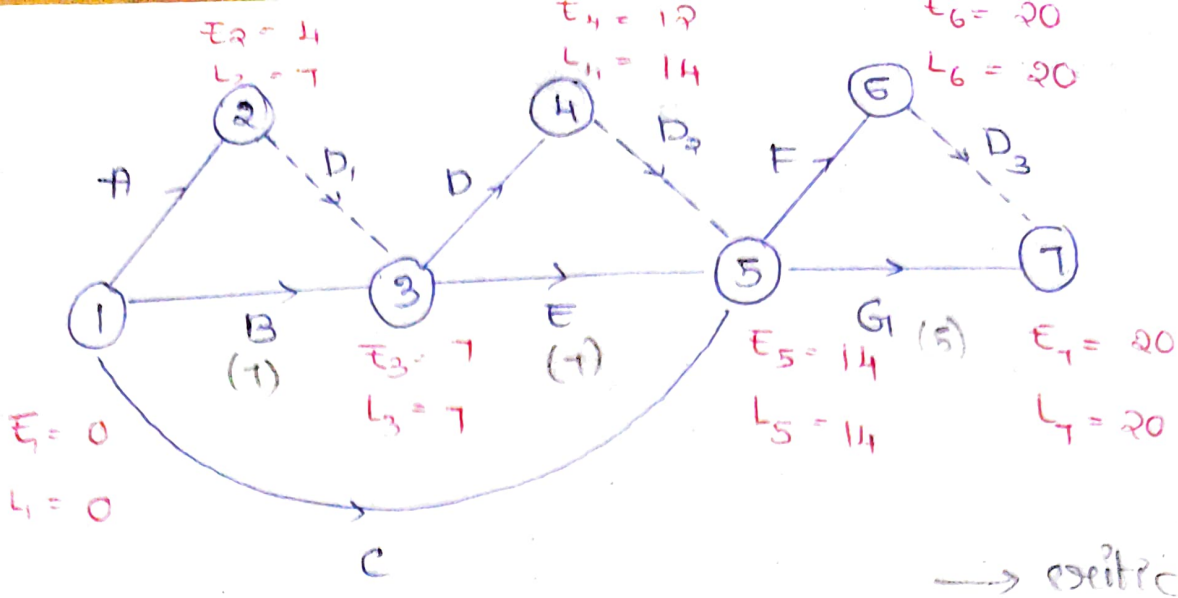


→ critical path happens when E and L are same

∴ Critical activities are.

B, E, F

which implies critical path:

B - E - F.

(B is a critical activity since $E_1 = L_1 = 0$ & $E_3 = L_3 = 7$

also since $E_3 - E_1 = L_3 - L_1 = 7$ (equal to duration of B)

F is a critical activity since $E_3 = L_3 = 7$ & $E_5 = L_5 = 14$

also since $E_5 - E_3 = L_5 - L_3 = 7$ (equal to duration of F)

G is not a critical path because.

$E_7 - E_5 = L_7 - L_5 = 6 \neq$ duration of G = 5)

(a) ∴ Project duration = 20

(b) To evaluate the critical node and total float the calculation in the network are displayed in the following table.

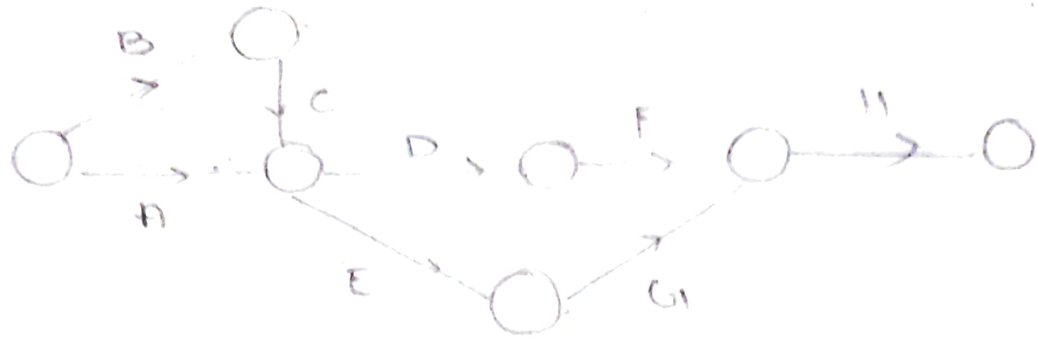
Activity	Normal activity t_{ij}	Earliest start time E_i	Earliest finish time $E_i + t_{ij}$	Latest ~~finish~~ ~~start~~ time L_j	Latest ~~start~~ ~~finish~~ time $L_j - k_{ij}$	Total float $L_j - (E_j + t_{ij})$	Remark
1-2	4	0	4	7	3	3	non critical
1-3	7	0	7	7	0	0	critical
1-5	6	0	6	14	8	8	non critical
3-4	5	7	12	14	9	2	non critical
3-5	7	7	14	14	7	0	critical
5-6	6	14	20	20	14	0	critical
5-7	5	14	19	20	15	1	non critical

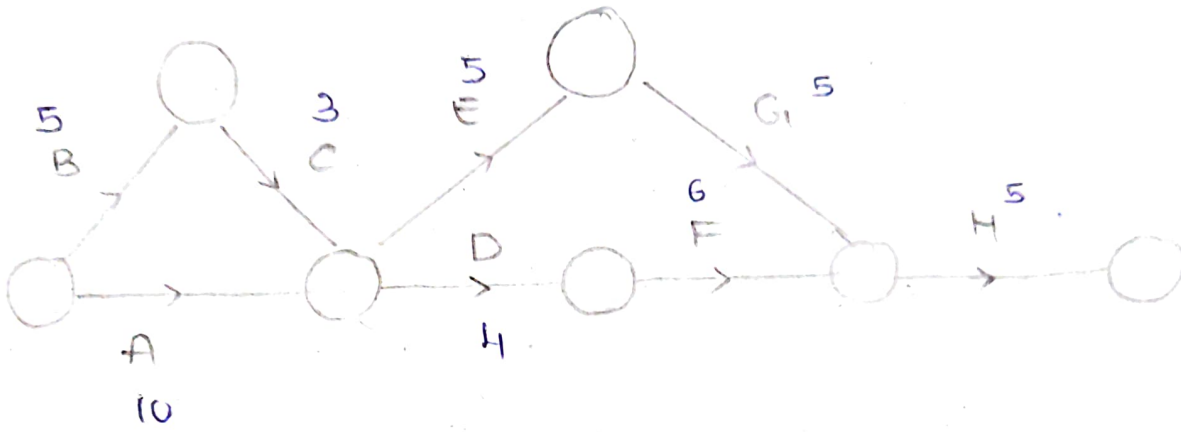
1-3, 3-5, 5-6 is the critical activity.

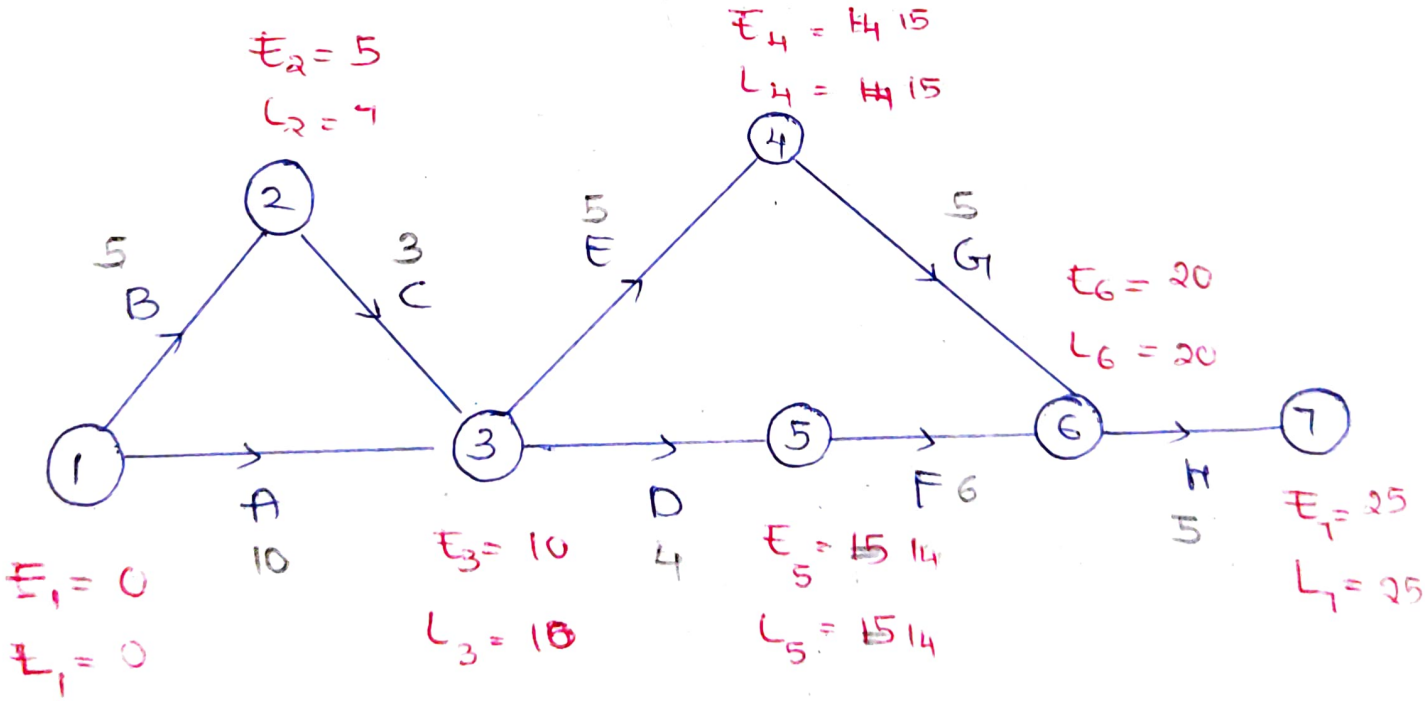
If u are asked to find free float draw a column with $E_j - (E_i + t_{ij})$

(2) Consider the project consisting of 8 jobs A, B, C, D, E, F, G, H. The sequencing and time elaveration are given below. Draw the network and calculate the earliest and latest time. Also find total float and critical path.

Job	Predecessor	Duration
A	-	10
B	-	5
C	B	3
D	A, C	4
E	A, C	5
F	D	6
G	E	5
H	F, G	5







Foreward passing

$E_1 = 10$

$E_2 = 0 + 5 = 5$

$E_3 = 0+10 = 10$

$E_4 =$

$E_3 = \max\{0+10, 5+3\}$

$= \max\{10, 8\}$

$= 10$

$E_4 = 10+5$

$= 15$ ~~14~~

$E_5 = 10+5^4 = 15$ 14

$E_6 = \max\{15+5, 15^4+6\}$

$= \max\{20, 20\}$

$= 20$

$E_7 = 20+5$

$= 25$

Backwaard passing.

$L_4 = 25$

$L_6 = 25-5 = 20$

$L_5 = 20-6 = 14$

$L_4 = 20-5 = 15$

$L_3 = \min\{14-4, 15-5\}$

$= 10$

$L_2 = 10-3 = 7$

$L_1 = \min\{7-5, 10-10\}$

$= 0$

∴ Critical activities are

A, D, E, F, G, H

∴ Critical path are

A-D-F-H.

A-E-G-H.

∴ Project duration = 25.

Activity	Normal time t_{ij}	Earliest start time F_i	Earliest Finish time $E_i + t_{ij}$	latest Finish time L_j	latest start time $L_j - t_{ij}$	Total float $L_j - (E_i + t_{ij})$	Remark
1-2	5	0	5	7	2	2	N.C
1-3	10	0	10	10	0	0	C.
2-3	3	5	8	10	7	2	N.C
3-4	5	10	15	15	10	0	C
3-5	4	10	14	14	10	0	C
4-6	5	15	20	20	15	0	C
5-6	6	~~15~~ 14	20	20	14	0	C
6-7	5	20	25	25	20	0	C

∴ Critical path

A - D - F - H

A - E - G - H.

Critical activities are

(1,3) (3,4) (3,5) (4,6) (5,6) (6,7)

Note:

Critical Activity

- An activity (i,j) with the time duration t_{ij} said to be critical an- if the following are satisfied

1. $E_i = L_i$ and $E_j = L_j$
2. $E_j - E_i = L_j - L_i = k_{ij}$.

hlw

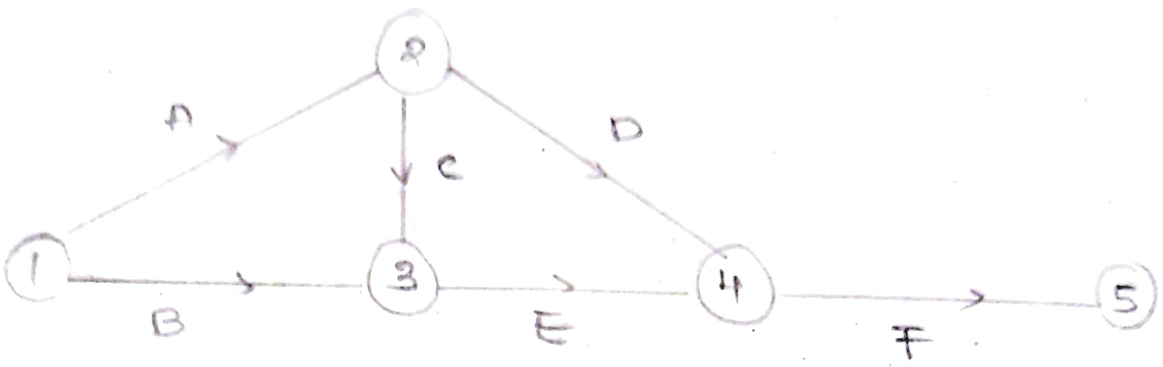
(1) The following are the details of estimated times of activities of a certain project

Activity	Predecessor	Normal time
A	-	16
B	-	20
C	A	8
D	A	10
E	B, C	6
F	D, E	12.

a) Find the critical path and expected time duration of project

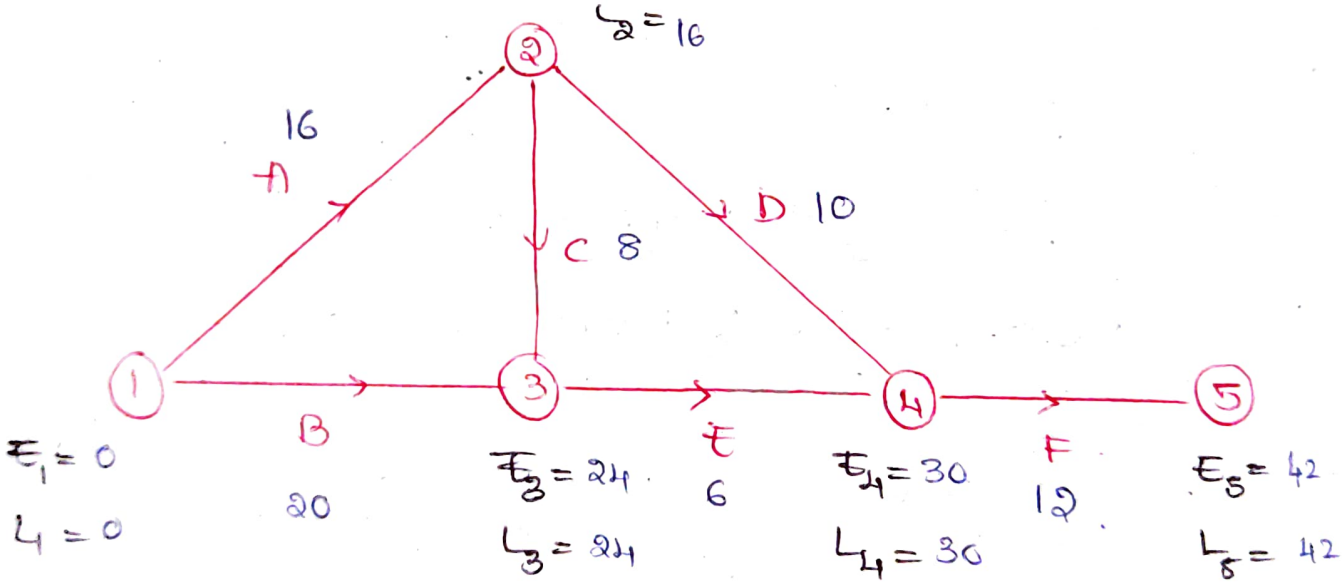
b) Find the total and freefloat for each activity

ans:



$E_2 = 16$

$L_2 = 16$



Forward Passing:

$E_1 = 0$

$E_2 = 16$

$E_3 = \max\{20, 24\} = 24$

$E_4 = \max\{26, 30\} = 30$

$E_5 = 42$

Backward Passing:

$L_1 = 0$

$L_2 = \min\{16, 20\} = 16$

$L_3 = 24$

$L_4 = 30$

$L_5 = 42.$

Critical activities are.

A B C D E F

Critical path are.

1 - C - E - F

Projection duration = 42.

Activity	Normal time t_{ij}	Earliest start E_i	Earliest finish $E_i + t_{ij}$	Latest finish L_j	Latest start $L_j - t_{ij}$	Total float $L_j - (E_i + t_{ij})$	Free float $E_j - (E_i + t_{ij})$	E_j	Remark
1-2	16	0	16	16	0	0	0	16	C
1-3	20	0	20	24	4	4	4	24	N.C
2-3	8	16	24	24	16	0	0	24	C.
2-4	10	16	26	30	20	4	4	30	N.C
3-4	6	24	30	30	24	0	0	30	C
4-5	12	30	42	42	30	0	0	42	C

∴ critical path

A C E F

critical activities are.

(1,2) (2,3) (3,4) (4,5)

Das.SP

21/10/21
Monday

PERT

Project Evaluation Review Technique.

① A statue is to be erected in a village square on a stone base which is to be build on a cement concrete foundation. The statue is made at another place and then move to the base and erected. The various operations of the entire project are given in following random order.

A → make statue.

B → lift statue into place.

C → construct concrete foundation

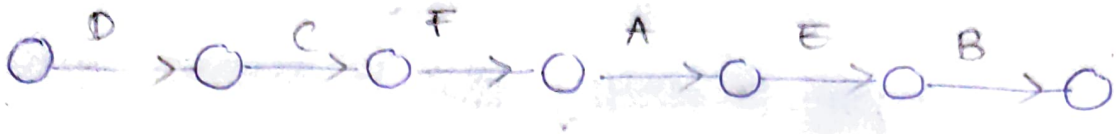
D → compact and levelled site.

E → move statue to the village square

F → construct strong base.

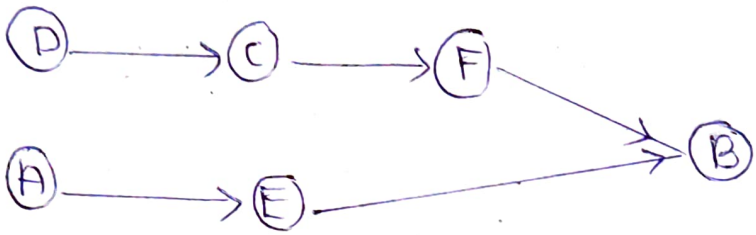
Draw the network.

ans: The simplest form is:



-OR-

We can redraw it as (complex form)



Because
Dcf and AE can be done simultaneously without affecting others work.

Diff b/w PERT and CPM

1) CPM gives more importance to operation of a project and assume that the time required to finish a task is deterministic (already fixed will not change)

In PERT, time is the most essential and basic variable and it is assumed that, there is always some factors of uncertainity in estimating time of any operation. This probabilistic nature is due to the following reasons

(1) Human skills in doing various jobs are subject to change due to ~~Pectic~~ fatigue.

(2) difference in nature of work

(3) Resource availability varies from time to time

(4) Due to changes in climate and local el. conditions.

(5) Activities based on research and development process are probabilistic.

mode' mean most frequent occurring dat

Due to the uncertainity factors we tried to find best estimate of time using some suitable statistical methods.

In PERT we have 3 values for each operation

1) optimistic time (t_o)

- is the shortest possible time to complete the activity if all goes well as planned

2) most likely time (t_m)

- It is the time which often required

if the activity is repeated a no: of times.

Eg: From the past example if the construction of statue required time 13, 15, 13, 14, 16, 14 days then the most likely time $t_m = 14$ days.

3) Pessimistic time. (t_p)

- it is the longest time for an activity under adverse conditions.

→ Exp-experience have shown that the best estimate of time of an activity / operations it is which is known as expected duration of activity is given by the weighted average of t_o, t_m, t_p. and the variance of expected time of an activity.

$$V(t_e) = \left[\frac{t_p - t_o}{6}\right]^2.$$

Expected time of an activity $t_e = \dfrac{t_o + 4t_m + t_p}{6}$

(1, 4, 1) is a std value based on past exp

① A mother notes that when her teenage son uses telephone, he takes no less that 10 min for a phone and sometimes as much as 1 hr, 90 min calls are more frequent than calls of any other duration. Treating his phone call as an activity in a PERT project compute the column.

1) what could be the expected duration of his phone call

2) what would be its variance

3) In scheduling the project how much time could be allocated for a phone call.

ans: Given,

$t_o = 10$

$t_p = 60$

$t_m = 90$

(1)

∴ Expected duration of his phone call,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$= \frac{10 + 80 + 20 \cancel{+ 60}}{6}$

$= \frac{150}{6} = \frac{50}{2} = \underline{25}$

(2) $\text{var}(te) = \left(\frac{tp - t_o}{6}\right)^2$

$= \left(\frac{60 - 10}{6}\right)^2 = \left(\frac{25}{3}\right)^2$

$= \frac{625}{9} = \underline{69.6}$

(3) The time allocated

= expected time of his call

= 25

(2) The following table gives the diff. activities and ~~these~~ different times t_o, t_m, t_p during the installation of a statue in a village square.

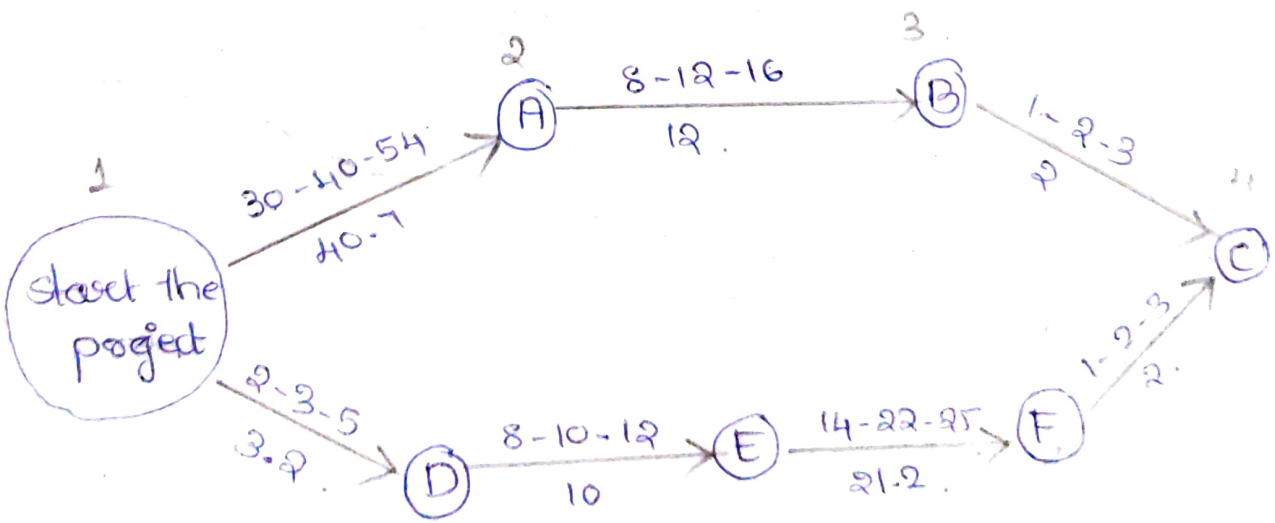
Compute the best estimate the completion and variance of each task. Draw its PERT diagram and find its critical path.

operation	Time to complete the activity (days)		
	t_o	t_m	t_p.
Statue (A) made	30	40	54
Statue (B) moved.	18	12	16
statue (C) lyted.	1	2	3
Site comp leted (D)	2	3	5
Foundation (E) constructed	8	10	12
Base const ructed (F)	14	22	25.

ans: Let A, B, C, D, E, F represents each of the activities in their given order.

A slight delay in an activity affecting the entire project is called critical activity

operation	Expected time duration $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $var(te) = \left(\frac{tp - t_o}{6}\right)^2$
A	$= \frac{244}{6} = 40.7$	16
B	$= \frac{82}{6} = 12.$	1.8
C	$= \frac{142}{6} = 2.$	0.12
D	$= \frac{19}{6} = 3.2$	0.25
E	$= \frac{60}{6} = 10$	0.45
F	$= \frac{127}{6} = 21.2$	3.36



This is called the PERT diagram.

- Time taken by

  path 1 ⟹ 40.7 + 12 + 2 = 54.7

  path 2 ⟹ 3.2 + 10 + 21.2 + 2 = 36.4.

- Maximum these two path is 54.7

Hence it is the critical path.

- ∴ Critical path 1 → 2 → 3 → 4
- Total time (estimated time) to finish the project

  = sum of time needed to finish each activity in the critical path

  = 40.7 + 12 + 2

  = 54.7

- Variance of the critical path.

  = sum of variances of each activity in the critical path

  = 16 + 1.8 + 0.12.

  = 17.92

Hence,

$E(T) = 54.7$

Var(critical path) = 17.92 days.

This means that, normally (in real world) we requires a total time of 54.7 with variation expected in 17.92 days (either take more days or less days).