Circuits and Transforms

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\, dt \tag{1.2}$$

2 Laplace Transform

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1\, \mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2\, \mu C$.

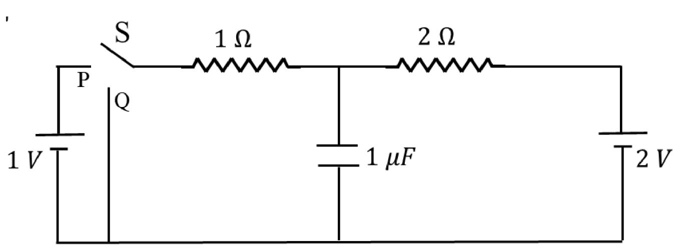

Fig. 2.1

2. Draw the circuit using latex-tikz.
Solution: The circuit drawn using the latex-tikz is Fig.(2.2)
When switch is closed at position P

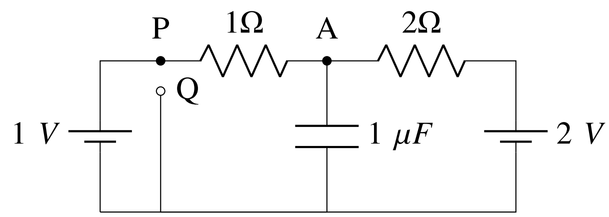

Fig. 2.2

3. Find q_1.
Solution: When the switch is closed for long time the circuit achives steady state condition. Then the equivalent circuit is given by

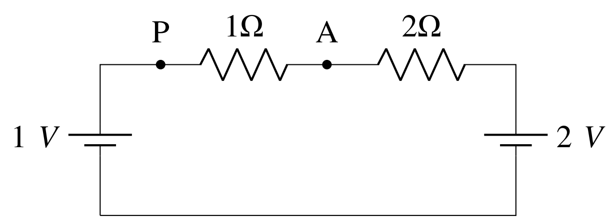

Fig. 2.3

consider the circuit as cells with internal resistors connected in series.

$$i = \frac{V}{R_{eq}} = \frac{1}{1+2} = \frac{1}{3}A \tag{2.1}$$

Now voltage across 2Ω resistro is given by

$$\implies \quad 2 - V_A = \frac{1}{3} \times 2 = \frac{2}{3} \tag{2.2}$$

$$V_A = 2 - \frac{2}{3} \tag{2.3}$$

$$= \frac{4}{3} \tag{2.4}$$

Now, charge

$$q_1 = C\Delta V \tag{2.5}$$

$$= 1 \times \frac{4}{3} \tag{2.6}$$

$$= \frac{4}{3} \tag{2.7}$$

charge on q_1 is $\frac{4}{3}\mu C$

4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
Solution:

$$\mathcal{L}[u(t)] = U(s) = \int_{-\infty}^{\infty} u(t)e^{-st}dt \tag{2.8}$$

$$U(s) = \int_{0}^{0} \frac{1}{2}e^{-st}dt + \int_{0}^{\infty} 1e^{-st}dt \tag{2.9}$$

$$= -\frac{1}{s}\left|e^{-st}\right|_{0}^{\infty} \tag{2.10}$$

$$= -\frac{1}{s}\left[e^{-s\infty} - e^{-s\times 0}\right] \tag{2.11}$$

The above term converges if and only if $e^{-st} \to 0$ as $t \to \infty$ only possible if $Re\{s\} > 0$.
hence

$$U(s) = \frac{1}{s} \qquad Re\{s\} > 0 \tag{2.12}$$

5. Show that

$$e^{-at}u(t) \overset{\mathcal{H}}{\longleftrightarrow} L\frac{1}{s+a}, \quad a > 0 \tag{2.13}$$

and find the ROC.
Solution: Let $x(t) = e^{-at}u(t)$

$$\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt \tag{2.14}$$

$$= \int_{-\infty}^{\infty} e^{-(s+a)t}u(t)dt \tag{2.15}$$

$$X(s) = \int_{0}^{0} \frac{1}{2}e^{-(s+a)t}dt + \int_{0}^{\infty} 1e^{-(s+a)t}dt \tag{2.16}$$

$$= -\frac{1}{s+a}\left|e^{-(s+a)t}\right|_{0}^{\infty} \tag{2.17}$$

$$= -\frac{1}{s+a}\left[e^{-(s+a)\infty} - e^{-(s+a)\times 0}\right] \tag{2.18}$$

The above term converges if and only if $e^{-(s+a)t} \to 0$ as $t \to \infty$ only possible if $Re\{s\} > -a$.
hence

$$X(s) = \frac{1}{s+a} \qquad Re\{s\} > -a \tag{2.19}$$

6. Now consider the following resistive circuit transformed from Fig. 2.1 where

$$u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_1(s) \tag{2.20}$$

$$2u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_2(s) \tag{2.21}$$

Find the voltage across the capacitor $V_{C_0}(s)$.

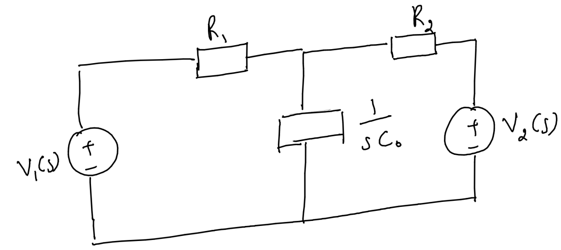

Fig. 2.4

Solution: We see that

$$V_1(s) = \frac{1}{s} \qquad V_2(s) = \frac{2}{s} \tag{2.22}$$

In Fig. 2.2, we use KCL at A.

$$\frac{V - \frac{1}{s}}{R_1} + \frac{V - \frac{2}{s}}{R_2} + sC_0V = 0 \tag{2.23}$$

$$V\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_0\right) = \frac{1}{s}\left(\frac{1}{R_1} + \frac{2}{R_2}\right) \tag{2.24}$$

$$V(s) = \frac{\frac{1}{R_1} + \frac{2}{R_2}}{s\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_0\right)} \tag{2.25}$$

$$= \frac{\frac{1}{R_1} + \frac{2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}\left(\frac{1}{s} - \frac{1}{\frac{1}{C_0}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + s}\right) \tag{2.26}$$

7. Find $v_{C_0}(t)$. Plot using python.
Solution: Taking the inverse Laplace transform in (2.26),

$$V(s) \overset{\mathcal{H}}{\longleftrightarrow} L\frac{2R_1 + R_2}{R_1 + R_2}u(t)\left(1 - e^{-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{t}{C_0}}\right) \tag{2.27}$$

$$= \frac{4}{3}\left(1 - e^{-\left(1.5\times 10^{6}\right)t}\right)u(t) \tag{2.28}$$

The following code will plot the graph Fig.(2.5)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Cktsig/codes/2.7.py
```

run the above code using the command

```
python3 2.7.py
```

8. Verify your result using ngspice.
Solution: The follwoing code will generate the text file of stimulation.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Cktsig/codes/2.8.cir
```

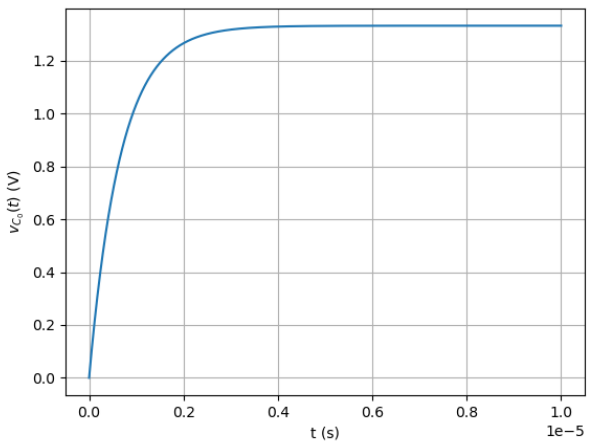

Fig. 2.5: $v_{C_0}(t)$ before the switch is flipped

The following code will plot the graph Fig.(2.6)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Cktsig/codes/2.8.py
```

run the above code using the command

```
python3 2.8.py
```

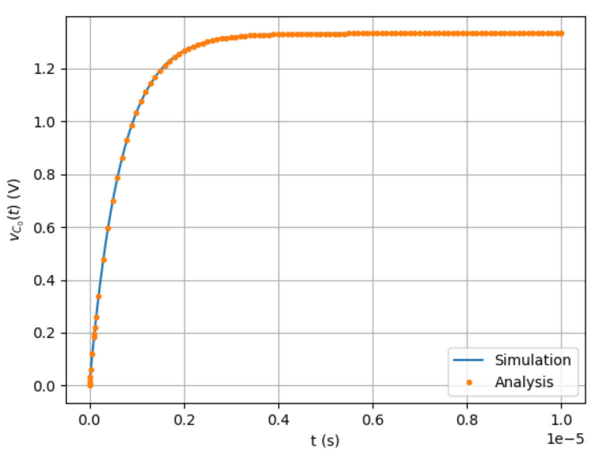

Fig. 2.6: $v_{C_0}(t)$ before the switch is flipped

9. Obtain Fig. 2.4 using the equivalent differential equation.

3 Initial Conditions

1. Find q_2 in Fig. 2.1.
Solution: When the switch is closed for long time the circuit achives steady state condition.

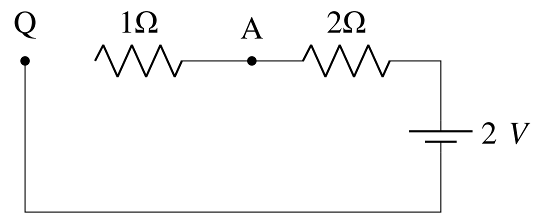

Fig. 3.1

Then the equivalent circuit is given by Since capacitor behaves as an open circuit, we use KCL at A.

$$\frac{V-0}{1}+\frac{V-2}{2}=0 \implies V=\frac{2}{3}V \tag{3.1}$$

and hence, $q_2 = \frac{2}{3}\mu C$.

2. Draw the equivalent s-domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements. Use latex-tikz.
Solution:

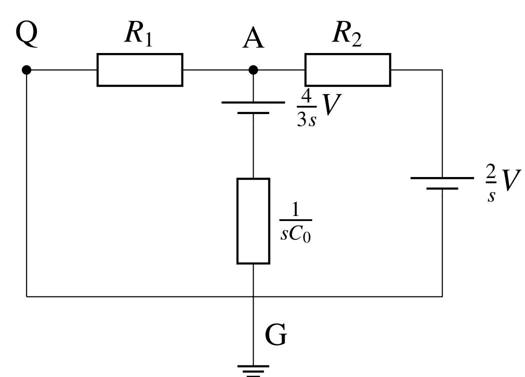

Fig. 3.2

3. $V_{C_0}(s) = ?$
Solution: Using KCL at node A in Fig. 3.2

$$\frac{V-0}{R_1}+\frac{V-\frac{2}{s}}{R_2}+sC_0\left(V-\frac{4}{3s}\right)=0 \tag{3.2}$$

$$\implies V_{C_0}(s)=\frac{\frac{2}{sR_2}+\frac{4C_0}{3}}{\frac{1}{R_1}+\frac{2}{R_2}+sC_0} \tag{3.3}$$

4. $v_{C_0}(t) = ?$ Plot using python.

Solution: From (3.3),

$$V_{C_0}(s) = \frac{4}{3}\left(\frac{1}{\frac{1}{C_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+s}\right) + \frac{2}{R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\left(\frac{1}{s} - \frac{1}{\frac{1}{C_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+s}\right) \tag{3.4}$$

Taking an inverse Laplace Transform,

$$v_{C_0}(t) = \frac{4}{3}e^{-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{t}{C_0}}u(t) + \frac{2}{R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\left(1 - e^{-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{t}{C_0}}\right)u(t) \tag{3.5}$$

Substituting values gives

$$v_{C_0}(t) = \frac{2}{3}\left(1 + e^{-\left(1.5\times10^6\right)t}\right)u(t) \tag{3.6}$$

The following code will plot the graph Fig.(3.3)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Cktsig/codes/3.4.py
```

run the above code using the command

```
python3 3.4.py
```

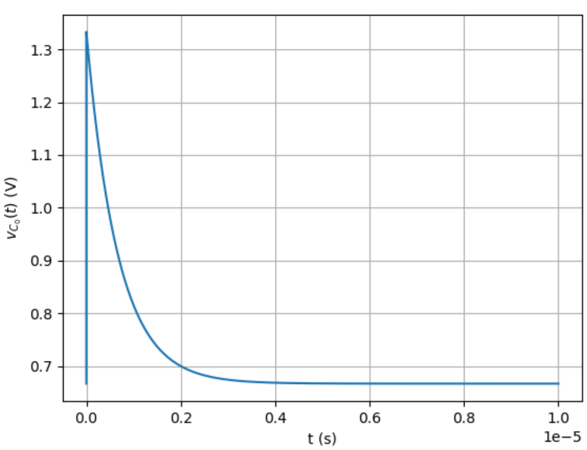

Fig. 3.3: $v_{C_0}(t)$ after the switch is flipped

5. Verify your result using ngspice.
Solution: The follwoing code will generate the text file of stimulation.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Cktsig/codes/3.5.cir
```

The following code will plot the graph Fig.(3.4)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Cktsig/codes/3.5.py
```

run the above code using the command

```
python3 3.5.py
```

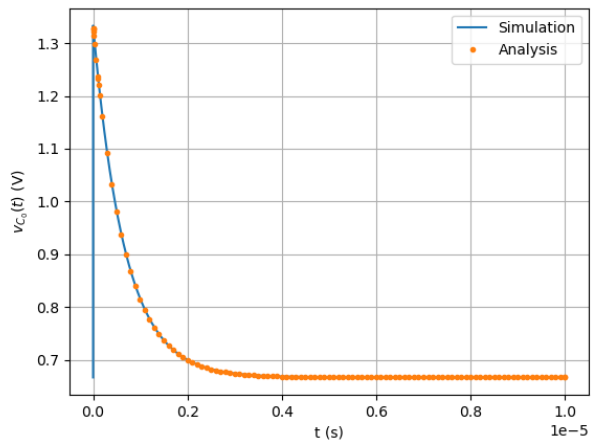

Fig. 3.4: $v_{C_0}(t)$ after the switch is flipped

6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.
Solution: From the initial conditions,

$$v_{C_0}(0-) = \frac{q_1}{C} = \frac{4}{3}V \tag{3.7}$$

$$v_{C_0}(0+) = \lim_{t\to0+} v_{C_0}(t) = \frac{4}{3}V \tag{3.8}$$

Using (3.6),

$$v_{C_0}(\infty) = \lim_{t\to\infty} v_{C_0}(t) = \frac{2}{3}V \tag{3.9}$$

7. Obtain the Fig. in problem 3.2 using the equivalent differential equation.