Homework Set 3, CPSC 8420, Fall 2023

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Due 11/17/2023, Friday, 11:59PM EST

Problem 1

Considering soft margin SVM, where we have the objective and constraints as follows:

$$\min \frac{1}{2} ||w||_2^2 + C \sum_{i=1}^m \xi_i$$
s.t. $y_i(w^T x_i + b) \ge 1 - \xi_i \ (i = 1, 2, ...m)$

$$\xi_i \ge 0 \ (i = 1, 2, ...m)$$
(1)

Now we formulate another formulation as:

$$\min \frac{1}{2} ||w||_2^2 + \frac{C}{2} \sum_{i=1}^m \xi_i^2$$
s.t. $y_i(w^T x_i + b) \ge 1 - \xi_i \ (i = 1, 2, ...m)$

- 1. Different from Eq. (1), we now drop the non-negative constraint for ξ_i , please show that optimal value of the objective will be the same when ξ_i constraint is removed.
- 2. What's the generalized Lagrangian of the new soft margin SVM optimization problem?
- 3. Now please minimize the Lagrangian with respect to w, b, and ξ .
- 4. What is the dual of this version soft margin SVM optimization problem? (should be similar to Eq. (10) in the slides)

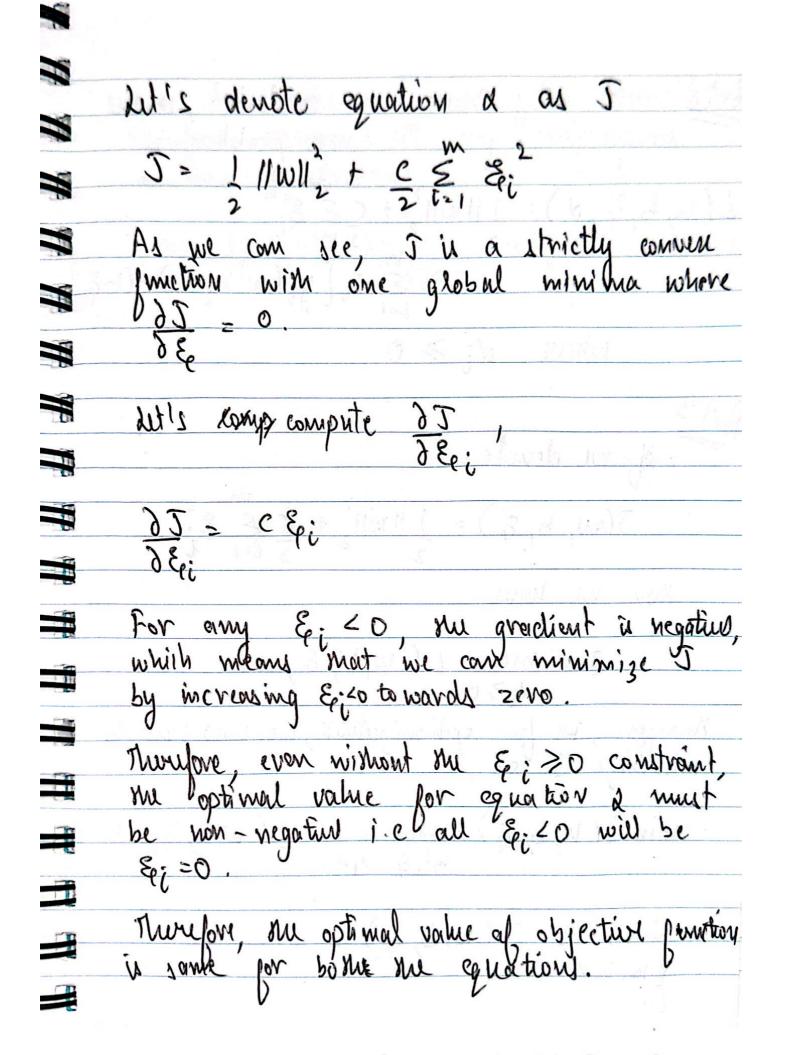
Problem 2

Recall vanilla SVM objective:

$$L(w, b, \alpha) = \frac{1}{2} ||w||_2^2 - \sum_{i=1}^m \alpha_i [y_i(w^T x_i + b) - 1] \quad s.t. \quad \alpha_i \ge 0$$
(3)

If we denote the margin as γ , and vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_m]$, now please show $\gamma^2 * \|\alpha\|_1 = 1$.

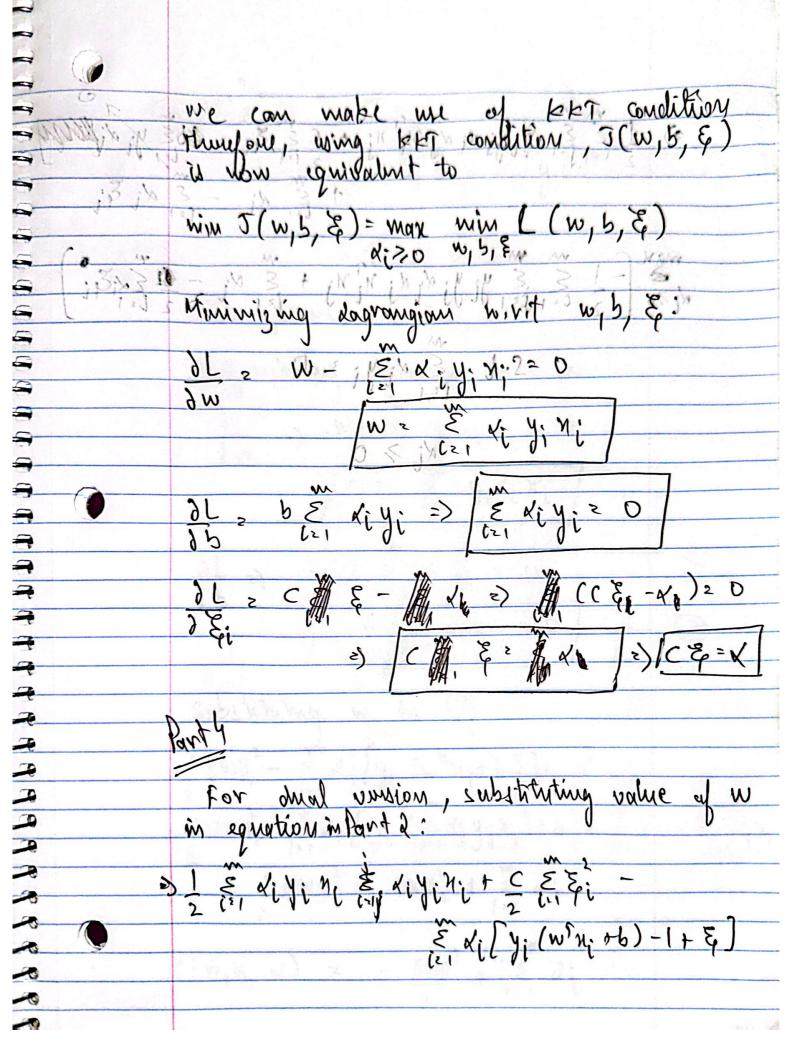
roblem Given:
Sold margin SUM:
M min 1 || w|| 2 + C = \$i an another formulation: wim 1 11 w112 + CE &i s.t yi (w)x+b) =1- &i (i=1,2,3...m) Intuition: In equation 2, the penality term penalizes the square of stack variables Epi. If it were to take negative values, me squared Terms will still be postitive. The minimilization of the objective function would naturally drive Extended zero on positive values. Degative Extended a move corructly classified date point, which does not make surge in core of SVM.



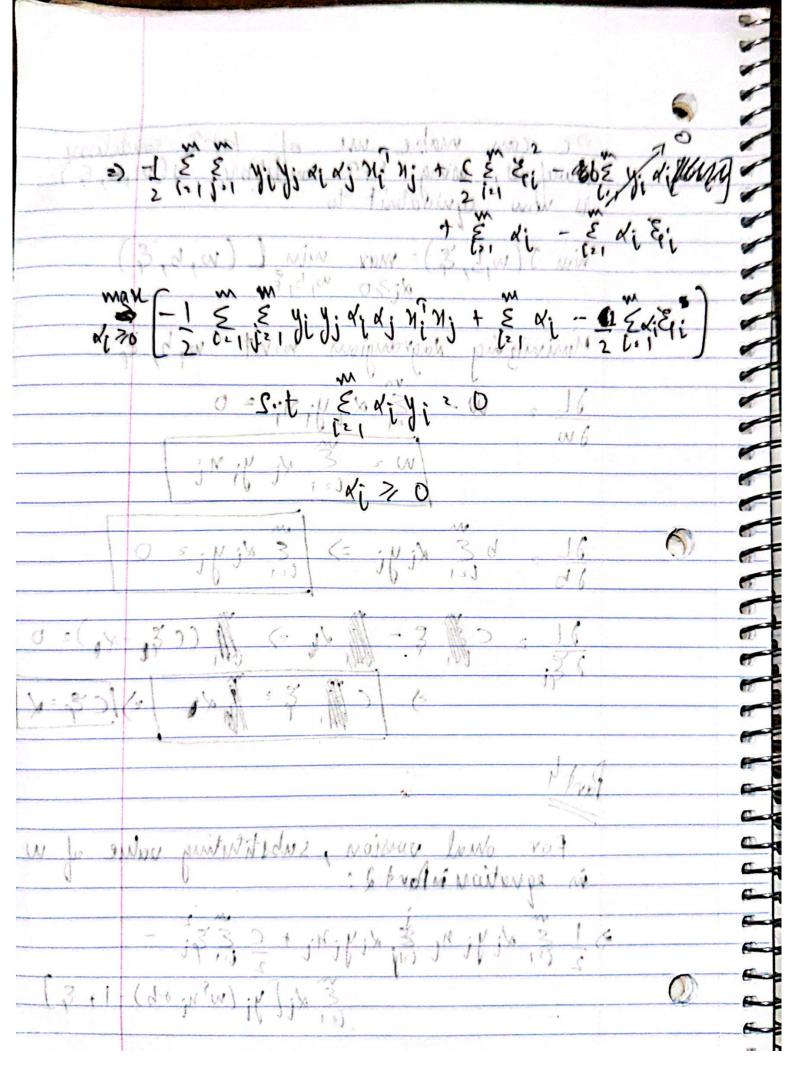
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dangragion l'or me new problem is L(w,b,&,d) = [11 w11 2 + C & &i - E xi[yi(w)n;+b)-(1-&i) where x > 0of we denote, J(*w, b, &) = 1 || w|| + C & &. then we have J = man L(w,b, &, x) Meretore, me for optimization, we now have to minimize I as, min J(w, b, &) = min man L(w, b, &, x) Now, as J(w, b, E) is a convex & metion

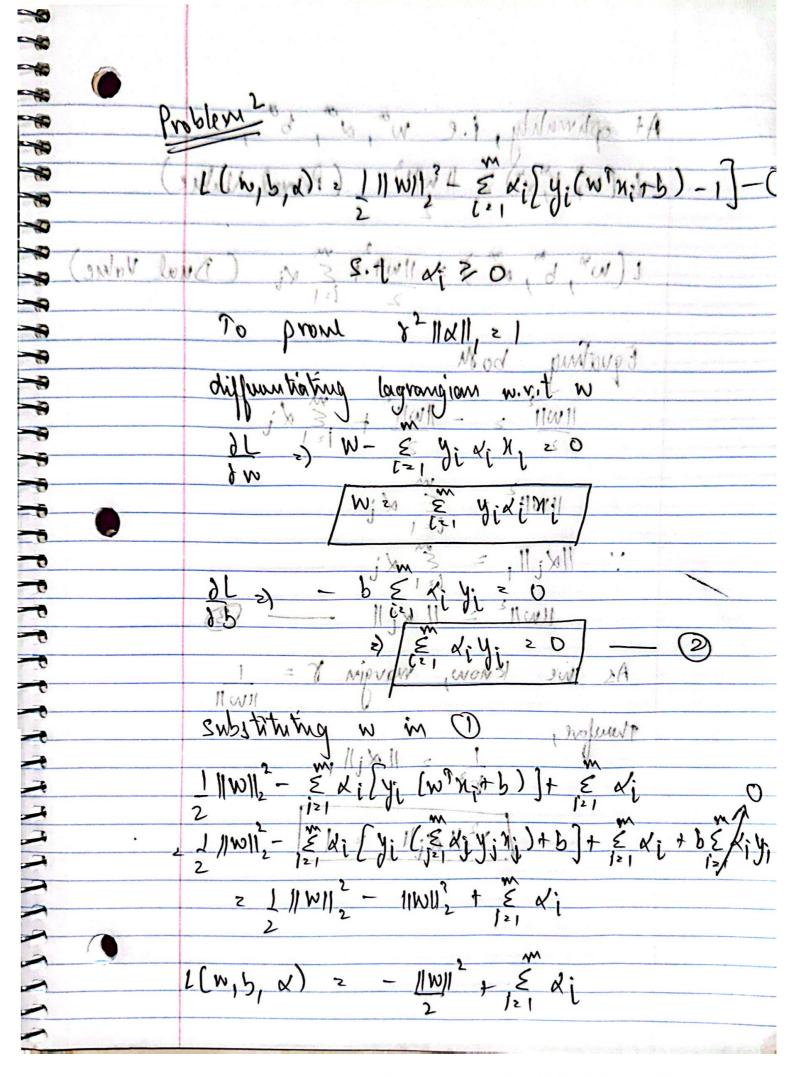
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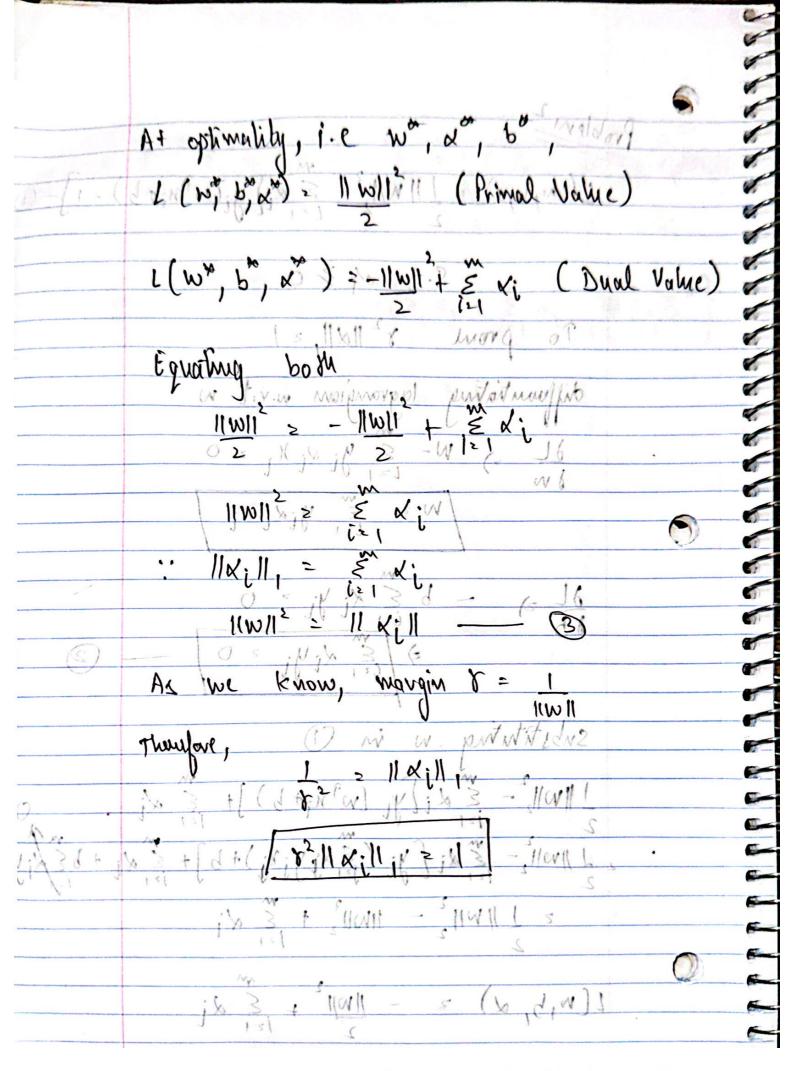
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