \tag{5.20}$$
$$C'_3 = 5.31\text{mF} \tag{5.21}$$
$$\tag{5.22}$$

The L-C network is shown in Fig. 5.3. This circuit is simulated in the ngspice

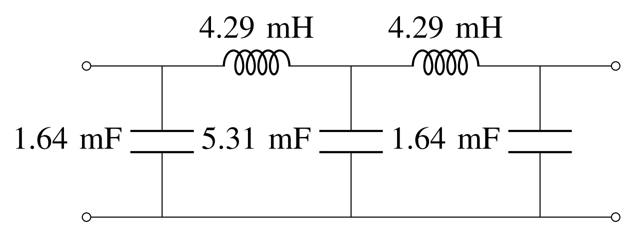

Fig. 5.3: L-C Butterworth Filter

code `codes/5_3.cir`. The Python code `codes/5_3.py` compares the amplitude response of the simulated circuit with the theoretical expression.

5.4 Design a circuit for your Chebyshev filter. **Solution:** Looking at the table of normalized element values of the Chebyshev filter for order 3 and 0.5 dB ripple, and de-nomrmalizing those values, taking $f_c = 50\text{Hz}$,

$$C'_1 = 4.43\text{mF} \tag{5.23}$$
$$L'_2 = 3.16\text{mH} \tag{5.24}$$
$$C'_3 = 6.28\text{mF} \tag{5.25}$$
$$L'_4 = 2.23\text{mH} \tag{5.26}$$

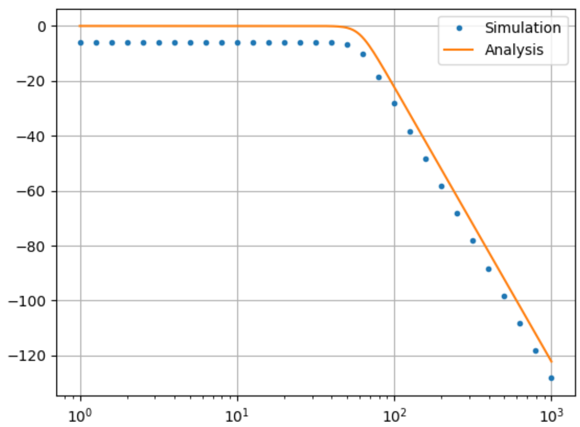

Fig. 5.4: Simulation of Butterworth filter.

The L-C network is shown in Fig. 5.4. This circuit is simulated in the ngspice

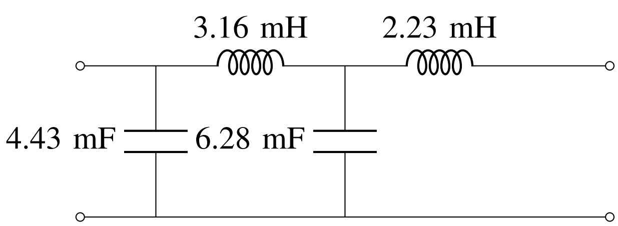

Fig. 5.4: L-C Chebyshev Filter

code `codes/5_4.cir`. The Python code `codes/5_4.py` compares the amplitude response of the simulated circuit with the theoretical expression.

5.5 Design a low pass digital Butterworth filter for your $H(f)$. **Solution:**

5.6 Design a low pass digital Chebyshev filter for your $H(f)$. **Solution:**

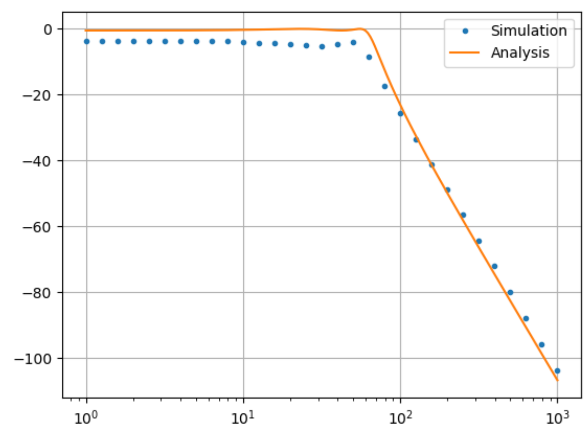

Fig. 5.4: Simulation of Chebyshev filter.