Possible errata in Physics from Symmetry (Second Edition)

Author: Juvid Aryaman
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1. $(\frac{1}{2}, \frac{1}{2})$ Representation (p82)

On p82, Eq.(3.225), we have

$$v \to v' = {v'}_{a\dot{b}} = \left(e^{i\vec{\theta}\frac{\vec{\sigma}}{2}+\vec{\phi}\frac{\vec{\sigma}}{2}}\right)_a^{\ c} v_{c\dot{d}} \left(\left(e^{-i\vec{\theta}\frac{\vec{\sigma}^*}{2}+\vec{\phi}\frac{\vec{\sigma}^*}{2}}\right)_{\dot{b}}^{\ \dot{d}}\right)^T$$

1.1. Suggested correction

I suggest this equation should read

$$v \to v' = {v'}_{a\dot{b}} = \left(e^{i\vec{\theta}\frac{\vec{\sigma}}{2}+\vec{\phi}\frac{\vec{\sigma}}{2}}\right)_a^{\ c} v_{c\dot{d}} \left(e^{i\vec{\theta}\frac{\vec{\sigma}}{2}-\vec{\phi}\frac{\vec{\sigma}}{2}}\right)^{\dot{d}}_{\ \dot{b}} \tag{1.1}$$

If Eq.(3.225) is indeed in error, then the error appears to propagate through the rest of Section 3.7.8.

1.2. Rationale

We found in the previous section that dotted and undotted indices transform differently, namely that

$$\chi'^{\dot{a}} = \Lambda^{\dot{a}}_{\ \dot{b}}\chi^{\dot{b}} \tag{1.2}$$

$$\Lambda^{\dot{a}}_{\ \dot{b}} = \left(e^{i\vec{\theta}\frac{\vec{\sigma}}{2}-\vec{\phi}\frac{\vec{\sigma}}{2}}\right)^{\dot{a}}_{\ \dot{b}} \tag{1.3}$$

(see Eq.(3.219) and Eq.(3.222) from Physics from Symmetry), and also

$$\chi'_a = \Lambda_a^{\ b}\chi_b \tag{1.4}$$

$$\Lambda_a^{\ b} = \left(e^{i\vec{\theta}\frac{\vec{\sigma}}{2}+\vec{\phi}\frac{\vec{\sigma}}{2}}\right)_a^{\ b} \tag{1.5}$$

(see Eq.(3.220) and Eq.(3.221) from Physics from Symmetry).

I assume that v' may be written explicitly as follows:

$${v'}_{a\dot{b}} = \Lambda_a^{\ c} v_{c\dot{d}} \Lambda^{\dot{d}}_{\ \dot{b}} \tag{1.6}$$

Substituting Eq.(1.5) and Eq.(1.3) into the above equation trivially yields Eq.(1.1).

I believe Eq.(1.6) and Eq.(3.225) from Physics from Symmetry are incompatible. There is no way to derive the complex conjugate $e^{+\vec{\phi}\frac{\vec{\sigma}^*}{2}}$ from $e^{-\vec{\phi}\frac{\vec{\sigma}}{2}}$ in the (dotted) right-chiral terms of the transformation.

2. Spinors and Charge Conjugation (p86)

On p86, Eq.(3.243) reads

$$\tilde{\Psi} \to \tilde{\Psi}' = \begin{pmatrix} e^{-\frac{\vec{\theta}}{2}\vec{\sigma}} & 0 \\ 0 & e^{\frac{\vec{\theta}}{2}\vec{\sigma}} \end{pmatrix} \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} \tag{2.1}$$

2.1. Suggested correction

I suggest this equation should read

$$\tilde{\Psi} \to \tilde{\Psi}' = \begin{pmatrix} e^{-\frac{\vec{\theta}}{2}\vec{\sigma}} & 0 \\ 0 & e^{\frac{\vec{\theta}}{2}\vec{\sigma}} \end{pmatrix} \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} \tag{2.2}$$

2.2. Rationale

$\tilde{\Psi}$ is defined as an object where the chirality of each component is naively flipped Eq.(3.241)

$$\tilde{\Psi} = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}. \tag{2.3}$$

Therefore, a Lorentz boost should operate on the same initial object. Such an object would not transform under $\Lambda_{(\frac{1}{2},0)\oplus(0,\frac{1}{2})}$; it would transform under $\Lambda_{(0,\frac{1}{2},0)\oplus(\frac{1}{2},0)}$.