Set Theory and Relations

SET Self Evaluation Test - 1

1. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are

[MNR 1998, 91; UPSEAT 1999, 2000; Kerala (Engg.) 2003, 05]

(a) 7, 6 (b) 6, 3
(c) 5, 1 (d) 8, 7

2. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then

(a) $d = bc$ (b) $c = bd$
(c) $b = cd$ (d) None of these

3. Suppose $A_1, A_2, A_3, \ldots\ldots\ldots A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots\ldots, B_n$ are n sets each with 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of the $A_i's$ and exactly 9 of the $B_j's$. Then n is equal to

(a) 15 (b) 3
(c) 45 (d) None of these

4. If $A = \{1, 2, 3, 4, 5\}$, then the number of proper subsets of A is

(a) 120 (b) 30
(c) 31 (d) 32

5. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then

(a) A is always a subset of the complement of B

(b) B is always a subset of A

(c) A and B are always disjoint

(d) A and the complement of B are always non-disjoint

6. If $A = [x : f(x) = 0]$ and $B = [x : g(x) = 0]$, then A B will be

(a) $[f(x)]^2 + [g(x)]^2 = 0$ (b) $\frac{f(x)}{g(x)}$

(c) $\frac{g(x)}{f(x)}$ (d) None of these

7. If $n(A) = 3$ and $n(B) = 6$ and $A \subseteq B$. Then the number of elements in $A \cap B$ is equal to

(a) 3 (b) 9
(c) 6 (d) None of these

8. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard:

1. 10% families own both a car and a phone
2. 35% families own either a car or a phone
3. 40,000 families live in the town

Which of the above statements are correct

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

9. Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it an equivalence relation is

(a) 5 (b) 6
(c) 7 (d) 8

10. Let $A = \{2, 4, 6, 8\}$. A relation R on A is defined by $R = \{(2, 4), (4, 2), (4, 6), (6, 4)\}$. Then R is

[Karnataka CET 1995]

(a) Anti-symmetric (b) Reflexive
(c) Symmetric (d) Transitive

11. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b + c) = bc(a + d)$, then R is **[Roorkee 1995]**

(a) Symmetric only (b) Reflexive only
(c) Transitive only (d) An equivalence relation

12. The solution set of $8x \equiv 6 (\text{mod } 14), x \in Z$, are

(a) [8] [6] (b) [8] [14]
(c) [6] [13] (d) [8] [6] [13]

13. If A = {1, 2, 3} , B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. The range of R is

(a) {1, 4, 6, 9} (b) {4, 6, 9}
(c) {1} (d) None of these

14. Let A = {p, q, r}. Which of the following is an equivalence relation on A

(a) R_1 = {(p, q), (q, r), (p, r), (p, p)}

(b) R_2 = {(r, q), (r, p), (r, r), (q, q)}

(c) R_3 = {(p, p), (q, q), (r, r), (p, q)}

(d) None of these

15. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then the relation R is

(a) Reflexive (b) Symmetric
(c) Transitive (d) Equivalence

Answers and Solutions

(SET - 1)

1. (b) Since $2^m - 2^n = 56 = 8\times 7 = 2^3\times 7$

$2^n(2^{m-n}-1) = 2^3\times 7$, $\therefore$ $n = 3$ and $2^{m-n} = 8 = 2^3$

$m - n = 3$ $m - 3 = 3$ $m = 6$; $\therefore m = 6, n = 3$.

2. (a) $bN =$ the set of positive integral multiples of b, $cN =$ the set of positive integral multiples of c.

$\therefore$ $bN \cap cN =$ the set of positive integral multiples of bc

$= b \subset N$, [$\because b, c$ are prime]

$\therefore$ $d = bc$.

3. (c) $O(S) = O\left(\bigcup_{i=1}^{30} A_i\right) = \frac{1}{10}(5\times 30) = 15$

Since, element in the union S belongs to 10 of A_i' s

Also, $O(S) = O\left(\bigcup_{j=1}^{n} B_j\right) = \frac{3n}{9} = \frac{n}{3}$,

$\frac{n}{3} = 15 \Rightarrow n = 45$.

4. (c) The number of proper subset $= 2^n - 1$

$= 2^5 - 1 = 32 - 1 = 31$.

5. (d) $\because$ A is not a subset of B

$\therefore$ Some point of A will not be a point of B, So that point will being to B^c . Hence A and complement of B are always non-disjoint.

6. (a) $A \cap B = \{x : x \in A$ and $x \in B\}$

$= [x : f(x) = 0$ and $g(x) = 0] = [f(x)]^2 + [g(x)]^2 = 0$.

7. (a) Since $A \subseteq B$, $A \cap B = A$

$n(A \cap B) = n(A) = 3$.

8. (c) $n(P) = 25\%, n(C) = 15\%$

$n(P^c \cap C^c) = 65\%, n(P \cap C) = 2000$

Since, $n(P^c \cap C^c) = 65\%$

$n(P \cup C)^c = 65\%$ and $n(P \cup C) = 35\%$

Now, $n(P \cup C) = n(P) + n(C) - n(P \cap C)$

$35 = 25 + 15 - n(P \cap C)$

$n(P \cap C) = 40 - 35 = 5$. Thus $n(P \cap C) = 5\%$

But $n(P \cap C) = 2000$

Total number of families $= \frac{2000\times 100}{5} = 40{,}000$

Since, $n(P \cup C) = 35\%$

and total number of families = 40,000

and $n(P \cap C) = 5\%$. (2) and (3) are correct.

9. (c) R is reflexive if it contains (1, 1), (2, 2), (3, 3)

$\because$ $(1, 2) \in R, (2, 3) \in R$

$\therefore$ R is symmetric if (2, 1), (3, 2) R.

Now, $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (2, 3), (1, 2)\}$

R will be transitive if (3, 1); (1, 3) R. Thus, R becomes an equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1) (3,2) (1, 3) (3, 1). Hence, the total number of ordered pairs is 7.

10. (c) Given $A = \{2, 4, 6, 8\}$; $R = \{(2, 4)(4, 2)$ (4, 6) (6, 4)}

(a, b) R (b, a) R and also $R^{-1} = R$.

Hence, R is symmetric.

11. (d) For (a, b), (c, d) $N \times N$

$(a, b)R(c, d) \Rightarrow ad(b + c) = bc(a + d)$

Reflexive : Since $ab(b + a) = ba(a + b) \forall ab \in N$,

$\therefore (a, b)R(a, b)$, $\therefore$ R is reflexive.

Symmetric : For $(a, b), (c, d) \in N \times N$, let $(a, b)R(c, d)$

$\therefore$ $ad(b + c) = bc(a + d)$ $bc(a + d) = ad(b + c)$

$cb(d + a) = da(c + b)$ $(c, d)R(a, b)$

$\therefore$ R is symmetric

Transitive : For $(a, b), (c, d), (e, f) \in N \times N$,

Let $(a, b)R(c, d), (c, d)R(e, f)$

$\therefore$ $ad(b + c) = bc(a + d)$, $cf(d + e) = de(c + f)$

$adb + adc = bca + bcd$ .....(i)

and $cfd + cfe = dec + def$ .....(ii)

(i) × ef + (ii) × ab gives,

$adbef + adcef + cfdab + cfeab$

$= bcaef + bcdef + decab + defab$

$adcf(b + e) = bcde(a + f)$ $af(b + e) = be(a + f)$

$(a, b)R(e, f)$.

$\therefore$ R is transitive. Hence R is an equivalence relation.

12. (c) $8x - 6 = 14P, (x \in Z)$

$x = \frac{1}{8}[14P + 6]$, $(x \in Z)$

$x = \frac{1}{4}(7P + 3)$ x = 6, 13, 20, 27, 34, 41, 48,.....

$\therefore$ Solution set = {6, 20, 34, 48,...} {13, 27, 41, ...}

* * *

= [6]   [13],

where [6], [13] are equivalence classes of 6 and 13 respectively.

13. (c) Here R is a relation A to B defined by ' x is greater than y'

$R = \{(2,1);(3,1)\}$

Hence, range of $R = \{1\}$.

14. (d) Here $A = \{p, q, r\}$

R_1 is not symmetric because $(p, q) \in R_1$ but $(q, p) \notin R_1$

R_2 is not symmetric because $(r, q) \in R_2$ but $(q, r) \notin R_2$

R_3 is not symmetric because $(p, q) \in R_3$ but $(q, p) \notin R_3$.

Hence, R_1, R_2, R_3 are not equivalence relation.

15. (a,b,c,d) Here $l_1 R l_2$

l_1 is parallel l_2 and also l_2 is parallel to l_1, so it is symmetric.

Clearly, it is also reflexive and transitive. Hence it is equivalence relation.