Assignment-2

JARPULA BHANU PRASAD - AI21BTECH11015

Download codes from:
Python code - python.
LaTeX code - LaTeX.

Contents

1 Problem-Oppenheim 2.10-b

1.1 Determine the output of a linear time-invariant system if the impulsive response $h[n]$ and the input $x[n]$ are as follows:

$$x[n] = u[n-4] \qquad \text{and} \qquad h[n] = 2^n u[-n-1].$$

2 Solution

2.1 **Solution:**
The output of linear time-invariant system is given by

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad (2.1)$$

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} x(k)h(n-k) && (2.2) \\ &= \sum_{k=-\infty}^{\infty} u[k-4]2^{n-k}u[-n+k-1] && (2.3) \\ &= \sum_{k=-\infty}^{\infty} 2^{n-k}u[k-4]u[k-(n+1)] && (2.4) \end{aligned}$$

Now we can define eqn(2.1) as

$$y(n) = \begin{cases} \sum\limits_{k=n+1}^{\infty} 2^{n-k} & n \geq 3 \\ \\ \sum\limits_{k=4}^{\infty} 2^{n-k} & n \leq 2 \end{cases}$$

case 1: $n \geq 3$

$$\begin{aligned} y(n) &= 2^n \sum_{k=n+1}^{\infty} 2^{-k} \\ &= 2^n \times \frac{2^{-(n+1)}}{1-2^{-1}} \\ &= 2^n \times \left(2 \times 2^{-(n+1)}\right) \\ y(n) &= 1 && (2.5) \end{aligned}$$

case 1: $n \leq 2$

$$\begin{aligned} y(n) &= 2^n \sum_{k=4}^{\infty} 2^{-k} \\ &= 2^n \times \frac{2^{-4}}{1-2^{-1}} \\ &= 2^n \times \left(2 \times 2^{-4}\right) \\ y(n) &= 2^{(n-3)} && (2.6) \end{aligned}$$

$$\therefore y(n) = \begin{cases} 1 & n \geq 3 \\ 2^{(n-3)} & n \leq 2 \end{cases} \quad (2.7)$$