$$-4x_2 + 3x_3 + 8x_5 + x_6 = 0$$

all $x_j = 0$

6. Find all the basic solutions of the following system of equations :

   $2x_1 + x_2 + 4x_3 = 11,\ 3x_1 + x_2 + 5x_3 = 14$ (June 2006)

7. Use simplex Method solve.

   $Max\ z = 3x_1 + 5x_2$

   $x_1 + x_2 \le 2,\ 2x_1 + 5x_2 \le 10,\ 8x_1 + 3x_2 \le 12, x_1, x_2 \ge 0$

Network Analysis

A project is composed of a number of jobs, activities that are related one to other and all of these should be completed in order to complete the project. An activity of a project can be start only at the completion of many other activities. A network is a combination of activities and events of a project.

Objectives of network analysis

1. Minimization of total cost of project
2. Minimization of total time of a project
3. Minimization of cost of a project for a given total time
4. Minimization of time of a project for a given total cost.
5. Minimization of idle resources
6. Minimization of production delays interruptions and conflicts
7. Planning, scheduling and controlling projects.

Uses of network techniques for management.

1) Network techniques help the management in planning the complicated projects, controlling working plan and also keeping the plan up to date.
2) Network techniques provide a number of checks and safeguards against going astray in developing the plan for the project.
3) Network techniques help the management in reaching the goal with minimum time and least cost and also in forecasting probable project duration and the associated cost.
4) Network techniques have resulted in better managerial control, better utilization of resources and better decision making.

5) Network techniques have resulted in saving of time or early completion of the project which in turn results in earlier return of revenue and increase in profit.

Application of network techniques

Network techniques are widely used in the following areas

1. Construction of building, bridges, factories and irrigation projects.
2. Administration
3. Manufacturing
4. Maintenance planning
5. Research and development
6. Inventory planning
7. Marketing

Fulkersons rule for numbering of events

The numbering of events is necessary in a network. Every activity has two events known as tail and head events. These two events are identified by the numbers given to them. The following steps may be adopted for numbering of events.

1. The initial event of the network diagram is numbered 1.
2. The arrows emerging from the events 1 are then considered. Those arrows end in new events treat them as initial events and number them as 2, 3, 4, ...From these new initial arrow emerge which end in new events. They may be treated as new initial events and number them as 5, 6, ... Follow step 2 until last events which has no emerging arrows.

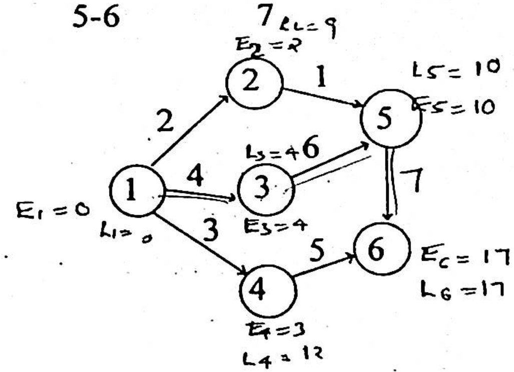
Rules for Constructing network diagram

1. Each activity is represented by one and only one arrow the network

2. No two activities can be identified by the same head and tail events.
3. Except the beginning and ending nodes every node must have atleast one activity proceeding it and atleast one following it.
4. Only one activity may connect any two nodes.

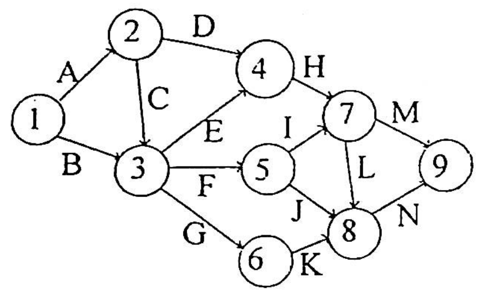
1. Draw the network diagram to the following activities.

Activity	Duration
1-2	2
1-3	4
1-4	3
2-5	1
3-5	6
4-6	5
5-6	7



2.

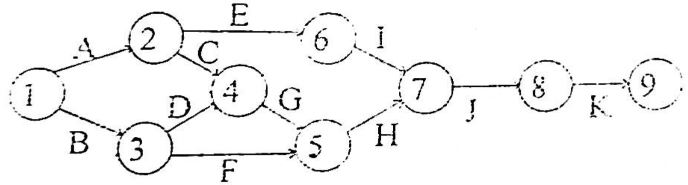
Activity	A	B	C	D	E	F	G
Preceding Activities	—	—	A	A	B&C	B&C	B&C
Activity	H	I	J	K	L	M	N
Preceding Activities	D&E	F	F	G	H&I	H&I	J,K&L



3.

Predecessor Immediate

Activity	A	B	C	D	E	F	G
Prerequisites	—	—	A	B	A	B	C,D
Activity	H	I	J	K			
Prerequisites	G,F	E	H.I	J			



Activities and Event

An activity is a task associated with a a project. It is physically identifiable past of a project which consumes time and resources. An activity is represented by an arrow, the tail of which represents its start and the head,its finish

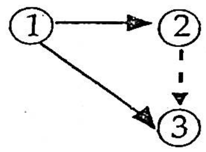
eg: (1) ⟶ (2)

Dummy Activity

Certain activities which does not take time or resources are called Dummy Activity. Dummy activities are used to represents a situation where one event cannot take place until a

previous event has taken place, Dummy activity is denoted by dotted arrows.

eg:-



Event

Event represents instants in time when certain activities have been started or completed ie event describes start or completion of a task. Event is a point in time does not consume any resources. There are two types of event namely tail event and head event. If an activity is2-3 then 2 is the tail event and 3 is the head event.

Earliest and Latest event time

Earliest event time T_E: The earliest occurrence time or earliest event time is the earliest at which an event can occur. Earliest occurrence of an event say 2 is denoted by E_2.

Latest even time, T_E : The latest event time is the latest time by which an event must occur to keep the project on schedule.

Earliest starting Time (EST) : The earliest starting time of an activity is the earliest time by which it can comments. This is naturally equal to the earliest event Wtime associated with the trial event of the activity.

eg., For the activity (2) ⟶ (3) EST is E_2.

Earliest Finish time (EFT) : If the activity preceeds at its early time and takes the estimated duration for completion, then it will have early finish.

EFT + EST + activity duration

Latest Finish time (LFT) : The latest finish time for an activity is the latest time by which an activity can finish without delaying the completion of a project. The latest finish time of an activities 2 3 is L_3

Latest starting Time (LST) : The latest starting time of an activity is the latest time by which an activities can be started without delaying the completion of a project. i.e., LST = LFT – activity duration.

Slack and Float

Slack is a term associate with events. It denotes the flexibility range within which an event can occur. i.e, slack of an event is the difference between the earliest event time and latest event time. ie, slack of the event 2 is L_2-E_2

The term float is associated with activitiy start time or its finish time may fluctuates without affecting completion of the project. Floats are of the following type.

(i). Total float

(ii). Free float

(iii). Independent float

(iv). Interfering float.

(i) Total Float

It is the time spent by which the starting (finishing) of an activity can be delayed without delaying the completion of the project.

Total float = LFT – EFT or

TF = LST - EST

(ii) Free Float (FF)

It is that portion of +ve total float that can be used by an activity without delaying any succeeding activity. The concept of free float is based on the possibility that all the events occur at their earliest time. Hence free float for an acitivity is the difference between its earliest finish time and the earliest start line of its successor activity ie.,

$$\text{Free float} = \text{EST}_{\text{successor}} - \text{EFT}_{\text{present activity}}.$$

(iii) Independent-float

Independent float is defined as the excess of minimum available time over the required activity duration ie., It is the amount of time an activity could be delayed if proceeding activities at their latest and subsequent activities start at their earliest. ie

IF = EST for subsequent activity – LFT for preceding activity.

(iv) Interfering float

It is just another name given to the head event slack specially in CPM network which are activity oriented. It is the difference between total float and free float.

Uses of float

Floats are useful to solve resources leveling and resource allocation problems.

Critical Activity

An activity is said to be critical if a delay in its start will cause a further delay in the completion of the entire project.

CRITICAL PATH METHOD (CPM)

The critical path method, known as CPM, is a network technique. It was originally discovered for applications to industrial situations like construction, manufacturing, maintenance etc. Since then it was found wide acceptance by construction industry with application to bridges dams, tunnels, buildings, highways, power plants etc.

CPM is a network technique which consists of (1) planning the sequence of activities to be performed in a net work (2) scheduling the time and resources to various operations and (3) controlling the performance so that they are not deviating from the plans.

CPM is generally used for repetitive type projects or for those projects for which fairly accurate estimate of time for completion of activity can be made and for which cost estimation can be made with fair degree of accuracy. The critical path method can be used effectively in production planning, road systems and traffic schedules, communication net work etc. CPM emphasizes the relationship between applying more resources to shorten the duration of given jobs in a project and increased cost of these additional resources.

Steps involved in Critical Path Method

1. List all the activities (tasks) and draw a network diagram.
2. Find the Earliest event time (T_E) and Latest event time (T_L) of each event and show in the network diagram.
3. Calculate Earliest start time, Earliest finish time, Latest start time and Latest finish time for each activity.

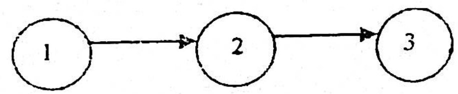
4. Determine the float for each activity.
5. Identify the critical activities (having zero floats)
6. Draw double lines in the network diagram passing through critical activities. The double lines show the critical path.
7. Calculate the total project duration.

Critical path

The sequence of critical activity in a network is called the critical path. It is the longest path in the network from the starting event to the ending event. It defines the minimum time required to complete the project. In the network critical path is denoted by double line.

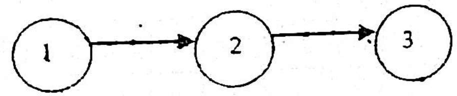
Forward Pass (Earliest start Time Rule)

1. Earliest occurrence of an event = 0, if there is no predecessor event.



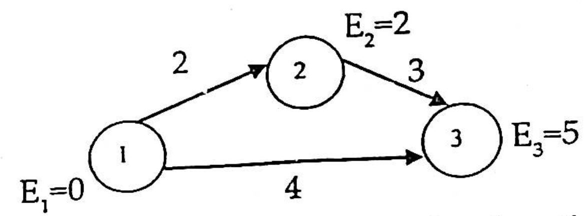
Eg: $E_1 = 0$

2. Earliest occurrence of an event = Earliest occurrence of predecessor event + duration of the predecessor activity.



Eg: E_2=E_1+ duration of the activity 1-2.

3. Earliest occurrence of an Event = Maximum value selected from all the earliest occurrences of every event + duration of each of the corresponding predecessor activity.



Eg: $E_3 = \max(E_2 + \text{duration of 2-3}, E_1 + \text{duration of 1-3})$
$= \max(2+3, 0+4) = \max(5,4) = 5$

Earliest start time (EST) of an activity = Earliest occurrence of the tail event

$\therefore$ EST of 2-3 = E_2.

Backward Pass (Latest finish Time Rule)

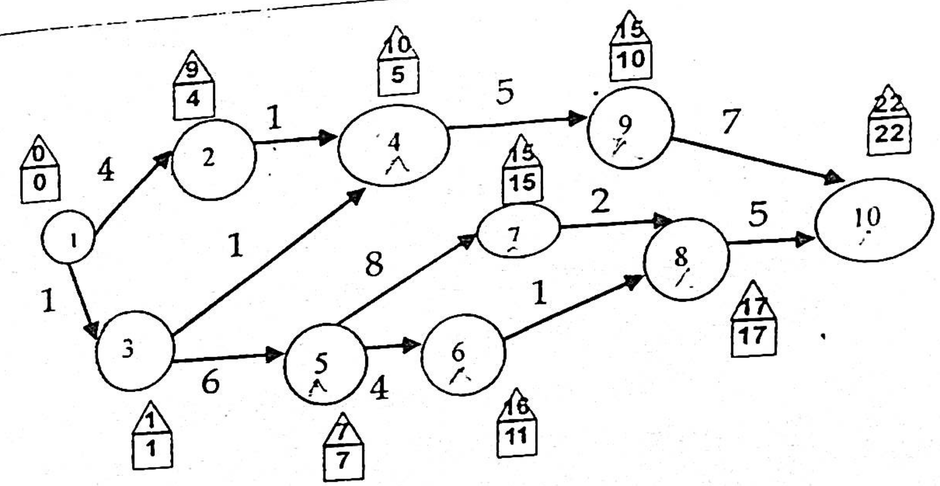
1. Latest occurrence of terminal event = Earliest occurrence of the terminal event, if there is no successor event.
2. Latest occurrence of an event = Latest occurrence of the successor event – duration of the successor activity.
3. Latest occurrence of an event = Minimum chosen from latest occurrence of all the successor events-durations of each of the corresponding successor activity.

***Problems* 1 :**

A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	7

1. Construct a network diagram
2. Compute LST, EST, LFT, EFT of all activities
3. **Find critical path & project duration.**
4. Find F.F, T.F, I.F for each activity



Earliest starting time (EST)

$ES_1 = 0$

$ES_2 = ES_1 + t_{1\text{-}2} = 0 + 4 = 4$

$ES_3 = ES_1 + t_{1\text{-}3} = 0 + 1 = 1$

$ES_4 = \text{Max}\{ES_2 + t_{2\text{-}4}, ES_3 + t_{3\text{-}4}\}$
$= \text{Max}\{4+1, 1+1\}$
$= 5$

$ES_5 = ES_3 = t_{3\text{-}5} = 1 + 6 = 7$

$ES_6 = ES_5 + t5\text{-}6 = 7 + 4 = 11$

$ES_7 = ES_5 + t_{5\text{-}7} = 7 + 8 = 15$

$ES_8 = \text{Max}\{ES_7 + t_{7\text{-}8}, ES_6 + t_{6\text{-}8}\}$
$= \text{Max}\{15+2, 11+1\}$
$= 17$

$ES_9 = ES4 + t4\text{-}9 = 5+5 = 10$

$$ES_{10} = \text{Max}\{ES_9 + t_{9\text{-}10}, ES_8 + t_{8\text{-}10}\}$$
$$= \text{Max}\{10 + 7, 17 + 5\}$$
$$= 22$$

Latest Finish Time

$L_{10} = 22$

$L_9 = L_{10} - t_{9\text{-}10} = 22\text{-}7 = 15$

$L_8 = L_{10} - t_{8\text{-}10} = 22\text{-}5 = 17$

$L_7 = L_8 - t_{7\text{-}8} = 17\text{-}2 = 15$

$L_6 = L_8 - t_{6\text{-}8} = 17\text{-}1 = 16$

$L_5 = \text{Min}\{L_7\text{-}t_{5\text{-}7}, L_6\text{-}t_{5\text{-}6}\}$

$= \text{Min}\{15\text{-}8, 16\text{-}4\} = 7$

$L_4 = L_9 - t_{4\text{-}9} = 15\text{-}5 = 10$

$L_3 = \text{Min}\{L_4\text{-}t_{3\text{-}4}, L_5 - t_{3\text{-}5}\}$

$= \text{Min}\{19\text{-}1, 7\text{-}6\} = 1$

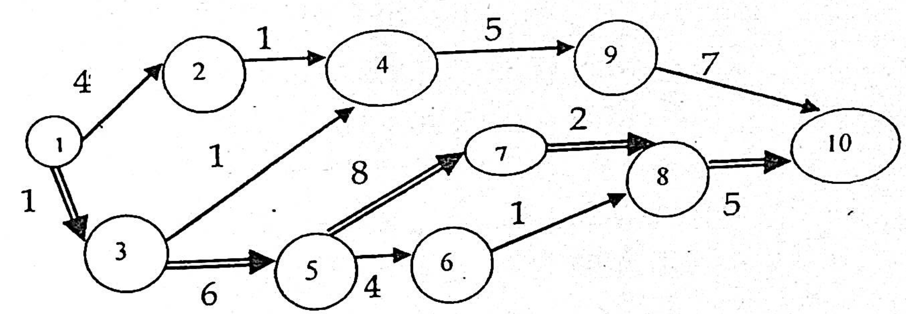
$L_2 = L_4 - t_{2\text{-}4} = 10\text{-}1 = 9$

$L_1 = \text{Min}\{L_2\text{-}t_{1\text{-}2}, L_3\text{-}t_{1\text{-}3}\}$

$= \text{Min}\{9\text{-}4, 1\text{-}1\}$

$= 0$

Activity	Duration	Earliest time		Latest Time		Total float
		EST	EFT $E_i + t_{ij}$	LST $L_j - t_{ij}$	LFT	LFT-EFT or LST -EST
1-2	4	0	4	5	9	5
1-3	1	0	1	0	1	0
2-4	1	4	5	9	10	5
3-4	1	1	2	9	10	8
3-5	6	1	7	1	7	0
4-9	5	5	10	10	15	5
5-6	4	7	11	12	16	5
5-7	8	7	15	7	15	0
6-8	1	11	12	16	17	5
7-8	2	15	17	15	17	0
8-10	5	17	22	17	22	0
9-10	7	10	17	15	22	5



The critical path is 1-3-5-7-8-10

The duration of the project is (project length)

= 1 + 6 8 + 2 + 5 = 22

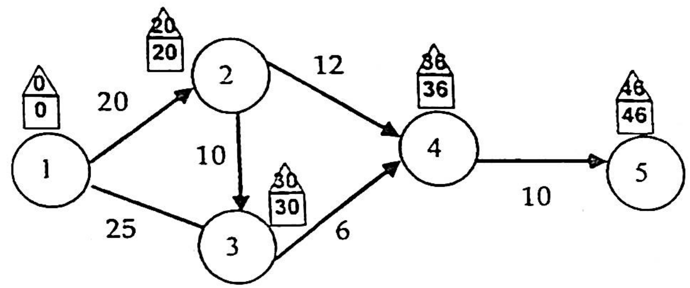
2.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Time	20	25	10	12	6	10

a) Draw the network for the project

b) Find free, total & independent float for each activity

c) Which are the critical activities.



Earliest starting Time

$ES_1 = 0$

$ES_2 = ES_1 + t_{1-2} = 0 + 20 = 20$

$ES_3 = \text{Max}\{ES_1 + t_{1-3}, ES_2 + t_{2-3}\}$

$= \text{Max}\{0 + 25, 20 + 10\}$

$= 30$

$ES_4 = \text{Max}\{ES_3 + t_{3-4}, ES_2 + t_{2-4}\}$

$= \text{Max}\{30 + 6, 20 + 12\}$

$= 36$

$ES_5 = ES_4 + t_{4-5} = 36 + 10$

$= 46$

Latest Finish Time

$L_5 = 46$

$L_4 = L_5 - t_{4-5} = 46 = 10 = 36$

$L_3 = L_4 - t_{3-4} = 36-6 = 30$

$L_2 = \text{Min}\{L_4 - t_{2-4}, L_3 - t_{2-3}\}$

$= \text{Min}\{36-12, 30-10\}$

$= 20$

$L_1 = \text{Min}\{L_2 - t_{1-2}, L_3 - t_{1-3}\}$

$= \text{Min}\{20-20, 390-25\}$

$= 0$

F.F = $E_j - E_i - t$

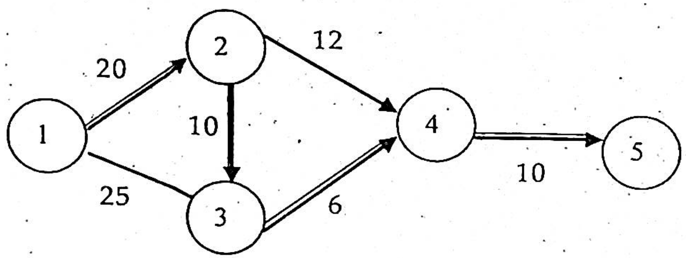
Indep F = $E_j - L_i - t_{ij}$

TF = LFT − EFT or LST − EST

Activity	Duration	Earliest Time EST (E_i)	EFT ($E_i + t_{ij}$)	Latest Time LST ($L_j - t_{ij}$)	LFT (L_j)	TF (LFT−EFT)	FF	I-F
1-2	20	0	20	00	20	0	0	0
1-3	25	0	25	05	30	5	5	5
2-3	10	20	30	20	30	0	0	0
2-4	12	20	36	24	36	4	4	4
3-4	06	30	36	30	36	0	0	0
4-5	10	36	46	36	46	0	0	0

I F

TF − FF



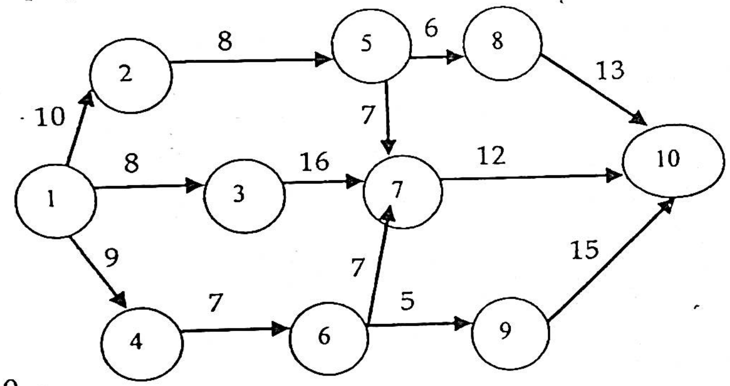
The critical path is 1-2-3-4-5

Critical activities are 1-2, 2-3,3-4, 4-5

3.

Activity	1-2	1-3	1-4	2-5	4-6	3-7	5-7	6-7	5-8	6-9	7-10	8-10	9-10
Time	10	8	9	8	7	16	7	7	6	5	12	13	15

a) Draw the network diagram
b) Identify the critical path
c) Find the project duration



EST

$ES_1 = 0$

$ES_2 = ES_1 + t_{1\text{-}2} = 0 + 10 = 10$

$ES_3 = ES_1 + t_{1\text{-}3} = 0 + 8 = 8$

$ES_4 = ES_1 + t_{1\text{-}4} = 0 + 9 = 9$

$ES_5 = ES_2 + t_{2\text{-}5} = 10 + 8 = 18$

$ES_6 = ES_4 + t_{4\text{-}6} = 9 + 7 = 16$

$ES_7 = \text{Max}\{ES_3 + t_{3\text{-}7}, ES_5 + t_{5\text{-}7}, ES_6 + t6\text{-}7\}$

$= \text{Max}\{8 + 16, 18 + 7, 16 + 7\} = 25$

$ES_8 = ES_5 + t_{5\text{-}8} = 18 + 6 = 24$

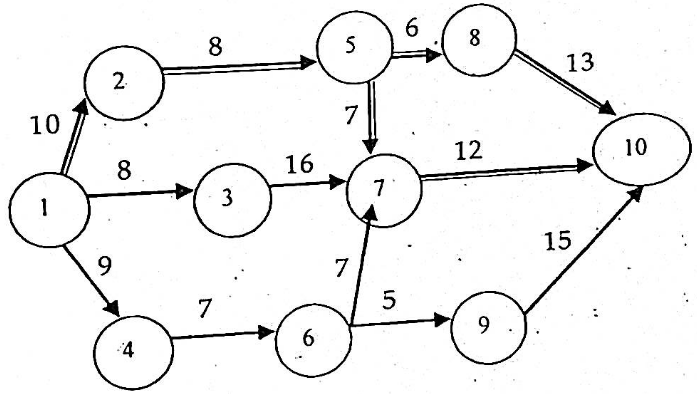
$ES_9 = ES_6 + t_{6\text{-}9} = 16 + 5 = 21$

$ES_{10} = \text{Max}\{ES_8 + t_{8\text{-}10}, ES_7 + t_{7\text{-}10}, ES_9 + t_{9\text{-}10}\}$

$= \text{Max}\{24 + 13, 25 + 12, 21 + 15\}$

$= 37$

Activity	Duration	Earliest time		Latest Time		TF
		EST	EFT	LST	LFT	LFT-TFT
1-2	10	0	10	0	10	0
1-3	8	0	8	0	9	1
1-4	9	0	9	1	10	1
2-5	8	10	18	10	18	0
4-6	7	9	16	10	17	1
3-7	16	7	24	9	25	1
5-7	7	18	25	18	25	0
6-7	7	16	23	18	25	2
5-8	6	18	24	18	24	0
6-9	5	16	21	17	22	1
7-10	12	25	37	25	37	0
8-10	13	24	37	24	37	0
9-10	15	21	36	22	37	1



The critical path is 1-2-5-7-10 and 1-2-5-8-10

Total project duration is 10+ 8 + 7 + 12 or 10 + 8 + 6 + 13 = 37

4. Tasks : A,B,C .... H, I constitute a project. The notation A<B means that the task A must be finished before B can begin,. With this notation

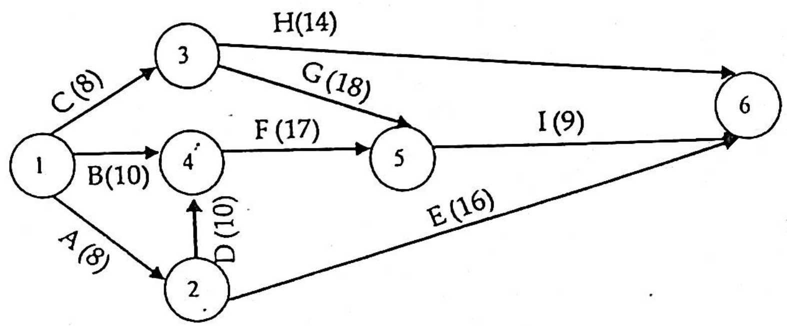
A<D A<E, B<F, C<G D<F, C<H, F<I, G<I

Draw a graph to represent the sequence of tasks and find the minimum time of completion of the project, when the time (in days) of completion of each task is as follows and minimum time for the completion of the project.

Tasks:	A	B	C	D	E	F	G	H	I
Time:	8	10	8	10	16	17	18	14	9

Find the critical path and minimum time for the completion of the project.

Ans: Network diagram



Calculation of T_E and T_L values for all events, we get

$E_1 = 0,\ E_2 = 0+8$

$E_3 = 0+8=8$

$E_4 = \max (10, 10+8) = 18,$

$E_5 = \max (18+ 17, 8+18) = 35$

$E_6 = \max (8+14, 35+9, 8+16) = 44$

$L_6 = E_6 = 44$

$L_5 = 44-9 = 35$

$L_4 = 35-17= 18$

$L_3 = \min (44-14, 35 - 18) = 17,$

$L_2 = \min (44-16, 18-10)= 8$

$L_1 = 0$

Task (I, j)	Duration	Total Float
1-2	8	8-0-8 = 0
1-3	8	17-0-8 = 9
1-4	10	18-0-10 = 0
2-4	10	18-8-10 = 0
2-6	16	44-8-16 = 20
3-5	18	35-8-18 = 9
4-5	17	35-18-17 = 0
3-6	14	44-8-14 = 22
5-6	9	44-35-9 = 0

Critical activities 1-2, 2-4., 4-5 and 5-6 which have zero float

Excersice

1. A project have the following timing schedule

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time	2	1	4	8	5	3	1	5	4	3	2

a) Construct a pert network and compute the total floatfor each activites.

b) Find critical path and duration of the project.

2. Construct the network for the project whose activities are given below and complete total free and independent float for each activity and hence find critical path and project duration.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-5	4-7	5-7	6-7
Time	3	8	12	6	3	3	8	5	3	8

3. Consider the following data for activities in a given project

Activity	A	B	C	D	E	F
Predecessor	-	A	-	B,C	C	D,E
Time	5	4	7	3	4	2

Draw the arrow diagram for the project. Compute the earliest and the latest event times. What is the minimum project completion time? List the activities on the critical path.

4) Task A, B, C ..... H, I constitute a project the rotation x<y means that task n must be finished before y can begin. With this rotation A<D, A<E, B<F, D<F, C<G, C<H, F<I, G<I. Draw a network for this project. Compute the total and free floats. Determine the critical path f project. Find also the minimum time of completion of project when the time (in days) of completion of each task as follows

Task	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

5. A project consists of 20 activities where time estimates (in days) are given below

Activity	Time
1 – 2	15
1 – 3	13
1 – 4	14
2 – 5	9
2 – 7	8
3 – 5	7
3 – 6	8
3 – 8	34
4 – 6	9
4 – 7	21
5 – 10	22
6 – 9	33
6 – 11	25
7 – 10	10
8 – 9	5
8 – 12	17
9 – 10	17
9 – 11	13
10 – 12	19
11 – 12	18

(a) Draw a network for the project
(b) Determine the critical path
(c) Compute the total and free float and summarise the critical path calculation in a tabular form.

5. A project has the following schedule

Activity	Time
1 – 2	2
1 – 3	2
1 – 4	1
2 – 5	4
3 – 6	8
3 – 7	5
4 – 6	3
5 – 8	1
6 – 9	5
7 – 8	4
8 – 9	3

Construct a PERT network and compute (i) the total float for each activity (ii) Critical path and duration?

Project Evaluation and Review Technique (PERT)

PERT is a management technique in which we try to exercise logical disciplines in planning and controlling projects. It is a network technique which uses a network diagram consisting of events. The successive events are jointed by arrows. The main objective of the PERT analysis is to find out whether a job could be finished on a given date. That is, to minimising the total time for a project.

PERT system is preferred for those projects or operations which are of non repetitive nature or for those projects in which precise time determination for various activities can not be made.

In such projects, managements cannot be guided by the past experience. For example, the project of launching a space craft involves the work never done before. For such research and development projects, the time estimates made for use may be little more than guesses. PERT system is best suited for such projects.

PERT is useful technique in project planning and control. It gives the planner a perfect idea about the sequence of activities and their times. It is a method of minimizing delay and interruptions. It helps in coordinating the various parts of the overall job and seeing that every predecessor activity is finished in time for the following activity to commence. It shows the way how a project can be finished earlier than the original schedule. For this, resources may be reallocated from activities with spare time to activities that have no spare time.

The main assumption in PERT is that activity durations are independent. That is, time required for one activity has nothing to do with the time for another activity.

Time Estimate

Time is the most essential and basic variable in pert calculation. Probabilistic approach is followed for time estimation. Here three time estimates are made for each activities.

1. Optimistic Time (to)
2. Pessimistic Time (tp)
3. Most likely time (tm)

The expected or average time based upon these 3 estimates is workout by the formula:

$$t_e = \frac{t_o + t_p + 4t_m}{6}$$

Optimistic Time

This is the shortest possible time in which an activity can be completed under ideal condition. This particular time estimates represents the time in which the activity or job can be completed if everything goes well with no problems.

Pessimistic Time

This is the maximum time that would be required to complete the activity. This particular time estimates represent the time it might take to complete a particular activity if every thing went wrong.

Most likely time

This is the time which the activity will take most frequently, if performed a number of times. This time estimates reflects a situation where condition are longer.

Steps Involved in PERT Calculations

1. Identify the events and activities and prepare a suitable network for the given problem.
2. Events are numbered in ascending order from left to right.
3. Obtain the various time estimates for each activity. They are most likely (t_m), the pessimistic (t_p) and the optimistic (t_0) estimates.
4. Compute the expected time (te) for each activity.

$$t_e = \frac{t_o + t_p + 4t_m}{6}$$

5. Using the expected activity time estimates, determine the earliest events time (T_E) and the latest event time (T_L) for each event.
6. Compute the float associated with each activity. The actives with zero float are the critical activities.
7. Find the total expected duration time (t_E), by adding the time estimates for various activities on the critical path.
8. Find the variance of the time estimates of all activities.

$$\text{Variance of an activity} = \left(\frac{t_p - t_o}{6}\right)^2$$

Variance of the project duration

(σ^2) = Sum of variances of time estimates of all critical activities.

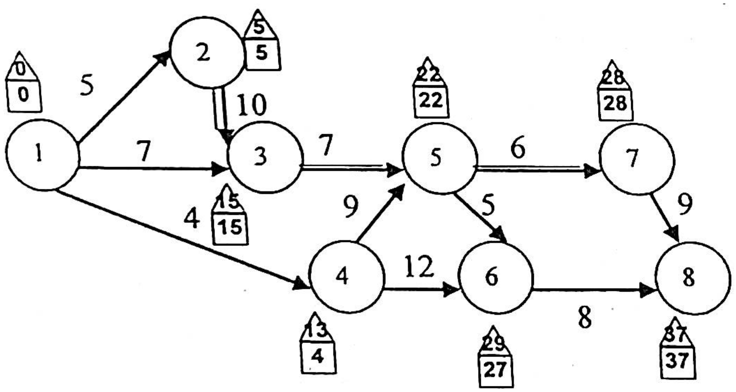
9. Find the probability of finishing the project on some fixed target by using the table of normal distribution. The value of Z is obtained by the equation.

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sigma}$$

Where σ = Standard deviation of the critical path

1. For the project given below find
   1) Expect time for each activities
   2) EST, EFT, LST, LFT
   3) Critical path
   4) Variance of the project
   5) Duration of the project

Task (tm)	Least time (to)	Greatest) time (tp	Most likely time
1-2	4	6	5
1-3	5	9	7
2-3	8	12	10
1-4	2	6	4
3-5	4	10	7
4-5	7	15	8
4-6	8	16	12
5-7	4	8	6
5-6	3	7	5
6-8	5	11	8
7-8	6	12	9



Task	T_E	σ^2	EST	LFT	LST	LFT	TF
1-2	5	1/9	0	5	0	5	0
1-3	7	4/9	0	7	8	15	8
2-3	10	4/9	5	15	5	15	0
1-4	4	4/9	0	4	9	13	9
3-5	7	1	15	22	15	22	0
4-5	9	16/9	4	13	13	22	9
4-6	12	16/9	4	16	17	29	13
5-7	6	4/9	22	28	22	28	0
5-6	5	4/9	22	27	24	29	2
6-8	8	1	27	35	29	37	2
7-8	9	1	28	37	28	37	0

Critical path is 1-2-3-5-7-8

Project duration = $5 + 10 + 7 + 6 + 9$

$= 37$

Total variants $= \frac{1}{9}+\frac{4}{9}+\frac{4}{9}+1=3$

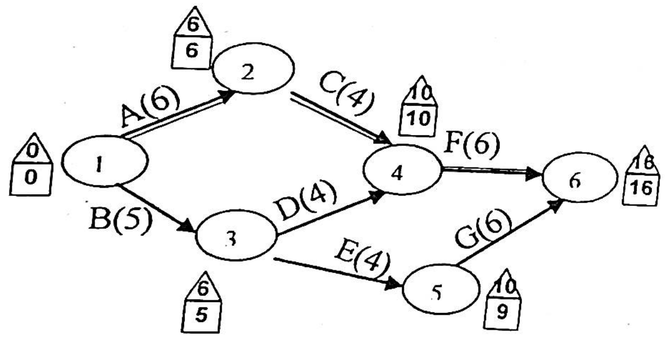
2.. Consider the following activities of the project

Time estimates in days

Activity	to	tm	tp	Predecessor
A	3	6	9	—
B	2	5	8	—
C	2	4	6	A
D	2	3	10	B
E	1	3	11	B
F	4	6	8	C,D
G	1	5	15	E

Find the path and standard deviation. Also find the probability of completing the project by 18 days.

Activity	t_e	σ^2
A 1-2	6	1
B 1-3	5	1
C 2-4	4	4/9
D 3-4	4	16/9
E 3-5	4	25/9
F 4-6	6	4/9
G 5-6	6	49/9



Project duration $= 6 + 4 + 6 = 16$

Variance σ^2 $= 1 + 4/9 + \frac{4}{9}$

$= 1.888$

$\sigma = 1.374$ (Standard deviation)

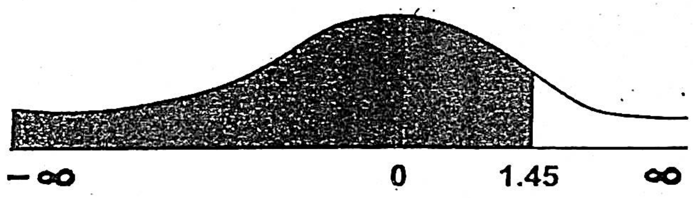
$$Z = \frac{\text{Due date} - \text{expected date}}{\sigma}$$

$$= \frac{18-16}{1.374}$$

$$= 1.45$$

Z – table

Z Table



Required probability (RP) $= P(z \le 1.45)$

$= 0.5 + 0.4265$

$= 0.9265$

Probability of completing the project by 18 beats is 92.65%

3. The following table shows the jobs of a network along with time estimates. The time estimates are in days

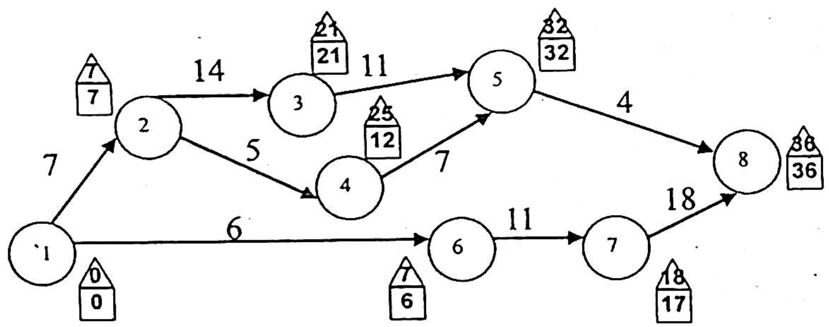
Jobs	(t_0)	(t_m)	(t_p)
1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
5-8	1	4	7
6-7	3	9	27
7-8	4	19	28

a) Draw the project network

b) Find the critical path

c) Find the probability that the project is completed in 31 days

Jobs	te	V
1-2	7	4
1-6	6	4
2-3	14	16
2-4	5	1
3-5	11	4
4-5	7	4
5-8	4	1
6-7	11	16
7-8	18	16



Project duration $= 7 + 14 + 11 + 4$

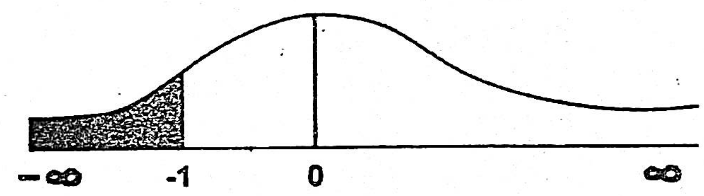
$= 36$

Total variant, $= 4 + 16 + 4 + 1$

$= 25$

$\sigma = 5$

$Z = \dfrac{D - E}{\sigma}$



$= \dfrac{31-36}{5} = -1$

Required probability $= P(Z \leq -1)$

$= 0.5 - p(0 \leq Z \leq 1)$

$= 0.5 - .3413$

$= 0.1587$

The probability that the project is completed in 31 days is 15.87%

Excercise:

1. Find the probability of meeting the schedule date as given for the network

Activity	1-2	1-3	2-4	3-4	4-5	3-5
Optimistic	2	9	5	2	6	8
Most likely	5	12	14	5	6	17
Pessimistic	14	15	17	12	12	20

Scheduled project completion date is 30 days. Also find the rate on which the project manager can complete theproject with a probability of .90.

2. The three estimate for the activeis of a project are given below. Find thd probability that theproject will be completed at least 15 days earlier than expected ?

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Optimistic	5	1	2	3	1	2	1
Most likely	6	1	4	6	1	2	4
Pessimistic	7	7	12	15	1	8	7

3. Draw the network diagram for the following project and find out:

(i) Earliest expected time and latest allowable time for completion different stages of the project

(ii) the critical path after calculating the slack values.

Activity	O	M	P
1 – 2	1	2	03
1 – 3	2	3	10
2 – 4	3	6	09
3 – 4	1	1	07
3 – 5	1	2	09
4 – 5	2	2	02
4 – 6	2	4	01
5 – 7	3	5	03
6 – 7	1	3	.05

Previous University Questions

1. Define the following:

(a) Most likely time

(b) Optimistic Time

(c) Pessimistic Time

(d) Expected Time

2. How CPM differ from PERT

3. Define Total float, free float, independent float and slack critical activity

4. Distinguish between an activity and an event .

5. What is the difference between the time estimates of CPM and PERT

6. Draw the network diagram for the following project and find out:

Activity:	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-7	6-7
O	01	02	03	01	01	02	02	03	01
M	02	03	06	01	02	02	04	05	03
P	03	10	09	07	09	02	06	13	05

(a) The earliest expect time

(b) The critical path after calculating the slack values

Where O = Optimistic time, M = most likely time,

P = pessimistic Time

7. A small project is composed of seven activities. Whose time estimate in week are given below:

Activity:	1-2	1-3	1-4	2-5	3-5	4-6	5-6
t_o	01	01	02	01	02	02	03
t_m	01	04	02	01	05	05	06
t_p	07	07	08	01	14	08	15

(i) Draw the project network

(ii) Find the expected duration and variance of each activity

(iii) Calculate the variance of the project length

(iv) If the project due date is 18 weeks. What is the probability of not meeting the due date.

8. Draw the network diagram from the following activities and find the critical path and floats.

Activity:	A	B	C	D	E	F	G	H	I	J
Time (weeks)	15	15	03	05	08	12	01	14	03	14
Proceeded by	-	-	A	A	B,C,	B,C	E	E	G	F,H,I

Module II