BSc. (Hons.) COMPUTER SCIENCE

GE III MATHS PRACTICAL

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Solution of Differential Equation by Variation of Parameter Method

Second order Differential equation of type – (Non-homogeneous Linear ODE)

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = f(x)$$

Where $P(x)$, $Q(x)$ and $f(x)$ are functions of x.

Simpler Case – when $f(x) = 0$. (Homogeneous Linear ODE)

The equation – $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$.

We can find solution to these equation using methods of Homogeneous linear ODE with constant and non-constant coefficients.

Now, there are two main methods to solve equations like

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = f(x).$$

Method 1. – ~~the~~ Method of Undetermined Coefficients

It only works when $f(x)$ is a polynomial, exponential, sine, cosine, or a linear combination of those, basically standard functions or equations.

Method 2. – Method of Variation of Parameters.

Consider the equation–

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = f(x).$$

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The complete solution to such an equation can be found by combining two types of solution:

- The general solution of the homogeneous equation
$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$
- Particular solutions of the non-homogeneous equation
$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = f(x).$$

complete solution = general solution of homog. eqn. + Particular solution non-homog. eqn.

* **Command** - (1) **kill**:

(a) kill(a_1, ---, a_n)
(b) kill(labels)
(c) kill(n)
(d) kill(infolist)
(e) kill([m, n])
(f) kill(all)
(g) kill(allbut(a_1 --- a_n))
(h) kill(symbol)

(2) depends(f_1, x_1 --- f_n, x_n).

depends(f, [x-1, ..., x-n]) is equivalent to
dependencies(f(x-1, ---, x-n)).

(3) diff(expr, x_1, n_1, ---, x_m, n_m).

(a) diff(expr, x, n)
(b) diff(expr, x)
(c) diff(expr).

(%i2) diff(y);

(%o2) $\left(\frac{d}{dx} y\right)$ del(x).

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(i) Ode2: solves an ODE of first or 2^{nd} order.
It takes 3 arguments an ODE, the dependent variable and the independent variable.
It returns either an explicit or implicit sol^n for dependent variable. %C is used to represent the integration constant in case of 1st order eqn. & K1 and K2 the const. for 2^{nd} order eqn.
For instance ode2(eq, y, x);

ODE | Dependent variable | Independent variable.

(ii) second(expr): This command returns the 2nd item of expression or list.
ex- In $y = ax + c$ expression
Second command will ~~require~~ return the part 'ax+c' of this equation.

(iii) first(expr): Returns the first part of expr.
ex- In $y = ax + c$ expression
first command will return the part 'y' of this equation.

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matrix--> Returns a rectangular matrix which has the rows row_1, …, row_n. Each row is a list of expressions. All rows must be the same length.

determinant--> Computes the determinant of M by a method similar to Gaussian elimination.

m: matrix ([a,b] , [diff(a,x), diff(b,x)]);

w: determinant(m);

$y_1 = \cos x$, $y_2 = \sin x$, $\boxed{r(x) = \frac{1}{\cos x}}$

$r(x)$ is not a standard fun.

$\therefore$ Apply VOP.

Wronskian $= W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x = \boxed{1}$

$y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$

$= -\cos x \int \frac{\sin x \, dx}{\cos x} + \sin x \int dx$

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In the variation of parameter method,
the sol^n of Equation is combination of two parts,

i.e sol^n = $y_c + y_p$

$y_c \Rightarrow$ General sol^n of {Equation = 0}

$y_p \Rightarrow$ Particular sol^n of {Equation = $f(x)$}

The formula to find the particular sol^n. is :-

$$y_p = -y_1 \int \frac{y_2 \cdot f(x)}{W} \cdot dx + y_2 \int \frac{y_1 \cdot f(x)}{W} \cdot dx$$

Here, W = Determinant of Wronskian Matrix

y_1 & y_2 are basis of equation (i.e whose sol^n we have to find)