Answers and Solutions

Solution of trigonometrical equations,

1. (b) $\sin\theta+\cos\theta=1 \Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta=\frac{1}{\sqrt{2}}$

Dividing by $\sqrt{1^2+1^2}=\sqrt{2}$,

we get $\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}$

$\Rightarrow \theta+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4} \Rightarrow \theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$.

2. (c) $\sin^2\theta=\frac{1}{4}=\sin^2\frac{\pi}{6} \Rightarrow \theta=n\pi\pm\frac{\pi}{6}$.

3. (b) $\cos^2\theta=\frac{3}{4}=\cos^2\left(\frac{\pi}{6}\right) \Rightarrow \theta=n\pi\pm\frac{\pi}{6}$.

4. (b) On simplification, it reduces to $\cos 2\theta=\sin 2\theta$

$\Rightarrow \tan 2\theta=\tan\frac{\pi}{4} \Rightarrow 2\theta=n\pi+\frac{\pi}{4} \Rightarrow \theta=\frac{n\pi}{2}+\frac{\pi}{8}$.

5. (d) $\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta=\frac{\sqrt{2}}{2}$ {dividing by $\sqrt{(\sqrt{3})^2+1^2}=2$}

$\Rightarrow \sin\left(\theta+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}=\sin\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{3}$.

6. (b) $1-\cos^2\theta-2\cos\theta+\frac{1}{4}=0$

$\Rightarrow \cos^2\theta+2\cos\theta-\frac{5}{4}=0$

$\Rightarrow \cos\theta=\frac{-2\pm\sqrt{4+5}}{2}=-1\pm\frac{3}{2}$

Since $|\cos\theta|\le 1$, hence $\cos\theta=-1-\frac{3}{2}$ is ruled out.

$\Rightarrow \cos\theta=-1+\frac{3}{2}=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right) \Rightarrow \theta=2n\pi\pm\frac{\pi}{3}$.

7. (c) $\sqrt{2}\sec\theta+\tan\theta=1 \Rightarrow \frac{\sqrt{2}}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=1$

$\Rightarrow \sin\theta-\cos\theta=-\sqrt{2}$

Dividing by $\sqrt{2}$ on both sides, we get

$\frac{1}{\sqrt{2}}\sin\theta-\frac{1}{\sqrt{2}}\cos\theta=-1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta=1 \Rightarrow \cos\left(\theta+\frac{\pi}{4}\right)=\cos(0)$

$\Rightarrow \theta+\frac{\pi}{4}=2n\pi\pm 0 \Rightarrow \theta=2n\pi-\frac{\pi}{4}$.

8. (c) $2\tan^2\theta=\sec^2\theta \Rightarrow 2\tan^2\theta=\tan^2\theta+1$

$\Rightarrow \tan^2\theta=1=\tan^2\left(\frac{\pi}{4}\right) \Rightarrow \theta=n\pi\pm\frac{\pi}{4}$.

9. (b) $2\sin\theta+\tan\theta=0$; $\sin\theta\left(2+\frac{1}{\cos\theta}\right)=0$

i.e., $\sin\theta=0 \Rightarrow \theta=n\pi$

or $\frac{1}{\cos\theta}=-2 \Rightarrow \cos\theta=-\frac{1}{2} \Rightarrow \theta=2n\pi\pm\left(\frac{2\pi}{3}\right)$.

10. (b) $\sqrt{3}\tan 2\theta+\sqrt{3}\tan 3\theta+\tan 2\theta\tan 3\theta=1$

$\Rightarrow \frac{\tan 2\theta+\tan 3\theta}{1-\tan 2\theta\tan 3\theta}=\frac{1}{\sqrt{3}}$ $\tan 5\theta=\tan\frac{\pi}{6}$

$\Rightarrow 5\theta=n\pi+\frac{\pi}{6} \Rightarrow \theta=\left(n+\frac{1}{6}\right)\frac{\pi}{5}$.

11. (a) $\tan 2\theta=\cot\theta$ $\tan 2\theta=\tan\left(\frac{\pi}{2}-\theta\right)$

$\Rightarrow 2\theta=n\pi+\frac{\pi}{2}-\theta \Rightarrow \theta=\frac{n\pi}{3}+\frac{\pi}{6}$.

12. (c) $\frac{1}{\sin\theta}=1+\frac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta+\cos\theta=1$

$\Rightarrow \cos\left(\theta-\frac{\pi}{4}\right)=\cos\frac{\pi}{4} \Rightarrow \theta-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$

Hence $\theta=2n\pi$ or $\theta=2n\pi+\frac{\pi}{2}$.

But $\theta=2n\pi$ is ruled out.

13. (d) $\frac{1-\cos 2\theta}{1+\cos 2\theta}=3$ $\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}=3$

$\Rightarrow \tan^2\theta=3 \Rightarrow \theta=n\pi\pm\frac{\pi}{3}$.

14. (c) $\sec^2\theta+\tan^2\theta=\frac{5}{3}$, also $\sec^2\theta-\tan^2\theta=1$

$\Rightarrow \tan^2\theta=\frac{1}{3}=\tan^2\left(\frac{\pi}{6}\right) \Rightarrow \theta=n\pi\pm\frac{\pi}{6}$.

15. (c) $\sin 4\theta=\cos\theta-\cos 7\theta$ $\sin 4\theta=2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta=0 \Rightarrow 4\theta=n\pi$ or $\sin 3\theta=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 3\theta=n\pi+(-1)^n\frac{\pi}{6} \Rightarrow \theta=\frac{n\pi}{4},\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$.

16. (a) $\frac{1-\tan^2\theta}{\sec^2\theta}=\frac{1}{2} \Rightarrow \cos^2\theta-\sin^2\theta=\frac{1}{2}$

$\Rightarrow \cos 2\theta=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta=2n\pi\pm\frac{\pi}{3} \Rightarrow \theta=n\pi\pm\frac{\pi}{6}$.

17. (d) $\cos^2\theta-\frac{5}{2}\cos\theta+1=0$

$\Rightarrow \cos\theta=\frac{(5/2)\pm\sqrt{(25/4)-4}}{2}=\frac{5\pm 3}{4}$

Rejecting (+) sign,

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$.

18. (c) $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$ $\quad \frac{2}{\sin\theta} = \frac{1}{\sin\theta\cos\theta}$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\sin\theta = 0 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$ or $\theta = n\pi$.

19. (a) $\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$

$\Rightarrow \tan\theta(\tan\theta - 1) - \sqrt{3}(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - \sqrt{3})\ (\tan\theta - 1) = 0 \Rightarrow \theta = n\pi + \frac{\pi}{3},\ n\pi + \frac{\pi}{4}$.

20. (a) It is obvious.

21. (a) $4 - 4\cos^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4\cos^2\theta - 2(\sqrt{3}+1)\cos\theta + \sqrt{3} = 0$

$\Rightarrow \cos\theta = \frac{2(\sqrt{3}+1) \pm \sqrt{4(\sqrt{3}+1)^2 - 16\sqrt{3}}}{8}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $1/2 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$ or $2n\pi \pm \pi/3$.

22. (d) $\cot\theta + \cot\left(\frac{\pi}{4} + \theta\right) = 2 \Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\cos\{(\pi/4)+\theta\}}{\sin\{(\pi/4)+\theta\}} = 2$

$\Rightarrow \sin\left(\frac{\pi}{4} + 2\theta\right) = 2\sin\theta\sin\left(\frac{\pi}{4} + \theta\right)$

$\Rightarrow \sin\left(\frac{\pi}{4} + 2\theta\right) + \cos\left(\frac{\pi}{4} + 2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

23. (a) $2\cos^2\theta - 1 + 3\cos\theta = 0$

$\cos\theta = \frac{-3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$

$\Rightarrow \theta = 2n\pi \pm \cos^{-1}\left(\frac{-3+\sqrt{17}}{4}\right)$, (Taking $+ve$ sign).

24. (a) $\tan m\theta = \tan n\theta \Rightarrow m\theta = p\pi + n\theta \Rightarrow \theta = \frac{p\pi}{(m-n)}$

Hence different values of θ are in A.P. with $\frac{\pi}{m-n}$ as common difference.

25. (d) $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3} \Rightarrow \sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$

$\Rightarrow \sin\left(\theta - \frac{\pi}{3}\right) = \sin\frac{\pi}{4} \Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}$.

26. (b) $\sin\theta + \cos\theta = \sqrt{2}\cos\alpha \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\alpha$

$\Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \alpha \Rightarrow \theta = 2n\pi + \frac{\pi}{4} \pm \alpha$.

27. (b) $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta\tan 2\theta\tan 3\theta$

$\tan 6\theta = \frac{\tan\theta + \tan 2\theta + \tan 3\theta - \tan\theta\tan 2\theta\tan 3\theta}{1 - \sum\tan\theta\tan 2\theta}$

= 0, (from the given condition)

$\Rightarrow 6\theta = n\pi \Rightarrow \theta = n\pi/6$.

Trick : In such type of problems, the general value of θ is given by $\frac{n\pi}{\text{sum of number of }\theta}$. So the general value of θ is $\frac{n\pi}{1+2+3} = \frac{n\pi}{6}$.

28. (a) $\frac{3\sin(A-15^o)}{\cos(A-15^o)} = \frac{\sin(A+15^o)}{\cos(A+15^o)}$

$3\sin(A-15^o)\cos(A+15^o) = \cos(A-15^o)\sin(A+15^o)$

$\Rightarrow 2\sin(A-15^o)\cos(A+15^o) = \frac{1}{2}$

$\Rightarrow \sin 2A - \sin 30^o = \frac{1}{2} \Rightarrow 2A = 2n\pi + \frac{\pi}{2}$

$\Rightarrow A = n\pi + \frac{\pi}{4}$.

29. (b) $\tan\theta + \frac{1}{\tan\theta} = 2 \Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{4}$.

30. (b) $2\cos^2\theta - (\sqrt{2}+1)\cos\theta - 1 + \frac{(\sqrt{2}+1)}{\sqrt{2}} = 0$

$\Rightarrow \cos\theta = \frac{(\sqrt{2}+1) \pm \sqrt{(\sqrt{2}+1)^2 - \frac{8}{\sqrt{2}}}}{4}$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{4}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}$.

Trick : Since $\theta = \frac{\pi}{4}$ satisfies the equation and therefore the general value should be $2n\pi \pm \frac{\pi}{4}$.

31. (a) $\tan\theta = \cot\alpha \Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} - \alpha\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{2} - \alpha$.

32. (a) $3(\sin\theta - \cos\theta) = 4\sin\theta\cos\theta$

$3(\sin\theta - \cos\theta) = 2\sin 2\theta$

Squaring both sides, we get $9(1-S) = 4S^2$, where $S = \sin 2\theta$ or $4S^2 + 9S - 9 = 0$.

$\therefore\ (S+3)(4S-3) = 0$ or $S = \frac{3}{4}$ as $S \neq -3$

or $\sin 2\theta = \frac{3}{4} = \sin\alpha$

$\therefore\ 2\theta = n\pi + (-1)^n\alpha$ or $\theta = \frac{1}{2}\left[n\pi + (-1)^n\sin^{-1}\left(\frac{3}{4}\right)\right]$.

33. (b) $\cos p\theta = \cos q\theta \Rightarrow p\theta = 2n\pi \pm q\theta \Rightarrow \theta = \frac{2n\pi}{p \pm q}$.

34. (b) $4 + 2\sin^2 x = 5$

$\Rightarrow \sin^2 x = \frac{1}{2} = \sin^2\frac{\pi}{4} \Rightarrow x = n\pi \pm \frac{\pi}{4}$.

35. (b) $3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$

$\therefore\ \sin^2 x = \left(\frac{\sqrt{3}}{2}\right)^2 \qquad \sin^2 x = \sin^2 \pi/3$

$x = n\pi \pm \pi/3$.

36. (a) We have $\frac{\pi}{4}\cot\theta = \frac{\pi}{2} - \frac{\pi}{4}\tan\theta \Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow\ \sin 2\theta = 1 = \sin\frac{\pi}{2} \Rightarrow \theta = n\pi + \frac{\pi}{4}$.

37. (d) $2\sin^2\theta - 3\sin\theta - 2 = 0 \Rightarrow\ (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow\ \sin\theta = -\frac{1}{2}\ ,\ (\because \sin\theta \neq 2) \Rightarrow\ \sin\theta = \sin\left(\frac{-\pi}{6}\right)$

$\Rightarrow\ \theta = n\pi + (-1)^n\left(\frac{-\pi}{6}\right) \Rightarrow \theta = n\pi + (-1)^{n+1}\frac{\pi}{6}$

$\Rightarrow\ \theta = n\pi + (-1)^n\frac{7\pi}{6}\ ,\ \left\{\because \frac{-\pi}{6}\ \text{is equivalent to}\ \frac{7\pi}{6}\right\}$.

38. (a) Let $\sqrt{3}+1 = r\cos\alpha$ and $\sqrt{3}-1 = r\sin\alpha$.

Then $r = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = 2\sqrt{2}$

$\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-(1/\sqrt{3})}{1+(1/\sqrt{3})} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \Rightarrow \alpha = \frac{\pi}{12}$

The given equation reduces to

$2\sqrt{2}\cos(\theta - \alpha) = 2 \Rightarrow \cos\left(\theta - \frac{\pi}{12}\right) = \cos\frac{\pi}{4}$

$\Rightarrow\ \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$.

39. (b) $\tan 3x = 1 \Rightarrow \tan 3x = \tan\frac{\pi}{4}$

$\Rightarrow 3x = n\pi + \frac{\pi}{4}\ \Rightarrow\ x = \frac{n\pi}{3} + \frac{\pi}{12}$.

40. (b) The given equation can be written as

$\Rightarrow\ \frac{\sin^2\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0 \Rightarrow\ \tan\theta\sin\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\sin\theta + \sqrt{3}) = 0\ \Rightarrow\ \tan\theta = 0\ \ \Rightarrow \theta = n\pi, n \in Z$.

41. (b) Using $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1+\tan^2\theta}{1-\tan^2\theta}$, we can write the given equation as $\tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$.

$\Rightarrow\ \tan^2\theta(1-\tan^2\theta) + 1 + \tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow\ 3\tan^2\theta - \tan^4\theta = 0 \Rightarrow \tan^2\theta(3 - \tan^2\theta) = 0$

$\Rightarrow\ \tan\theta = 0$ or $\tan\theta = \pm\sqrt{3}$

Now $\tan\theta = 0 \Rightarrow \theta = m\pi$, where m is an integer and $\tan\theta = \pm\sqrt{3} = \tan(\pm\pi/3) \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$, where n is an integer. Thus $\theta = m\pi, n\pi \pm \frac{\pi}{3}$, where m and n are integers.

42. (d) $\cos 2\theta = \cos\left(\frac{\pi}{2} - \alpha\right) \Rightarrow 2\theta = 2n\pi \pm \left(\frac{\pi}{2} - \alpha\right)$

$\Rightarrow\ \theta = n\pi \pm \left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$.

43. (a) $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow\ \sin 4\theta(2\cos 2\theta + 1) = 0$

$\Rightarrow\ 2\cos 2\theta = -1\ \Rightarrow\ \cos 2\theta = -\frac{1}{2}$

$\Rightarrow\ 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

and $\sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$

$\theta = \frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$.

44. (c) $\sin^2\theta + \sin\theta - 2 = 0 \Rightarrow (\sin\theta - 1)(\sin\theta + 2) = 0$

$\Rightarrow\ \sin\theta \neq -2\ ,\ \ \therefore \sin\theta = 1 = \sin\pi/2$

$\Rightarrow\ \theta = n\pi + (-1)^n\frac{\pi}{2}$.

45. (a) $\tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)\ \Rightarrow\ 5\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\Rightarrow\ 7\theta = n\pi + \frac{\pi}{2}\ \Rightarrow\ \theta = \frac{n\pi}{7} + \frac{\pi}{14}$.

46. (b) $3\sin^2 x + 10\cos x - 6 = 0$

$3(1-\cos^2 x) + 10\cos x - 6 = 0$

On solving, $(\cos x - 3)(3\cos x - 1) = 0$

Either $\cos x = 3$, (which is not possible) or $\cos x = \frac{1}{3}\ \Rightarrow\ x = 2n\pi \pm \cos^{-1}(1/3)$.

47. (a,b) $\cos\theta + \cos 2\theta + \cos 3\theta = 0$

$(\cos\theta + \cos 3\theta) + \cos 2\theta = 0$

$2\cos 2\theta\cos\theta + \cos 2\theta = 0$

$\cos 2\theta(2\cos\theta + 1) = 0 \qquad \cos 2\theta = 0 = \cos\frac{\pi}{2}$

$\theta = \frac{\pi}{4} \qquad \theta = 2m\pi \pm \frac{\pi}{4}$

or $\cos\theta = \frac{-1}{2} = \cos\frac{2\pi}{3} \qquad \theta = 2m\pi \pm \frac{2\pi}{3}$.

48. (c) $2\sqrt{3}\cos^2\theta = \sin\theta \qquad 2\sqrt{3}\sin^2\theta + \sin\theta - 2\sqrt{3} = 0$

$\sin\theta = \frac{-1 \pm 7}{4\sqrt{3}} \Rightarrow \sin\theta = \frac{-8}{4\sqrt{3}}$, (Impossible)

and $\sin\theta = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \qquad \theta = n\pi + (-1)^n\frac{\pi}{3}$.

49. (a) On expanding determinant,

$\cos^2(A+B) + \sin^2(A+B) + \cos 2B = 0$

$1 + \cos 2B = 0$ or $\cos 2B = \cos\pi$

or $\ 2B = 2n\pi + \pi$ or $B = (2n+1)\frac{\pi}{2}$, $n \in Z$.

50. (a) $\sin 2\theta = \cos 3\theta\ \Rightarrow\ 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow\ \theta = \frac{2n\pi}{5} + \frac{\pi}{10}$ or $\theta = 2n\pi - \frac{\pi}{2}$.

Since θ is acute $\theta = \frac{\pi}{10}$ $\sin\theta = \frac{\sqrt{5}-1}{4}$.

51. (a) The given equation can be put in the form $4\sin^4 x = 1-\cos^4 x = (1-\cos^2 x)(1+\cos^2 x)$

$\Rightarrow \sin^2 x[4\sin^2 x - 1 - (1-\sin^2 x)] = 0$

$\Rightarrow \sin^2 x[5\sin^2 x - 2] = 0 \Rightarrow \sin x = 0$ or $\sin x = \pm\sqrt{2/5}$.

Hence $x = n\pi$ is the required answer.

52. (a) We have $\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$

$\Rightarrow 1+\cos 3x + 1 + \sin\left(2x - \frac{7\pi}{6}\right) = 0$

$\Rightarrow (1+\cos 3x) + 1 - \cos\left(2x - \frac{2\pi}{3}\right) = 0$

$\Rightarrow 2\cos^2\frac{3x}{2} + 2\sin^2\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ and $\sin\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \frac{3x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$ and $x - \frac{\pi}{3} = 0, \pi, 2\pi \ldots\ldots \Rightarrow x = \frac{\pi}{3}$

Therefore, the general solution of $\cos\frac{3x}{2} = 0$ and $\sin\left(x-\frac{\pi}{3}\right) = 0$ is $x = 2k\pi + \frac{\pi}{3} = \frac{\pi}{3}(6k+1)$, where $k \in Z$.

53. (b) After solving the determinant $2\cos\theta = 0$

$\theta = 2n\pi \pm \frac{\pi}{2}$.

54. (a) $\tan(3x - 2x) = \tan x = 1$ $x = n\pi + \frac{\pi}{4}$

But this value does not satisfy the given equation. Hence option (a) is the correct answer.

55. (c) Given relation is

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$ $\tan 3\theta = \tan(\pi/3)$

$3\theta = n\pi + \frac{\pi}{3}$ $\theta = (3n+1)\frac{\pi}{9}$.

56. (b) We have, $1 - \cos\theta = \sin\theta.\sin\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2} = 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\sin\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2}\left[1 - \cos\frac{\theta}{2}\right] = 0$ $\sin\frac{\theta}{2} = 0$ or $2\sin^2\frac{\theta}{4} = 0$

$\sin\frac{\theta}{2} = 0$ or $\sin\frac{\theta}{4} = 0$ $\frac{\theta}{2} = k\pi$ or $\frac{\theta}{4} = k\pi$.

Hence, $\theta = 2k\pi$ or $\theta = 4k\pi$, $k \in I$.

57. (b) $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$

$\frac{\tan 3\theta - \tan(\pi/4)}{1 + \tan 3\theta.\tan(\pi/4)} = \sqrt{3}$ $\tan\left(3\theta - \frac{\pi}{4}\right) = \tan\frac{\pi}{3}$

$3\theta - (\pi/4) = n\pi + (\pi/3)$

$3\theta = n\pi + \frac{7\pi}{12}$ $\theta = \frac{n\pi}{3} + \frac{7\pi}{36}$.

58. (a) $2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0 \Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$ *i.e.*, $30^\circ, 150^\circ, 90^\circ$.

59. (d) No solution as $|\sin x| \le 1, |\cos x| \le 1$ and both of them do not attain their maximum value for the same angle.

Aliter : Since the maximum value of $(\sin x + \cos x) = \sqrt{1^2 + 1^2} = \sqrt{2}$.

Hence there is no 'x' satisfying $\sin x + \cos x = 2$.

60. (a) $2 - 2\cos^2\theta = 4 + 3\cos\theta \Rightarrow 2\cos^2\theta + 3\cos\theta + 2 = 0$

$\Rightarrow \cos\theta = \frac{-3 \pm \sqrt{9-16}}{4}$,

which is imaginary, hence no solution.

61. (d) $\sin x\cos x = 2$ or $\sin 2x = 4$, which is impossible.

62. (c) $\sec\theta + \tan\theta = \sqrt{3}$ ....(i)

Also we have $\sec^2\theta - \tan^2\theta = 1$ .....(ii)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ .....(iii)

Now (i) and (iii) gives,

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$.

$\therefore$ Solutions for $0 \le \theta \le 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.

Hence there are two solutions.

63. (c) $\sin 5x + \sin 3x + \sin x = 0$

$\Rightarrow -\sin 3x = \sin 5x + \sin x = 2\sin 3x\cos 2x$

$\Rightarrow \sin 3x = 0 \Rightarrow x = 0$

or $\cos 2x = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow 2x = 2n\pi \pm \left(\pi - \frac{\pi}{3}\right) \Rightarrow x = n\pi \pm \left(\frac{\pi}{3}\right)$

For x lying between 0 and $\frac{\pi}{2}$, we get $x = \frac{\pi}{3}$.

Trick : Check with options.

64. (a) $f(x) = \cos x - x + \frac{1}{2}$, $f(0) = \frac{3}{2} > 0$

$f\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} + \frac{1}{2} = \frac{1-\pi}{2} < 0$, $\left(\because \pi = \frac{22}{7} \text{ nearly}\right)$

One root lies in the interval $\left[0, \frac{\pi}{2}\right]$.

65. (c) $\sec x \cos 5x = -1 \Rightarrow \cos 5x = -\cos x$

$\Rightarrow 5x = 2n\pi \pm (\pi - x) \Rightarrow x = \frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{4}$

Hence $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6}, \frac{11\pi}{6}$.

66. (c) $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$

$(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x + \sin 2x + \alpha = 0$

$\sin^2 2x - 2\sin 2x - 2 - 2\alpha = 0$

Let $\sin 2x = y$. Then the given equation becomes $y^2 - 2y - 2(1+\alpha) = 0$,

where $-1 \le y \le 1$, $(\because -1 \le \sin 2x \le 1)$

For real, discriminant $\ge 0 \Rightarrow 3 + 2\alpha \ge 0 \Rightarrow \alpha \ge -\frac{3}{2}$

Also $-1 \le y \le 1 \Rightarrow -1 \le 1 - \sqrt{3+2\alpha} \le 1$

$\Rightarrow 3 + 2\alpha \le 4 \Rightarrow \alpha \le \frac{1}{2}$. Thus $-\frac{3}{2} \le \alpha \le \frac{1}{2}$.

67. (b) $3\cos\theta + 4\sin\theta = 5\left[\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right] = 5\cos(\theta - \alpha)$

where $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$

Now $3\cos\theta + 4\sin\theta = k$

$5\cos(\theta - \alpha) = k \Rightarrow \cos(\theta - \alpha) = \pm 1$

$\Rightarrow \theta - \alpha = 0^o, 180^o \Rightarrow \theta = \alpha, 180^o + \alpha$.

68. (c) $3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow 3\sin(\sin x - 2) - (\sin x - 2) = 0$

$\Rightarrow (3\sin x - 1)(\sin x - 2) = 0 \Rightarrow \sin x = \frac{1}{3}$ or 2

$\Rightarrow \sin x = \frac{1}{3}$, $(\because \sin x \ne 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha$, $0 < \alpha < \frac{\pi}{2}$ are the solutions in $[0, 5\pi]$. Then α, $\pi - \alpha$, $2\pi + \alpha$, $3\pi - \alpha$, $4\pi + \alpha$, $5\pi - \alpha$ are the solutions in $[0, 5\pi]$.

$\therefore$ Required number of solutions = 6.

69. (d) Given equation is $\sqrt{3}\sin x + \cos x = 4$ which is of the form $a\sin x + b\cos x = c$ with $a = \sqrt{3}, b = 1, c = 4$.

Here $a^2 + b^2 = 3 + 1 = 4 < c^2$, therefore the given equation has no solution.

70. (d) $3\cos x + 4\sin x = 6$

$\frac{3}{5}\cos x + \frac{4}{5}\sin x = \frac{6}{5}$ $\qquad \cos(x - \theta) = \frac{6}{5}$,

[where $\theta = \cos^{-1}(3/5)$]

So, that equation has no solution.

71. (a) Given $\sin x + \sin y + \sin z = -3$ is satisfied only when $x = y = z = \frac{3\pi}{2}$, for $x, y, z \in [0, 2\pi]$.

72. (d) $\sin 2\theta = \cos\theta \Rightarrow \cos\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right) \Rightarrow \theta \pm 2\theta = 2n\pi \pm \frac{\pi}{2}$

i.e., $3\theta = 2n\pi + \frac{\pi}{2} \Rightarrow \theta = \frac{1}{3}\left(2n\pi + \frac{\pi}{2}\right)$

and $-\theta = 2n\pi - \frac{\pi}{2} \Rightarrow \theta = -\left(2n\pi - \frac{\pi}{2}\right)$

Hence value of θ between 0 and π are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

i.e., $30^o, 90^o, 150^o$.

73. (b) $2 - 2\cos^2\theta = 3\cos\theta$

$2\cos^2 + 3\cos\theta - 2 = 0$

$\cos\theta = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$

Neglecting (–) sign, we get

$\cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$.

The values of θ between 0 and 2π are $\frac{\pi}{3}, \frac{5\pi}{3}$.

74. (d) $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

$\Rightarrow 2\cos^2 3\theta + 2\cos 3\theta . \cos\theta = 0$

$\Rightarrow 4\cos 3\theta \cos 2\theta \cos\theta = 0$

$\Rightarrow 3\theta = (2n+1)\frac{\pi}{2}$; $2\theta = (2n+1)\frac{\pi}{2}$ and $\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = 30^o, 90^o, 150^o, 45^o, 135^o$.

75. (d) $\operatorname{cosec}\theta + 2 = 0$

$\Rightarrow \sin\theta = -\frac{1}{2} \Rightarrow \theta = 210^o$ or 330^o.

76. (b) We have $1 - \cos 2x + 1 - \cos^2 2x = 2$
or $\cos 2x(\cos 2x + 1) = 0$

$\therefore \cos 2x = 0, -1$, $\therefore 2x = \left(n + \frac{1}{2}\right)\pi$ or $(2n+1)\pi$

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $(2n+1)\frac{\pi}{2}$

Now put $n = -2, -1, 0, 1, 2$

$\therefore x = \frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

Since $-\pi \le x \le \pi$, therefore $x \pm \frac{\pi}{4}, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}$ only.

77. (a) $\sin 7\theta + \sin\theta - \sin 4\theta = 0$

$\Rightarrow \quad 2\sin 4\theta \cos 3\theta - \sin 4\theta = 0$

$\Rightarrow \quad \sin 4\theta(2\cos 3\theta - 1) = 0 \Rightarrow \sin 4\theta = 0,\ \cos 3\theta = \frac{1}{2}$

Now $\sin 4\theta = 0 \ \Rightarrow\ 4\theta = \pi \ \Rightarrow\ \theta = \frac{\pi}{4}$.

and $\cos 3\theta = \frac{1}{2} \ \Rightarrow\ 3\theta = \frac{\pi}{3} \ \Rightarrow\ \theta = \frac{\pi}{9}$.

78. (c) The expression is $\frac{(1+\tan x+\tan^2 x)(1+\tan^2 x-\tan x)}{\tan^2 x}$

$$= \frac{(1+\tan^2 x)^2 - \tan^2 x}{\tan^2 x}$$

Obviously, $1+\tan^2 x \ge \tan^2 x,\ \forall x$. Hence it is positive for all value of *x*.

79. (d) $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$

$\Rightarrow \quad 5(2\cos^2\theta - 1) + (1+\cos\theta) + 1 = 0$

$\Rightarrow \quad 10\cos^2\theta + \cos\theta - 3 = 0$

$\Rightarrow \quad (5\cos\theta + 3)(2\cos\theta - 1) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2}, \cos\theta = -\frac{3}{5} \Rightarrow \theta = \frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$.

80. (c) Given, $\cos\theta = \frac{-1}{2}$ and $0^o < \theta < 360^o$. We know that $\cos 60^o = \frac{1}{2}$ and $\cos(180^o - 60^o)$ $= -\cos 60^o = -\frac{1}{2}$ or $\cos 120^o = -\frac{1}{2}$.

Similarly $\cos(180^o + 60^o)$ $= -\cos 60^o = -\frac{1}{2}$

or $\cos 240^o = -\frac{1}{2}$. Therefore $\theta = 120^o$ and 240^o.

81. (b) $(2\cos x - 1)(3 + 2\cos x) = 0$

Then $\cos x = \frac{1}{2}$ as $\cos x \ne \frac{-3}{2}$

$$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}; \left\{\begin{array}{l} \text{for } n=0,\ x = \frac{\pi}{3}, \frac{5\pi}{3} \\ \text{for } n=1,\ x = \frac{5\pi}{3} \end{array}\right\}$$

82. (a) We have, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

Now check by options, put $x = \frac{\pi}{6}$

then $(81)^{\sin^2 \pi/6} + (81)^{\cos^2 \pi/6} = 30$

$(81)^{1/4} + (81)^{3/4} = 30 \quad 30 = 30$

Hence (a) is the correct answer.

83. (b) $\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = n\pi + \frac{\pi}{3}$

For $-\pi < \theta < 0$

Put $n = -1$, we get $\theta = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$ or $\frac{-4\pi}{6}$.

84. (d) We have, $\tan\theta + \frac{1}{\sqrt{3}} = 0$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$\because\ \theta$ lies in between 0° and 360°

$\therefore\ \theta = 150^\circ$ and 330°.

85. (d) We have, $\cos^2\theta + \sin\theta + 1 = 0$

$1 - \sin^2\theta + \sin\theta + 1 = 0$

$\sin^2\theta - \sin\theta - 2 = 0 \qquad (\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = 2$, which is not possible and $\sin\theta = -1$.

Therefore, solution of given equation lies in the interval $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$.

86. (a) We know $\frac{5^x + 5^{-x}}{2} \ge 1$, (using A.M. $\ge$ G.M.)

But since $\cos(e^x) \le 1$

So, there does not exist any solution.

87. (c) $\tan\theta = -1 = \tan\left(2\pi - \frac{\pi}{4}\right)$, $\cos\theta = \frac{1}{\sqrt{2}} = \cos\left(2\pi - \frac{\pi}{4}\right)$

Hence general value is $2n\pi + \left(2\pi - \frac{\pi}{4}\right) = 2n\pi + \frac{7\pi}{4}$.

88. (d) $\sin\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$

$\tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) \Rightarrow \theta = \left(\pi + \frac{\pi}{6}\right)$

Hence general value of θ is $2n\pi + \frac{7\pi}{6}$.

89. (b) $2\sin^2 x + \sin^2 2x = 2$ ......(i)

and $\sin 2x + \cos 2x = \tan x$ .....(ii)

Solving (i), $\sin^2 2x = 2\cos^2 x$

$2\cos^2 x \cos 2x = 0 \qquad x = (2n+1)\frac{\pi}{2}$ or $x = (2n+1)\frac{\pi}{4}$

$\therefore$ Common roots are $(2n \pm 1)\frac{\pi}{4}$

Solving (ii), $\frac{2\tan x + 1 - \tan^2 x}{1 + \tan^2 x} = \tan x$

$\Rightarrow\ \tan^3 x + \tan^2 x - \tan x - 1 = 0$

$\Rightarrow\ (\tan^2 x - 1)(\tan x + 1) = 0 \ \Rightarrow\ x = m\pi \pm \frac{\pi}{4}$

Trick : For $n = 0$, option (a) gives $\theta = -\frac{\pi}{2}$ which satisfies the equation (i) but does not satisfy the (ii). Now option (b) gives $\theta = \frac{\pi}{4}$ which satisfies both the equations.

90. (b) Eliminating r, we get $\therefore \sin\theta = \frac{1}{2}, -\frac{3}{2}$ (rejected)

$\Rightarrow \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

91. (b) $\cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}, \frac{5\pi}{4}$; $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore$ The general value is $2n\pi + \frac{5\pi}{4}$ or $(2n+1)\pi + \frac{\pi}{4}$.

92. (a) $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$

$\Rightarrow A+B = \frac{\pi}{2}$ and $A-B = \frac{\pi}{6} \Rightarrow A = \frac{\pi}{3}, B = \frac{\pi}{6}$.

93. (a) $2 - 2\cos^2\theta + \sqrt{3}\cos\theta + 1 = 0$

$\Rightarrow 2\cos^2\theta - \sqrt{3}\cos\theta - 3 = 0$

$\Rightarrow \cos\theta = \frac{\sqrt{3} \pm \sqrt{3+24}}{4} = \frac{\sqrt{3}(1\pm 3)}{4} = \sqrt{3}\left(-\frac{1}{2}\right)$

$\Rightarrow \theta = \frac{5\pi}{6}$.

94. (b) $\cot\theta = \sin 2\theta$, $(\theta \neq n\pi) \Rightarrow 2\sin^2\theta\cos\theta = \cos\theta$

$\Rightarrow \cos\theta = 0$ or $\sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}$ or $\theta = n\pi \pm \frac{\pi}{4}$

$\Rightarrow \theta = 90^o$ and 45^o.

95. (a) $\sin\left(\theta + \frac{\pi}{6}\right) = 1 = \sin\left(\frac{\pi}{2}\right) \Rightarrow \theta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.

96. (a) $\cos A \sin\left(A - \frac{\pi}{6}\right) = \frac{1}{2}\left[\sin\left(2A - \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\right]$

But $\sin\left(2A - \frac{\pi}{6}\right) - \frac{1}{2}$ attain maximum value at $2A - \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow A = \frac{\pi}{3}$.

97. (a) Here $\cos\theta = 1 - 2\cos^2 40^o = -(2\cos^2 40^o - 1)$

$= -\cos(2\times 40^o) = -\cos 80^o$

$= \cos(180^o + 80^o) = \cos(180^o - 80^o)$

Hence, $\cos 260^o$ and $\cos 100^o$ *i.e.*, $\theta = 100^o$ and 260°.

98. (a) Since A.M. ≥ G.M. $\frac{1}{2}(2^{\sin x} + 2^{\cos x}) \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1+\frac{\sin x+\cos x}{2}}$

and we know that $\sin x + \cos x \geq -\sqrt{2}$

$\therefore 2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$, for $x = \frac{5\pi}{4}$.

99. (b) $(1+\tan\theta)(1+\tan\phi) = 2 \Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta+\phi) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4} = 45^o$.

100. (a) $\tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\therefore \sin\theta + \cos\theta = \frac{1}{2} \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$.

101. (c) $\tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\therefore \sin\theta + \cos\theta = \frac{1}{2}$ $\quad \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$.

102. (a) The given determinant
(Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$)

reduces to $\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$

$\Rightarrow 1 + 4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$

(By expanding along R_1)

$4\sin 4\theta = -2 \quad \sin 4\theta = \frac{-1}{2}$

$4\theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, $\quad (0 < 4\theta < 2\pi)$

Since, $0 < \theta < \frac{\pi}{2} \quad 0 < 4\theta < 2\pi \quad \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$.

103. (a) Given, cot $(\alpha+\beta) = 0 \Rightarrow \cos(\alpha+\beta) = 0$

$\alpha + \beta = (2n+1)\frac{\pi}{2}, n \in I$

$\sin(\alpha + 2\beta) = \sin(2\alpha + 2\beta - \alpha) = \sin[(2n+1)\pi - \alpha]$

$= \sin(2n\pi + \pi - \alpha) = \sin(\pi - \alpha) = \sin\alpha$.

104. (c) Given equation is, $\cos x - \sin x = \frac{1}{\sqrt{2}}$

Dividing equation by $\sqrt{2}$,

$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{2}$

$\cos\left(\frac{\pi}{4} + x\right) = \cos\frac{\pi}{3}$. Hence, $\frac{\pi}{4} + x = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$

or $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}$.

105. (b) $\sin x\frac{1}{\sqrt{2}} - \cos x\frac{1}{\sqrt{2}} = 1 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) = -1$

$x + \frac{\pi}{4} = 2n\pi \pm \pi \Rightarrow 2n\pi + \frac{3\pi}{4}$ or $2n\pi - \frac{5\pi}{4}$.

106. (c) $12\cot^2\theta - 31\text{cosec}\,\theta + 32 = 0$

$12(\text{cosec}^2\theta - 1) - 31\text{cosec}\,\theta + 32 = 0$

$12\text{cosec}^2\theta - 31\text{cosec}\,\theta + 20 = 0$

$12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$(4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3}$; $\sin\theta = \frac{4}{5}, \frac{3}{4}$.

Periodic functions

1. (b) Period of $|\sin 2x|$. Period of $\sin 2x = \frac{2\pi}{2} = \pi$ and period of $|\sin 2x| = \frac{\pi}{2}$.

2. (b) Since $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$. Hence period $= \frac{2\pi}{2} = \pi$.

3. (c) $\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta} = \frac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\left(\frac{3\theta}{2}\right)$

Hence period $= \frac{2\pi}{3}$.

4. (c) It is obvious.

5. (d) $\sin\theta - \sqrt{3}\cos\theta = 2\sin\left(\theta - \frac{\pi}{3}\right)$, hence period $= 2\pi$.

6. (d) Period of $\sin\frac{x}{2}$ is 4π and period of $\cos\frac{x}{3}$ is 6π. Hence period of expression is 12π (L.C.M.).

7. (c) Period of $\cot 3x$ is $\frac{\pi}{3}$ and period of $\cos(4x+3)$ is $\frac{\pi}{2}$ $\Rightarrow$ L.C.M. is π.

8. (d) Period of $2\sin 3\theta$ is $\frac{2\pi}{3}$ and period of $4\cos 3\theta$ is $\frac{2\pi}{3}$. Therefore period of the expression is $\frac{\pi}{3}$.

9. (a) Let $f(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1 - \frac{4\sin^2 x\cos^2 x}{2} = 1 - \frac{\sin^2 2x}{2}$

$= 1 - \frac{1}{4}(2\sin^2 2x) = 1 - \left(\frac{1-\cos x}{4}\right) = \frac{3}{4} + \frac{1}{4}\cos 4x$

Hence the period of function $= \frac{2\pi}{4} = \frac{\pi}{2}$.

10. (d) Period of $\sin(\theta/3) = 6\pi$ and period of $\cos(\theta/2) = 4\pi$

L.C.M. of 6π and $4\pi = 12\pi$.

11. (d) $\sin\left(\frac{x}{n}\right) = \sin\left(2\pi + \frac{x}{n}\right) = \sin\left(\frac{1}{n}(2n\pi + x)\right)$

Period of the function $\sin\left(\frac{x}{n}\right)$ is $2n\pi$.

$2n\pi = 4\pi \Rightarrow n = 2$.

12. (a) $\sin^2 x = \frac{1-\cos 2x}{2}$ Period $= \frac{2\pi}{2} = \pi$.

13. (b) $\because$ Period of $\sin(ax+b) = \frac{2\pi}{|a|}$

Period of $\sin 2x = \frac{2\pi}{|2|} = \pi$.

14. (c) The period of the function in option (a) is 2.
The period of the function in option (b) is 24.
The period of the function in option (c) is 2π.

15. (d) Period of $\sin\left(\frac{2x}{3}\right) = \frac{2\pi}{2/3} = 3\pi$

Period of $\sin\left(\frac{3x}{2}\right) = \frac{2\pi}{3/2} = \frac{4\pi}{3}$

L.C.M. of 3π and $\frac{4\pi}{3} = 12\pi$. Hence period is 12π.

16. (a) Let $f(x)$ be periodic with period λ, then
$\sin(x+\lambda) + \cos p(x+\lambda) = \sin x + \cos px, \forall x \in R$
Putting $x = 0$ and replace λ by $-\lambda$, we have $\sin\lambda + \cos p\lambda = 1$ and $-\sin\lambda + \cos p\lambda = 1$
Solving these, we get $\sin\lambda = 0$ so $\lambda = n\pi$ and $\cos p\lambda = 1$ so $p\lambda = 2m\pi$. As $\lambda \neq 0$, m and n are non-zero integers. Hence $p = \frac{2m\pi}{\lambda}$, which is rational.

17. (a) Period of $\sin\left(\frac{\pi x}{2}\right) = \frac{2\pi}{\pi/2} = 4$

Period of $\cos\left(\frac{\pi x}{2}\right) = \frac{2\pi}{\pi/2} = 4$

$\therefore$ Period of $\sin\frac{\pi x}{2} + \cos\frac{\pi x}{2} =$ L.C.M. of (4, 4)=4.

18. (d) Period of $\sin\frac{\pi x}{2} = \frac{2\pi}{\pi/2} = 4$

Period of $\cos\frac{\pi x}{3} = \frac{2\pi}{\pi/3} = 6$

Period of $\tan\frac{\pi x}{4} = \frac{\pi}{\pi/4} = 4$

$\therefore$ Period of $f(x) =$ L.C.M. of (4, 6, 4) = 12.

19. (d) Period of $|\sin\pi x| = \frac{\pi}{\pi} = 1$.

20. (c) $f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$

Period of $\sin\left(\frac{\pi x}{n-1}\right) = \frac{2\pi}{\left(\frac{\pi}{n-1}\right)} = 2(n-1)$

and period of $\cos\left(\frac{\pi x}{n}\right) = \frac{2\pi}{\left(\frac{\pi}{n}\right)} = 2n$

Hence period of $f(x)$ is L.C.M. of $2n$ and $2(n-1) \Rightarrow 2n(n-1)$.

Relation between sides and angles, Solutions of triangles

1. (b) $\frac{\sin A}{a}=\frac{\sin B}{b} \Rightarrow \frac{3}{4\times 5}=\frac{\sin B}{7} \Rightarrow \sin B=\frac{21}{20}$,

which is not possible.

2. (a) $\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=1 = \tan\left(\frac{\pi}{4}\right)$, from given data. Hence $C=90^o$.

3. (a) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$

$\Rightarrow bc\sin^2\frac{A}{2}=(s-b)(s-c)$

Hence $x=bc$.

4. (b) A, B, C are in A. P. then angle $B=60^o$,

$\cos B=\frac{a^2+c^2-b^2}{2ac}$, $\left\{\begin{matrix}\text{since } A+B+C=180^o \text{ and}\\ A+C=2B \Rightarrow B=60^o\end{matrix}\right\}$

$\frac{1}{2}=\frac{a^2+c^2-b^2}{2ac} \Rightarrow a^2+c^2-b^2=ac$

$b^2=a^2+c^2-ac$.

5. (c) $(b+c)\cos A+(c+a)\cos B+(a+b)\cos C=a+b+c$

From expanding and collecting terms using projection rule, $a=b\cos C+c\cos B$ etc.

6. (b) $\frac{\sin B}{\sin(A+B)}=\frac{\sin B}{\sin C}=\frac{b}{c}$.

7. (a) $\frac{\sin(A-B)}{\sin(A+B)}=\frac{\sin A\cos B-\sin B\cos A}{\sin C}$

$=\frac{a}{c}\cos B-\frac{b}{c}\cos A$

But $\cos B=\frac{a^2+c^2-b^2}{2ac}, \cos A=\frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \frac{a}{c}\cos B-\frac{b}{c}\cos A=\frac{1}{2c^2}$

$(a^2+c^2-b^2-b^2-c^2+a^2)$

$=\frac{a^2-b^2}{c^2}$.

8. (c) $\cot B+\cot C-\cot A=\frac{\cos B}{\sin B}+\frac{\cos C}{\sin C}-\cot A$

$=\frac{\sin C\cos B+\cos C\sin B}{\sin B\sin C}-\cot A=\frac{\sin(B+C)}{\sin B\sin C}-\frac{\cos A}{\sin A}$

$=\frac{\sin^2 A-\sin B\sin C\cos A}{\sin A\sin B\sin C}=\frac{a^2-bc\cos A}{k(abc)}$

Since $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$ (say)

and $\cos A=\frac{b^2+c^2-a^2}{2bc}$ $=\frac{a^2-bc\frac{(b^2+c^2-a^2)}{2bc}}{(abc)k}$

$=\frac{(a^2-a^2)}{abck}=0, \left\{\text{As } \frac{b^2+c^2-a^2}{2}=\frac{3a^2-a^2}{2}=\frac{2a^2}{2}=a^2\right\}$.

9. (c) $\cos B=\frac{c^2+a^2-b^2}{2ac} \Rightarrow \cos B=\frac{1}{2}$ *i.e.*, $B=\frac{\pi}{3}$.

10. (d) $\left\{\cot\frac{A}{2}+\cot\frac{B}{2}\right\}\left\{a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right\}$

$=\left\{\frac{\cos\frac{C}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}\right\}\left\{a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right\}$

$=\left\{\cos\frac{C}{2}\right\}\left\{a\frac{\sin\frac{B}{2}}{\sin\frac{A}{2}}+b\frac{\sin\frac{A}{2}}{\sin\frac{B}{2}}\right\}$

$=\sqrt{\frac{s(s-c)}{ab}}\left\{a\frac{\sqrt{\frac{(s-a)(s-c)}{ac}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}}+b\frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}\right\}$

$=\sqrt{\frac{s(s-c)}{ab}}\left\{\sqrt{\left(\frac{s-a}{s-b}\right)ab}+\sqrt{\left(\frac{s-b}{s-a}\right)ab}\right\}$

$=\sqrt{s(s-c)}\left\{\frac{s-a+s-b}{\sqrt{(s-a)(s-b)}}\right\}=\sqrt{s(s-c)}\left\{\frac{2s-a-b}{\sqrt{(s-a)(s-b)}}\right\}$

$=c\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}=c\cot\frac{C}{2}$.

Trick : Such type of unconditional problems can be checked by putting the particular values for $a=1$, $b=\sqrt{3}$, $c=2$ and $A=30^o$, $B=60^o, C=90^o$.

Hence expression is equal to 2 which is given by (d).

11. (c) $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in A. P.

$\frac{1}{\sin^2\frac{C}{2}}-\frac{1}{\sin^2\frac{B}{2}}=\frac{1}{\sin^2\frac{B}{2}}-\frac{1}{\sin^2\frac{A}{2}}$

$\frac{ab}{(s-a)(s-b)}-\frac{ac}{(s-a)(s-c)}$

$=\frac{ac}{(s-a)(s-c)}-\frac{bc}{(s-b)(s-c)}$

$\left(\frac{a}{s-a}\right)\left(\frac{b(s-c)-c(s-b)}{(s-b)(s-c)}\right)=\left(\frac{c}{s-c}\right)\left(\frac{a(s-b)-b(s-a)}{(s-a)(s-b)}\right)$

$abs-abc-acs+abc=acs-abc-bcs+abc$

$ab-ac=ac-bc \Rightarrow ab+bc=2ac$

or $\frac{1}{c}+\frac{1}{a}=\frac{2}{b}$, *i.e.*, a,b,c are in H. P.

Note : Students should remember this question as a fact.

12. (c) $(a^2+b^2-2ab)\cos^2\frac{C}{2}+(a^2+b^2+2ab)\sin^2\frac{C}{2}$

$=a^2+b^2+2ab\left(\sin^2\frac{C}{2}-\cos^2\frac{C}{2}\right)$

$=a^2+b^2-2ab\cos C=a^2+b^2-(a^2+b^2-c^2)=c^2$.

13. (a) $2s=a+b+c$; $\cos\frac{B}{2}=\sqrt{\frac{30\times 6}{320}}=\frac{3}{4}$.

14. (a) $\cos A+\cos C=4\sin^2\frac{1}{2}B$

$2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=4\sin^2\frac{B}{2}$

$\cos\frac{A+C}{2}\cos\frac{A-C}{2}=2\sin^2\frac{B}{2}$

$\cos\left(\frac{A-C}{2}\right)=2\sin\frac{B}{2}$

$\cos\frac{A}{2}\cos\frac{C}{2}+\sin\frac{A}{2}\sin\frac{C}{2}=2\sin\frac{B}{2}$

$\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-c)}{ab}}+\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-b)}{ab}}$

$=2\sqrt{\frac{(s-a)(s-c)}{ac}}$

$\frac{\sqrt{(s-a)(s-c)}}{ac}+\frac{s-b}{b}\sqrt{\frac{(s-c)(s-a)}{ac}}=2\sqrt{\frac{(s-a)(s-c)}{ac}}$

$\frac{s}{b}+\frac{s-b}{b}=2$ $\quad a+c=2b$ $\quad a,b,c$ are in A. P.

15. (a) $1-\tan\frac{A}{2}\tan\frac{B}{2}=\frac{\cos\frac{A}{2}\cos\frac{B}{2}-\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}$

$=\frac{\cos\left(\frac{A}{2}+\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}=\frac{\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}$

$=\left[\frac{(s-a)(s-b)bc\,ac}{ab\,s(s-a)s(s-b)}\right]^{1/2}=\frac{c}{s}=\frac{2c}{a+b+c}$.

16. (a) $b^2\cos 2A-a^2\cos 2B=b^2(1-2\sin^2 A)-a^2(1-2\sin^2 B)$

$=b^2-a^2-2(b^2\sin^2 A-a^2\sin^2 B)=b^2-a^2$.

17. (a) $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)$

$=k(\Sigma\sin A\sin(B-C))=k\{\Sigma\sin(B+C)\sin(B-C)\}$

$=k\left\{\Sigma\frac{1}{2}(\cos 2C-\cos 2B)\right\}=0$.

Note: Students should note here that most of the expressions containing the cyclic factor associating with '–' reduces to 0.

18. (c) $\cot A,\cot B$ and $\cot C$ are in A. P.

$\cot A+\cot C=2\cot B$ $\quad \frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}=\frac{2\cos B}{\sin B}$

$\frac{b^2+c^2-a^2}{2bc(ka)}+\frac{a^2+b^2-c^2}{2ab(kc)}=2\frac{a^2+c^2-b^2}{2ac(kb)}$

$a^2+c^2=2b^2$. Hence a^2,b^2,c^2 are in A. P.

Note : Students should remember this question as a fact.

19. (a) $(a+c)^2-b^2=3ac\Rightarrow a^2+c^2-b^2=ac$

But $\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{1}{2}\Rightarrow B=\frac{\pi}{3}$.

20. (c) On putting the values of $\cos A,\cos B$ and $\cos C$, we get the required result *i.e.*, $a^2+b^2+c^2$.

21. (c) $\left(\frac{b}{a}\cos C+\frac{c}{a}\cos B\right)=1$; (by projection rule).

22. (b) It is obvious.

Trick: Obviously it is not an equilateral triangle because $A=B=C=60^o$ does not satisfy the given condition. But $B=90^o$ then $\sin^2 B=1$ and

$\cos^2 A+\cos^2 C=\cos^2 A+\cos^2\left(\frac{\pi}{2}-A\right)$

$=\cos^2 A+\sin^2 A=1$

Hence this satisfy the condition, so it is a right angle triangle but not necessarily isosceles.

23. (c) $x+2x+7x=180^o\Rightarrow x=18^o$

Hence the angles are $18^o,36^o,126^o$

Greatest side $\propto\sin(126^o)$

Smallest side $\propto$ $\sin(18^o)$ and

ratio $=\frac{\sin 126^o}{\sin(18^o)}=\frac{\sqrt{5}+1}{\sqrt{5}-1}$.

24. (c) $\cos C=\frac{\pi}{3}\Rightarrow a^2+b^2-c^2=ab$

$b^2+bc+a^2+ac=ab+ac+bc+c^2$

$b(b+c)+a(a+c)=(a+c)(b+c)$

Divide by $(a+c)(b+c)$ and add 2 on both sides

$1+\frac{b}{a+c}+1+\frac{a}{b+c}=3$ $\quad \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$.

25. (d) $\tan\frac{A}{2}\tan\frac{C}{2}=\frac{1}{2}$ $\quad \sqrt{\frac{(s-b)(s-c)}{s(s-a)}\frac{(s-a)(s-b)}{s(s-c)}}=\frac{1}{2}$

$\frac{s-b}{s}=\frac{1}{2}\Rightarrow 2s-2b-s=0$ $\quad a+c-3b=0$.

26. (b) $\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}}=\sqrt{\frac{ac(s-b)(s-c)(s-b)(s-a)}{(s-a)(s-c)bc\times ab}}=\frac{s-b}{b}$

But a, b and c are in A. P. $\quad 2b=a+c$

Hence $\frac{s-b}{b}=\frac{\frac{3b}{2}-b}{b}=\frac{1}{2}$.

27. (c) $\tan\frac{B-C}{2} = x\cot\frac{A}{2} \Rightarrow x = \frac{b-c}{b+c}$.

28. (c) $\cos B = \frac{9+25-16}{2.3.5} = \frac{18}{2.3.5} = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}$

Therefore $\sin 2B = 2.\frac{4}{5}.\frac{3}{5} = \frac{24}{25}$.

29. (b) $\cos\theta = \frac{4+6-(\sqrt{3}+1)^2}{2.2.\sqrt{6}} \Rightarrow \theta = 75^o$.

30. (b) $a^3\cos(B-C) + b^3\cos(C-A) + c^3\cos(A-B)$

$= k^3\sin^3 A\cos(B-C) + k^3\sin^3 B\cos(C-A) + k^3\sin^3 C\cos(A-B)$

$= \frac{1}{2}k^3[\sin^2 A(\sin 2B + \sin 2C) + \sin^2 B(\sin 2C + \sin 2A) + \sin^2 C(\sin 2A + \sin 2B)]$

$= k^3[\sin A\sin B(\sin A\cos B + \cos A\sin B) + \sin B\sin C(\sin B\cos C + \cos B\sin C) + \sin C\sin A(\sin C\cos A + \cos C\sin A)]$

$= k^3[\sin A\sin B\sin C + \sin B\sin C\sin A + \sin C\sin A\sin B]$

$= 3k^3\sin A\sin B\sin C = 3abc$.

31. (b) $\cos\theta = \frac{49+16.3-13}{2.7.4\sqrt{3}} \Rightarrow \theta = 30^o$.

32. (c) Let sides be $a-d, a, a+d$ and as it is a right angled triangle $(a-d)^2 + a^2 = (a+d)^2$

$a^2 + d^2 - 2ad + a^2 = a^2 + d^2 + 2ad$

$a = 4d \Rightarrow d = \frac{a}{4}$.

Hence the sides are $\frac{3a}{4}, a, \frac{5a}{4}$ *i.e.*, in ratio 3 : 4 : 5.

33. (d) $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\frac{\sqrt{3}}{2} = \frac{(47)^2 + (94)^2 - c^2}{2\times 47\times 94} \Rightarrow c = 58.24$

$\angle B = 124^o$. Hence obtuse angled triangle.

34. (a) It is a fundamental property.

35. (b) $\angle C = 90^o, \angle A = 30^o, c = 20$,

then $a = \frac{c\sin A}{\sin C} = 10$ and $b = \frac{c\sin B}{\sin C} = 10\sqrt{3}$.

Trick : Since the angles are $30^o, 60^o, 90^o$, therefore sides must be $1 : \sqrt{3} : 2$. Hence $a = 10, b = 10\sqrt{3}$.

36. (b) $c\cos(A-\alpha) + a\cos(C+\alpha) = c(\cos A\cos\alpha + \sin A\sin\alpha) + a(\cos C\cos\alpha - \sin C\sin\alpha)$

$= \cos\alpha(c\cos A + a\cos C) + c\sin A\sin\alpha - a\sin C\sin\alpha$

$b\cos\alpha + kac\sin\alpha - kac\sin\alpha = b\cos\alpha$.

37. (b) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$= \frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2abc}$

$= \frac{a^2+b^2+c^2}{2abc}$.

38. (a) $\Sigma a^2(\cos^2 B - \cos^2 C) = \Sigma a^2(\sin^2 C - \sin^2 B)$

$= k^2\Sigma a^2(c^2 - b^2) = 0$.

39. (d) $\frac{1+\cos C\cos(A-B)}{1+\cos(A-C)\cos B} = \frac{1-\cos(A+B)\cos(A-B)}{1-\cos(A-C)\cos(A+C)}$

$\frac{1-\cos^2 A + \sin^2 B}{1-\cos^2 A + \sin^2 C} = \frac{\sin^2 A + \sin^2 B}{\sin^2 A + \sin^2 C} = \frac{a^2+b^2}{a^2+c^2}$.

40. (b) $\frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{\sin\frac{B+C}{2}\sin\frac{A}{2}} = \frac{\sin B + \sin C}{\sin A} = \frac{b+c}{a}$.

41. (a)

$(b^2-c^2)\cot A = (b^2-c^2)\frac{\cos A}{\sin A} = \frac{(b^2-c^2)(b^2+c^2-a^2)}{2bcka}$

Hence L.H.S. $= \frac{1}{2k\,abc}$

$[(b^4-c^4) + (c^4-a^4) + (a^4-b^4) - \{a^2(b^2-c^2) + b^2(c^2-a^2) + c^2(a^2-b^2)\}] = 0$.

42. (d) $\frac{\sin 3B}{\sin B} = \frac{3\sin B - 4\sin^3 B}{\sin B} = 3 - 4\sin^2 B$

$= 3 - 4 + 4\cos^2 B = -1 + \frac{4(a^2+c^2-b^2)^2}{4(ac)^2}$

$= -1 + \frac{\left(\frac{a^2+c^2}{2}\right)^2}{(ac)^2} = -1 + \frac{(a^2+c^2)^2}{4(ac)^2}$

$= \frac{(a^2+c^2)^2 - 4a^2c^2}{4(ac)^2} = \left(\frac{c^2-a^2}{2ac}\right)^2$.

43. (c) Let $\cot\frac{A}{2}, \cot\frac{B}{2}$ and $\cot\frac{C}{2}$ be in A.P. ,

then $2\cot\frac{B}{2} = \cot\frac{C}{2} + \cot\frac{A}{2}$

$2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} + \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$

R.H.S $= \sqrt{\frac{s}{(s-b)}}\left(\sqrt{\frac{(s-c)}{(s-a)}} + \sqrt{\frac{(s-a)}{(s-c)}}\right)$

$= \sqrt{\frac{s}{s-b}}\left(\frac{s-c+s-a}{\sqrt{(s-a)(s-c)}}\right) = \sqrt{\frac{s}{s-b}}\left(\frac{2s-a-c}{\sqrt{(s-a)(s-c)}}\right)$

$= 2\sqrt{\frac{s}{(s-b)}}\sqrt{\frac{(s-b)^2}{(s-a)(s-c)}}$,

$\{\because a+c = 2b,\ a+b+c = 2s$ *i.e.*, $2(s-b) = 2s-a-c\}$

$= 2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$ =L.H.S.

Note : Students should remember this question as a fact.

44. (b) Angles are $x+2x+3x=180^o$ or 30°, 60° and 90°, therefore sides are in ratio of $\sin 30^o:\sin 60^o:\sin 90^o$

$=\frac{1}{2}:\frac{\sqrt{3}}{2}:1=1:\sqrt{3}:2$.

Note: This is a fact. We have used it in so many questions.

45. (c) $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ca}$

$\frac{2(b^2+c^2-a^2)}{2abc}+\frac{a^2+c^2-b^2}{2abc}+\frac{2(a^2+b^2-c^2)}{2abc}=\frac{a}{bc}+\frac{b}{ca}$

$\frac{3b^2+c^2+a^2}{2abc}=\frac{a}{bc}+\frac{b}{ca}$

$\frac{3b}{2ac}+\frac{c}{2ab}+\frac{a}{2bc}=\frac{a}{bc}+\frac{b}{ca}$

$b^2+c^2=a^2$. Hence $\angle A=90^o$.

46. (d) $\cos C=\frac{2}{3}=\frac{81+64-x^2}{2.9.8}\Rightarrow x^2=49\Rightarrow x=7$.

47. (a) $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$ $\quad\tan\left(\frac{90^o}{2}\right)=\frac{\sqrt{3}-1}{\sqrt{3}+1}\cot\frac{A}{2}$

$\tan\left(\frac{A}{2}\right)=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{3+1-2\sqrt{3}}{2}=2-\sqrt{3}$

$\frac{A}{2}=15^o\Rightarrow A=30^o$.

48. (a) $a\frac{s(s-c)}{ab}+c\frac{s(s-a)}{bc}=\frac{3b}{2}$

$2s(s-c+s-a)=3b^2 \quad 2s(b)=3b^2\Rightarrow 2s=3b$

$a+b+c=3b\Rightarrow a+c=2b\Rightarrow a,b,c$ are in A.P.

49. (d) Since the angles are in A.P., therefore $B=60^o$ and $\frac{b}{c}=\frac{\sin B}{\sin C}=\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{3}}{2\sin C}=\frac{\sqrt{3}}{\sqrt{2}}$

$C=45^o$ so that $A=180^o-60^o-45^o=75^o$.

50. (c) Use $\cos A=\frac{b^2+c^2-a^2}{2bc}$.

51. (b) $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{a-b}{a+b}\tan\left(\frac{A+B}{2}\right)$

$\tan\frac{A-B}{2}\cot\frac{A+B}{2}=\frac{a-b}{a+b}$.

52. (a) Since A, B and C are in A.P., therefore $B=60^o$ and $b^2=ac$.

$\cos B=\frac{a^2+c^2-b^2}{2ac}\Rightarrow\frac{1}{2}=\frac{a^2+c^2-b^2}{2b^2}$, $(\because b^2=ac)$

$b^2=a^2+c^2-b^2\Rightarrow a^2+c^2=2b^2$.

53. (b) Largest side is $\sqrt{p^2+pq+q^2}$. If largest angle is θ, then

$\cos\theta=\frac{p^2+q^2-p^2-pq-q^2}{2pq}=-\frac{1}{2}=\cos\left(\frac{2\pi}{3}\right)$

$\theta=\frac{2\pi}{3}$.

54. (b) $\sqrt{\frac{b+c}{4c}}=\sqrt{\frac{\sin 3C+\sin C}{4\sin C}}$

$\sqrt{\frac{2\sin 2C\cos C}{4\sin C}}=\cos C$

$\frac{b-c}{2c}=\frac{\sin 3C-\sin C}{2\sin C}=\frac{2\cos 2C\sin C}{2\sin C}=\cos 2C=\sin\frac{A}{2}$.

55. (a) $(b-c)\cot\frac{A}{2}=k(\sin B-\sin C)\cot\frac{A}{2}$

$=2k\cos\frac{B+C}{2}\sin\frac{B-C}{2}\cot\frac{A}{2}$

$=2k\sin\frac{A}{2}.\sin\frac{B-C}{2}.\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$=2k\sin\left(\frac{B-C}{2}\right)\sin\left(\frac{B+C}{2}\right)=2k\left(\sin^2\frac{B}{2}-\sin^2\frac{C}{2}\right)$

or we get L.H.S. $=\Sigma 2k\left(\sin^2\frac{B}{2}-\sin^2\frac{C}{2}\right)=0$.

56. (a) $2a^2b^2+2b^2c^2=a^4+b^4+c^4$

Also, $(a^2-b^2+c^2)^2=$

$a^4+b^4+c^4-2(a^2b^2+b^2c^2-c^2a^2)$

$(a^2-b^2+c^2)^2=2c^2a^2 \quad \frac{a^2-b^2+c^2}{2ca}=\pm\frac{1}{\sqrt{2}}=\cos B$

$B=45^o$ or 135^o.

57. (b) $\Delta=10\sqrt{3}$

$\Delta=\frac{1}{2}ab\sin C\Rightarrow ab=20\sqrt{3}\frac{2}{\sqrt{3}}=40$ .....(i)

Also $a+b+c=20$ or $a+b=(20-c)$

Now, $\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$

$a^2+b^2-c^2=ab \quad (a+b)^2-c^2=ab+2ab=3ab$

$(20-c)^2-c^2=3(40) \quad -40c+400=120\Rightarrow c=7$.

58. (a) $A+C=2B\Rightarrow B=60^o$, $\cos B=\frac{a^2+c^2-b^2}{2ac}$

Since $B=60^o\Rightarrow ac=a^2+c^2-b^2$

$b^2=a^2+c^2-ac$

Therefore $\frac{a+c}{\sqrt{a^2-ac+c^2}}=\frac{a+c}{b}=\frac{\sin A+\sin C}{\sin B}$

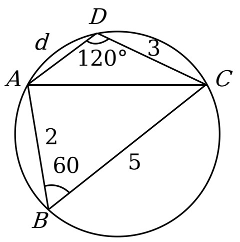
$=\frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{2\sin\frac{B}{2}\sin\frac{A+C}{2}}=\frac{\cos\frac{A-C}{2}}{\sin\frac{B}{2}}$

$$= \frac{\cos\frac{A-C}{2}}{\sin 30^o} \Rightarrow 2\cos\frac{A-C}{2}.$$

59. (a) Let the fourth side be of 'd' length.

We know that $AB^2 + BC^2 - 2AB.BC\cos 60^o = AC^2$

$= CD^2 + DA^2 - 2CD.DA\cos 120^o$ (by cosine rule)



or $4 + 25 - 2.2.5.\frac{1}{2} = 9 + d^2 + 3d$

$d^2 + 3d - 10 = 0 \Rightarrow d = -5$ or $d = 2$; $d = 2$.

60. (a) $\cos A = \frac{(b^2 + c^2 - a^2)}{2bc}$

$\Rightarrow \cos 60^o = \frac{1}{2} = \frac{9 + c^2 - 16}{2.3c}$ $c^2 - 3c - 7 = 0$.

61. (d) $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = -1$

$\angle C = 180^o$, which is inadmissible in a triangle.

62. (b) Here $b\ \sin A = 8\ \sin 30^o = 4, a = 7$

Thus, we have $b > a > b\sin A$.

Hence angle B has two solutions.

63. (d) Hence $c\sin B = 4\sin\frac{\pi}{3} = 2\sqrt{3} > b(=3)$

Thus, we have $b < c\sin B$.

Hence no triangle is possible *i.e.*, the number of triangles that can be constructed is nil.

64. (c) $\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$

$\sin(B+A)\sin(B-A) = \sin(C+B)\sin(C-B)$

or $\sin C(\sin B\cos A - \cos B\sin A)$

$= \sin A(\sin C\cos B - \cos C\sin B)$

Divide by $\sin A\sin B\sin C$

$\therefore \cot A - \cot B = \cot B - \cot C$. Hence the result.

65. (d) Let the sides of ΔABC be $a = n, b = n+1, c = n+2$, where n is a natural number. Then C is the greatest and A the least angle. As given $C = 2A$.

$\sin C = \sin 2A = 2\sin A\cos A$

$kc = 2ka\frac{b^2 + c^2 - a^2}{2bc}$ or $bc^2 = a(b^2 + c^2 - a^2)$

Substituting the values of *a, b, c,* we get

$(n+1)(n+2)^2 = n[(n+1)^2 + (n+2)^2 - n^2]$

or $(n+1)(n+2)^2 = n(n^2 + 6n + 5) = n(n+1)(n+5)$

Since $n \neq -1$, we can cancel $n+1$.

Thus $(n+2)^2 = n(n+5)$ or $n^2 + 4n + 4 = n^2 + 5n$

This gives $n = 4,$ Hence the sides are 4, 5 and 6.

66. (a) From the given relation $\sin C = \frac{1 - \cos A\cos B}{\sin A\sin B} \le 1$ .....(i)

$1 \le \cos A\cos B + \sin A\sin B$

$\cos(A-B) \ge 1$; $\because \cos\theta \not> 1$ .....(ii)

$A - B = 0$ or $A = B$

Hence from (i), $\sin C = \frac{1 - \cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$

$C = 90^o \Rightarrow A + B = 90^o$ or $A = B = 45^o$ {by (ii)}

Hence, $a : b : c = \sin A : \sin B : \sin C = 1 : 1 : \sqrt{2}$.

67. (d) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} \Rightarrow \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$

$\cot A = \cot B = \cot C \Rightarrow A = B = C = 60^o$

ΔABC is equilateral.

$\Delta = \frac{\sqrt{3}}{4}a^2 = \sqrt{3}$.