68. (a) We have $a+b+c=\dfrac{6(\sin A+\sin B+\sin C)}{3}$

$k(\sin A+\sin B+\sin C)=2(\sin A+\sin B+\sin C)$,

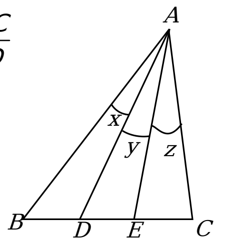
where $k=\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

$k=2$, $(\because \sin A+\sin B+\sin C\neq 0)$

$\therefore\ \dfrac{a}{\sin A}=2 \qquad \sin A=\dfrac{1}{2}$, $(\because\ a=1)$

$A=\dfrac{\pi}{6}$.

69. (c) From ΔADC, $\dfrac{\sin(y+z)}{DC}=\dfrac{\sin C}{AD}$



From ΔABD, $\dfrac{\sin x}{BD}=\dfrac{\sin B}{AD}$

From ΔAEC, $\dfrac{\sin z}{EC}=\dfrac{\sin C}{AE}$

From ΔABE, $\dfrac{\sin(x+y)}{BE}=\dfrac{\sin B}{AE}$

Therefore $\dfrac{\sin(x+y)\sin(y+z)}{\sin x\sin z}$

$=\dfrac{BE}{AE}\times\dfrac{DC}{AD}\times\dfrac{AD}{BD}\times\dfrac{AE}{EC}=\dfrac{2BD\times 2EC}{BD\times EC}=4$.

70. (a) $\cos A+2\cos B+\cos C=2$

$\cos A+\cos C=2(1-\cos B)$

$2\cos\dfrac{A+C}{2}\cos\dfrac{A-C}{2}=4\sin^2\dfrac{B}{2}$

$2\cos\left(\dfrac{A-C}{2}\right)=4\sin\dfrac{B}{2}$

$2\cos\dfrac{B}{2}\cos\left(\dfrac{A-C}{2}\right)=2\left(2\sin\dfrac{B}{2}\cos\dfrac{B}{2}\right)$

$2\sin\left(\dfrac{A+C}{2}\right)\cos\left(\dfrac{A-C}{2}\right)=2\left(2\sin\dfrac{B}{2}\cos\dfrac{B}{2}\right)$

$\sin A+\sin C=2\sin B\Rightarrow a+c=2b$

a, b, c are in A.P.

71. (c) Since $\cos 3A+\cos 3B+\cos 3C=1$

$4\sin\dfrac{3A}{2}\sin\dfrac{3B}{2}\sin\dfrac{3C}{2}=0$

Either $\dfrac{3A}{2}=180^o$ or $\dfrac{3B}{2}=180^o$ or $\dfrac{3C}{2}=180^o$

Either $A=120^o$ or $B=120^o$ or $C=120^o$.

72. (b) Since A, B, C are in A.P., therefore $B=60^o$

$[\because A+B+C=180$ and $A+C=2B]$

Now, $\sin(2A+B)=\dfrac{1}{2}$ (given)

$2A+B=30^o$ or 150^o

But as $B=60^o$, $2A+B\neq 30^o$.

Hence $2A+B=150^o\Rightarrow A=45^o$

Hence $A=45^o$, $B=60^o, C=75^o$.

73. (d) We have, $\dfrac{\tan\left(\frac{B}{2}\right)}{\cot\left(\frac{C-A}{2}\right)}=\dfrac{\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)}{\cos\frac{B}{2}\cos\left(\frac{C-A}{2}\right)}$

$=\dfrac{\sin C-\sin A}{\sin C+\sin A}=\dfrac{kc-ka}{kc+ka}=\dfrac{c-a}{c+a}=\dfrac{a}{3a}=\dfrac{1}{3}, \{\because c=2a\}$.

74. (b) $B=60^o$, $C=75^o \qquad A=180^o-60^o-75^o=45^o$

Now $\dfrac{b}{\sin B}=\dfrac{a}{\sin A} \qquad \dfrac{b}{\sin 60^o}=\dfrac{2}{\sin 45^o}\Rightarrow b=\sqrt{6}$.

75. (c) $\dfrac{\sin B}{b}=\dfrac{\sin A}{a} \qquad \sin B=\dfrac{b\sin A}{a}=\dfrac{8\sin 30^o}{6}=\dfrac{2}{3}$.

76. (c) We have $\dfrac{b}{\sin B}=\dfrac{c}{\sin C} \qquad \sin B=\dfrac{b\sin C}{c}$

$\sin B=\dfrac{2\sin 60^o}{\sqrt{6}}=\dfrac{2}{\sqrt{6}}.\dfrac{\sqrt{3}}{2}=\dfrac{1}{\sqrt{2}} \qquad B=45^o$,

$(\because B\neq 135^o)$

$A=180^o-(B+C)=75^o$

Now, $\dfrac{\sin A}{a}=\dfrac{\sin B}{b} \qquad a=\dfrac{b\sin A}{\sin B}=\dfrac{2\sin 75^o}{\sin 45^o}=\sqrt{3}+1$.

77. (a) We have $\tan\left(\dfrac{A-B}{2}\right)=\sqrt{\dfrac{1-\cos(A-B)}{1+\cos(A-B)}}$

$=\sqrt{\dfrac{1-(31/32)}{1+(31/32)}}=\dfrac{1}{\sqrt{63}} \qquad \dfrac{a-b}{a+b}\cot\dfrac{C}{2}=\dfrac{1}{\sqrt{63}}$

$\dfrac{1}{9}\cot\dfrac{C}{2}=\dfrac{1}{\sqrt{63}}\Rightarrow\tan\dfrac{C}{2}=\dfrac{\sqrt{7}}{3}$

Now $\cos C=\dfrac{1-\tan^2(C/2)}{1+\tan^2(C/2)} \qquad \cos C=\dfrac{1-(7/9)}{1+(7/9)}=\dfrac{1}{8}$

$c^2=a^2+b^2-2ab\cos C$

$c^2=25+16-40\times\dfrac{1}{8}=36\Rightarrow c=6$.

78. (b) We have $\tan\dfrac{C-B}{2}=\dfrac{c-b}{c+b}\cot\dfrac{A}{2}$

$\tan\left(\dfrac{C-B}{2}\right)=\dfrac{\sqrt{3}+1-2}{\sqrt{3}+1+2}\cot 15^o=\dfrac{1}{\sqrt{3}}$

$\dfrac{C-B}{2}=30^o$.

79. (c) Let $A=6+\sqrt{12}, b=\sqrt{48}, c=\sqrt{24}$. Clearly, c is the smallest side. Therefore, the smallest angle C is given by

$\cos C=\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{\sqrt{3}}{2}\Rightarrow C=\dfrac{\pi}{6}$.

80. (b) Since $C=90^o$

Hence, $a=\dfrac{c\sin A}{\sin C}=\dfrac{7\sqrt{3}\sin 30^o}{\sin 90^o}=\dfrac{7\sqrt{3}}{2}$.

81. (a) Obviously, the angles are $30^o, 45^o, 105^o$.

$\therefore\ a:b:c=\sin 30^o:\sin 45^o:\sin 105^o$

$$= \frac{1}{2} : \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}} = \sqrt{2} : 2 : (\sqrt{3}+1).$$

82. (b) $a = 2, b = \sqrt{6}, c = \sqrt{3}+1$

$$\cos A = \frac{6+3+1+2\sqrt{3}-4}{2\sqrt{6}(\sqrt{3}+1)} \Rightarrow A = 45^o.$$

83. (b) $$\frac{\tan\frac{A}{2} - \tan\frac{B}{2}}{\tan\frac{A}{2} + \tan\frac{B}{2}} = \frac{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} - \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}$$

$$= \frac{(s-b)\sqrt{s(s-c)} - (s-a)\sqrt{s(s-c)}}{(s-b)\sqrt{s(s-c)} + (s-a)\sqrt{s(s-c)}}$$

$$= \frac{\sqrt{s(s-c)}(s-b-s+a)}{\sqrt{s(s-c)}\,(s-b+s-a)} = \frac{a-b}{c}.$$

84. (b) $$\Delta = \frac{1}{2}\frac{a^2 \sin B \sin C}{\sin(B+C)} = \frac{1}{2}\frac{(\sqrt{3}+1)^2 . \frac{1}{2} \times \frac{1}{\sqrt{2}}}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \frac{\sqrt{3}+1}{2}.$$

85. (a) If x is length of perpendicular drawn to it from opposite vertex of a right angled triangle,

So, length of diagonal $AB = y_1 + y_2$ .....(i)

From $\Delta OCB, y_2 = x\cot\theta$

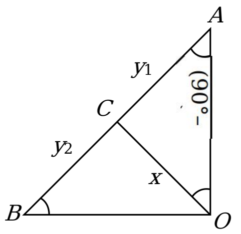
From ΔOCA, $y_1 = x\tan\theta$

Put the values in equation (i), then

$AB = x(\tan\theta + \cot\theta)$ .....(ii)

$\because$ Length of hypotenuse = 4 (length of perpendicular)

$$x(\tan\theta + \cot\theta) = 4x \Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta} = 4$$



$$\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = 30^o \text{ or } \theta = 15^o.$$

Trick : $$\frac{(\text{length of hypotenuse})}{(\text{length of perpendicular drawn from opposite vertex to hypotenuse})} = \frac{2}{\sin 2\theta}$$

$$4 = \frac{2}{\sin 2\theta} \Rightarrow \sin 2\theta = \frac{1}{2} = \sin 30^o \Rightarrow \theta = 15^o.$$

86. (b) $A = 45^o, C = 60^o, A+B+C = \pi \Rightarrow B = 75^o$

$$a + c\sqrt{2} = k\sin A + k\sin C(\sqrt{2})$$

$$= k\sin 45^o + \sqrt{2}k\sin 60^o$$

$$= 2k\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = 2k\sin 75^o = 2k\sin B$$

$$a + c\sqrt{2} = 2b.$$

87. (c) Let sides of triangle a, b, c are respectively 3, 5 and 7.

$$\therefore \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{9+25-49}{2\times3\times5} = \frac{-15}{30} = -\frac{1}{2}$$

$\angle C = \frac{2\pi}{3}$ (largest angle).

88. (b) Given, $\angle C = 45^o$

$$A + B = 180^o - 45^o = 135^o$$

$$\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$$

$$\cot(135^o) = -1 = \frac{\cot A\cot B - 1}{\cot A + \cot B}$$

$$\cot A + \cot B = 1 - \cot A\cot B$$

$$1 + \cot A + \cot B + \cot A\cot B = 2$$

$$(1+\cot A)(1+\cot B) = 2.$$

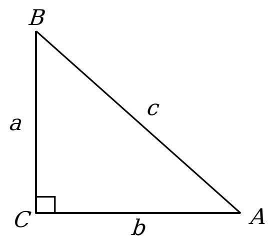
89. (a) Applying sine rule, $\frac{\sin B}{b} = \frac{\sin A}{a}$ or $\frac{\sin B}{8} = \frac{5/13}{3}$

or $\sin B = \frac{40}{39} > 1$, which is not possible. Hence no triangle can be formed by the given conditions.

90. (b) $$2ac\sin\frac{A-B+C}{2} = 2ac\sin\frac{\pi-2B}{2} = 2ac\cos B$$

$$= 2ac\frac{c^2+a^2-b^2}{2ca} = c^2+a^2-b^2.$$

91. (d) Given, A right–angled triangle *ABC* with right angled at *C*.



Let *a*, *b* and *c* be the lengths of sides *BC*, *CA* and *AB* respectively. We know from the Pythagoras theorem that $c^2 = a^2 + b^2$ and $\tan A = \frac{a}{b}$.

Similarly, $\tan B = \frac{b}{a}$.

Therefore, $\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab} = \frac{c^2}{ab}$.

92. (c) $\because$ $A : B : C = 3 : 5 : 4$

$$A + B + C = 12x = 180^o \Rightarrow x = 15^o$$

$\therefore \quad A = 45^o, B = 75^o, C = 60^o$

$$\frac{a}{\sin 45^o} = \frac{b}{\sin 75^o} = \frac{c}{\sin 60^o} = K \quad \text{(say)}$$

$$\therefore a = \frac{1}{\sqrt{2}}K, \quad b = \frac{\sqrt{3}+1}{2\sqrt{2}}K, \quad c = \frac{\sqrt{3}}{2}K$$

$\therefore a + b + c\sqrt{2} = 3b.$

93. (b) $\dfrac{\cos C + \cos A}{c+a} + \dfrac{\cos B}{b}$

$$= \frac{(b\cos C + c\cos B) + (b\cos A + a\cos B)}{b(c+a)}$$

$= \dfrac{a+c}{b(c+a)}$, (using Projection formulae)

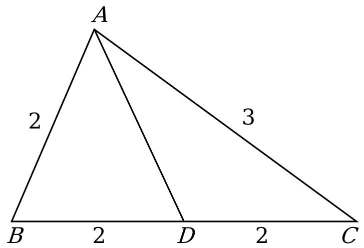
$= \dfrac{1}{b}$.

94. (a) Let $x + 3x + 5x = 180^o \qquad 9x = 180^o$

$x = 20^o = \dfrac{\pi}{9}$

$\therefore$ Greatest angle is $\dfrac{5\pi}{9}$.

95. (d) $\cos B = \dfrac{2^2 + 4^2 - 3^2}{2 \times 2 \times 4} = \dfrac{11}{16}$



$$\frac{11}{16} = \frac{2^2 + 2^2 - AD^2}{2 \times 2 \times 2} \Rightarrow AD^2 = 2.5\,.$$

96. (a) $4x + x + x = 180^o \Rightarrow 6x = 180^o \Rightarrow x = 30^o$

$$\frac{\sin 120^o}{a} = \frac{\sin 30^o}{b} = \frac{\sin 30^o}{c}$$

$$\therefore a : (a+b+c) = (\sin 120^o) : (\sin 120^o + \sin 30^o + \sin 30^o)$$

$$= \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+2}{2} = \sqrt{3} : \sqrt{3}+2.$$

97. (b) **Trick:** Take $A = B = C = 60^o$, then $\cot\dfrac{A}{2}, \cot\dfrac{B}{2}$ and $\cot\dfrac{C}{2}$ are in A.P. with common difference zero.

Now option (b) satisfies.

98. (a) Smallest angle is opposite to smaller side.

$$\therefore \cos C = \frac{b^2 + a^2 - c^2}{2ab} = \frac{49 + 48 - 13}{2 \times 7 \times 4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\angle C = 30^o$.

99. (b) $\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13} = \lambda$ (Let)

$\therefore b + c = 11\lambda$ ....(i)

$c + a = 12\lambda$ ....(ii)

and $a + b = 13\lambda$ ....(iii)

From (i) + (ii) + (iii), $2(a + b + c) = 36$,

$\therefore a + b + c = 18\lambda$ ....(iv)

Now, (iv) – (i) gives, $a = 7\lambda$

(iv) – (ii) gives, $b = 6\lambda$

(iv) – (iii) gives, $c = 5\lambda$

Now, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{(7\lambda)^2 + (6\lambda)^2 - (5\lambda)^2}{2 \times (7\lambda) \times (6\lambda)}$

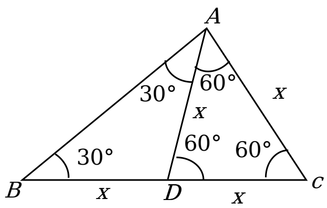
$$= \frac{49\lambda^2 + 36\lambda^2 - 25\lambda^2}{84\lambda^2} = \frac{60\lambda^2}{84\lambda^2} = \frac{5}{7}.$$

100. (b) $a^2 = b^2 + c^2 - 2bc\cos A$

$$a^2 = (20)^2 + (21)^2 - 2.20.21.\frac{4}{5} = 169 \qquad a = 13.$$

101. (b) $\cos 120^o = \dfrac{x^2 + x^2 - AB^2}{2x^2}$

$$\frac{2x^2 - AB^2}{2x^2} = \frac{-1}{2} \qquad 4x^2 - 2AB^2 = -2x^2$$



$3x^2 = AB^2 \qquad AB = x\sqrt{3}$

$a^2 : b^2 : c^2 = (2x)^2 : x^2 : (x\sqrt{3})^2$

$= 4x^2 : x^2 : 3x^2 = 4 : 1 : 3.$

102. (d) We have , $a : b : c = 1 : \sqrt{3} : 2$

i.e, $a = \lambda, b = \sqrt{3}\lambda, c = 2\lambda$

$$\cos A = \frac{3\lambda^2 + 4\lambda^2 - \lambda^2}{2(\sqrt{3}\lambda)\,(2\lambda)} = \frac{6\lambda^2}{4\sqrt{3}\lambda^2} = \frac{\sqrt{3}}{2}$$

$\cos A = \dfrac{\sqrt{3}}{2} \qquad A = 30^o$

Similarly, $\cos B = \dfrac{1}{2} \Rightarrow B = 60^o$,

$\cos C = 0 \Rightarrow C = 90^o$.

Hence $A : B : C = 1 : 2 : 3$.

103. (d) We have, $b = \sqrt{3}, c = 1, \angle A = 30^o$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{\sqrt{3}}{2} = \frac{(\sqrt{3})^2 + 1^2 - a^2}{2.\sqrt{3}.1}$$

$\therefore \; a = 1, \; b = \sqrt{3}, \; c = 1.$

b is the largest side. Therefore, the largest angle B is given by

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{1+1-3}{2.1.1} = -\frac{1}{2}$$

$B = 120^\circ$.

104. (c) Let $a = \alpha - \beta, b = \alpha + \beta, c = \sqrt{3\alpha^2+\beta^2}$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{\alpha^2+\beta^2-2\alpha\beta+\alpha^2+\beta^2+2\alpha\beta-3\alpha^2-\beta^2}{2(\alpha^2-\beta^2)}$$

$$\cos C = -\frac{(\alpha^2-\beta^2)}{2(\alpha^2-\beta^2)} = \cos\left(\frac{2\pi}{3}\right)$$

$\angle C = \frac{2\pi}{3}$, (largest angle).

105. (b) We have, $a = 4cm,\ b = 5cm,\ c = 6cm$.

Semi-perimeter (s) $= \frac{a+b+c}{2} = \frac{4+5+6}{2} = \frac{15}{2}$ cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{15}{2}\left(\frac{15}{2}-4\right)\left(\frac{15}{2}-5\right)\left(\frac{15}{2}-6\right)} = \frac{15}{4}\sqrt{7}cm^2.$$

106. (a) We have, $\sin^2\alpha + \sin^2\beta + \sin^2\gamma - 2\cos\alpha\cos\beta\cos\gamma$

$= 3 - [\cos^2\alpha + \cos^2\beta + \cos^2\gamma] - 2\cos\alpha\cos\beta\cos\gamma$

$= 3 - \left[\frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2}\right]$

$-2\cos\alpha\cos\beta\cos\gamma$

$= 3 - \frac{1}{2}[3+\cos 2\alpha + \cos 2\beta + \cos 2\gamma] - 2\cos\alpha\cos\beta\cos\gamma$

$= 3 - \frac{3}{2} - \frac{1}{2}(\cos 2\alpha + \cos 2\beta) - \frac{1}{2}\cos 2\gamma - 2\cos\alpha\cos\beta\cos\gamma$

$= \frac{3}{2} - \frac{1}{2}[-2\cos\gamma\cos(\alpha-\beta)] - \frac{1}{2}[2\cos^2\gamma - 1]$

$-2\cos\alpha\cos\beta\cos\gamma$

$= \frac{3}{2} + \cos\gamma\cos(\alpha-\beta) - \cos^2\gamma + \frac{1}{2} - 2\cos\alpha\cos\beta\cos\gamma$

$= 2$.

107. (c) We have, $a = 6, b = 3, \cos(A-B) = \frac{4}{5}$

Let $t = \tan\left(\frac{A-B}{2}\right)$

$$\cos(A-B) = \frac{1-t^2}{1+t^2} \Rightarrow \frac{4}{5} = \frac{1-t^2}{1+t^2} = t = \frac{1}{3}$$

So, $\tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$. Then,

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\frac{1}{3} = \frac{6-3}{6+3}\cot\frac{C}{2} \Rightarrow C = 90^\circ$$

Hence, $\Delta = \frac{1}{2}(6)(3)\sin 90^\circ = 9$ *square unit.*

108. (b) $\cos^2\frac{A}{2} - \sin^2\frac{A}{2} = \cos\frac{2A}{2} = \cos A$.

109. (c) Given, $C = 60^o, a = 2, b = 4$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$ or $ab = a^2+b^2-c^2$

$8 = 4+16-c^2 \quad c^2 = 12 \Rightarrow c = \sqrt{12} = 2\sqrt{3}$.

$$\sin A = \frac{a\sin C}{c} = \frac{2.\frac{\sqrt{3}}{2}}{2\sqrt{3}} = \frac{1}{2} \Rightarrow A = \frac{\pi}{6}$$

and $\sin B = \frac{b\sin C}{c} = \frac{4.\frac{\sqrt{3}}{2}}{2\sqrt{3}} = 1 \quad B = \frac{\pi}{2}$.

110. (b) It is a fundamental concept.

111. (c) $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2}$

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B}$$

$$\frac{\sin(A-B)}{\sin C} = \frac{\sin(A-B)\sin(A+B)}{\sin^2 A + \sin^2 B}$$

$$\sin(A-B)\left[\frac{1}{\sin C} - \frac{\sin C}{\sin^2 A + \sin^2 B}\right] = 0$$

Either $\sin(A-B) = 0 \Rightarrow A = B$ *i.e.* isosceles or $\sin^2 A + \sin^2 B = \sin^2 C$ or $a^2+b^2 = c^2$ *i.e.*, right angled triangle.

112. (c) $b = 4cm \Rightarrow a = 1\times 2, c = 3\times 2$

Hence perimeter $= 2+4+6 = 12$ *cm*.

113. (c) Let sides be $5x, 12x, 13x$.

Obviously the triangle is right angled.

Hence $\Delta = \frac{1}{2}(12x)(5x) \Rightarrow 30x^2 = 270 \Rightarrow x = 3$

Hence sides are 15, 36, 39.

114. (b) $\cos A = \frac{\sin B}{2\sin C} \Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{b}{2c}$

$b^2+c^2-a^2-b^2 = 0 \quad c^2 = a^2$.

115. (b) $\Delta = \frac{1}{2}bc\sin A \Rightarrow 9 = \frac{1}{2}.36\sin A$

$\sin A = \frac{1}{2} \Rightarrow A = \frac{\pi}{6} = 30^\circ$.

116. (a) $\cos\theta = \frac{36+100-(14)^2}{2.6.10} \Rightarrow \theta = 120^o$.

117. (b) $\sin A\cos B - \cos A\sin B = 0$

$\sin(A-B) = 0 \quad A = B$ *i.e.*, isosceles triangle.

118. (c) $a^2 = b^2 \Rightarrow a = b$.

119. (b) Putting $\cos A = \frac{b^2+c^2-a^2}{2bc}$ in given relation, we get

$$\frac{b^2+c^2-a^2}{2bc} + \frac{c^2+a^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab} = \frac{3}{2}$$

$$a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)$$

$$= a^3+b^3+c^3+3abc$$

$$(b-c)^2(b+c-a)+(c-a)^2(c+a-b)$$

$$+(a-b)^2(a+b-c)=0 \quad .....(i)$$

In triangle, $b+c-a>0$ etc. and hence (i) will hold good if each factor is zero so that $a=b=c$.

120. (d) $\tan A = \tan(\pi - B - C)$; $\infty = -\frac{\tan B + \tan C}{1-\tan B \tan C}$

$\Rightarrow 1-\tan B \tan C = 0 \qquad \tan B \tan C = 1$.

121. (b) Given that $2b = a+c$ and $c = 7cm$.

$$\cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow -\frac{1}{2} = \frac{a^2 + \frac{a^2+c^2+2ac}{4} - c^2}{2a\frac{(a+c)}{2}}$$

On simplification and putting the value of c, we get $a=3$ and $b=5$. Hence the area is $\frac{15\sqrt{3}}{4}cm^2$.

122. (c) $\Delta = \frac{1}{2}bc\sin A \Rightarrow \frac{1}{2}k^2 \sin B \sin C \sin A = \Delta$

$$a^2\sin 2B + b^2 \sin 2A = 2(a^2 \sin B \cos B + b^2 \sin A \cos A)$$

$$= 2k^2(\sin^2 A \sin B \cos B + \sin^2 B \sin A \cos A)$$

$$= 2k^2(\sin A \sin B)(\sin C) = 2k^2(\sin A \sin B \sin C) = 4\Delta.$$

123. (a) $\tan\alpha = \frac{a/2}{a} = \frac{1}{2} \Rightarrow \cot\alpha = 2$.

124. (b) It is obvious.

125. (c) $ab^2\cos A + ba^2 \cos B + ac^2 \cos A + ca^2 \cos C$

$$+ bc^2 \cos B + b^2 c \cos C$$

$$= ab(b\cos A + a\cos B) + ac(c\cos A + a\cos C)$$

$$+ bc(c\cos B + b\cos C)$$

$$= abc + abc + abc = 3abc.$$

126. (c) $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{a^2-(b^2-c^2)}{2ac}$

Now, $AD = \frac{abc}{b^2-c^2}$; $\qquad \cos B = \frac{a^2 - \frac{abc}{AD}}{2ac}$

Also, $AD = b\sin 23^o$; $\qquad \cos B = \frac{a - \frac{c}{\sin 23^o}}{2c}$

By sine formula, $\frac{a}{c} = \frac{\sin(B+23^o)}{\sin 23^o}$

$$\cos B = \left(\frac{\sin(B+23^o)}{\sin 23^o} - \frac{1}{\sin 23^o}\right) \div 2$$

$$\sin(23^o - B) = -1 = \sin(-90^o)$$

$23^o - B = -90^o$ or $B = 113^o$.

127. (d) $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\sin B = \frac{(b\sin A)}{a} = \frac{(4\sqrt{3}\sin 60^o)}{5} = \frac{6}{5} > 1,$$

which is inadmissible.

128. (b) $\Delta = 2bc - (b^2+c^2-a^2)$

$$\Delta = 2bc(1-\cos A) = 2bc 2\sin^2\frac{A}{2} \qquad .....(i)$$

$$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}(bc)2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\Delta = bc\sin\frac{A}{2}\cos\frac{A}{2} \qquad .....(ii)$$

$\tan\frac{A}{2} = \frac{1}{4} = t$, {by (i) and (ii)}

$$\tan A = \frac{2t}{1-t^2} = \frac{8}{15}.$$

129. (d) Δ is right angled, $\angle C = 90^o$

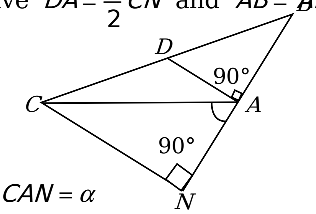
$$\therefore 4\Delta^2 = 4\left(\frac{1}{2}ab\right)^2 = a^2b^2.$$

130. (b) We have $\frac{1}{2}ap_1 = \Delta, \frac{1}{2}bp_2 = \Delta, \frac{1}{2}cp_3 = \Delta$

$$p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$$

$$\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2+b^2+c^2}{4\Delta^2}.$$

131. (c) We have $BD = DC$ and $\angle DAB = 90^o$. Draw CN perpendicular to BA produced, then in ΔBCN, we have $DA = \frac{1}{2}CN$ and $AB = AN$



Let $\angle CAN = \alpha$

$$\because \tan A = \tan(\pi - \alpha) = -\tan\alpha = -\frac{CN}{NA} = -2\frac{AD}{AB} = -2\tan B$$

$$\tan A + 2\tan B = 0.$$

132. (a) We have, $2s = a+b+c$, $A^2 = s(s-a)(s-b)(s-c)$

$\because$ A.M. $\geq$ G.M.

$$\frac{s-a+s-b+s-c}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)}$$

$$\frac{3s-2s}{3} \geq \frac{(A^2)^{1/3}}{s^{1/3}} \Rightarrow \frac{s^3}{27} \geq \frac{A^2}{s} \Rightarrow A \leq \frac{s^2}{3(\sqrt{3})}.$$

133. (b) $s-a=3 \Rightarrow b+c-a=6 \qquad ....(i)$

$s-c=2 \Rightarrow a+b-c=4 \qquad ....(ii)$

Adding these two equations, we get $b=5$

Since B is a right angle

$b^2 = a^2+c^2 \Rightarrow a^2+c^2 = 25 \qquad ....(iii)$

Solving, we get $a=3, c=4$.

134. (c) It is given that area of ΔABC = Area of ΔDEF

$$\frac{1}{2}(AB)(AC)\sin A = \frac{1}{2}(DE)(EF)\sin E$$

$\sin A = \sin E \Rightarrow \sin 2E = \sin E \Rightarrow 2E = \pi - E$

$E = \frac{\pi}{3} \Rightarrow A = 2E = \frac{2\pi}{3}$.

135. (a) $\frac{\sin B}{b} = \frac{\sin C}{c}$ $\sin C = \frac{c}{b}\sin B > 1$

$(\because b < c\sin B)$, which is impossible.

Hence no triangle is possible.

136. (c) Let $a = 3x + 4y, b = 4x + 3y$ and $c = 5x + 5y$.

Clearly, c is the largest side and thus the largest angle C is given by

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{-2xy}{2(12x^2 + 25xy + 12y^2)} < 0$$

$\Rightarrow$ C is an obtuse angle.

Trick : Check by putting $x = 1$, $y = 1$.

137. (b) We have $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow$ $b^2 - 2bc\cos A + (c^2 - a^2) = 0$

It is given that b_1 and b_2 are roots of this equation.

Therefore $b_1 + b_2 = 2c\cos A$ and $b_1 b_2 = c^2 - a^2$

$\Rightarrow$ $3b_1 = 2c\cos A$, $2b_1^2 = c^2 - a^2$, $(\because b_2 = 2b_1$ given)

$\Rightarrow$ $2\left(\frac{2c}{3}\cos A\right)^2 = c^2 - a^2 \Rightarrow 8c^2(1 - \sin^2 A) = 9c^2 - 9a^2$

$\Rightarrow$ $\sin A = \sqrt{\frac{9a^2 - c^2}{8c^2}}$.

138. (a) We have $\cos A = \frac{c^2 + b^2 - a^2}{2bc}$

$\Rightarrow c^2 - 2bc\cos A + (b^2 - a^2) = 0$

It is given that c_1 and c_2 are roots of this equation.

Therefore $c_1 + c_2 = 2b\cos A$ and $c_1 c_2 = b^2 - a^2$

$\Rightarrow$ $k(\sin C_1 + \sin C_2) = 2k\sin B\cos A$

$\Rightarrow$ $\sin C_1 + \sin C_2 = 2\sin B\cos A$

$\Rightarrow$ Now sum of the areas of two triangles

$= \frac{1}{2}ab\sin C_1 + \frac{1}{2}ab\sin C_2 = \frac{1}{2}ab(\sin C_1 + \sin C_2)$

$= \frac{1}{2}ab(2\sin B\cos A) = ab\sin B\cos A$

$= b(a\sin B)\cos A = b(b\sin A)\cos A = \frac{1}{2}b^2\sin 2A$.

139. (c) $2\cos A = \frac{\sin B}{\sin C} \Rightarrow \frac{2(c^2 + b^2 - a^2)}{2bc} = \frac{b}{c}$

$\Rightarrow$ $c^2 = a^2 \Rightarrow$ $c = a$.

140. (b) $\cos C = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5} \Rightarrow C = 120^\circ$.

141. (c) It is conditional identity.

142. (a) Given $\tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$ and $\tan\frac{P}{2}\tan\frac{Q}{2} = \frac{c}{a}$

or $\tan\frac{\alpha}{2} + \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) = -\frac{b}{a}$, $\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) = \frac{c}{a}$

$\therefore$ $\tan\frac{\alpha}{2} + \frac{1 - \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}} = -\frac{b}{a} \Rightarrow \frac{\tan^2\frac{\alpha}{2} + 1}{1 + \tan\frac{\alpha}{2}} = -\frac{b}{a}$ .....(i)

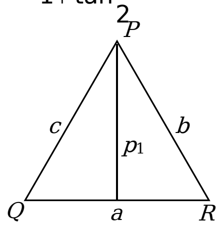
Similarly, $\frac{\tan\frac{\alpha}{2}\left(1 - \tan\frac{\alpha}{2}\right)}{1 + \tan\frac{\alpha}{2}} = \frac{c}{a}$

$\Rightarrow$ $\frac{\tan\frac{\alpha}{2} - \tan^2\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}} = \frac{c}{a}$ ......(ii)

By adding (i) and (ii), we get $\frac{1 + \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}} = -\frac{b}{a} + \frac{c}{a}$

$\Rightarrow$ $-b + c = a$ $\Rightarrow$ $c = a + b$.

143. (b) $\sin P, \sin Q, \sin R$ are in A.P.



$\Rightarrow$ a, b, c are in A.P.

$\therefore$ $\frac{\sin P}{a} = \frac{\sin Q}{b} = \frac{\sin R}{c} = \lambda$

Let p_1, p_2, p_3 be altitudes from P, Q, R

$p_1 = c\sin Q = \lambda bc$,

$p_2 = a\sin R = \lambda ac$,

$p_3 = b\sin P = \lambda ab$

Since a, b, c are in A.P. Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P.

$\Rightarrow$ $\frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in H.P. $\Rightarrow$ bc, ac, ab are in H.P. $\Rightarrow$ $\lambda bc, \lambda ac, \lambda ab$ are in H.P. $\Rightarrow$ p_1, p_2, p_3 are in H.P.

144. (a) $\frac{\sin A}{\sin C} = \frac{\sin A\cos B - \cos A\sin B}{\sin B\cos C - \cos B\sin C}$

$\frac{a}{c} = \frac{a\cos B - b\cos A}{b\cos C - c\cos B}$, (Using sine formula)

$ab\cos C - ac\cos B = ac\cos B - bc\cos A$

$ab\cos C + bc\cos A = 2ac\cos B$

$\frac{a^2 + b^2 - c^2}{2} + \frac{b^2 + c^2 - a^2}{2} = \frac{c^2 + a^2 - b^2}{1}$

$b^2 = c^2 + a^2 - b^2$ $b^2 = \frac{c^2 + a^2}{2}$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

145. (a) From cosine formula, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or $b^2 - (2c\cos A)b + (c^2 - a^2) = 0$, which is quadratic equation in b. $\because c\sin A < a < c$

$\therefore$ Two triangles will be obtained, but this is possible when two values of third side are also obtained. Clearly two values of side b will be b_1 and b_2 . Let these are roots of above equation.

$\therefore$ Sum of roots $= b_1 + b_2 = 2c\cos A$.

146. (d) $a^2\sin 2C + c^2\sin 2A = a^2(2\sin C\cos C) + c^2(2\sin A\cos A)$

$$= 2a^2\left(\frac{2\Delta}{ab}\cos C\right) + 2c^2\left(\frac{2\Delta}{bc}\cos A\right)$$

$$(\because \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A, \therefore \sin C = \frac{2\Delta}{ab}, \sin A = \frac{2\Delta}{bc})$$

$$= 4\Delta\left\{\frac{a\cos C + c\cos A}{b}\right\} = 4\Delta\left(\frac{b}{b}\right) = 4\Delta .$$

147. (c) We have, $a^2 + b^2 + c^2 - ac - ab\sqrt{3} = 0$

$$\frac{a^2}{4} - ac + c^2 + \frac{3a^2}{4} + b^2 - ab\sqrt{3} = 0$$

$$\left[\frac{a}{2} - c\right]^2 + \left[\frac{\sqrt{3}a}{2} - b\right]^2 = 0$$

i.e, $a = 2c$ and $2b = \sqrt{3}a$ *i.e.*, $b^2 + c^2 = a^2$

Hence triangle is right angled.

148. (c) We have, $a = 1, b = 2, \angle C = 60^o$

Area of triangle $= \frac{1}{2}ab\sin C = \frac{1}{2}(1)(2)\sin 60^o = \frac{\sqrt{3}}{2}$.

149. (b) We have, $b + c = 2a$ .....(i)

$$\cos 60^o = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{1}{2} = \frac{4a^2 - 2bc - a^2}{2bc} \qquad \frac{1}{2} = \frac{3a^2}{2bc} - 1$$

$$\frac{3}{2} = \frac{3a^2}{2bc} \qquad bc = a^2 \qquad ......(ii)$$

From (i) and (ii), $b = c = 0$ *i.e.*, triangle is equilateral.

150. (d) $\frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P. $\quad a, b, c$ are in A.P.

$\sin A, \sin B, \sin C$ are in A.P.

151. (a) $a^4 + b^4 + c^4 - 2a^2c^2 - 2b^2c^2 + 2a^2b^2 = 2a^2b^2$

$$(a^2 + b^2 - c^2)^2 = (\sqrt{2}ab)^2 \Rightarrow a^2 + b^2 - c^2 = \pm\sqrt{2}ab$$

$$\frac{a^2 + b^2 - c^2}{2ab} = \pm\frac{\sqrt{2}ab}{2ab} = \pm\frac{1}{\sqrt{2}}$$

$\cos C = \cos 45^o$ or $\cos 135^o \Rightarrow C = 45^o$ or 135^o .

152. (a) $a = 3, b = 5, c = 4, \ s = \frac{a + b + c}{2} = \frac{12}{2} = 6$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}} = \sqrt{\frac{2.3}{12}} = \sqrt{\frac{1}{2}}$$

$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}} = \sqrt{\frac{6.1}{12}} = \sqrt{\frac{1}{2}}$$

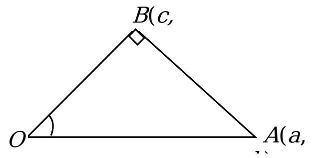
$$\sin\frac{B}{2} + \cos\frac{B}{2} = \frac{2}{\sqrt{2}} = \sqrt{2} .$$

153. (c) $\frac{b - c}{a} = \frac{\sin B - \sin C}{\sin A} = \frac{2\sin\frac{B - C}{2}\cos\frac{B + C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\sin\frac{B - C}{2}}{\cos\frac{A}{2}}$

$$(b - c)\cos\frac{A}{2} = a\sin\frac{B - C}{2} .$$

154. (b) Here $(AB)^2 = (a - c)^2 + (b - d)^2$

$(OA)^2 = (a - 0)^2 + (b - 0)^2 = a^2 + b^2$ and $(OB)^2 = c^2 + d^2$



Now from triangle $AOB, \cos\theta = \frac{(OA)^2 + (OB)^2 - (AB)^2}{2OA.OB}$

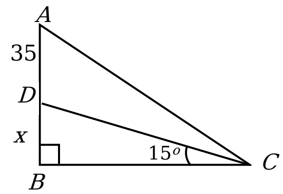
$$= \frac{a^2 + b^2 + c^2 + d^2 - \{(a - c)^2 + (b - d)^2\}}{2\sqrt{a^2 + b^2}.\sqrt{c^2 + d^2}}$$

$$= \frac{ac + bd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}} .$$

155. (a) $\angle DCB = 15^o$

$\angle CAB = 45^o$ and $\angle CDB = 75^o$

Let $BD = x$ and $AD = 35\,cm.$



$$\tan\angle CAB = \frac{CB}{AB} \qquad \tan 45^o = \frac{CB}{35 + x}$$

$$\tan 75^o = \frac{CB}{DB} = \frac{CB}{x} \qquad CB = x\tan 75^o$$

$$CB = (35 + x)\tan 45^o = x\tan 75^o$$

$$x = \frac{35\tan 45^o}{\tan 75^o - \tan 45^o} = \frac{35}{\tan 75^o - 1}$$

But $\cos 75^o = \frac{x}{CD}$

$$CD = \frac{x}{\cos 75^o} = \frac{1}{\cos 75^o} \times \frac{35}{\tan 75^o - 1} = \frac{35}{\sin 75^o - \cos 75^o}$$

$$= \frac{35}{\frac{\sqrt{3} + 1}{2\sqrt{2}} - \frac{\sqrt{3} - 1}{2\sqrt{2}}} = \frac{35}{\frac{2}{2\sqrt{2}}} = 35\sqrt{2}\ cm.$$

156. (d) $\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$; $\frac{5}{\sin\left(\frac{\pi}{2}+B\right)}=\frac{4}{\sin B}$

$\frac{5}{\cos B}=\frac{4}{\sin B}$, $\quad \tan B=\frac{4}{5}$

$\tan A=\tan\left(\frac{\pi}{2}+B\right)=-\cot B=\frac{-5}{4}$

$\tan C=\tan(\pi-(A+B))=-\tan(A+B)$, $[A+B+C=\pi]$

$=-\frac{(\tan A+\tan B)}{1-\tan A.\tan B}=\frac{-\left(-\frac{5}{4}+\frac{4}{5}\right)}{1+1}=\frac{9}{40}$

$C=\tan^{-1}\left(\frac{\left(2.\frac{1}{9}\right)}{1-\left(\frac{1}{9}\right)^2}\right)$; $\quad C=2\tan^{-1}\left(\frac{1}{9}\right)$.

Circle connected with triangle

1. (c) $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{1}{2R}\Rightarrow \sin A=\frac{a}{2R}$ etc.

Therefore $2R^2\sin A\sin B\sin C=2R^2\frac{a}{2R}.\frac{b}{2R}.\frac{c}{2R}$

$=\frac{abc}{4R}=\Delta$.

2. (c) $\cos A=0\Rightarrow 36+64-a^2=0\Rightarrow a=10\Rightarrow R=\frac{a}{2\sin A}=\frac{5}{1}$.

3. (b) Let $a=3k, b=7k, c=8k$

$\therefore s=\frac{1}{2}(a+b+c)=9k$

Then $\frac{R}{r}=\frac{abc}{4\Delta}.\frac{s}{\Delta}=\frac{abcs}{4s(s-a)(s-b)(s-c)}$

$=\frac{3k.7k.8k}{4.6k.2k.k}=\frac{7}{2}$

i.e., $R:r=7:2$.

4. (c) $s=\frac{1}{2}(a+b+c)=21$

$\Delta=\sqrt{[s(s-a)(s-b)(s-c)]}=84$; $\therefore r=\frac{\Delta}{s}=4$.

5. (a) $r=\frac{\Delta}{s}=\sqrt{\frac{8}{7}}$.

6. (c) $a=b=c=2\sqrt{3}$

$\Delta=\left(\frac{\sqrt{3}a^2}{4}\right)=3\sqrt{3}\,sq\,cm$, $\therefore R=\frac{abc}{4\Delta}=2cm$.

7. (b) $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow r=4R\sin^3 30^o$, $\{\because A=B=C=60^o\}$

$\Rightarrow r=\frac{R}{2}$.

8. (c) $a\cot A+b\cot B+c\cot C$

$=2R(\sin A\cot A+\sin B\cot B+\sin C\cot C)$

$=2R(\cos A+\cos B+\cos C)$

$=2R\left(1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)=2R\left(1+\frac{r}{R}\right)=2(R+r)$.

9. (d) In ΔPQR, the radius of circumcircle is $PQ=PR$

$\therefore PQ=PR=\frac{PQ}{2\sin R}=\frac{QR}{2\sin P}=\frac{PR}{2\sin Q}$

$\Rightarrow \sin R=\sin Q=\frac{1}{2}\Rightarrow \angle R=\angle Q=\frac{\pi}{6}$

$\Rightarrow \angle P=\pi-\angle R-\angle Q=\frac{2\pi}{3}$.

10. (b) We have $R=\frac{abc}{4\Delta}$ and $r=\frac{\Delta}{s}$

$\Rightarrow \frac{R}{r}=\frac{abc}{4\Delta}.\frac{s}{\Delta}=\frac{abc}{4(s-a)(s-b)(s-c)}$

Since $a:b:c=4:5:6$; $\frac{a}{4}=\frac{b}{5}=\frac{c}{6}=k$(say)

Thus $\frac{R}{r}=\frac{(4k)(5k)(6k)}{4\left(\frac{15k}{2}-4k\right)\left(\frac{15k}{2}-5k\right)\left(\frac{15k}{2}-6k\right)}=\frac{16}{7}$.

11. (c) $\because a=2R\sin A, b=2R\sin B, c=2R\sin C$

$\therefore a\cos A+b\cos B+c\cos C$

$=R[(2\sin A\cos A)+(2\sin B\cos B)+(2\sin C\cos C)]$

$=R(\sin 2A+\sin 2B+\sin 2C)=4R\sin A\sin B\sin C$.

12. (a) Let area of triangle be Δ, then according to question, $\Delta=\frac{1}{2}ax=\frac{1}{2}by=\frac{1}{2}cz$

$\therefore \frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}=\frac{b}{c}\left(\frac{2\Delta}{a}\right)+\frac{c}{a}\left(\frac{2\Delta}{b}\right)+\frac{a}{b}\left(\frac{2\Delta}{c}\right)$

$=\frac{2\Delta(b^2+c^2+a^2)}{abc}=\frac{2(a^2+b^2+c^2)}{abc}.\frac{abc}{4R}=\frac{a^2+b^2+c^2}{2R}$.

13. (b) Sides are 3, 4, 5 since $3^2+4^2=5^2$
So, triangle is a right angle triangle.
Hence, $R=5/2=2.5$.

14. (c) Area of the triangle *ABC* $(\Delta)=\frac{bc}{2}\sin A$. From the sine formula, $a=2R\sin A$ or $\sin A=\frac{a}{2R}$.

$\Rightarrow \Delta=\frac{1}{2}bc.\frac{a}{2R}=\frac{abc}{4R}$ or $R=\frac{abc}{4\Delta}$.

15. (b) $\tan\left(\frac{\pi}{n}\right)=\frac{a}{2r}$ and $\sin\left(\frac{\pi}{n}\right)=\frac{a}{2R}$

$r+R=\frac{a}{2}\left[\cot\frac{\pi}{n}+\text{cosec}\frac{\pi}{n}\right]=\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$.

16. (a) In a ΔABC, $r_1<r_2<r_3$

$\frac{1}{r_1}>\frac{1}{r_2}>\frac{1}{r_3}\Rightarrow \frac{s-a}{\Delta}>\frac{s-b}{\Delta}>\frac{s-c}{\Delta}$

$s-a > s-b > s-c \Rightarrow -a > -b > -c \quad a < b < c$.

17. (c) $r = \dfrac{\text{Area of triangle}}{s} = \dfrac{\Delta}{s}$

$s = \dfrac{a+b+c}{2} = \dfrac{18+24+30}{2}$, $s = 36$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{36(36-18)(36-24)(36-30)}$

$\Delta = \sqrt{36 \times 18 \times 12 \times 6} = 216$

So, radius of incircle $= \dfrac{216}{36} = 6\,cm$.

18. (a) $a = 5k, b = 6k$ and $c = 5k$

$s = \dfrac{a+b+c}{2} = \dfrac{5k+6k+5k}{2} = 8k$

$r = \dfrac{\Delta}{s} = \sqrt{\dfrac{s(s-a)(s-b)(s-c)}{s}}$

$r = \sqrt{\dfrac{8k(8k-5k)(8k-6k)(8k-5k)}{8k}}$

$r = \dfrac{3k}{2} \Rightarrow k = \dfrac{2r}{3} = \dfrac{2 \times 6}{3} = 4$.

19. (b) $\dfrac{a\cos A + b\cos B + c\cos C}{a+b+c}$

$= \dfrac{k[\sin A\cos A + \sin B\cos B + \sin C\cos C]}{k(\sin A + \sin B + \sin C]}$

$= \dfrac{1}{2}\dfrac{(\sin 2A + \sin 2B + \sin 2C)}{(\sin A + \sin B + \sin C)}$

$= \dfrac{1}{2}\left[\dfrac{2\sin(A+B)\cos(A-B) + 2\sin C\cos C}{2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right) + 2\sin\dfrac{C}{2}\cos\dfrac{C}{2}}\right]$

$= \dfrac{1}{2}\left[\dfrac{\sin C\{\cos(A-B) - \cos(A+B\})}{\cos\dfrac{C}{2}\left\{\cos\left(\dfrac{A-B}{2}\right) + \cos\left(\dfrac{A+B}{2}\right)\right\}}\right]$

$= \dfrac{1}{2}\left[\dfrac{\sin C(2\sin A\sin B)}{\cos\dfrac{C}{2}\left(2\cos\dfrac{A}{2}\cos\dfrac{B}{2}\right)}\right]$

$= \dfrac{1}{2}\left[\dfrac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}.2\sin\dfrac{B}{2}\cos\dfrac{B}{2}.2\sin\dfrac{C}{2}\cos\dfrac{C}{2}}{\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}}\right]$

$= 4\sin\dfrac{A}{2}.\sin\dfrac{B}{2}.\sin\dfrac{C}{2} = \dfrac{r}{R}, \left[\because r = 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right]$.

20. (c) Radius of circum-circle (R)

$= \dfrac{a}{2\sin A} = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C}$

$R = \dfrac{b}{2\sin B} = \dfrac{2}{2\sin 30^\circ} = 2$

Now, area of circle $= \pi R^2 = 4\pi$.

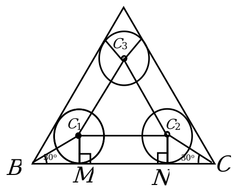
21. (b) $R = \dfrac{abc}{4\Delta}$, where $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$a = 13, b = 12, c = 5, s = \dfrac{30}{2} = 15$

$\Delta = \sqrt{15(2)(3)10} = 3 \times 2 \times 5 = 30$

$\therefore R = \dfrac{13 \times 12 \times 5}{4 \times 30} = \dfrac{13}{2}$.

22. (a) In ΔBC_1M; $BM = (C_1M).\cot 30$



$BM = \sqrt{3}$

Similarly, $CN = \sqrt{3}$ and $MN = C_1C_2 = 1 + 1 = 2$

Hence, side $BC = \sqrt{3} + \sqrt{3} + 2 = 2(1+\sqrt{3})$

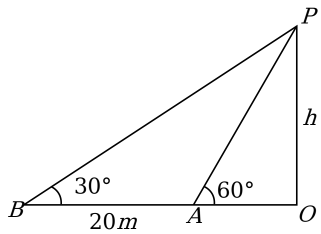
Area of equilateral triangle

$= \dfrac{\sqrt{3}}{4}[2(1+\sqrt{3})]^2 = 6 + 4\sqrt{3}$ *sq units.*

Height and Distance

1. (c) $OA = h\cot 60^o, OB = h\cot 30^o$,

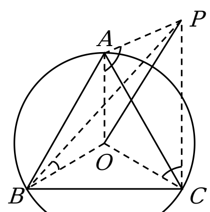
$OB - OA = 20 = h(\cot 30^o - \cot 60^o)$



$\Rightarrow \quad h = \dfrac{20}{\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)} = \dfrac{20\sqrt{3}}{2} = 10\sqrt{3}$.

2. (c) $20\cot 30^o = d \Rightarrow d = 20\sqrt{3}$.

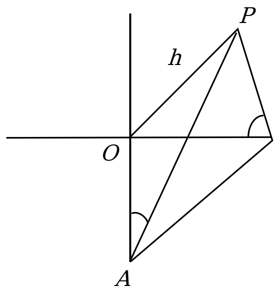
3. (d) Since the tower makes equal angles at the vertices of the triangle, therefore foot of the tower is at the circumcentre.



From ΔOAP, we have $\tan\alpha = \dfrac{OP}{OA}$

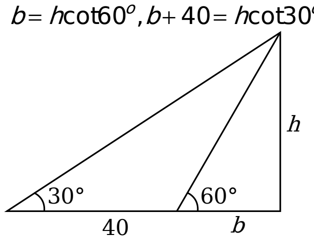
$\Rightarrow \quad OP = OA\tan\alpha \Rightarrow OP = R\tan\alpha$.

4. (c) $OB = h\cot\beta, OA = h\cot\alpha$



$$h^2 = \frac{d^2}{\cot^2\beta + \cot^2\alpha} \Rightarrow h = \frac{d}{\sqrt{\cot^2\beta + \cot^2\alpha}}.$$

5. (a) $b = h\cot 60^o, b + 40 = h\cot 30^o$

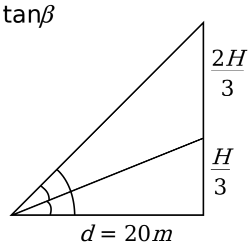


$$\Rightarrow \frac{b}{b+40} = \frac{\cot 60^o}{\cot 30^o} \Rightarrow b = 20m.$$

6. (b) $\frac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$

or $\frac{H}{3d} = \tan\alpha$ and $\frac{H}{d} = \tan\beta$

$$\tan(\beta - \alpha) = \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1 + \frac{H^2}{3d^2}}$$



$$\Rightarrow 1 + \frac{H^2}{3d^2} = \frac{4H}{3d}$$

$$\Rightarrow H^2 - 4dH + 3d^2 = 0$$

$$\Rightarrow H^2 - 80H + 3(400) = 0$$

$$\Rightarrow H = 20 \text{ or } 60m.$$