67. (b) $BA=\begin{bmatrix}2\\3\\4\end{bmatrix}_{3\times 1}[1\ 2\ 3]_{1\times 3}=\begin{bmatrix}2&4&6\\3&6&9\\4&8&12\end{bmatrix}_{3\times 3}$

$AB=[1 2 3]_{1\times 3}\begin{bmatrix}2\\3\\4\end{bmatrix}_{3\times 1}=[20]_{1\times 1}$.

So, AB and BA are defined.

68. (d) It is a property.

69. (d) Given, A and B are square matrices of order $n \times n$. We know that $(A-B)^2=(A-B)(A-B)$

$$= A^2 - AB - BA + B^2$$

Note that $AB \neq BA$ in general.

70. (a) We know that every identity matrix is a scalar matrix.

71. (b) $AB=\begin{bmatrix}1&0\\2&0\end{bmatrix}\begin{bmatrix}0&0\\1&12\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O$

while $BA=\begin{bmatrix}0&0\\1&12\end{bmatrix}\begin{bmatrix}1&0\\2&0\end{bmatrix}=\begin{bmatrix}0&0\\25&0\end{bmatrix}\neq O$.

72. (c) We know that if all the elements below the diagonal in the matrix are zero, then it is an upper triangular matrix.

73. (a) $A.A=\begin{bmatrix}-1&2i\\0&-1\end{bmatrix}$, $A^4=\begin{bmatrix}1&-4i\\0&1\end{bmatrix}$.

74. (c) $\begin{bmatrix}1\\-1\\2\end{bmatrix}[2\ \ 1\ \ -1]=\begin{bmatrix}2&1&-1\\-2&-1&1\\4&2&-2\end{bmatrix}$.

75. (c) For subtraction of two matrix, they should be of the same order *i.e.* $p=r$, $q=s$.

76. (b) $A=\begin{bmatrix}1&3\\2&1\end{bmatrix}$

$A^2=\begin{bmatrix}1&3\\2&1\end{bmatrix}\begin{bmatrix}1&3\\2&1\end{bmatrix}=\begin{bmatrix}7&6\\4&7\end{bmatrix}$

and $A^2-2A=\begin{bmatrix}5&0\\0&5\end{bmatrix}$, $\det(A^2-2A)=\begin{vmatrix}5&0\\0&5\end{vmatrix}=25$.

77. (c) $AB=O \Rightarrow |AB|=0$

$|A|.|B|=0$ $\quad$ $|A|=0$ or $|B|=0$

When $AB=O$, neither A nor B may be O.

For example if $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ and $B=\begin{bmatrix}0&0\\1&0\end{bmatrix}$, then

$AB=\begin{bmatrix}0&0\\0&0\end{bmatrix}$.

78. (b) $a_{ij}=\frac{1}{2}(3i-2j)$

$a_{11}=1/2$, $a_{12}=-1/2$ and $a_{21}=2$, $a_{22}=1$

$\therefore\ A=[a_{ij}]_{2\times 2}=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}$

$\therefore\ A=\begin{bmatrix}1/2&-1/2\\2&1\end{bmatrix}$.

79. (c) $2X-\begin{bmatrix}1&2\\7&4\end{bmatrix}=\begin{bmatrix}3&2\\0&-2\end{bmatrix}$

$2X=\begin{bmatrix}3&2\\0&-2\end{bmatrix}+\begin{bmatrix}1&2\\7&4\end{bmatrix}$

$2X=\begin{bmatrix}4&4\\7&2\end{bmatrix}$ $\quad X=\begin{bmatrix}2&2\\7/2&1\end{bmatrix}$.

80. (a) $A^2=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&4\\0&1\end{bmatrix}$ and $A^3=A^2A$.

$=\begin{bmatrix}1&4\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&6\\0&1\end{bmatrix}$ and so on.

$\therefore\ A^n=\begin{bmatrix}1&2n\\0&1\end{bmatrix}$.

81. (c) Given, $kA=\begin{bmatrix}0&3a\\2b&24\end{bmatrix}$ $\quad k\begin{bmatrix}0&2\\3&-4\end{bmatrix}=\begin{bmatrix}0&3a\\2b&24\end{bmatrix}$

$2k=3a, 3k=2b, -4k=24$

$a=\frac{2k}{3}$, $b=\frac{3k}{2}$, $k=-6$

$\Rightarrow$ $k=-6$, $a=-4$, $b=-9$.

82. (b) Given, $\begin{vmatrix}2+x&3&4\\1&-1&2\\x&1&-5\end{vmatrix}=0$

$(2+x)(5-2)-3(-5-2x)+4(1+x)=0$

$6+3x+15+6x+4+4x=0$

$13x=-25 \Rightarrow x=-\frac{25}{13}$.

83. (b) $A^2=AA=\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}=\begin{bmatrix}2&3&1\\5&6&2\\3&4&1\end{bmatrix}$

$A^3=A^2A=\begin{bmatrix}2&3&1\\5&6&2\\3&4&1\end{bmatrix}\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}=\begin{bmatrix}7&9&3\\15&19&6\\9&12&4\end{bmatrix}$

Here, $A^3-3A^2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$ $\quad A^3-3A^2-I=0$.

84. (a) The given matrix $A=\begin{bmatrix}2&\lambda&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$ is non singular, if $|A|\neq 0$

$|A|=\begin{vmatrix}2&\lambda&-4\\-1&3&4\\1&-2&-3\end{vmatrix}=\begin{vmatrix}1&\lambda+3&0\\-1&3&4\\1&-2&-3\end{vmatrix}$, $[R_1\to R_2+R_1]$

$=\begin{vmatrix}1&\lambda+3&0\\0&1&1\\0&-\lambda-5&-3\end{vmatrix}$ $\quad \begin{bmatrix}R_2\to R_2+R_3\\R_3\to R_3-R_1\end{bmatrix}$

$= 1(-3+\lambda+5) \neq 0$

$\Rightarrow \lambda+2 \neq 0 \Rightarrow \lambda \neq -2.$

85. (b) Given, $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix}$$

$$A^2+B^2 = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$$

Also, $(A+B)^2 = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$

$$(A+B)^2 = \begin{bmatrix} (1+a)^2 & 0 \\ (2+b)(1+a)-2(2+b) & 4 \end{bmatrix}$$

$$(A+B)^2 = A^2+B^2$$

$$\begin{bmatrix} (1+a)^2 & 0 \\ (2+b)(a-1) & 4 \end{bmatrix} = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$$

By equating, $a-1=0 \Rightarrow a=1$ and $b=4$.

86. (a) It is obvious.

87. (b) $A^2 = A.A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$ and $5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$

$$A^2 - 5A = 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 14I.$$

88. (d) Given, Matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

We know that

$$A^2 = A.A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Therefore

$$A^{16} = (A^2)^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^8 = \begin{bmatrix} (-1)^8 & 0 \\ 0 & (-1)^8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

89. (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^3 = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad A^{100} = 2^{99}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

90. (b) Matrix multiplication distributive and associative not commutative.

91. (d) $\because$ On expansion, $|A| = k^2+1$, which can be never zero. Hence matrix A is invertible for all real k.

92. (a) Given, $x+y=4$ .....(i)

and $x-y=0$ ....(ii)

After solving (i) and (ii), $x=2, y=2$

$2x+z=7 \quad z=3$ and $2z+w=10 \quad w=4$.

93. (b) Put $a=1$; $\therefore |A| = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4-4=0$

Hence, A is a singular matrix for $a = 1$.

94. (a) $A^2 = A.A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$$4A-3I = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}.$$

95. (b) First note that PQ must be of order 3 × 2 and its (1, 1)th entry is $i(-i)+0-i(i)=2$.

96. (b) Determinants of unit matrix of any order = 1.

97. (c) It is obvious.

98. (a) $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1\times1+2\times2+(-1)(0) & 1\times0+2\times1+(-1)(1) & 1\times0+2\times0+(-1)(3) \\ 3\times1+0\times2+2\times0 & 3\times0+0\times1+2\times1 & 3\times0+0\times0+2\times3 \\ 4\times1+5\times2+0\times0 & 4\times0+5\times1+0\times1 & 4\times0+5\times0+0\times3 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}.$$

99. (c) Given $AB=A$, $\quad B=I \quad BA=B$, $\therefore A = I$.

Hence, $A^2 = A$ and $B^2 = B$.

100. (d) $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Clearly, no real value of .

101. (a) $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 35 \\ 40 \end{bmatrix}; \qquad \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}.$

102. (a) Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

Check by options.

(i) $A^2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

$$A^2=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I$$

(ii) $(-1)I=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}\neq A$.

(iii) $|A|=1\neq 0\Rightarrow A^{-1}$ exists.

(iv) Clearly A, is not a zero matrix.

103. (d) We have $A=\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}$ and $B=\begin{bmatrix}-5&7&1\\1&-5&7\\7&1&-5\end{bmatrix}$

$$\therefore\ AB=\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}\begin{bmatrix}-5&7&1\\1&-5&7\\7&1&-5\end{bmatrix}$$

$$AB=\begin{bmatrix}18&0&0\\0&18&0\\0&0&18\end{bmatrix}=18\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$AB=18I_3$.

104. (a) $2X=\begin{bmatrix}3&8\\7&2\end{bmatrix}-\begin{bmatrix}1&2\\3&4\end{bmatrix}$

$$2X=\begin{bmatrix}2&6\\4&-2\end{bmatrix}\Rightarrow X=\begin{bmatrix}1&3\\2&-1\end{bmatrix}.$$

105. (c) $2A+2B=\begin{bmatrix}2&0\\2&2\end{bmatrix}$, $A-2B=\begin{bmatrix}-1&1\\0&-1\end{bmatrix}$

On adding, we get $3A=\begin{bmatrix}1&1\\2&1\end{bmatrix}$

$$\Rightarrow A=\begin{bmatrix}1/3&1/3\\2/3&1/3\end{bmatrix}.$$

106. (b) Let $\begin{bmatrix}a&a\\a&a\end{bmatrix}$ be the identity element then

$$\begin{bmatrix}x&x\\x&x\end{bmatrix}\begin{bmatrix}a&a\\a&a\end{bmatrix}=\begin{bmatrix}x&x\\x&x\end{bmatrix}$$

i.e., $2ax=x\Rightarrow a=\frac{1}{2}$, $(\because x\neq 0)$

Identity element $=\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}$.

107. (c) $A^2=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}1&0\\1&1\end{bmatrix}=\begin{bmatrix}1&0\\2&1\end{bmatrix}$

$$A^3=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}1&0\\1&1\end{bmatrix}=\begin{bmatrix}1&0\\3&1\end{bmatrix}$$

$$A^n=\begin{bmatrix}1&0\\n&1\end{bmatrix}$$

$$nA=\begin{bmatrix}n&0\\n&n\end{bmatrix},(n-1)I=\begin{bmatrix}n-1&0\\0&n-1\end{bmatrix}$$

$$nA-(n-1)I=\begin{bmatrix}1&0\\n&1\end{bmatrix}=A^n.$$

Special types of matrices, Transpose, Adjoint and inverse of matrices

1. (c) Let $A=\begin{bmatrix}3&-2&-1\\-4&1&-1\\2&0&1\end{bmatrix}$,

then $|A|=\begin{vmatrix}3&-2&-1\\-4&1&-1\\2&0&1\end{vmatrix}=1$

The matrix of cofactors of A

$$=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}=\begin{bmatrix}1&2&-2\\2&5&-4\\3&7&-5\end{bmatrix}$$

Therefore, $adj(A)=\begin{bmatrix}1&2&3\\2&5&7\\-2&-4&-5\end{bmatrix}$

$$\therefore\ A^{-1}=\frac{1}{|A|}.adjA=\begin{bmatrix}1&2&3\\2&5&7\\-2&-4&-5\end{bmatrix},\quad (\because |A|=1).$$

2. (d) $(AB)^{-1}=B^{-1}A^{-1}$.

3. (a) The cofactors of $N=\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&3\end{bmatrix}$ are

$C_{11}=-4, C_{12}=1, C_{13}=4$; $C_{21}=-3, C_{22}=0, C_{23}=4$

$C_{31}=-3, C_{32}=1, C_{33}=3$

$$\therefore\ adjN=\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&3\end{bmatrix}=N.$$

4. (b) It is obvious.

5. (a) $(I-A)(I+A)=I-A^2=O$,

{Since A is involuntory, therefore $A^2=I$}.

6. (b) Let $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, then $kI=\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}$

$$\Rightarrow\ adj(kI)=\begin{bmatrix}k^2&0&0\\0&k^2&0\\0&0&k^2\end{bmatrix}=k^2I.$$

7. (b) We know by the fundamental concept that $adj(adjA)=|A|^{n-2}A$.

8. (b) For $A=\begin{bmatrix} i & 0 \\ 0 & i/2 \end{bmatrix}$, $adj(A)=\begin{bmatrix} i/2 & 0 \\ 0 & i \end{bmatrix}$ and $|A|=-\frac{1}{2}$.

$\therefore \quad A^{-1}=\frac{1}{\Delta}(adjA)=\frac{1}{-1/2}\begin{bmatrix} i/2 & 0 \\ 0 & i \end{bmatrix}=\begin{bmatrix} -i & 0 \\ 0 & -2i \end{bmatrix}$.

9. (c) $A(adjA)=A.A^{-1}|A|=|A|I$.

10. (b) In A^{-1}, the element of 2nd row and 3rd column is the c_{32} element of the matrix (c_{ij}) of cofactors of element of A, (due to transposition) divided by $\Delta=|A|=-2$.

$\therefore$ Required element = $\frac{(-1)^{3+2}M_{32}}{-2}=\frac{-(-2)}{-2}=-1$,

where $M_{32}=$ minor of c_{32} in $A=\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}=0-2=-2$.

11. (c) $R(s)R(t)=\begin{bmatrix} \cos s & \sin s \\ -\sin s & \cos s \end{bmatrix}\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

$=\begin{bmatrix} \cos(s+t) & \sin(t+s) \\ -\sin(s+t) & \cos(t+s) \end{bmatrix}=R(s+t)$.

12. (b) Since A, B are symmetric $\Rightarrow$ $A=A'$ and $B=B'$

$\therefore \ (AB-BA)'=(AB)'-(BA)'=B'A'-A'B'$

$=-(A'B'-B'A')=-(AB-BA)$

$\Rightarrow$ $(AB-BA)$ is skew-symmetric.

13. (a) $(M'AM)'=M'A'M=M'AM$

{A is symmetric. Hence $M'AM$ is a symmetric matrix).

14. (b) A square matrix is to be orthogonal matrix if $A'A=I=AA'$

$\Rightarrow \ A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, $A'=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow \ AA'=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A'A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \ AA'=A'A=I$.

15. (a) $A^{-1}=\frac{adjA}{|A|}$

But $|A|=\begin{vmatrix} a & c \\ d & b \end{vmatrix}=ab-cd$ and $adjA=\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$

therefore $A^{-1}=\frac{1}{ab-cd}\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$.

16. (b) It is obvious.

17. (a) Let $A=\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$, $\therefore |A|=\begin{vmatrix} 2 & -3 \\ -4 & 2 \end{vmatrix}=4-12=-8$

The matrix of cofactors of the elements of A viz.

$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}=\begin{bmatrix} 2 & -(-4) \\ -(-3) & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$\therefore$ $adjA=$ transpose of the matrix of cofactors of elements of $A=\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

$\therefore \ A^{-1}=\frac{1}{\Delta}adjA=\frac{1}{-8}.\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$.

18. (d) $A=\begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ -1 & 6 & 1 \end{bmatrix}$

$\Rightarrow \ adj(A)=\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{bmatrix}^T=\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$.

19. (a) $A(adjA)=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}.\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}=\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

Aliter : $A(adjA)=|A|\ I=10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

20. (d) Since $(A+A')'=A'+A=A+A'$, so it is symmetric.

$(AA')'=(A')'A'=AA'$, so it is symmetric.

$(A'A)'=A'(A')'=A'A$, so it is symmetric.

But $(A-A')'=A'-A\neq A-A'$. Hence it is not symmetric.

21. (b) Let $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

The matrix of cofactors of the elements of A,

$=\begin{bmatrix} \cos\alpha & -(-\sin\alpha) \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore$ $adjA=$ the transpose of matrix of cofactors of A

$=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore \ A\,adjA=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ (as given) $\Rightarrow$ $k=1$.

22. (a) $3A^3+2A^2+5A+I=0\Rightarrow I=-3A^3-2A^2-5A$

$\Rightarrow IA^{-1}=-3A^2-2A-5I \ \Rightarrow \ A^{-1}=-(3A^2+2A+5I)$

23. (b) It is obvious.

24. (d) All the given statements are true.

25. (b) As $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

26. (a) Let A be a symmetric matrix.

Then $AA^{-1}=I\Rightarrow(AA^{-1})^T=I$

$\Rightarrow \ (A^{-1})^TA^T=I\Rightarrow(A^{-1})^T=(A^T)^{-1}$

$\Rightarrow \ (A^{-1})^T=(A)^{-1}$, $(\because A^T=A)$

$\Rightarrow \ A^{-1}$ is a symmetric matrix.

27. (a) Since A is symmetric, therefore $A^T=A$.

Now $(A^n)^T=(A^T)^n=(A)^n$

$\therefore \ A^n$ is also a symmetric matrix.

28. (d) Since A is a skew-symmetric matrix, therefore

$A^T = -A \Rightarrow (A^T)^n = (-A)^n \Rightarrow (A^n)^T = \begin{cases} A^n, \text{if } n \text{ is even} \\ -A^n, \text{if } n \text{ is odd} \end{cases}$

29. (a) Since $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

30. (b) Let, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$; $adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$\Rightarrow A_{11} = 3, A_{12} = -9, A_{13} = -5$

$A_{21} = -4, A_{22} = 1, A_{23} = 3$

$A_{31} = -5, A_{32} = 4, A_{33} = 1$

$\Rightarrow Adj(A) = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$.

31. (b) Since $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

32. (b) Since the given matrix is symmetric, therefore $a_{12} = a_{21} \Rightarrow x + 2 = 2x - 3 \Rightarrow x = 5$.

33. (a) Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \Rightarrow |A| = 14$

$\therefore adjA = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \frac{4}{14} & \frac{2}{14} \\ \frac{-1}{14} & \frac{3}{14} \end{bmatrix}$.

34. (c) It is skew-symmetric.

35. (a) $A = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$, $A + A^T = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$.

36. (a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = -1(1+0) = -1$

$adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$\Rightarrow A_{11} = 0, A_{12} = -1, A_{13} = 0$

$A_{21} = -1, A_{22} = 0, A_{23} = 0$

$A_{31} = 0$, $A_{32} = 0, A_{33} = -1$

$\Rightarrow A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$.

37. (a) Since $|adjA| = |A|^{n-2}$, therefore $|A| = 0$

$|adjA| = 0 \Rightarrow adjA$ is also singular.

38. (b) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = 1(1+0) = 1$

$Adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$Adj(A) = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{Adj(A)}{|A|} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

39. (c) If $A' = A$, then order of A' will be same to order of A. So it is a square matrix.

40. (c) $A.\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$.

41. (a) Every skew symmetric matrix of odd order is singular. So option (a) is incorrect.

42. (b) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$,

$A_{11} = 1, A_{21} = -2, A_{31} = 4$

$A_{12} = 4, A_{22} = 1, A_{32} = -2$

$A_{13} = -2, A_{23} = 4, A_{33} = 1$

$Adj(A) = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$.

43. (b) It is a concept.

44. (c) It is obvious.

45. (a) $|A| = 3, AdjA = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$; $\therefore A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$

$\Rightarrow (A^{-1})^3 = \frac{1}{27}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}^3 = \frac{1}{27}\begin{pmatrix} 1 & -26 \\ 0 & 27 \end{pmatrix}$.

46. (b) The matrix is not invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(2-5) - a(1-10) + 2(1-4) = 0$

$\Rightarrow -3 + 9a - 6 = 0 \Rightarrow a = 1$.

47. (b) $A(adj.A) = |A| I \Rightarrow \begin{vmatrix} 10 & 0 \\ 0 & 10 \end{vmatrix} = 10.\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$.

48. (d) $a_{ij} = i^2 - j^2$ is a square matrix. For a skew symmetric matrix $a_{ij} = -a_{ji}$

$a_{ij} = i^2 - j^2$ and $a_{ji} = j^2 - i^2$

$a_{ij} + a_{ji} = 0 \Rightarrow a_{ij} = -a_{ji}$.

Hence, a_{ij} is a skew symmetric matrix.

49. (a) $A^{-1} = \frac{Adj(A)}{|A|}$; $A^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

50. (c) Here, $C_{11} = 1, C_{12} = -2, C_{13} = -2$

$C_{21}=-1, C_{22}=3, C_{23}=3$

$C_{31}=0, C_{32}=-4, C_{33}=-3$

$$\det A=|A|=\begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}=1$$

$$A^{-1}=\frac{1}{|A|}.(Adj A)=\frac{1}{1}.\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Now, $A^2=\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

and $A^3=A^2.A=\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\times\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}=A^{-1}.$$

51. (b) $|Adj A|=|A|^{n-1}=d^{n-1}$.

52. (b) It is obvious.

53. (d) Let $A=\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $|A|=1$

$$adj(A)=\begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ 7 & -2 & 1 \end{bmatrix}^T.$$

Hence, $A^{-1}=\frac{adj(A)}{|A|}$

$A^{-1}=\begin{bmatrix} 1 & -2 & 7 \\ 2 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$. Hence, element $A_{13}=7$.

54. (b) We have, $(AA^T)=(A^T)^T A^T=AA^T$ (by reversal law)

AA^T is symmetric matrix.

55. (b) $\begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix}\neq 0 \Rightarrow \lambda \neq -17.$

56. (a) It is obvious.

57. (c) As $I_3 I_3=I_3$, therefore $I_3^{-1}=I_3$.

58. (b) The given matrix is a skew-symmetric matrix $[\because A'=-A]$.

59. (b) $AB=\begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} -3 & -2 \\ 10 & 7 \end{pmatrix}$

$(AB)^T=\begin{pmatrix} -3 & 10 \\ -2 & 7 \end{pmatrix}.$

60. (b) *adj* (*A*) can be obtained by changing the diagonal element and changing the sign of off diagonal elements. Here, $adj(A)=\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$.

61. (b) $A+A^T$ is a square matrix.

$(A+A^T)^T=A^T+(A^T)^T=A^T+A$

Hence *A* is a symmetric matrix.

62. (b) $|A|=(ad-bc)$

$$\therefore A^{-1}=\frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

63. (d) $|A|=1\neq 0$, therefore *A* is invertible.

Thus (d) is not correct.

64. (c) $A^2-A+I=0$

$I=A-A^2 \Rightarrow I=A(I-A)$

$A^{-1}I=A^{-1}(A(I-A)) \Rightarrow A^{-1}=I-A$.

65. (a) $(B^{-1}A^{-1})^{-1}=(A^{-1})^{-1}(B^{-1})^{-1}=AB$

(Reversal law of inverses)

$$=\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}.$$

66. (b) When $a_{ij}=0$ for $i\neq j$ and a_{ij} is constant for $i=j$, then the matrix $[a_{ij}]_{n\times n}$ is called a scalar matrix.

67. (b) Given, Square matrices *A* and *B* of same order. We know that if *A* and *B* are non-singular matrices of the same orders, then $(AB)^{-1}=B^{-1}A^{-1}$.

68. (b) $A=\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$AX=B \Rightarrow X=A^{-1}B$

$A^{-1}=\frac{adjA}{|A|}$

$A^{-1}=\frac{-1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

and $X=A^{-1}B=\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$; $X=\frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$.

69. (b) $A=\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$

$adjA=\begin{bmatrix} -5 & -2 \\ -3 & +1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -11$

$\therefore A^{-1} = \frac{1}{11}\begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{bmatrix}$.

70. (d) Given, $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$, we know that $A^{-1} = \frac{adjA}{|A|}$.

Therefore, $|A| = [12 - 12] = 0$. Since $|A|$ is zero, therefore inverse of A does not exist.

71. (b) $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$

$adj\ A = \begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix}$

$|adjA| = (4 \times 4) - (-3 \times -2) = 16 - 6$

$|adjA| = 10$.

72. (a) Since $A^2 = O$ (Zero matrix) and 2 is the least $+ve$ integer for which $A^2 = O$. Thus, A is nilpotent of index 2.

73. (d) Since for $A = \begin{bmatrix} i & 1-2i \\ -1-2i & 0 \end{bmatrix}$ $(\overline{A})^T = -A$.

Thus, A is skew hermitian.

74. (b) Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Then, $A_{11} = 1, A_{12} = -2, A_{13} = -2$

$A_{21} = -1$, $A_{22} = 3$, $A_{23} = 3$

$A_{31} = 0$, $A_{32} = -4$, $A_{33} = -3$

$adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$.

75. (a) $K = [|A|]^{-1} = \frac{-1}{6}$.

76. (b) It is obvious.

77. (a) $A.(adj(A)) = |A| I$

Here $|A| = \cos^2 x + \sin^2 x = 1$.

Hence, $A.(adj(A)) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

78. (a) $A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{|A|}.adj(A)$

$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$; $|A| = 0 - 1(1-9) + 2(1-6) = 8 - 10$

$|A| = -2 \neq 0$

$Adj\ A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$A_{11} = (-1)^{1+1}[(2)(1) - (3)(1)] = -1$

$A_{12} = 8$, $A_{13} = -5$, $A_{21} = 1$, $A_{22} = -6$

$A_{23} = 3$, $A_{31} = -1$, $A_{32} = 2$, $A_{33} = -1$

$\therefore A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$.

79. (d) From option check $AA^{-1} = I$.

80. (a) It is obvious.

81. (d) $K = |A|$; $|A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 11$.

82. (c) $A[adj(A)] = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A| I$

$[\because |A| = 21 - 20 = 1]$.

83. (a) $A^{-1} = A^2$, because $A^3 = I$.

84. (d) Since $A.A = I$, therefore $A^{-1} = A$.

85. (a) Let $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

$|A| = 4 + 6 = 10 \neq 0$

Now, $A_{11} = 4$, $A_{12} = -3$, $A_{21} = -(-2) = 2$, $A_{22} = 1$

$\therefore adj(A) = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$.

Trick : Check from the options $AA^{-1} = I$

$AA^{-1} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} \frac{4}{10} & \frac{2}{10} \\ \frac{-3}{10} & \frac{1}{10} \end{bmatrix} = \begin{bmatrix} \frac{10}{10} & 0 \\ 0 & \frac{10}{10} \end{bmatrix}$

$= AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$.

86. (d) Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$, $|A| = 1$

$adj(A) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

87. (a) Given, $\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} = 10A^{-1}$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$

$\Rightarrow -5 + \alpha = 0 \Rightarrow \alpha = 5$

(Equating the element of 2nd row and first column).

88. (b) We have, $A(adjA) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

or $A(adjA) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 10I$ .....(i)

and $A^{-1} = \dfrac{1}{|A|}(adjA)$

$A(adjA) = |A|\, I$ .....(ii)

$\therefore$ From equation (i) and (ii), we get $|A| = 10$.

89. (b) It is obvious that $(ABC)' = C'B'A'$.

90. (c) By fundamental property, $adj(\lambda X) = \lambda^{n-1}(adjX)$.

Here $n = 3$

$adj(\lambda X) = \lambda^{3-1}(adjX)$

$adj(\lambda X) = \lambda^{2}(adjX)$.

91. (c) $X = \begin{bmatrix} -x & -y \\ z & t \end{bmatrix}$; $adj\,X = \begin{bmatrix} t & y \\ -z & -x \end{bmatrix}$

Transpose of $(adj\,(X)) = \begin{bmatrix} t & -z \\ y & -x \end{bmatrix}$.

92. (a) $A = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$

$|A| = 11, A_{11} = 1, A_{12} = -3, A_{21} = 2, A_{22} = 5$

$A^{-1} = \dfrac{1}{11}\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$.

93. (c) Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $A^{-1} = \dfrac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$cA = \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix};\ dI = \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}$$

$\therefore$ By $A^{-1} = \dfrac{1}{6}[A^2 + cA + dI]$

$\Rightarrow 6 = 1 + c + d$, (By equality of matrices)

(–6,11) satisfy the relation.

94. (a) If $Q = PAP^T$

$P^TQ = AP^T$, (as $PP^T = I$)

$P^TQ^{2005}P = AP^TQ^{2004}P$

$= A^2P^TQ^{2003}P = A^3P^TQ^{2002}P = A^{2004}P^T(QP)$

$= A^{2004}P^T(PA)\ (Q = PAP^T \Rightarrow QP = PA) = A^{2005}$

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$.

95. (d) $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 1[3] + 1[6] + 1[-4] = 5$

$B = adj\ A = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

$adj\ B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 5A$ and $C = 5A$

$C = adj\ B$; $|C| = |adj\ B|$; $\dfrac{|adj B|}{|C|} = 1$.

96. (a) If A is a singular matrix of order n, then $A(adjA) = (adjA)A = 0 =$ zero matrix.

97. (c) It is obvious.

Relation between determinants and matrices, Rank of matrices and Solution of the equations

1. (a) Since det $(-A) = (-1)^3 \det A = -\det A$.

2. (a) We know that if A, B are n square matrices, then $|AB| = |A||B|$.

3. (a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & -6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} x + 2y + 3z = 6 \\ 3x + y + 2z = -6 \\ 2x + 3y + z = 0 \end{matrix}$

On Simplification the above equation, we get the required result *i.e.,* $x = -4, y = 2, z = 2$.

4. (d) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow A^{-1}\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$\therefore\ AX = B \Rightarrow X = A^{-1}B\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

Aliter : $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x + 0y + z \\ -x + y + 0z \\ 0x - y + z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}\quad \begin{matrix} x + z = 1 \\ -x + y = 1 \\ z - y = 2 \end{matrix}$

$\Rightarrow (x, y, z) = (-1, 0, 2)$.

5. (a) Let A be a skew-symmetric matrix of odd order, say $(2n+1)$ .Since A is skew-symmetric, therefore $A^T = -A$.

$\Rightarrow |A^T| = |-A| \Rightarrow |A^T| = (-1)^{2n+1}|A|$

$\Rightarrow |A^T| = -|A| \Rightarrow |A| = -|A|$

$\Rightarrow 2|A| = 0 \Rightarrow |A| = 0$.

6. (a) Here $|A| \neq 0$. Hence unique solution.

7. (c) $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=2I$

$\therefore\ AB=2IB=2B=\begin{bmatrix}2&4&6\\0&2&6\\0&0&4\end{bmatrix}$

Therefore $|AB|=\begin{vmatrix}2&4&6\\0&2&6\\0&0&4\end{vmatrix}=2(8)=16$

Aliter : $|A|=2\times2\times2=8$, $|B|=1\times1\times2=2$

$\therefore\ |AB|=|A||B|=2\times8=16$.

8. (b) $|A|=-1, |B|=3 \Rightarrow |AB|=-3$

$|3AB|=(3)^3(-3)=-81$.

9. (c) Form (ii) equation, $2(x+y)=3$ or $2.2=3$ or $4=3$

Which is not feasible, so given equation has no solution.

10. (c) $A=\begin{bmatrix}1&2&3\\1&4&9\\1&8&27\end{bmatrix}$

Let c_{ij} be co-factor of a_{ij} in A.

Then co-factor of elements of A are given by

$C_{11}=\begin{vmatrix}4&9\\8&27\end{vmatrix}=36, C_{21}=\begin{vmatrix}2&3\\8&27\end{vmatrix}=-30,\ C_{31}=\begin{vmatrix}2&3\\4&9\end{vmatrix}=6$

$C_{12}=\begin{vmatrix}1&9\\1&27\end{vmatrix}=-18, C_{22}=\begin{vmatrix}1&3\\1&27\end{vmatrix}=24,\ C_{32}=\begin{vmatrix}1&3\\1&9\end{vmatrix}=-6$

$C_{13}=\begin{vmatrix}1&4\\1&8\end{vmatrix}=4, C_{23}=\begin{vmatrix}1&2\\1&8\end{vmatrix}=-6,\ C_{33}=\begin{vmatrix}1&2\\1&4\end{vmatrix}=2$

$Adj(A)=\begin{bmatrix}36&-30&6\\-18&24&-6\\4&-6&2\end{bmatrix}$

$|Adj(A)|=36(48-36)+30(-36+24)+6(108-96)$

$|Adj(A)|=144$.

11. (c) $|A|.adj(A)=|A|^3$ for order n, $DD'=D^n$.

12. (a) $D=\begin{vmatrix}0&1&-1\\-1&0&2\\1&-2&0\end{vmatrix}=0$

$D_1=\begin{vmatrix}1&1&-1\\-2&0&2\\3&-2&0\end{vmatrix}=14\Rightarrow D_1\neq0$

$\because\ D=0$ and $D_1\neq0$,hence the system is inconsistent, so it has no solution.

13. (a) $|A|=\begin{vmatrix}1&0&1\\2&1&0\\3&2&1\end{vmatrix}=1(1-0)+0+1(4-3)=2$.

14. (a) Given $A=\begin{bmatrix}2&3&1&4\\0&1&2&-1\\0&-2&-4&2\end{bmatrix}$, $(R_2\to2R_2+R_3)$

$A=\begin{bmatrix}2&3&1&4\\0&0&0&0\\0&-2&-4&2\end{bmatrix}$

Since every minor of order 3 in A is 0 and there exists a minor order 3 *i.e.* $\begin{bmatrix}2&3\\0&-2\end{bmatrix}$ in A which is non-zero. Thus, rank = 2.

15. (d) $\Delta=\begin{vmatrix}2&1&-1\\1&-3&2\\1&4&-3\end{vmatrix}$

$=2(9-8)-1(-3-2)-1(4+3)=7-7=0$

Hence, number of solutions is zero.

16. (b) $A=\begin{bmatrix}2&4&5\\4&8&10\\-6&-12&-15\end{bmatrix}_{3\times3}$

$|A|=0$, then rank cannot be 3.

Considering a 2×2 minor, $\begin{bmatrix}2&4\\4&8\end{bmatrix}$ its determinant is zero.

Similarly considering

$\begin{bmatrix}4&5\\8&10\end{bmatrix},\begin{bmatrix}4&8\\-6&-12\end{bmatrix},\begin{bmatrix}8&10\\-12&15\end{bmatrix},\begin{bmatrix}2&5\\4&10\end{bmatrix},\begin{bmatrix}4&10\\-6&-15\end{bmatrix}$

their determinants is zero. Each rank can not be 2. Thus rank = 1.

17. (a) $|A^3|=125;\ |A|^3=125=5^3$

$\Rightarrow\ |A|=5\Rightarrow\alpha^2-4=5\Rightarrow\alpha=\pm3$.

18. (d) We have, $\begin{bmatrix}1&1&1\\1&-2&-2\\1&3&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\3\\4\end{bmatrix}$

$x+y+z=0$ ......(i)

$x-2y-2z=3$ ......(ii)

$x+3y+z=4$ .....(iii)

On solving $x=1, y=2, z=-3$ *i.e.,* $\begin{bmatrix}1\\2\\-3\end{bmatrix}$.

19. (a) $A\neq O$ and $B\neq O$

$\therefore\ AB=0$

Hence, Det $(A)=0$ or Det $(B)=0$.

20. (b) Let $\frac{x^2}{a^2}=X, \frac{y^2}{b^2}=Y$ and $\frac{z^2}{c^2}=Z$, then the given system of equations is $X+Y-Z=1$, $X-Y+Z=1$, $-X+Y+Z=1$.

The coefficient matrix is $A=\begin{bmatrix}1&1&-1\\1&-1&1\\-1&1&1\end{bmatrix}$

Clearly $|A| \neq 0$. So the given system of equations has unique solution.

21. (b) Since A and B are square matrix

$|AB| = |A||B|$; $|A| = -10$; $|B| = -10$

$|AB| = 100$.

22. (a) Given $|A| = 6$ and $B = 5A^2$

$|B| = 5|A|^2 = 5 \times 36 = 180$.

23. (b) $|A_i| = \begin{vmatrix} a^i & b^i \\ b^i & a^i \end{vmatrix} = (a^i)^2 - (b^i)^2$, $|a| < 1, |b| < 1$

$$\sum_{i=1}^{\infty} |A_i| = (a^2 - b^2) + (a^4 - b^4) + (a^6 - b^6) + \ldots\ldots$$

$$= (a^2 + a^4 + a^6 + \ldots\ldots) - (b^2 + b^4 + b^6 + \ldots\ldots)$$

$$= \frac{a^2}{1-a^2} - \frac{b^2}{1-b^2} = \frac{a^2 - a^2b^2 - b^2 + a^2b^2}{(1-a^2)(1-b^2)}$$

$$= \frac{a^2 - b^2}{(1-a^2)(1-b^2)}.$$

24. (c) $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, (By $R_3 \to R_3 - 2R_2$)

$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, (By $C_1 \to C_1 - 4C_2 - 3C_3$)

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

(Replace C_1 by C_2 and then Replace C_2 by C_3)

Hence rank of matrix is 2.

25. (c) $A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

$nA = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix}, (n-1)I = \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$

$nA - (n-1)I = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$.

Critical Thinking Questions

1. (b) $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$; $\alpha = a^2 + b^2; \beta = 2ab$

2. (d) $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}, \text{ by } \begin{array}{l} C_1 \to \frac{1}{a}C_1 \\ C_2 \to \frac{1}{b}C_2 \\ C_3 \to \frac{1}{c}C_3 \end{array}$$

$$= abc \begin{vmatrix} \left(1+\Sigma\frac{1}{a}\right) & \frac{1}{b} & \frac{1}{c} \\ \left(1+\Sigma\frac{1}{a}\right) & \frac{1}{b}+1 & \frac{1}{c} \\ \left(1+\Sigma\frac{1}{a}\right) & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} \text{ by}$$

$C_1 \to C_1 + C_2 + C_3$

$$= abc\left(1+\Sigma\frac{1}{a}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

[By taking $\left(\Sigma\frac{1}{a}+1\right)$ as common

$$\Delta = abc\left(1+\Sigma\frac{1}{a}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}, \text{ by } \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).1$, (by expansion along C_1)

Therefore, $\Delta = 0$ $\Rightarrow$ Either $abc = 0$ or $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$

But a, b, c are non-zero and hence the product abc cannot be zero. So the only alternative is that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = -1$.

3. (c) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc - a^2) - b(b^2 - ca) + c(ab - c^2)$

$= -a^3 - b^3 - c^3 + 3abc = -1[a^3 + b^3 + c^3 - 3abc]$

$= -[(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)]$ $\quad k = -1$.

4. (b) $\Delta = \begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix} = 0$ Applying $R_2 \to R_2 - R_1$

$= \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0$

On expansion we get,

$$p(q-b)(r-c)-b(a-p)(r-c)-c(q-b)(a-p)=0$$

$$(p-a)(q-b)(r-c)\left[\frac{p}{(p-a)}+\frac{b}{(q-b)}+\frac{c}{(r-c)}\right]=0$$

$$(p-a)(q-b)(r-c)\left[\frac{p}{(p-a)}+\frac{q}{(q-b)}-1+\frac{r}{(r-c)}-1\right]=0$$

$\because\ p\neq a, q\neq b, r\neq c$

$\therefore\ \frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

5. (b) $\Delta=\begin{vmatrix}1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha)\\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta)\\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1\end{vmatrix}$

$$=\begin{vmatrix}\cos^2\alpha+\sin^2\alpha & \cos\beta\cos\alpha+\sin\beta\sin\alpha & \cos\alpha\cos\gamma+\sin\alpha\sin\gamma\\ \cos\alpha\cos\beta+\sin\alpha\sin\beta & \cos^2\beta+\sin^2\beta & \cos\beta\cos\gamma+\sin\beta\sin\gamma\\ \cos\alpha\cos\gamma+\sin\alpha\sin\gamma & \cos\beta\cos\gamma+\sin\beta\sin\gamma & \cos^2\beta+\sin^2\beta\end{vmatrix}$$

$$=\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ \cos\beta & \sin\beta & 0\\ \cos\gamma & \sin\gamma & 0\end{vmatrix}.\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ \cos\beta & \sin\beta & 0\\ \cos\gamma & \sin\gamma & 0\end{vmatrix}=\begin{vmatrix}\sin\alpha & \cos\alpha & 0\\ \sin\beta & \cos\beta & 0\\ \sin\gamma & \cos\gamma & 0\end{vmatrix}^2.$$

6. (a) Equation given, $\begin{vmatrix}x+\alpha+\beta+\gamma & \beta & \gamma\\ x+\alpha+\beta+\gamma & x+\beta & \alpha\\ x+\alpha+\beta+\gamma & \beta & x+\gamma\end{vmatrix}=0,$

$[C_1\to C_1+(C_2+C_3)]$

or $(x+\alpha+\beta+\gamma)\begin{vmatrix}1 & \beta & \gamma\\ 1 & x+\beta & \alpha\\ 1 & \beta & x+\gamma\end{vmatrix}=0$

or $(x+\alpha+\beta+\gamma)\begin{vmatrix}1 & \beta & \gamma\\ 0 & x & \alpha-\gamma\\ 0 & 0 & x\end{vmatrix}=0$,

$\begin{bmatrix}R_2\to R_2-R_1\\ R_3\to R_3-R_1\end{bmatrix}$

or $(x+\alpha+\beta+\gamma)[x^2-0]=0$

or $x^2(x+\alpha+\beta+\gamma)=0$

$x=0$ or $x=-(\alpha+\beta+\gamma)$.

7. (d) Given, One root = 5 and equation $\begin{vmatrix}x & 3 & 7\\ 2 & x & -2\\ 7 & 8 & x\end{vmatrix}=0$.

Expanding the given equation, we get

$$x[x^2-(-16)]-3[2x-(-14)]+7[16-7x]=0$$

$$x^3+16x-6x-42+112-49x=0$$

$$x^3-39x+70=0$$

Since 5 is the one root of given equation, therefore $x^3-5x^2+5x^2-25x-14x+70=0$

$$x^2(x-5)+5x(x-5)-14(x-5)=0$$

$$(x-5)(x^2+5x-14)=0$$

$(x-5)(x-2)(x+7)=0$ or $x=5,2$ and -7.

8. (b) We can write the given determinant as a product of two determinants as follows $\Delta=0.0=0$ (on simplification), which is independent of a, b, c and d.

9. (a) Applying $C_1\to C_1+C_2+C_3$, we get

$$\Delta=\begin{vmatrix}1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n}\\ 1+\omega^n+\omega^{2n} & 1 & \omega^n\\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1\end{vmatrix}=\begin{vmatrix}0 & \omega^n & \omega^{2n}\\ 0 & 1 & \omega^n\\ 0 & \omega^{2n} & 1\end{vmatrix}=0,$$

($\because\ 1+\omega^n+\omega^{2n}=0$, if n is not multiple of 3).

10. (d) Given, $\Delta=\begin{vmatrix}a & b & 0\\ 0 & a & b\\ b & 0 & a\end{vmatrix}=0.$

Expanding the given determinant, we get $a(a^2-0)-b(0-b^2)=0$ or $a^3+b^3=0$.

This equation may be written as $\left(\frac{a}{b}\right)^3=-1$.

Therefore, $\left(\frac{a}{b}\right)$ is one of the cube roots of – 1.

11. (a) Given, in ΔABC $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix}=0$

$$1(c^2-ab)-a(c-a)+b(b-c)=0$$

$$a^2+b^2+c^2-ab-bc-ca=0$$

$$2a^2+2b^2+2c^2-2ab-2bc-2ca=0$$

$$(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)=0$$

$$(a-b)^2+(b-c)^2+(c-a)^2=0$$

Here, sum of squares of three members can be zero if and only if $a=b=c$

ΔABC is equilateral $\angle A=\angle B=\angle C=60^o$

$\therefore \sin^2 A+\sin^2 B+\sin^2 C=(\sin^2 60^o+\sin^2 60^o+\sin^2 60^o)$

$$=3\times\left(\frac{\sqrt{3}}{2}\right)^2=\frac{9}{4}.$$

12. (a) $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix}$

$=(1-\log_z y\log_y z)-\log_x y(\log_y x-\log_z x\log_y z)$

$+\log_x z(\log_y x\log_z y-\log_z x)$

$=(1-1)-(1-\log_x y\log_y x)+(\log_x z\log_z x-1)=0$

{Since $\log_x y.\log_y x=1$}.

13. (d) Let A be the first term and R be the common ratio of the G.P. then,

$l=AR^{p-1}\Rightarrow \log l=\log A+(p-1)\log R$ .....(i)

$m=AR^{q-1}\Rightarrow \log m=\log A+(q-1)\log R$ .....(ii)

$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$ .....(iii)

Multiplying (i), (ii) and (iii) by $(q-r), (r-p)$ and $(p-q)$ respectively and adding we get, $\log l(q-r) + \log m(r-p) + \log n(p-q) = 0$

$\therefore \Delta = 0$.

14. (d) Given $x^a y^b = e^m, x^c y^d = e^n$

$\Rightarrow a\log x + b\log y = m$ and $c\log x + d\log y = n$

By Cramer's rule, $\log x = \frac{\Delta_1}{\Delta_3}$ and $\log y = \frac{\Delta_2}{\Delta_3}$

$x = e^{\Delta_1/\Delta_3}$ and $y = e^{\Delta_2/\Delta_3}$.

15. (a) $\Delta = -(a^3 + b^3 + c^3 - 3abc)$

$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$,

which is clearly negative because of the given conditions.

16. (c) The system of homogeneous equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

has a non-trivial solution (since x, y, z are not all zero)

If $\Delta = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

i.e., if $(1-a^2) + c(-c-ab) - b(ac+b) = 0$

i.e., if $a^2 + b^2 + c^2 + 2abc = 1$.

17. (a) If A is square matrix of order 3, then

$|-2A| = (-2)^3 |A| = -8|A|$.

18. (c) As the system of equations has a non-trivial solution

$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$, by $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$\Rightarrow a(b-1)(c-1) - 1.(1-a)(c-1) - 1.(1-a)(b-1) = 0$

$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\Rightarrow \frac{1}{1-a} - 1 + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$.

19. (d) We know $A.adj(A) = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$|A|.|adj(A)| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$

$|A|.adj|A| = |A|^3$

Now question gives $|A| = 8$

$8.adj|A| = 8^3$ or $adj|A| = 8^2 = (2^3)^2 = 2^6$.

20. (b,d) $A = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & a+6 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$

(Operating $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - 2R_3$)

$= \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$ (Operating $R_1 \to R_1 + R_2$)

When $a = -6, A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -2 & -5 \end{vmatrix}$, $\therefore \rho(A) = 1$

Where $\rho(A) =$ number of non-zero rows

When $a = 6, A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -12 \\ 1 & -2 & 7 \end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 1, A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -7 \\ 1 & -2 & 2 \end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 2, A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & -2 & 3 \end{vmatrix}$, $\therefore \rho(A) = 2$.

21. (c) Since $A^2 = A.A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$.

22. (d) $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ $A.A = A^2 = \begin{bmatrix} 6 & 11 & 7 \\ -11 & 4 & -11 \\ 7 & 11 & 12 \end{bmatrix}$,

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then, $A^2 + 9I = \begin{bmatrix} 15 & 11 & 7 \\ -11 & 13 & -11 \\ 7 & 11 & 21 \end{bmatrix}$.

23. (b) $|A| = 1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$

$AB = I \Rightarrow B = IA^{-1}$

$$\frac{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix}}{\sec^2\frac{\theta}{2}} = \cos^2\frac{\theta}{2}.A^T.$$

24. (b) $(A-2I)(A-3I)=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O.$

25. (d) $A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$

$$A^2 = A.A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}=\begin{bmatrix}4 & 3 & 0\\ -3 & 2 & -2\\ 6 & 4 & 5\end{bmatrix}$$

$$A.A^2=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}\begin{bmatrix}4 & 3 & 0\\ -3 & 2 & -2\\ 6 & 4 & 5\end{bmatrix}=\begin{bmatrix}4 & 11 & 1\\ -9 & -2 & -7\\ 21 & 11 & 7\end{bmatrix}$$

$$A^3-3A^2-A+9I_3=0.$$

26. (c) $\begin{matrix}3X+2Y=I\\ 2X-Y=O\end{matrix} \Rightarrow \begin{matrix}3X+2Y=I\\ 4X-2Y=O\end{matrix} \Rightarrow \begin{matrix}7X=I\\ X=\frac{1}{7}I\end{matrix}$

(Solving simultaneously)

Therefore from (i), $2Y=I-\frac{3}{7}I=\frac{4}{7}I \Rightarrow Y=\frac{2}{7}I.$

27. (c) It is obvious.

28. (a) $A_{3\times 4} \Rightarrow A'_{4\times 3}$; Now $A'B$ defined $\Rightarrow$ B is $3\times p$

Again $B_{3\times p}A'_{4\times 3}$ defined $\Rightarrow$ $p=4$

$\therefore$ B is 3×4.

29. (c) $A'=[1\ 2\ 3]$ therefore

$$AA'=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}[1\ 2\ 3]=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{bmatrix}.$$

30. (b) It is a fundamental concept.

31. (c) Since

$$A^2=A.A=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}=\begin{bmatrix}a^2 & 0 & 0\\ 0 & b^2 & 0\\ 0 & 0 & c^2\end{bmatrix}$$

And $A^3=\begin{bmatrix}a^3 & 0 & 0\\ 0 & b^3 & 0\\ 0 & 0 & c^3\end{bmatrix}$,.....

$$A^n=A^{n-1}.A=\begin{bmatrix}a^{n-1} & 0 & 0\\ 0 & b^{n-1} & 0\\ 0 & 0 & c^{n-1}\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$$

$$=\begin{bmatrix}a^n & 0 & 0\\ 0 & b^n & 0\\ 0 & 0 & c^n\end{bmatrix}.$$

Note: Students should remember this question as a formula.

32. (d) Let $A=\begin{bmatrix}3 & 5 & 7\\ 2 & -3 & 1\\ 1 & 1 & 2\end{bmatrix}$, $|A|=3(-7)-5(3)+7(5)=-1$

$$Adj(A)=\begin{bmatrix}-7 & -3 & 26\\ -3 & -1 & 11\\ 5 & 5 & -19\end{bmatrix}$$

$$A^{-1}=\frac{Adj(A)}{|A|}$$

$$A^{-1}=\begin{bmatrix}7 & 3 & -26\\ 3 & 1 & -11\\ -5 & -5 & 19\end{bmatrix}.$$

33. (c) $A(z)=A\left(\frac{x+y}{1+xy}\right)=\left[\frac{1+xy}{(1-x)(1-y)}\right]$

$$\begin{bmatrix}1 & -\left(\frac{x+y}{1+xy}\right)\\ -\left(\frac{x+y}{1+xy}\right) & 1\end{bmatrix}$$

$\therefore$ $A(x).A(y)=A(z)$.

34. (c) $|A|=-20$

$\therefore$ $a_{23}=\frac{\text{Cofactor of }6}{-20}=\frac{-8}{20}=\frac{-2}{5}$.

35. (a) We have

$$F(\alpha)F(-\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$\therefore$ $F(-\alpha)=[F(\alpha)]^{-1}$.

36. (b) Given, $B=-A^{-1}BA$

$AB=-AA^{-1}BA=-IBA=-BA$

$AB=-BA$

Now $(A+B)^2=(A+B)(A+B)=A^2+AB+BA+B^2$

$=A^2+B^2$ $[\because BA=-AB]$

Thus, $(A+B)^2=A^2+B^2$.

37. (d) Given, Square matrices of 2 × 2 over the real numbers. We know that as inverse axiom may not exist for all 2 × 2 matrices, therefore the set of all 2 × 2 matrices over the real numbers is not a group.

38. (b) $AB=AC \Rightarrow B=C$

If A^{-1} exists $\Leftrightarrow$ A is a non-singular matrix.

39. (a) In a skew-symmetrix matrix $a_{ij} = -a_{ji} \forall i, j = 1, 2, 3$ for $j = i$, $a_{ii} = -a_{ji}$ each $a_{ii} = 0$.

Hence the matrix $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ is skew-symmetric.

40. (c) $(A - A^T)^T = A^T - (A^T)^T$

$= A^T - A \qquad [\because (A^T)^T = A]$

$= -(A - A^T)$

So, $A - A^T$ is a skew symmetric matrix.

41. (c) $A^2 = A.A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore$ Matrix A is nilpotent of order 2.

42. (a) Since for given $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$

$AA^T = A^T A = I_{(3\times 3)}$. Thus A is orthogonal.

43. (a) As we know, a square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$.

Such as, $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 5 \end{bmatrix}_{4\times 4}$

Number of zeros $= \dfrac{4(4-1)}{2} = 6 = \dfrac{n(n-1)}{2}$.

44. (c) Let $A = \begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

and $A^T = \begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

and $AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$\therefore |A| = \pm 1$.

45. (d) $n = 2 \times 3 \times 4 = 24$.