58. Out of the following which one is not true [Orissa JEE
(a) $\mathbf{a}.(\mathbf{b}\times\mathbf{c})$ (b) $(\mathbf{b}\times\mathbf{c}).\mathbf{a}$
(c) $(\mathbf{a}\times\mathbf{b}).\mathbf{c}$ (d) $(\mathbf{a}.\mathbf{c})\times\mathbf{b}$

59. If $\mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{c}, |\mathbf{a}|=2, |\mathbf{b}|=3$, $|\mathbf{c}|=4$ and the angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{2\pi}{3}$, then $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$ is equal to **[Kerala (Engg.) 2005]**
(a) $4\sqrt{3}$ (b) $6\sqrt{3}$
(c) $12\sqrt{3}$ (d) $18\sqrt{3}$
(e) $8\sqrt{3}$

60. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors and λ is a real number then $[\lambda(\mathbf{a}+\mathbf{b})\ \ \lambda^2\mathbf{b}\ \ \lambda\mathbf{c}] = [\mathbf{a}\ \ \mathbf{b}+\mathbf{c}\ \ \mathbf{b}]$ for **[AIEEE**
(a) Exactly three values of λ
(b) Exactly two values of λ
(c) Exactly one value of λ
(d) No value of λ

61. If the vectors $2\mathbf{i}+\mathbf{j}-\mathbf{k}, -\mathbf{i}+2\mathbf{j}+\lambda\mathbf{k}$ and $-5\mathbf{i}+2\mathbf{j}-\mathbf{k}$ are coplanar, then the value of λ is equal **[J & K 200**
(a) – 13 (b) 13/9
(c) – 13/9 (d) – 9/13

62. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-zero, non-coplanar vectors and
$\mathbf{b}_1 = \mathbf{b} - \frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}, \mathbf{b}_2 = \mathbf{b} + \frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$, $\mathbf{c}_1 = \mathbf{c} - \frac{\mathbf{c}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{c}.\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}$,
$\mathbf{c}_2 = \mathbf{c} - \frac{\mathbf{c}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{c}.\mathbf{b}_1}{|\mathbf{b}_1|^2}\mathbf{b}_1$, $\mathbf{c}_3 = \mathbf{c} - \frac{\mathbf{c}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{c}.\mathbf{b}_2}{|\mathbf{b}_2|^2}\mathbf{b}_2$,
$\mathbf{c}_4 = \mathbf{a} - \frac{\mathbf{c}.\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$. Then which of the following is a set of mutually orthogonal vectors is
(a) $\{\mathbf{a}, \mathbf{b}_1, \mathbf{c}_1\}$ (b) $\{\mathbf{a}, \mathbf{b}_1, \mathbf{c}_2\}$
(c) $\{\mathbf{a}, \mathbf{b}_2, \mathbf{c}_3\}$ (d) $\{\mathbf{a}, \mathbf{b}_2, \mathbf{c}_4\}$

63. If a vector lie in the plane and then which is correct **[Orissa JEE 2005]**
(a) $[\alpha\ \beta\ \gamma]=0$ (b) $[\alpha\ \beta\ \gamma]=1$
(c) $[\alpha\ \beta\ \gamma]=3$ (d) $[\beta\ \gamma\ \alpha]=1$

Vector triple product

1. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is coplanar with
(a) $\mathbf{b}$ and $\mathbf{c}$ (b) $\mathbf{c}$ and $\mathbf{a}$
(c) $\mathbf{a}$ and $\mathbf{b}$ (d) $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$

2. If $\mathbf{u} = \mathbf{i}\times(\mathbf{a}\times\mathbf{i}) + \mathbf{j}\times(\mathbf{a}\times\mathbf{j}) + \mathbf{k}\times(\mathbf{a}\times\mathbf{k})$, then
[RPET 1989, 97; MNR 1986, 93; MP PET 1987, 98, 99, 2004; UPSEAT 2000, 02; Kerala (Engg.) 2002]
(a) $\mathbf{u}=0$ (b) $\mathbf{u}=\mathbf{i}+\mathbf{j}+\mathbf{k}$
(c) $\mathbf{u}=2\mathbf{a}$ (d) $\mathbf{u}=\mathbf{a}$

3. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}-2\mathbf{k}, \mathbf{b}=2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+3\mathbf{j}-\mathbf{k}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is equal to **[RPET 1989]**
(a) $20\mathbf{i}-3\mathbf{j}+7\mathbf{k}$ (b) $20\mathbf{i}-3\mathbf{j}-7\mathbf{k}$
(c) $20\mathbf{i}+3\mathbf{j}-7\mathbf{k}$ (d) None of these

4. If $\boldsymbol{\alpha}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}, \boldsymbol{\beta}=-\mathbf{i}+2\mathbf{j}-4\mathbf{k}$ and $\boldsymbol{\gamma}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, then $(\boldsymbol{\alpha}\times\boldsymbol{\beta}).(\boldsymbol{\alpha}\times\boldsymbol{\gamma})$ is equal to
[MNR 1984; UPSEAT 2000; Orissa JEE 2005]
(a) 60 (b) 64
(c) 74 (d) – 74

5. If $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\mathbf{0}$, then **[RPET 1995]**
(a) $|\mathbf{a}|=|\mathbf{b}|.|\mathbf{c}|=1$ (b) $\mathbf{b}\,||\,\mathbf{c}$
(c) $\mathbf{a}\,||\,\mathbf{b}$ (d) $\mathbf{b}\perp\mathbf{c}$

6. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is equal to
[RPET 1995; Kurukshetra CEE 1998; MP PET 2003]
(a) $(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{a})\mathbf{b}$ (b) $(\mathbf{a}.\mathbf{c})\mathbf{a}-(\mathbf{b}.\mathbf{c})\mathbf{a}$
(c) $(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{a}.\mathbf{b})\mathbf{c}$ (d) $(\mathbf{a}.\mathbf{b})\mathbf{c}-(\mathbf{a}.\mathbf{c})\mathbf{b}$

7. If $\mathbf{a}\times\mathbf{b}=\mathbf{c}, \mathbf{b}\times\mathbf{c}=\mathbf{a}$ and a, b, c be moduli of the vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively, then
(a) $a=1, b=c$ (b) $c=1, a=1$
(c) $b=2, c=2a$ (d) $b=1, c=a$

8. If $\mathbf{a}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, $\mathbf{b}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{c}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}$, then $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is equal to
(a) $24\mathbf{i}+7\mathbf{j}-5\mathbf{k}$ (b) $7\mathbf{i}-24\mathbf{j}+5\mathbf{k}$
(c) $12\mathbf{i}+3\mathbf{j}-5\mathbf{k}$ (d) $\mathbf{i}+\mathbf{j}-7\mathbf{k}$

9. $\mathbf{i}\times(\mathbf{j}\times\mathbf{k})=$ **[RPET 1988; MP PET 1997]**
(a) 1 (b) 0
(c) – 1 (d) None of these

10. If three unit vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are such that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\frac{\mathbf{b}}{2}$, then the vector $\mathbf{a}$ makes with $\mathbf{b}$ and $\mathbf{c}$ respectively the angles
[MP PET 1998]
(a) $40^o, 80^o$ (b) $45^o, 45^o$
(c) $30^o, 60^o$ (d) $90^o, 60^o$

11. Let $\mathbf{a}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+\mathbf{j}$. If $\mathbf{c}$ is a vector such that $\mathbf{a}.\mathbf{c}=|\mathbf{c}|, |\mathbf{c}-\mathbf{a}|=2\sqrt{2}$ and the angle between $(\mathbf{a}\times\mathbf{b})$ and $\mathbf{c}$ is 30^o, then $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=$
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 2 (d) 3

12. $\mathbf{i}\times(\mathbf{j}\times\mathbf{k})+\mathbf{j}\times(\mathbf{k}\times\mathbf{i})+\mathbf{k}\times(\mathbf{i}\times\mathbf{j})$ equals **[RPET 1999]**
(a) $\mathbf{i}$ (b) $\mathbf{j}$
(c) $\mathbf{k}$ (d) 0

13. $[\mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}\ \ \mathbf{a}\times\mathbf{b}]$ is equal to **[MP PET 2004]**
(a) $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ (b) $2[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$
(c) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^2$ (d) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

14. Given three unit vector $\mathbf{a}, \mathbf{b}, \mathbf{c}$ such that $\mathbf{a}\perp\mathbf{b}$ and $\mathbf{a}\,||\,\mathbf{c}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is **[AMU 1999]**
(a) $\mathbf{a}$ (b) $\mathbf{b}$
(c) $\mathbf{c}$ (d) $\mathbf{0}$

15. If $\mathbf{a}=\mathbf{i}+\mathbf{j}-\mathbf{k}, \mathbf{b}=\mathbf{i}-\mathbf{j}+\mathbf{k}, \mathbf{c}=\mathbf{i}-\mathbf{j}-\mathbf{k}$, then $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is

[MP PET 2000]

(a) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (b) $2\mathbf{i} - 2\mathbf{j}$
(c) $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ (d) $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

16. If $\vec{A} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}, \vec{B} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \vec{C} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, then $(\vec{A} \times \vec{B}) \times \vec{C}$ is **[MP PET 2001]**

(a) $5(-\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ (b) $4(-\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$
(c) $5(-\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})$ (d) $4(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$

17. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) =$ **[RPET 2003]**

(a) 0 (b) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$
(c) $\mathbf{a} + \mathbf{b} + \mathbf{c}$ (d) $3[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

18. Let **a,b,c** be three vectors from $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$, if **[Orissa JEE 2003]**

(a) $\mathbf{b} \times (\mathbf{a} \times \mathbf{c}) = 0$ (b) $\mathbf{a}(\mathbf{b} \times \mathbf{c}) = 0$
(c) $\mathbf{c} \times \mathbf{a} = \mathbf{a} \times \mathbf{b}$ (d) $\mathbf{c} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$

19. If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{b} = \mathbf{i} + \mathbf{j}, \mathbf{c} = \mathbf{i}$ and $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \lambda\,\mathbf{a} + \mu\,\mathbf{b}$, then $\lambda + \mu =$ **[EAMCET 2003]**

(a) 0 (b) 1
(c) 2 (d) 3

20. If the vectors **a** and **b** are mutually perpendicular, then $\mathbf{a} \times \{\mathbf{a} \times \{\mathbf{a} \times (\mathbf{a} \times \mathbf{b})\}\}$ is equal to

(a) $|\mathbf{a}|^2\ \mathbf{b}$ (b) $|\mathbf{a}|^3\ \mathbf{b}$
(c) $|\mathbf{a}|^4\ \mathbf{b}$ (d) None of these

21. If **a**, **b**, **c**, **d** are coplanar vectors, then $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) =$ **[MP PET 1998]**

(a) $|\mathbf{a} \times \mathbf{c}|^2$ (b) $|\mathbf{a} \times \mathbf{d}|^2$
(c) $|\mathbf{b} \times \mathbf{c}|^2$ (d) **0**

22. $\mathbf{a} \times [\mathbf{a} \times (\mathbf{a} \times \mathbf{b})]$ is equal to **[AMU 2001]**

(a) $(\mathbf{a} \times \mathbf{a}).(\mathbf{b} \times \mathbf{a})$ (b) $\mathbf{a}.(\mathbf{b} \times \mathbf{a}) - \mathbf{b}.(\mathbf{a} \times \mathbf{b})$
(c) $[\mathbf{a}.(\mathbf{a} \times \mathbf{b})]\mathbf{a}$ (d) $(\mathbf{a}.\mathbf{a})(\mathbf{b} \times \mathbf{a})$

23. Given the following simultaneous equations for vectors **x** and **y**

$\mathbf{x} + \mathbf{y} = \mathbf{a}$ .....(i)
$\mathbf{x} \times \mathbf{y} = \mathbf{b}$ .....(ii)
$\mathbf{x}.\mathbf{a} = 1$ .....(iii)

Then $\mathbf{x} = ...,\ \mathbf{y} = ...$ **[Roorkee 1994]**

(a) $\mathbf{a}, \mathbf{a} - \mathbf{x}$ (b) $\mathbf{a} - \mathbf{b}, \mathbf{b}$
(c) $\mathbf{b}, \mathbf{a} - \mathbf{b}$ (d) None of these

24. $(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a}) =$ **[MP PET 1997]**

(a) $[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]\ \mathbf{a}$ (b) $[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]\ \mathbf{b}$
(c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\ \mathbf{c}$ (d) $[\mathbf{a}\ \mathbf{c}\ \mathbf{b}]\ \mathbf{b}$

Application of vectors in three dimensional geometry

1. The locus of a point equidistant from two given points **a** and **b** is given by

(a) $[\mathbf{r} - \frac{1}{2}(\mathbf{a} + \mathbf{b})].(\mathbf{a} - \mathbf{b}) = 0$
(b) $[\mathbf{r} - \frac{1}{2}(\mathbf{a} - \mathbf{b})].(\mathbf{a} + \mathbf{b}) = 0$
(c) $[\mathbf{r} - \frac{1}{2}(\mathbf{a} + \mathbf{b})].(\mathbf{a} + \mathbf{b}) = 0$
(d) $[\mathbf{r} - \frac{1}{2}(\mathbf{a} - \mathbf{b})].(\mathbf{a} - \mathbf{b}) = 0$

2. If the non-zero vectors **a** and **b** are perpendicular to each other, then the solution of the equation $\mathbf{r} \times \mathbf{a} = \mathbf{b}$ is given by

(a) $\mathbf{r} = x\mathbf{a} + \frac{1}{\mathbf{a}.\mathbf{a}}(\mathbf{a} \times \mathbf{b})$ (b) $\mathbf{r} = x\mathbf{b} - \frac{1}{\mathbf{b}.\mathbf{b}}(\mathbf{a} \times \mathbf{b})$
(c) $\mathbf{r} = x\mathbf{a} \times \mathbf{b}$ (d) $\mathbf{r} = x\mathbf{b} \times \mathbf{a}$

3. If **r** be position vector of any point on a sphere and **a**, **b** are respectively position vectors of the extremities of a diameter, then

(a) $\mathbf{r}.(\mathbf{a} - \mathbf{b}) = 0$ (b) $\mathbf{r}.(\mathbf{r} - \mathbf{a}) = 0$
(c) $(\mathbf{r} + \mathbf{a}).(\mathbf{r} + \mathbf{b}) = 0$ (d) $(\mathbf{r} - \mathbf{a}).(\mathbf{r} - \mathbf{b}) = 0$

4. Angle between the line $\mathbf{r} = (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$ and the normal to the plane $\mathbf{r}.(2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 4$ is **[MP PET 19**

(a) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (b) $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$
(c) $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (d) $\cot^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

5. If the equation of a line through a point **a** and parallel to vector **b** is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where t is a parameter, then its perpendicular distance from the point **c** is **[MP PET 1998]**

(a) $|(\mathbf{c} - \mathbf{b}) \times \mathbf{a}| \div |\mathbf{a}|$ (b) $|(\mathbf{c} - \mathbf{a}) \times \mathbf{b}| \div |\mathbf{b}|$
(c) $|(\mathbf{a} - \mathbf{b}) \times \mathbf{c}| \div |\mathbf{c}|$ (d) $|(\mathbf{a} - \mathbf{b}) \times \mathbf{c}| \div |\mathbf{a} + \mathbf{c}|$

6. If $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ are two vectors, then the point of intersection of two lines $\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$ is **[RPET 2000]**

(a) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ (b) $\mathbf{i} - \mathbf{j} + \mathbf{k}$
(c) $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ (d) $3\mathbf{i} - \mathbf{j} + \mathbf{k}$

7. If **a**, **b**, **c** are three non-coplanar vectors, then the vector equation $\mathbf{r} = (1 - \mathbf{p} - \mathbf{q})\mathbf{a} + p\mathbf{b} + q\mathbf{c}$ represents a **[EAMCET 2003]**

(a) Straight line
(b) Plane
(c) Plane passing through the origin
(d) Sphere

8. The vector equation of the line joining the points $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $-2\mathbf{j} + 3\mathbf{k}$ is **[MP PET 2003]**

(a) $\mathbf{r} = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$
(b) $\mathbf{r} = t_1(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t_2(3\mathbf{k} - 2\mathbf{j})$
(c) $\mathbf{r} = (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(2\mathbf{k} - \mathbf{i})$
(d) $\mathbf{r} = t(2\mathbf{k} - \mathbf{i})$

9. The spheres $\mathbf{r}^2 + 2\mathbf{u}_1.\mathbf{r} + 2d_1 = 0$ and $\mathbf{r}^2 + 2\mathbf{u}_2.\mathbf{r} + 2d_2 = 0$ cut orthogonally, if

(a) $\mathbf{u}_1.\mathbf{u}_2 = 0$
(b) $\mathbf{u}_1 + \mathbf{u}_2 = 0$

(c) $\mathbf{u}_1.\mathbf{u}_2 = d_1 + d_2$

(d) $(\mathbf{u}_1 - \mathbf{u}_2).(\mathbf{u}_1 + \mathbf{u}_2) = d_1^2 + d_2^2$

10. A tetrahedron has vertices at $O(0,0,0)$, $A(1,2,1), B(2,1,3)$ and $C(-1,1,2)$. Then the angle between the faces OAB and ABC will be **[MNR 1994; UPSEAT 2000; AIEEE 2003]**

(a) $\cos^{-1}\left(\frac{19}{35}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) 30^o (d) 90^o

11. A vector $\mathbf{n}$ of magnitude 8 units is inclined to x-axis at 45^o, y-axis at 60^o and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\mathbf{n}$, then its equation in vector form is

(a) $\mathbf{r}.(\sqrt{2}\mathbf{i} + \mathbf{j} + \mathbf{k}) = 4$ (b) $\mathbf{r}.(\sqrt{2}\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2$

(c) $\mathbf{r}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 4$ (d) None of these

12. The vector equation of a plane, which is at a distance of 8 *unit* from the origin and which is normal to the vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, is

(a) $\mathbf{r}.(2\mathbf{i} + \mathbf{j} + \mathbf{k}) = 24$ (b) $\mathbf{r}.(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 24$

(c) $\mathbf{r}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 24$ (d) None of these

13. The distance of the point $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ from the plane $\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) = 9$ is

(a) $\frac{13}{\sqrt{21}}$ (b) $\frac{3}{\sqrt{21}}$

(c) $\frac{13}{21}$ (d) $\frac{13}{3\sqrt{21}}$

14. The centre of the circle given by $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 15$ and $|\mathbf{r} - (\mathbf{j} + 2\mathbf{k})| = 4$ is

(a) (0, 1, 2) (b) (1, 3, 4)

(c) (–1, 3, 4) (d) None of these

15. A vector **r** is equally inclined with the co-ordinate axes. If the tip of **r** is in the positive octant and |**r**| = 6, then **r** is

(a) $2\sqrt{3}(\mathbf{i} - \mathbf{j} + \mathbf{k})$ (b) $2\sqrt{3}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$

(c) $2\sqrt{3}(\mathbf{i} + \mathbf{j} - \mathbf{k})$ (d) $2\sqrt{3}(\mathbf{i} + \mathbf{j} + \mathbf{k})$

16. The position vectors of two points P and Q are $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ respectively. The equation of the plane through Q and perpendicular to PQ is

(a) $\mathbf{r}.(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) = 28$ (b) $\mathbf{r}.(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) = 32$

(c) $\mathbf{r}.(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) + 28 = 0$ (d) None of these

17. The vector equation of the plane passing through the origin and the line of intersection of the plane $\mathbf{r}.\mathbf{a} = \lambda$ and $\mathbf{r}.\mathbf{b} = \mu$ is

(a) $\mathbf{r}.(\lambda\mathbf{a} - \mu\mathbf{b}) = 0$ (b) $\mathbf{r}.(\lambda\mathbf{b} - \mu\mathbf{a}) = 0$

(c) $\mathbf{r}.(\lambda\mathbf{a} + \mu\mathbf{b}) = 0$ (d) $\mathbf{r}.(\lambda\mathbf{b} + \mu\mathbf{a}) = 0$

18. The position vectors of points A and B are $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ respectively. The equation of a plane is $\mathbf{r}.(5\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) + 9 = 0$. The points A and B

(a) Lie on the plane

(b) Are on the same side of the plane

(c) Are on the opposite side of the plane

(d) None of these

19. The vector equation of the plane through the point $2\mathbf{i} - \mathbf{j} - 4\mathbf{k}$ and parallel to the plane $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) - 7 = 0$ is

(a) $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) = 0$ (b) $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) = 32$

(c) $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) = 12$ (d) None of these

20. The vector equation of the plane through the point (2, 1, –1) and passing through the line of intersection of the plane $\mathbf{r}.(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 0$ and $\mathbf{r}.(\mathbf{j} + 2\mathbf{k}) = 0$ is

(a) $\mathbf{r}.(\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}) = 0$ (b) $\mathbf{r}.(\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}) = 6$

(c) $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} - 13\mathbf{k}) = 0$ (d) None of these

21. The vector equation of the plane through the point $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and perpendicular to the line of intersection of the planes $\mathbf{r}.(3\mathbf{i} - \mathbf{j} + \mathbf{k}) = 1$ and $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k} = 2$ is

(a) $\mathbf{r}.(2\mathbf{i} + 7\mathbf{j} - 13\mathbf{k}) = 1$ (b) $\mathbf{r}.(2\mathbf{i} - 7\mathbf{j} - 13\mathbf{k}) = 1$

(c) $\mathbf{r}.(2\mathbf{i} + 7\mathbf{j} + 13\mathbf{k}) = 0$ (d) None of these

22. The equation of the plane containing the lines $\mathbf{r} = \mathbf{a}_1 + \lambda\mathbf{a}_2$ and $\mathbf{r} = \mathbf{a}_2 + \lambda\mathbf{a}_1$ is

(a) $[\mathbf{r}\ \mathbf{a}_1\ \mathbf{a}_2] = 0$ (b) $[\mathbf{r}\ \mathbf{a}_1\ \mathbf{a}_2] = \mathbf{a}_1.\mathbf{a}_2$

(c) $[\mathbf{r}\ \mathbf{a}_2\ \mathbf{a}_1] = \mathbf{a}_1.\mathbf{a}_2$ (d) None of these

23. The vector equation of the plane containing the lines $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \mu(-\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ is

(a) $\mathbf{r}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$ (b) $\mathbf{r}.(\mathbf{i} - \mathbf{j} - \mathbf{k}) = 0$

(c) $\mathbf{r}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 3$ (d) None of these

24. The cartesian equation of the plane $\mathbf{r} = (1 + \lambda - \mu)\mathbf{i} + (2 - \lambda)\mathbf{j} + (3 - 2\lambda + 2\mu)\mathbf{k}$ is

(a) $2x + y = 5$ (b) $2x - y = 5$

(c) $2x + z = 5$ (d) $2x - z = 5$

25. The length of the perpendicular from the origin to the plane passing through three non-collinear points $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is

(a) $\frac{[\mathbf{abc}]}{|\mathbf{a}\times\mathbf{b} + \mathbf{c}\times\mathbf{a} + \mathbf{b}\times\mathbf{c}|}$ (b) $\frac{2[\mathbf{abc}]}{|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|}$

(c) $[\mathbf{abc}]$ (d) None of these

26. The length of the perpendicular from the origin to the plane passing through the point **a** and containing the line $\mathbf{r} = \mathbf{b} + \lambda\mathbf{c}$ is

(a) $\frac{[\mathbf{abc}]}{|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|}$ (b) $\frac{[\mathbf{abc}]}{|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c}|}$

(c) $\frac{[\mathbf{abc}]}{|\mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|}$ (d) $\frac{[\mathbf{abc}]}{|\mathbf{c}\times\mathbf{a} + \mathbf{a}\times\mathbf{b}|}$

27. The position vector of a point at a distance of $3\sqrt{11}$ units from $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ on a line passing through the points $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ is

(a) $10\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ (b) $-8\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

(c) $8\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ (d) $-10\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$

28. The line joining the points $6\mathbf{a}-4\mathbf{b}+4\mathbf{c},-4\mathbf{c}$ and the line joining the points $-\mathbf{a}-2\mathbf{b}-3\mathbf{c},\mathbf{a}+2\mathbf{b}-5\mathbf{c}$ intersect at

(a) $-4\mathbf{a}$ (b) $4\mathbf{a}-\mathbf{b}-\mathbf{c}$
(c) $4\mathbf{c}$ (d) None of these

29. Angle between the line $\mathbf{r}=(2\mathbf{i}-\mathbf{j}+\mathbf{k})+\lambda(-\mathbf{i}+\mathbf{j}+\mathbf{k})$ and the plane $\mathbf{r}.(3\mathbf{i}+2\mathbf{j}-\mathbf{k})=4$ is

(a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (b) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (d) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

30. The line through $\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and perpendicular to the lines $\mathbf{r}=(\mathbf{i}+2\mathbf{j}-\mathbf{k})+\lambda(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ and $\mathbf{r}=(2\mathbf{i}+6\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ is

(a) $\mathbf{r}=(\mathbf{i}+2\mathbf{j}-\mathbf{k})+\lambda(-\mathbf{i}+5\mathbf{j}-3\mathbf{k})$
(b) $\mathbf{r}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}-5\mathbf{j}+3\mathbf{k})$
(c) $\mathbf{r}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}+3\mathbf{k})$
(d) $\mathbf{r}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}+5\mathbf{j}-3\mathbf{k})$

31. The distance from the point $-\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ to the straight line through the point (2, 3, –4) and parallel to the vector $6\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ is

(a) 7 (b) 10
(c) 9 (d) None of these

32. The position vector of the point in which the line joining the points $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $3\mathbf{k}-2\mathbf{j}$ cuts the plane through the origin and the points $4\mathbf{j}$ and $2\mathbf{i}+\mathbf{k}$, is

(a) $6\mathbf{i}-10\mathbf{j}+3\mathbf{k}$ (b) $\frac{1}{5}(6\mathbf{i}-10\mathbf{j}+3\mathbf{k})$
(c) $-6\mathbf{i}+10\mathbf{j}-3\mathbf{k}$ (d) None of these

33. The distance between the planes given by $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-2\mathbf{k})+5=0$ and $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-2\mathbf{k})-8=0$ is

(a) 1 *unit* (b) $\frac{13}{3}$ *unit*
(c) 13 *unit* (d) None of these

34. The equation of the plane containing the line $\mathbf{r}=\mathbf{i}+\mathbf{j}+\lambda(2\mathbf{i}+\mathbf{j}+4\mathbf{k})$ is

(a) $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-\mathbf{k})=3$ (b) $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-\mathbf{k})=6$
(c) $\mathbf{r}.(-\mathbf{i}-2\mathbf{j}+\mathbf{k})=3$ (d) None of these

35. The equation $|\mathbf{r}|^2-\mathbf{r}.(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})-10=0$ represents a

(a) Circle (b) Plane
(c) Sphere of radius 4 (d) Sphere of radius 3
(e) None of these

36. The centre of the sphere $\alpha\,\mathbf{r}-2\mathbf{u}.\mathbf{r}=\beta,(\alpha\neq 0)$ is **[AMU 1999]**

(a) $-\mathbf{u}/\alpha$ (b) $\mathbf{u}/\alpha$
(c) $\alpha\mathbf{u}/\beta$ (d) $\frac{\alpha+\beta}{\alpha}\mathbf{u}$

37. The shortest distance between the lines $\mathbf{r}=(3\mathbf{i}-2\mathbf{j}-2\mathbf{k})+\mathbf{i}t$ and $\mathbf{r}=\mathbf{i}-\mathbf{j}+2\mathbf{k}+\mathbf{j}s$ (t and s being parameters) is **[AMU 1999]**

(a) $\sqrt{21}$ (b) $\sqrt{102}$
(c) 4 (d) 3

38. The equation of the line passing through the points $a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$ is **[RPET 2002]**

(a) $(a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k})+t(b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k})$
(b) $(a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k})-t(b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k})$
(c) $a_1(1-t)\mathbf{i}+a_2(1-t)\mathbf{j}+a_3(1-t)\mathbf{k}+(b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k})t$
(d) None of these

39. The distance between the line $\mathbf{r}=2\mathbf{i}-2\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+4\mathbf{k})$ and the plane $\mathbf{r}.(\mathbf{i}+5\mathbf{j}+\mathbf{k})=5$ is **[AIEEE 2005]**

(a) $\frac{3}{10}$ (b) $\frac{10}{3}$
(c) $\frac{10}{9}$ (d) $\frac{10}{3\sqrt{3}}$

40. The image of the point with position vector $\mathbf{i}+3\mathbf{k}$ in the plane $\mathbf{r}.(\mathbf{i}+\mathbf{j}+\mathbf{k})=1$ is

(a) $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ (b) $\mathbf{i}-2\mathbf{i}+\mathbf{k}$
(c) $-\mathbf{i}-2\mathbf{j}+\mathbf{k}$ (d) $\mathbf{i}+2\mathbf{j}-\mathbf{k}$

41. The equation of the plane passing through the points $(-1,-2,0),(2,3,5)$ and parallel to the line $\mathbf{r}=-3\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+5\mathbf{j}-\mathbf{k})$ is **[J & K 2005]**

(a) $\mathbf{r}.(-30\mathbf{i}+13\mathbf{j}+5\mathbf{k})=4$ (b) $\mathbf{r}.(30\mathbf{i}+13\mathbf{j}+5\mathbf{k})=4$
(c) $\mathbf{r}.(30\mathbf{i}+13\mathbf{j}-5\mathbf{k})=4$ (d) $\mathbf{r}.(30\mathbf{i}-13\mathbf{j}-5\mathbf{k})=4$

42. The shortest distance between the lines
$\mathbf{r}_1=4\mathbf{i}-3\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}-4\mathbf{j}+7\mathbf{k})$
and $\mathbf{r}_2=\mathbf{i}-\mathbf{j}-10\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j}+8\mathbf{k})$ is **[J & K 2005]**

(a) 3 (b) 1
(c) 2 (d) 0

43. The position vector of the point where the line $\mathbf{r}=\mathbf{i}-\mathbf{j}+\mathbf{k}+t(\mathbf{i}+\mathbf{j}+\mathbf{k})$ meets the plane $\mathbf{r}.(\mathbf{i}+\mathbf{j}+\mathbf{k})=5$ is **[Kerala (Engg.) 2005]**

(a) $5\mathbf{i}+\mathbf{j}-\mathbf{k}$ (b) $5\mathbf{i}+3\mathbf{j}-3\mathbf{k}$
(c) $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$ (d) $5\mathbf{i}+\mathbf{j}+\mathbf{k}$
(e) $4\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

44. A Plane meets the co-ordinate axes at P, Q and R such that the position vector of the centroid of ΔPQR is $2\mathbf{i}-5\mathbf{j}+8\mathbf{k}$. Then the equation of the plane is **[J & K 2005]**

(a) $\mathbf{r}.(20\mathbf{i}-8\mathbf{j}+5\mathbf{k})=120$ (b) $\mathbf{r}.(20\mathbf{i}-8\mathbf{j}+5\mathbf{k})=1$
(c) $\mathbf{r}.(20\mathbf{i}-8\mathbf{j}+5\mathbf{k})=2$ (d) $\mathbf{r}.(20\mathbf{i}-8\mathbf{j}+5\mathbf{k})=20$

45. The line of intersection of the planes $\mathbf{r}.(\mathbf{i}-3\mathbf{j}+\mathbf{k})=1$ and $\mathbf{r}.(2\mathbf{i}+5\mathbf{j}-3\mathbf{k})=2$ is parallel to the vector

(a) $-4\mathbf{i}+5\mathbf{j}+11\mathbf{k}$ (b) $4\mathbf{i}+5\mathbf{j}+11\mathbf{k}$
(c) $4\mathbf{i}-5\mathbf{j}+11\mathbf{k}$ (d) $4\mathbf{i}-5\mathbf{j}-11\mathbf{k}$

46. The equation of plane passing through a point $A(2,-1,3)$ and parallel to the vectors $\mathbf{a}=(3,0,-1)$ and $\mathbf{b}=(-3,2,2)$ is **[Orissa JEE 2005]**

(a) $2x-3y+6z-25=0$ (b) $2x-3y+6z+25=0$

(c) $3x-2y+6z-25=0$ (d) $3x-2y+6z+25=0$

47. If the position vectors of two point P and Q are respectively $9\mathbf{i}-\mathbf{j}+5\mathbf{k}$ and $\mathbf{i}+3\mathbf{j}+5\mathbf{k}$, and the line segment PQ intersects the YOZ plane at a point R, the $PR:RQ$ is equal to

(a) 9 : 1 (b) 1 : 9

(c) –1 : 9 (d) – 9 : 1

Critical Thinking

Objective Questions

1. Three forces of magnitudes 1, 2, 3 dynes meet in a point and act along diagonals of three adjacent faces of a cube. The resultant force is

(a) 114 dyne (b) 6 dyne

(c) 5 dyne (d) None of these

2. The vectors **b** and **c** are in the direction of north-east and north-west respectively and $|\mathbf{b}|=|\mathbf{c}|=4$. The magnitude and direction of the vector **d** = **c** – **b**, are **[Roorkee 2000]**

(a) $4\sqrt{2}$, towards north (b) $4\sqrt{2}$, towards west

(c) 4, towards east (d) 4, towards south

3. If **a, b** and **c** are unit vectors, then $|\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{c}-\mathbf{a}|^2$ does not exceed **[IIT Screening 2001]**

(a) 4 (b) 9

(c) 8 (d) 6

4. The vectors $\overrightarrow{AB}=3\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ and $\overrightarrow{AC}=5\mathbf{i}-5\mathbf{j}+2\mathbf{k}$ are the sides of a triangle ABC. The length of the median through A is **[UPSEAT 2004]**

(a) $\sqrt{13}$ *unit* (b) $2\sqrt{5}$ *unit*

(c) 5 *unit* (d) 10 *unit*

5. Let the value of $\mathbf{p}=(x+4y)\mathbf{a}+(2x+y+1)\mathbf{b}$ and $\mathbf{q}=(y-2x+2)\mathbf{a}+(2x-3y-1)\mathbf{b}$, where **a** and **b** are non-collinear vectors. If $3\mathbf{p}=2\mathbf{q}$, then the value of x and y will be **[RPET 1984; MNR 1984]**

(a) – 1, 2 (b) 2, – 1

(c) 1, 2 (d) 2, 1\

6. The points D, E, F divide BC, CA and AB of the triangle ABC in the ratio 1 : 4, 3 : 2 and 3 : 7 respectively and the point K divides AB in the ratio $1:3$, then $(\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}):\overrightarrow{CK}$ is equal to

(a) 1 : 1 (b) 2 : 5

(c) 5 : 2 (d) None of these

7. If two vertices of a triangle are $\mathbf{i}-\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$, then the third vertex can be

(a) $\mathbf{i}+\mathbf{k}$ (b) $\mathbf{i}-2\mathbf{j}-\mathbf{k}$

(c) $\mathbf{i}-\mathbf{k}$ (d) $2\mathbf{i}-\mathbf{j}$

(e) All the above

8. If **a** of magnitude 50 is collinear with the vector $\mathbf{b}=6\mathbf{i}-8\mathbf{j}-\frac{15\mathbf{k}}{2}$, and makes an acute angle with the positive direction of z-axis, then the vector **a** is equal to **[Pb. CET 2004]**

(a) $24\mathbf{i}-32\mathbf{j}+30\mathbf{k}$ (b) $-24\mathbf{i}+32\mathbf{j}+30\mathbf{k}$

(c) $16\mathbf{i}-16\mathbf{j}-15\mathbf{k}$ (d) $-12\mathbf{i}+16\mathbf{j}-30\mathbf{k}$

9. If three non-zero vectors are $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$ and $\mathbf{c}=c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$. If **c** is the unit vector perpendicular to the vectors **a** and **b** and the angle between **a** and **b** is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is **[IIT 1986]**

(a) 0 (b) $\frac{3(\Sigma a_1^2)(\Sigma b_1^2)(\Sigma c_1^2)}{4}$

(c) 1 (d) $\frac{(\Sigma a_1^2)(\Sigma b_1^2)}{4}$

10. Let the unit vectors **a** and **b** be perpendicular and the unit vector **c** be inclined at an angle to both **a** and **b**. If $\mathbf{c}=\alpha\mathbf{a}+\beta\mathbf{b}+\gamma(\mathbf{a}\times\mathbf{b})$, then

(a) $\alpha=\beta=\cos\theta,\ \gamma^2=\cos 2\theta$

(b) $\alpha=\beta=\cos\theta,\ \gamma^2=-\cos 2\theta$

(c) $\alpha=\cos\theta,\ \beta=\sin\theta,\ \gamma^2=\cos 2\theta$

(d) None of these

11. The vector $\mathbf{a}+\mathbf{b}$ bisects the angle between the vectors **a** and **b,** if

(a) $|\mathbf{a}|=|\mathbf{b}|$

(b) $|\mathbf{a}|=|\mathbf{b}|$ or angle between **a** and **b** is zero

(c) $|\mathbf{a}|=m|\mathbf{b}|$

(d) None of these

12. The points O, A, B, C, D are such that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$, $\overrightarrow{OC}=2\mathbf{a}+3\mathbf{b}$ and $\overrightarrow{OD}=\mathbf{a}-2\mathbf{b}$. If $|\mathbf{a}|=3|\mathbf{b}|$, then the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) None of these

13. If $\vec{A}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\vec{B}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\vec{C}=3\mathbf{i}+\mathbf{j}$, then the value of t such that $\vec{A}+t\vec{B}$ is at right angle to vector $\vec{C}$, is **[RPET 2002]**

(a) 2 (b) 4

(c) 5 (d) 6

14. Let $\mathbf{b}=4\mathbf{i}+3\mathbf{j}$ and **c** be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along **b** and **c** respectively, are given by **[IIT 1987]**

(a) $2\mathbf{i}-\mathbf{j},\ \frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$ (b) $2\mathbf{i}+\mathbf{j},\ -\frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$

(c) $2\mathbf{i}+\mathbf{j},-\frac{2}{5}\mathbf{i}-\frac{11}{5}\mathbf{j}$ (d) $2\mathbf{i}-\mathbf{j},\ -\frac{2}{5}\mathbf{i}+\frac{11}{5}\mathbf{j}$

15. Let $\mathbf{a}=2\mathbf{i}-\mathbf{j}+\mathbf{k},\ \mathbf{b}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+\mathbf{j}-2\mathbf{k}$ be three vectors. A vector in the plane of **b** and **c** whose projection on **a** is of magnitude $\sqrt{2/3}$ is **[IIT 1993; Pb. CET 2004]**

(a) $2\mathbf{i}+3\mathbf{j}-3\mathbf{k}$ (b) $2\mathbf{i}+3\mathbf{j}+3\mathbf{k}$

(c) $-2\mathbf{i}-\mathbf{j}+5\mathbf{k}$ (d) $2\mathbf{i}+\mathbf{j}+5\mathbf{k}$

16. A vector **a** has components $2p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the anti-clockwise sense. If **a** has components $p+1$ and 1 with respect to the new system, then

(a) $p=0$ (b) $p=1$ or $-\frac{1}{3}$

(c) $p=-1$ or $\frac{1}{3}$ (d) $p=1$ or -1

17. If $\mathbf{u}=2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{v}=6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, then a unit vector perpendicular to both **u** and **v** is

(a) $\mathbf{i}-10\mathbf{j}-18\mathbf{k}$ (b) $\frac{1}{\sqrt{17}}\left(\frac{1}{5}\mathbf{i}-2\mathbf{j}-\frac{18}{5}\mathbf{k}\right)$

(c) $\frac{1}{\sqrt{473}}(7\mathbf{i}-10\mathbf{j}-18\mathbf{k})$ (d) None of these

18. If $\mathbf{a}=2\mathbf{i}+\mathbf{k},\ \mathbf{b}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=4\mathbf{i}-3\mathbf{j}+7\mathbf{k}$. If $\mathbf{d}\times\mathbf{b}=\mathbf{c}\times\mathbf{b}$ and $\mathbf{d}.\mathbf{a}=0$, then **d** will be **[IIT 1990]**

(a) $\mathbf{i}+8\mathbf{j}+2\mathbf{k}$ (b) $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$

(c) $-\mathbf{i}+8\mathbf{j}-\mathbf{k}$ (d) $-\mathbf{i}-8\mathbf{j}+2\mathbf{k}$

19. If $\mathbf{a}\times\mathbf{r}=\mathbf{b}+\lambda\mathbf{a}$ and $\mathbf{a}\,.\,\mathbf{r}=3$, where $\mathbf{a}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=-\mathbf{i}-2\mathbf{j}+\mathbf{k}$, then **r** and are equal to

(a) $\mathbf{r}=\frac{7}{6}\mathbf{i}+\frac{2}{3}\mathbf{j},\ \lambda=\frac{6}{5}$ (b) $\mathbf{r}=\frac{7}{6}\mathbf{i}+\frac{2}{3}\mathbf{j},\ \lambda=\frac{5}{6}$

(c) $\mathbf{r}=\frac{6}{7}\mathbf{i}+\frac{2}{3}\mathbf{j},\ \lambda=\frac{6}{5}$ (d) None of these

20. Let the vectors **a**, **b**, **c** and **d** be such that $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=0$. Let P_1 and P_2 be planes determined by pair of vectors **a**, **b** and **c**, **d** respectively. Then the angle between P_1 and P_2 is **[IIT Screening 2000; MP PET 2004]**

(a) 0^o (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

21. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k},\ \mathbf{a}.\mathbf{b}=1$ and $\mathbf{a}\times\mathbf{b}=\mathbf{j}-\mathbf{k}$, then $\mathbf{b}=$ **[IIT Screening 2004]**

(a) $\mathbf{i}$ (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$

(c) $2\mathbf{j}-\mathbf{k}$ (d) $2\mathbf{i}$

22. The position vectors of the vertices of a quadrilateral *ABCD* are **a,b,c** and **d** respectively. Area of the quadrilateral formed by joining the middle points of its sides is **[Roorkee 2000]**

(a) $\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|$

(b) $\frac{1}{4}|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{a}\times\mathbf{d}+\mathbf{b}\times\mathbf{a}|$

(c) $\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|$

(d) $\frac{1}{4}|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{b}|$

23. The moment about the point $M(-2,4,-6)$ of the force represented in magnitude and position by $\overrightarrow{AB}$ where the points *A* and *B* have the co-ordinates $(1,2,-3)$ and $(3,-4,2)$ respectively, is

(a) $8\mathbf{i}-9\mathbf{j}-14\mathbf{k}$ (b) $2\mathbf{i}-6\mathbf{j}+5\mathbf{k}$

(c) $-3\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ (d) $-5\mathbf{i}+8\mathbf{j}-8\mathbf{k}$

24. If the vectors $a\mathbf{i}+\mathbf{j}+\mathbf{k},\ \mathbf{i}+b\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+c\mathbf{k}$ $(a\neq b\neq c\neq 1)$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$ **[BIT Ranchi 1988; RPET 1987; IIT 1987; DCE 2001; MP PET 2004; Orissa JEE 2005]**

(a) -1 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) 1

25. If $\alpha(\mathbf{a}\times\mathbf{b})+\beta(\mathbf{b}\times\mathbf{c})+\gamma(\mathbf{c}\times\mathbf{a})=\mathbf{0}$ and at least one of the numbers α,β and γ is non-zero, then the vectors **a, b** and **c** are

(a) Perpendicular (b) Parallel

(c) Coplanar (d) None of these

26. The volume of the tetrahedron, whose vertices are given by the vectors $-\mathbf{i}+\mathbf{j}+\mathbf{k},\ \mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}-\mathbf{k}$ with reference to the fourth vertex as origin, is

(a) $\frac{5}{3}$ *cubic unit* (b) $\frac{2}{3}$ *cubic unit*

(c) $\frac{3}{5}$ *cubic unit* (d) None of these

27. Let $\mathbf{a}=\mathbf{i}-\mathbf{j},\ \mathbf{b}=\mathbf{j}-\mathbf{k},\ \mathbf{c}=\mathbf{k}-\mathbf{i}$. If $\hat{\mathbf{d}}$ is a unit vector such that $\mathbf{a}.\hat{\mathbf{d}}=0=[\mathbf{b}\ \mathbf{c}\ \hat{\mathbf{d}}]$, then $\hat{\mathbf{d}}$ is equal to **[IIT 1995]**

(a) $\pm\frac{\mathbf{i}+\mathbf{j}-\mathbf{k}}{\sqrt{3}}$ (b) $\pm\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$

(c) $\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$ (d) $\pm\,\mathbf{k}$

28. The value of 'a' so that the volume of parallelopiped formed by $\mathbf{i}+a\mathbf{j}+\mathbf{k},\mathbf{j}+a\mathbf{k}$ and $a\mathbf{i}+\mathbf{k}$ becomes minimum is **[IIT Screening 2003]**

(a) – 3 (b) 3

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

29. If **b** and **c** are any two non-collinear unit vectors and **a** is any vector, then $(\mathbf{a}.\mathbf{b})\mathbf{b}+(\mathbf{a}.\mathbf{c})\mathbf{c}+\frac{\mathbf{a}.(\mathbf{b}\times\mathbf{c})}{|\mathbf{b}\times\mathbf{c}|}(\mathbf{b}\times\mathbf{c})=$ **[IIT 1996]**

(a) **a** (b) **b**

(c) **c** (d) **0**

30. If **a, b, c** are non-coplanar unit vectors such that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\frac{\mathbf{b}+\mathbf{c}}{\sqrt{2}}$, then the angle between **a** and **b** is **[IIT 1995]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) π

31. $[(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]=$

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^3$

(c) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^4$ (d) None of these

32. Unit vectors **a, b** and **c** are coplanar. A unit vector **d** is perpendicular to them. If $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\frac{1}{6}\mathbf{i}-\frac{1}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$ and the angle between **a** and **b** is 30^o, then **c** is **[Roorkee Qualifying 1998]**

(a) $\frac{(\mathbf{i}-2\mathbf{j}+2\mathbf{k})}{3}$ (b) $\frac{(2\mathbf{i}+\mathbf{j}-\mathbf{k})}{3}$

(c) $\frac{(-\mathbf{i}+2\mathbf{j}-2\mathbf{k})}{3}$ (d) $\frac{(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}{3}$

33. The radius of the circular section of the sphere $|\mathbf{r}|=5$ by the plane $\mathbf{r}.(\mathbf{i}+\mathbf{j}+\mathbf{k})=3\sqrt{3}$ is

(a) 1 (b) 2

(c) 3 (d) 4

34. If **x** is parallel to **y** and **z** where $\mathbf{x}=2\mathbf{i}+\mathbf{j}+\alpha\mathbf{k}$, $\mathbf{y}=\alpha\mathbf{i}+\mathbf{k}$ and $\mathbf{z}=5\mathbf{i}-\mathbf{j}$, then α is equal to **[J & K 200**

(a) $\pm\sqrt{5}$ (b) $\pm\sqrt{6}$

(c) $\pm\sqrt{7}$ (d) None of these

35. The vector **c** directed along the internal bisector of the angle between the vectors $\mathbf{a}=7\mathbf{i}-4\mathbf{j}-4\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}-\mathbf{j}+2\mathbf{k}$ with $|\mathbf{c}|=5\sqrt{6}$, is

(a) $\frac{5}{3}(\mathbf{i}-7\mathbf{j}+2\mathbf{k})$

(b) $\frac{5}{3}(5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$

(c) $\frac{5}{3}(\mathbf{i}+7\mathbf{j}+2\mathbf{k})$

(d) $\frac{5}{3}(-5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$

36. The distance of the point $B(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ from the line which is passing through $A(4\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ and which is parallel to the vector $\vec{C}=2\mathbf{i}+3\mathbf{j}+6\mathbf{k}$ is

(a) 10 (b) $\sqrt{10}$

(c) 100 (d) None of these

37. Let **a, b, c** are three non-coplanar vectors such that $\mathbf{r}_1=\mathbf{a}-\mathbf{b}+\mathbf{c}$, $\mathbf{r}_2=\mathbf{b}+\mathbf{c}-\mathbf{a}$, $\mathbf{r}_3=\mathbf{c}+\mathbf{a}+\mathbf{b}$, $\mathbf{r}=2\mathbf{a}-3\mathbf{b}+4\mathbf{c}$. If $\mathbf{r}=\lambda_1\mathbf{r}_1+\lambda_2\mathbf{r}_2+\lambda_3\mathbf{r}_3$, then

(a) $\lambda_1=7$ (b) $\lambda_1+\lambda_3=3$

(c) $\lambda_1+\lambda_2+\lambda_3=4$ (d) $\lambda_3+\lambda_2=2$

38. Let $\mathbf{a}=2\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and a unit vector **c** be coplanar. If **c** is perpendicular to **a**, then **c** = **[IIT 1999; Pb. CET 2003; DCE 2005]**

(a) $\frac{1}{\sqrt{2}}(-\mathbf{j}+\mathbf{k})$ (b) $\frac{1}{\sqrt{3}}(-\mathbf{i}-\mathbf{j}-\mathbf{k})$

(c) $\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$ (d) $\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}-\mathbf{k})$

39. Let **p, q, r** be three mutually perpendicular vectors of the same magnitude. If a vector **x** satisfies equation $\mathbf{p}\times\{(\mathbf{x}-\mathbf{q})\times\mathbf{p}\}+\mathbf{q}\times\{(\mathbf{x}-\mathbf{r})\times\mathbf{q}\}+\mathbf{r}\times\{(\mathbf{x}-\mathbf{p})\times\mathbf{r}\}=0$, then **x** is given by **[IIT 1997 Cancelled]**

(a) $\frac{1}{2}(\mathbf{p}+\mathbf{q}-2\mathbf{r})$

(b) $\frac{1}{2}(\mathbf{p}+\mathbf{q}+\mathbf{r})$

(c) $\frac{1}{3}(\mathbf{p}+\mathbf{q}+\mathbf{r})$

(d) $\frac{1}{3}(2\mathbf{p}+\mathbf{q}-\mathbf{r})$

40. The point of intersection of $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$ and $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$, where $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{k}$ is

(a) $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ (b) $3\mathbf{i}-\mathbf{k}$

(c) $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ (d) None of these

Answers

Modulus of vector, Algebra of vectors

1	a	2	d	3	d	4	a	5	c
6	c	7	b	8	d	9	c	10	c
11	a	12	b	13	b	14	a	15	d

16	a	17	a	18	c	19	b	20	d
21	b	22	a	23	c	24	c	25	b
26	b	27	b	28	b	29	b	30	c
31	b	32	d	33	d	34	c	35	c
36	c	37	d	38	c	39	b	40	b
41	d	42	c	43	b	44	b	45	c
46	d	47	d	48	a	49	c	50	b
51	c	52	b	53	a	54	a	55	b
56	c	57	b	58	a	59	a	60	a
61	b	62	a	63	d	64	b	65	b
66	d	67	a	68	b	69	c	70	d
71	b	72	a	73	a	74	a	75	b
76	b	77	d	78	b	79	c	80	c
81	a	82	c	83	a	84	b	85	a
86	a	87	a	88	c	89	d	90	c
91	a	92	a	93	d	94	d	95	d
96	a	97	a	98	d	99	a	100	c
101	c	102	c	103	d	104	b	105	a
106	d	107	b	108	d	109	c	110	c
111	a	112	a	113	c	114	b	115	c
116	a	117	c	118	a	119	a		

Scalar or Dot product of two vectors and its applications

1	a	2	d	3	c	4	b	5	c
6	c	7	a	8	c	9	d	10	b
11	d	12	a	13	a	14	d	15	d
16	b	17	d	18	b	19	c	20	c
21	b	22	d	23	b	24	c	25	c
26	a	27	c	28	d	29	a	30	a
31	d	32	c	33	d	34	a	35	a
36	a	37	d	38	a	39	a	40	d
41	c	42	c	43	a	44	b	45	c
46	b	47	b	48	b	49	b	50	b
51	d	52	b	53	a	54	d	55	d
56	b	57	b	58	c	59	c	60	b
61	c	62	d	63	c	64	d	65	b
66	a	67	a	68	b	69	a	70	a
71	a	72	b	73	d	74	d	75	c
76	b	77	b	78	c	79	b	80	a
81	a	82	c	83	c	84	a,c,	85	d

							d		
86	d	87	d	88	a	89	c	90	c
91	d	92	a	93	b	94	b	95	b
96	b	97	b	98	c	99	a	100	b
101	a	102	a	103	a	104	a	105	c
106	d	107	c	108	b	109	c	110	a
111	b	112	c	113	b	114	d	115	d

Vector or Cross product of two vectors and its applications

1	d	2	c	3	b	4	a	5	c
6	c	7	b	8	c	9	b	10	d
11	a	12	c	13	b	14	a	15	c
16	a,c	17	b	18	a	19	c	20	d
21	a	22	c	23	c	24	c	25	a
26	d	27	b	28	a	29	c	30	d
31	b	32	c	33	a	34	b	35	b
36	b	37	c	38	a	39	b	40	c
41	b	42	d	43	a	44	a	45	c
46	b	47	b	48	b	49	a	50	b
51	c	52	c	53	a	54	c	55	b
56	b	57	a	58	d	59	b	60	b
61	b	62	b	63	c	64	c	65	b
66	d	67	c	68	c	69	c	70	c
71	c	72	b	73	b	74	c	75	d
76	a	77	a	78	a	79	d	80	c
81	d	82	b	83	b	84	d	85	a
86	c	87	c	88	d				

Scalar triple product and their applications

1	a	2	b	3	c	4	c	5	a
6	d	7	d	8	a	9	a	10	c
11	b	12	c	13	a	14	b	15	d
16	c	17	a	18	a	19	b	20	d
21	b	22	d	23	a	24	c	25	d
26	b	27	c,d	28	c	29	c	30	d
31	c	32	a,c	33	b	34	b	35	c
36	b	37	a	38	b	39	a	40	d
41	d	42	c	43	b	44	b	45	c
46	a	47	d	48	d	49	b	50	d
51	c	52	b	53	c	54	c	55	d
56	c	57	c	58	d	59	c	60	d

61	c	62	b	63	a				

Vector triple product

1	a	2	c	3	a	4	d	5	b
6	c	7	d	8	a	9	b	10	d
11	b	12	d	13	c	14	b	15	b
16	a	17	a	18	a	19	a	20	c
21	d	22	d	23	d	24	c		

Application of vectors in three dimensional geometry

1	a	2	a	3	d	4	a	5	b
6	c	7	b	8	c	9	c	10	a
11	b	12	b	13	a	14	b	15	d
16	c	17	b	18	c	19	b	20	a
21	b	22	a	23	b	24	c	25	a
26	c	27	b	28	d	29	d	30	d
31	a	32	b	33	b	34	a	35	c
36	d	37	c	38	c	39	d	40	c
41	a	42	d	43	b	44	a	45	b
46	a	47	d						

Critical Thinking Questions

1	c	2	b	3	b	4	c	5	b
6	b	7	e	8	b	9	d	10	b
11	b	12	d	13	c	14	d	15	a,c
16	b	17	b	18	d	19	b	20	a
21	a	22	c	23	a	24	d	25	c
26	b	27	c	28	c	29	a	30	c
31	c	32	a,c	33	d	34	c	35	a
36	b	37	b,c	38	a	39	b	40	a