Assignment 7
AI1110: Probability and Random Variables

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Question: The random variable X has the Erlang density $f(x)$ $c^4.x^3.e^{-cx}$. We observe the samples $X_i = 3.1, 3.4.3.3$ Find the ML estimate c.

Solution: Lets us generalize and find ML.Lets take for n random values the p.d.f will be

$$f(x,c) = c^4.x^3.e^{-cx} \tag{1}$$

$$f(x_1, x_2, x_3, ..x_n, c) = c^{4n}.(x_1..x_n)^3.e^{-nc\hat{x}} \tag{2}$$

Where $\hat{x}$ is the mean of random variable X.
Now to find ML of this function partially differentiate it w.r.t c.

$$\frac{\partial f(X,c)}{\partial c} = \frac{\partial c^{4n}.(x_1..x_n)^3.e^{-n^c\hat{x}}}{\partial c} \tag{3}$$

$$\frac{\partial f(X,c)}{\partial c} = n.c^{4n-1}.x^3e^{-cn\hat{x}}(4 - c\hat{x}) \tag{4}$$

for ML equate partial differentiation to zero and that value is the estimate of c,

$$n.c^{4n-1}.x^3e^{-cn\hat{x}}(4 - c\hat{x}) = 0 \tag{5}$$

$$4 - c\hat{x} = 0 \tag{6}$$

$$c = \frac{4}{\hat{x}} \tag{7}$$

given that,$X_i = 3.1, 3.4.3.3,$

$$\hat{x} = \frac{3.1 + 3.3 + 3.4}{3} \tag{8}$$

$$\hat{x} = 3.27 \tag{9}$$

Now,We can Assume that

$$c = \frac{4}{\hat{x}} \tag{10}$$

$$c = \frac{4}{3.27} \tag{11}$$

$$c = 1.224. \tag{12}$$