18.S097 PS2 (in Swift)

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Question 1.

Rephrased, Question 1 is asking for the cardinality of the objects in the functor category, $[\mathbf{Set}, \mathbf{3}]$, i.e. $|\mathbf{Obj}([\mathbf{Set}, \mathbf{3}])|$. To start, we have three functors into $\mathbf{3}$ at hand $K_1, K_2, K_3 \in \mathbf{Obj}([\mathbf{Set}, \mathbf{3}])$ (borrowing notation from Question 2).

In asking if there are any functors beyond these, we need to make an observation about $\mathbf{Set}$'s hom-sets. $\forall S_1, S_2 \in \mathbf{Obj}(\mathbf{Set}), S_1 \neq \emptyset, S_2 \neq \emptyset : \mathrm{Hom}(S_1, S_2) \neq \emptyset$. Home-sets in $\mathbf{Set}$ are non-empty for all *non-empty* set pairings. There will always be functions between them. Now, let's turn our attention to the empty set. Its self-hom-set only contains the identity morphism, $\mathrm{id}_\emptyset$. And all of its originating hom-sets—$\mathrm{Hom}(\emptyset, S)$ for $S \in \mathbf{Obj}(\mathbf{Set}), S \neq \emptyset$—are empty since there we can't construct functions from non-empty sets into the empty set.

We can lean on the above to make sure connections (morphisms) aren't broken across our functors.

Let's make this more precise.

The remaining functors need to map $\emptyset$ to an object in $\mathbf{3}$ that *only* has outbound morphisms. Then, all other objects in $\mathbf{Set}$ must be mapped in the usual way. Listing out the steps for our next candidate functor F_1,

(a) Map $\emptyset$ and $\mathrm{id}_\emptyset$ to 1 and id_1, respectively.

(b) Map all $\emptyset$-originating morphisms to a.

(c) Map all non-empty sets, their identities, and morphisms between non-empty sets to 2 and id_2 (akin to K_2).

We have some degrees of freedom in carrying out (a)–(c). That is, we can form another functor F_2 by, substituting in 2 for all mentions of 1 in (a). b for a in (b). And 3 for all occurrences of 2 in (c).

Similarly, to form an F_3. Leave (a) as is. $b \circ a$ for a in (b). And 3 for all occurrences of 2 in (c).

$K_1, K_2, K_3, F_1, F_2, F_3$ exhaust all possible structure preserving mappings between **Set** and **3**.

Question 2.

(a)

(b) (Assume Bow is imported for all snippets.)

Question 3.

Question 4.

Question 5.

Question 6.

Question 7.

Question 8.