

Notes on Physics from Symmetry

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This document contains my personal notes on Jakob Schwichtenberg's Physics from Symmetry ([Schwichtenberg, 2015](#)).

1. Special relativity

1.1. Definitions and postulates

In special relativity, **inertial frames of reference** are coordinate systems moving with constant velocity relative to each other. Special relativity has two basic postulates:

1. **The principal of relativity:** The laws of physics are the same in all inertial frames of reference.
2. **The invariance of the speed of light:** The velocity of light has the same value c in all inertial frames of reference.

Theorem 1.1 (Invariant of special relativity). *Consider two events A and B in an inertial observer O 's frame of reference. Let the time interval measured by O between the two events be (Δt) , and the three spatial intervals be (Δx) , (Δy) , (Δz) . Then, the quantity*

$$(\Delta s)^2 := (\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad (1.1)$$

is invariant between all frames of reference. I.e.

$$(\Delta s')^2 = (\Delta s)^2 \quad (1.2)$$

for any inertial frame of reference O' .

Theorem 1.1 follows directly from the invariance of the speed of light.

Definition 1.1 (Proper time). *Proper time, τ , is the time measured by an observer in the special frame of reference where the object in question is at rest. In this frame of reference,*

$$(\Delta s)^2 = (c\Delta\tau)^2. \quad (1.3)$$

In the infinitesimal limit

$$(ds)^2 = (cd\tau)^2. \quad (1.4)$$

Physically, Defn. 1.1 means that all observers agree on the time interval between events for an observer who travels with the object in question. However, different observers **do not** in general agree on the time interval between events generally: $(\Delta t) \neq (\Delta t')$ – this is called **time dilation**.

1.2. c is an upper speed limit

All observers agree on the value of $(ds)^2 = (cd\tau)^2$. Furthermore, we commonly assume that there exists a minimal proper time of $\tau = 0$ for two events if $\Delta s^2 = 0$. We can therefore write that when $\tau = 0$

$$c^2 = \frac{(dx)^2 + (dy)^2 + (dz)^2}{(dt)^2} \quad (1.5)$$

between two events with an infinitesimal distance. We can equate the right-hand side with a squared velocity, and hence

$$\tau = 0 \implies c^2 = v^2 \quad (1.6)$$

so

$$(ds)^2 \geq 0 \implies c^2 \geq v^2 \tag{1.7}$$

for **any** pair of events (which are causally connected, although how this follows is not immediately clear to me right now).

References

Schwichtenberg, J., 2015 *Physics from symmetry*. Springer.