Multivariate Statistical Analysis

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1 Homework Assignment 3: Maximizing Variance

Let x denote a p-variate random vector with a finite mean vector μ and a finite full- rank covariance matrix Σ. Let $y_k = \gamma_k^T(x - \mu)$ denote the kth principal component of x. Let $b \in \mathbb{R}^p$ such that $b^T b = 1$. Assume that b^T is uncorrelated with first $k - 1$ principal components of x. Read lecture slides 2 carefully and give detailed proofs for the following.

1.1

$b = d_1\gamma_1 + ... + d_p\gamma_p$. Show that $d_i = 0$, when $i < k$

$b^T x = b^T \Sigma b = \sum_{i=1}^p d_i \gamma_i^T (\sum_{k=1}^p \lambda_k \gamma_k \gamma_k^T) \sum_{j=1}^p d_j \gamma_j$

$= \sum_{i=1}^p \lambda_i d_i^2$

$y_k = \gamma_k^T (x - \mu)$

$$\gamma_k = \frac{(x - \mu)}{y_k}$$

$$b = d_1 \frac{(x - \mu)}{y_1} + ... + d_p \frac{(x - \mu)}{y_p}$$

$var(\gamma_k) \geq var(b^T x) = \sum_{i=1}^p d_i \gamma_i^T (\sum_{k=1}^p \lambda_k \gamma_k \gamma_k^T) \sum_{j=1}^p d_j \gamma_j = \sum_{i=1}^p \lambda_i d_i^2$
Since $b^T b = 1$, $\sum_{i=1}^p d_i^2 = 1$.