

A Simple Problem On Definition of Cumulative Distribution Function

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Outline

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Question

Q15 [12th Papoulis Textbook Exercise 4]:

Show that, if $a \leq x(\zeta) < b$ for every $\zeta \in S$, then $F(x) = 1$ for $x > b$ and $F(x) = 0$ for $x < a$.

Solution

1) for $x > b$

$$F(x) = \Pr(\mathbf{x} \leq x) \quad (1)$$

$$x(\zeta) < b, \forall \zeta \quad (2)$$

$$\implies x(\zeta) < b < x, \forall \zeta \quad (3)$$

$$\implies \{\mathbf{x} \leq x\} = S \quad (4)$$

$$\implies F(x) = \Pr(S) = 1 \quad (5)$$

$\therefore F(x) = 1, \forall x > b.$

Solution

2) for $x < a$

$$F(x) = \Pr(\mathbf{x} \leq x) \quad (6)$$

$$a \leq x(\zeta), \forall \zeta \quad (7)$$

$$\implies x < a \leq x(\zeta), \forall \zeta \quad (8)$$

$$\implies \{\mathbf{x} < x\} = \phi \quad (9)$$

$$\implies F(x) = \Pr(\phi) = 0 \quad (10)$$

$\therefore F(x) = 0, \forall x < a.$