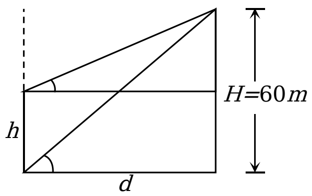
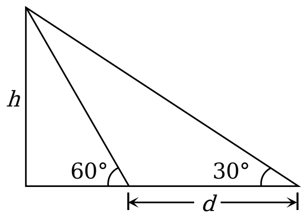
7. (d) $H = d\tan\beta$ and $H - h = d\tan\alpha$

$\Rightarrow \dfrac{60}{60-h} = \dfrac{\tan\beta}{\tan\alpha} \Rightarrow -h = \dfrac{60\tan\alpha - 60\tan\beta}{\tan\beta}$



$\Rightarrow h = \dfrac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\,\dfrac{\sin\beta}{\cos\beta}} \Rightarrow x = \cos\alpha\sin\beta$.

8. (c) $d = h\cot 30^o - h\cot 60^o$ and time $= 3$ *min.*



$\therefore$ Speed $= \dfrac{h(\cot 30^o - \cot 60^o)}{3}$ *per minute*

It will travel distance $h\cot 60^o$ in

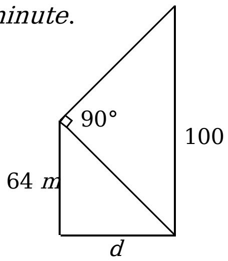
$\dfrac{h\cot 60^o \times 3}{h(\cot 30^o - \cot 60^o)} = 1.5$ *minute.*

9. (a) $64\cot\theta = d$

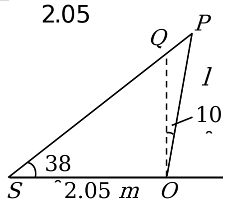
Also $(100-64)\tan\theta = d$

or $(64)(36) = d^2$,

$\therefore$ $d = 8\times 6 = 48\,m$.



10. (a) $\dfrac{\sin 38^o}{l} = \dfrac{\sin(SPO)}{2.05}$

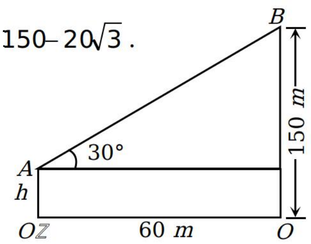


$= \dfrac{\sin(180^o - 38^o - 90^o - 10^o)}{2.05} \Rightarrow l = \dfrac{2.05\sin 38^o}{\sin 42^o}$.

11. (b) $\tan 45^o = \dfrac{h}{20} \Rightarrow h = 20m$.

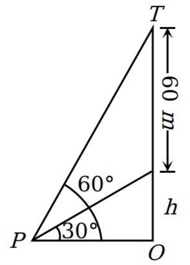
12. (c) $(150-h)\cot 30^o = 60$

$h = 150 - 20\sqrt{3}$.

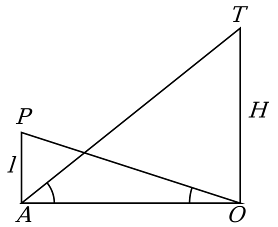


13. (b) Required distance = $60\cot 15^o = 60\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$.

14. (a) $(60+h)\cot 60^o = h\cot 30^o \Rightarrow h = 30m$



15. (b) From figure, we can deduce $H = l\tan\alpha\cot\beta$.

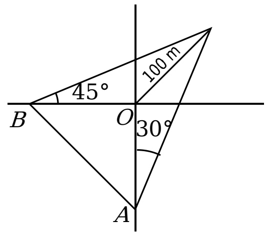


16. (b) $OB = 100\cot 45^o$

$OA = 100\cot 30^o$

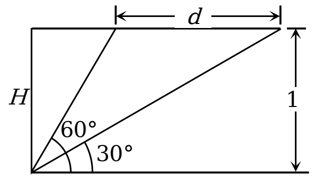
$AB = \sqrt{(OA^2 + OB^2)}$

$= 200m$.



17. (b) $d = H\cot 30^o - H\cot 60^o$;

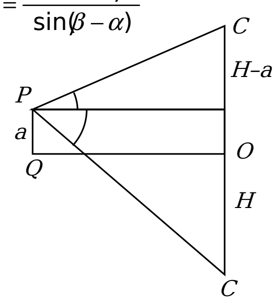
Time taken = 10 *second*



Speed = $\dfrac{\cot 30^o - \cot 60^o}{10} \times 60 \times 60 = 240\sqrt{3}$.

18. (b) $(H+a)\cot\beta = (H-a)\cot\alpha$

$H = \dfrac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$

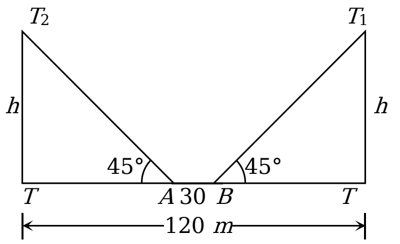


Using $\cot\alpha + \cot\beta = \dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$

and $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$.

19. (c) It is a fundamental concept.

20. (b) $\tan 45^o = 1 = \frac{h}{T_2A} \Rightarrow T_2A = h$

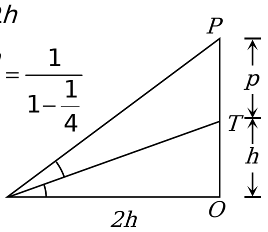


Hence $120 = h + 30 + h \Rightarrow h = 45\,m$.

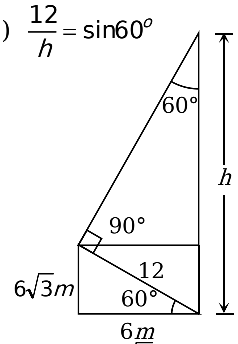
21. (a) $\tan\alpha = \frac{1}{2}$ and $\tan 2\alpha = \frac{p+h}{2h}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{p+h}{2h} = \frac{1}{1-\frac{1}{4}}$

$\Rightarrow \frac{p+h}{2h} = \frac{4}{3} \Rightarrow p = \frac{5h}{3}$.



22. (b) $\frac{12}{h} = \sin 60^o$

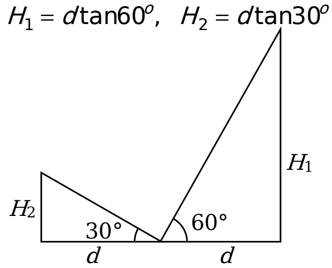


$\Rightarrow h = 8\sqrt{3}\ m$.

23. (b) $\tan\alpha = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^o$.

24. (b) Length of ladder = $\frac{6\sqrt{3}}{\sin 60^o} = 12m$.

25. (c) $H_1 = d\tan 60^o$, $H_2 = d\tan 30^o$

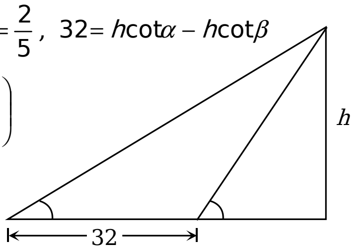


$\frac{H_1}{H_2} = \frac{\tan 60^o}{\tan 30^o} = \frac{3}{1}$.

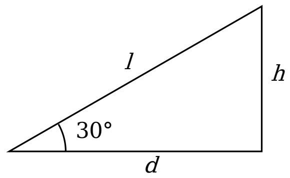
26. (a) $\cot\alpha = \frac{3}{5}$, $\cot\beta = \frac{2}{5}$, $32 = h\cot\alpha - h\cot\beta$

$h = \left(\frac{32}{\cot\alpha - \cot\beta}\right)$

$= \frac{32}{1/5} = 160m$.



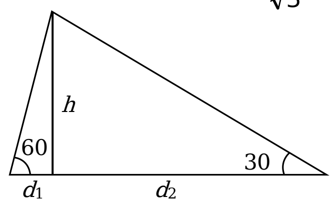
27. (c) $H = 20 = l + h, l = \frac{d}{\cos 30^o}, h = d\tan 30^o$



$\therefore\ d = \frac{20}{(\sec 30^o + \tan 30^o)} = \frac{20}{\sqrt{3}}$

and hence $h = d\tan 30^o = \frac{20}{3}m$.

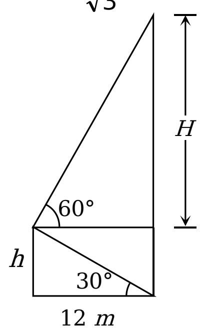
28. (b) $d_2 = h\cot 30^o = 500\sqrt{3}, d_1 = \frac{500}{\sqrt{3}}$



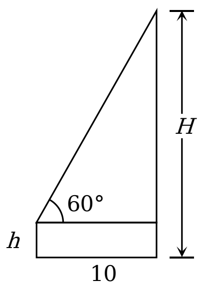
Diameter $D = 500\sqrt{3} + \frac{500}{3}\sqrt{3} = \frac{2000}{\sqrt{3}}m$.

29. (b) $h = 12\tan 30^o = \frac{12}{\sqrt{3}}$ and $H = 12\tan 60^o + \frac{12}{\sqrt{3}}$

$= 12\sqrt{3} + \frac{12}{\sqrt{3}} = 16\sqrt{3}m$.

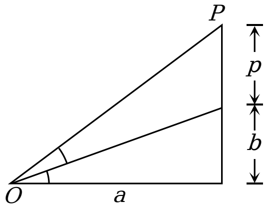


30. (a) $H = (10\tan 60^o + 1.5) = (10\sqrt{3} + 1.5)m$



31. (a) **Trick:** From $H = l\tan\alpha.\tan\beta$, the height of tower is $h\tan 30^o\cot 60^o$ or $\frac{h}{3}$.

32. (b) $\tan\alpha = \frac{b}{a}$, $\tan 2\alpha = \frac{2(b/a)}{1-(b/a)^2} = \frac{p+b}{a}$



$\Rightarrow \frac{2ba}{a^2-b^2} = \frac{p+b}{a} \Rightarrow \frac{2ba^2 - a^2b + b^3}{a^2-b^2} = p$

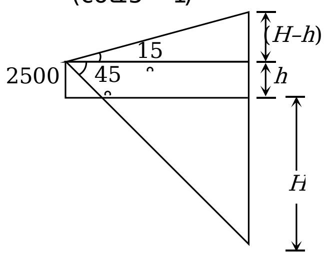
$\Rightarrow p = \frac{b(a^2+b^2)}{(a^2-b^2)}$.

33. (c) Obviously, the length of the tree is equal to $10 + 10\sqrt{2} = 10(1+\sqrt{2})m$.

34. (c) $\tan 60^o = \frac{30}{x} \Rightarrow x = 10\sqrt{3}m$.

35. (a) $(H-h)\cot 15^o = (H+h)\cot 45^o$

or $H = \frac{h(\cot 15^o + 1)}{(\cot 15^o - 1)}$

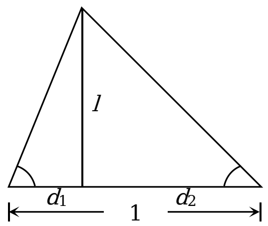


Since $h = 2500$ and substitute

$\cot 15^o = 2 + \sqrt{3}$, we get, $H = 2500\sqrt{3}$.

36. (d) $d_1 = h\cot\alpha$ and $d_2 = h\cot\beta$

$d_1 + d_2 = 1$ *mile* $= h(\cot\alpha + \cot\beta)$

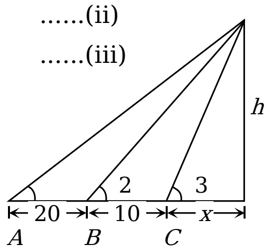


$\Rightarrow h = \frac{1}{(\cot\alpha + \cot\beta)} = \frac{\tan\alpha.\tan\beta}{\tan\alpha + \tan\beta}$.

37. (d) $x = h\cot 3\alpha$ .....(i)

$(x+100) = h\cot 2\alpha$ ......(ii)

$(x+300) = h\cot\alpha$ ......(iii)



From (i) and (ii), $-100 = h(\cot 3\alpha - \cot 2\alpha)$,

From (ii) and (iii), $-200 = h(\cot 2\alpha - \cot\alpha)$,

$100 = h\left(\frac{\sin\alpha}{\sin 3\alpha \sin 2\alpha}\right)$ and $200 = h\left(\frac{\sin\alpha}{\sin 2\alpha \sin\alpha}\right)$

or $\frac{\sin 3\alpha}{\sin\alpha} = \frac{200}{100} \Rightarrow \frac{\sin 3\alpha}{\sin\alpha} = 2$

$\Rightarrow 3\sin\alpha - 4\sin^3\alpha - 2\sin\alpha = 0$

$\Rightarrow 4\sin^3\alpha - \sin\alpha = 0 \Rightarrow \sin\alpha = 0$

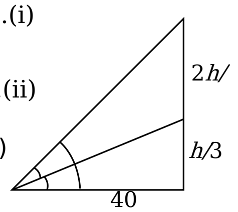
or $\sin^2\alpha = \frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right) \Rightarrow \alpha = \frac{\pi}{6}$

Hence, $h = 200\sin\frac{\pi}{3} = 200\frac{\sqrt{3}}{2} = 100\sqrt{3}$, {from (ii)}.

38. (b) Obviously, from figure

$\tan\alpha = \frac{h/3}{40} = \frac{h}{120}$ .....(i)

$\tan\beta = \frac{h}{40} = \frac{3h}{120}$ .....(ii)



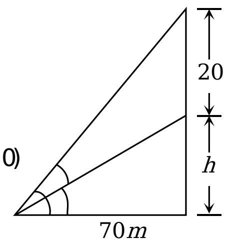
Therefore $\tan\theta = \tan(\beta - \alpha)$

$\Rightarrow \frac{1}{2} = \frac{\frac{3h}{120} - \frac{h}{120}}{1 + \frac{3h^2}{14400}} \Rightarrow h = 120, 40$

But $h = 40$ cannot be taken according to the condition, therefore $h = 120 ft$.

39. (c) $\tan\alpha = \tan(\phi - \theta)$

$\tan\alpha = \frac{1}{6} = \frac{\frac{20+h}{70} - \frac{h}{70}}{1 + \frac{(20+h)h}{(70)^2}}$



$\Rightarrow (70)^2 + 20h + h^2 = (6)(70)(20)$

$\Rightarrow h^2 + 20h + 70(70-120) = 0$

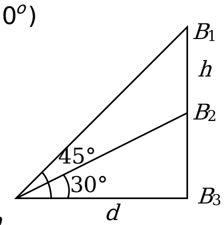
$\Rightarrow h^2 + 20h - (50)(70) = 0$

$\Rightarrow h = \frac{-20 \pm \sqrt{400 + (4)(50)(70)}}{2} = 50m$.

40. (b) $B_1B_2 = h = (d\tan 45^o - d\tan 30^o)$

Time taken = 10 *min*

Rate = $4 = \frac{d}{10}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

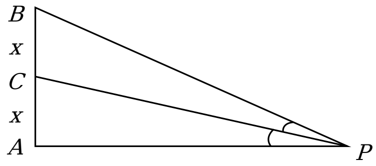


$\Rightarrow d = \frac{40\sqrt{3}}{\sqrt{3}-1} = 20(3+\sqrt{3})m$.

41. (b) $\tan 45^o = \frac{h}{x} \Rightarrow x = h$.

42. (b) Let $AC = x = CB, AP = 3AB = 6x$. Let $\angle CPA = \alpha$

In $\Delta ACP, \tan\alpha = \frac{x}{6x} = \frac{1}{6}$



In ΔABP, $\tan(\alpha+\beta)=\frac{2x}{6x}=\frac{1}{3}$

Now $\tan\beta=\tan\{(\alpha+\beta)-\alpha\}=\frac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha}$

$$=\frac{\frac{1}{3}-\frac{1}{6}}{1+\frac{1}{3}\cdot\frac{1}{6}}=\frac{1}{6}\times\frac{18}{19}=\frac{3}{19}.$$

43. (d) Let the two roads intersect at A. If the bus and the car are at B and C on the two roads respectively, then $c=AB=2km, b=AC=3\ km$.

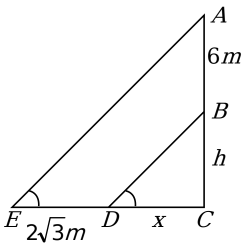
The distance between the two vehicles $=BC=a\,km$

Now $\cos A=\cos 60^o=\frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \frac{1}{2}=\frac{3^2+2^2-a^2}{2.3.2}\Rightarrow a=\sqrt{7}\ km$

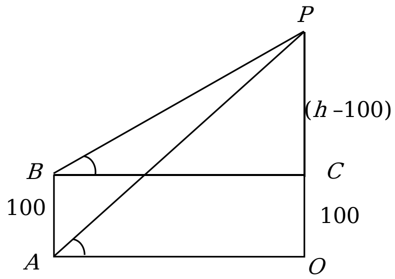
44. (a) Accordingly, $\tan\theta=\frac{h}{x}=\frac{h+6}{x+2\sqrt{3}}=\frac{6}{2\sqrt{3}}\Rightarrow\theta=60^o$

[Since the triangles AEC and BDC are similar].



45. (c) If $OP=h$, then $CP=h-100$

Now equate the values of OA and BC.

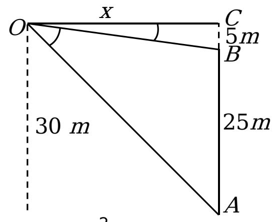


$h\cot\alpha=(h-100)\cot\beta$

$\therefore\ h=\frac{100\cot\beta}{\cot\beta-\cot\alpha}$

46. (b) We have $\tan\alpha=\frac{5}{x}$ and $\tan 2\alpha=\frac{30}{x}$

$\therefore\ \tan 2\alpha=\frac{30}{5\cot\alpha}\Rightarrow\tan 2\alpha=6\tan\alpha$

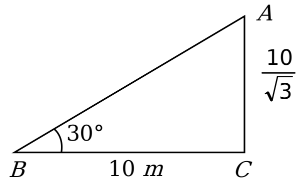


$\Rightarrow\ 3-3\tan^2\alpha=1\Rightarrow\tan\alpha=\sqrt{\frac{2}{3}}$

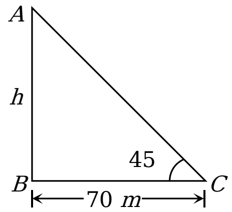
$\therefore\ x=5\cot\alpha=5\sqrt{\frac{3}{2}}$.

47. (c) Height of tree is

$AB+AC=\frac{10}{\sqrt{3}}+\frac{20}{\sqrt{3}}=\frac{30}{\sqrt{3}}=10\sqrt{3}=17.32m$



48. (a) Let height of tower is h.



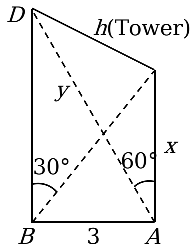
$\therefore\ \tan 45^o=\frac{h}{70}$, $\therefore\ h=70m$.

49. (d) From $\Delta\, CDA, x=h\cot 60^o=\frac{h}{\sqrt{3}}$

From $\Delta\, CDB, y=h\cot 30^o=\sqrt{3}h$

From ΔABC, by Pythagoras theorem,

$x^2+3^2=y^2$



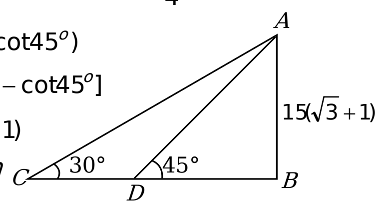
$\Rightarrow\ \left(\frac{h}{\sqrt{3}}\right)+3^2=(\sqrt{3}h)^2\Rightarrow h=\frac{3\sqrt{6}}{4}\ km.$

50. (a) $CD=h(\cot 30^o-\cot 45^o)$

$15(\sqrt{3}+1)[\cot 30^o-\cot 45^o]$

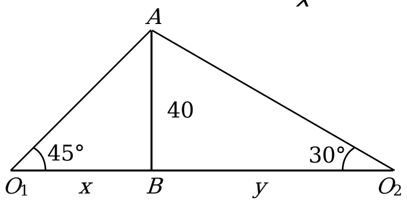
$=15(\sqrt{3}+1)(\sqrt{3}-1)$

$=\ 15(3-1)=30m$



Speed $=\frac{30}{1000}\times\frac{60\times 60}{3}km/hr=\ 36km/hr$.

51. (d) From $\Delta\, O_1AB$, $\tan 45^o = \frac{40}{x} \Rightarrow x = 40m$



From ΔAO_2B, $\cot 30^o = \frac{y}{40}$

$\Rightarrow$ $y = 40\ \cot\ 30^o = 40\sqrt{3}$

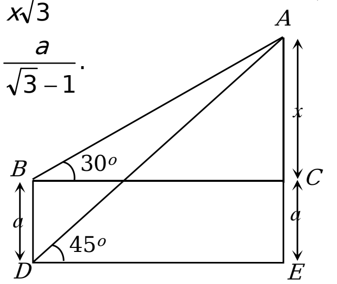
Distance between the men = $40 + 40\sqrt{3} = 109.28\ m$.

52. (c) In ΔABC, $\tan 30^o = \frac{AC}{BC}$ or $\frac{1}{\sqrt{3}} = \frac{x}{BC}$, where $AC = x$

or $BC = x\sqrt{3}$ and in ΔADE, $\tan 45^o = \frac{a+x}{DE}$ or

$1 = \frac{a+x}{x\sqrt{3}}$ or $x\sqrt{3} = a + x$, $x(\sqrt{3}-1) = a$ or

$x = \frac{a}{\sqrt{3}-1}$.

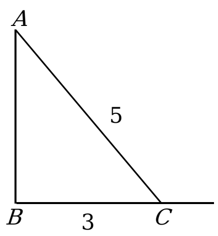


Therefore height of the tower,

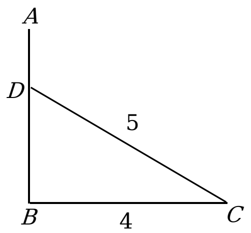
$a + x = a + \frac{a}{\sqrt{3}-1}$ $= a\left[\frac{\sqrt{3}-1+1}{\sqrt{3}-1}\right]$

$= \frac{a\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{a(3+\sqrt{3})}{2}$.

53. (a) From first case, From second case,



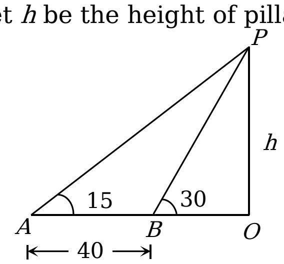
$AB = 4m$



$BD =$

$\therefore\ AD = 4 - 3 = 1m$

54. (b) Let h be the height of pillar

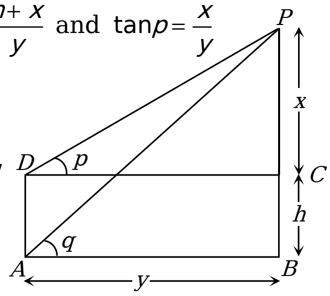


$OB = h\cot 30^o$ and $OA = h\cot 15^o$

$AB = OA - OB = h(\cot 15^o - \cot 30^o)$

$h = \frac{40}{\cot 15^o - \cot 30^o}$ = 20 *metre*.

55. (b) Let AD be the building of height h and BP be the hill then $\tan q = \frac{h+x}{y}$ and $\tan p = \frac{x}{y}$



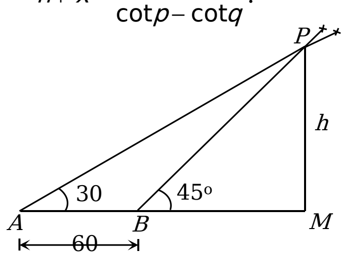
$\tan q = \frac{h+x}{x\cot p}$

$\Rightarrow x\cot p = (h+x)\cot q$

$x = \frac{h\cot q}{\cot p - \cot q}$

$h + x = \frac{h\cot p}{\cot p - \cot q}$.

56. (d)



$\because AB = AM - BM \Rightarrow \frac{AB}{h} = \frac{AM}{h} - \frac{BM}{h}$

$\frac{AB}{h} = \cot 30^o - \cot 45^o \Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1}$

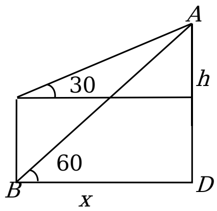
$h = 30(\sqrt{3}+1)\,m$.

57. (b) Let the height be h,

$\therefore \tan 30^o = \frac{h}{500}$ $\qquad h = \frac{500}{\sqrt{3}}$

58. (a) Total distance from temple = $\sqrt{x^2 + (240)^2}$

where $x = \frac{h}{\tan 60^o}$ = $\frac{h}{\sqrt{3}}$



So distance $= \sqrt{\frac{h^2}{3} + (240)^2}$

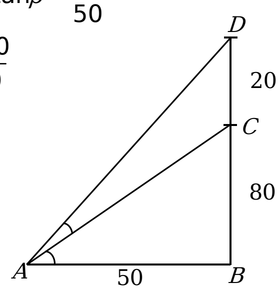
but $\frac{h}{\sqrt{\frac{h^2}{3} + (240)^2}} = \frac{1}{\sqrt{3}}$ $\qquad \frac{h^2}{\frac{h^2}{3} + (240)^2} = \frac{1}{3}$

After solving, $h = 60\sqrt{6}\ m$

59. (b) Let $\angle BAC = \beta$, $\therefore \tan\beta = \frac{80}{50}$

Now $\tan(\alpha + \beta) = \frac{100}{50}$

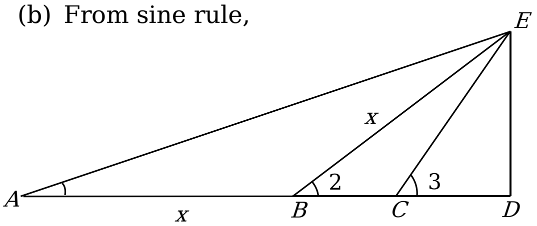
$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2$



$$\frac{\tan\alpha + \frac{8}{5}}{1 - \frac{8}{5}\tan\alpha} = 2$$

$\Rightarrow \tan\alpha = \frac{2}{21}$.

60. (b) From sine rule,



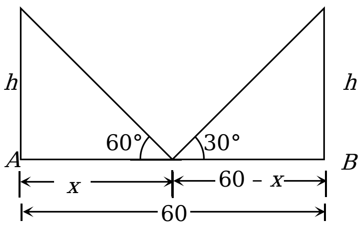
$$\frac{BE}{\sin(180^o - 3\alpha)} = \frac{BC}{\sin\alpha}$$

$\frac{AB}{\sin 3\alpha} = \frac{BC}{\sin\alpha}$ (Since $BE = AB$)

$$\frac{AB}{BC} = \frac{\sin 3\alpha}{\sin\alpha} = 3 - 4\sin^2\alpha$$

$$= 3 - 2(1 - \cos 2\alpha) = 1 + 2\cos 2\alpha.$$

61. (a)



$\tan 60^o = \frac{h}{x} \Rightarrow \frac{\sqrt{3}}{1} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ .....(i)

$\tan 30^o = \frac{h}{60 - x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60 - x} \Rightarrow 60 - x = \sqrt{3}h$ .....(ii)

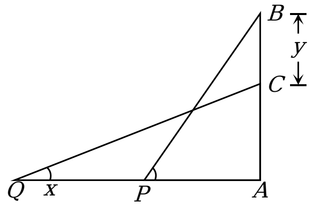
From equation (i) and (ii), $60 - x = \sqrt{3}(\sqrt{3}x)$

$$\frac{60}{4} = x \Rightarrow x = 15$$

Then $h = \sqrt{3}x \Rightarrow h = 15\sqrt{3}$ *metre.*

62. (a) $PB = QC = l$ (Length of ladder)

$\Rightarrow PA = l\cos\alpha, QA = l\cos\beta$



$\Rightarrow AC = l\sin\beta, AB = l\sin\alpha$

$\Rightarrow CB = AB - AC = l(\sin\alpha - \sin\beta)$

$\Rightarrow y = l(\sin\alpha - \sin\beta)$

and $QP = x = AQ - AP = l, (\cos\beta - \cos\alpha)$

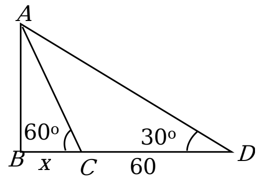
$\Rightarrow$

$$\frac{CB}{QP} = \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{y}{x} = \frac{2\sin\left(\frac{\alpha - \beta}{2}\right)\cos\left(\frac{\alpha + \beta}{2}\right)}{2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)}$$

$\Rightarrow \frac{y}{x} = \cot\left(\frac{\alpha + \beta}{2}\right) \Rightarrow x = y\tan\left(\frac{\alpha + \beta}{2}\right)$.

63. (e) $\tan 30^o = \frac{h}{x + 60}, \frac{1}{\sqrt{3}} = \frac{h}{x + 60}$

$x + 60 = \sqrt{3}h, \; x = \sqrt{3}h - 60$



$\tan 60^o = \frac{h}{x}, \; x = \frac{h}{\sqrt{3}}$

$\sqrt{3}h - 60 = \frac{h}{\sqrt{3}}$ $\quad 3h - 60\sqrt{3} = h$

$h = \frac{60\sqrt{3}}{2} = 30\sqrt{3} = 51.96 \approx 52m.$

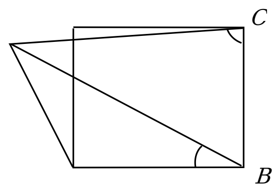
64. (a) Let AE is a vertical lamp-post. Given, $AE = 12m$

$\tan 45^o = \frac{AE}{AC}$

$AC = AE = 12m$

$\tan 60^o = \frac{AE}{AB}$

$AB = \frac{AE}{\sqrt{3}} = 4\sqrt{3}$



$BC = \sqrt{AC^2 - AB^2} = \sqrt{144 - 48} = \sqrt{96} = 4\sqrt{6}$

Area $= AB \times BC = 4\sqrt{3} \times 4\sqrt{6} = 48\sqrt{2}$ *sq. cm.*

Critical Thinking Questions

1. (b) $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x - 2\cos 2x\cos x + 3\cos 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$\Rightarrow (\sin 2x - \cos 2x)(2\cos x - 3) = 0 \Rightarrow \sin 2x = \cos 2x$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$ *i.e.,* $x = \frac{n\pi}{2} + \frac{\pi}{8}$.

2. (b) $5 - 5\sin^2\theta + 7\sin^2\theta = 6 \Rightarrow 2\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

3. (c) Combining θ and 7θ, 3θ and 5θ, we get

$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$4\cos 4\theta\cos 2\theta\cos\theta = 0$

$4\dfrac{1}{2^3\sin\theta}\ (\sin 2^3\theta)=0$; $\sin 8\theta=0$.

Hence $\theta=\dfrac{n\pi}{8}$.

4. (b) $\tan\theta=\dfrac{-1}{\sqrt{3}}=\tan\left(\pi-\dfrac{\pi}{6}\right)$, $\sin\theta=\dfrac{1}{2}=\sin\left(\pi-\dfrac{\pi}{6}\right)$ and $\cos\theta=\dfrac{-\sqrt{3}}{2}=\cos\left(\pi-\dfrac{\pi}{6}\right)$

Hence principal value is $\theta=\dfrac{5\pi}{6}$.

5. (d) $\dfrac{a}{\sqrt{a^2+b^2}}\cos x+\dfrac{b}{\sqrt{a^2+b^2}}\sin x=\dfrac{c}{\sqrt{a^2+b^2}}$

$\Rightarrow\ \cos\left(x-\cos^{-1}\dfrac{a}{\sqrt{a^2+b^2}}\right)=\dfrac{c}{\sqrt{a^2+b^2}}$

$\Rightarrow\ x-\cos^{-1}\dfrac{a}{\sqrt{a^2+b^2}}=\cos^{-1}\dfrac{c}{\sqrt{a^2+b^2}}$

General solution is,

$x-\cos^{-1}\dfrac{a}{\sqrt{a^2+b^2}}=2n\pi\pm\cos^{-1}\dfrac{c}{\sqrt{a^2+b^2}}$

or $x=2n\pi\pm\cos^{-1}\dfrac{c}{\sqrt{a^2+b^2}}+\cos^{-1}\dfrac{a}{\sqrt{a^2+b^2}}$

$x=\ 2n\pi+\tan^{-1}\dfrac{b}{a}\pm\cos^{-1}\dfrac{c}{\sqrt{a^2+b^2}}$.

Trick : Put $a=b=c=1$, then $\cos\left(x-\dfrac{\pi}{4}\right)=\cos\dfrac{\pi}{4}$

$\Rightarrow\ x=2n\pi+\dfrac{\pi}{4}\pm\dfrac{\pi}{4}$ which is given by option (d).

6. (c) $\sec 4\theta-\sec 2\theta=2$

$\Rightarrow\ \cos 2\theta-\cos 4\theta=2\cos 4\theta\cos 2\theta$

$\Rightarrow\ -\cos 4\theta=\cos 6\theta\ \Rightarrow\ 2\cos 5\theta\cos\theta=0$

$\Rightarrow\ \theta=n\pi+\dfrac{\pi}{2}$ or $\dfrac{n\pi}{5}+\dfrac{\pi}{10}$.

7. (a) $2\sin 3x\cos x-2\sin 3x=0$, $\therefore\ \sin 3x=0$, $\cos x=1$

$\Rightarrow\ 3x=n\pi$ or $x=\dfrac{n\pi}{3}$ and $x=2n\pi$

The second value $x=2n\pi$ is included in the value given by $x=\dfrac{n\pi}{3}$.

8. (b) $\tan(\cot x)=\cot(\tan x)\ \Rightarrow\ \tan(\cot x)=\tan\left(\dfrac{\pi}{2}-\tan x\right)$

$\Rightarrow\ \cot x=n\pi+\dfrac{\pi}{2}-\tan x\Rightarrow \cot x+\tan x=n\pi+\dfrac{\pi}{2}$

$\Rightarrow\ \dfrac{2}{\sin 2x}=n\pi+\dfrac{\pi}{2}\Rightarrow \sin 2x=\dfrac{2}{n\pi+\dfrac{\pi}{2}}=\dfrac{4}{(2n+1)\pi}$.

9. (c) $(5+4\cos\theta)(2\cos\theta+1)=0$

$\cos\theta=-5/4$, which is not possible.

$\therefore 2\cos\theta+1=0$ or $\cos\theta=-1/2$

$\theta=\dfrac{2\pi}{3},\dfrac{4\pi}{3}$. Solution set is $\left\{\dfrac{2\pi}{3},\dfrac{4\pi}{3}\right\}\in[0,2\pi]$.

10. (d) $1+\sin x+\sin^2 x+\ldots\infty=4+2\sqrt{3}$

$\Rightarrow\ \dfrac{1}{1-\sin x}=4+2\sqrt{3}\ \Rightarrow\ \sin x=1-\dfrac{1}{4+2\sqrt{3}}$

$\Rightarrow\ \sin x=1-\dfrac{(4-2\sqrt{3})}{4}=\dfrac{2\sqrt{3}}{4}=\dfrac{\sqrt{3}}{2}$

$\Rightarrow\ x=\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$.

11. (b) Given $\cos p\theta=-\cos q\theta=\cos(\pi+q\theta)$

$p\theta=2n\pi\pm(\pi+q\theta), n\in I$

$\theta=\dfrac{(2n+1)\pi}{p-q}$ or $\dfrac{(2n-1)\pi}{p+q}$, $n\in I$

Both the solutions form an A.P. $\theta=\dfrac{(2n+1)\pi}{p-q}$ gives us an A.P. with common difference $\dfrac{2\pi}{p-q}$ and $\theta=\dfrac{(2n-1)\pi}{p+q}$ gives us an A.P. with common difference $=\dfrac{2\pi}{p+q}$. Certainly, $\dfrac{2\pi}{p+q}<\left|\dfrac{2\pi}{p-q}\right|$.

12. (d) $\dfrac{a}{\sqrt{a^2+b^2}}\sin x+\dfrac{b}{\sqrt{a^2+b^2}}\cos x=\dfrac{c}{\sqrt{a^2+b^2}}$

$\sin(x+\alpha)=\dfrac{c}{\sqrt{a^2+b^2}}>1$, (as given)

Hence there is no solution.

13. (c) The first equation can be written as

$2\sin\dfrac{1}{2}(x+y)\cos\dfrac{1}{2}(x-y)=2\sin\dfrac{1}{2}(x+y)\cos\dfrac{1}{2}(x+y)$

$\therefore$ Either $\sin\dfrac{1}{2}(x+y)=0$ or $\sin\dfrac{1}{2}x=0$ or $\sin\dfrac{1}{2}y=0$

Thus $x+y=-1, x-y=-1$.

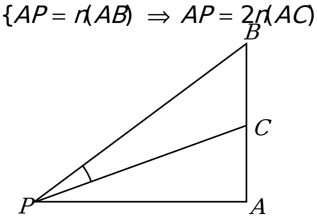
When $x+y=0$, we have to reject $x+y=1$ and check with the options or $x+y=-1$ and solve it with $x-y=1$ or $x-y=-1$ which gives $\left(\dfrac{1}{2},-\dfrac{1}{2}\right)$ or $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$ as the possible solution. Again solving with $x=0$, we get $(0,\pm 1)$ and solving with $y=0$, we get $(\pm 1, 0)$ as the other solution. Thus we have six pairs of solution for x and y.

14. (d) **Trick:** Let $a=2, b=3, c=4$ and check with the options.

15. (b) $a(b\cos C-c\cos B)=ab\cos C-ac\cos B$

$=\dfrac{a^2+b^2-c^2}{2}-\dfrac{a^2+c^2-b^2}{2}=b^2-c^2$.

16. (d) $\tan\alpha = \dfrac{-\dfrac{AC}{AP}+\dfrac{AB}{AP}}{1+\dfrac{AC}{AP}\cdot\dfrac{AB}{AP}}$ $\{AP = n(AB) \Rightarrow AP = 2n(AC)\}$



$\tan\alpha = \dfrac{-\dfrac{1}{2n}+\dfrac{1}{n}}{1+\dfrac{1}{2n^2}}$

$\Rightarrow \dfrac{n}{(2n^2+1)} = \tan\alpha \Rightarrow n = (2n^2+1)\tan\alpha$.

17. (c) $\dfrac{1}{a}\cos^2\dfrac{A}{2}+\dfrac{1}{b}\cos^2\dfrac{B}{2}+\dfrac{1}{c}\cos^2\dfrac{C}{2}$

$= \sum \dfrac{s(s-a)}{abc} = \dfrac{s(s-a+s-b+s-c)}{abc} = \dfrac{s^2}{abc}$.

18. (a) $\cos\dfrac{A}{2} = \sqrt{\dfrac{b+c}{2c}} \Rightarrow \sqrt{\dfrac{b+c}{2c}} = \sqrt{\dfrac{s(s-a)}{bc}}$

or $b^2+bc = 2s(s-a)$

$\Rightarrow b^2+bc = (a+b+c)\left(\dfrac{b+c-a}{2}\right) \Rightarrow a^2+b^2 = c^2$.

19. (d) Here $\tan\dfrac{A}{2}\tan\dfrac{C}{2} = \dfrac{s-b}{s}$

$\dfrac{5}{6}\cdot\dfrac{2}{5} = \dfrac{s-b}{s} \Rightarrow 3s-3b = s \Rightarrow 2s = 3b$

$\Rightarrow a+b+c = 3b$ or $a+c = 2b$.

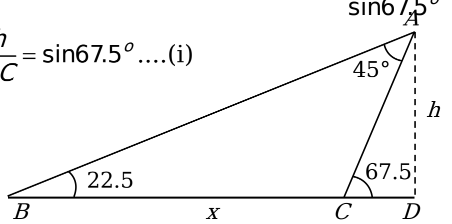
$\therefore$ a, b, c are in A.P., also $\sin A$, $\sin B$, $\sin C$ are in A.P.

20. (b) $(a+b+c)(a+b-c) = 3ab$

$\dfrac{a^2+b^2-c^2}{2ab} = \dfrac{1}{2} \Rightarrow \cos C = \cos\dfrac{\pi}{3} \Rightarrow \angle C = \pi/3$

21. (a) In $\Delta ACD, \dfrac{h}{\sin 67.5^o} = \dfrac{AC}{\sin 90^o}$

$\Rightarrow \dfrac{h}{AC} = \sin 67.5^o$ ....(i)



In $\Delta ABC, \dfrac{AC}{\sin 22.5^o} = \dfrac{x}{\sin 45^o} \Rightarrow \dfrac{AC}{x} = \sqrt{2}\sin 22.5^o$ .....(ii)

From (i) and (ii), $\dfrac{h}{x} = \dfrac{1}{2}$.

22. (c) $a\cos x + b\sin x = c$

$\Rightarrow a\left(\dfrac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right)+\dfrac{2b\tan(x/2)}{1+\tan^2(x/2)} = c$

$\Rightarrow (a+c)\tan^2\dfrac{x}{2} - 2b\tan\dfrac{x}{2} + (c-a) = 0$

This equation has roots $\tan\dfrac{\alpha}{2}$ and $\tan\dfrac{\beta}{2}$.

$\therefore \tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2} = \dfrac{2b}{a+c}$ and $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2} = \dfrac{c-a}{a+c}$

Now $\tan\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right) = \dfrac{\tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2}}{1-\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}} = \dfrac{\dfrac{2b}{a+c}}{1-\dfrac{c-a}{a+c}} = \dfrac{b}{a}$.

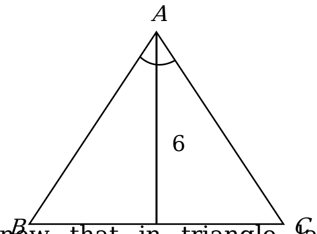
23. (c) $\dfrac{\cos A}{\cos B} = \dfrac{a}{b} = \dfrac{\sin A}{\sin B}$ $\sin A\cos B = \sin B\cos A$

$\sin(A-B) = 0$ $\sin(A-B) = \sin 0$

$A-B = 0$ $A = B$

Similarly, $A = B = C$. Hence it is an equilateral triangle.

24. (c) Obviously, $\tan A = \dfrac{\dfrac{2}{6}+\dfrac{3}{6}}{1-\dfrac{2}{6}\cdot\dfrac{3}{6}} = 1$ $A = \dfrac{\pi}{4}$.



25. (c) We know that in triangle larger the side larger the angle. Since angles $\angle A, \angle B$ and $\angle C$ are in AP.

Hence $\angle B = 60^o$

$\cos B = \dfrac{a^2+c^2-b^2}{2ac} \Rightarrow \dfrac{1}{2} = \dfrac{100+a^2-81}{20a}$

$\Rightarrow a^2+19 = 10a \Rightarrow a^2-10a+19 = 0$

$a = \dfrac{10\pm\sqrt{100-76}}{2} \Rightarrow a+c\sqrt{2} = 5\pm\sqrt{6}$.

26. (d) $\angle C = 180^o - 45^o - 75^o = 60^o$

Therefore $a+c\sqrt{2} = k(\sin A+\sqrt{2}\sin C)$ $=$

$k\left(\dfrac{1}{\sqrt{2}}+\dfrac{\sqrt{3}}{2}\sqrt{2}\right) = k\left(\dfrac{1+\sqrt{3}}{\sqrt{2}}\right)$

and $k = \dfrac{b}{\sin B} \Rightarrow a+c\sqrt{2} = \dfrac{b}{\sin 75^o}\left(\dfrac{1+\sqrt{3}}{\sqrt{2}}\right) = 2b$.

27. (b) $\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \sin(B+C)\sin(B-C) = \sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$\Rightarrow 2\sin^2 B = \sin^2 A + \sin^2 C \Rightarrow 2b^2 = a^2 + c^2$

Hence a^2, b^2, c^2 are in A.P.

28. (a) We have $\dfrac{\sin A}{4} = \dfrac{\sin B}{5} = \dfrac{\sin C}{6} \Rightarrow \dfrac{a}{4} = \dfrac{b}{5} = \dfrac{c}{6} = \lambda$ (say)

$\therefore a = 4\lambda, b = 5\lambda, c = 6\lambda$

Now $\cos A + \cos B + \cos C$

$= \frac{b^2+c^2-a^2}{2bc} + \frac{c^2+a^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab}$

$= \frac{1}{240\lambda^3}\{4\lambda(45\lambda^2 + 5\lambda(27\lambda^2) + 6\lambda(5\lambda^2)\} = \frac{69}{48}$.

29. (b) $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2} \times 2x \times 2y \times \frac{\sqrt{3}}{2} = xy\sqrt{3}$.

30. (b) Sides are $(x^2+x+1), (2x+1), (x^2-1)$. The greatest side subtends the greatest angle. Hence x^2+x+1 is the greatest side.

Now $\cos\theta = \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2(2x+1)(x^2-1)}$

$\Rightarrow \theta = 120^o$.

31. (b) $\cot\frac{B}{2}.\cot\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)} \cdot \frac{s(s-c)}{(s-a)(s-b)}}$

$= \frac{s}{s-a}$, {Since $3a = b+c$ or $a+b+c = 2s = 4a$}

$= 2a/a = 2$.

32. (a) $8R^2 = a^2+b^2+c^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow (\cos^2 A - \sin^2 C) + \cos^2 B = 0$

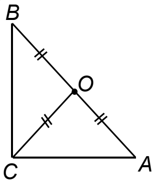
$\Rightarrow \cos(A-C)\cos(A+C) + \cos^2 B = 0$

$2\cos A \cos B \cos C = 0$

So that, $\cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

$\Rightarrow A = \frac{\pi}{2}$ or $B = \frac{\pi}{2}$ or $C = \frac{\pi}{2}$.

33. (a) Here, $R = OA = OB = OC = \frac{1}{2}AB = \frac{c}{2}$



$r = \frac{\Delta}{s} = \frac{\frac{1}{2}ab}{\frac{1}{2}(a+b+c)} = \frac{ab}{a+b+c}$

$\therefore r + R = \frac{ab}{a+b+c} + \frac{c}{2} = \frac{2ab + c(a+b+c)}{2(a+b+c)}$

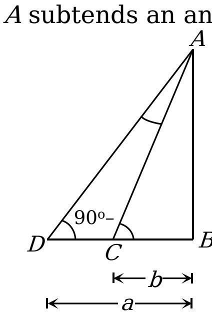
$= \frac{2ab + ca + bc + a^2 + b^2}{2(a+b+c)}$, $(\because c^2 = a^2 + b^2)$

$= \frac{(a+b)^2 + c(a+b)}{2(a+b+c)} = \frac{(a+b)(a+b+c)}{2(a+b+c)}$

$\therefore 2(r+R) = a+b$.

34. (c) Let there are two points C and D on horizontal line passing from point B of the base of the tower AB. The distance of these points are b and a from B respectively.

$\therefore BD = a$ and $BC = b$. $\because$ line CD, on the top of tower A subtends an angle θ, $\therefore \angle CAD = \theta$



According to question, on point C and D, the elevation of top are α and $90^o - \alpha$.

$\therefore \angle BCA = \alpha$ and $\angle BDA = 90^o - \alpha$.

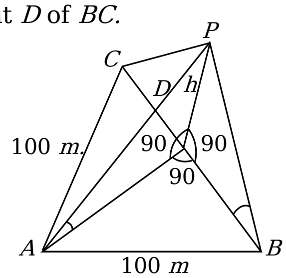
In ΔABC, $AB = BC\tan\alpha = b\tan\alpha$ ..... (i)

and in ΔABD, $AB = BD\tan(90^o - \alpha) = a\cot\alpha$ .....(ii)

Multiplying equation (i) and (ii),

$(AB)^2 = (b\tan\alpha)(a\cot\alpha) = ab$, $\therefore AB = \sqrt{(ab)}$.

35. (b) DP is a clock tower standing at the middle point D of BC.



$\angle PAD = \alpha = \cot^{-1} 3.2 \Rightarrow \cot\alpha = 3.2$

and $\angle PBD = \beta = \text{cosec}^{-1} 2.6 \Rightarrow \text{cosec}\beta = 2.4$

$\therefore \cot\beta = \sqrt{(\text{cosec}^2\beta - 1)} = \sqrt{(5.76)} = 2.4$

In the triangles PAD and PBD,

$AD = h\cot\alpha = 3.2h$ and $BD = h\cot\beta = 2.4h$

In the right angled ΔABD, $AB^2 = AD^2 + BD^2$

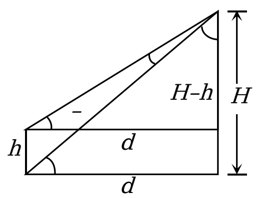
$\Rightarrow 100^2 = [(3.2)^2 + (2.4)^2]h^2 = 16h^2 \Rightarrow h = 25m$.

36. (b) $d = H\cot\alpha$

$d = (H-h)\cot(\alpha - \beta)$

$\Rightarrow H\cot\alpha = (H-h)\cot(\alpha - \beta)$

or $H = \frac{h\cot(\alpha-\beta)}{\cot(\alpha-\beta) - \cot\alpha}$.



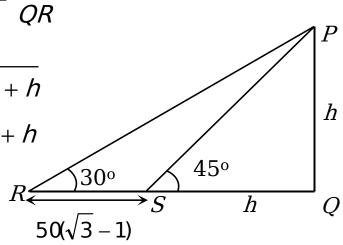
37. (b) In ΔPQR, $\tan 30^o = \frac{PQ}{QR}$

$\frac{1}{\sqrt{3}} = \frac{h}{50(\sqrt{3}-1) + h}$

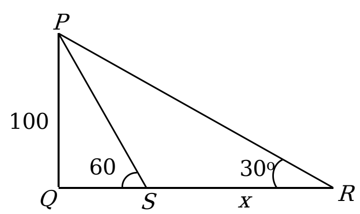
$\sqrt{3}h = 50(\sqrt{3}-1) + h$

$(\sqrt{3}-1)h = 50(\sqrt{3}-1)$

$h = 50 metre$.

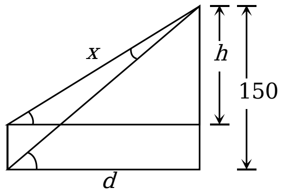


38. (b) $x = QR - QS = 100\cot 30^o - 100\cot 60^o$



$$= 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}.$$

39. (d) $d = 150\cot\phi = 60m$, Also $h = 60\tan\theta = 80m$.



Hence $x = \sqrt{80^2 + 60^2} = 100m$.

40. (a) For , $n = 2$, $f(x) = \frac{\sin 2x}{\sin\left(\frac{x}{2}\right)} = \frac{4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\cos x}{\sin\left(\frac{x}{2}\right)}$

$$= 4\cos\left(\frac{x}{2}\right)\cos x$$

The period of $\cos x = 2\pi$ and that of $\cos\frac{x}{2}$ is 4π, so period of $\frac{\sin 2x}{\sin\left(\frac{x}{2}\right)}$ is 4π.

For $n = 3$, $\frac{\sin\{3(x+4\pi)\}}{\sin\left\{\frac{(x+4\pi)}{3}\right\}} = \frac{\sin 3x}{\sin\left(\frac{x}{3} + \frac{4\pi}{3}\right)} \neq \frac{\sin 3x}{\sin\left(\frac{x}{3}\right)}$

So, 4π is not the period for $n = 3$.

Similarly, we can see that 4π is not the period of $\frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ for $n = 4$ and 5 also.