# Mathematical Models of Hybrid Vehicle Powertrain Performance

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**ABSTRACT.** The structure of the hybrid powertrain includes an internal combustion engine, the electric motor/generator, electric drive, electric power converter. Electric motors of conventional design and power converting devices are described in the paper. In this paper an attention paid to the mathematical description of an internal combustion engines, as a part of hybrid powertrain component. Following the paper provides brief mathematical description of galvanic energy storage of hybrid powertrain.

**Introduction.** ICE (internal combustion engine) are the most common type of heat engines, in which the heat released during the combustion of fuel is converted into mechanical energy.

On the fig. 1 the conventional ICE scheme is represented. The pedal is mechanically connected to the throttle. In this case, the driver controls the throttle position and thus the amount of air supplying the engine. In general, the torque depends on this parameter.

Advanced technologies of ICE now have been developed to improve the efficiency of the engine and reduce emissions. Most of these technical solutions can be divided into two categories. The first category:

– Mechanical throttle compounded with fuel supply system. Allows reducing emission through fuel ratio (14:1);

– Fully electronically driven throttle and fuel supply system, which obtains the data from multiple sensors and adjusts fuel ratio and gearbox parameters in order to achieve maximum performance and reduce emissions.

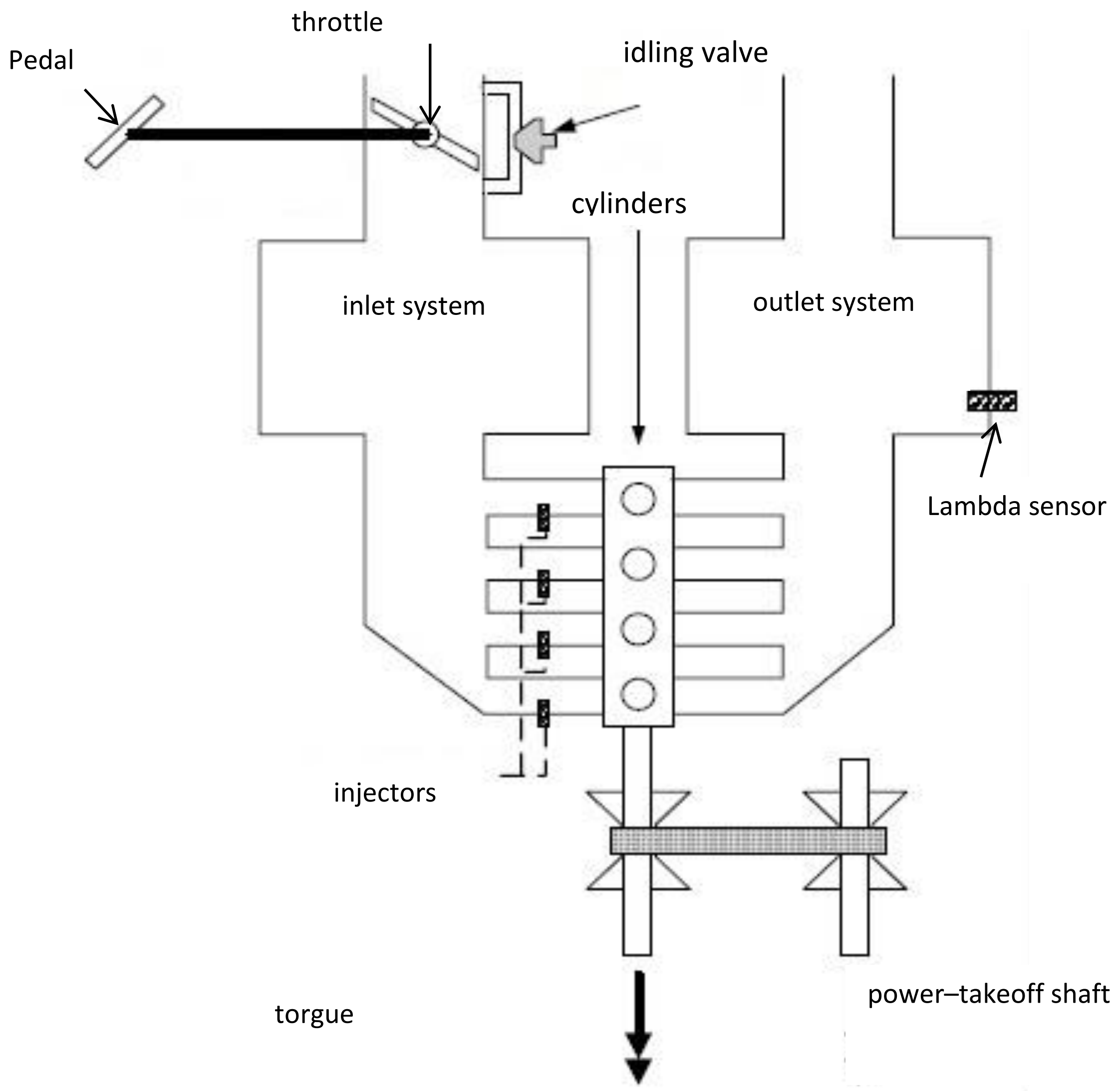


*Fig. 1. The scheme of the internal combustion engine.*

The mathematical model of heat engine subsystem includes a working fluid dynamics and crankshaft subsystem dynamics.

Number of air mass flowing into the inlet system $m_a$ is a function of the pressure in the system $p_m$ and throttle position angle $\theta$ [3]:

$$m_a' = f(\theta) \cdot g(p_m). \tag{1}$$

Each of the components of this equation can be represented as follows:

$$f(\theta) = k_{th0} + k_{th1} \cdot \theta + k_{th2} \cdot \theta^2 + k_{th3} \cdot \theta^3, \tag{2}$$

$$g(p_m) = \begin{cases} 1, \; p_m \le 0.5 \cdot p_{atm} \\ \frac{2}{p_{atm}} \cdot \sqrt{p_{atm} \cdot p_m - p_m{}^2}, \; p_m > 0.5 \cdot p_{atm} \end{cases}, \tag{3}$$

where $k_{th0\,..\,3}$ – constant equation;

$\theta$ – throttle position angle;

$p_{atm}$ –inlet pressure for naturally aspirated ICE;

$p_m$ – pressure in the inlet system.

The dynamics of the working fluid in the inlet system can be described by the differential equation of the first order:

$$p_m' = \frac{R \cdot T_m}{V_m} \cdot (m_{ai}' - m_{a0}') \tag{4}$$

where $R$ – gas constant;

$V_m$ – volume of inlet system;

$T_m$ – the temperature in the inlet system.

Airflow entering the cylinders from the inlet system, $m_{ao}$ 'is a function of the pressure in the inlet system $p_m$ and speed $n$ of internal combustion engine:

$$m_{a0}' = k_{mo0} + k_{mo1} \cdot n \cdot p_m + k_{mo2} \cdot n \cdot {p_m}^2 + k_{mo3} \cdot n^2 \cdot p_m \tag{5}$$

where $k_{mo0\,...\,3}$ – constant equation;

$n$ – rotating speed.

Block diagram corresponding to equation (1) – (4), shown in Fig. 2.

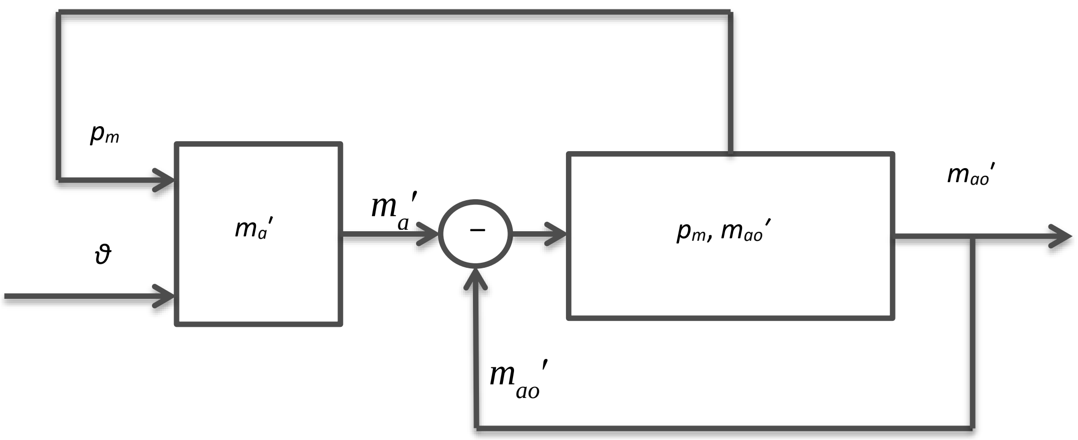


*Fig. 2. Structural scheme of ICE inlet system.*

Equation of the speed of crankshaft will be written as follows:

$$J \cdot n' = T_{eng} - T_l \tag{5}$$

where $T_{eng}$ – engine torque;

$T_l$ – reaction torque;

$J$ – inertia momentum of the engine.

The moment of the internal combustion engine can be described by the following empirical function [3]:

$$T_{eng} = k_{e0} + k_{e1} \cdot m_a + k_{e2} \cdot (AFR) + k_{e3} \cdot (AFR)^2 + k_{e4} \cdot \sigma + k_{e5} \cdot \sigma^2 + k_{e6} \cdot n + k_{e7} \cdot n^2 + k_{e8} \cdot n \cdot \sigma + k_{e9} \cdot \sigma \cdot m_a + k_{e10} \cdot \sigma^2 \cdot m_a \quad (6)$$

where $ke_{0 \dots 10}$ – constant equation;

$m_a$ – the number of the working fluid in the cylinder;

$AFR$ – the ratio of air / fuel;

$\sigma$ – ignition timing.

Variable $m_a$ is a mass of air, entering the cylinder during the inlet, which is $\pi$ radians in the first four cycles of the crankshaft. Thus, $m_a$ can be obtained by integration of air masses moving from the inlet system and resetting the integrator at the end of each cycle. Time reset integrator is variable, depending on the speed of the crankshaft. We know that in real engine there is time lag between the working fluid inlet and obtaining the moment, so the delay can be included in the model that is equal to $\pi$ to the speed of the crankshaft [4].

However, with varying integrator reset time can be approximated by the following expression:

$$m_a = \frac{m_{a0}' \cdot \pi}{n} \quad (7)$$

where $m_a$ – air mass entering the cylinder, g;

$m_{ao}$ – air mass flowing from the inlet system, g / s;

The block diagram is based on the equations of combustion engines, shown in the figure below. On the block diagram shows that the model of the internal combustion engine is complex and nonlinear.

**Simulation of internal combustion engines and engine performance.**

The external characteristics of the engine is the dependence of the effective power, momentum and other indicators of the engine crankshaft rotational speed at full throttle in a gasoline engine.

To construct the external characteristics of the engine can be used any known empirical expression [4].

Taking some arbitrary values of speed, we can calculate the value of the effective power of the engine at different values of speed, which get a few points characteristics. It is recommended in the calculation and construction of high–speed external characteristics (as well as the performance further traction calculation) to select the frequency of the crankshaft of the engine in at least eight points. These points must present:

- minimum steady speed, which can be taken as 800 ... 1000 rpm / min for gasoline engines;
- rated speed corresponding to the maximum capacity of the engine;
- speed corresponding to the maximum speed of the vehicle;
- speed corresponding to the maximum motor torque.

The coefficients *a*, *b*, *c* included in the formula of Leiderman vary depending on the type and parameters of a particular engine. These values give very good agreement forms the estimated external speed characteristics of a pilot for many existing engines.

In general, the coefficients *a*, *b* and *c* depends on the ratio of rotational speed at maximum power (nominal) and the rotational speed at maximum moment [5].

Effective power can be calculated by the following formula

$$N_e = N_{emax} \cdot \left[a \cdot \frac{n}{n_N} + b \cdot \left(\frac{n}{n_N}\right)^2 - c \cdot \left(\frac{n}{n_N}\right)^3\right], \quad (8)$$

where $n$ – rotational speed of the crankshaft, rev / min

$n_n$ – rated speed, rev / min

$a$, $b$, $c$ – coefficients of equation

$N_{emax}$ – power corresponding rated speed kW.

The torque can be find out by the formula

$$M_к = 9550 \cdot \frac{N_e}{n}.$$

The full–load curve of the engine shown in Figure 4.

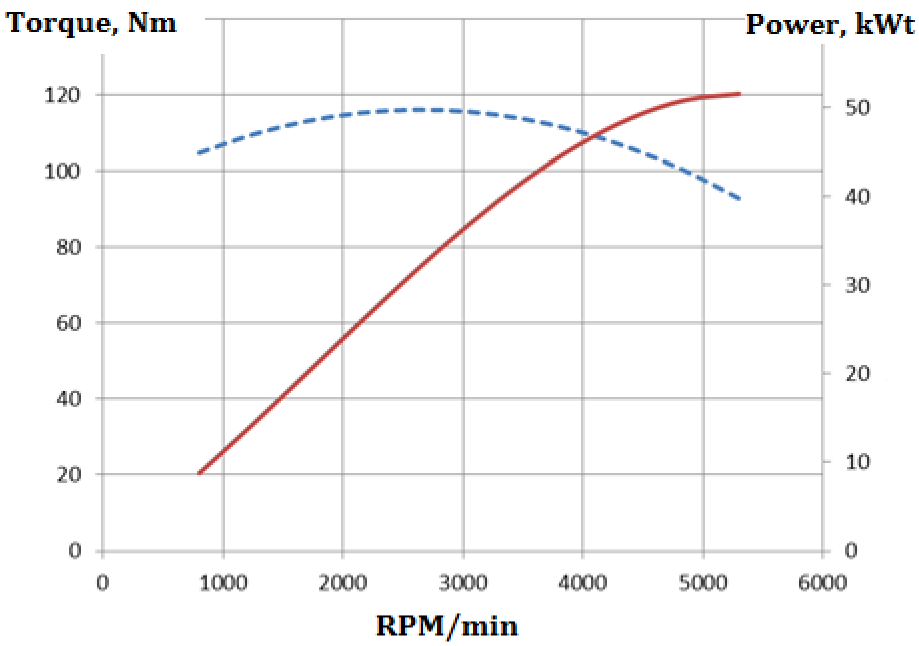


*Fig. 3. The full–load curve of the engine.*

After calculating the external speed characteristics ICE must calculate the equations for the mathematical model of the engine. The simulation results are shown in the figure below.

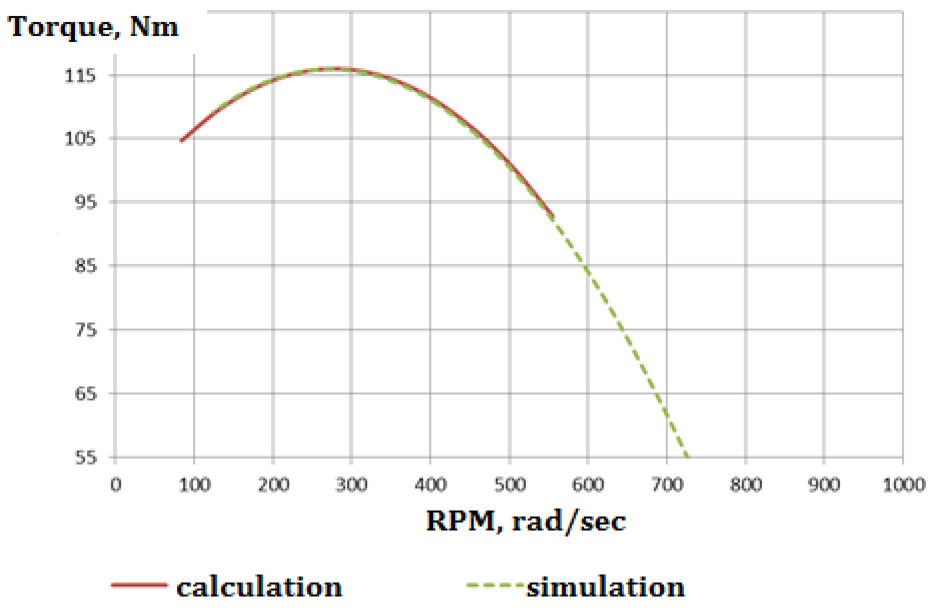


*Fig. 4. The full–load curve of the ICE (simulated).*

It is evident that the design characteristics and external characteristics model engine speed virtually identical. Conclusion – this mathematical model can be used for the synthesis of the regulatory system.

Energy model of the internal combustion engine

One of the main indicators of ICE is fuel efficiency of the engine. Fuel efficiency is the set of properties that determine fuel consumption when performing transport vehicle in various conditions.

The fuel efficiency of the car is largely determined by performance of the engine, as the clock fuel *GT* kg / h – mass of the fuel consumed in one hour and specific fuel consumption $g_e$ (g / kW × h) – the mass of the fuel consumed by one hour unit of engine power.

The main meter fuel economy vehicle in our country and most European countries have fuel consumption in liters per 100 kilometers traveled path (track consumption) $Q_s$ *l.*

The initial schedule for determining fuel consumption $g_e$ and GT are loading characteristic charts dependencies GT = $f(P_e)$ and $g_e = f(P_e)$ when $n$ = const. These dependencies are building sustainable mode of the engine with the same configuration it adopted high–speed characteristics [4, 5].

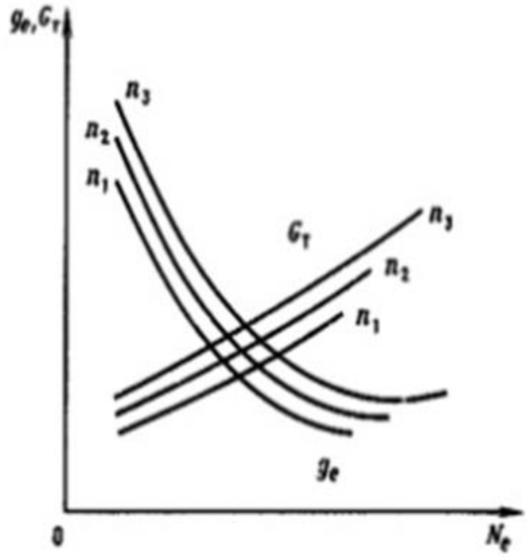


*Fig. 5. Loading characteristics of the engine.*

To calculate the cost $Q_s$ sometimes convenient to use the plot of *g*, by utilization of engine power. It can be obtained by loading and external characteristics of the engine.

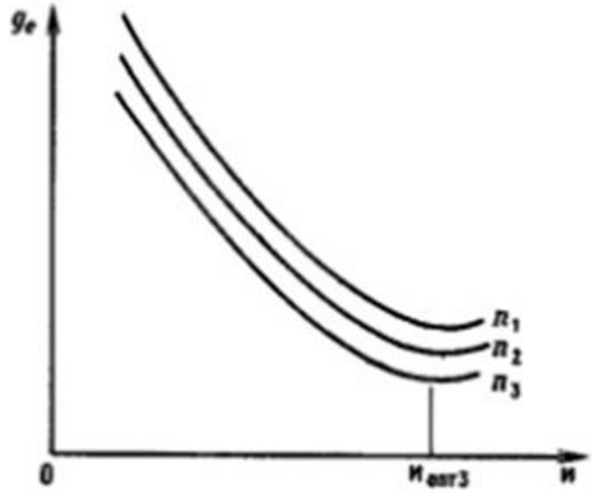


*Fig. 6. The dependence of the specific fuel consumption on engine performance.*

For each frequency *n* consumption, $g_e$ is minimum at the value and close to 100%. At low values of the coefficient and specific consumption increases by reducing engine efficiency and the deterioration of the combustion conditions and at large and (in gasoline engines) – in connection with an enrichment fuel mixture economizer.

For gasoline engines at low values of the coefficient using motor power consumption ge increased compared with the minimum several times, and at a 100% increase =10...15%.

Without depending $g_e = f(N_e, n)$ use different approximate methods. Schlippe has proposed the following formula [1]:

$$g_e = g_N \cdot k_и \cdot k_w$$

where $g_e$ – specific fuel consumption at Nemaks

$k_y$ – factor that takes into account the dependence ge = f (n)

$k_W$ – coefficient taking into account the dependence ge = f (n).

To determine the approximate coefficients ky kW and can use the graphs below [6].

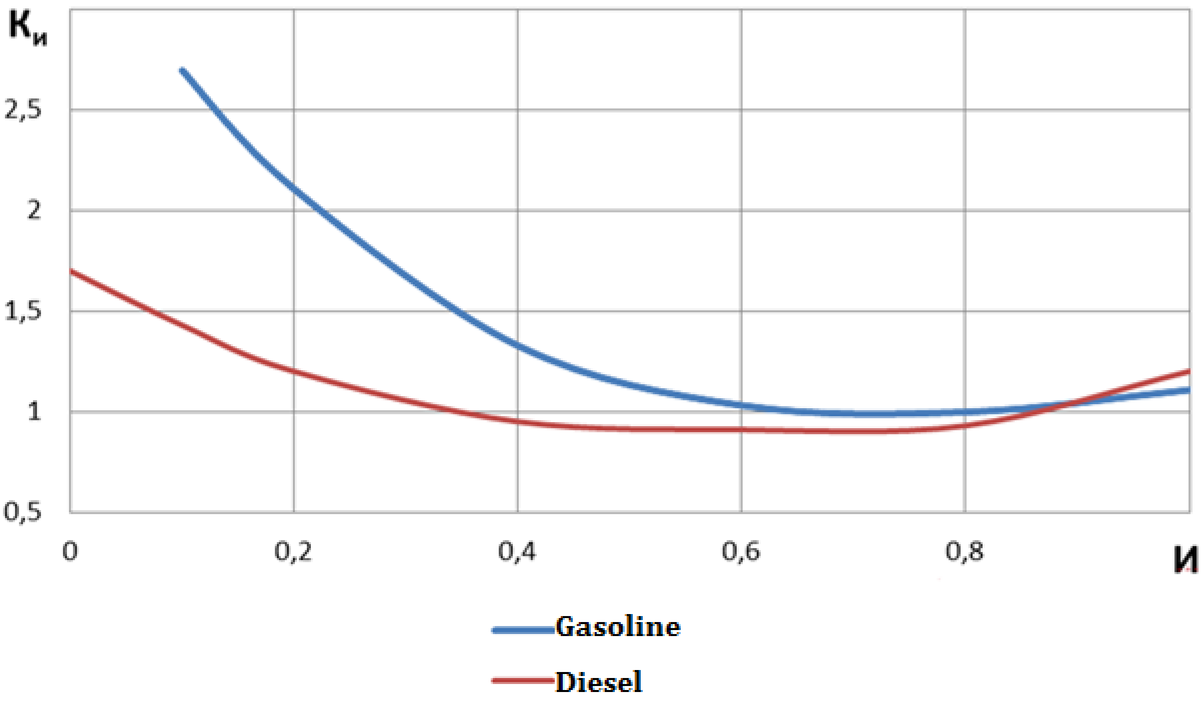


*Fig. 7. Specific fuel consumption at different loads*

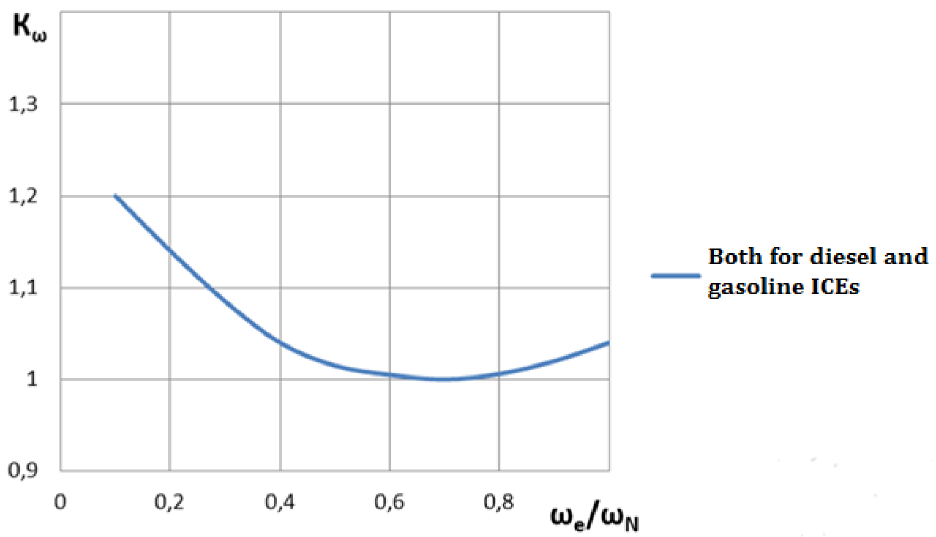


*Fig. 8. Specific fuel consumption at different speeds.*

The specific fuel consumption can be found using the formula below:

$$k_w = 0{,}8 \cdot \left(\frac{n}{n_N}\right)^2 - \frac{n}{n_N} + 1{,}2,$$

$$k_u = 2{,}85 \cdot u^2 - 4{,}35 \cdot u + 2{,}52.$$

The load on the drive system depends on the resistance of the vehicle. This force depends on vehicle speed, wind speed and slope of the road, as shown below.

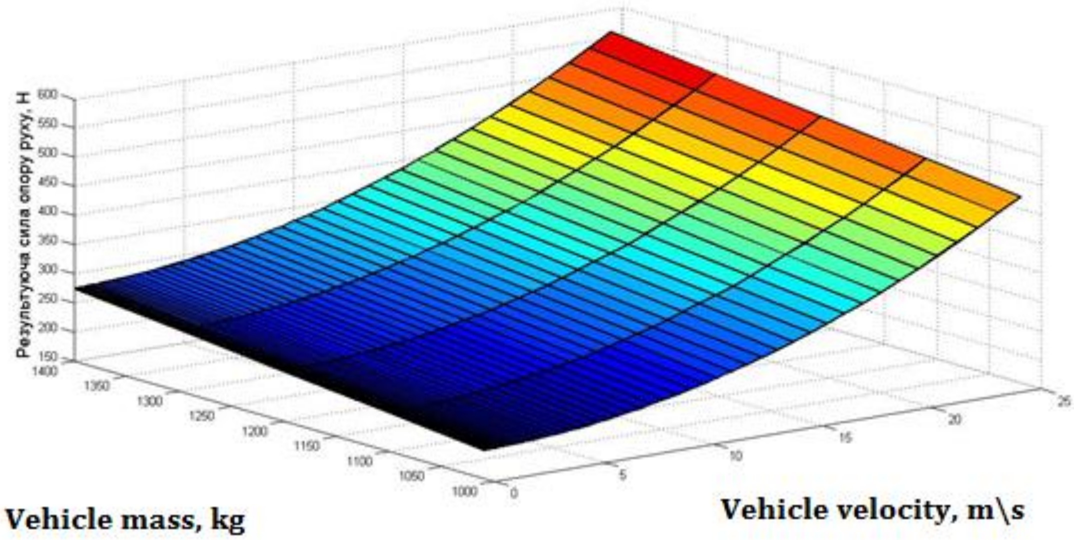


*Fig. 9. Dependence of motion resistance on mass and velocity of vehicle.*

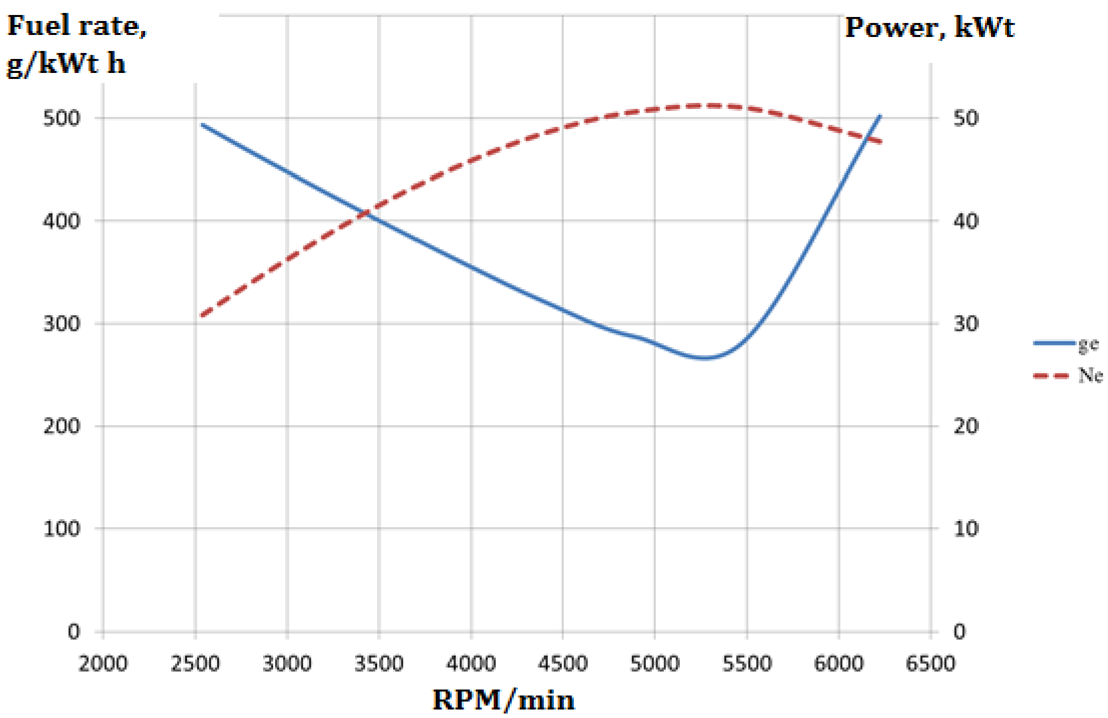


*Fig. 10. Fuel consumption simulation results.*

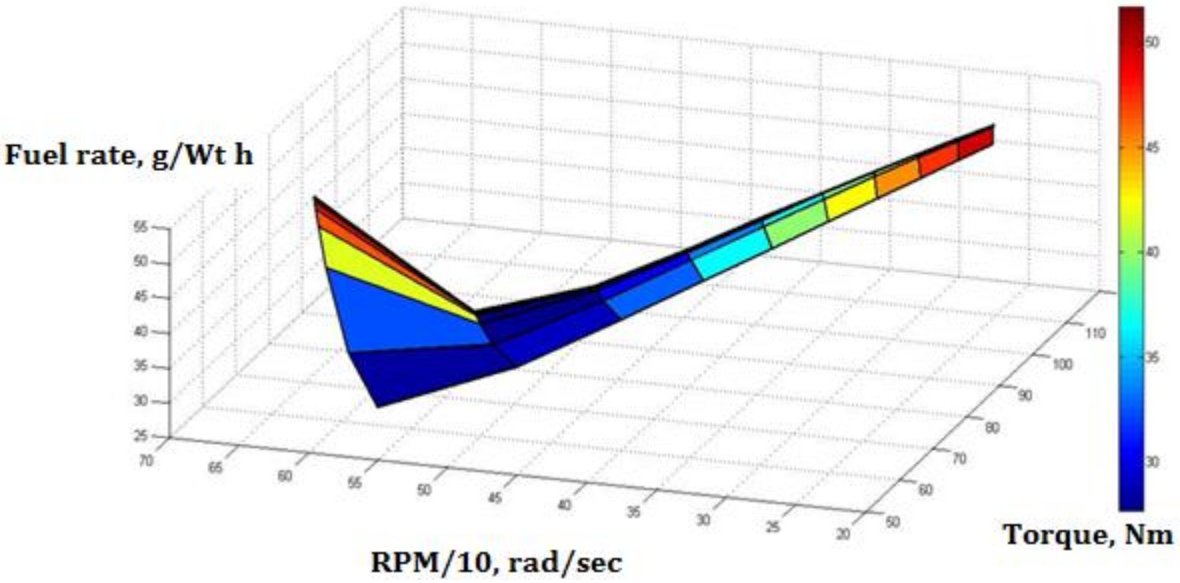


*Fig. 11. The change in fuel consumption and torque during changes in throttle position.*

Map effectiveness of the internal combustion engine is presented in the figure below. Specific efficient fuel consumption is limited to the mechanical characteristics of internal combustion engines at full throttle flap.

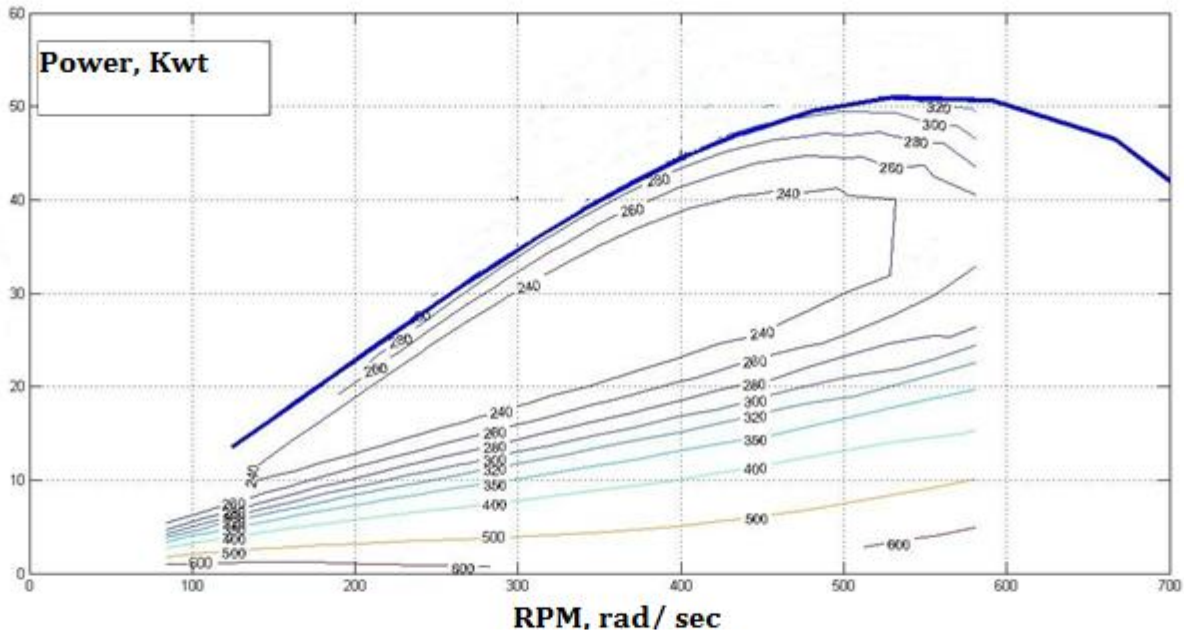


*Fig. 12. ICE efficiency map.*

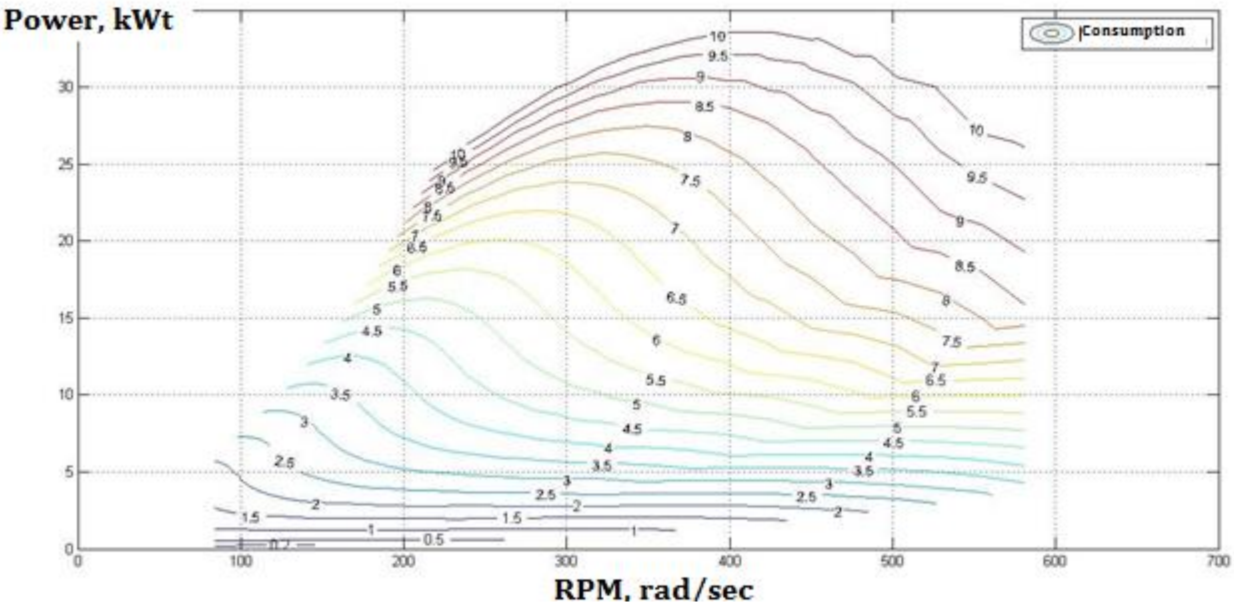


*Fig. 13. Fuel consumption.*

**Summary.** Obtained the dynamic and energetic models of internal combustion engine can be shared for the synthesis of the managing system of electric hybrid vehicle.

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