Page No : 1
Date :

29/4/21

⇒ Random Experiment:-
It is an experiment, trial, or obsrvⁿ that can be repeated numerous times under the same conditions.

⇒ Sample Space:-
Set of all possible outcomes of a random experiment. Denoted by S.

⇒ Sample Point:-
Elements of a sample space is known as sample point.

⇒ Sample space is divided into 2:

1. Discrete:-
If sample space S contains finite no. of points or infinite no. of points which may be arranged in some order and counted then the S is called Discrete.
eg:- Sample space for Dice, $S = \{1, 2, 3, 4, 5, 6\}$
Sample space for Coin, $S = \{H, T\}$.

2. Continuous:-
If sample space S contains continuous points or set of intervals, then that S is called continuous sample space.
eg: life length of an electric bulb.

Page No :
Date :

⇒ Events:-
Any subset of a sample space.
eg: When a coin is tossed, events are
$\phi, \{H\}, \{T\}, \{H, T\}$.

⇒ RANDOM VARIABLES:-

A real valued funcⁿ defined over a sample space of a random experiment is called a random variable.
Denoted by capital letters X, Y, Z..
Elements of random variables are $x_1, x_2 \dots x_n$;
ie, $X = \{x_1, x_2 \dots x_n\}$
eg: When a die is thrown, random variables are:
$X = \{1, 2, 3, 4, 5, 6\}$
• When a coin is tossed, random variables are $X = \{0, 1\}$ $0 \to H$, $1 \to T$

• 2 types of random variables:
1. Discrete RV:
Its a variable which takes only discrete values. ie; Its a variable whose value is obtained by counting.
eg:- no. of students in a class.

Page No :
Date :

2. Continuous RV:-
Its a variable whose value is obtained by measuring.
eg:- height of students in a class.

⇒ Probability Distribution:-

Let X be a discrete RV, where $X = \{x_1, x_2 \dots x_n\}$. Then pb of an event $(X = x)$ is called probability mass funcⁿ or pb density funcⁿ or pb distrbⁿ funcⁿ [pdf]. Denoted by $f(x)$.
ie : $f(x) = P(X = x)$

× Properties:-
1. $f(x) \geq 0$
2. $\sum_x f(x) = 1$
3. $P(a \leq X \leq b) = \sum_{x=a}^{b} f(x)$.

?
$$f(x) = \begin{cases} 1/2 & ; x = 1 \\ 2/3 & ; x = 0 \\ 1/4 & ; x = 2 \\ 0 & ; \text{elsewhere} \end{cases}$$
Can the following funcⁿ a pdf ?
→ • $f(x) \geq 0$ for $x = 0, 1, 2$.
• $\sum f(x) = \frac{1}{2} + \frac{2}{3} + \frac{1}{4} + 0 = \frac{17}{12} \neq 1$
$\therefore f(x)$ is not a pdf.

Page No :

Date :

Q. $f(x) = \begin{cases} -1/2 \; ; & x = 2 \\ 1/2 \; ; & x = 3 \\ 1/2 \; ; & x = 4 \\ 0 \; ; & \text{elsewhere} \end{cases}$

Check $f(x)$ is a pdf or not ?

→ $f(x) < 0$ for $x = 2$

$\therefore f(x)$ is not a pdf.

Q. $f(x) = \begin{cases} 1/3 \; ; & x = -1 \\ 1/3 \; ; & x = 0 \\ 1/3 \; ; & x = 5 \\ 0 \; ; & \text{elsewhere} \end{cases}$ . Is $f(x)$ a pdf ?

→ • $f(x) \geq 0$ for all '$x$'

• $\Sigma f(x) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + 0 = 1$

$\therefore f(x)$ is a pdf.

⇒ Distribution func^n / Cumulative Distribution func^n [CDF] :-

Let $X$ be a discrete RV where $X = \{x_1, x_2 .. x_n\}$. Then pb of an event $(X \leq x)$ is called CDF, denoted by $F(x)$.

ie; $F(x) = P(X \leq x) = \sum_{i=1}^{n} f(x_i)$.

× Properties :-

1. $F(x)$ is a increasing func^n
2. $F(-\infty) = 0$ ; $F(+\infty) = 1$.



⇒ Mean & Variance :-

Let $X$ be a discrete RV where $X = \{x_1, x_2 ... x_n\}$ and corr. pdf are $f(x_1), f(x_2) ... f(x_n)$. Then mean of $X$,

$\mu = E(x)$ ; $E(x)$ - Expectation of $X$.

$= \Sigma x_i f(x_i)$

Variance of $X$,

$\sigma^2 = E(x^2) - [E(x)]^2$

$= \Sigma x_i^2 f(x_i) - \mu^2$

$SD = \sqrt{Variance}$

- $E(c) = c$ ; $c \to$ constant
- $E(aX + b) = aE(x) + b$ ; $a, b \to$ constants
- $E[E(x)] = E(x)$
- $Var(ax) = a^2 Var(x)$.

Q.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | $k/2$ | $k/3$ | $\frac{k+1}{3}$ | $\frac{2k-1}{6}$ |

Find $k$, mean, variance.

→ We know that

$\Sigma f(x) = 1$ —— ①

$\therefore \Sigma f(x) = \frac{k}{2} + \frac{k}{3} + \frac{k+1}{3} + \frac{2k-1}{6}$

$= \frac{3k + 2k + 2(k+1) + 2k - 1}{6}$

$= \frac{5k + 2k + 2 + 2k - 1}{6}$

Page No :
Date :

$\Sigma f(x) = \frac{9k+1}{6}$ — (2)

(1) & (2) ⇒

$\frac{9k+1}{6} = 1$

$9k+1 = 6 \Rightarrow 9k = 5$

$\therefore k = \frac{5}{9}$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | $\frac{5}{18}$ | $\frac{5}{27}$ | $\frac{14}{27}$ | $\frac{1}{54}$ |

$\mu = \Sigma x \cdot f(x)$

$= \left(0 \times \frac{5}{18}\right) + \left(1 \times \frac{5}{27}\right) + \left(2 \times \frac{14}{27}\right) + \left(3 \times \frac{1}{54}\right)$

$= \frac{5}{27} + \frac{28}{27} + \frac{3}{54}$

$= \frac{10+56+3}{54} = \frac{69}{54}$

$\sigma^2 = \Sigma x_i^2 f(x_i) - \mu^2$

$= \left[0 + \left(1^2 \times \frac{5}{27}\right) + \left(4 \times \frac{14}{27}\right) + \left(9 \times \frac{1}{54}\right)\right] - \left(\frac{69}{54}\right)^2$

$= \left[\frac{5}{27} + \frac{56}{27} + \frac{9}{54}\right] - \frac{4761}{2916}$

$= \frac{10+112+9}{54} - \frac{4761}{2916} = \frac{131}{54} - \frac{4761}{2916}$

$= \frac{257}{324}$  0.7932

Page No :
Date :

3/5/21

Q) If the random variable X takes values 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$. Find the pb distrb^n and cdf of $x$.

→ Given $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$

Let $P(X=3) = 30k$

Since $2P(X=1) = 30k \Rightarrow P(X=1) = 15k$

Similarly, $3P(X=2) = 30k \Rightarrow P(X=2) = 10k$

$5P(X=4) = 30k \Rightarrow P(X=4) = 6k$

Since $\Sigma F(x) = 1$

$\Rightarrow 15k + 10k + 30k + 6k = 1$

$\Rightarrow 61k = 1$

$\Rightarrow k = 1/61$

The distrb^n of X is given as:

| X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X=i) = f(x_i)$ | $\frac{15}{61}$ | $\frac{10}{61}$ | $\frac{30}{61}$ | $\frac{6}{61}$ |

when $x = 1$,

$F(x) = \sum_{i=1} f(x_i) = f(x_1) = \frac{15}{61}$

when $x = 2$,

$F(x) = \sum_{i=1}^{2} f(x_i) = f(x_1) + f(x_2)$

$= \frac{15}{61} + \frac{10}{61} = \frac{25}{61}$

when $x = 3$,

$F(x) = \sum_{i=1}^{3} f(x_i) = f(x_1) + f(x_2) + f(x_3)$

$= \frac{15}{61} + \frac{10}{61} + \frac{30}{61} = \frac{55}{61}$

when $x = 4$,

$F(x) = \sum_{i=1}^{4} f(x_i) = f(x_1) + f(x_2) + f(x_3) + f(x_4)$

$= \frac{55}{61} + \frac{6}{61} = \frac{61}{61} = 1$

9. A random variable $x$ has following pb distrbn:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $P(x)$ | $0.1$ | $k$ | $0.2$ | $2k$ | $0.3$ | $3k$ |

a) find $k$.

b) Evaluate $P(x<2)$ and $P(-2<x<2)$.

c) find cdf of $X$

d) Evaluate mean of $x$ and variance of $x$.

→ a) Since $\sum P(x) = 1$

$\Rightarrow 0.1 + k + 0.2k + 2k + 0.3k + 3k = 1$

$\Rightarrow 6k + 0.6 = 1$

$\Rightarrow 6k = 1 - 0.6 = 0.4$

$\Rightarrow k = \frac{0.4}{6} = \frac{1}{15}$

∴ The pb distribution becomes:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $P(x)$ | $1/10$ | $1/15$ | $1/5$ | $2/15$ | $3/10$ | $1/5$ |



b) $P(X<2) = P(X = -2, -1, 0, 1)$

$= P(X=-2) + P(X=-1) + P(X=0) + P(X=1)$

$= \frac{1}{10} + \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{1}{2}$

$P(-2<x<2) = P(X = -1, 0, 1)$

$= P(X=-1) + P(X=0) + P(X=1)$

$= \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{2}{5}$

c) cdf:-

when $x = -2$;

$F(x) = \sum_{i=-2} P(x_i) = P(x_1) = \frac{1}{10}$

when $x = -1$,

$F(x) = \sum_{i=-2}^{-1} P(x_i) = P(x_1) + P(x_2)$

$= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$

when $x = 0$,

$F(x) = \sum_{i=-2}^{0} P(x_i) = P(x_1) + P(x_2) + P(x_3)$

$= \frac{1}{10} + \frac{1}{15} + \frac{1}{5} = \frac{11}{30}$

when $x = 1$,

$F(x) = \sum_{i=-2}^{1} P(x_i) = P(x_1) + P(x_2) + P(x_3) + P(x_4)$

$= \frac{11}{30} + \frac{2}{15} = \frac{1}{2}$

when $x = 2$,

$F(x) = \sum_{i=-2}^{2} P(x_i) = P(x_1) + P(x_2) + P(x_3) + P(x_4) + P(x_5)$

$= \frac{1}{2} + \frac{3}{10} = \frac{4}{5}$

when $n=3$,

$F(x) = \sum_{i=-2}^{3} P(x_i) = P(x_1)+P(x_2)+P(x_3)+P(x_4)+P(x_5)+P(x_6)$

$= \frac{4}{5} + \frac{1}{5} = \frac{5}{5} = 1$

d) Mean of $x$ is defined by:

$\mu = E(x) = \sum x\, p(x)$.

$= \left(-2 \times \frac{1}{10}\right) + \left(-1 \times \frac{1}{15}\right) + \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{2}{15}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{5}\right)$

$= \frac{-1^{\times 3}}{5} + \frac{-1}{15} + 0 + \frac{2}{15} + \frac{3^{\times 3}}{5} + \frac{3^{\times 3}}{5} = \frac{14}{15}$

Variance of $X$,

$\sigma^2 = E(x^2) - [E(X)]^2 = E(x^2) - \mu^2$.

$E(x^2) = \sum x^2 p(x)$

$= \left(4 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{15}\right) + 0 + \left(1 \times \frac{2}{15}\right) + \left(4 \times \frac{3}{10}\right) + \left(9 \times \frac{1}{5}\right)$

$= \frac{2^{\times 3}}{5} + \frac{1}{15} + \frac{2}{15} + \frac{6^{\times 3}}{5} + \frac{9^{\times 3}}{5} = \frac{54}{15}$

$\therefore \sigma^2 = \frac{54}{15} - \left(\frac{14}{15}\right)^2 = 2.462$.

Q.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 0 | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

a) find value of a.

b) Evaluate $P(X<3)$

c) find $\mu, \sigma^2$, cdf.

a) we know that $\sum f(x) = 1$

$\Rightarrow 0+3a+5a+7a+9a+11a+13a+15a+17a=1$

$\Rightarrow 80a = 1 \Rightarrow a = \frac{1}{80}$.

$\therefore$ The pb distribution becomes:

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 0 | 3/80 | 5/80 | 7/80 | 9/80 | 11/80 | 13/80 | 15/80 | 17/80 |

b) $P(X<3) = P(X=0,1,2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= 0 + \frac{5}{80} + \frac{5}{80} = \frac{8}{80} = \frac{1}{10}$

c) $\mu = \sum x\, f(x)$

$= (0\times 0) + \left(1 \times \frac{3}{80}\right) + \left(2 \times \frac{5}{80}\right) + \left(3 \times \frac{7}{80}\right) + \left(4 \times \frac{9}{80}\right) + \left(5 \times \frac{11}{80}\right) + \left(6 \times \frac{13}{80}\right) + \left(7 \times \frac{15}{80}\right) + \left(8 \times \frac{17}{80}\right)$

$= \frac{3+10+21+36+55+78+105+136}{80} = \frac{444}{80} = 5.55$

$\sigma^2 = E(x^2) - [E(x)]^2$

$E(x^2) = \sum x^2 \cdot f(x)$

$= \left(1 \times \frac{3}{80}\right) + \left(4 \times \frac{5}{80}\right) + \left(9 \times \frac{7}{80}\right) + \left(16 \times \frac{9}{80}\right) + \left(25 \times \frac{11}{80}\right.$

$$+\left(\frac{36\times 13}{80}\right)+\left(\frac{49\times 15}{80}\right)+\left(\frac{64\times 17}{80}\right)$$

$$= \frac{3+20+63+144+275+468+735+1088}{80}$$

$$= \frac{2796}{80} = 34.95$$

$\therefore \sigma^2 = 34.95 - (5.55)^2$

$= 4.1475$

cdf:-

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 3/80 | 5/80 | 7/80 | 9/80 | 11/80 | 13/80 | 15/80 | 17/80 |
| $F(x)$ | 0 | 3/80 | 8/80 | 15/80 | 24/80 | 35/80 | 48/80 | 63/80 | 80/80 |

Q

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

find

a) value of k.

b) $P(X<6)$; $P(X\geq 6)$; $P(0<X<5)$

c) If $P(X\leq a)>1/2$ find min value of 'a'.

d) Determine the distribution func$^n$ of X.

e) $P(1.5<X<4.5/X>2)$

→ a) Since $\sum_{x=0}^{7} P(x) = 1$

$\Rightarrow k+2k+2k+3k+k^2+2k^2+7k^2+k=1$



$\boxed{P(X\geq a) = 1-P(X<a)}$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow (10k-1)(k+1)=0$

$\Rightarrow k=\frac{1}{10}$ or $k=-1$

Since P(x) cannot be -ve, k = -1 is rejected $\therefore k = 1/10$

$\therefore$ Pb distrib$^n$ becomes:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | 1/10 | 2/10 | 2/10 | 3/10 | 1/100 | 2/100 | 17/100 |

b) $P(X<6) = P(X=0,1,2,3,4,5)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0+1/10+2/10+2/10+3/10+1/100$

$= \frac{81}{100}$

$P(X\geq 6) = 1-P(X<6)$

$= 1-\frac{81}{100} = \frac{19}{100}$

$P(0<X<5) = P(X=1,2,3,4)$

$= 1/10+2/10+2/10+3/10$

$= \frac{8}{10} = \frac{4}{5}$

c) $P(X\leq a)>1/2$.

By trial, we set

$a=4$.

$P(X\leq 0)=0$; $P(X\leq 1)=1/10$; $P(X\leq 2)=3/10$; $P(X\leq 3)=5/10=0.5$; $P(X\leq 4)=8/10=0.8$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Page No :

Date :

d) cdf :-

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | $1/10$ | $2/10$ | $2/10$ | $3/10$ | $1/100$ | $2/100$ | $17/100$ |
| $F(x)$ | 0 | $1/10$ | $3/10$ | $5/10$ | $8/10$ | $81/100$ | $83/100$ | 1 |

e) $P(1.5 < x < 4.5 / x > 2)$

$= \frac{P((1.5 < x < 4.5) \cap (X > 2))}{P(X > 2)}$

$= \frac{P(x = 3) + P(x = 4)}{P(x = 3, 4, 5, 6, 7)}$

$= \frac{P(X = 3) + P(x = 4)}{P(x = 3) + P(x = 4) + P(X = 5) + P(x = 6) + P(x = 7)}$

$= \frac{2/10 + 3/10}{2/10 + 3/10 + 1/100 + 2/100 + 17/100}$ $\quad \frac{5}{10} + \frac{20}{100}$

$= \frac{5/10}{7/10} = \frac{5}{7}$

c) $P(x \le a) > 1/2 = 0.5$

$P(x \le 0) = 0.$

$P(x \le 1) = P(x = 0) + P(x = 1) = 0 + 1/10 = 0.1$

$P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2) = 3/10 = 0.3$

$P(x \le 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$= 5/10 = 0.5$

$P(x \le 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) = 8/10 = 0.8$

$\therefore a = 4$

5/8/21

Page No :

Date :

$P(x \le a) > 1/2 \Rightarrow$ satisfy when $a = 4$.

9. If the pb mass func of a random variable $x$ is given by

$P(x = r) = kr^3 \; ; r = 1, 2, 3, 4.$

a) find value of $k$.

b) $P((1/2 < X < 5/2) / (X > 1))$

c) find $\mu$ and $\sigma^2$

d) find distrbn func of $x$.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | $k$ | $8k$ | $27k$ | $64k$ |

a) $\Sigma f(x) = 1$

$\Rightarrow k + 8k + 27k + 64k = 1$

$\Rightarrow 100k = 1 \Rightarrow k = 1/100$

$5/2 = 2.5$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | $1/100$ | $8/100$ | $27/100$ | $64/100$ |

b) $P(1/2 < x < 5/2 / x > 1) = \frac{P(1/2 < x < 5/2 \cap x > 1)}{P(x > 1)}$

$= \frac{P(1 < x < 5/2)}{P(x > 1)}$

$= \frac{P(x = 2)}{P(x = 2, 3, 4)}$

$= \frac{8/100}{8/100 + 27/100 + 64/100} = \frac{8}{99}$

c) $\mu = \sum x \cdot f(x)$

$= \left(1 \times \frac{1}{100}\right) + \left(2 \times \frac{8}{100}\right) + \left(3 \times \frac{27}{100}\right) + \left(4 \times \frac{64}{100}\right)$

$= \frac{1}{100} + \frac{16}{100} + \frac{81}{100} + \frac{256}{100} = \frac{354}{100} = 3.54$

$\sigma^2 = E(x^2) - [E(x)]^2$

$E(x^2) = \sum x^2 \cdot f(x)$

$= \left(1 \times \frac{1}{100}\right) + \left(4 \times \frac{8}{100}\right) + \left(9 \times \frac{27}{100}\right) + \left(16 \times \frac{64}{100}\right)$

$= \frac{1}{100} + \frac{32}{100} + \frac{243}{100} + \frac{1024}{100}$

$= \frac{1300}{100} = 13$

$\sigma^2 = 13 - (3.54)^2 = 0.4684$

d) cdf :-

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | $1/100$ | $8/100$ | $27/100$ | $64/100$ |
| $F(x)$ | $1/100$ | $9/100$ | $36/100$ | $100/100 = 1$ |

⇒ Uniform Distribution (discrete) :-

A random variable $x$ is said to follow the uniform distrib$^n$ of the discrete type if its pdf is:



$$f(x) = \begin{cases} 1/n \ ; & x = x_1, x_2 \ldots x_n \\ 0 \ ; & \text{elsewhere.} \end{cases}$$

eg:- Let an unbiased die is thrown and x denote no. of throws. Then the pdf of X is

$$f(x) = \begin{cases} 1/6 \ ; & x = 1, 2, 3, 4, 5, 6 \\ 0 \ ; & \text{elsewhere.} \end{cases}$$

Mean & Variance :-

Mean $= E(X)$

$= \sum_x x \cdot f(x)$

$= \sum_x x \cdot 1/n$

$= \frac{x_1 + x_2 + \ldots + x_n}{n}$

$= \bar{x}$

$E(x^2) = \sum x^2 \cdot f(x)$

$= \sum x^2 \cdot 1/n$

$= \sum_{i=1}^{n} \frac{x_i^2}{n}$

∴ Variance, $Var(x) = E(x^2) - [E(x)]^2$

$= \frac{\sum x_i^2}{n} - (\bar{x})^2$

Q. An unbiased die is thrown. Let x denote no. of throws. Find mean and variance of X.

Page No :
Date :

→ Let x denote no. of throws. Then the pdf,

$f(x) = \begin{cases} 1/6 ; & x = 1, 2, 3, 4, 5, 6 \\ 0 ; & \text{elsewhere.} \end{cases}$

Mean $= \sum_{i=1}^{6} x \cdot f(x)$

$= \sum_{1}^{6} x \cdot 1/6$

$= \frac{1}{6}(1+2+3+4+5+6)$

$= \frac{21}{6} = \frac{7}{2}$

$1+2+\cdots+n = \frac{n(n+1)}{2}$

$\frac{1}{6} \times \frac{6(6+1)}{2}$

$1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Variance $= E(x^2) - [E(x)]^2$

$E(x^2) = \sum_{1}^{6} x^2 \cdot f(x) = \sum^{6} x^2 \cdot \frac{1}{6}$

$= \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2)$

$= \frac{1}{\cancel{6}} \frac{\cancel{6}(6+1)(2\times 6+1)}{6}$

$= \frac{91}{6} = 15.17$

$Var(x) = 15.17 - (\frac{7}{2})^2 = 2.92$

Q. Let an unbiased coin is thrown. Let x denote no. of throws. Find mean and variance of x.

Page No :
Date :

→ Let x denote no. of throws. Then pdf is

$f(x) = \begin{cases} 1/2 ; & x = 0, 1 \\ 0 ; & \text{elsewhere} \end{cases}$

Mean $= \sum x \cdot f(x) = \sum_{x=0}^{1} x \cdot 1/2$

$= \frac{1}{2}(1+0) = 1/2.$

$Var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \sum_{x=0}^{1} x^2 \cdot f(x) = \sum_{x=0}^{1} x^2 \cdot 1/2$

$= 1/2 (0+1) = 1/2$

$Var(x) = \frac{1}{2} - (\frac{1}{2})^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Q. From an urn containing lots numbers 1 to 10. One is randomly chosen and x denote no. of throws. Write down the pdf of x and find its mean and standard deviation.

→ $f(x) = \begin{cases} \frac{1}{10} ; & x = 1, 2, 3 \ldots 10 \\ 0 ; & \text{elsewhere} \end{cases}$

$\mu = \frac{1}{n} \cdot \sum x = \frac{1}{10} \times (1+2+\cdots+10)$

$= \frac{1}{10} \frac{10(10+1)}{2} = \frac{1}{\cancel{10}} \times \frac{\cancel{10} \times 11}{2} = \frac{11}{2} = 5.5$

$SD = \sigma = \sqrt{Var\ x}$

Page No :

Date :

$Var(x) = E(x^2) - [E(x)]^2$

$Var(x) = \frac{1}{n}\sum x^2 - (\bar{x})^2$

$= \frac{1}{n}(1^2+2^2+\dots+10^2) - (5.5)^2$

$= \frac{1}{10}\cdot\frac{10(10+1)(2\times10+1)}{6} - (5.5)^2$

$= \frac{11 * 21}{6} - (5.5)^2$

$= \frac{231}{6} - (5.5)^2 = \underline{8.25}$

$\sigma = \sqrt{Var(x)} = \sqrt{8.25} = \underline{2.87}$

⇒ **Geometric Distribution [discrete]:-**

A random variable $x$ is said to be geometric if its pdf is

$$f(x) = \begin{cases} q^x p & ; q = 1-p;\ 0<p<1 \ (p+q=1) \\ 0 & ; \text{elsewhere} \end{cases}$$

Mean & Variance:-

Mean $= E(x)$

$= \sum_{x=0}^{\infty} x\cdot f(x) = \sum_{x=0}^{\infty} x\cdot pq^x$

$(1-x)^{-2} = 1+2x+3x^2+\dots$

$(1-x)^{-3} = 1+3x+6x^2+\dots$

Page No : 6

Date :

$= p[0+q+2q^2+3q^3+4q^4+\dots]$

$= pq[1+2q+3q^2+4q^3+\dots]$

$= pq[1-q]^{-2}$

$= \frac{pq}{(1-q)^2} = \frac{pq}{p^2} = \frac{q}{p}$ $\because 1-q=p$

$E(x^2) = \sum x^2 f(x)$

$= \sum_{0}^{\infty} x^2 pq^x$

$= p\sum_{0}^{\infty}(x(x-1)+x)q^x$

$= p\sum_{0}^{\infty} x(x-1)q^x + p\sum_{0}^{\infty} xq^x$

$= p(0+0+2\cdot1\times q^2+3\cdot2q^3+4\cdot3q^4+\dots) + q/p$

$= p\cdot2q^2[1+3q+6q^2+\dots] + q/p$

$= 2pq^2(1-q)^{-3} + \frac{q}{p}$

$= \frac{2pq^2}{(1-q)^3} + \frac{q}{p}$

$= \frac{2pq^2}{p^3} + \frac{q}{p} = \frac{2q^2}{p^2} + \frac{q}{p}$

$Var(x) = \frac{2q^2}{p^2} + \frac{q}{p} - \left(\frac{q}{p}\right)^2$

$= \frac{2q^2}{p^2} - \frac{q^2}{p^2} + \frac{q}{p} = \frac{q^2}{p^2} + \frac{q}{p}$

$= \frac{q}{p}\left(\frac{q}{p}+1\right) = \frac{q}{p}\left(\frac{q+p}{p}\right) = \frac{q}{p}\times\frac{1}{p} = \frac{q}{p^2}$

$$\boxed{\begin{aligned}\text{Mean} &= q/p \\ \text{Var} &= q/p^2\end{aligned}}$$

## Distribution func$^n$:-

$F(x) = P(X \le x)$ [by def$^n$]

$$= \sum_{x=0}^{\infty} f(x) = \sum_{x=0}^{\infty} q^x p$$

$$= p(1+q+q^2+\cdots)$$

$$= p(1-q)^{-1}$$

$$= \frac{p}{1-q} = \frac{p}{p} = 1.$$

10/5/2021

## ⇒ Binomial Distribution:-

A random variable '$x$' is binomial distrib$^n$, then its pdf,

$$f(x) = \begin{cases} nC_x\, p^x q^{n-x} & ;\ 0<p<1;\ q=1-p. \\ & \ \ x = 0,1,2\ldots n \\ 0 & ;\ \text{elsewhere.} \end{cases}$$

* $(p+q)^n = 1$

Proof:-

$$(p+q)^n = \sum_0^n f(x) = \sum_0^n nC_x\, p^x q^{n-x}$$

$$= nC_0 p^0 q^{n-0} + nC_1 p^1 q^{n-1} + nC_2 p^2 q^{n-2} + \cdots + nC_n p^n q^{n-n}$$

$$\left(1+\frac{p}{q}\right)^n = \left(\frac{q+p}{q}\right)^n = (1/q)^n = 1/q^n$$

$$q^n \times 1/q^n = 1.$$

$$= q^n\left[1 + nC_1(p/q) + \cdots + nC_n(p/q)^n\right]$$

$$= q^n\left[1+p/q\right]^n = 1 \;//$$

## Mean & Variance:-

* Mean, $\mu = E(x)$

$$= \sum_0^n x f(x) = \sum_0^n nC_x\, p^x q^{n-x}$$

$$= \sum_0^n x \cdot \frac{n!}{(x)!\,(n-x)!} \cdot p^x q^{n-x}$$

$p^x = p^{x+1-1}$

$$= \sum_0^n \frac{x \cdot n!}{x(x-1)!\,(n-x)!}\, p^x q^{n-x}$$

$$= \sum_0^n \frac{n(n-1)!}{(x-1)!\,(n-x)!}\, p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_0^n \frac{(n-1)!}{(x-1)!\,(n-x)!}\, p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1}$$

$$= np \cdot 1^{n-1} = np$$

* Variance, $V(X) = E(X^2) - [E(X)]^2$

$$V(X) = \sum_0^n x^2 \cdot f(x) - (np)^2 \quad \text{—(1)}$$

$$E(X^2) = \sum_0^n x^2 \cdot f(x) = \sum_0^n \left(x(x-1)+x\right) \cdot f(x)$$

$$x(x-1)+x = x^2 - x + x = x^2$$

Page No :

Date :

$$= \sum_{0}^{n} x(x-1) f(x) + \sum_{0}^{n} x \cdot f(x)$$

$$= \sum_{0}^{n} x(x-1)\, nC_x\, p^x q^{n-x} + np.$$

$$= \sum_{0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np.$$

$p^x = p^{x+2-2} = p^{x-2} \cdot p^2$

$$= \sum_{0}^{n} \frac{\cancel{x(x-1)}\, n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!\,(n-x)!} p^2 \cdot p^{x-2} \cdot q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{0}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} \cdot q^{n-x} + np.$$

$$= np^2(n-1)(q+p)^{n-2} + np$$

$$= n(n-1)p^2 \times 1^{n-2} + np$$

$$= n(n-1)p^2 + np$$

$$\therefore V(X) = n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p)$$

$$= npq$$

$$\boxed{\begin{array}{l} \text{Mean, } \mu = np. \\ \text{Variance } \sigma^2 = npq \end{array}}$$



Date :

Q. Find the binomial distribution when mean $= 16$; $Var(X) = 8$; $P(x=0)$; $P(x=1)$; $P(x \geq 2)$.

→ Given, mean $= 16 = np$ —— ①.

Variance $= 8 = npq$ —— ②

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{8}{16}$$

$$q = 1/2$$

$p = 1 - q$

$p = 1 - 1/2$

$= 1/2$

① $\Rightarrow np = 16 \Rightarrow n \times \frac{1}{2} = 16.$

$\Rightarrow n = 32$

Binomial distrib$^n$,

$$f(x) = nC_x\, p^x q^{n-x}$$

$$= 32C_x (1/2)^x (1/2)^{32-x}.$$

- $P(x=0)$

$$f(x_0) = 32C_0 (1/2)^0 (1/2)^{32} = (1/2)^{32}$$

- $P(x=1)$

$$f(x_1) = 32C_1 (1/2)^1 (1/2)^{31}$$

- $P(x \geq 2) = 1 - P(x < 2)$

$$= 1 - P(x = 0, 1)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - [32C_0 (1/2)^0 (1/2)^{32} + 32C_1 (1/2)^1 (1/2)^{31}]$$

Page No :

Date :

Q. Write down the pdf of a binomial distrib$^n$ for which mean is 4 and variance is 3.

→ Given $\mu = np = 4$ —— ①

$\sigma^2 = npq = 3$ —— ②

$\frac{②}{①} \Rightarrow \frac{npq}{np} = \frac{3}{4} \Rightarrow q = 3/4$

$p = 1 - q = 1 - \frac{3}{4} = 1/4$

$① \Rightarrow n \times \frac{1}{4} = 4 \Rightarrow n = 16$

$f(x) = nC_x \, p^x q^{n-x}$

$= \begin{cases} 16C_x (1/4)^x (3/4)^{16-x} & ; x = 0, 1, 2 \cdots 16 \\ 0 & ; \text{elsewhere.} \end{cases}$

Q. Find binomial distrib$^n$ for which mean = 5 & variance = 15/4

→ $\mu = np = 5$ —— ①

$\sigma^2 = npq = \frac{15}{4}$ —— ②

$\frac{②}{①} \Rightarrow \frac{npq}{np} = \frac{15/4}{5} \Rightarrow q = 3/4$

$\frac{15}{4} \times \frac{1}{5}$

$p = 1 - q \Rightarrow p = 1/4$

$① \Rightarrow n \times 1/4 = 5 \Rightarrow n = 20$

Binomial distrib$^n$,

$f(x) = nC_x \, p^x q^{n-x}$

$= \begin{cases} 20C_x (1/4)^x (3/4)^{16-x} & ; x = 0, 1, 2 \cdots 16 \\ 0 & ; \text{elsewhere.} \end{cases}$

Q. A fair coin is tossed 3 times. Find the pb that

(i) 3 tails occur.

(ii) 2 tails and no head occur.

→ The occurrence of tail considered as a success.

$\therefore p = 1/2$ and $q = 1/2$ ; $n = 3$.

$\therefore f(x) = nC_x \, p^x q^{n-x}$.

$= 3C_x (1/2)^x (1/2)^{3-x}$ ; $x = 0, 1, 2, 3$.

(i) $P(\text{3 tails occur}) = P(x = 3)$

$= 3C_3 (1/2)^3 (1/2)^{3-3}$

$= 1 \times (1/2)^3 \times 1 = 1/8$

(ii) $P(\text{2 tail and no head occur})$

$= P(x = 2)$

$= 3C_2 (1/2)^2 (1/2)^{3-2}$

$= \frac{3 \cdot 2}{1 \cdot 2} \times 1/4 \times 1/2 = 3/8$

Page No :

Date :

Q. Out of 800 families of 5 children each, how many would to expect to have (i) 3 boys (iii) 5 girls.

→ Required number = N x pb.

Here $n = 5$ ; $N = 800$.

$p$ = pb for a boy = $1/2$

$q = 1/2$

$P(x) = P(x \text{ boys})$

$= nC_x \, p^x \, q^{n-x}$

$= 5C_x (1/2)^x (1/2)^{5-x}$ ; $x = 0, 1, 2, 3, 4, 5$

(i) $P(3 \text{ boys}) = 5C_3 (1/2)^3 (1/2)^{5-3}$

$= 0.3125$

Required no. of boys $= N \times P(3)$

$= 800 \times 0.3125$

$= 250$

(ii) $P(5 \text{ girls}) = P(\text{no boys})$

$= P(x = 0) = P(0)$

$= 5C_0 (1/2)^0 (1/2)^{5-0}$

$= 1 \times 1 \times (1/2)^5$

$= (1/2)^5 = 0.03125$

required no. of girls $= N \times P(0)$

$= 800 \times 0.03125$

$= 25$

$1 - P(x$

Page No : 8

Date :

Q. Out of 800 families with 4 children each, how many families would be expected to have

i) 2 boys & 2 girls

ii) atleast 1 boy

iii) atmost 2 girls

iv) children of both sexes.

Assume equal pb for boys and girls.

→ Considering each child as a trial, $n = 4$

Assuming birth of a boy is a success.

$P = 1/2$ ; $q = 1/2$ ; $N = 800$

[N → total no. of families]

$P(x) = P(x \text{ boys}) = nC_x \, p^x \, q^{n-x}$

$= 4C_x (1/2)^x (1/2)^{4-x}$ ; $x = 0, 1, 2, 3, 4$

(i) $P(2 \text{ boys } \& \text{ 2 girls}) = P(x = 2)$

$= 4C_2 (1/2)^2 (1/2)^{4-2}$

$= 3/8$

Required number $= N \times P(x = 2)$

$= 800 \times 3/8 = 300$

(ii) $P(\text{atleast one boy}) = P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \{4C_0 (1/2)^0 (1/2)^{4-0}\}$

$= 1 - 1/16 = \frac{15}{16}$

Required number $= N \times P(x \geq 1)$

$= 800 \times 15/16$

$= 750$

$\frac{1}{16} + \frac{6}{16} + \frac{4}{16}$

$\frac{11}{16}$

(iii) P(atmost 2 girls) $= P(x \leq 2)$

$= P(x = 0, 1, 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 1/16 + \{4C_1 (1/2)^1 (1/2)^{4-1}\} + 3/8$

$= 1/16 + 3/8 + 1/4$

$= 11/16$

Required number $= N \times P(x \leq 2)$

$= 800 \times 11/16$

$= 550$

(iv) P(children of both sexes)

$= 1 - P(\text{children of same sex})$

$= 1 - \{P(\text{all are boys}) + P(\text{all are girls})\}$

$= 1 - \{P(x=4) + P(x=0)\}$

$= 1 - \{4C_4 (1/2)^4 (1/2)^{4-4} + 4C_0 (1/2)^0 (1/2)^{4-0}\}$

$= 1 - 1/8 = 7/8$

Required number $= 800 \times 7/8$

$= 700$.

Q. An irregular 6 faced die is such that the pb that it gives 3. Even numbers in 5 throws is twice the pb that it give 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets?

→ Let x denote no. of even numbers obtained 5 trials.

Given $N = 2500$

$P(x=3) = 2P(x=2)$

$5C_3 p^3 q^{5-3} = 2 \cdot 5C_2 p^2 q^{5-2}$

$5C_3 p \cdot \cancel{p^2} \cancel{q^2} = 2 \times 5C_2 \cancel{p^2} q \cancel{q^2}$

$p = 2q$

$p = 2(1-p)$

$\Rightarrow p = 2 - 2p \Rightarrow 3p = 2 \Rightarrow p = 2/3$

$\therefore q = 1 - p = 1 - 2/3 = 1/3$

P(getting no even number) $= P(x=0)$

$= 5C_0 p^0 q^{5-0}$

$= (1/3)^5 = 1/243$

Required number of sets $= N \times P(x=0)$

$= 2500 \times \frac{1}{243} = 10$.

Q. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find parameter 'p' of the distribution.

→ Here $n = 5$

$p(x) = {}^nC_x\, p^x q^{n-x} = 5C_x\, p^x q^{5-x}$.

Given, $p(1) = 0.4096$

$\Rightarrow 5C_1\, p^1 q^4 = 0.4096$

$p(2) = 0.2048$

$\Rightarrow 5C_2\, p^2 q^3 = 0.2048$

$\frac{P(1)}{P(2)} = \frac{5 \cdot p \cdot q^4}{2\cancel{10} \cdot p^2 \cdot q^3} = \frac{q}{2p}$

$\frac{q}{2p} = \frac{0.4096}{0.2048}$

$\frac{q}{2p} = 2 \Rightarrow 4p = q$

$4p = 1 - p$

$5p = 1 \Rightarrow p = 1/5$

? With usual notations, find 'p' for a binomial variate X, if $n = 10$ and $9P(X=4) = P(X=2)$

→ $f(x) = nC_x\, p^x q^{n-x} = 10C_x\, p^x q^{10-x}$.

Given $9 \cdot P(x=4) = P(x=2)$

? For a binomial distribution, the mean is 6 and std devn is $\sqrt{2}$. Write out all terms of this distribution.

→ For a binomial distribn,

$f(x) = nC_x\, p^x q^{n-x}$.

mean $= np = 6$

$SD = \sqrt{2} \Rightarrow Var = 2$

$Var = npq = 2$.

$\therefore \frac{npq}{np} = \frac{2}{6} \Rightarrow q = 1/3$

$p = 1 - q = 1 - 1/3 = 2/3$

$np = 6 \Rightarrow n \times \frac{2}{3} = 6 \Rightarrow n = \cancel{6}^{3} \times \frac{3}{\cancel{2}} = 9$

$\therefore f(x) = nC_x\, p^x q^{n-x}$

$= 9C_x\, (2/3)^x (1/3)^{9-x}$ ; $x = 0, 1, 2 \ldots 9$.

?. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the no. of success.

→ $P(\text{success}) = P(\text{getting 1 or 6})$

$= 1/6 + 1/6 = 2/6 = 1/3$

$q = 1 - p = 1 - 1/3 = 2/3$

$n = 3$

Mean = $np = 3 \times 1/3 = 1$

Variance = $npq$

$= 1 \times 2/3 = 2/3$

Q. In a sampling, a large no. of parts manufactured by a machine, the mean no. of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain

i) no defective

ii) exactly 3 defectives

iii) not more than 3 defectives

iv) atleast 3 defectives

v) atmost 3 defectives.

$n = 20, N = 1000$

Here $np = 2 \Rightarrow 20p = 2$

$\Rightarrow p = 2/20 = 1/10 = 0.1$

$q = 1 - p = 1 - 0.1 = 0.9$

$p(x) = P(x \text{ defective})$

$= 20_{C_x} p^x q^{n-x} = 20_{C_x} (0.1)^x (0.9)^{20-x}$,

$x = 0, 1, \ldots 20.$

i) $P(\text{no defective}) = P(0)$

$= 20_{C_0} (\cdot 1)^0 (\cdot 9)^{20-0} = (0.9)^{20}$

$= 0.1216$

Required numbers = $N \times P(0) = 1000 \times 0.1216 = 122$

ii) $P(\text{exactly 3 defective}) = P(3)$

$= 20C_3 (0.1)^3 (0.9)^{20-3} = 0.1901$

Required number = $1000 \times 0.1901 = 190$

iii) $P(\text{not more than } 3) = P(x \leq 3) = P(x = 0, 1, 2, 3)$

$= P(0) + \{P(1) + P(2) + P(3)$

$= (0.9)^{20} + 20(\cdot 1)(\cdot 9)^{19} + 190(\cdot 1)^2 (\cdot 9)^{18} + 1140 (\cdot 1)^3 (\cdot 9)^{17}$

$= 0.1215 + 0.2702 + 0.2852 + 0.1901 = 0.867$

Required number = $1000 \times P(x \leq 3)$

$= 1000 \times 0.867 = 867$

⇒ **Fitting of a binomial distbⁿ :-**

Consider a frequency table for which $x$ taken values $0, 1, 2, \ldots n$. The corresponding frequencies are $f_0, f_1, \ldots f_n$ and $N = \Sigma f_i$. If it assume that $X$ follows the binomial distribution with parameters $n$ and $p$.

The pb that $X = x$ is

$f(x) = P(X = x) = {}^nC_x p^x q^{n-x}$

and the total number of observation is $N$, we expect that $X = x$ is

$N \times {}^nC_x p^x q^{n-x}$

this is called the theoretical frequency of $X = x$

Here we have to determine n and p

the mean of the binomial distribution is $np$.

from this given observation, mean $= \bar{x}$

$\therefore$ Equate, $\bar{x} = np$

$$\text{ie } p = \frac{\bar{x}}{n}$$

$$\therefore f(x) = n_{C_x}\left(\frac{\bar{x}}{n}\right)^x\left(1-\frac{\bar{x}}{n}\right)^{n-x},\ x = 0,1,2,\dots n$$

And the theoretical frequencies are $N f(0), N f(1), \dots, N f(n)$

? fit a binomial distribution of the following data

| x: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f: | 8 | 32 | 34 | 24 | 5 |

→ Sol

| x | f | xf |
|---|---|---|
| 0 | 8 | 0 |
| 1 | 32 | 32 |
| 2 | 34 | 68 |
| 3 | 24 | 72 |
| 4 | 5 | 20 |
| | $\Sigma fi = 103$ | 192 |

$N = \Sigma fi = 103$

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{192}{103} = 1.86$$

Here $n = 4$

for a binomial distribution,

$$\bar{x} = np \Rightarrow p = \frac{\bar{x}}{n} = \frac{1.86}{4} = 0.46$$

$$q = 1-p = 1-0.46 = 0.54$$

$$f(x) = 4_{C_x}(.46)^x(.54)^{4-x},\ x = 0,1,2,3,4$$

To find the theoretical frequencies

$f(0) = 4C_0\,(.46)^0\,(.54)^{4-0} = 1\times1\times(.54)^4 = 0.085$

$N f(0) = 103\times0.085 = 8.75 \approx 8.8 \approx 9$

$f(1) = 4C_1\,(0.46)^1\,(0.54)^{4-1}$

$= 4\times0.46\times(0.54)^3 = 2.89$

$N.f(1) = 103\times2.89 = 29.84 \approx 30$

$f(2) = 4C_2(0.46)^2(0.54)^2$

$= 6\times(0.46)^2(0.54)^2 = 0.37$

$N.f(2) = 103\times0.37 = 38.13 \approx 38$

$f(3) = 4C_3(0.46)^3(0.54)$

$= 0.21$

$N\times f(3) = 103\times0.21 = 21.66 \approx 22$

$f(4) = 4C_4\,(0.46)^4\times(0.54)^0$

$= 0.044$

$N\times f(4) = 103\times0.044 \Rightarrow 4.61 \approx 4$

| | | | | | | Total |
|---|---|---|---|---|---|---|
| f | 8 | 32 | 34 | 24 | 5 | 103 = $\Sigma f$ |
| T.f | 9 | 30 | 38 | 22 | 4 | $\Sigma N \times f(x) = 103$ |

Q. Fit a binomial distrbⁿ for the following data:

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|---|
| f | 5 | 18 | 28 | 12 | 7 | 6 | 4 | 80 |
| fx | 0 | 18 | 56 | 36 | 28 | 30 | 24 | 192 |

$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{192}{80} = 2.4$

$np = 2.4 \Rightarrow 6p = 2.4$

$\Rightarrow p = \frac{2.4}{6} = 0.4$

$q = 1 - p = 0.6$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Tf | 3.73 | 14.93 | 24.88 | 22.12 | 11.06 | 2.95 | 0.33 |
| ≈TF | 4 | 15 | 25 | 22 | 11 | 3 | 0 |

Q. 6 dies are thrown 729 times. How many times do you expect atleast 3 dies to show a five or six.

→ $n = 6$ ; $N = 729$

$p = P(\text{getting 5 or 6})$

$= 1/6 + 1/6 = 2/6 = 1/3$

$\therefore q = 2/3$



$P(X \geq 3) = P(3) + P(4) + P(5) + P(6)$

$= \frac{233}{729}$

Required number $= N \times P(x \geq 3)$

$= 729 \times \frac{233}{729} = \underline{\underline{233}}$

Q. For a binomial distrbⁿ, mean = 6 and SD = $\sqrt{2}$. Write down pdf of this binomial distrbⁿ.

→ $f(x) = nC_x \, p^x \, q^{n-x}$

mean $= np = 6$

$SD = \sqrt{2} \Rightarrow var = npq = 2$

$\frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$

$\therefore p = 2/3$

$np = 6 \Rightarrow n \times 2/3 = 6 \Rightarrow n = 6 \times \frac{3}{2} = 9$

$f(x) = nC_x \, p^x \, q^{n-x}$

$= 9C_x \, (2/3)^x \, (1/3)^{9-x}$