Answers and Solutions

Definition of permutation, Number of permutations with or without repetition, Conditional permutation

1. (b) If the best and the worst appear always together, the number of ways $5! \times 2$. Therefore required number of ways $6! - 5! \times 2 = 480$.

2. (c) To be divisible by 5, the digit 5 must be at unit place, so now to be between 3000 and 4000, 3 must be at thousand place. Hence the required number of ways $= {}^4P_2$.

3. (a) It is obvious.

4. (d) Number of 1 digit numbers $= {}^4P_1$

   Number of 2 digit numbers $= {}^4P_2$

   Number of 3 digit numbers $= {}^4P_3$

   Number of 4 digit numbers $= {}^4P_4$

   Hence the required number of ways

   $= {}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$.

5. (b) 1^{st} man can vote in 3 ways. 2^{nd} man can vote in 3 ways.

   Similarly all other men can vote in 3 ways individually, hence the total number of ways $3 \times 3 \times 3 \times \; 3 \times 3 \times 3 \times 3 = 3^7$ ways.

6. (a) Since the man can go in 4 ways and can back in 3 ways.

   Therefore total number of ways are $4 \times 3 = 12$ ways.

7. (b) $\dfrac{n!}{(n-5)!} \times \dfrac{(n-3)!}{n!} = 20$

   $\Rightarrow (n-3)(n-4) = 20 \Rightarrow n = -1, 8$

   But -1 is not exceptable.

8. (a) Required number of words $= {}^9P_3 = 504$.

9. (c) $\dfrac{n!}{(n-4)!} \times \dfrac{(n-5)!}{n!} = \dfrac{1}{2} \Rightarrow n - 4 = 2 \Rightarrow n = 6$.

10. (c) Required number of ways are n^{mn} since each letter may be posted in n ways.

11. (d) Required number of ways are $2^{10} = 1024$, because every question may be answered in 2 ways.

12. (a) The number will be even if last digit is 2, 4, 6 or 8 *i.e.,* the last digit can be filled in 4 ways and remaining two digits can be filled in ${}^8P_2$ ways. Hence required number of numbers are ${}^8P_2 \times 4 = 224$.

13. (d) $\dfrac{{}^nP_5}{{}^{n-1}P_4} = 9 \Rightarrow \dfrac{n!}{(n-5)!} \times \dfrac{(n-5)!}{(n-1)!} = 9 \Rightarrow n = 9$.

14. (a) ${}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

    $= \dfrac{(n-1)!}{(n-1-r)!} + r\dfrac{(n-1)!}{(n-r)!}$ $\quad \left(\because \; {}^nP_r = \dfrac{n!}{(n-r)!}\right)$

    $= \dfrac{(n-1)!}{(n-1-r)!}\left\{1 + r.\dfrac{1}{n-r}\right\}$

    $= \dfrac{(n-1)!}{(n-1-r)!(n-r)!}\left(\dfrac{n}{n-r}\right) = \dfrac{n!}{(n-r)!} = {}^nP_r$.

    Aliter : We know that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$

    $\dfrac{{}^{n-1}P_r}{r!} + \dfrac{{}^{n-1}P_{r-1}}{(r-1)!} = \dfrac{{}^nP_r}{r!}$ $\quad {}^{n-1}P_r + r.{}^{n-1}P_{r-1} = {}^nP_r$.

15. (a) There are 10 digits in all *viz.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The required 9 digit numbers = (Total number of 9 digit numbers including those numbers which have 0 at the first place) – (Total number of those 9 digit numbers which have 0 at the first place)

    $= {}^{10}P_9 - {}^9P_8 = \dfrac{10!}{1!} - \dfrac{9!}{1!} = 10! - 9! = (10-1)9! = 9.9!$.

16. (c) The number of possible outcomes with 2 on at least one die = (The total number of outcomes) – (The number of outcomes in which 2 does not appear on any die)

    $6^4 - 5^4 = 1296 - 625 = 671$.

17. (c) Required number of ways are 5^4.

    {Since each parcel can be registered in 5 ways}.

18. (a) Required number of ways are $4^5 = 1024$.

    {Since each prize can be distributed in 4 ways}.

19. (d) Required number of ways $= {}^5P_3 = 60$.

20. (a) It is obvious.

21. (c) Required sum $= 3!(3+4+5+6) = 6 \times 18 = 108$.

    [If we fix 3 of the unit place, other three digits can be arranged in $3!$ ways similarly for 4, 5, 6.]

22. (a) Required number of ways $= \dfrac{6!}{3!3!} = \dfrac{720}{6 \times 6} = 20$.

    [No. of heads $= 3$, no. of tails =3 and coins are identical]

23. (c) Required number of ways $= 5! - 4! - 3!$

    $120 - 24 - 6 = 90$.

    [Number will be less than 56000 only if either 4 occurs on the first place or 5, 4 occurs on the first two places].

24. (c) A gets 2, B gets 8; $\dfrac{10!}{2!8!} = 45$

A gets 8, B gets 2; $\frac{10!}{8!2!} = 45$

$\therefore 45 + 45 = 90$.

25. (a) Sum of the digits in the unit place is $6(2+4+6+8) = 120$ units. Similarly, sum of digits in ten place is 120 tens and in hundredth place is 120 hundreds etc. Sum of all the 24 numbers is

$120(1+10+10^2+10^3) = 120 \times 1111 = 133320$.

26. (a) The man can go in 5 ways and he can return in 5 ways. Hence, total number of ways are $5 \times 5 = 25$.

27. (d) Total arrangements of 5 papers are $= 5! = 120$

When physics and chemistry come together $= 4! \times 2! = 48$

Hence required number of ways are $120 - 48 = 72$.

28. (b) First prize can be given in 5 ways. Then second prize can be given in 4 ways and the third prize in 3 ways.

(Since a competitior cannot get two prizes)

and hence the no. of ways $= 5 \times 4 \times 3 = 60$ ways.

29. (b) Extreme left place can be filled in 6 ways, the middle place can be filled in 6 ways and extreme right place in only 3 ways.

($\because$ number to be formed is odd)

Required number of numbers = 6 × 6 × 3 = 108.

30. (a) Numbers are 2, 0, 4, 3, 8

Numbers can be formed

= (Total) – (those beginning with 0)

= 5! – 4! = 120 – 24 = 96.

31. (c) Since ${}^{12}P_3 = 1320$, $\therefore r = 3$.

32. (a) It is obvious.

33. (c) Total number of arrangements are $\frac{6!}{2!} = 360$.

The number of ways in which $O's$ come together $= 5! = 120$.

Hence required number of ways $= 360 - 120 = 240$.

34. (a) $\bullet A \bullet I \bullet U \bullet$

The pointed places to be filled by $MXMM$.

Hence required number of ways $3! \times \frac{4!}{3!} = 4!$

{Since three vowels can be arranged in $3!$ ways also}.

35. (b) Total number of arrangements of n books $= n!$.

If two specified books always together then number of ways $= (n-1)! \times 2$

Hence required number of ways $= n! - (n-1)! \times 2$

$= n(n-1)! - (n-1) \times 2 = (n-1)!(n-2)$.

36. (a) There must be 5 at hundred place, now 2 numbers to be chosen from 5 numbers *i.e.*, ${}^5P_2 = 5 \times 4 = 20$.

37. (b) Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of remaining places.

After fixing 1^{st} place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4^{th} place can be filled up in 5 ways. Thus there will be $5 \times 5 \times 5 = 125$ ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and *i.e.* 4000. Hence the required numbers are $124 + 125 + 125 + 1 = 375$ ways.

38. (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\frac{4!}{2!2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\frac{3!}{2!} = 3$ ways. Hence the required number of ways $= 6 \times 3 = 18$.

39. (c) Since the 5 boys can sit in $5!$ ways. In this case there are 6 places are vacant in which the girls can sit in ${}^6P_3$ ways. Therefore required number of ways are ${}^6P_3 \times 5!$.

40. (a) Number of 1 digit numbers $= {}^6P_1$

Number of 2 digit numbers $= {}^6P_2$

Number of 3 digit numbers $= {}^6P_3$

The required number of numbers $= 6 + 30 + 120 = 156$.

41. (a) Since C and Y are fixed now remaining letters are 6 that can be arranged in $6!$ ways.

42. (b) Since L is fixed now 4 letter can be arranged in $4! = 24$ ways.

43. (b) Required number of ways are $\frac{5!}{2!} = 60$.

44. (b) Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box. Hence required ways = 60.

45. (d) Using the digits 0, 1, 2, ..., 9 the number of five digit telephone numbers which can be formed is 10^5 (since repetition is allowed)

The number of five digit telephone, numbers which have none of the digits repeated = ${}^{10}P_5 = 30240$.

$\therefore$ The required number of telephone numbers $= 10^5 - 30240 = 69760$.

46. (c) Since there are 2 M's, 2 A's and 2T's.

$\therefore$ Required number of ways are $\frac{11!}{2!2!2!}$.

47. (b) Required number of ways $= \frac{8!}{2!2!2!} = 5040$.

48. (a) Required number of ways

$= {}^6P_3 - {}^5P_2 = 120 - 20 = 100$.

{Since between 99 and 1000, the numbers are of 3 digits, but those numbers should be omit in which ‘0’ comes at hundred place}.

49. (b) At first we have to accommodate those 5 animals in cages which can not enter in 4 small cages, therefore number of ways are ${}^6P_5$. Now after accommodating 5 animals we left with 5 cages and 5 animals, therefore number of ways are $5!$. Hence required number of ways $= {}^6P_5 \times 5! = 86400$.

50. (b) It is obvious.

51. (b) 3 must be at thousand place and since the number should be divisible by 5, so 5 must be at unit place. Now we have to filled two place (ten and hundred) *i.e.*, ${}^4P_2 = 12$.

52. (c) Words start with D are $6! = 720$, start with E are 720, start with MD are $5! = 120$ and start with ME are 120. Now the first word starts with MO is nothing but MODESTY. Hence rank of MODESTY is 1681.

53. (b) We have $a = {}^{x+2}P_{x+2} = (x+2)!$, $b = {}^xP_{11} = \frac{x!}{(x-11)!}$

and $c = {}^{x-11}P_{x-11} = (x-11)!$

Now $a = 182bc \Rightarrow (x+2)! = 182.\frac{x!}{(x-11)!}(x-11)!$

$\Rightarrow (x+2)! = 182x! \Rightarrow (x+2)(x+1) = 182 \Rightarrow x = 12$.

54. (c) In forming even numbers, the position on the right can be filled either 0 or 2. When 0 is filled, the remaining positions can be filled in $3!$ ways and when 2 is filled, the position on the left can be filled in 2 ways (0 cannot be used) and the middle two positions in $2!$ ways (0 can be used). Therefore the number of even numbers formed $= 3! + 2(2!) = 10$.

55. (d) Without any restriction the 10 persons can be ranked among themselves in $10!$ ways; but the number of ways in which A_1 is above A_{10} and the number of ways in which A_{10} is above A_1 make up $10!$. Also the number of ways in which A_1 is above A_{10} is exactly same as the number of ways in which A_{10} is above A_1.

Therefore the required number of ways $= \frac{1}{2}(10!)$.

56. (c) First, we arrange 3 consonants in $3!$ ways and then at four places (two places between them and two places on two sides) 3 vowels can be placed in ${}^4P_3 \times \frac{1}{2!}$ ways.

Hence the required number $= 3! \times {}^4P_3 \times \frac{1}{2!} = 72$.

57. (c) 5 boys can be stand in a row $5!$ ways

Now, two girls can't stand in a row together in ${}^6P_5$ ways.

Total no. of required arrangement $= 5! \times {}^6P_5 = 5! \times 6!$.

58. (a) Total no. of permutations $= \frac{6!}{3!2!} = 60$.

59. (b) The word MOBILE has three even places and three odd places. It has 3 consonants and 3 vowels. In three odd places we have to fix up 3 consonants which can be done in ${}^3P_3$ ways.

Now, remaining three places we have to fix up remaining three places we have to fix up remaining three which can be done in ${}^3P_3$ ways.

The total number of ways $= {}^3P_3 \times {}^3P_3 = 36$.

60. (c) The digits are 1, 2, 3, 4, 5. We have to form number greater than 24000.

Required number will be = (Total) – (Those beginning with 1) – (Those beginning with 21) – (Those beginning with 23)

$= 5! - 4! - 3! - 3! = 120 - 24 - 6 - 6 = 84$.

61. (b) Numbers which are divisible by 5 have ‘5’ fixed in extreme right place

3 Digit Numbers			4 Digit Numbers			
H	T	U	Th	H	T	U
×	×	5	×	×	×	5
${}^3P_2$ ways			${}^3P_3$ ways			
$= \frac{3!}{1!} = 3 \times 2$			$= \frac{3!}{0!} = 3 \times 2$			

$\Rightarrow$ Total ways = 12.

62. (a) Required number $= \frac{7!}{3!2!} = \frac{5040}{6 \times 2} = 420$.

($\because$ 3 is repeated thrice and 2 is repeated twice).

63. (d) After fixing 1 at one position out of 4 places 3 places can be filled by ${}^7P_3$ ways. But some numbers whose fourth digit is zero, so such type of ways $= {}^6P_2$

Total ways = ${}^7P_3 - {}^6P_2 = 480$.

64. (c) The units place can be filled in 4 ways as any one of 0, 2, 4 or 6 can be placed there. The remaining three places can be filled in with remaining 6 digits in ${}^6P_3$ = 120 way. So, total number of ways = 4 × 120 = 480. But, this includes those numbers in which 0 is fixed in extreme left place. Numbers of such numbers $= 3 \times {}^5P_2 = 3 \times 5 \times 4 = 60$

0	×	×	×
Fix	${}^5P_2$ ways		3 ways (only 2, 4 or 6)

Required number of ways = 480 – 60 = 420.

65. (d) 0, 1, 2, 3, 5, 7 : Six digits

The last place can be filled in by 1, 3, 5, 7. *i.e.*, 4 ways as the number is to be odd. We have to fill in the remaining 3 places of the 4 digit number *i.e.* I, II, III place. Since repetition is allowed each place can be filled in 6 ways. Hence the 3 place can be filled in 6 × 6 × 6 = 216 ways.

But in case of 0 = 216 – 36 = 180 ways.

Hence by fundamental theorem, the total number will be = 180 × 4 = 720.

66. (a) Required number of arrangements

= (Total number of arrangements)

– (Number of arrangements in which N'*s* are together)

$= \frac{6!}{2! \times 3!} - \frac{5!}{3!} = 60 - 20 = 40$.

67. (a) First, we seat 5 boys, which can be done in 5! ways B × B × B × B × B. Three girls can be seated in places marked '$\times$' in ${}^4P_3$ ways.

Total ways = $5! . {}^4P_3 = 2880$

68. (c) Possible ways = $5! 4! 3! 2!$.

69. (c) Out of 7 places, 4 places are odd and 3 even. Therefore 3 vowels can be arranged in 3 even places in ${}^3P_3$ ways and remaining 4 consonants can be arranged in 4 odd places in ${}^4P_4$ ways.

Hence required no. of ways = ${}^3P_3$ × ${}^4P_4$ = 144.

70. (a) Total ways $= \frac{9!}{(3!)^3} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{3 \times 2 \times 3 \times 2} = 1680$.

71. (c) There are five seats in a bus are vacant. A man sit on any one of 5 seats in 5 ways. After the man is seated, his wife can be seated in any of 4 remaining seats in 4 ways. Hence total number of ways of seating them = $5 \times 4 = 20$.

72. (c) Words starting with A, C, H, I, N are each equals to 5!

Total words = 5×5! =600

The first word starting with S is *SACHIN*.

SACHIN appears in dictionary at serial number 601.

73. (c) Given set of numbers is {1, 2,....11} in which 5 are even six are odd, which demands that in the given product it is not possible to arrange to subtract only even number from odd numbers. There must be at least one factor involving subtraction of an odd number form another odd number. So at least one of the factors is even. Hence product is always even .

74. (c) Total number of ways to distribute one Rs. 100 note and five other notes = 3^6.

75. (c) The numbers between 999 and 10000 are of four digit numbers.

The four digit numbers formed by digits 0, 2,3,6,7,8 are ${}^6P_4 = 360$.

But here those numbers are also involved which begin from 0. So we take those numbers as three digit numbers.

Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2,3,6,7,8 are ${}^5P_3 = 60$

So the required numbers = 360-60 = 300.

Circular permutations

1. (c) Required number of ways $9! \times 2$.

{By fundamental property of circular permutation}.

2. (a) Required number of ways = $\frac{1}{2}(5-1)! = \frac{4!}{2}$.

{Since clockwise and anticlockwise are same in case of ring}.

3. (b) Since total number of ways in which boys can occupy any place is $(5-1)! = 4!$ and the 5 girls can be sit accordingly in $5!$ ways.

Hence required number of ways are $4! \times 5!$.

4. (d) It is obvious by fundamental property of circular permutations.

5. (a) Since total members are 15, but one is to left, because of circular condition, therefore remaining members are 14 but three special member constitute a member. Therefore required number of arrangements are $12! \times 2$, because, chairman remains between the two specified persons and the person can sit in two ways.

6. (d) A garland can be made from 10 flowers in $\frac{1}{2}(9!)$ ways.

{ $\because$ n flowers' garland can be made in $\frac{1}{2}(n-1)!$ ways}

7. (b) There are 20 + 1 = 21 persons in all. The two particular persons and the host be taken as one unit so that these remain 21 – 3 + 1 = 19 persons to be arranged in 18 ! ways. But the two person on either side of the host can themselves be arranged in 2! ways. Hence there are 2 ! 18 ! ways or $2.18!$ ways.

8. (a) The number of ways in which 5 beads of different colours can be arranged in a circle to form a necklace are $(5-1)! = 4\,!$.

But the clockwise and anticlockwise arrangement are not different (because when the necklace is turned over one gives rise to another)

Hence the total number of ways of arranging the beads $= \frac{1}{2}(4\,!) = 12$.

9. (b) It is a fundamental concept.

10. (a) Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be $^5P_2$. Hence by fundamental theorem the total number of ways is

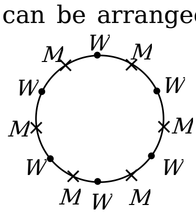
$= 4! \times {}^5P_2$ = 24 × 20 = 480 ways.

11. (b) Fix up 1 man and the remaining 6 men can be seated in 6! ways. Now no two women are to sit together and as such the 7 women are to be arranged in seven empty seats between two consecutive men and number of arrangement will be 7!. Hence by fundamental theorem the total number of ways = 7! × 6!.

12. (d) Required number = $(n-1)!$.

13. (a) 8 different beads can be arranged in circular form in (8 – 1)! = 7! ways. Since there is no distinction between the clockwise and anticlockwise arrangement. So the required number of arrangements $= \frac{7!}{2} = 2520$

14. (a) No. of ways in which 6 men can be arranged at a round table = (6 – 1)!



Now women can be arranged in 6! ways.

Total Number of ways = 6! × 5!

Definition of combination, Conditional combinations, Division into groups, Derangements

1. (a) It is obvious.

2. (c) Required number of ways = $2^7 - 1 = 127$.

{Since the case that no friend be invited *i.e.*, $^7C_0$ is excluded}.

3. (d) Required number of ways = $^{11}C_8 = 165$.

{Since, captain already be chosen, so now from 11 players 8 are to be chosen}.

4. (a) Required number of ways

$= {}^{15}C_1 \times {}^8C_1 = 15 \times 8$.

5. (a) $^{15}C_{3r} = {}^{15}C_{r+3} \Rightarrow {}^{15}C_{15-3r} = {}^{15}C_{r+3}$

$\Rightarrow 15 - 3r = r + 3 \Rightarrow r = 3$.

6. (c) $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4 + {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$.

7. (c) On simplifying you will get $\frac{n-r+1}{r}$.

8. (b) $\frac{(2n)\,!}{(2n-3)\,!\,.\,3\,!} \times \frac{2\,! \times (n-2)\,!}{n!} = \frac{44}{3}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3n(n-1)} = \frac{44}{3}$

$\Rightarrow 4(2n-1) = 44 \Rightarrow 2n = 12 \Rightarrow n = 6$

Now $^6C_r = 15 \Rightarrow {}^6C_r = {}^6C_2$ or $^6C_4 \Rightarrow r = 2, 4$.

9. (b) By inspection $n = 10$.

10. (d) $^{n^2-n}C_2 = {}^{n^2-n}C_{10} \Rightarrow {}^{n^2-n}C_{n^2-n-2} = {}^{n^2-n}C_{10}$

$\Rightarrow n^2 - n - 2 = 10$ or $n = 4, -3$.

11. (c) Here $\frac{^nC_{r-1}}{^nC_r} = \frac{36}{84}$ and $\frac{^nC_r}{^nC_{r+1}} = \frac{84}{126}$.

$3n - 10r = -3$ and $4n - 10r = 6$

On solving, we get $n = 9$, $r = 3$.

12. (c) $^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2} = {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$.

13. (d) Total number of shake hands when each person shake hands with the other once only = $^8C_2 = 28$ ways.

14. (a) It is a fundamental property

15. (b) $^8C_r = {}^8C_{r+2} \Rightarrow 8 - r = r + 2 \Rightarrow r = 3$

Hence $^3C_2 = 3$.

16. (d) No value can satisfy.

17. (a) $^{15}C_3 + {}^{15}C_{13} = {}^{15}C_3 + {}^{15}C_2 = {}^{16}C_3$.

18. (b) $^nC_2 = 66 \Rightarrow n(n-1) = 132 \Rightarrow n = 12$.

19. (c) It is obvious and can be checked by putting the values. Since other three sets do not hold good.

20. (a) Since $^nC_r = {}^nC_{n-r}$ and $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$ we have

$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r = {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + ... + {}^{n+m}C_m$

$= [1+(n+1)]+{}^{n+2}C_2+{}^{n+3}C_3+.......+{}^{n+m}C_m$

$={}^{n+m+1}C_{n+1}$, $\quad [\because {}^nC_r={}^nC_{n-r}]$.

21. (b) ${}^nC_2=153 \Rightarrow \frac{n(n-1)}{2}=153 \Rightarrow n=18$.

22. (d) Each question can be answered in 4 ways and all questions can be answered correctly in only one way, so required number of ways $=4^3-1=63$.

23. (d) $\alpha={}^mC_2 \Rightarrow \alpha=\frac{m(m-1)}{2}$

$\therefore {}^{\alpha}C_2={}^{m(m-1)/2}C_2=\frac{1}{2}.\frac{m(m-1)}{2}\left\{\frac{m(m-1)}{2}-1\right\}$

$=\frac{1}{8}m(m-1)(m-2)(m+1)$

$=\frac{1}{8}(m+1)\,m(m-1)(m-2)=3.\,{}^{m+1}C_4$

24. (b) $2.\,{}^{20}C_2$ {Since two students can exchange cards each other in two ways}.

25. (c) We have 32 places for teeth. For each place we have two choices either there is a tooth or there is no tooth. Therefore the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32}-1$.

26. (b) $\left(\frac{(2n)!}{2!(2n-2)!}\right)2=\left(\frac{n!}{2!(n-2)!}\right)9$

$\Rightarrow (2n)(2n-1)2=9n(n-1) \Rightarrow n=5$

Now ${}^5C_r=10 \Rightarrow r=2$.

27. (d) ${}^{10}C_r={}^{10}C_{r+2} \Rightarrow r+r+2=10 \Rightarrow r=4$

$\therefore {}^5C_r={}^5C_4=\frac{5!}{1!4!}=5$.

28. (b) $\frac{n-r+1}{r}=\frac{84}{36}=\frac{7}{3}$ and $\frac{n-r}{r+1}=\frac{126}{84}=\frac{3}{2}$

$\therefore \frac{7}{3}r-1=n-r=\frac{3}{2}(r+1)$

or $14r-6=9r+9$ or $r=3$. So, $n=9$.

29. (a) ${}^nC_3+{}^nC_4>{}^{n+1}C_3$

${}^{n+1}C_4>{}^{n+1}C_3$ $(\because {}^nC_r+{}^nC_{r+1}={}^{n+1}C_{r+1})$

$\frac{{}^{n+1}C_4}{{}^{n+1}C_3}>1 \qquad \frac{n-2}{4}>1 \Rightarrow n>6$.

30. (c) Either $r+3=2r-6$

or $r+3+2r-6=15$, $(\because {}^nC_r={}^nC_{n-r})$

$r=9$ or $r=6$.

31. (c) ${}^{n+1}C_3=2.\,{}^nC_2$

$\frac{(n+1)!}{3!.(n-2)!}=2.\frac{n!}{2!.(n-2)!} \qquad \frac{n+1}{3.2!}=\frac{2}{2!}$

$n+1=6 \Rightarrow n=5$.

32. (d) $\binom{n}{n-r}+\binom{n}{r+1}={}^nC_{n-r}+{}^nC_{r+1}$

$\Rightarrow {}^nC_r+{}^nC_{r+1}={}^{n+1}C_{r+1}=\binom{n+1}{r+1}$.

33. (a) ${}^nC_5+{}^nC_6>{}^{n+1}C_5 \qquad {}^{n+1}C_6>{}^{n+1}C_5$

$\frac{(n+1)!}{6!.(n-5)!}.\frac{5!.(n-4)!}{(n+1)!}>1 \qquad \frac{(n-4)}{6}>1$

$n-4>6 \Rightarrow n>10$

Hence according to options $n=11$.

34. (b) Total number of handshakes $={}^{15}C_2$.

35. (d) ${}^nC_{r-1}+{}^nC_r={}^{n+1}C_r$ is a standard formula.

36. (a) Given, ${}^{43}C_{r-6}={}^{43}C_{3r+1}$

$r-6=3r+1$ or $r-6+3r+1=43$

$r=-\frac{7}{2}$ (impossible) or $r=12$.

37. (c) Required number of ways $=\frac{6!}{2!\,2!\,2!}=90$.

38. (c) Required number of ways $={}^8C_1+{}^8C_2+{}^8C_3+{}^8C_4+{}^8C_5$

$=8+28+56+70+56=218$

{Since voter may vote to one, two, three, four or all candidates}.

39. (c) Let there are n candidates then

${}^nC_1+{}^nC_2+......+{}^nC_{n-1}=254 \Rightarrow 2^n-2=254$

$\Rightarrow 2^n=2^8 \Rightarrow n=8$.

40. (a) $\bullet E\bullet E\bullet E\bullet......\bullet E\bullet$

According to condition there are 22 vacant places for Hindi books hence total number of ways are $={}^{22}C_{19}=1540$, {Since books are same}.

41. (b) ${}^rC_r+{}^{r+1}C_r+{}^{r+2}C_r......+{}^{n-1}C_r+{}^nC_r$

$={}^{r+1}C_{r+1}+{}^{r+1}C_r+{}^{r+2}C_r+.....+{}^{n-1}C_r+{}^nC_r$

$={}^{r+2}C_{r+1}+{}^{r+2}C_r+.....+{}^{n-1}C_r+{}^nC_r$

$={}^{r+3}C_{r+1}+......+{}^{n-1}C_r+{}^nC_r$.

On solving similar way, we get

${}^{n-1}C_{r+1}+{}^nC_r+{}^nC_r={}^nC_{r+1}+{}^nC_r={}^{n+1}C_{r+1}$.

42. (d) The letters can be select in ${}^5C_3\times{}^4C_2$ ways.

Therefore the number of arrangements are $({}^5C_3\times{}^4C_2)\,5!$.

43. (b) Since 5 are always to be excluded and 6 always to be included, therefore 5 players to be chosen from 14. Hence required number of ways are ${}^{14}C_5=2002$.

44. (a) Required number of ways

$={}^{12}C_1+{}^{12}C_2+{}^{12}C_3+.......+{}^{12}C_{12}=2^{12}-1$

$=4096-1=4095.$

45. (c) The required number of ways are

$(10+1)(9+1)(7+1)-1=11\times 10\times 8-1=879.$

46. (c) Since number of selections are ${}^{n-p}C_{r-p}$. Therefore the arrangement of r things can be done in $r!$ ways. Hence the total permutations are ${}^{n-p}C_{r-p}\,r!$

47. (a) 26 cards can be chosen out of 52 cards, in ${}^{52}C_{26}$ ways. There are two ways in which each card can be dealt, because a card can be either from the first pack or from the second. Hence the total number of ways $={}^{52}C_{26}\,.\,2^{26}$.

48. (b) Required number of ways

$={}^{5}C_3\times{}^{2}C_1\times{}^{9}C_7=10\times 2\times 36=720.$

49. (b) Required number of ways

$={}^{6}C_1+{}^{6}C_2+{}^{6}C_3+{}^{6}C_4+{}^{6}C_5+{}^{6}C_6=2^6-1=63.$

50. (a) It is a fundamental concept.

51. (a) Required number of ways

$={}^{52}C_{13}\times{}^{39}C_{13}\times{}^{26}C_{13}\times{}^{13}C_{13}$

$=\frac{52!}{39!\times 13!}\times\frac{39!}{26!\times 13!}\times\frac{26!}{13!\times 13!}\times\frac{13!}{13!}=\frac{52!}{(13!)^4}$.

52. (b) Required number of words $={}^{6}C_4\times{}^{5}C_3\times 7!=756000$

{Selection can be made in ${}^{6}C_4\times{}^{5}C_3$ while the 7 letters can be arranged in $7!$}.

53. (c) The number of ways can be given as follows

2 bowlers and 9 other players $={}^{4}C_2\times{}^{9}C_9$

3 bowlers and 8 other players $={}^{4}C_3\times{}^{9}C_8$

4 bowlers and 7 other players $={}^{4}C_4\times{}^{9}C_7$

Hence required number of ways

$=6\times 1+4\times 9+1\times 36=78.$

54. (c) The arrangement can be make as $.+.+.+.+.+.+.$ *i.e.*, the (–) signs can be put in 7 vacant (pointed) place.

Hence required number of ways $={}^{7}C_4=35$.

55. (b) At least one green ball can be selected out of 5 green balls in 2^5-1 *i.e.*, in 31 ways. Similarly at least one blue ball can be selected from 4 blue balls in $2^4-1=15$ ways. And at least one red or not red can be select in $2^3=8$ ways.

Hence required number of ways = $31\times 15\times 8=3720.$

56. (c) Required number of ways

$={}^{4}C_1\times{}^{8}C_5+{}^{4}C_2\times{}^{8}C_4+{}^{4}C_3\times{}^{8}C_3+{}^{4}C_4\times{}^{8}C_2$

$=4\times 56+6\times 70+4\times 56+1\times 28=896.$

57. (a) The selection can be made in ${}^{5}C_3\times{}^{22}C_9$.

{Since 3 vacancies filled from 5 candidates in ${}^{5}C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

58. (d) A voter can vote in ${}^{5}C_1+{}^{5}C_2+{}^{5}C_3=25$ ways.

59. (a) 5 persons are to be seated on 8 chairs *i.e.* ${}^{8}C_3\times 5!$ or 6720.

{Since 5 chairs can be select in ${}^{8}C_5$ and then 5 persons can be arranged in $5!$ ways}.

60. (c) 1 girl and 6 boys $={}^{4}C_1\times{}^{6}C_6=4$

2 girls and 5 boys $={}^{4}C_2\times{}^{6}C_5=36$

3 girls and 4 boys $={}^{4}C_3\times{}^{6}C_4=60$

Hence total ways $60+36+4=100.$

61. (b) First omit two particular persons, remaining 8 persons may be 4 in each boat. This can be done in ${}^{8}C_4$ ways. The two particular persons may be placed in two ways one in each boat. Therefore total number of ways are $=2\times{}^{8}C_4$.

62. (b) Required number of ways $={}^{30}C_3-{}^{15}C_3=3605.$

63. (c) The required number of words is

$({}^{2}C_1\times{}^{4}C_2+{}^{2}C_2\times{}^{4}C_1)3!=96.$

64. (c) Four letters can be selected in the following ways

(i) All different *i.e.* C, O, R, G.

(ii) 2 like and 2 different.

(iii) 3 like and 1 different *i.e.* three O and 1 from R, G and C.

The number of ways in (i) is ${}^{4}C_4=1$

The number of ways in (ii) is $1\,.\,{}^{3}C_2=3$

The number of ways in (iii) is $1\times{}^{3}C_1=3$

Therefore, required number of ways = $1+3+3=7.$

65. (a) There can be two types of numbers :

(i) Any one of the digits 1, 2, 3, 4 repeats thrice and the remaining digits only once *i.e.* of the type 1, 2, 3, 4, 4, 4 etc.

(ii) Any two of the digits 1, 2, 3, 4 repeat twice and the remaining two only once *i.e.* of the type 1, 2, 3, 3, 4, 4 etc.

Now number of numbers of the (i) type

$=\frac{6!}{3!}\times{}^{4}C_1=480$

Number of numbers of the (ii) type

$=\frac{6!}{2!\,2!}\times{}^{4}C_2=1080$

Therefore the required number of numbers

$=480+1080=1560.$

66. (b) The total number of two factor products $={}^{200}C_2$. The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore total number of two factor products which are not multiple of 5 is ${}^{160}C_2$.

Hence the required number of factors
$={}^{200}C_2-{}^{160}C_2=7180.$

67. (a) Since the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$.

Therefore the required number $={}^{3+6-1}C_6=28$.

68. (b) The required number $={}^{3+35-1}C_{3-1}={}^{37}C_2=666$

Aliter : The required number = coefficient of x^{35} in $(1+x+x^2+......+x^{35})^3$.

69. (c) The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore the required number $={}^8C_3=56$.

70. (b) Required number $={}^{10}C_5\times{}^8C_4$.

71. (b) ${}^{14}C_4+{}^{14}C_3+{}^{15}C_3+{}^{16}C_3+{}^{17}C_3={}^{18}C_4$.

72. (d) Word 'MATHEMATICS' has 2*M*, 2*T*, 2*A*, *H*, *E*, *I*, *C*, *S*. Therefore 4 letters can be chosen in the following ways.

Case I : 2 alike of one kind and 2 alike of second kind *i.e.*, ${}^3C_2\Rightarrow$ No. of words $={}^3C_2\dfrac{4!}{2!\,2!}=18$

Case II : 2 alike of one kind and 2 different *i.e.*, ${}^3C_1\times{}^7C_2\Rightarrow$ No.of words $={}^3C_1\times{}^7C_2\times\dfrac{4!}{2!}=756$

Case III : All are different *i.e.*, ${}^8C_4\Rightarrow$ No. of words $={}^8C_4\times 4!=1680.$

Hence total number of words are 2454.

73. (a) Number of words of 5 letters in which letters have been repeated any times $=10^5$

But number of words on taking 5 different letters out of $10={}^{10}C_5=252$

Required number of words $=10^5-252=99748$.

74. (a) There can be two types of committees

(i) Containing 3 men and 3 ladies

(ii) Containing 2 men and 4 ladies

Required number of ways

$=({}^8C_3\times{}^4C_3)+({}^8C_2\times{}^4C_4)=252.$

75. (a) Since the person is allowed to select at most n coins out of $(2n+1)$ coins, therefore in order to select one, two, three, ..., n coins. Thus, if T is the total number of ways of selecting one coin, then

$T={}^{2n+1}C_1+{}^{2n+1}C_2+......+{}^{2n+1}C_n=255$ .....(i)

Again the sum of binomial coefficients

$={}^{2n+1}C_0+{}^{2n+1}C_1+{}^{2n+1}C_2+.....+{}^{2n+1}C_n+{}^{2n+1}C_{n+1}$
$+{}^{2n+1}C_{n+2}+.....+{}^{2n+1}C_{2n+1}=(1+1)^{2n+1}=2^{2n+1}$

${}^{2n+1}C_0+2\left({}^{2n+1}C_1+{}^{2n+1}C_2+...+{}^{2n+1}C_n\right)+{}^{2n+1}C_{2n+1}=2^{2n+1}$

$1+2(T)+1=2^{2n+1}\Rightarrow 1+T=\dfrac{2^{2n+1}}{2}=2^{2n}$

$1+255=2^{2n}\Rightarrow 2^{2n}=2^8\Rightarrow n=4$.

76. (d) Required number of ways = $2^{10}-1$

(Since the case that no friend be invited *i.e.*, ${}^{10}C_0$ is excluded).

77. (b) As for given question two cases are possible.

(i) Selecting 4 out of first 5 questions and 6 out of remaining 8 questions $={}^5C_4\times{}^8C_6=140$ choices.

(ii) Selecting 5 out of first 5 questions and 5 out of remaining 8 questions $={}^5C_5\times{}^8C_5=56$ choices.

Total no. of choices = 140 + 56 = 196.

78. (b) Expression = ${}^nC_{r+1}+{}^nC_{r-1}+{}^nC_r+{}^nC_r$

$={}^nC_{r+1}+{}^{n+1}C_r+{}^nC_r={}^{n+1}C_{r+1}+{}^{n+1}C_r={}^{n+2}C_{r+1}$.

79. (b) Since the student is allowed to select at most n books out of $(2n+1)$ books, therefore in order to select one book he has the choice to select one, two, three, ..., n books.

Thus, if T is the total number of ways of selecting one book then

$T={}^{2n+1}C_1+{}^{2n+1}C_2+...+{}^{2n+1}C_n=63$ .....(i)

Again the sum of binomial coefficients

${}^{2n+1}C_0+{}^{2n+1}C_1+{}^{2n+1}C_2+.....+{}^{2n+1}C_n+{}^{2n+1}C_{n+1}$
$+{}^{2n+1}C_{n+2}+....+{}^{2n+1}C_{2n+1}=(1+1)^{2n+1}=2^{2n+1}$

or

${}^{2n+1}C_0+2({}^{2n+1}C_1+{}^{2n+1}C_2+..+{}^{2n+1}C_n)+{}^{2n+1}C_{2n+1}=2^{2n+1}$

$1+2(T)+1=2^{2n+1}$ $1+T=\dfrac{2^{2n+1}}{2}=2^{2n}$

$1+63=2^{2n}\Rightarrow 2^6=2^{2n}\Rightarrow n=3$.

80. (d) We have $\dfrac{(n-1)!}{(n-r-1)!\,r!}=\dfrac{(k^2-3)\,n!}{(n-r-1)!\,(r+1)!}$, $0\le r\le n-1$

$\Rightarrow\quad k^2=\dfrac{r+1}{n}+3,\dfrac{1}{n}\le\dfrac{r+1}{n}\le 1$

$k^2\in\left[\dfrac{1}{n}+3,4\right],n\ge 2$

$k\in\left[-2,-\sqrt{\dfrac{1}{n}+3}\right]\cup\left[\sqrt{\dfrac{1}{n}+3},2\right];n\ge 2$.

81. (b) $\displaystyle\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}=\sum_{r=0}^{n-1}\frac{1}{1+\dfrac{{}^nC_{r+1}}{{}^nC_r}}=\sum_{r=0}^{n-1}\frac{1}{1+\dfrac{n-r}{r+1}}$

$$=\sum_{r=0}^{n-1}\frac{r+1}{n+1}=\frac{1}{n+1}\sum_{r=0}^{n-1}(r+1)=\frac{1}{(n+1)}[1+2+\ldots+n]=\frac{n}{2}.$$

82. (a) Number of ways = coefficient of x^{15} in the expansion $(1+x+x^2+x^3+x^4+x^5)$

$$(1+x+x^2+\ldots\ldots+x^{10})$$
$$(1+x+x^2+\ldots\ldots+x^{15})$$
$$(1+x+x^2+x^3+x^4+x^5)(1+x+x^2+\ldots..+x^{10})$$
$$(1+x+x^2+\ldots+x^{15})=(1-x^6-x^{11})(1+{}^3C_1x+{}^4C_2x^2$$
$$+\ldots\ldots+{}^6C_4x^4+{}^{11}C_9x^9+{}^{17}C_{15}x^{15}+\ldots\ldots\ldots.)$$
$$=\ldots\ldots.+\ldots\ldots.+x^{15}(-{}^{11}C_9-{}^6C_4+{}^{17}C_{15})$$
$$=\ldots\ldots.+\ldots\ldots+x^{15}(-55-15+136)=x^{15}\times 66$$

Coefficient of $x^{15}=66$.

83. (b) ${}^{50}C_4+\left({}^{50}C_3+{}^{51}C_3+{}^{52}C_3+\ldots\ldots{}^{55}C_3\right)$. Taking first two terms together and adding them and following the same pattern, we get ${}^{56}C_4$, $[As\ {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r]$.

84. (b) Given ${}^nC_{12}={}^nC_6$

$12+6=n\Rightarrow n=18$

${}^{18}C_2=\frac{18\times 17}{2}=9\times 17=153.$

85. (a) Number of ways for select 4 Questions out of 5 Questions.

Number of ways = ${}^5C_4=5$

Remaining Questions =8

Remaining Questions for solving =6

Number of ways ${}^8C_6=28$

Number of total ways $={}^5C_4\times{}^8C_6=5\times 28=140$.

Geometrical problems

1. (c) The required number of ways $={}^8C_3-{}^5C_3-{}^3C_3$.

{Since total points are 8, but 5 are collinear and other three are also collinear}.

2. (b) Required number of ways are ${}^8C_2-8=20$.

3. (b) Since ${}^nC_2-n=44\Rightarrow n=11$.

4. (a) Required number of ways = ${}^4C_3=4$.

5. (a) The number of triangles are ${}^9C_3=84$.

6. (c) Required number of diagonals = ${}^mC_2-m$

$$=\frac{m(m-1)}{2!}-m=\frac{m}{2!}(m-3).$$

7. (d) Required number of ways ${}^8C_2=28$.

8. (a) Required number of ways $={}^{12}C_3-{}^7C_3$

$=220-35=185$.

9. (b) Required number of triangles

$={}^{10}C_3-{}^4C_3=120-4=116$.

10. (a) The number of lines are

${}^{16}C_2-{}^6C_2+1=120-15+1=106$.

11. (b) Total number of points are $m+n+k$, the $\Delta's$ formed by these points $={}^{m+n+k}C_3$

Joining 3 points on the same line gives no triangle, such $\Delta's$ are ${}^mC_3+{}^nC_3+{}^kC_3$

Required number $={}^{m+n+k}C_3-{}^mC_3-{}^nC_3-{}^kC_3$.

12. (b) Required number of ways $={}^4C_2\times{}^3C_2=18$.

13. (c) Number of lines from 6 points $={}^6C_2=15$. Points of intersection obtained from these lines $={}^{15}C_2=105$. Now we find the number of times, the original 6 points come.

Consider one point say A_1 . Joining A_1 to remaining 5 points, we get 5 lines, and any two lines from these 5 lines give A_1 as the point of intersection.

$\therefore$ A_1 come ${}^5C_2=10$ times in 105 points of intersections.

Similar is the case with other five points.

$\therefore$ 6 original points come $6\times 10=60$ times in points of intersection.

Hence the number of distinct points of intersection $=105-60+6=51$.

14. (a) **Case I :** When A is excluded.

Number of triangles = selection of 2 points from AB and one point from $AC+$ selection of one point from AB and two points from AC

$$={}^mC_2{}^nC_1+{}^mC_1{}^nC_2=\frac{1}{2}(m+n-2)mn \quad \ldots..(i)$$

Case II : When A is included.

The triangles with one vertex at $A=$ selection of one point from AB and one point from $AC=mn$.

$\therefore$ Number of triangles

$$=mn+\frac{1}{2}mn(m+n-2)=\frac{1}{2}mn(m+n) \quad \ldots..(ii)$$

$\therefore$ Required ratio $=\frac{(m+n-2)}{(m+n)}$.

15. (c) Since no two lines are parallel and no three are concurrent, therefore n straight lines intersect at ${}^nC_2=N$ (say) points. Since two points are required to determine a straight line, therefore the total number of lines obtained by joining N points ${}^NC_2$. But in this each old line has been counted ${}^{n-1}C_2$ times, since on each old line there will be $n-1$ points of intersection made by the remaining $(n-1)$ lines.

Hence the required number of fresh lines is

$${}^NC_2 - n.\,{}^{n-1}C_2 = \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{{}^nC_2({}^nC_2-1)}{2} - \frac{n(n-1)(n-2)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}.$$

16. (c) Each set is having $m+2$ parallel lines and each parallelogram is formed by choosing two straight lines from the first set and two straight lines from the second set. Two straight lines from the first set can be chosen in ${}^{m+2}C_2$ ways and two straight lines from the second set can be chosen in ${}^{m+2}C_2$ ways.

Hence the total number of parallelograms formed $= {}^{m+2}C_2\,.\,{}^{m+2}C_2 = \left({}^{m+2}C_2\right)^2$.

17. (a) The number of points of intersection of 37 straight lines is ${}^{37}C_2$. But 13 of them pass through the point A. Therefore instead of getting ${}^{13}C_2$ points we get merely one point.

Similarly 11 straight lines out of the given 37 straight lines intersect at B. Therefore instead of getting ${}^{11}C_2$ points, we get only one point. Hence the number of intersection points of the lines is ${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$.

18. (d) The required number of points

$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + \left({}^8C_1 \times {}^4C_1\right) \times 2$

$= 28 + 12 + 32 \times 2 = 104$

19. (c,b) 18 points, 5 collinear :

(i) Number of lines $= {}^{18}C_2 - {}^5C_2 + 1 = 153 - 10 + 1 = 144$

(ii) Number of $\Delta's = {}^{18}C_3 - {}^5C_3 = 816 - 10 = 806$.

20. (a) ${}^{16}C_3 - {}^8C_3 = 504$.

21. (b) Clearly, ${}^nC_3 = T_n$.

So, ${}^{n+1}C_3 - {}^nC_3 = 21$ $\quad ({}^nC_3 + {}^nC_2) - {}^nC_3 = 21$

${}^nC_2 = 21$ or $n(n-1) = 42 = 7.6$ $\therefore$ $n = 7$.

22. (a) Number of triangles $= {}^{10}C_3 - {}^6C_3 = 120 - 20 = 100$.

23. (c) Given, total number of points = n and number of collinear points = p. We know that one line has two end points. Therefore total number of lines = ${}^nC_2$. Since p points are collinear, therefore total number of lines drawn from collinear points = ${}^pC_2$. We also know that, corresponding to the line of collinearity, one will also be added.

Therefore number of lines = ${}^nC_2 - {}^pC_2 + 1$.

24. (b) No. of triangles = ${}^6C_3 - 6$.

25. (c) Since ${}^nC_2 - n = 35$ $\qquad \dfrac{n!}{2!\,(n-2)!} - n = 35$

$n(n-1) - 2n = 70 \qquad n^2 - 3n = 70$

$n^2 - 3n - 70 = 0 \qquad (n+7)(n-10) = 0 \qquad n = 10$.

26. (d) Required number $= {}^{20}C_2 - {}^4C_2 + 1$

$= \dfrac{20 \times 19}{2} - \dfrac{4 \times 3}{2} + 1 = 190 - 6 + 1 = 185$.

Multinomial theorem, Number of divisors, Miscellaneous problems

1. (a) The conditions provided that $n - r = r - 1$ $\quad r = \dfrac{n+1}{2}$.

So if we put $n = 3$, then $r = 2$ satisfies the conditions.

2. (d) On simplification you get required result.

3. (a) The total number of 4 digits are $9999 - 999 = 9000$.

The numbers of 4 digits number divisible by 5 are $90 \times 20 = 1800$. Hence required number of ways are $9000 - 1800 = 7200$.

{Since there are 20 numbers in each hundred (1 to 100) divisible by 5 and from 999 to 9999 there are 90 hundreds, hence the results}.

4. (d) $\dfrac{{}^nP_r}{{}^nC_r} = 24 \Rightarrow r! = 24 \Rightarrow r = 4$

$\therefore {}^nC_4 = 35 \Rightarrow n = 7$.

5. (a) By inspection $n = 5$.

6. (b) Required number of ways

= Coefficient of x^{16} in $(x^3 + x^4 + x^5 + ... + x^7)^4$

= Coefficient of x^{16} in $x^{12}(1 + x + x^2 + ... x^4)^4$

= Coefficient of x^{16} in $x^{12}(1 - x^5)^4(1 - x)^{-4}$

= Coefficient of x^4 in $(1 - x^5)^4(1 - x)^{-4}$

= Coefficient of x^4 in $(1 - 4x^5 + ...)$

$$\left[1 + 4x + ... + \frac{(r+1)(r+2)(r+3)}{3!}x^r\right]$$

$= \dfrac{(4+1)(4+2)(4+3)}{3!} = 35$.

Aliter : Remaining 4 rupees can be distributed in ${}^{4+4-1}C_{4-1}$ *i.e.*, 35 ways.

7. (d) The number of sub-sets of the set which contain at most n elements is

${}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n = S$ (Say)

Then $2S = 2({}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n)$

$= ({}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}) + ({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + ...$

$.... + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1})$

$\{\because {}^nC_r = {}^nC_{n-r}\}$

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow \quad S = 2^{2n}$.

8. (c) Since $9600 = 2^7 \times 3 \times 5^2$

Hence, number of divisors $= (7+1)(1+1)(2+1) = 48$.

9. (c) The number of selections = coefficient of x^8 in $(1+x+x^2+........+x^8)(1+x+x^2+......+x^8).(1+x)^8$

= coefficient of x^8 in $\dfrac{(1-x^9)^2}{(1-x)^2}(1+x)^8$

= coefficient of x^8 in $(1+x)^8(1-x)^{-2}$

= coefficient of x^8 in $({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + ... + {}^8C_8x^8)$

$\times(1+2x+3x^2+4x^3+......+9x^8+.....)$

$= 9\,.\,{}^8C_0 + 8\,.\,{}^8C_1 + 7\,.\,{}^8C_2 + ... + 1\,.\,{}^8C_8$

$= C_0 + 2C_1 + 3C_2 + ... + 9C_8 \qquad [C_r = {}^8C_r]$

Now $C_0x + C_1x^2 + ... + C_8x^9 = x(1+x)^8$

Differentiating with respect to x, we get

$C_0 + 2C_1x + 3C_2x^2 + ...9C_8x^8 = (1+x)^8 + 8x(1+x)^7$

Putting $x = 1$, we get $C_0 + 2C_1 + 3C_2 + ... + 9C_8$

$= 2^8 + 8\,.\,2^7 = 2^7.(2+8) = 10\,.\,2^7$.

10. (c) $\dfrac{{}^nP_4}{{}^nC_5} = 30 \Rightarrow \dfrac{n!}{(n-4)!} \times \dfrac{5!\,.\,(n-5)!}{n!} = 30$

$\Rightarrow (n-4) = 4 \Rightarrow n = 8$.

11. (b) The number of triplets of positive integers which are solutions of $x+y+z = 100$.

= coefficient of x^{100} in $(x+x^2+x^3+.....)^3$

= coefficient of x^{100} in $x^3(1-x)^{-3}$

= coefficient of x^{100} in

$x^3\left(1+3x+6x^2+....+\dfrac{(n+1)(n+2)}{2}x^n+.....\right)$

$= \dfrac{(97+1)(97+2)}{2} = 49 \times 99 = 4851$.

12. (d) The numbers of divisors

$= (1+1)(2+1)(2+1)(1+1)(1+1) - 1 = 71$.

13. (b) Since at any place, any of the digits 2, 5 and 7 can be used, total number of such positive n-digit numbers are 3^n. Since we have to form 900 distinct numbers, hence $3^n \ge 900 \Rightarrow n = 7$.

14. (a) Since, 38808 = 8 × 4851

$= 8 \times 9 \times 539 = 8 \times 9 \times 7 \times 7 \times 11 = 2^3 \times 3^2 \times 7^2 \times 11$

So, number of divisors

= (3 + 1) (2 + 1) (2 + 1) (1 + 1) = 72.

This includes two divisors 1 and 38808. Hence, the required number of divisors = 72 – 2 = 70.

15. (c) Given, ${}^nP_4 = 24.{}^nC_5$.

Therefore, $n(n-1)(n-2)(n-3)$

$= 24 \times \dfrac{n(n-1)(n-2)(n-3)(n-4)}{5.4.3.2.1}$

$\Rightarrow 1 = \dfrac{(n-4)}{5} \qquad n-4 = 5 \qquad n = 9$.

16. (a) ${}^nP_r = 720\,{}^nC_r$

${}^nP_r \div {}^nC_r = 720 \qquad r! = 720 = 6! \qquad r = 6$.

17. (b) For $m = 5$, $\displaystyle\sum_{i=0}^{5}\binom{10}{i}\binom{20}{5-i}$

$= \binom{10}{0}\binom{20}{5} + \binom{10}{1}\binom{20}{4} + ... + \binom{10}{5}\binom{20}{0}$,

for $m = 10$, $\displaystyle\sum_{i=0}^{10}\binom{10}{i}\binom{20}{10-i}$

$= \binom{10}{0}\binom{20}{10} + \binom{10}{1}\binom{20}{9} + \binom{10}{2}\binom{20}{8} + ... + \binom{10}{10}\binom{20}{0}$,

for $m = 15$, $\displaystyle\sum_{i=0}^{15}\binom{10}{i}\binom{20}{15-i}$

$= \binom{10}{0}\binom{20}{15} + \binom{10}{1}\binom{20}{14} + \binom{10}{2}\binom{20}{13} + .. + \binom{10}{10}\binom{20}{5}$

and for $m = 20$, $\displaystyle\sum_{i=0}^{20}\binom{10}{i}\binom{20}{20-i}$

$= \binom{10}{0}\binom{20}{20} + \binom{10}{1}\binom{20}{19} + ... + \binom{10}{10}\binom{20}{10}$

Clearly, the sum is maximum for $m = 15$.

Note that ${}^{10}C_r$ is maximum for $r = 5$ and ${}^{20}C_r$ is maximum for $r = 10$. Note that the single term ${}^{10}C_5 \times {}^{20}C_{10}$ (in case $m = 15$) is greater than the sum

${}^{10}C_0\,{}^{20}C_{10} + {}^{10}C_1\,{}^{20}C_9 + {}^{10}C_2\,{}^{20}C_8 + ...$

${}^{10}C_8\,{}^{20}C_2 + {}^{10}C_9\,{}^{20}C_1 + {}^{10}C_{10}\,{}^{20}C_0$ (in case $m = 10$).

Also the sum in case $m = 10$ is same as that in case $m = 20$.

18. (a) Given number is 960, we know that $960 = 2^6 \times 3^1 \times 5^1$. Therefore bases are $p_1 = 2, p_2 = 3$ and $p_3 = 5$. and powers $a_1 = 6$, $a_2 = 1$ and $a_3 = 1$. Thus sum of all the positive divisors of 960

$= \left(\dfrac{p_1^{a_1+1}-1}{p_1-1}\right)\left(\dfrac{p_2^{a_2+1}-1}{p_2-1}\right)\left(\dfrac{p_3^{a_3+1}-1}{p_3-1}\right)$

$= \left(\dfrac{2^{6+1}-1}{2-1}\right)\left(\dfrac{3^{1+1}-1}{3-1}\right)\left(\dfrac{5^{1+1}-1}{5-1}\right) = (127)(4)(6) = 3048$.

19. (b)

3 men and 2 women equal to 5. A group of 5 members make 5! permutation with each other.

The number of ways to sit 5 members = 5!

6 Places are filled by 5 members by ${}^6C_5$ ways

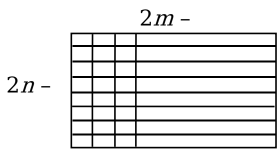
The total number of ways to sit 5 members on 6 seats of a bus = ${}^6C_5 \times 5!$.

20. (d) Along horizontal side one unit can be taken in (2*m*–1) ways and 3 unit side can be taken in $2m-3$ ways.

The number of ways of selecting a side horizontally is

$(2m-1+2m-3+2m-5+....+3+1)$

Similarly the number of ways along vertical side is $(2n-1+2n-3+....+5+3+1)$.



Total number of rectangles

$=[1+3+5+.....+(2m-1)]\times[1+3+5+....+(2n-1)]$

$=\frac{m(1+2m-1)}{2}\times\frac{n(1+2n-1)}{2}=m^2n^2$.

21. (b) $P(n,r)=1680 \frac{n!}{(n-r)!}=1680$ .....(i)

$C(n,r)=70 \quad \frac{n!}{r!(n-r)!}=70$ .....(ii)

$\frac{1680}{r!}=70$, [From (i) and (ii)]

$r!=\frac{1680}{70}=24 \quad r=4 \quad \because P(n,4)=1680$

$\because n(n-1)(n-2)(n-3)=1680 \quad n=8$

$\because \ 8\times7\times6\times5=1680$

Now $69n+r!=69\times8+4!=552+24=576$.

Critical Thinking Questions

1. (a) $1.3.5......(2n-1)2^n=\frac{1.2.3.4.5.6....(2n-1)(2n)2^n}{2.4.6....2n}$

$=\frac{(2n)!\,2^n}{2^n(1.2.3.....n)}=\frac{(2n)!}{n!}$.

2. (c) The number of ways that the candidate may select 2 questions from A and 4 from $B={}^5C_2\times{}^5C_4$

3 questions from A and 3 from $B={}^5C_3\times{}^5C_3$

4 questions from A and 2 from $B={}^5C_4\times{}^5C_2$

Hence total number of ways are 200.

3. (b) The total number between 10 and 1000 are 989 but we have to form the numbers by using numerals $1,2,......9$, *i.e.* 0 is not occuring so the numbers containing any '0' would be excluded *i.e.*,

Required number of ways

$$=989-\left\{\begin{array}{l}20,30,40,..............100=9\\101,102,................200=19\\201,.....................300=19\\.....................................\\.....................................\\901,.....................990=18\end{array}\right\}$$

$=989-(9+18+19\times8)=810$.

Aliter : Between 10 and 1000, the numbers are of 2 digits and 3 digits.

Since repetition is allowed, so each digit can be filled in 9 ways.

Therefore number of 2 digit numbers $=9\times9=81$

and number of 3 digit numbers $9\times9\times9=729$

Hence total ways $=81+729=810$.

4. (c) $\bullet T\bullet R\bullet N\bullet G\bullet L$

Three vowels can be arrange at 6 places in ${}^6P_3=120$ ways. Hence the required number of arrangements $=120\times5!=14400$.

5. (c) Since number of derangements in such a problems is given by

$n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-.......(-1)^n\frac{1}{n!}\right\}$

$\therefore$ Number of derangements are

$=4!\left\{\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right\}=12-4+1=9$.

6. (b) $\frac{56!}{(50-r)!}\times\frac{(51-r)!}{54!}$

$=\frac{30800}{1}\Rightarrow 56\times55\times(51-r)=30800 \Rightarrow r=41$.

7. (a) Number of words in which all the 5 letters are repeated $=10^5=100000$ and the number of words in which no letter is repeated are ${}^{10}P_5=30240$.

Hence the required number of ways are

100000 – 30240 = 69760.

8. (a) For the first set number of ways ${}^{52}C_{17}$. Now out of 35 cards left 17 cards can be put for second in ${}^{35}C_{17}$ ways similarly for 3rd in ${}^{18}C_{17}$. One card for the last set can be put in only one way. Therefore the required number of ways for the proper distribution

$=\frac{52!}{35!17!}\times\frac{35!}{18!17!}\times\frac{18!}{17!1!}\times1!=\frac{52!}{(17!)^3}$.

9. (b) The word ARRANGE, has AA, RR, NGE letters, that is two A' s, two R's and N, G, E one each.

$\therefore$ The total number of arrangements

$= \frac{7!}{2!2!1!1!1!} = 1260$

But, the number of arrangements in which both RR are together as one unit $= \frac{6!}{2!1!1!1!1!} = 360$

$\therefore$ The number of arrangements in which both RR do not come together $= 1260 - 360 = 900$.

10. (a) A selection of 3 balls so as to include at least one black ball, can be made in the following 3 mutually exclusive ways

(i) 1 black ball and 2 others $= {}^3C_1 \times {}^6C_2 = 3 \times 15 = 45$

(ii) 2 black balls and one other $= {}^3C_2 \times {}^6C_1 = 3 \times 6 = 18$

(iii) 3 black balls and no other $= {}^3C_3 = 1$

$\therefore$ Total numbers of ways = 45 + 18 + 1 = 64.

11. (a) First arrange m men, in a row in m ! ways. Since $n < m$ and no two women can sit together, in any one of the $m!$ arrangement, there are $(m+1)$ places in which n women can be arranged in ${}^{m+1}P_n$ ways.

$\therefore$ By the fundamental theorem, the required number of arrangements of m men and n women $(n < m)$

$= m! . {}^{m+1}P_n = \frac{m!.(m+1)!}{\{(m+1)-n\}!} = \frac{m!(m+1)!}{(m-n+1)!}$.

12. (a) We know that a five digit number is divisible by 3, if and only if sum of its digits (= 15) is divisible by 3, therefore we should not use 0 or 3 while forming the five digit numbers. Now, (i) In case we do not use 0 the five digit number can be formed (from the digit 1, 2, 3, 4, 5) in ${}^5P_5$ ways.

(ii) In case we do not use 3, the five digit number can be formed (from the digit 0, 1, 2, 4, 5) in ${}^5P_5 - {}^4P_4 = 5! - 4! = 120 - 24 = 96$ ways.

$\therefore$ The total number of such 5 digit number

$= {}^5P_5 + ({}^5P_5 - {}^4P_4) = 120 + 96 = 216$.

13. (b) Since the number of students giving wrong answers to at least i question $(i = 1, 2, ..., n) = 2^{n-i}$.

The number of students answering exactly $i(1 \le i \le -1)$ questions wrongly = {the number of students answering at least i questions wrongly, $i = 1, 2, ...n)$} – {the number of students answering at least $(i+1)$ questions wrongly $(2 \le i+1 \le n)$} $= 2^{n-i} - 2^{n-(i+1)} (1 \le i \le n-1)$.

Now, the number of students answering all the n questions wrongly $= 2^{n-n} = 2^0$.

Thus the total number of wrong answers

$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4})$

$+ ....+ (n-1)(2^1 - 2^0) + n(2^0)$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + ...+ 2^0 = 2^n - 1$

($\because$ Its a G.P.)

$\therefore$ As given

$2^n - 1 = 2047 \Rightarrow 2^n = 2048 = 2^{11} \Rightarrow n = 11$

14. (b) To find the number of times 3 occurs in listing the integer from 1 to 999. (since 3 does not occur in 1000). Any number between 1 to 999 is a 3 digit number xyz where the digit x, y, z are any digits from 0 to 9.

Now, we first count the numbers in which 3 occurs once only. Since 3 can occur at one place in ${}^3C_1$ ways, there are ${}^3C_1 . (9 \times 9) = 3 . 9^2$ such numbers.

Again, 3 can occur in exactly two places in ${}^3C_1(9)$ such numbers. Lastly 3 can occur in all the three digits in one such number only 3337.

$\therefore$ The number of times 3 occurs is equal to

$1 \times (3 \times 9^2) + 2 \times (3 \times 9) + 3 \times 1 = 300$.

15. (a) For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in ${}^{10}C_3$ ways.

The remaining 7 persons can speak in $7!$ ways. Hence, the number of ways in which all the 10 person can speak is ${}^{10}C_3 . 7! = \frac{10!}{3!} . = \frac{10!}{6}$.

16. (a) Since the minimum marks to any question is two, the maximum marks that can be assigned to any questions is $16 (= 30 - 2 \times 7), n_1 + n_2 + ...+ n_8 = 30$. If n_i are the marks assigned to i^{th} questions, then $n_1 + n_2 + ...+ n_8 = 30$ with $2 \le n_i \le 16$ for $i = 1, 2, ...8$. Thus the required number of ways

= the coefficient of x^{30} in $(x^2 + x^3 + ...+ x^{16})^8$

= the coefficient of x^{30} in $x^{16}(1 + x + ...x^{14})^8$

= the coefficient of x^{30} in $x^{16}\left(\frac{1-x^{15}}{1-x}\right)^8$

= the coefficient of x^{14} in $(1-x)^{-8} . (1-x^{15})^8$

= the coefficient of x^{14} in

$\left\{1 + \frac{8}{1!}x + \frac{8.9}{2!}x^2 + \frac{8.9.10}{3!}x^3 + ...\right\}(1 - {}^8C_1 x^{15} + ...)$

= the coefficient x^{14} in $\{1 + {}^8C_1 x + {}^9C_2 x^2 + {}^{10}C_3 x^3 + ...\}$

since the second bracket has powers of x^0, x^{15} etc.

$= {}^{21}C_{14} = {}^{21}C_7$.

17. (a) Required number of ways are $\frac{8!}{2!3!} \times \frac{5!}{4!} = 16800$.

{Since IEEEENDPNDNC = 8 letters}.

18. (c) Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and in all five balls have to put in. There will be two possibilities.

(i) Any two containing one and 3^{rd} containing 3.

$A(1)\ B(1)\ C\ (3)$

${}^5C_1 . {}^4C_1 . {}^3C_3 = 5.4.1 = 20$.

Since the box containing 3 balls could be any of the three boxes A, B, C.

Hence the required number is $= 20 \times 3 = 60$.

(ii) Any two containing 2 each and 3^{rd}containing 1.

$A(2)\ B(2)\ C\ (1)$

${}^5C_2 . {}^3C_2 . {}^1C_1 = 10 \times 3 \times 1 = 30$

Since the box containing 1 ball could be any of the three boxes A, B, C.

Hence the required number is $= 30 \times 3 = 90$.

Hence total number of ways are $= 60 + 90 = 150$.

19. (b) The number of ways can be deduce as follows :

1 woman and 4 men $= {}^4C_1 \times {}^6C_4 = 60$

2 women and 3 men $= {}^4C_2 \times {}^6C_3 = 120$

3 women and 2 men $= {}^4C_3 \times {}^6C_2 = 60$

4 women and 1 man $= {}^4C_4 \times {}^6C_1 = 6$

Required number of ways $= 60 + 120 + 60 + 6 = 246$.

20. (c) The total number of words $= \frac{6!}{2!} = 360$.

The number of words in which BH come together are

$\frac{5!}{2!} \times 2! = 120$.

Hence required number of ways in which B and H never came together are $360 - 120 = 240$.

21. (d) Out of 10 persons, A is in and G and H are out of the team, so we have to select 4 more from 7 remaining. This can be done in ${}^7C_4$ ways. These 5 persons can be arranged in a line in $5!$ ways. Hence the number of possible arrangements is ${}^7C_4 . 5! = {}^7C_3 .(5!)$.

22. (c) Since 5 does not occur in 1000, we have to count the number of times 5 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form $xyz, 0 \le x, y, z \le 9$. The numbers in which 5 occurs exactly once $= ({}^3C_1).9 \times 9 = 243$.

The numbers in which 5 occurs exactly twice $= ({}^3C_2 .9) = 27$

The numbers in which 5 occurs in all three digits $= 1$

Hence, the number of times 5 occurs is

$1 \times 243 + 2 \times 27 + 3 \times 1 = 300$.

23. (c) Let $E(n)$ denote the exponent of 3 in n. The greatest integer less than 100 divisible by 3 is 99.

We have $E(100!) = E(1.2.3.4....99.100)$

$= E(3.6.9....99) = E[(3.1)(3.2)(3.3)........(3.33)]$

$= 33 + E(1.2.3.....33)$

Now $E(1.2.3.....33) = E(3.6.9....33)$

$= E[(3.1)(3.2)(3.3)........(3.11)]$

$= 11 + E(1.2.3.....11)$

and

$E(1.2.3....11) = E(3.6.9) = E[(3.1)(3.2)(3.3)]$

$3 + E(1.2.3) = 3 + 1 = 4$

Thus $E(100!) = 33 + 11 + 4 = 48$.

24. (c) Here we have $1M, 4I, 4S$ and $2P$.

Therefore total number of selections of one or more letters $= (1+1)(4+1)(4+1)(2+1) - 1 = 149$.

25. (c) The required number

= coefficient of x^{2m} in $(x^0 + x^1 + ...+ x^m)^4$

= coefficient of x^{2m} in $\left(\frac{1-x^{m+1}}{1-x}\right)^4$

= coefficient of x^{2m} in $(1-x^{m+1})^4(1-x)^{-4}$

= coefficient of x^{2m} in $(1 - 4x^{m+1} + 6x^{2m+2} + ...)$

$\left(1 + 4x + ...+ \frac{(r+1)(r+2)(r+3)}{3!}x^r + ...\right)$

$= \frac{(2m+1)(2m+2)(2m+3)}{6} - 4m\frac{(m+1)(m+2)}{6}$

$= \frac{(m+1)(2m^2+4m+3)}{3}$.

26. (c) Let there be n men participants. Then the number of games that the men play between themselves is $2.{}^nC_2$ and the number of games that the men played with the women is $2.(2n)$.

$\therefore 2.\,{}^nC_2 - 2.2n = 66$ (By hypothesis)

$\Rightarrow n^2 - 5n - 66 = 0 \Rightarrow n = 11$

$\therefore$ Number of participants $= 11\text{ men} + 2\text{ women} = 13$.

27. (b) Each child will go as often as he (or she) can be accompanied by two others.

Therefore the required number is ${}^7C_2 = 21$.

28. (b) Total number of books $= a + 2b + 3c + d$

Since there are b copies of each of two books, c copies of each of three books and single copies of d books. Therefore the total number of arrangements is $\dfrac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$.

29. (b) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.

Therefore the required number of ways in which the car can be filled is ${}^5C_2 \times {}^2C_1 = 20$.

30. (d) Since the balls are to be arranged in a row so that the adjacent balls are of different colours, therefore we can begin with a white ball or a black ball. If we begin with a white ball, we find that $(n+1)$ white balls numbered 1 to $(n+1)$ can be arranged in a row in $(n+1)!$ ways. Now $(n+2)$ places are created between $n+1$ white balls which can be filled by $(n+1)$ black balls in $(n+1)!$ ways.

So the total number of arrangements in which adjacent balls are of different colours and first ball is a white ball is $(n+1)! \times (n+1)! = [(n+1)!]^2$. But we can begin with a black ball also. Hence the required number of arrangements is $2[(n+1)!]^2$.

31. (a) 12 persons can be seated around a round table in 11! ways. The total number of ways in which 2 particular persons sit side by side is $10! \times 2!$. Hence the required number of arrangements $= 11! - 10! \times 2! = 9 \times 10!$.

32. (a) A number between 5000 and 10,000 can have any of the digits 5, 6, 7, 8, 9 at thousand's place. So thousand's place can be filled in 5 ways. Remaining 3 places can be filled by the remaining 8 digits in ${}^8P_3$ ways. Hence required number $= 5 \times {}^8P_3$.

33. (c) The result is trivially true for $r = 1, 2$. It can be easily proved by the principle of mathematical induction that the result is true for r also.

34. (c) We have got $2P^s, 2R^s, 3O^s, 1I, 1T, 1N$ *i.e.* 6 types of letters. We have to form words of 4 letters. We consider four cases

(i) All 4 different : Selection ${}^6C_4 = 15$

Arrangement $= 15.4! = 15 \times 25 = 360$

(ii) Two different and two alike :

P^s, R^s and O^s in ${}^3C_1 = 3$ ways. Having chosen one pair we have to choose 2 different letters out of the remaining 5 different letters in ${}^5C_2 = 10$ ways. Hence the number of selections is $10 \times 3 = 30$. Each of the above 30 selections has 4 letters out of which 2 are alike and they can be arranged in $\dfrac{4!}{2!} = 12$ ways.

Hence number of arrangements is $12 \times 30 = 360$.

(iii) 2 like of one kind and 2 of other :

Out of these sets of three like letters we can choose 2 sets in ${}^3C_2 = 3$ ways. Each such selection will consist of 4 letters out of which 2 are alike of one kind, 2 of the other. They can be arranged in $\dfrac{4!}{2!2!} = 6$ ways.

Hence the number of arrangements is $3 \times 6 = 18$.

(iv) 3 alike and 1 different :

There is only one set consisting of 3 like letters and it can be chosen in 1 way. The remaining one letter can be chosen out of the remaining 5 types of letters in 5 ways.

Hence the number of selection $= 5 \times 1$. Each consists of 4 letters out of which 3 are alike and each of them can be arranged in $\dfrac{4!}{3!} = 4$ ways.

Hence the number of arrangements is $5 \times 4 = 20$.

From (i), (ii), (iii) and (iv), we get

Number of selections $= 15 + 30 + 3 + 5 = 53$

Number of arrangements

$= 360 + 360 + 18 + 20 = 758$.

35. (d) In MORADABAD, we have 6 different types of letters $3A^s$, $2D^s$ and rest four different. We have to form words of 4 letters.

(i) All different $^6P_4 = 6 \times 5 \times 4 \times 3 = 360$.

(ii) Two different two alike $^2C_1 \times {}^5C_2 \times \frac{4\,!}{2\,!} = 240$

(iii) 3 alike 1 different $^1C_1 \times {}^5C_1 \times \frac{4\,!}{3\,!} = 20$

(iv) 2 alike of one type and 2 alike of other type

$^2C_2 \times \frac{4\,!}{2\,!\,2\,!} = 6$

Therefore total number of words

$= 360 + 240 + 20 + 6 = 626$.

36. (b) Seven boys can be seated in a row in $7\,!$ ways. Hence the total no. of arrangement such that no two girls seated together $= 7\,! \times {}^8P_3$.

37. (d) Expression $= {}^nC_r + 2.\,{}^nC_{r-1} + {}^nC_{r-2}$

$= ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$.

38. (b) We have, 30 = 2 × 3 × 5. So, 2 can be assigned to either *a* or *b* or *c* *i.e.* 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the no. of solutions are 3 × 3 × 3 = 27.

39. (c) There are four even places for the four odd digits 3, 3, 5, 5.

The required no. of ways $= \frac{4!}{2!2!} . \frac{5!}{2!\,3!} = 60$.

40. (a) The number of words before the word CRICKET is $4 \times 5! + 2 \times 4! + 2! = 530$.