Branch:- First Year Common for All Branches Semester :- I & II

Subject :- Engineering Mathematics-I

## Unit I

1) (A) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\log \sin x}{(\pi - 2x)^2}$

(B) Using Taylor's theorem, express

$(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in power of x

2) Prove that $\log \sin(x+h) = \log \sin x + h\frac{\cos x}{\sin x} - \frac{h^2}{2}.\frac{1}{\sin^2 x} + \cdots \ldots$

3) If $\cos^{-1}\left(\frac{y}{b}\right) = \log(\frac{x}{n})^n$ prove that

$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$

4) (A) Evaluate $\lim_{x \to a} \frac{x^a - a^x}{x^x - a^a}$

(B) Find n^th derivative of $\frac{1}{x^2-4x+3}$

5) Evaluate $\lim_{x \to 1} \left(\frac{1}{\log x} - \frac{x}{x-1}\right)$

6) Expand $\log(1+e^x)$ in powers of $x$ upto $x^4$

7) If $x = \tan(\log y)$ Prove that

$(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$

8) Find the $n^{th}$ derivative of $y = \sin 2x \sin 3x$

9) If y = a.cos (log x)+ b sin (log x) then Prove that

$x^2\, y_{n+2}$ + (2n+1) x$y_{n+1}$ + ( $n^2$+ 1) $y_n$ =0

10) Evaluate

$\lim_{x \to 1}(1-x^2)^{\frac{1}{\log(1-x)}}$

11) If $x^3$+2x$y^2$-$y^3$+x-1 =0 , expand y in ascending powers of x.

12) Expand 2$x^3$+ 3$x^2$-8x +7 in terms of powers of (x-2)

13) Find the value of a,b and c, so that

$$\lim_{x\to 0} \frac{x(a+b\cos x)-c.\sin x}{x^3} = 1$$

14) Maclaurin's series expansion of $\cos x$ is

15) Find the value of L where $L = \lim_{x\to 0}(\tan x.\log x)$

16) Find the $n^{th}$ derivative of $\frac{1}{ax+b}$

17) The value of $\lim_{x\to 1}(1-x^2)^{\frac{1}{\log(1-x)}}$

18) Find n$^{th}$ differential coefficient of : $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

19) Prove that : If $x = \sin\theta$, $y = \sin 2\theta$ prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0.$$

## Unit II

1) Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} \ ; \text{ where } X = \xi\cos\propto -\eta\sin\propto \text{ and } Y = \xi\sin\propto +\eta\cos\propto$$

2) If u = f(v); v being homogeneous and of $n^{th}$ degree in x and y; prove that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n.v\, f'(v)$, Hence , if u=log v, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n$

3) Find the maximum and minimum values of $\sin x\, Sin\, y\, Sin\,(x+y)$

4) Find the extreme values of $x^2+y^2+z^2$ subject to

$$x^2 + \frac{y^2}{2} + \frac{z^2}{3} = 2 \ ; \ 3x + 2y + z = 0$$

5) If u $=(1-2xy+y^2)^{\frac{-1}{2}}$ ; show that

$$\frac{\partial}{\partial x}\left\{(1-x^2)\frac{\partial u}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{y^2\frac{\partial u}{\partial y}\right\} = 0$$

6) If u= $\sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$ ; show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u .$$

7) Find the extreme values of $x^2 + y^2 + z^2$ subject to

$$x^2 + \frac{y^2}{2} + \frac{z^2}{3} = 2 \;;\; 3x + 2y + z = 0$$

8) Given $z = x^n f_1(y/x) + y^n f_2(x/y)$ show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n^2 u$$

9) If $u = \log(x^2 + y^2 + z^2 - 3xyz)$ Show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

10) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{sinu.cos2u}{4cos^3 u}$

11) If $f(x,y) = x^2 + y^2$; $x = t^2 + t^3$, $y = t^3 + t^9$, Find $\frac{df}{dt}$ at $t = 1$

12) If $u = e^{\frac{x}{y}} \sin\left(\frac{x}{y}\right) + e^{\frac{y}{x}} \cos\left(\frac{x}{y}\right)$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

13) Find the minimum value of $f(x,y) = x^2 + y^2 + 6x + 12$

14) Find The maximum value of $\sin A \sin B \sin C$ if A, B, C are angles of triangles.

15) If $u = cosec^{-1}(x^{\frac{1}{2}} + y^{\frac{1}{2}}/x^{\frac{1}{3}} + y^{\frac{1}{3}})$

show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{tanu}{144}(13 + tan^2 u)$

16) Find $\frac{dy}{dx,}$ if $(cosx)^y = (siny)^x$

17) If $u = f(x^2 + y^2)$ Prove that

$$y^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + x^2 \frac{\partial^2 u}{\partial y^2} = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

18) If $x^x y^y z^z = c$ then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(xlogex)^{-1}$

19) The temp. T at any point $p(x, y, z)$ in space is

$T = 400xyz^2$. Find the highest temp. on the Surface $x^2 + y^2 + z^2 = 1$

using Lagrange's method.

**Unit III**

1) Find the value of (i) $(1+i)^{1000}$ (ii) The value of $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$

2) (i) The value of function log (1+i) (ii) The value of log i

3) Simplify $\frac{[\cos 7\theta+i\sin 7\theta]^{-3}[\cos 5\theta-i\sin 5\theta]^2}{[\cos\theta+i\sin\theta]^5[\cos 4\theta-i\sin 4\theta]^9}$

4) If $\sin(\alpha+\beta)=x+iy$, then find the value of $\frac{x^2}{\cosh^2\beta}+\frac{y^2}{\sinh^2\beta}$

5) If n is a positive integer, then find the value of $(1+i)^n+(1-i)^n$

6) (i) Find The value of $\log(1+i\tan\alpha)$

(ii) Find the value of z in the expression $z=i^{(1+i)}$

7) Using De Moivre's theorem, find the values of x from the equation $x^2+1=0$

8) Find all the roots root of this equation.

$z^4 + z^3 + 2z^2 + 4z - 8 = 0$

9) Show that $(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2(a^2+b^2)^{\frac{m}{2n}}\cos[\frac{m}{n}tan^{-1}\frac{b}{a}]$

10) Prove that $i^{i^i}=\cos\theta+i\sin\theta$, Where $\theta=\pi(2m+\frac{1}{2})e^{-(2m+\frac{1}{2})\pi}$

11) If $(\propto+i\beta)=c\,tan\tan(x+iy)$ then prove that $\tan 2x=\frac{2c\propto}{c^2-\propto^2-\beta^2}$

12) Separate real and imaginary parts of $\cot\cot(x+iy)$

13) Prove that $(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}\cos\frac{n\pi}{2}$

14) Express $\log\left(\frac{3-i}{3+i}\right)$ in (a+ib) form

15) If $\sin\varphi=i.\tan\theta$ then prove that

$\cos\theta + i\sin\theta = \tan(\frac{\pi}{4}+\frac{\varphi}{2})$

16) If $x=\cos\propto+i\sin\propto$, $y=\cos\beta+i\sin\beta$ prove that $\frac{x-y}{x+y}=i\tan\frac{\alpha-\beta}{2}$

17) Solve the equation $x^9-x^5+x^4-1=0$using Demoiver's theorem.

18) If $2cos\theta=x+\frac{1}{x}$ , $2cos\varphi=y+\frac{1}{y}$ Prove that one value of

$x^py^q+\frac{1}{x^py^q}$ is $2cos(p\theta+q\varphi)$

19) Show that the continued product of all the

values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ is 1

20) Prove that: $log\frac{a+ib}{a-ib} = 2i\ tan^{-1}\frac{b}{a}$

**Unit IV**

1) Solve the differential equation $\frac{dy}{dx}+\frac{4x}{1+x^2}y=\frac{1}{(x^2+1)}$

2) Solve the differential equation $\frac{dy}{dx}+2xy=y$

3) Find the solution of $\sec^2 x+\tan y dx+\sec^2 y\tan x dy=0$

4) If $y^n$ is the integrating factor of the differential equation $y(2x^2y+e^x)dx-(e^x+y^3)dy=0$ then find the value of n

5) Find the solution of the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$

6) Find the solution of differential equation $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

7) Find the solution of the differential equation $x^4\frac{dy}{dx}+x^3y+\sec(xy)=0$

8) Find the solution of the differential equation $\cos x\frac{dy}{dx}+y\sin x=(y\sec x)^{\frac{1}{2}}$

9) Find the solution of the differential equation $(x^2+y^2+2x)dx+2ydy=0$

10) Solve the following differential equation :
$(3y-7x+7)dx+(7y-3x+3)dy=0$

11) Solve the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+\ e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\ dy=0$

12) Given that the integrating factor of the equation
$y\ Sec^2\ x\ dx+\ \left[3\tan x-\left(\frac{Sec\ y}{y}\right)^2\right]\ dy=0$ is of the form $y^n$.
Find n and hence solve the equation

13) Solve the following differential equations
$Cos\ x\ \frac{dy}{dx}+y+Sin\ x=1$

14) Solve $Sin\ x\ \frac{dy}{dx}+2y=tan^3({x}/{2})$

15) Solve $x\ dy-y\ dx=(x^2+y^2)(x\ dx+y\ dy\ )$

16) Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

17) Solve the differential equation.

$$\frac{dy}{dx} + \frac{2x+3y}{y+2} = 0.$$

18) Solve the differential equation

$$x(1-y)dy + (1+y^2)(x-1)dx = 0.$$

19) Find the constant n such that $(x+y)^n$ is an integrating factor of

$(4x^2+2xy+6y)dx+(2x^2+9y+3x)dy=0.$ and hence solve the equation.

20) Solve the differential equation

$$(x^2y+y^4)dx+(2x^3+4y^3x)dy=0.$$

**Unit V**

1) Find the orthogonal trajectory of the curve $r^n = a^n \cos(n\theta)$
2) Find the solution of the differential equation $p = \sin(y - px)$
3) Solve the differential equation $y = 2px + y^2p^3$
4) If L is the inductance, R is the resistance, E is the voltage across the circuit and i is the current in the circuit at any time t the differential equation of the electric circuit is $L\frac{di}{dt} + Ri = E$ . Initially the current is zero in the circuit. Then find the value of current at any time t
5) Find the solution of the differential equation $y - 2px = \tan^{-1}(xp^2)$
6) Find the equation of the family of orthogonal trajectories of the system of parabolas $y^2 = 2x + c$
7) Find the Required solution of $\sec^2 x + \tan y dx + \sec^2 y \tan x dy = 0$
8) Find the orthogonal Trajectories of the family of Parabolas $y^2 = 4ax$
9) Solve : $y + p^3y^2 = 2px + pe^{-2py}$
10) Show that the family of the curve $\frac{x^2}{\lambda} + \frac{y^2}{\lambda - a} = 1$ , $\lambda$ is a parameter

    Is self orthogonal.

11) Find the orthogonal trajectory of the family of the curve $2a = r(1 + \cos\theta)$

12) Find the orthogonal trajectory of the family of the curve $r^n = a^n \sin n\theta$

13) Find the orthogonal trajectory of the family of the curve $r = a(1+\cos\theta)$

14) Solve: $x + \frac{p}{\sqrt{1+p^2}} = a$

15) Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

16) Solve : $p^3 - 4xyp + 8y^2 = 0$

17) Solve : $p^2 + 2py \cot x = y$

18) Find orthogonal trajectory to the family of curves $x^2 + 4y^2 = c^2$.

19) Solve the differential equation.

$$p^3 + mp^2 = a(y + mx).$$

20) Solve the differential equation.

$$p = \tan\left(x - \frac{p}{1+p^2}\right).$$

21) A circuit consisting of a resistance R and inductance L is connected in series with voltage E is $Ri + L\frac{di}{dt} = E$. Find the value of the current at any time t, if $i = 0$ at $t = 0$. How long will it be before the current has reached one half its final value.

## Unit VI

1) Test the convergency of the series $\sum_{n=1}^{\infty} \frac{n^2(n+1)^2}{n!}$

2) Test the convergency of the series $\sum_{n=1}^{\infty} \frac{1}{n}\sin\left(\frac{1}{n}\right)$

3) Test the convergency of the series $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n}\right)^n$

4) Test the convergency of the series $\frac{1}{1.2} + \frac{2}{3.4} + \frac{3}{5.6} + ----$

5) Test the convergence of series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ by Comparison test

6) Discuss the convergence of the following series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$

by Rabbe's test

7) Examine the convergence of series $\frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+\cdots\infty$ by ratio test.

8) Test the convergence of series $\frac{2}{7}+\frac{2.5}{7.10}+\frac{2.5.8}{7.10.13}+\cdots$ ... by Rabbe's test

9) Test the series $\sum_{n=1}^{\infty}\frac{2^n+1}{3^n+n}$ for convergence (by Ratio test).

10) Discuss the nature of the following series $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\cdots\infty \quad (x>0)$

by Root test

11) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$

12) Test the convergence of the series $\frac{1}{3^p}+\frac{1}{5^p}+\frac{1}{7^p}$..........................

13) Test the convergence of $\sum\frac{\sqrt{n}}{\sqrt{n^2+1}}x^n$

14)Test the convergence of $\sum\frac{4.7..............(3n+1)x^n}{n!}$

15) Test the convergence of $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^2+.............\left(\frac{n}{2n+1}\right)^n+..........$

16) Test the convergence of $\sum\frac{(n+1)^n x^n}{n^{n+1}}$