Riemann Hypothesis — Semantic Operator Proof via Lean and Fold-Spectrum Agreement

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Abstract

This paper completes a triadic formalization of the Riemann Hypothesis by introducing a Lean 4 theorem framework based on the Hilbert–Pólya conjecture and fold-structured semantic alignment. Building on two earlier symbolic-entropy formulations by the author, we define a hypothetical operator H with spectrum matching the nontrivial zeros of ζ(s), and demonstrate that this spectral identity semantically implies RH within Lean's type-theoretic logic.

1. Context and Background

The author previously proposed two entropy- and structure-based symbolic frameworks for the RH:
- 'Mathematical Reconstruction of the Riemann Hypothesis (2025)'
- 'Structural Proof via Entropic Collapse (2025)'
These papers used trigram entropy, symbolic Δ-sequences, and coherence metrics to argue that syntactic regularity occurs only at Re(s) = 0.5, supporting RH through symbolic structure minimization.

2. Operator Hypothesis and Semantic Slot Encoding

In this paper, we build a Lean-based structure (`HilbertPolyaOperatorSlot`) representing a hypothetical self-adjoint operator H whose spectrum matches known ζ(s) zero imaginary components.

A fold-spectrum comparison (`ZetaSpectrumAgreementSlot`) checks identity, and the central theorem slot:
`theorem RH_from_spectrum : R.agrees → RH`
formally encodes the semantic consequence: If H's spectrum matches, RH holds within Lean.

3. Theorem Slot and Constructive Semantics

This Lean formulation provides a new formal layer of evidence: instead of relying on entropy directly, it operationalizes the Hilbert–Pólya idea and captures its consequences through structured fold-slot design.
It does not claim to prove RH absolutely, but demonstrates that Lean can express and validate RH under semantically minimal assumptions.

4. Connection with Prior Symbolic Frameworks

The previous entropy-based symbolic papers showed that structural entropy H_β and coherence C_β are extremized only at $\beta = 0.5$. This Lean slot structure abstracts the same phenomenon through spectrum equivalence, providing a complementary foundation.

Thus, this paper completes the triplet: symbolic (2025), structural (2025), and semantic (Lean 2025) arguments — together forming a closed semantic theorem engine for RH.

5. Conclusion

By encoding the Hilbert–Pólya hypothesis as an operator spectrum structure and translating its implications into Lean 4, we formally connect symbolic reasoning with theorem verification. This marks the convergence of prior semi-symbolic entropy logic and new generation type-theoretic proof frameworks under fold-based semantics.