Random Numbers

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Contents

***Abstract*—This manual provides a simple introduction to digital signal processing.**

1 Software installation

1.1 Run the following commands.

```
sudo apt−get update
sudo apt−get install libffi−dev libsndfile1
    python3−scipy python3−numpy python3−
    matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/soundfiles/
    Sound_Noise.wav
```

2.2 You will find a spectogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in problem 2.1 in the spectrogram and play. Observe the spectogram. What do you find?
Solution: There are a lot of yellow lines betweeen 440Hz to 5.1KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band nosie and execute the code. **Solution:** Download and run the following code.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/2.3_noise.
    py
```

run the above code using the command

```
python3 2.3_noise.py
```

2.4 The output of the python scripy in problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectogram in problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also the signal is blank for frequencies above 5.1KHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{3.1}$$

Sketch x(n).
Solution: Download and run the following code.Below code plots fig(3.1)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/3.1.py
```

run the above code using the command

```
python3 3.1.py
```

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2), y(n) = 0, n < 0 \tag{3.2}$$

Sketech y(n).
Solution: Download and run the following code.Below code plots fig(3.2)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/3.2.py
```

run the above code using the command

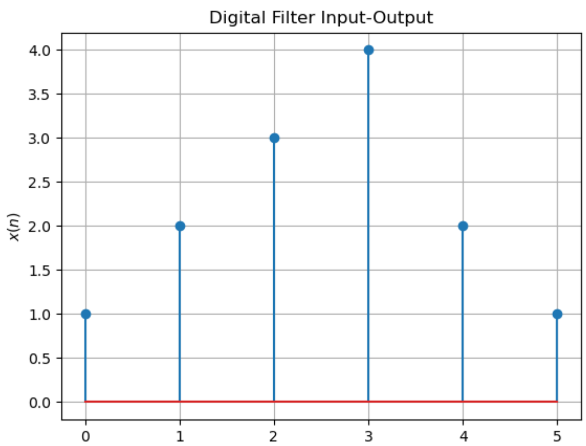

Fig. 3.1: Sketch of $x(n)$

```
python3 3.2.py
```

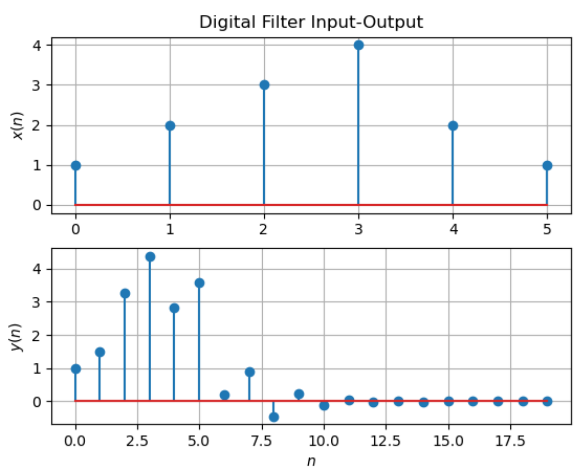

Fig. 3.2: Sketch of $x(n)$ and $y(n)$

4 Z-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: From (4.1)

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.5}$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{4.6}$$

4.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.7}$$

from (3.2) assuming that the Z-transform is a linear operation.

Solution: Applying (4.6) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.8}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{4.9}$$

4.3 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.10}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.11}$$

is

$$U(z) = \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.12}$$

Solution: It is easy to show that

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{4.13}$$

and from (4.11),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.14}$$

$$= \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.15}$$

using the fomula for the sum of an infinite geometric progression.

4.4 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1-az^{-1}} \quad |z| > |a| \tag{4.16}$$

Solution:

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \sum_{n=0}^{\infty} (az^{-1})^n \tag{4.17}$$

$$= \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.18}$$

4.5 Let

$$H\left(e^{\jmath\omega}\right) = H\left(z = e^{\jmath\omega}\right). \tag{4.19}$$

Plot $\left|H\left(e^{\jmath\omega}\right)\right|$. Comment. $H(e^{\jmath\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$.
Solution: Download and run the following code. The following code plots Fig. 4.5.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/4.5.py
```

run the above code using the command

```
python3 4.5.py
```

We observe that $\left|H\left(e^{\jmath\omega}\right)\right|$ is periodic with fundamental period 2π.

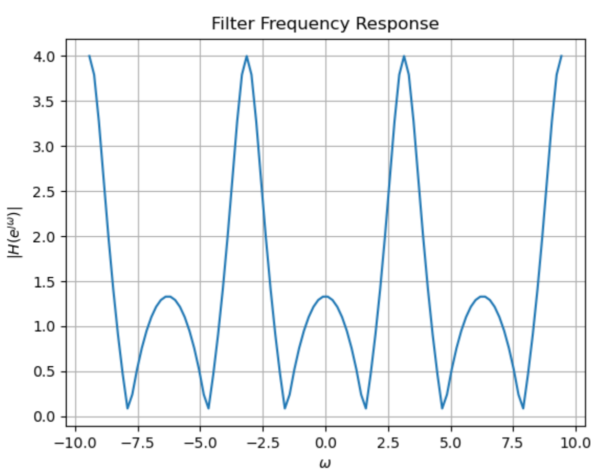

Fig. 4.5: $\left|H\left(e^{\jmath\omega}\right)\right|$

5 Impulse Response

5.1 Find an expression for $h(n)$ using $H(z)$, given that

$$h(n) \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{5.1}$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined by (3.2).
Solution: From (4.9),

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{5.2}$$

$$\implies h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{5.3}$$

using (4.16) and (4.6).

5.2 Sketch $h(n)$. Is it bounded? Convergent?
Solution: Download and run the following code.The following code plots Fig. 5.2.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.2.py
```

run the above code using the command.

```
python3 5.2.py
```

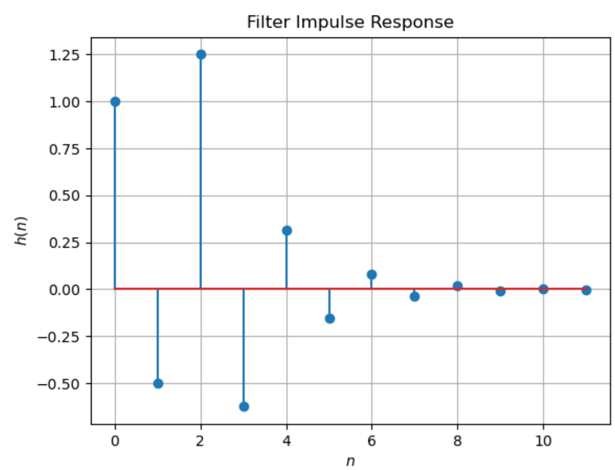

Fig. 5.2: $h(n)$ as the inverse of $H(z)$

$h(n)$ is bounded and convergent.

5.3 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \tag{5.4}$$

Is the system defined by (3.2) stable for the impulse response in (5.1)?
Solution: Note that

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{5.5}$$

$$= 2\left(\frac{1}{1 + \frac{1}{2}}\right) = \frac{4}{3} \tag{5.6}$$

Thus, the given system is stable.

5.4 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \tag{5.7}$$

This is the definition of $h(n)$.

Solution: The following code plots Fig. 5.4. Note that this is the same as Fig. 5.2.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/5.4.py
```

run the above code using the command.

```
python3 5.4.py
```

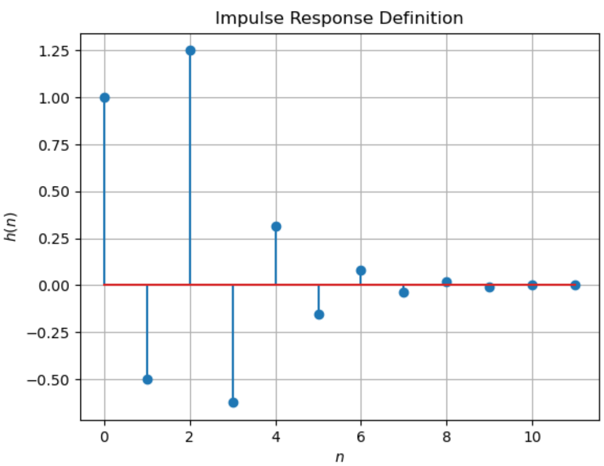

Fig. 5.4: $h(n)$ from the definition

5.5 Compute

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \tag{5.8}$$

Comment. The operation in (5.8) is known as *convolution*.

Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/5.5.py
```

run the above code using the command.

```
python3 5.5.py
```

5.6 Show that

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-k)h(k) \tag{5.9}$$

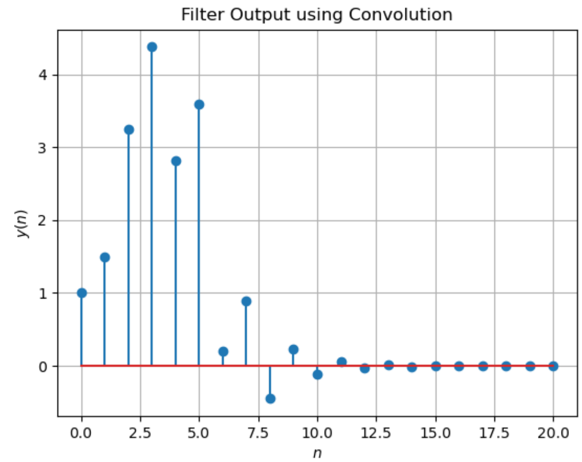

Fig. 5.5: $y(n)$ from the definition of convolution

Solution: from 5.8, we substitute $k := n - k$ to get

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.10}$$

$$= \sum_{n-k=-\infty}^{\infty} x(n-k)h(k) \tag{5.11}$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k) \tag{5.12}$$

6 DFT and FFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-\jmath 2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{6.1}$$

and $H(k)$ using $h(n)$.

Solution: The following code plots Fig. 6.1.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/6.1.py
```

run the above code using the command.

```
python3 6.1.py
```

6.2 Compute

$$Y(k) = X(k)H(k) \tag{6.2}$$

Solution: Download and run the following code.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/6.2.py
```

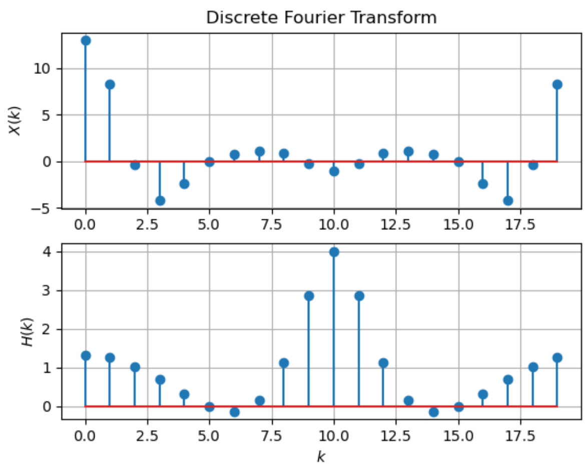

Fig. 6.1: Plots of the real parts of the DFT of $x(n)$ and $h(n)$

run the above code using the command.

```
python3 6.2.py
```

6.3 Compute

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\cdot e^{\jmath 2\pi kn/N}, \quad n = 0, 1, \dots, N-1 \tag{6.3}$$

Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.3.py
```

run the above code using the command.

```
python3 6.3.py
```

6.4 Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT. **Solution:** Download the code from

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.4.py
%
```

and execute it using

```
$ python3 6_4.py
```

Observe that Fig. (6.4) is the same as $y(n)$ in Fig. (3.2).

6.5 Wherever possible, express all the above equations as matrix equations.

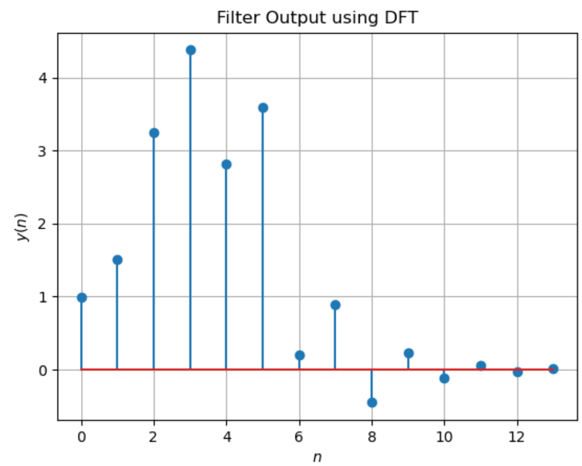

Fig. 6.3: $y(n)$ from the DFT

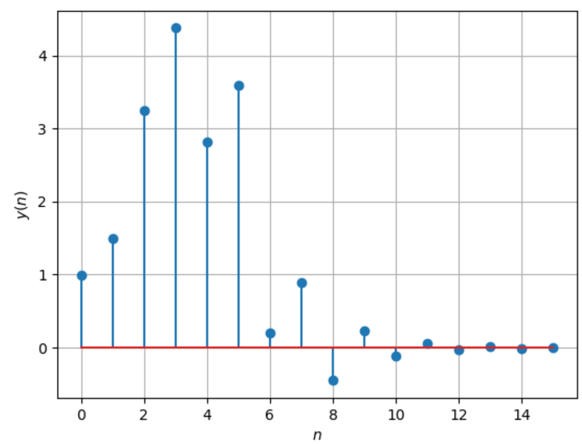

Fig. 6.4: $y(n)$ using FFT and IFFT

Solution: We use the DFT Matrix, where $\omega = e^{-\frac{j2k\pi}{N}}$, which is given by

$$\mathbf{W} = \begin{pmatrix} \omega^0 & \omega^0 & \dots & \omega^0 \\ \omega^0 & \omega^1 & \dots & \omega^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{N-1} & \dots & \omega^{(N-1)(N-1)} \end{pmatrix} \tag{6.4}$$

i.e. $W_{jk} = \omega^{jk}$, $0 \leq j, k < N$. Hence, we can write any DFT equation as

$$\mathbf{X} = \mathbf{W}\mathbf{x} = \mathbf{x}\mathbf{W} \tag{6.5}$$

where

$$\mathbf{x} = \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(n-1) \end{pmatrix} \tag{6.6}$$

Using (6.3), the inverse Fourier Transform is given by

$$\mathbf{x} = \mathcal{F}^{-1}(\mathbf{X}) = \mathbf{W}^{-1}\mathbf{X} = \frac{1}{N}\mathbf{W}^{\mathbf{H}}\mathbf{X} = \frac{1}{N}\mathbf{X}\mathbf{W}^{\mathbf{H}} \tag{6.7}$$

$$\implies \mathbf{W}^{-1} = \frac{1}{N}\mathbf{W}^{\mathbf{H}} \tag{6.8}$$

where H denotes hermitian operator. We can rewrite (6.2) using the element-wise multiplication operator as

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} = (\mathbf{W}\mathbf{h}) \cdot (\mathbf{W}\mathbf{x}) \tag{6.9}$$