

BSc. (Hons.) COMPUTER SCIENCE

GE III MATHS PRACTICAL

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Solution of Differential Equation by Variation of Parameter Method

Second order Differential equation of type - (Non-homogeneous Linear ODE)

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = f(x)$$

Where $P(x)$, $Q(x)$ and $f(x)$ are functions of x .Simplest Case - when $f(x) = 0$. (Homogeneous Linear ODE)

$$\text{The equation - } \frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = 0$$

We can find solution to these equation using methods of Homogeneous linear ODE with constant and non-constant coefficients.

Now, there are two main methods to solve equations like

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = f(x)$$

Method 1 - The Method of Undetermined CoefficientsIt only works when $f(x)$ is a polynomial, exponential, sine, cosine, or a linear combination of those, basically standard functions or equations.Method 2 - Method of Variation of Parameters -

Consider the equation -

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = f(x)$$

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The complete solution to such an equation can be found by combining two types of solutions:

- The general solution of the homogeneous equation

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = 0$$

- Particular solutions of the non-homogeneous equation

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = f(x)$$

complete solution = general solution of homog. eqn. + particular solution of non-homog. eqn.

* Command - kill:-

- (a) kill(a_1, \dots, a_n)
- (b) kill(labels)
- (c) kill(n)
- (d) kill(infolist)
- (e) kill($[m, n]$)
- (f) kill(all)
- (g) kill(allbut(a_1, \dots, a_n))
- (h) kill(symbol)

(2) depends($f_1, x_1, \dots, f_m, x_m$).

depends($f, [x_1, \dots, x_m]$) is equivalent to
dependencies($f(x_1, \dots, x_m)$).

(3) diff(expr, $x_1, m_1, \dots, x_m, m_m$).

- (a) diff(expr, x, m)
- (b) diff(expr, x)
- (c) diff(expr).

(7.92) diff(y);

(7.02) $\left(\frac{d}{dx} y\right) del(x)$.

PYUSH DEEP

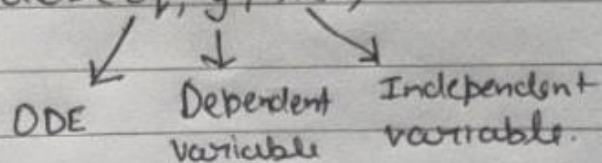
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(i) Ode2: solves an ODE of first or 2nd order.

It takes 3 arguments an ODE, the dependent variable and the independent variable.

It returns either an explicit or implicit solⁿ for dependent variable. %C is used to represent the integration constant in case of 1st order eqn. & K1 and K2 the const. for 2nd order eqn.

For instance `ode2(eq, y, x);`



(ii) Second(expr): This command returns the 2nd item of expression or list.

ex- In $y = ax + c$ expression

Second command will ~~require~~ return the part 'ax+c' of this equation.

(iii) first(expr): Returns the first part of expr.

ex- In $y = ax + c$ expression

first command will return the part 'y' of this equation.

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matrix--> Returns a rectangular matrix which has the rows row_1, ..., row_n. Each row is a list of expressions. All rows must be the same length.

determinant--> Computes the determinant of M by a method similar to Gaussian elimination.

m: matrix ([a,b] , [diff(a,x), diff(b,x)]);

w: determinant(m);

$y_1 = \cos x, y_2 = \sin x, r(x) = \frac{1}{\cos x}$
 $r(x)$ is not a standard fun.
 \therefore Apply VOP.
 Wronskian $w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$
 $= \cos^2 x + \sin^2 x = 1$
 $y_p = -y_1 \int \frac{y_2 r}{w} dx + y_2 \int \frac{y_1 r}{w} dx$
 $= -\cos x \int \frac{\sin x dx}{\cos x} + \sin x \int dx$

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In the variation of parameter method,
the solⁿ of Equation is combination of two parts,
i.e. $\text{sol}^n = y_c + y_p$
 $y_c \Rightarrow$ General solⁿ of {Equation = 0}
 $y_p \Rightarrow$ Particular solⁿ of {Equation = $f(x)$ }
The formula to find the particular solⁿ is:-
$$y_p = -y_1 \int \frac{y_2 f(x)}{W} dx + y_2 \int \frac{y_1 f(x)}{W} dx$$

Here, W = Determinant of Wronskian matrix
 y_1 & y_2 are basis of equation (i.e. whose solⁿ
we have to find)