

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

Base case: Suppose $n=1$. We have that $1^3 = 1$ and that $\frac{1^2(1+1)^2}{4} = 1$. So the base case holds.

Inductive step: Assume $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$. We must show that $1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3 = \frac{(n+1)^2((n+1)+1)^2}{4}$. We have that:

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3 &= \frac{n^2(n+1)^2}{4} + (n+1)^3 \\ &= \frac{n^2(n+1)^2 + 4(n+1)^3}{4} \\ &= \frac{(n+1)^2(n^2 + 4(n+1))}{4} \\ &= \frac{(n+1)^2(n^2 + 4n + 4)}{4} \end{aligned}$$

$$= \frac{(n+1)^2(n+2)^2}{4} = \frac{(n+1)^2((n+1)+1)^2}{4}$$

So the inductive step holds.