Jarrod Wilcox

# DEVELOPING PRACTICAL INVESTMENT RESILIENCE

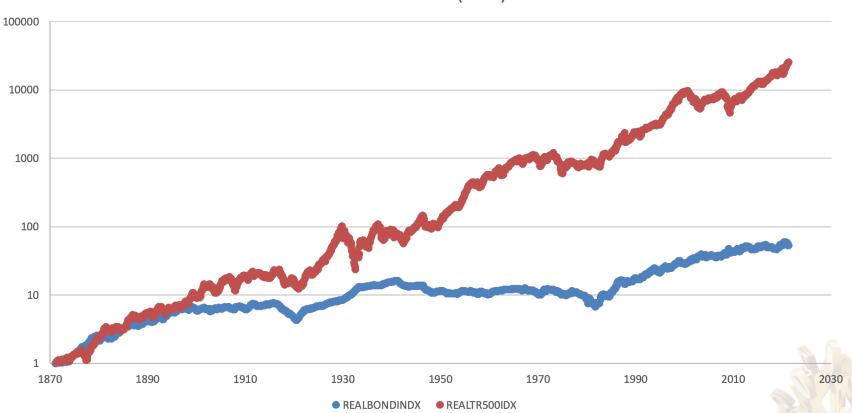
JULY 20, 2021

RESOURCE: GITHUB.COM/JARRODWILCOX/RESILIENCE\_LAB.



# DIFFERENT RETURN CHARACTERISTICS AT DIFFERENT SCALES

REAL TOTAL RETURN INDEXES Source: Shiller (2021)



# WHY STUDY RESILIENT INVESTING?

### What we face:

- Markets are never at more than momentary equilibrium.
- Returns are not i.i.d. Parameters are uncertain.
- Investors differ more widely than we assume.
- Continuous time finance is an heroic assumption.
- Market disruptions awaken us to our deeper need for better asset allocation methods.

## GOING BACK TO EXPECTED UTILITY

- A single period decision...
- Responding to multiple future time points,
- Multiple return probability regimes,
- And investors with widely different needs.

Investor's		Investment					
Probability of				Trial			
Consequent Utility Need		Probability Distributions		Allocation			
otime, recu		DISTIBUTIONS					
		Pa1 Outcome	Χ	Utility a1			
Pa	{	Pa2 Outcome	Χ	Utility a2			
		Pa3 Outcome	Χ	Utility a3			
		Pb1 Outcome	Χ	Utility b1			
Pb	{	Pb2 Outcome	Χ	Utility b2			
	·	Pb3 Outcome	Χ	Utility b3			
		Pz1 Outcome	Χ	Utility z1			
Pz	{	Pz2 Outcome	Χ	Utility z2			
	ι	Pz2 Outcome	Χ	Utility z3			
		Expected Utility					

<u>ID</u>	NAME	<b>CLUSTER</b>
EFA	iShares MSCI EAFE ETF	C11111
EZU	iShares MSCI Eurozone ETF	C11111
EST_EEM	iShares MSCI Emerging Markets ETF	C11112
EWJ	iShares MSCI Japan ETF	C1112
IWR	iShares Russell Mid-Cap ETF	C11211111
IWS	iShares Russell Mid-Cap Value ETF	C11211111
IWP	iShares Russell Mid-Cap Growth ETF	C11211112
SPY	SPDR S&P 500 ETF Trust	C1121112
XLK	Technology Select Sector SPDR Fund	C112112
XLE	Energy Select Sector SPDR Fund	C11212
VWEHX	Vanguard High-Yield Corporate Fund	C1122
IYR	iShares U.S. Real Estate ETF	C1122
XLP	Consumer Staples Select Sector SPDR Fund	C1211
DIA	SPDR Dow Jones Industrial Average ETF Trust	C1212
XLY	Consumer Discretionary Select Sector SPDR Fund	C1212
IBB	iShares Nasdaq Biotechnology ETF	C122
XLV	Health Care Select Sector SPDR Fund	C122
IEF	iShares 7-10 Year Treasury Bond ETF	C21111
VFITX	Vanguard Intermediate-Term Treasury Fund	C21111
VFIIX	Vanguard GNMA Fund Investor Shares	C21112
VUSTX	Vanguard Long-Term Treasury Fund	C21121
TLT	iShares 20+ Year Treasury Bond ETF	C21121
VWESX	Vanguard Long-Term Investment-Grade Fund	C21122
EST_GLD	SPDR Gold Shares	C212
VWSTX	Vanguard Short-Term Tax-Exempt Fund	C2211
VWAHX	Vanguard High-Yield Tax-Exempt Fund	C2212
VWLTX	Vanguard Long-Term Tax-Exempt Fund	C2212
LQD	iShares iBoxx \$ Investment Grade Corporate Bond ETF	C222

### THE SIMPLEST BAYESIAN POSTERIOR

(real returns would have been an improvement)

	<u>N</u>	<u>MEAN</u>	STD DEV
VFINX (stocks)	540	0.99%	4.49%
VWESX (bonds)	540	0.53%	2.52%
VWSTX (cash)	540	0.39%	0.32%
PGOLD (gold)	276	-0.01%	5.10%

Let N be the number of observations, and V their variance, with subscripts *h* signifying the prior history and *o* signifying the new observations.

$$P_h = N_h / V_h$$
,  $P_o = N_o / V_o$  (the precisions)  
 $W_h = P_h / (P_h + P_o)$ ,  $W_o = 1 - W_h$  (the weights)

For a conjugate normal with known variance approximation, the shrinkage mean  $M_S$  is calculated as:  $M_S = W_h M_h + W_o M_o$ 



# A Little Utility Math

- Mean-variance:  $MV = \mu L \sigma^2/2$ 
  - Ensemble statistics  $\mu$ ,  $\sigma^2$
- Rubinstein utility: GLUM=In(1+Lr)
  - Each outcome portfolio return r has a well-defined utility
- Let Q =  $L\sigma/(1+L\mu)$ , then:
- Expected GLUM =  $ln(1+L\mu) Q^2/2 + SQ^3/3 KQ^4/4 + ...$
- Limit  $\Delta t \rightarrow 0$  of Expected GLUM = L (MV)

# WHERE IS SIMPLEST TAIL RISK?

- A. Return Distribution Skew and Kurtosis
- **B.** Utility Function
- C. Risk Aversion

In this case:

Utility = ln(1+Lr)

L is risk aversion r is investment return

The rightmost 3 columns represent change in utility from skewness and kurtosis.

MONTHLY RETURN						TAIL RISK	BY RISK A	VERSION
<u>ID</u>	<u>MEAN</u>	<u>SMEAN</u>	STD DEV	<u>SKEW</u>	<u>KURT</u>	<u>L:2</u>	<u>L:4</u>	<u>L:8</u>
EFA	0.73%	0.93%	4.97%	-0.54	4.8	0.000	-0.003	-0.032
EZU	0.76%	0.95%	6.23%	-0.38	4.6	0.000	-0.006	-0.066
EST_EEM	1.06%	1.00%	6.28%	-0.34	4.3	0.000	-0.005	-0.062
EWJ	0.63%	0.89%	4.57%	-0.32	3.5	0.000	-0.001	-0.017
IWR	1.07%	1.01%	4.85%	-0.81	6.7	0.000	-0.004	-0.040
IWS	0.99%	0.99%	4.96%	-0.98	8.0	0.000	-0.005	-0.053
IWP	1.16%	1.03%	4.93%	-0.65	5.8	0.000	-0.003	-0.036
SPY	0.93%	0.97%	4.16%	-0.56	4.8	0.000	-0.002	-0.017
XLK	1.26%	1.06%	4.90%	-0.36	3.4	0.000	-0.002	-0.020
XLE	0.75%	0.96%	7.31%	-0.16	7.6	-0.001	-0.013	-0.178
VWEHX	0.57%	0.74%	2.29%	-1.82	16.3	0.000	-0.001	-0.007
IYR	0.93%	0.98%	6.20%	-0.61	9.5	-0.001	-0.011	-0.127
XLP	0.82%	0.92%	3.26%	-0.61	4.1	0.000	-0.001	-0.007
DIA	0.89%	0.96%	4.00%	-0.56	4.6	0.000	-0.001	-0.014
XLY	1.16%	1.03%	5.07%	-0.10	4.9	0.000	-0.002	-0.026
IBB	1.26%	1.04%	5.92%	-0.21	3.7	0.000	-0.003	-0.040
XLV	0.90%	0.96%	3.84%	-0.31	3.7	0.000	-0.001	-0.009
IEF	0.42%	0.48%	1.82%	0.25	4.3	0.000	0.000	0.000
VFITX	0.24%	0.36%	1.37%	-0.44	5.9	0.000	0.000	0.000
VFIIX	0.29%	0.34%	0.81%	-0.38	6.5	0.000	0.000	0.000
VUSTX	0.43%	0.51%	3.34%	0.36	4.1	0.000	0.000	-0.002
TLT	0.61%	0.54%	3.77%	0.46	4.9	0.000	0.000	-0.004
VWESX	0.55%	0.53%	2.78%	0.07	5.3	0.000	0.000	-0.002
EST_GLD	0.88%	0.40%	4.92%	-0.02	3.2	0.000	-0.001	-0.015
VWSTX	0.14%	0.28%	0.22%	0.37	6.8	0.000	0.000	0.000
VWAHX	0.42%	0.47%	1.45%	-1.06	7.2	0.000	0.000	-0.001
VWLTX	0.38%	0.44%	1.39%	-0.92	6.0	0.000	0.000	-0.001
LQD	0.49%	0.52%	2.13%	0.08	11.9	0.000	0.000	-0.002

# CLUSTERS TO IDENTIFY STRUCTURE

- Top-Down
   Hierarchical
   Clusters,
   McQuitty &
   Clark 1968.
- Identify
   Structure-Based
   Modeling
   Potential.

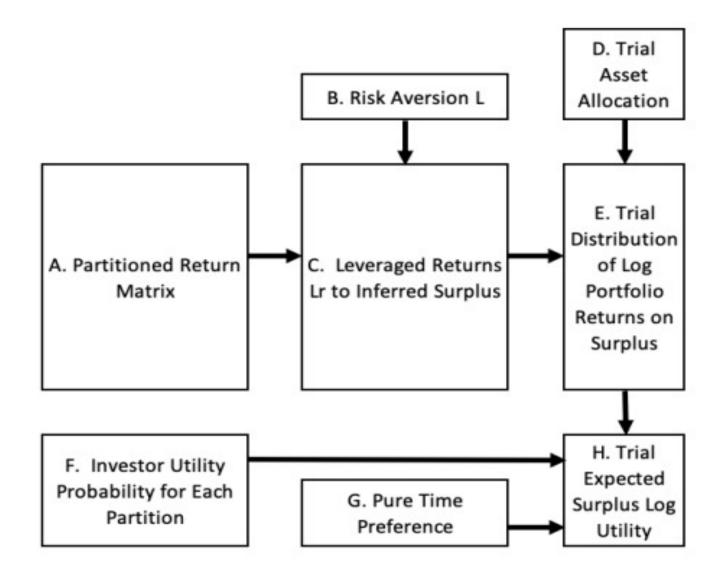
HISTORY	1-MONTH	CI	RISIS	6-MONTH		
<u>ID</u>	CLUSTER	<u>ID</u>	CLUSTER	<u>ID</u>	CLUSTER	
EFA	C11111	EFA	C1111111	EFA	C111111	
EZU	C11111	EZU	C1111111	EZU	C111111	
EST_EEM	C11112	EWJ	C1111112	EST_EEM	C111112	
EWJ	C1112	XLP	C1111121	EWJ	C11112	
IWR	C11211111	SPY	C1111122	XLE	C1112	
IWS	C11211111	DIA	C1111122	IWR	C11211	
IWP	C11211112	IWS	C1111211	IWS	C11211	
SPY	C1121112	IWR	C1111211	IWP	C11212	
XLK	C112112	XLY	C1111212	VWEHX	C1122	
XLE	C11212	IYR	C111122	IYR	C1122	
VWEHX	C1122	XLE	C11121	XLP	C1211	
IYR	C1122	IWP	C1112211	XLY	C121211	
XLP	C1211	XLK	C1112211	SPY	C121212	
DIA	C1212	EST_EEM	C1112212	DIA	C121212	
XLY	C1212	VWEHX	C111222	XLK	C12122	
IBB	C122	XLV	C112	IBB	C122	
XLV	C122	IBB	C112	XLV	C122	
IEF	C21111	VWAHX	C12	IEF	C21111	
VFITX	C21111	VWLTX	C12	TLT	C21112	
VFIIX	C21112	IEF	C21111	VUSTX	C21112	
VUSTX	C21121	VUSTX	C21112	VFIIX	C2112	
TLT	C21121	TLT	C21112	VFITX	C2112	
VWESX	C21122	VFIIX	C2112	EST_GLD	C212	
_EST_GLD	C212	VFITX	C2112	VWSTX	C212	
VWSTX	C2211	VWESX	C212	VWAHX	C221	
VWAHX	C2212	_EST_GLD	C212	VWLTX	C221	
VWLTX	C2212	LQD	C22	LQD	C222	
LQD	C222	VWSTX	C22	VWESX	C222	

## THE INVESTOR JOINS THE DECISION TREE

(But how should valuation probabilities be set?)

						LOG	
					VALUATION	TIME	
	<u>Date</u>	<u>EFA</u>	<u>IEF</u>		<u>PROBABILITY</u>	<u>DISCOUNT</u>	
History Fragment	Sep-20	-1.84%	0.40%	•••			
	Oct-20	-3.35%	-1.31%	•••			
	Nov-20	14.47%	0.40%		54%	-0.0025	
		•••	•••				
Crisis Fragment	Jul-07	-0.63%	2.26%	•••			
	Aug-07	5.32%	0.08%	•••			
	Sep-07	4.25%	1.10%		6%	-0.0025	
			•••				
6 Month Fragment	Sep-20	22.19%	1.29%	•••			
	Oct-20	11.39%	-0.38%				
	Nov-20	20.72%	-0.38%		40%	-0.0148	
		•••	•••				

## SEARCH FOR BEST ALLOCATION



# DISRUPTION CREATES MORE HIGH MOMENT THREAT

Mixing pdf's increases tail risk.

 This improves the benefit of Rubinstein utility.

**Utility Difference:** 

### 0.8 MONTHLY HISTORY, 0.2 CRISIS

### MEAN-VARIANCE ALLOCATION

	<u>L:1</u>	<u>L:2</u>	<u>L:4</u>	<u>L:8</u>	<u>L:16</u>
Utility:	0.0049	0.0105	0.0196	0.0345	NaN
Allocation Weights:					
IEF	0	0	0	41%	49%
VWSTX	0	0	0	0	5%
VWEHX	0	0	14%	21%	23%
XLP	38%	67%	51%	28%	16%
VWAHX	0	0	0	0	1%
IBB	57%	33%	18%	9%	4%
TLT	0	0	17%	0	0
XLK	6%	0	1%	1%	2%

### **EXPECTED SURPLUS GROWTH ALLOCATION**

	<u>L:1</u>	<u>L:2</u>	<u>L:4</u>	<u>L:8</u>	<u>L:16</u>
Utility:	0.0049	0.0108	0.0206	0.0371	0.0599
Allocation Weights:					
IEF	0	0	43%	61%	32%
VWSTX	0	0	0	0	9%
VWEHX	0	0	0	12%	5%
XLP	33%	39%	30%	15%	8%
VFIIX	0	0	0	0	36%
IBB	67%	38%	22%	12%	8%
VWLTX	0	0	0	0	2%
TLT	0	16%	0	0	0
EST_GLD	0	8%	5%	0	0

0.0003

0.0010

0.0026

+Inf

0.0000

# WHICH PORTFOLIO WOULD YOU CHOOSE TODAY?

- Base your answer just on holdings.
- If the investor has said she will have little turnover?
- Does the utility comparison tell you anything?

### **HISTORY, 1-MONTH AHEAD**

#### MEAN-VARIANCE ALLOCATION

	<u>L:1</u>	<u>L:2</u>	<u>L:4</u>	<u>L:8</u>	<u>L:16</u>
Utility:	0.007	0.0147	0.0269	0.0413	NaN
Allocation Weights:					
IEF	0	0	0	1%	44%
EWJ	0	0	1%	2%	2%
VWEHX	0	0	0	28%	26%
XLP	0	34%	44%	29%	15%
IBB	22%	12%	4%	1%	0
XLV	0	11%	19%	12%	6%
TLT	0	0	10%	19%	0
DIA	0	0	0	0	3%
XLK	79%	43%	23%	10%	5%

### 0.4 MONTHLY HISTORY, 0.2 CRISIS, 0.4 6-MONTH AHEAD

### **EXPECTED SURPLUS GROWTH ALLOCATION**

		=			
	<u>L:1</u>	<u>L:2</u>	<u>L:4</u>	<u>L:8</u>	<u>L:16</u>
Utility:	0.0175	0.0381	0.0702	0.1186	0.1863
Allocation Weigh	ts:				
IEF	0	0	0	50%	63%
VWSTX	0	0	0	0	4%
VWEHX	0	0	0	3%	4%
XLP	66%	85%	66%	37%	17%
IBB	7%	10%	11%	8%	7%
TLT	0	0	22%	0	0
XLK	27%	4%	2%	3%	5%
Utility Delta?:	0.011	0.023	0.043	0.077	+Inf

## RESILIENT INVESTORS CAN

- Trigger better tail-risk review with:
  - Return mean shrinkage to make history usable,
  - Quantify return higher moment threats to surplus,
  - Cluster analysis to reveal structure change and outliers.
- Manage tail risk better using return matrices,
   Rubinstein utility, and scenarios.
- Reflect future trends, reversals and drift vs noise with multiple time point evaluations.