MODULE IV THEORY OF PRODUCTION & COSTS

- PRODUCTION FUNCTION
- SHORT – RUN PRODUCTION FUNCTION
- LONG - RUN PRODUCTION FUNCTION
- LAW OF VARIABLE PROPORTIONS
- LAST OF RETURNS TO SCALE
- LAW OF RETURNS TO FACTOR
- ISOQUANTS / EQUAL PRODUCT CURVES
- ISOCOST CURVE
- COBB _ DOUGLAS PRODUCTION FUNCTION
- ECONOMIES OF SCALE & DISECONOMIES OF SCALE
- INTERNAL ECONOMIES & EXTERNAL ECONOMIES
- TRADITIONAL THEORY OF COSTS (LONG RUN COSTS & SHORT RUN COSTS)
- THE SHAPE OF COST CURVES
- RELATION BETWEEN AC & MC AND SMC & SAC

Production Function

In economics, production function relates physical output of a production process to physical inputs or factors of production.

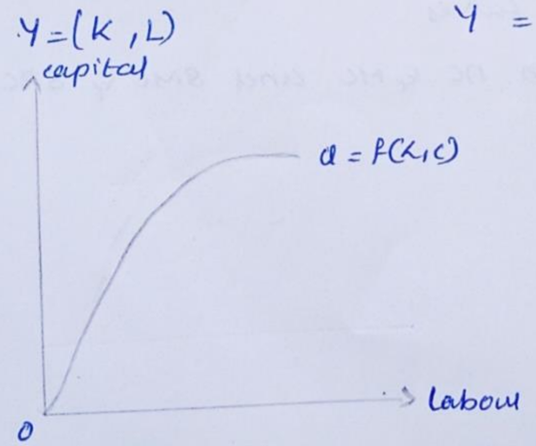
*It is a mathematical function that relates the maximum amount of output that can be obtained from a given number of inputs- generally capital & labour.

$Q = f(L, C, N)$
$Q = f(L, C)$

Q = Output
L = Labour
C = Capital
N = Land (irrelevant)

$Y = (K, L)$

Y = Output



Short - Run Production Function

In the short run, the output quantity can be increased (or decreased) by increasing (or decreasing) the quantities used for variable inputs. This functional relationship between the variable input quantities and the output quantity is called the short-run production function. $\left(Q = f[L, \bar{K}, \bar{T}]\right)$

* Capital, Organization & lands stands as fixed costs in long-run production in usual cases. Labour is the most commonly changed variable.
* In short-run, the firm uses a particular combination of fixed inputs and its short-run production function is obtained in respect of that combination.

Long - Run Production Function

In long-run production function, all inputs used by the firm, the variable inputs & the so-called fixed inputs, all are variables and the firm's production is a function of all these inputs.

This functional relation of dependence between all the inputs used by the firm and the quantity of its outputs is called the long run production function of the firm.

Law of Variable Propositions

Law of variable proportions states that as the quantity of one factor is increased, keeping the other factors fixed, the marginal product of that factor will eventually decline. This law is based on short-run production function.

* Upto the use of a certain amount of variable factor, marginal product of the factor may increase and after a certain stage, it starts diminishing. When the variable factor becomes relatively abundant, the marginal produc may become negative.

Assumptions :-

1) Constant technology - This law assumes that technology does not change throughout the operation of the law.

2) Fixed amount of some factors - one factor of production has to be fixed for this law.

3) Possibility of varying factor proportions - this law assumes that valiable factors can be - changed in the short-run.

→ Total Product - It's the total of output, resulting from efforts of all factors of production.

$TP = P*Q$

→ Avelage Product - It's the total product per unit of the valiable factor.

$AP = TP/N$

→ Malginal Product - It's the addition made to the Total product as a result of production of one more unit of output.

This law has 3 stages :-

1. Increasing Returns
2. Diminishing Returns
3. Negative Returns

Increasing Returns

* Average product, Marginal ~~proportions~~ Product and Total product increases.
* TP increases at more proportionate rate.
* This stage of increasing output by increasing labour doesn't last for a long time, TP will start falling after a point.
* Marginal product curve of a variable factor rises and then falls.
* AP curve rises throughout & remains below the MP curve. MP reaches maximum in this stage.

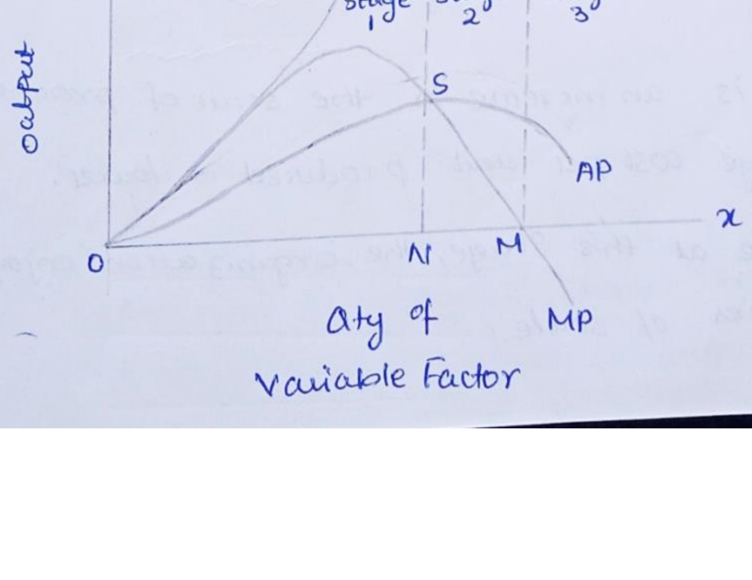
Diminishing Returns

* Most important stage in the production function.
* TP continues to increase at a diminishing rate until it reaches its maximum point where this stage ends.
* MP & AP of the variable factor are diminishing but positive.
* When TP is maximum, MP is 0. MP intersects the x axis in this stage.

As more and more variable factors are used on fixed factor, ~~most~~ MP & AP begins to decrease. Factors of production are indivisible. Economically this is the most viable area of production.

Negative Returns

* In the 3rd Stage, TP decreases. TP curve slopes downward.
* MP curve falls to 0 ~~at point~~ and then is negative. When we increase the labour even after MP became 0, the MP becomes negative and goes below the x axis. This is the most unviable region.



Law of Returns to Scale (long-run)

The Law of Returns to scale explains the proportional change in output with respect to proportional change in inputs.

* The degree of change in output varies with change in the amount of inputs.

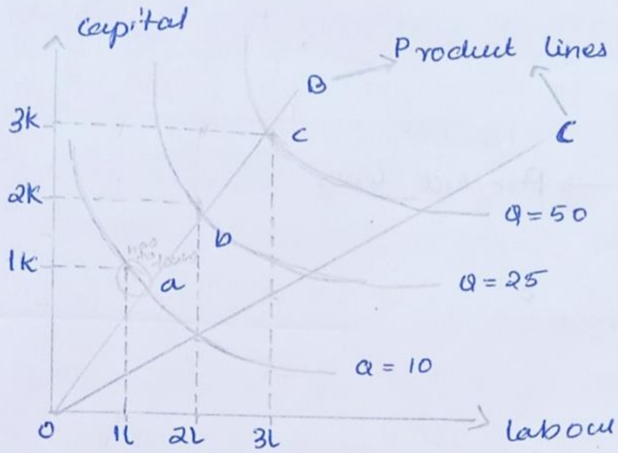
Law of returns can be classified into 3 categories:-

(1) Increasing returns to scale
(2) Constant returns to scale
(3) Diminishing returns to scale

(1) Increasing Returns to Scale

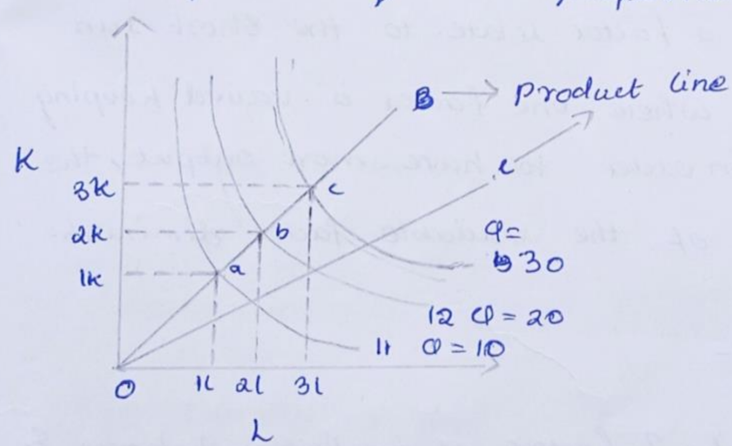
If the proportional change in the output of and organization is greater than the proportional change in inputs, the production is said to reflect increasing returns to scale.

When there is an increase in the scale of production, the average cost per unit produced is lower. This is because at this Stage, the organization enjoys high economies of scale.



(2) Constand Returns to Scale

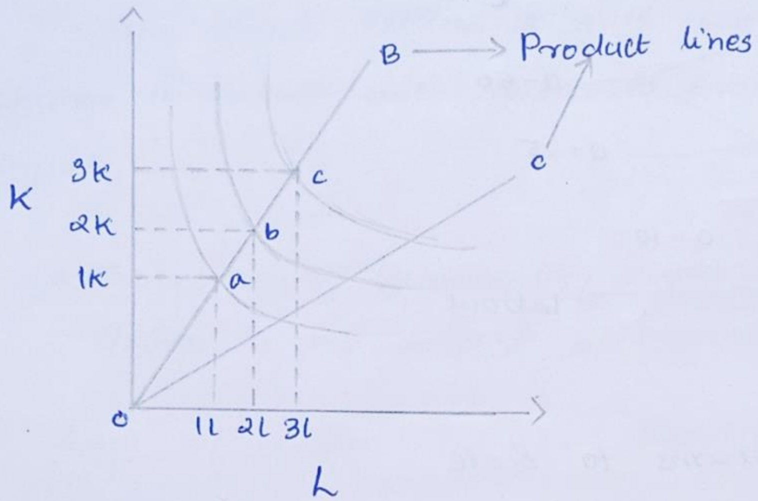
The production is said to generate constant returns to scale when the proportionate change in input is equal to proportionate change in output.



(3) Diminishing Returns to scale

Diminishing returns to scale refers to a situation when the propotionate change in input is ~~less~~ more than the propotionate change in output.

Diagram:-



Returns to a Factor (short run)

Returns to a factor relate to the short-run production function where one factor is varied keeping the other fixed in order to have more output, the marginal returns of the variable factor diminish.

Assumptions:-

(1) There are only 2 factors of production, labour & capital.

(2) Labour is the variable factor & capital the fixed

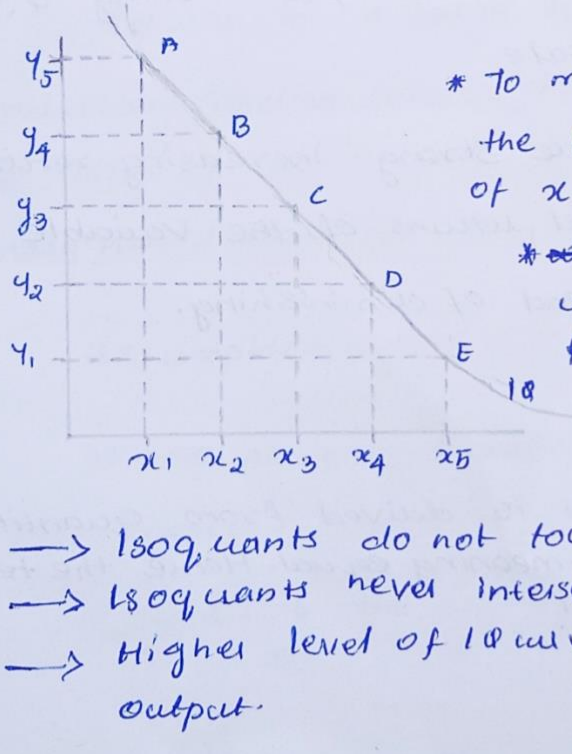
In microeconomics, Isoquant is a contour line drawn through the set of points at which the same quantity of output is produced while changing the quantities of two or more inputs.

* Isoquant is also called 'equal Product curve'.

Units	x	y	output
1	x_5	y_1	100
2	x_4	y_2	100
3	x_3	y_3	100
4	x_2	y_4	100
5	x_1	y_5	100

* Factor 1 is x
* Facto 2 is y

To use more of an input in production, with a limited budget, producer must forgo some units of the other product.



* To make use of more y, the produces curtails more units of x.

* ~~when~~ To produce 1 more unit of y, the producer ha forgone 1 unit of x.

$y_1 \longrightarrow y_2$

$x_5 \longrightarrow x_4$

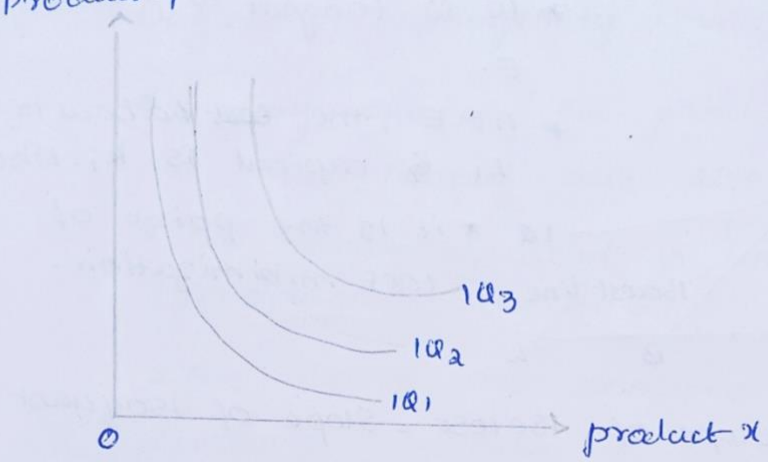
→ Isoquants do not touch the origin. (convex to O)

→ Isoquants never intersect each other.

→ Higher level of IQ curve show higher level of output.

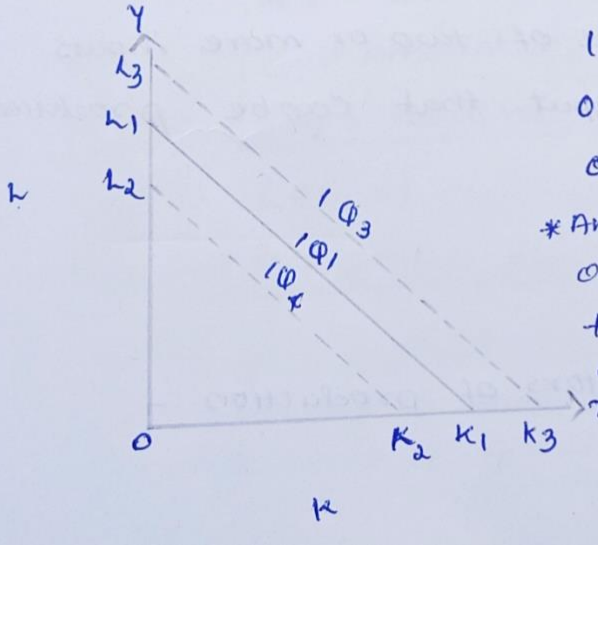
Any point on a IQ curve gives the same level of output.

→ A collection of isoquants is called isoquant map



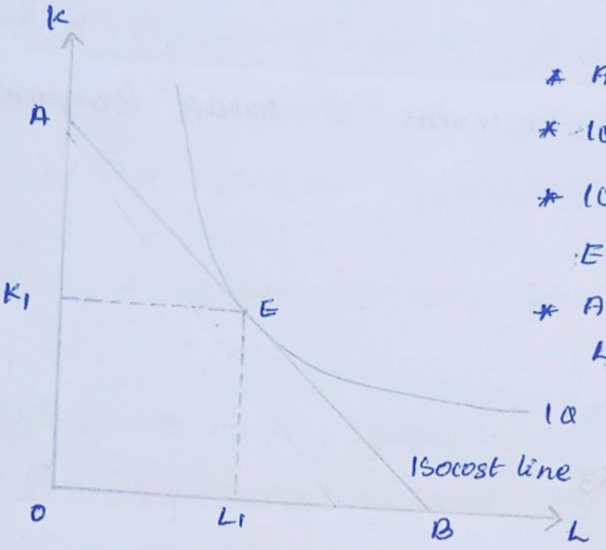
Isocost Curve

Isocost curve is the contour line drawn through the set of points at which the ~~same~~ to producer receives the same level of cost while changing the quantities of 2 or more outputs.



IQ1 is the original cost curve
ox denotes cost of capital &
oy denotes cost of labour.

* Any point on IQ1 is a combination of labour & capital that incurs the produce same level of cost.
* A collection of isocost is given here.



* AB is the iso cost line.
* IQ is the isoquant curve.
* IQ is tangent to AB at E.
* At E, the ~~cost~~ labour is L_1 & capital is K_1. Also,
* it is the point of cost minimization.

At E, slope of isocost = slope of isoquant

Cobb - Douglas Production Function

The Cobb-Douglas production function is a particular functional form of the production function, widely used to represent the technological relationship between the amounts of two or more inputs and the amount of output that can be produced by these inputs.

Assumptions:-

1. There are only 2 factors of production - labour & capital.

2. The share of labour in the process of production is 3 quaters. (3/4)

3. The share of capital in the process of production is 1 quater. (1/4)

4. Degree of homogenity is equal to 1.

* If any one of the production factors became 0, the output will also become 0.

* The theory is called a 'linear homogenous production function'.

* The theory was developed by a mathematician named C.W Cobb and an economist named Paul S Douglas. The theory was based on their observations at American manufacturing industry.

$Q = KL^{a} C^{1-a}$

Q = Qty. of output

L = Amount of labour

C = Amount of capital

K, a = Positive constants (based on the changes in K & L)

$L + C = 1$

$\frac{3}{4} + \frac{1}{4} = 1$ or

$Q = A L^{\alpha} K^{\beta}$

α = no. of Labour units

β = no. of capital units

$\alpha + \beta = 1$

K = capital

L = labour

Economies and Diseconomies of Scale

Economies of scale refers to the situation where, as the qty of output goes up, the cost per unit goes down. It results in a fall of Avg. cost curve.

* This is the idea behind "warehouse stores" like Costco or Walmart.

Simply, a larger factory can produce at a lower average cost than a smaller factory.

* Economies of scale are advantages as a result of increase is scales of production.

Factors that enable economies of scale in firms:-

1. Efficient production
2. Bulk buying
3. Low selling cost
4. cheaper capital
5. Cost Reduction.

As a result, consumers can enjoy lower price of commodities.

Types of Economies:-

1. Internal Economies
2. External Economies

1. Internal Economies

This refers to economies that are unique to a firm. It includes the managerial efficiency of the firm, technological advancements, financial ability, monopsony power and network.

eg:- A firm may hold a patent over a mass production machine, which allows it to lower its average cost of production more than other firms in the industry.

2. External Economies

This refers to the economies that all the firms in the industry enjoy. This can be due to geographical location or govt. intervention most of the times.

eg:- ~~Suppose~~ The Govt. wants to increase steel production. In order to do so, the govt.

announces that all steel producers who employ more than 10,000 workers will be given a 20% tax reduction.

Thus, firms employing less than 10,000 workers can potentially lower their avg. costs by employing more workers.

Diseconomies of Scale

Diseconomies of scale are the cost disadvantages that economic actors accrue due to an increase in organizational size or in output, resulting in production of goods & services at increased per-unit costs. This concept is the opposite of economies of scale.

Traditional Theory of Costs

Traditional theory analyses the behaviour of the cost curves in the short run & long run. Both the short run cost curves and long-run cost curves are 'U' shaped.

* Long run cost curves are flatter than short-run cost curves.

Short - Run Cost of Traditional Theory

$$\boxed{TC = TFC + TVC}$$

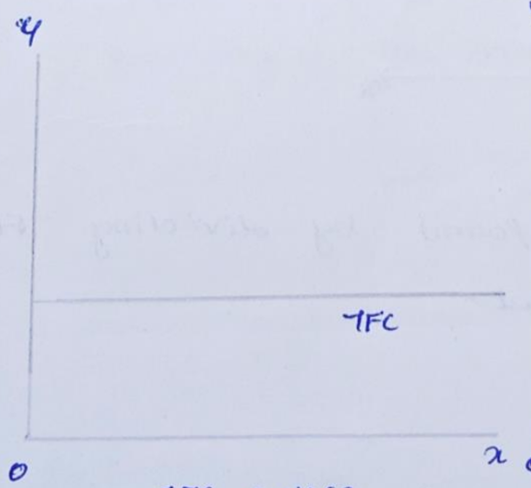
TC = Total cost

TFC = Total fixed cost

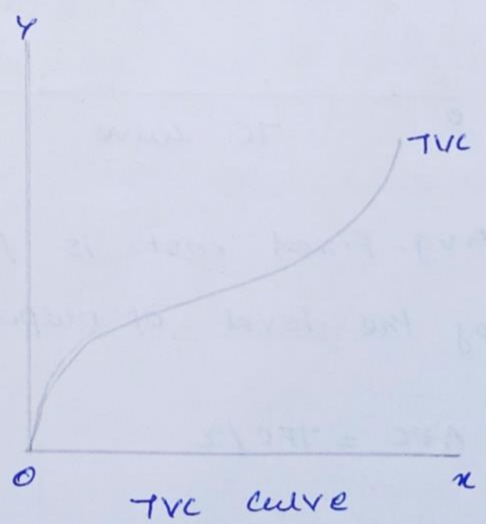
TVC = Total variable cost

* Total fixed cost is graphically denoted by a straight line parallel to to the output axis.

* The total variable cost has broadly an inverse 'S' shape which refers to Law of variable Proportions. (At the initial stages of production with a given plant, as more of the variable factors is employed, productivity increases & the avg. variable cost (AVC) falls.



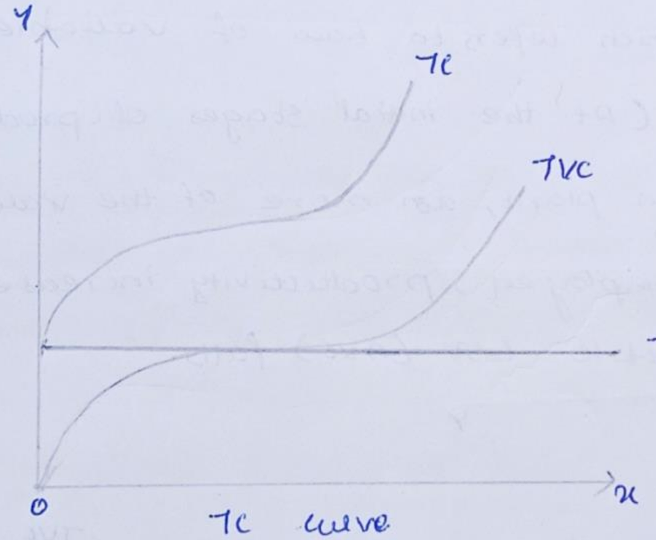
TFC curve



TVC curve

* Once the optimal combination of fixed & variable factors is reached, as increased quantities of variable factors are combined with the fixed factors, the productivity of variable factors declines & AVC rises.

By adding TFC & TVC, we can obtain TC of the firm. AVC can be obtained from TC curves.

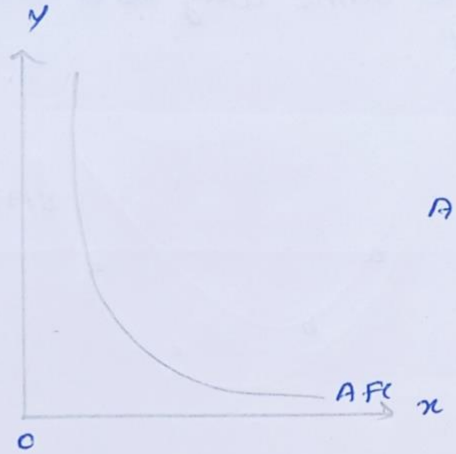


TC curve

* Avg. fixed cost is found by dividing TFC by the level of output

$AFC = TFC / x$

* Graphically, AFC is a rectangular hyperbola & all its points show the same magnitude or level of TFC.

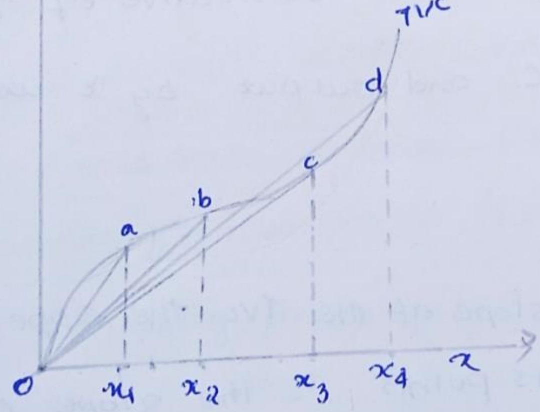


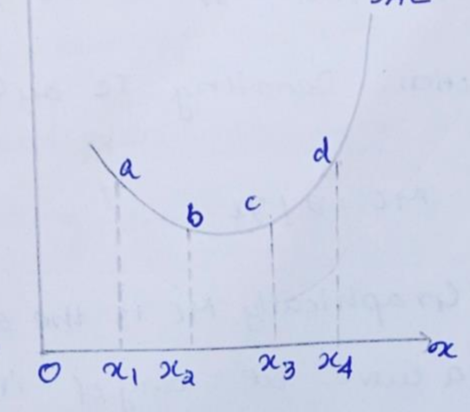
AFC curve

→ AVC is obtained by dividing TVC (corresponding) with level of input.

AVC = TVC / x

* Graphically AVC at each point of output is derived from the slope of a line drawn from origin to the point on TVC curve corresponding to the particular level of ~~income~~ output.

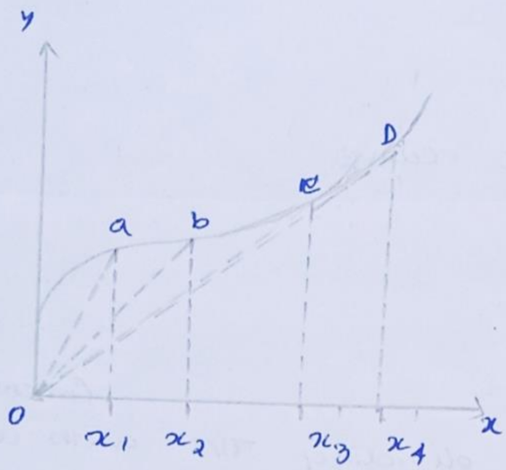


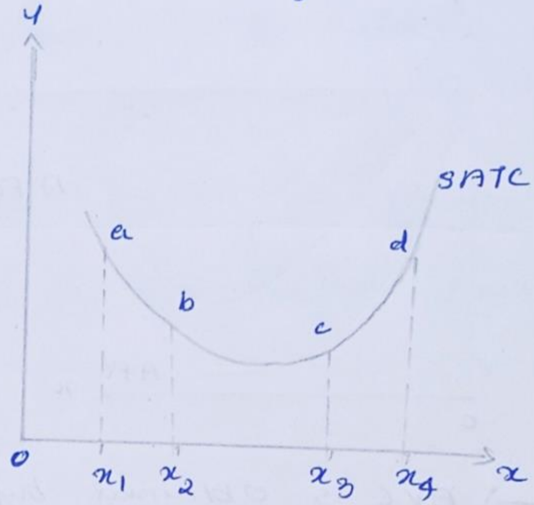


→ $ATC = TC/x = TFC + TVC/x = AFC + AVC$

$ATC = AFC + AVC$

ATC curve is derived in the same way as SAVC.





The U shape of both the AVC & the ATC reflects the law of variable proportions or law of eventually decreasing returns to to the variable factors of production

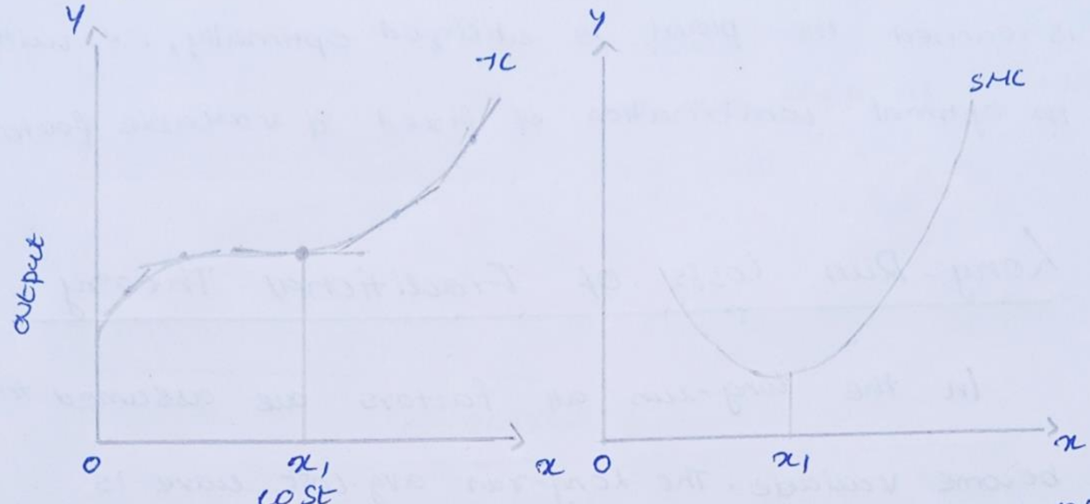
The MC is defined as the change in TC resulted from a unit change in output.

Mathematically, MC is the first derivative of TC function. Denoting TC by C and output by x we have

$MC = \partial C / \partial x$

* Graphically, MC is the slope of the TVC. The slope of a curve at any of its points is the slope of the

tangent at that point. With an inverse-S shape of TC & TVC the MC curve will be U-shaped.



In the figure, we observe that the slope of the tangent to ~~the~~ the TC curve declines gradually, until it becomes parallel to x- axis (with its slope = 0) and then starts raising. Accordingly MC curve is pictured as U-shaped.

Conclusion : The traditional theory of cost postulates that in short run the cost curves (AVC, ATC & MC) are U-shaped, reflecting the law of variable proportions. In short run with a fixed plant there is a phase of increasing productivity (falling unit costs) & a phase of decreasing productivity.

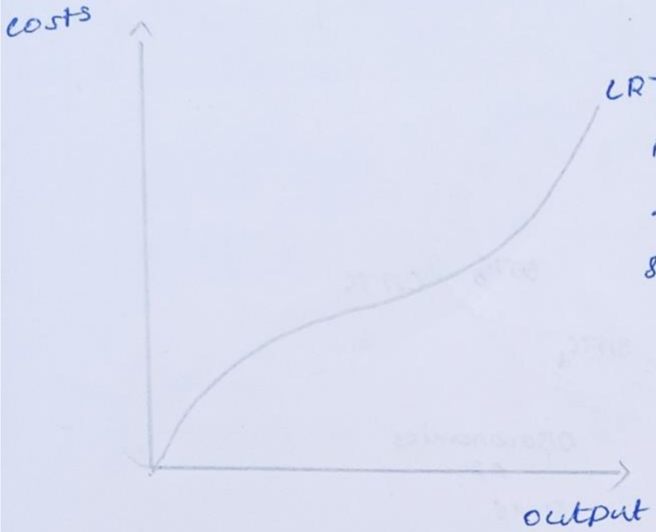
Between these is a single point at which unit costs are at a minimum. When this point on the SATC is reached the plant is utilized optimally, i.e, with th optimal combination of fixed & variable factors.

Long-Run Costs of Traditional Theory

In the long-run all factors are assumed to become variable. The long-run avg. cost curve is derived from short-run cost curves. Each point on the LAC corresponds to a point on a short-run cost curve, which is tangent to the LAC at that point.

Long-Run Total cost

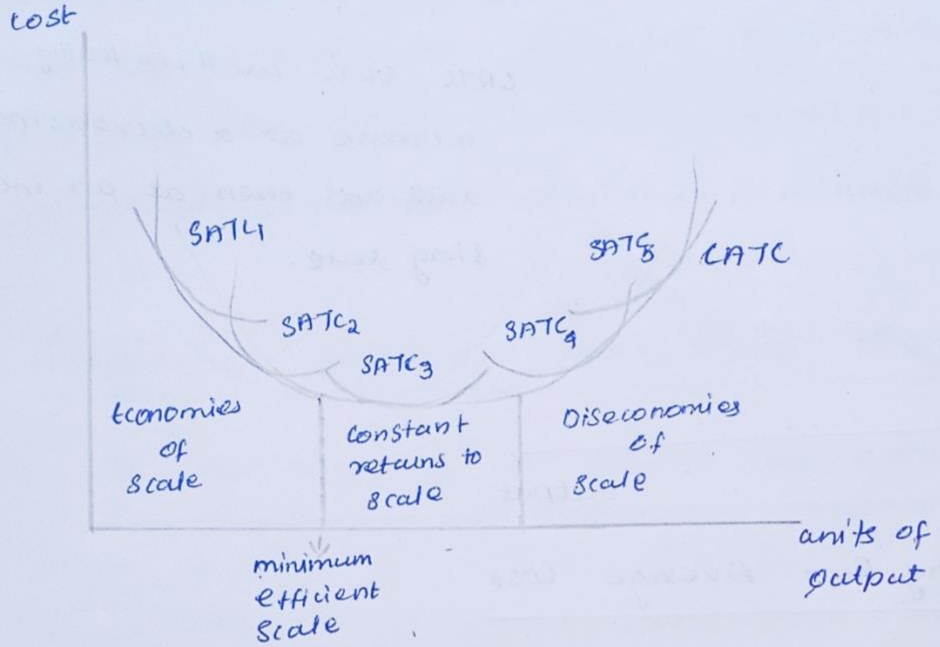
Long run total cost (LTC) refers to the minimum cost at which given level of output can be produced. LTC is always less than or equal to short run total cost, but it is never more that short run cost



LRTC will initially increase at a decreasing rate and then at an increasing rate.

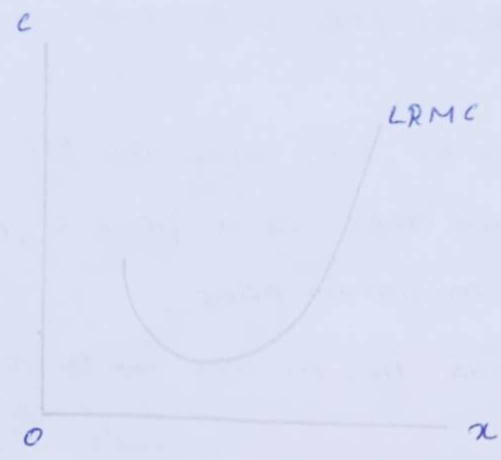
Long Run Average Cost

Long run Average cost (LAC) is equal to long run total costs divided by the level of output. The derivation of long run avg. costs is done from the short run avg. cost curves. In the short run, plant is fixed and each short run avg. costs curve is also called planning curve or envelope curve as it helps in making organizational plans for expanding production & acheiving minimum cost.



Long Run Marginal Cost

Long run marginal cost (LMC) is defined as added cost of producing an additional unit of a commodity when all inputs are variable. This cost is derived from short-run marginal cost. On the graph, the LMC is derived from the points of tangency between LAC & SAC

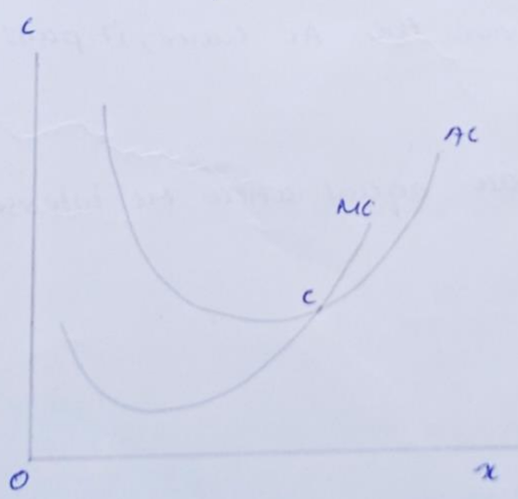


Relationship Between AC & MC

There exists a close relationship between AC & MC.

(i) Both AC & MC derive from TC.

(ii) Both AC & MC curves are U-shaped because of the Law of Variable Proportions.



* when MC < AC, AC falls
* when MC = AC, AC is constant & at it minimum point (C)
* when MC > AC, AC rises

(1) when MC is less than AC, AC falls with increase in output.

(2) when MC is equal to AC, i.e, when the MC & AC curves intersect each other at a point C, AC is constant & at its minimum point.

(3) when MC is more than AC, AC rises with increase in output.

(4) Thereafter both AC & MC rise, but MC increase at a faster rate compared to AC. As a result, MC curve is steeper than AC curve.

→ AC depends on the nature of MC

* when MC curve lies below AC curve, it pulls the latter downwards

* when MC curve lies above the AC curve, it pulls the latter upwards

* consequently, MC & AC are equal where MC intersects AC curve.

Relation Between SMC & SAC

If SMC = SAC, when quantity of output increase SAC remains unchanged.

If smc > SAC, the SAC would increase.

(i) If the output increases, SMC remains the same as SAC then SAC would remain unchanged. Conversely, if SAC remains unchanged as q_x increases, then SMC would be equal to SAC.

(ii) if, as q_x increases, SMC is less than SAC, SAC would fall. If SAC falls as q_x increases, then SMC must be smaller than SAC.

(iii) If SMC becomes greater than SAC as q_x increases, SAC would rise and vice versa.

Graphically explaining,

(i) At the minimum point on SAC curve, SAC would be equal to SMC. The point would the point of intersection between SAC & SMC curves.

(ii) At any point on the SAC curve to the left of its minimum point, SAC decreases at q_x increases. i.e, to the left of minimum point SMC < SAC. SMC curve would lie below SAC curve.

(iii) At any point on the SAC curve to right of its minimum point, SAC increases as q_x increases. ∴ SMC > SAC. SMC curve would lie above SAC curve.

Key points :-

(i) If M (marginal...) < A (Avg....) then A would fall. So when A falls, we have M < A.

(ii) If M > A, then A would rise. When A rises, M > A.

(iii) If M = A, A would remain unchanged. If A remains unchanged, we have M = A.