Answers and Solutions

Expansion of determinants, Solution of equation in the form of determinants and properties of determinants

1. (b) $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0$

[by $C_1 \to C_1 + C_2 + C_3$]

2. (a) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2-ab \end{vmatrix}$

by $\begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2-ab \end{vmatrix} = 0, \{\because R_1 \equiv R_2\}.$

3. (d) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix} = xy,$

$\begin{pmatrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{pmatrix}.$

4. (c) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$, by $\begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$, by $R_1 \to R_1 - R_2$

$= (a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

$= (a-b)(b-c)(a-c).(-1) = (a-b)(b-c)(c-a).$

5. (b) $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 6 & 15 \\ 0 & -2-2x & 5(1-x^2) \\ 1 & 2x & 5x^2 \end{vmatrix} = 0 \quad \begin{pmatrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{pmatrix}$

$\Rightarrow 3.2.5.\begin{vmatrix} 0 & 1 & 1 \\ 0 & -(1+x) & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$

$\Rightarrow (1+x)\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1-x \\ 1 & x & x^2 \end{vmatrix} = 0$

$\Rightarrow x+1=0$ or $x-2=0 \Rightarrow x=-1, 2.$

Trick: Obviously by inspection, $x=-1, 2$ satisfy the equation.

At $x=-1$, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & -2 & 5 \end{vmatrix} = 0$ as $R_2 \equiv R_3$

At $x=2$, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 4 & 20 \end{vmatrix} = 0$ as $R_1 \equiv R_3$.

6. (d) $\Delta = \begin{vmatrix} 1 & 5 & \pi \\ 1 & 5 & \sqrt{5} \\ 1 & 5 & e \end{vmatrix} = 0 \quad (\because \log_a a = 1 \text{ and } 5C_1 \equiv C_2)$

7. (a) Obviously, on putting $x=0$, we observe that the determinant becomes

$\Delta_{x=0} = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = a(bc) - b(ac) = 0$

$\therefore x=0$ is a root of the given equation.

Aliter : Expanding Δ, we get

$\Delta \equiv -(x-a)[-(x+b)(x-c)] + (x-b)[(x+a)(x+c)] = 0$

$\Rightarrow 2x^3 - (2\Sigma ab)x = 0$

$\Rightarrow$ Either $x=0$ or $x^2 = \sum ab$ (*i.e.*, $x = \pm\sqrt{\sum ab}$)

Again $x=0$ satisfies the given equation.

8. (a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 4 & 3 & 6 \end{vmatrix}$ by $C_1 \to C_1 + C_2$

$= \begin{vmatrix} 1 & 2 & 1 \\ 1 & 5 & 3 \\ 1 & 9 & 6 \end{vmatrix}$, by $C_2 \to C_2 + C_3$

$= \begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 3 \\ 10 & 3 & 6 \end{vmatrix}$, by $C_1 \to C_1 + C_2 + C_3$.

But $\neq \begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}$.

9. (b) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0.$

10. (c) $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}=0$

$\Rightarrow (x-\sum a)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}=0$

$\Rightarrow x=\sum a=0$ (by hypothesis)

or

$1 \quad \{(b-x)(c-x)-a^2\}-c\{c-x-a\}+b\{a-b+x\}=0$

by expanding the determinant.

or $x^2-(a^2+b^2+c^2)+(ab+bc+ca)=0$

or $x^2-\left(\sum a^2\right)-\frac{1}{2}\left(\sum a^2\right)=0$

$\left\{\because a+b+c=0 \Rightarrow (a+b+c)^2=0\right.$

$\left.\Rightarrow \sum a^2+2\sum ab=0 \Rightarrow \sum ab=-\frac{1}{2}\sum a^2\right\}$

or $x=\pm\sqrt{\frac{3}{2}\sum a^2}$

$\therefore$ The solution is $x=0$ or $\pm\sqrt{\frac{3}{2}\sum a^2}$.

Trick: Put $a=1, b=-1$ and $c=0$ so that they satisfy the condition $a+b+c=0$. Now the determinant becomes $\begin{vmatrix} 1-x & 0 & -1 \\ 0 & -1-x & 1 \\ -1 & 1 & -x \end{vmatrix}=0$

$\Rightarrow (1-x)\{x(1+x)-1\}+1(1+x)=0$

$\Rightarrow (1-x)\{x^2+x-1\}+x+1=0 \quad \Rightarrow x(x^2-3)=0$

Now putting these in the options, we find that option (c) gives the same values *i.e.,* 0, $\pm\sqrt{3}$.

11. (b) $\Delta=(2+i)\begin{vmatrix} 1 & 1 & i \\ 1 & 1+2i & 1+i \\ 1 & 2 & 1-i \end{vmatrix}$

$=(2+i)\begin{vmatrix} 0 & -2i & -1 \\ 0 & -1+2i & 2i \\ 1 & 2 & 1-i \end{vmatrix}$ by $\begin{matrix} R_1 \to R_1-R_2 \\ R_2 \to R_2-R_3 \end{matrix}$

$=(2+i)\{-4i^2+(-1+2i)\}=(2+i)(4-1+2i)$

$=(2+i)(3+2i)=4+7i$.

12. (d) By $C_1 \to C_1+C_2+C_3$,

we have $(9+x)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}=0$

$\Rightarrow (x+9)\begin{vmatrix} 0 & 1-x & 0 \\ 0 & -(1-x) & 1-x \\ 1 & 3 & x+4 \end{vmatrix}=0$

$\Rightarrow (x+9)\,(1-x)^2\begin{vmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 3 & x+4 \end{vmatrix}=0$

$\Rightarrow x=1,1,-9$, (Since the determinant = 1).

13. (b) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$=\begin{vmatrix} -\Sigma a & 0 & 2a \\ \Sigma a & -\Sigma a & 2b \\ 0 & \Sigma a & c-a-b \end{vmatrix}$, $\begin{pmatrix} C_1 \to C_1-C_2 \\ C_2 \to C_2-C_3 \end{pmatrix}$

$=(\Sigma a)^2\begin{vmatrix} -1 & 0 & 2a \\ 1 & -1 & 2b \\ 1 & 1 & c-a-b \end{vmatrix}=(\Sigma a)^3$, (on expansion)

$=(a+b+c)^3$.

14. (d) $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix}=\begin{vmatrix} a+b & a+2b & a+3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix}=0$

$\left\{\text{by } \begin{matrix} R_2 \to R_2-R_1 \\ R_3 \to R_3-R_2 \end{matrix}\right\}$

Trick: Putting $a=1=b$. The determinant will be $\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix}=0$. Obviously answer is (d)

Note : Students remember while taking the values of a, b, c,....... that for there values, the options (a), (b), (c) and (d) should not be identical.

15. (d) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}=\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

{by $R_1 \to R_1-(R_2+R_3)$}

$=2c.b(a+b-c)-2b.c(b-c-a)=4abc$.

16. (b) $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} 3+x & 0 & 1 \\ 3+x & x & 1 \\ 3+x & -x & 1+x \end{vmatrix}=0$, $\begin{pmatrix} C_1 \to C_1+C_2+C_3 \\ C_2 \to C_2-C_3 \end{pmatrix}$

$\Rightarrow (x+3)\begin{vmatrix} 1 & 0 & 1 \\ 1 & x & 1 \\ 1 & -x & 1+x \end{vmatrix}=0$

$\Rightarrow (x+3)\begin{vmatrix} 1 & 0 & 1 \\ 0 & x & 0 \\ 0 & -x & x \end{vmatrix} = 0$, $\begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{pmatrix}$

$\Rightarrow (x+3)x^2 = 0 \Rightarrow x = 0, 0, -3$.

Trick : Obviously the equation is of degree three, therefore it must have three solutions. So check for option (b).

17. (d) $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+c & a \\ 1 & a & x+b \end{vmatrix} = 0$, $(C_1 \to C_1 + C_2 + C_3)$

$\Rightarrow$ $x = -(a+b+c)$ is one of the root of the equation.

18. (b) $\Delta = \begin{vmatrix} -1 & -2 & x+4 \\ -2 & -3 & x+8 \\ -3 & -4 & x+14 \end{vmatrix}$, by $\begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$

$= \begin{bmatrix} -1 & -1 & x \\ -2 & -1 & x \\ -3 & -1 & x+2 \end{bmatrix}$, by $\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 + 4C_1 \end{matrix}$

$= -(-x-2+x) + 1.(-2x-4+3x) + x(2-3)$

$= 2 + x - 4 - x = -2$.

Trick : Put $x = 1$. Then $\begin{vmatrix} 2 & 3 & 5 \\ 4 & 6 & 9 \\ 8 & 11 & 15 \end{vmatrix} = -2$

Note : Since there is a option "None of these", therefore we should check for one more different value of *x*. Put $x = -1$.

$\begin{vmatrix} 0 & 1 & 3 \\ 2 & 4 & 7 \\ 6 & 9 & 13 \end{vmatrix} = -1(26-42) + 3(18-24) = -2$

Therefore answer is (b).

19. (a) $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} = 1(1+c^2) - a(-a+bc) + b(ac+b)$

$= 1 + a^2 + b^2 + c^2$.

20. (c) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ vanishes when $a = b, b = c, c = a$.

Hence $(a-b), (b-c), (c-a)$ are factors of Δ. Since Δ is symmetric in a, b, c and of 4th degree, $(a+b+c)$ is also a factor, so that we can write

$\Delta = k(a-b)(b-c)(c-a)(a+b+c)$ ......(i)

Where by comparing the coefficients of the leading term bc^3 on both the sides of identity (i). We get $1 = k(-1)(-1) \Rightarrow k = 1$

$\therefore \Delta = (a-b)(b-c)(c-a)(a+b+c)$.

Trick : Put $a = 1, b = 2, c = 3$, so that determinant

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 8 & 27 \end{vmatrix} = 1(30) - 1(24) + 1(8-2) = 12$ which is given by (c). *i.e.* $(1+2+3)(1-2)(2-3)(3-1) = 12$.

21. (c) $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0$ (Since value of determinant of skew-symmetric matrix of odd orders is 0).

22. (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$, $(R_1 \to R_1 + R_2 + R_3)$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & b-c & c-a \\ c & c-a & a-b \end{vmatrix}$

$= 3abc - a^3 - b^3 - c^3$, (After simplification).

23. (c) $\Delta = (b-a)(b-a).\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$

$= (a-b)^2\begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} = 0$, [by $C_2 \to C_2 + C_3$].

24. (d) $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix} = 0$

25. (c) $\Delta = \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

$= -2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$, by $R_1 \to R_1 - (R_2 + R_3)$

$= -2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}$, by $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= -2\{-c^2(b^2a^2) + b^2(-c^2a^2)\} = 4a^2b^2c^2$.

Trick: Put $a = 1, b = 2, c = 3$, so that the option give different values.

26. (a) $\Delta = xyz\begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z} \end{vmatrix}$

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix}1 & 1 & 1\\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y}\\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z}\end{vmatrix},$$

by $R_1 \to R_1 + R_2 + R_3$

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix}1 & 0 & 0\\ 1/y & 1 & 0\\ 1/z & 0 & 1\end{vmatrix}, \text{ by}$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right).$$

Trick: Put $x=1, y=2$ and $z=3$, then

$$\begin{vmatrix}2 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 4\end{vmatrix} = 2(11)-1(3)+1(1-3)=17$$

Option (a) gives, $1\times 2\times 3\left(1+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right)=17$.

27. (d) $\Delta = \begin{vmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{vmatrix}$

$$= \begin{vmatrix}x+1+\omega+\omega^2 & \omega & \omega^2\\ x+1+\omega+\omega^2 & x+\omega^2 & 1\\ x+1+\omega+\omega^2 & 1 & x+\omega\end{vmatrix}, (C_1 \to C_1 + C_2 + C_3)$$

$$= x\begin{vmatrix}1 & \omega & \omega^2\\ 1 & x+\omega^2 & 1\\ 1 & 1 & x+\omega\end{vmatrix} \qquad (\because 1+\omega+\omega^2=0)$$

$$= x[1\{(x+\omega^2)(x+\omega)-1\}+\omega\{1-(x+\omega)\} + \omega^2\{1-(x+\omega^2)\}]$$

$$= x(x^2+\omega x+\omega^2 x+\omega^3-1+\omega-\omega x-\omega^2 + \omega^2-\omega^2 x-\omega^4)$$

$$= x^3, \quad (\because \omega^3=1).$$

28. (b) $\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix} = (x+y+z)\begin{vmatrix}2 & 1 & 1\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$

by $R_1 \to R_1 + R_2 + R_3$

$$= (x+y+z)\begin{vmatrix}1 & 1 & 1\\ x & z & x\\ x & y & z\end{vmatrix}; \text{ by } C_1 \to C_1 - C_2$$

$$= (x+y+z).\{(z^2-xy)-(xz-x^2)+(xy-xz)\}$$

$$= (x+y+z)(x-z)^2 \Rightarrow k=1.$$

Trick : Put $x=1, y=2$, $z=3$, then

$$\begin{vmatrix}5 & 1 & 2\\ 4 & 3 & 1\\ 3 & 2 & 3\end{vmatrix} = 5(7)-1(12-3)+2(8-9)$$

$= 35-9-2=24$ & $(x+y+z)(x-z)^2=(6)(-2)^2=24$

$\therefore k=\frac{24}{24}=1.$

29. (a) $\begin{vmatrix}x & 3 & 7\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}=0 \Rightarrow (x+9)\begin{vmatrix}1 & 1 & 1\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}=0,$

by $R_1 \to R_1 + R_2 + R_3$

$\Rightarrow (x+9)\{(x^2-12)-(2x-14)+(12-7x)\}=0$

$\Rightarrow (x+9)(x^2-9x+14)=0$

$\Rightarrow (x+9)(x-2)(x-7)=0$

Hence the other two roots are $x=2,7$.

30. (d) It is obvious.

31. (b) $\Delta = \begin{vmatrix}2(a+b+c) & 0 & a+b+c\\ c+a & b-c & b\\ a+b & c-a & c\end{vmatrix}$

by $R_1 \to R_1 + R_2 + R_3$

$$\Delta = (a+b+c).\begin{vmatrix}2 & 0 & 1\\ c+a & b-c & b\\ a+b & c-a & c\end{vmatrix}$$

On expanding,

$-(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -(a^3+b^3+c^3-3abc)=3abc-a^3-b^3-c^3$.

Trick : Put $a=1, b=2, c=3$ and check it.

32. (a) Splitting the determinant into two determinants, we get

$$\Delta = \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix} + abc\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=0$$

$= (1+abc)[(a-b)(b-c)(c-a)]=0$

Because a, b, c are different, the second factor cannot be zero. Hence, option (a), $1+abc=0$, is correct.

33. (a) $\Delta \equiv \begin{vmatrix}2 & 2\omega & -\omega^2\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix} = \begin{vmatrix}2+2\omega+2\omega^2 & 2\omega & -\omega^2\\ 1+1-2 & 1 & 1\\ 1-1-0 & -1 & 0\end{vmatrix}$

$(C_1 \to C_1 + C_2 - 2C_3)$

$$= \begin{vmatrix}0 & 2\omega & -\omega^2\\ 0 & 1 & 1\\ 0 & -1 & 0\end{vmatrix}=0.$$

34. (a) $\begin{vmatrix}19 & 17 & 15\\ 9 & 8 & 7\\ 1 & 1 & 1\end{vmatrix}=19-34+15=0.$

35. (a) As given $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix} = 0$

$= \begin{vmatrix} -1 & -1 & x+3 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0$, by $\begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$

$\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0$, by $R_1 \to R_1 - R_2$

$\Rightarrow (-1)(-b+c+a-b) = 0$

$\Rightarrow 2b-a-c=0 \Rightarrow a+c=2b$ *i.e.*, a, b, c are in A.P.

Trick : In such type of problem, put any suitable value of x *i.e.* 0, so that the determinant.

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ a & b & c \end{vmatrix} = 0$

$\Rightarrow 1(3c-4b) - 2(2c-4a) + 3(2b-3a) = 0$

$\Rightarrow -c+2b-a=0 \Rightarrow 2b = a+c$. Hence the result.

36. (a) $\begin{vmatrix} 1 & \omega & -\omega^2/2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\frac{1}{2}\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$

$= -\frac{1}{2}\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} = 0$,(Apply $C_1 \to C_1 + C_2 + C_3$).

37. (b) Since it is an identity in λ so satisfied by every value of λ. Now put $\lambda = 0$ in the given equation, we have

$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -4 \\ -3 & 4 & 0 \end{vmatrix} = -12 + 30 = 18$.

38. (d) $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 5 & 0 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ {Apply $R_1 \to R_1 + R_3$}

$= -5(1+4) = -25$.

39. (c) $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$

$(C_2 \to C_2 + C_3)$

$= 0$, $(\because C_1 \equiv C_2)$.

40. (d) Multiplying R_1 by a, R_2 by b and R_3 by c, we have $\Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ a^2bc^2 & abc & bc+ab \\ a^2b^2c & abc & ac+bc \end{vmatrix}$

$= \frac{a^2b^2c^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ac & 1 & bc+ab \\ ab & 1 & ac+bc \end{vmatrix} = abc\begin{vmatrix} bc & 1 & \Sigma ab \\ ac & 1 & \Sigma ab \\ ab & 1 & \Sigma ab \end{vmatrix}$

{by $C_3 \to C_3 + C_1$}

$= abc\Sigma ab\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} = 0$, [Since $C_2 \equiv C_3$].

Trick : Put $a=1, b=2, c=3$ and check it.

41. (b) $\Delta \equiv \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

$= \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -(a\alpha^2+2b\alpha+c) \end{vmatrix}$, by $R_3 \to R_3 - \alpha R_1 - R_2$

$= a\{-c(a\alpha^2+2b\alpha+c)-0\} - b\{-b(a\alpha^2+2b\alpha+c)-0\}$

by expanding along C_1

$= (b^2-ac)(a\alpha^2+2b\alpha+c)$

Thus, $\Delta = 0$, if either $b^2-ac=0$ or $a\alpha^2+2b\alpha+c=0$

i.e., a, b, c in G.P. or $a\alpha^2+2b\alpha+c=0$.

Trick: Put $\alpha = 0$, then the determinant

$\begin{vmatrix} a & b & b \\ b & c & c \\ b & c & 0 \end{vmatrix} = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ b & c & -c \end{vmatrix} = -c(ac-b^2) = 0$. Hence the result.

42. (b) $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix} = \begin{vmatrix} 31 & 37 & 92 \\ 0 & 21 & -21 \\ 0 & 47 & -47 \end{vmatrix}$;by $\begin{matrix} R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1 \end{matrix}$

$= \begin{vmatrix} 31 & 129 & 92 \\ 0 & 0 & -21 \\ 0 & 0 & -47 \end{vmatrix} = 0$; (by $C_2 \to C_2 + C_3$).

43. (c) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix} = 2 - 8 + 6 = 0$.

44. (d) $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0 \Rightarrow k = \frac{33}{8}$.

45. (d) $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b+c-a & c+a-b & a+b-c \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 1 \\ b-a & c-b & a+b \\ 2(b-a) & 2(c-b) & a+b-c \end{vmatrix} = 0$.

46. (d) $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix} = k^3\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = k^3\Delta$.

47. (d) $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix} = \begin{vmatrix} a & a & bc \\ b & b & ca \\ c & c & ab \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$= -\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = -\begin{vmatrix} a & a^2 & 1 \\ b-a & b^2-a^2 & 0 \\ c-a & c^2-a^2 & 0 \end{vmatrix}$

[By $R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$]

$= -(a-b)(b-c)(c-a)$.

48. (a) $\begin{vmatrix} a_1 & ma_1 & b_1 \\ a_2 & ma_2 & b_2 \\ a_3 & ma_3 & b_3 \end{vmatrix} = m\begin{vmatrix} a_1 & a_1 & b_1 \\ a_2 & a_2 & b_2 \\ a_3 & a_3 & b_3 \end{vmatrix} = 0$, $\{\because C_1 \equiv C_2\}$.

49. (a) $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix} = \begin{vmatrix} 25 & 21 & 219 \\ 15 & 27 & 198 \\ 21 & 17 & 181 \end{vmatrix}$

{Applying $C_1 \to C_1 - C_2; C_2 \to C_2 - C_3$}

$= \begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 4 & 17 & 11 \end{vmatrix}$

{Applying $C_1 \to C_1 - C_2; C_3 \to C_3 - 10C_2$}

$= \begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 0 & -4 & 2 \end{vmatrix}$ {Applying $R_3 \to R_3 - R_1$}

$= 4\begin{vmatrix} 1 & 21 & 9 \\ -3 & 27 & -72 \\ 0 & -4 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & 21 & 9 \\ 0 & 90 & -45 \\ 0 & -4 & 2 \end{vmatrix}$ by $R_2 \to 3R_1 + R_2$

$= 4(90 \times 2 - 45 \times 4) = 0$.

50. (b) **Trick :** Put $x = 1$, then we have

$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = A - 12 \Rightarrow \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 5 & 1 & 1 \end{vmatrix} = A - 12$

{Apply $C_1 \to C_1 - C_2$ **}**

$\Rightarrow -2 + (-1)(-14) = A - 12 \Rightarrow A = 24$.

51. (a) We first operating $R_3 - 2R_2$ and $R_2 - 3R_1$ in given determinant, then we get

$= a[a^2 + ab - 2a^2 - ab] = -a^3 = i$.

52. (b) $\begin{vmatrix} 2 & 8 & 4 \\ -5 & 6 & -10 \\ 1 & 7 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 8 & 2 \\ -5 & 6 & -5 \\ 1 & 7 & 1 \end{vmatrix} = 0$

(Taking 2 common from C_3).

53. (b) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

$\Rightarrow 6i(-3+3) + 3i(4i+20) + 1(12-60i) = x + iy$

$\Rightarrow 0 + 60i - 12 + 12 - 60i = x + iy \Rightarrow x = 0, y = 0$.

54. (b) $\Delta = \frac{1}{abc}\begin{vmatrix} a^3+ax & a^2b & a^2c \\ ab^2 & b^3+bx & b^2c \\ c^2a & c^2b & c^3+cx \end{vmatrix}$

$= \begin{vmatrix} a^2+x & a^2 & a^2 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$

$= (a^2+b^2+c^2+x) \times \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$

{Applying $R_1 \to R_1 + R_2 + R_3$}

$= (a^2+b^2+c^2+x)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & x & 0 \\ c^2 & 0 & x \end{vmatrix} \begin{Bmatrix} \text{Applying } C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{Bmatrix}$

$= x^2(a^2+b^2+c^2+x)$.

Hence Δ is divisible by x^2 as well as by x.

55. (a) We have $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$

$= pqr(a^3+b^3+c^3) - abc(p^3+q^3+r^3)$

$= pqr(3abc) - abc(3pqr) = 0$,

$\begin{pmatrix} \because p+q+r=0, \therefore p^3+q^3+r^3 = 3pqr \\ \because a+b+c=0, \therefore a^3+b^3+c^3 = 3abc \end{pmatrix}$.

56. (d) $D' = D + pqrD = D(1+pqr)$.

Trick : Check by putting $a_1 = b_2 = c_3 = 1$ and all other zero.

57. (b) $\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$

By expanding, we get $-x(x^2 - 144) = 0$

$x = 0$ or $x^2 = 144 \Rightarrow x = \pm 12$

So, $x = 0, 12, -12$.

58. (c) Since $x = \frac{5}{2}$ satisfies the given determinant.

59. (b) The determinant can be written sum of $2 \times 2 \times 2 = 8$ determinants of which 6 are reduces to zero because of their two rows are identical. Hence proceed.

60. (a) Since determinant of a skew-symmetric matrix of odd order is zero.

61. (a) Apply $R_2 - R_3$ and note that $(x+y)^2 - (x-y)^2 = 4xy$

$\therefore \Delta = 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} \quad \{\text{Applying } R_3-(R_1-2R_2)\}.$$

62. (b) Apply $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$.

63. (d) Apply $C_1 \to C_1 + C_3$ and take $x+y+z$ common from C_1 and 4 from C_2 to make first two columns identical.

64. (d) Apply $R_1 \to R_1 + R_2$ and then expand along R_1.

65. (c) Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 3 \\ 1 & 3 & -6-x \end{vmatrix} = 0$$

$$\Rightarrow -x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

$$\Rightarrow -x(9-x)(-9-x) = 0 \Rightarrow x = 0, 9, -9.$$

Trick : Check by assuming the values of x from the given options.

66. (a) Applying $C_1 \to C_1 + C_2$, we get $\begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix} = 0$.

67. (b) We have $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0,$$

$\{\text{Applying } C_1 \to C_1 + C_2 + C_3\}$

$$\Rightarrow (x+1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+1)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0$$

$\{\text{Applying } R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$

$$\Rightarrow (x+1)(x-2)^2 = 0 \Rightarrow x = -1, 2.$$

68. (c) **Trick :** Put $a = 1, b = -1, c = 0$

$a' = 2, b' = 2, c' = 1$

Then the determinant is $\begin{vmatrix} 0 & -1 & 2 \\ 0 & 1 & 2 \\ -1 & 0 & 4 \end{vmatrix} = 4$

Option (c) also gives the same value.

69. (a) Obviously, the determinant is satisfied for $x = a, b$.

70. (a) We have $2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2-bc & b^2-ac & c^2-ab \end{vmatrix}$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} - \frac{2}{abc}\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ abc & abc & abc \end{vmatrix}$$

Applying $C_1(a), C_2(b), C_3(c)$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} - \frac{2}{abc}(abc)\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

71. (d) $D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix}, D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}$

$$D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix}, D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}$$

$$D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$$

$$D_1 + D_2 + D_3 + D_4 + D_5 = \begin{vmatrix} 15 & 75 & 40 \\ 55 & 175 & 45 \\ 225 & 125 & 50 \end{vmatrix}$$

$$= 15(3125) - 75(-7375) + 40(-32500)$$

$$= 46875 + 553125 - 1300000 = -700000.$$

72. (b) Operating $C_1 \to C_1 + C_2 + C_3$. We get the value of given determinant as

$$\begin{vmatrix} 3a+3b & a+b & a+2b \\ 3a+3b & a & a+b \\ 3a+3b & a+2b & a \end{vmatrix}$$

$$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix}$$

Operate $R_3 \to R_3 - R_1$, $R_2 \to R_2 - R_1$

$$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$$

$$= 3(a+b)(2b^2 + b^2) = 9b^2(a+b).$$

73. (b) $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0 \quad \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$

$$abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$(abc-1)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

Since a, b, c are different, so $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$

Hence $abc-1=0$ *i.e.*, $abc=1$.

74. (c) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

$$=(abc)(abc)\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = a^2b^2c^2(-1)(-4)$$

$=4a^2b^2c^2 = Ka^2b^2c^2$, (given) $K=4$.

75. (b) Applying $C_3 \to C_3 - C_1$ and $C_2 \to C_2 - C_1$, we get $\begin{vmatrix} 1 & ac & bc \\ 1 & ad & bd \\ 1 & ae & be \end{vmatrix} = ab\begin{vmatrix} 1 & c & c \\ 1 & d & d \\ 1 & e & e \end{vmatrix} = 0$, $\{\because C_2 \equiv C_3\}$.

76. (d) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -x & x \\ 1 & 0 & y \end{vmatrix} = -xy.$

77. (a) Applying $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$, we get $\begin{vmatrix} 13 & 3 & 3 \\ 14 & 3 & 3 \\ 15 & 3 & 3 \end{vmatrix} = 0$, $\{\because C_2 \equiv C_3\}$.

78. (b) $\begin{vmatrix} a & b & a\alpha-b \\ b & c & b\alpha-c \\ 2 & 1 & 0 \end{vmatrix} = 0$

$$a[-(b\alpha-c)]-b[-2(b\alpha-c)]+[a\alpha-b)(b-2c)]=0$$

$$-ab\alpha+ac+2b^2\alpha-2bc+ab\alpha-2ac\alpha-b^2+2bc=0$$

$$ac+2b^2\alpha-2ac\alpha-b^2=0$$

$$(ac-b^2)-2\alpha(ac-b^2)=0$$

$ac-b^2=0$ or $1-2\alpha=0 \Rightarrow b^2=ac$ or $\alpha=\dfrac{1}{2}$

$\because \alpha \neq \dfrac{1}{2}$ (As given in question)

So, $b^2=ac$ *i.e*, a, b, c are in G.P.

79. (c) $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$

$$=\begin{vmatrix} 17 & 9 \\ 9 & 5 \end{vmatrix} = (3x-2)-(x+6)$$

$85-81=2x-8 \quad 4+8=2x \quad x=6$.

80. (b) $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$, we get

$$(3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$(3x-2)\begin{vmatrix} 0 & -3x+11 & 0 \\ 0 & 3x-11 & -3x+11 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$(3x-2)\left[(-3x+11)^2\right]=0$$

$x=\dfrac{2}{3}$ or $x=\dfrac{11}{3} \Rightarrow x=\dfrac{2}{3}, \dfrac{11}{3}$.

81. (d) Let $A=\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, we get,

$$A=\begin{vmatrix} x+2 & 1 & x+a \\ x+4 & 1 & x+b \\ x+6 & 1 & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$A=\begin{vmatrix} x+2 & 1 & x+a \\ 2 & 0 & b-a \\ 4 & 0 & c-a \end{vmatrix} = -1(2c-2a-4b+4a)$$

$= 2(2b-c-a)$

$\because$ a, b, c are in A.P. $A=0$.

82. (b) Let $\Delta_1 = \begin{vmatrix} x & 2y & z \\ 2p & 4q & 2r \\ a & 2b & c \end{vmatrix} = 4\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = 4\Delta$.

(Taking common '2' from IInd row and '2' from IInd column).

83. (c) $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a-6 & 0 & 0 \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$ $[R_1 \to R_1 - 2R_2]$

$$(a-6)(b^2-ac)=0 \Rightarrow b^2-ac=0, \ (\because a \neq 6)$$

$\therefore$ $ac=b^2 \Rightarrow abc=b^3$.

84. (b) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$,

$$\begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix}$$

On expanding *w.r.t.* R_3,

$ab+bc+ca+abc=\lambda$ ....(i)

Given, $a^{-1}+b^{-1}+c^{-1}=0$

$\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=0 \quad ab+bc+ca=0$

$\lambda=abc$, (From equation (i)).

85. (d) $\begin{vmatrix} a^2+x^2 & ab & ca \\ ab & b^2+x^2 & bc \\ ca & bc & c^2+x^2 \end{vmatrix}$

Multiply C_1, C_2, C_3 by a, b, c respectively and hence divide by abc

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+x^2) & ab^2 & c^2a \\ a^2b & b(b^2+x^2) & bc^2 \\ ca^2 & b^2c & c(c^2+x^2) \end{vmatrix}$$

Now take out a, b and c common from R_1, R_2 and R_3,

$$\Delta = \begin{vmatrix} a^2+x^2 & b^2 & c^2 \\ a^2 & b^2+x^2 & c^2 \\ a^2 & b^2 & c^2+x^2 \end{vmatrix}$$

Now applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = (a^2+b^2+c^2+x^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+x^2 & c^2 \\ 1 & b^2 & c^2+x^2 \end{vmatrix}$$

$$\Delta = x^4(a^2+b^2+c^2+x^2)$$

Hence, it is divisible by x^2.

86. (b) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$

Applying $C_1 \to C_1 + C_3 - (2\cos x)C_2$

$$\Delta = \begin{vmatrix} 2(1-\cos x) & 1 & 1 \\ 0 & \cos(n+1)x & \cos(n+2)x \\ 0 & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

$$\Delta = 2(1-\cos x)[\cos(n+1)x\sin(n+2)x - \cos(n+2)x\sin(n+1)x]$$

$$\Delta = 2(1-\cos x)[\sin(n+2-n-1)x] = 2\sin x(1-\cos x)$$

i.e., Δ is independent of n.

87. (c) We know that the row to row multiplication of a determinant is always equal to the value of the determinant *i.e.,* $|A|$.

88. (c) $\Delta = 1[100-98] + 2[56-60] + 3[42-40]$

$\Delta = 2 - 8 + 6 = 0$.

89. (b) Let a, b, c are in G.P. and assume $a = 1, b = 2, c = 4$

$$\therefore A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 0 \end{vmatrix} = 0.$$

90. (a) Applying $C_1 \to C_1 + C_2 + C_3$

$$(a+b+c-x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$$

$$(a+b+c-x)[\{(b-x)(c-x)-a^2\} + c(a-c+x)\} + \{b(a-b+x)\}] = 0$$

$$(a+b+c-x)[bc - cx - bx + x^2 - a^2 + ca - c^2 + cx + ab - b^2 + bx] = 0$$

$$(a+b+c-x)[x^2 - (a^2+b^2+c^2) + ab + bc + ca] = 0$$

$$(a+b+c-x)[x^2 - (a^2+b^2+c^2)] = 0$$

$x = a+b+c$ and $(a^2+b^2+c^2)^{1/2}$.

91. (d) $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} = 2 \times 3\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = 6k.$

92. (b) B is obtained from A by the operations $R_1 \leftrightarrow R_3$, $R_3 \to 2R_3$ and $R_2 \to 2R_2$.

Hence, $B = (-1)4A = -4A$.

93. (b) Required determinant

$|adj A| = |A|^{3-1}$, where $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$= 5^2 = 25$, $(\because |adj A| = |a|^{n-1})$

94. (b) Operating $C_1 \to C_1 + C_2 + C_3$ we find that $x + a + b + c$ is a factor.

95. (b) Taking out 5 from R_2 makes $R_2 = R_1$.

96. (a) $\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$

Check at $x = 0$, we get $\begin{vmatrix} \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \\ 1 & \omega & \omega^2 \end{vmatrix}$

$= \omega^2(\omega^4 - \omega) - \omega(\omega^3 - 1) + 1(\omega^2 - \omega^2)$

$= \omega^2(\omega - \omega) - \omega(1-1) + 0 = 0$ Or

$$\Delta = \begin{vmatrix} 1+\omega+\omega^2+x & \omega & 1 \\ 1+\omega+\omega^2+x & \omega^2 & 1+x \\ 1+\omega+\omega^2+x & x+\omega & \omega^2 \end{vmatrix}$$

by $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x & \omega & 1 \\ x & \omega^2 & 1+x \\ x & x+\omega & \omega^2 \end{vmatrix}, \ (\because 1+\omega+\omega^2 = 0)$$

$= 0$, if $x = 0$.

97. (b) $\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1-\omega^2 & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$ $(C_1 \to C_1 + C_2 + C_3)$

$(\because 1+\omega+\omega^2 = 0)$

$= 3[\omega.\omega - \omega^4] = 3(\omega^2 - \omega) = 3\omega(\omega - 1)$.

98. (c) Operate $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$ and take out $a+b+c$ from C_2 as well as from C_3 to get

$$\Delta = (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

(Operate $R_1 \to R_1 - R_2 - R_3$)

$$= (a+b+c)^2 \begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

(Operate $C_2 \to C_2 + \frac{1}{b}C_1$ and $C_3 \to C_3 + \frac{1}{c}C_1$)

$$= (a+b+c)^2 \begin{vmatrix} 2bc & 0 & 0 \\ b^2 & c+a & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$

$= (a+b+c)^2[2bc\{(a+b)(c+a)-bc\}] = 2abc(a+b+c)^3$.

99. (c) Given, $\Delta = \begin{vmatrix} 41 & 42 & 43 \\ 44 & 45 & 46 \\ 47 & 48 & 49 \end{vmatrix}$.

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$ we get

$\Delta = \begin{vmatrix} 41 & 1 & 1 \\ 44 & 1 & 1 \\ 47 & 1 & 1 \end{vmatrix}$. Since two columns (C_2 and C_3) are identical, therefore $\Delta = 0$.

100. (b) Given, Angles of a triangle = A, B and C. We know that as $A + B + C =$, therefore $A + B = \pi - C$

or $\cos(A+B) = \cos(\pi - C) = -\cos C$

or $\cos A \cos B - \sin A \sin B = -\cos C$

$\cos A \cos B + \cos C = \sin A \sin B$

and $\sin(A+B) = \sin(\pi - C) = \sin C$.

Expanding the given determinant, we get

$\Delta = -(1-\cos^2 A) + \cos C(\cos C + \cos A \cos B)$
$+ \cos B(\cos B + \cos A \cos C)$

$= -\sin^2 A + \cos C(\sin A \sin B) + \cos B(\sin A \sin C)$

$= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C)$

$= -\sin^2 A + \sin A \sin(B+C) = -\sin^2 A + \sin^2 A = 0$.

101. (a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 3(\omega - \omega^2)$

$= 3\left[\frac{-1+\sqrt{3}i}{2} - \frac{-1-\sqrt{3}i}{2}\right] = 3\sqrt{3}\, i$.

102. (a) $\Delta = \frac{1}{a}[ab - ca] + 1\left[ca\frac{1}{c} - \frac{1}{b}.ab\right] + bc\left[\frac{1}{b} - \frac{1}{c}\right]$

$\Delta = (b-c) + 1(a-a) + (c-b)$ $\quad \Delta = 0$.

103. (a) Put $x = 0$, which gives answer (a).

104. (a) On expanding, $-a(b-c) + 2b(b-c) + (a-b)(b-2c) = 0$

$-ab + ac + 2b^2 - 2bc + ab - 2ac - b^2 + 2bc = 0$

$b^2 - ac = 0$ $\quad b^2 = ac$.

105. (a) From options put $x = 0, 6$ and –6. Then only option (a) is satisfying.

106. (a) $A = \begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix}$

$|A| = 0$ for $x = 1$ and 2. So option (a) is correct.

107. (d) On solving the determinant,

$1(1-\cos^2\beta) - \cos(\alpha-\beta)[\cos(\alpha-\beta) - \cos\alpha\cos\beta]$
$+ \cos\alpha[\cos\beta\cos(\alpha-\beta) - \cos\alpha]$

$= 1 - \cos^2\beta - \cos^2\alpha - \cos^2(\alpha-\beta)$
$+ 2\cos\alpha\cos\beta\cos(\alpha-\beta)$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos(\alpha-\beta)$
$(2\cos\alpha\cos\beta - \cos(\alpha-\beta))$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos(\alpha-\beta)$
$[\cos(\alpha+\beta) + \cos(\alpha-\beta) - \cos(\alpha-\beta)]$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\beta - \cos^2\alpha(1-\cos^2\beta) - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\beta - \cos^2\alpha\sin^2\beta - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\beta - \sin^2\beta = 0$.

108. (b) $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$ {Operate $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$ }

$= \begin{vmatrix} 1 & 4 & 9 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{vmatrix} = 1(45-49) - 4(27-35) + 9(21-25)$

$= -4 + 32 - 36 = -8$.

109. (d) **Trick:** Putting $x = 0$ and $x = 3a$, the value of determinant becomes zero.

110. (d) $\because$ Given equation reduces to $(x-1)(6x-38) = 0$

$3x^2 - 22x + 19 = 0 \Rightarrow (x-1)(3x-19) = 0$

$x = 1, 19/3$.

111. (a) We have $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$

$\therefore \quad \Delta = x(x-0) - 0(4x-6) + 8(0-2)$

or $x^2 - 16 = 0 \Rightarrow x = 4, -4$.

112. (b) We have $\Delta = \begin{vmatrix} -x & 1 & 0 \\ 1 & -x & 1 \\ 0 & 1 & -x \end{vmatrix}$

$\Delta = -x(x^2-1) - 1(-x-0) = 0 \Rightarrow x = \pm\sqrt{2}$.

113. (d) We have $\Delta = \begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\therefore \; -10x + 90 - 42 - 81 + 42 + 9x = 0$ or $x = 9$.

114. (c) We have $\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$

$$= \begin{vmatrix} a(1+\omega) & b\omega^2 & a\omega \\ b(\omega+\omega^2) & c & b\omega^2 \\ c(\omega^2+1) & a\omega & c \end{vmatrix}, \{C_1 \to C_1 + C_3\}$$

$$= \begin{vmatrix} -a\omega^2 & b\omega^2 & a\omega \\ -b & c & b\omega^2 \\ -c\omega & a\omega & c \end{vmatrix} = \omega^2\omega \begin{vmatrix} -a & b & a\omega^2 \\ -b & c & b\omega^2 \\ -c & a & c\omega^2 \end{vmatrix}$$

$$= \omega^2 \begin{vmatrix} -a & b & a \\ -b & c & b \\ -c & a & c \end{vmatrix} = -\omega^2 \begin{vmatrix} a & b & a \\ b & c & b \\ c & a & c \end{vmatrix} = 0.$$

115. (c) $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ c(b-a) & a(c-b) & ab \\ b-a & c+a & a+b \end{vmatrix}$

$\{C_1 \to C_1 - C_2, C_2 \to C_2 - C_3\}$

$$= (b-a)(c-b) \begin{vmatrix} 0 & 0 & 1 \\ c & a & ab \\ 1 & 1 & a+b \end{vmatrix} = (b-a)(c-a)(c-a)$$

$= (a-b)(b-c)(c-a)$.

116. (b) $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = \begin{vmatrix} -1 & -1 & 443 \\ -1 & -1 & 447 \\ -1 & -1 & 451 \end{vmatrix} = 0$

$\{C_1 \to C_1 - C_2, C_2 \to C_2 - C_3\}$

117. (a) $\begin{bmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{bmatrix} = 0$

or $\begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$

or $abc\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ a-b & a^3-b^3 & 0 \\ a-c & a^3-c^3 & 0 \end{vmatrix} = 0$

or

$$abc\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & a^2-b^2 & a^3-b^3 \\ 0 & a^2-c^2 & a^3-c^3 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ a-b & a^3-b^3 & 0 \\ (a-c) & (a^3-c^3) & 0 \end{vmatrix} = 0$$

or $(abc)(a-b)(a-c)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & a+b & a^2+b^2+ab \\ 0 & a+c & a^2+c^2+ac \end{vmatrix} +$

$$(a-b)(a-c)\begin{vmatrix} a & a^3 & -1 \\ 1 & a^2+b^2+ab & 0 \\ 1 & a^2+c^2+ac & 0 \end{vmatrix}$$

or $(a-b)(a-c)[(abc)[(a+b)(a^2+c^2+ac)-$

$(a+c)(a^2+b^2+ab)]] + (-1)(a-b)(a-c)$

$[a^2+c^2+ac-a^2-b^2-ab] = 0$

$= (abc)[(a-b)(a-c)(c-b)(ac+ab+bc)]$

$+(-1)(a-b)(a-c)(c-b)(a+b+c) = 0$

$\Rightarrow (abc)(ac+ab+bc) = a+b+c$.

118. (b) Applying $C_1 \to C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}, (\because a^2+b^2+c^2+2=0)$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2.$$

Hence degree of $f(x) = 2$.

119. (d) $\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix} = x^4(14+x^2) = x.x^3(14+x^2)$

Hence, the determinant is divisible by x, x^3 and $(14+x^2)$, but not divisible by x^5.

120. (c) $\begin{vmatrix} 0 & b^3-a^3 & c^3-a^3 \\ a^3-b^3 & 0 & c^3-b^3 \\ a^3-c^3 & b^3-c^3 & 0 \end{vmatrix}$

$$(b^3-a^3)(c^3-a^3)\begin{vmatrix} 0 & 1 & 1 \\ a^3-b^3 & 1 & 1 \\ a^3-c^3 & 1 & 1 \end{vmatrix} = 0$$

[$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$] and then taking out common (b^2-a^3) from IInd column and (c^3-a^3) from IIIrd column].

121. (a) $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0 \quad x(5x-2x) - 2(2x+x) - 1(4+5) = 0$

$3x^2 - 6x - 9 = 0$, $x^2 - 2x - 3 = 0$, $(x+1)(x-3) = 0$

$\Rightarrow x = -1, 3$.

122. (c) $\begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$

Using $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\begin{vmatrix} 1 & 0 & \sin^2\theta \\ -1 & 1 & \cos^2\theta \\ 0 & -1 & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$2(1+2\sin 4\theta) = 0 \Rightarrow \sin 4\theta = \frac{-1}{2}.$$

123. (b) $f(x) = 2(x-3)(x-5);\ \begin{vmatrix} 1 & x+3 & 3(x^2+3x+9) \\ 1 & x+5 & 4(x^2+5x+25) \\ 1 & 1 & 3 \end{vmatrix}$

(Taking out $(x-3),(x-5)$ and 2 from I[st] row, II[nd] row and II[rd] column respectively)

$f(x) = 2(x-3)(x-5)$

$$\begin{vmatrix} 0 & (x+2) & 3(x^2+3x+8) \\ 0 & 2 & x^2+11x+73 \\ 1 & 1 & 3 \end{vmatrix},$$

$(R_1 \to R_1 - R_3 \text{ and } R_2 \to R_2 - R_1)$

$= 2(x-3)(x-5)[1(x+2)\,(x^2+11x+73) - 6(x^2+3x+8)]$

$= 2(x^2-8x+15)(x^3+13x^2+95x+146-6x^2-18x-48)$

$= 2(x^2-8x+15)(x^3+7x^2+77x+98)$

$= 2(x^5 - x^4 + 36x^3 - 413x^2 + 371x + 1470)$

$f(1) = 2928,\ f(3) = 0,\ f(5) = 0$

$f(1).f(3) + f(3).f(5) + f(5).f(1) = 0+0+0 = 0 = f(3).$

124. (d) $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = \begin{vmatrix} y+z & x-z & x-y \\ 2y & 2x & 0 \\ 2z & 0 & 2x \end{vmatrix}$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$$= 4\begin{vmatrix} y+z & x-z & x-y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$$

$= 4[(y+z)(x^2) - (x-z)(xy) + (x-y)(-zx)]$

$= 4[x^2y + zx^2 - x^2y + xyz - zx^2 + xyz] = 8xyz$

Hence, $k = 8$.

Minors and Co-factors, Product of determinants

1. (b) The cofactor of element 4, in the 2^{nd} row and 3^{rd} column is

$= (-1)^{2+3}\begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = -\{1(-2) - 3(8-0) + 1.16\}$

$= 10.$

2. (b) We know that $\Delta\Delta' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} . \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

$$= \begin{vmatrix} \Sigma a_1A_1 & 0 & 0 \\ 0 & \Sigma a_2A_2 & 0 \\ 0 & 0 & \Sigma a_3A_3 \end{vmatrix} = \begin{vmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{vmatrix} = \Delta^3$$

$\Delta' = \Delta^2$.

3. (d) It is a fundamental concept.

4. (b) Since $\Delta = \omega^2 - 2\omega^2 = -\omega^2$. Therefore $\Delta^2 = \omega^4 = \omega$.

5. (b) $\Delta_2\Delta_1 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}\begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ c+ad & bd \end{vmatrix} = bd.$

6. (a) $B_2 = \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} = a_1c_3 - c_1a_3$

$C_2 = -\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} = -(a_1b_3 - a_3b_1)$

$B_3 = -\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = -(a_1c_2 - a_2c_1)$

$C_3 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

$$\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} = \begin{vmatrix} a_1c_3 - a_3c_1 & -(a_1b_3 - a_3b_1) \\ -(a_1c_2 - a_2c_1) & a_1b_2 - a_2b_1 \end{vmatrix}$$

$$= \begin{vmatrix} a_1c_3 & -a_1b_3 \\ -a_1c_2 & a_1b_2 \end{vmatrix} + \begin{vmatrix} a_1c_3 & a_3b_1 \\ -a_1c_2 & -a_2b_1 \end{vmatrix}$$

$$+ \begin{vmatrix} -a_3c_1 & -a_1b_3 \\ a_2c_1 & a_1b_2 \end{vmatrix} + \begin{vmatrix} -a_3c_1 & a_3b_1 \\ a_2c_1 & -a_2b_1 \end{vmatrix}$$

$= a_1^2(b_2c_3 - b_3c_2) + a_1b_1(-c_3a_2 + a_3c_2)$

$+ a_1c_1(-a_3b_2 + a_2b_3) + c_1b_1(a_3a_2 - a_2a_3) = a_1\Delta.$

7. (b) It is a fundamental concept

8. (b) $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

$$= \left(\frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4} \times \frac{\log 8}{\log 3}\right)$$

$$\times \left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} - \frac{\log 3}{\log 8} \times \frac{\log 4}{\log 3}\right)$$

$$= \left(\frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} - \frac{\log 2^3}{\log 2^2}\right) \times \left(\frac{\log 2^2}{\log 2} - \frac{\log 2^2}{\log 2^3}\right)$$

$$= \left(\frac{9 \times 2}{2} - \frac{3}{2}\right)\left(2 - \frac{2}{3}\right) = \frac{15}{2} \times \frac{4}{3} = 10.$$

9. (c) $C_{21} = (-1)^{2+1}(18+21) = -39$

$C_{22} = (-1)^{2+2}(15+12) = 27$

$C_{23} = (-1)^{2+3}(-35+24) = 11.$

10. (b) Minor of $-4 = \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = -42,\ 9 = \begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$

and cofactor of $-4 = (-1)^{2+1}(-42) = 42$,

cofactor of $9 = (-1)^{3+3}(-3) = -3$.

System of linear equations, Some special determinants, differentiation and integration of determinants

1. (d) The system of equations has infinitely many (non-trivial) solution, if $\Delta = 0$ *i.e.*, if
$$\begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$
$3(42-30) - \lambda(6-2) + 1(-30+14) = 0 \quad \lambda = 5$.

2. (c) For the given set of equation, by Cramer's Rule
$$x = \frac{D_x}{D} = \begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}.$$

3. (c) It has a non-zero solution if
$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0 \Rightarrow -6k + 6 = 0 \Rightarrow k = 1.$$

4. (d) For the system of given homogeneous equations
$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & -1 \\ 1 & -3 & 1 \end{vmatrix} = 1(-1-3) - 1(3+1) - 1(-9+1)$$
$= -4 - 4 + 8 = 0$. There are infinite number of solutions.

5. (d) The given system of homogeneous equations has a non-zero solution if, $\Delta = 0$

i.e., $\begin{vmatrix} 1 & 1 & -1 \\ 3 & -\alpha & -3 \\ 1 & -3 & 1 \end{vmatrix} = -2\alpha - 6 = 0$, *i.e.* if $\alpha = -3$.

6. (b) The given system of homogeneous equations has $\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 3 & -4 & -1 \\ 1 & -3 & 1 \end{vmatrix} = 1(-4-3) - 4(3+1) - 1(-9+4)$

$= -7 - 16 + 5 \neq 0$.

There exists only one trivial solution.

7. (b) $$\frac{d^n}{dx^n}[\Delta(x)] = \begin{vmatrix} \frac{d^n}{dx^n} x^n & \frac{d^n}{dx^n} \sin x & \frac{d^n}{dx^n} \cos x \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$
$$= \begin{vmatrix} n! & \sin\left(x + \frac{n\pi}{2}\right) & \cos\left(x + \frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$
$$\Rightarrow [\Delta^n(x)]_{x=0} = \begin{vmatrix} n! & \sin\left(0 + \frac{n\pi}{2}\right) & \cos\left(0 + \frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix} = 0$$
{Since $R_1 \equiv R_2$ }.

8. (a) The system will have a non-zero solution, if
$$\Delta \equiv \begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$
$$\Rightarrow \begin{vmatrix} a^3 & 3a^2 + 3a + 1 & 3(a+1)^2 + 3(a+1) + 1 \\ a^2 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$
by $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$

$3a^2 + 3a + 1 - \{3(a+1)^2 + 3(a+1) + 1\}$ (expanding along R_3)

$-6(a+1) = 0 \Rightarrow a = -1$.

9. (d) It is based on fundamental concept.

10. (d) Given set of equations will have a non trivial solution if the determinant of coefficient of *x*, *y*, *z* is zero

i.e., $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow 2k - 33 = 0$ or $k = \frac{33}{2}$.

11. (a) For the equation to be inconsistent $D = 0$

$\therefore D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3$

and $D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$

So that system is inconsistent for $k = -3$.

12. (d) For non-trivial solution $\Delta = 0$
$$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0 \qquad k = 1, -1.$$

13. (b) $a + b - 2c = 0$

$2a - 3b + c = 0$

$a - 5b + 4c = \alpha$

System is consistent, if $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ 1 & -5 & 4 \end{vmatrix} = 0$

and $D_1 = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -3 & 1 \\ \alpha & -5 & 4 \end{vmatrix} = 0$ and D_2 also zero.

Hence, value of α is zero.

14. (c) We have, $x_1 + 2x_2 + 3x_3 = c$

$2ax_1 + 3x_2 + x_3 = c$

$3bx_1 + x_2 + 2x_3 = c$

Let $a = b = c = 1$.

Then $D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1(5) - 2(1) + 3(-7) = -18 \neq 0$

$D_x = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -3$

Similarly $D_y = D_z = -3$. Now, $x = \frac{D_x}{D} = \frac{1}{6}$, $y = z = \frac{1}{6}$

Hence $D \neq 0$, $x = y = z$, *i.e.,* unique solution.

15. (a) Accordingly, $\begin{vmatrix} \lambda & 1 & 1 \\ -1 & \lambda & 1 \\ -1 & -1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda^3 + 3\lambda = 0$

Therefore $\lambda = 0$, since $\lambda = i\sqrt{3}$ does not exist.

16. (a) $\sum_{n=1}^{N} U_n = \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left\{\frac{N(N+1)}{2}\right\}^2 & 3N^2 & 3N \end{vmatrix}$

$= \frac{N(N+1)}{12} \begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$

$= \begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix} = 0$, {Applying $C_3 \to C_3 + C_2$}.

17. (c) If r is the common ratio, then $a_n = a_1 r^{n-1}$ for all $n \geq 1 \Rightarrow \log a_n = \log a_1 + (n-1)\log r$

$= A + (n-1)R$, where $\log a_1 = A$ and $\log r = R$.

Thus in Δ, on applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$, we obtain C_2 and C_3 are identical.

Thus $\Delta = 0$.

18. (c) $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 2.3^{r-1} & \sum_{r=1}^{n} 4.5^{r-1} \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

Since we know that $\sum_{r=1}^{n} 2^{r-1} = \frac{2^n - 1}{2-1} = 2^n - 1$,

$2\sum_{r=1}^{n} 3^{r-1} = 2\frac{3^n - 1}{3-1} = 3^n - 1$

and $4\sum_{r=1}^{n} 5^{r-1} = 4\frac{5^n - 1}{5-1} = 5^n - 1$

$\Rightarrow \sum_{r=1}^{n} D_r = 0$, $(\because R_1 \equiv R_3)$.

19. (c) $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $[\because |A| = |A'|]$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$.

20. (d) If the given system of equations has a non-trivial solution, then $\begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 29$.

21. (a) The given system of equations has a unique solution if $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$.

22. (c)

$D = \begin{vmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1[-1-2] - 1[6-3] + 1[3+3] = 0$

and $D_1 = \begin{vmatrix} 2 & -1 & 1 \\ -6 & -1 & 2 \\ -18 & 1 & 1 \end{vmatrix} = 2(-1-2) - 1(-36+6) + 1(-6-18)$

$= -6 + 30 - 24 = 0$

Also, $D_2 = 0; D_3 = 0$

So the system is consistent $(D = D_1 = D_2 = D_3 = 0)$

i.e. system has infinite solution.

23. (b) For infinitely many solutions, the two equations must be identical

$\Rightarrow \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$

$\Rightarrow (k+1)(k+3) = 8k$ and $8(3k-1) = 4k(k+3)$

$\Rightarrow k^2 - 4k + 3 = 0$ and $k^2 - 3k + 2 = 0$.

By cross multiplication, $\frac{k^2}{-8+9} = \frac{k}{3-2} = \frac{1}{-3+4}$

$k^2 = 1$ and $k = 1$; $k = 1$.

24. (b) $(1+ax)[(1+b_1x)(1+c_2x) - (1+b_2x)(1+c_1x)]$

$+ (1+bx)[(1+c_1x)(1+a_2x) - (1+a_1x)(1+c_2x)]$

$+ \ (1+cx)[(1+a_1x)(1+b_2x)-(1+b_1x)(1+a_2x)]$

$= A_0 + A_1x + A_2x^2 + A_3x^3$

After solving, the coefficient of x is 0.

25. (a) For unique solution of the given system $D \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} \neq 0.$$

So this depends on μ only.

26. (a) Given system of equation can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

On solving the above system we get the unique solution $x = -10, y = -4, z = 16$.

27. (c) Let $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

Applying $R_3 \to R_3 - xR_1 - R_2$; we get

$$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

$\Delta = (b^2-ac)(ax^2+2bx+c)$

Now, $b^2 - ac < 0$ and $a > 0$

Discriminant of $ax^2+2bx+c$ is $-ve$ and $a > 0$

$(ax^2+2bx+c) > 0$ for all $x \in R$

$\Delta = (b^2-ac)(ax^2+2bx+c) < 0$, *i.e.* $-ve$.

28. (d) Given system will be inconsistent when $D = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ -1 & 2-\lambda & \lambda \end{vmatrix} = 0 \quad -1(2-\lambda)-1 = 0 \Rightarrow \lambda = 3.$$

29. (b) $\Delta = \begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$

$$= x!(x+1)!(x+2)!\begin{vmatrix} 1 & (x+1) & (x+2)(x+1) \\ 1 & (x+2) & (x+3)(x+2) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$$

Applying $R_1 \to R_2 - R_1, R_2 \to (R_3 - R_2)$ we get

$$= x!(x+1)!(x+2)!\begin{vmatrix} 0 & 1 & 2(x+2) \\ 0 & 1 & 2(x+3) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$$

$= 2x!(x+1)!(x+2)!$, (on simplification).

Trick: Put $x = 1$ and then match the alternate.

30. (a) $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a^2) = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1.$

31. (d) Put the value $(x, y, z) = (1, 2, -1)$, which satisfies the equation. Hence, (d) is correct.

32. (d) The coefficient determinant $D = \begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$

$= 3\lambda - 6$

For no solution, the necessary condition is $D = 0$

i.e., $-3\lambda - 6 = 0 \Rightarrow \lambda = -2$

It can be seen that for $\lambda = -2$, there is no solution for the given system or equations.

33. (a) By Cramer's Rule, $x = \frac{D_1}{D}$,

$\therefore$ (a) is the correct option.

34. (b) The value of the determinant will vanish if $\lambda = 3$ and $\Delta_1 \neq 0$, $\therefore \mu \neq 10$.

35. (c) Let first term $= A$ and common difference $= D$

$\therefore a = A + (p-1)D,\ b = A + (q-1)D,\ c = A + (r-1)D$

$$\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix} = \begin{vmatrix} A+(p-1)D & p & 1 \\ A+(q-1)D & q & 1 \\ A+(r-1)D & r & 1 \end{vmatrix}$$

Operate $C_1 \to C_1 - DC_2 + DC_3$

$$= \begin{vmatrix} A & p & 1 \\ A & q & 1 \\ A & r & 1 \end{vmatrix} = A\begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} = 0.$$

36. (c) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0, \quad [C_2 \to C_2 - 2C_3]$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad ,$$

$[R_3 \to R_3 - R_2, R_2 \to R_2 - R_1]$

$$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0\ ;\ b(c-b) - (b-a)(2c-b) = 0$$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

a, b, c are in Harmonic progression.

37. (c) For no solution or infinitely many solutions

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0 \Rightarrow \alpha = 1, \alpha = -2.$$

But for $\alpha = 1$, clearly there are infinitely many solutions and when we put $\alpha = -2$ in given system of equations and adding them together L.H.S ≠ R.H.S. *i.e.*, No solution.

38. (d) For constant solution $|A| = 0$

i.e., $\begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ \alpha+1 & \alpha+2 & -(\alpha+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$

$6\alpha + 12 = 0 \Rightarrow \alpha = -2$.

Types of matrices, Algebra of matrices

1. (b) It is obvious.

2. (d) $M^2 - \lambda M - I_2 = 0$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 5-\lambda & 8-2\lambda \\ 8-2\lambda & 13-3\lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5 - \lambda = 1, 8 - 2\lambda = 0, 13 - 3\lambda = 1$

$\lambda = 4$, which satisfies all the three equations.

3. (c) Clearly, $AB = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = BA$ (verify).

4. (c) $\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = -1 \neq 0$, hence matrix is non-singular.

5. (a) $A^2 = A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

6. (a) $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$,

and $A^3 = A^2.A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^n = A^{n-1}.A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$.

7. (d) Since $AB = O$, even if $A \neq O$ and $B \neq O$.

8. (a)

$A^2 = \begin{bmatrix} 2 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 2 & 2 \\ a & b \end{bmatrix} = \begin{bmatrix} 4+2a & 4+2b \\ 2a+ab & 2a+b^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow 4 + 2a = 0, 4 + 2b = 0, 2a + ab = 0,$

$2a + b^2 = 0$ must be consistent.

$\Rightarrow a = -2, b = -2$.

9. (b) It is obvious that $(m,n) = (3, 4)$.

10. (a) By inspection, A^2 and A matrix is of order 3×3, while B matrix is of order 3×2. Therefore, $A^2 + 2B - 2A$ is not defined.

11. (b) Relation $A^2 = B^2$ is true because $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ have same matrices.

12. (b) The matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular,

If $\begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$

$1(40 - 40) - 3(20 - 24) + (\lambda + 2)(10 - 12) = 0$

$2(\lambda + 2) = 12 \Rightarrow \lambda = 4$.

13. (a) $A^2 = A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = O$

$\Rightarrow A^3 = A.A^2 = 0$ and $A^n = 0$, for all $n \geq 2$.

14. (b) $AB = \begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3-2x \end{bmatrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (As given)

$\Leftrightarrow 3 - 2x = 1$ or $x = 1$.

15. (d) It is obvious.

16. (b)

$A^2 = A.A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix} = \begin{bmatrix} \lambda^2-1 & 0 \\ 0 & -1+\lambda^2 \end{bmatrix} = 0$ (As given)

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$.

17. (b) $A' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -5 \\ -2 & 5 & 0 \end{bmatrix} = -A$.

18. (c) $A^2 - 6A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} - 6\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 19 & 6 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 24 & 6 \\ 18 & 12 \end{bmatrix} \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} = -5I$.

19. (c) By inspection, A' is a matrix of order 3×3 and B' is a matrix of order 3×2. Therefore multiplication of these matrices is defined.

20. (c) $AB = [1\ 2\ 3]\begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix} = [-2\ -1\ 4]$.

21. (b) It is obvious.

22. (c) $A' = [1\ 2\ 3]$, therefore

$AA' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[1\ 2\ 3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.

23. (c) $\begin{bmatrix}2 & -3\\4 & 0\end{bmatrix}-\begin{bmatrix}a & c\\b & d\end{bmatrix}=\begin{bmatrix}1 & 4\\2 & -5\end{bmatrix}$

$\begin{bmatrix}a & c\\b & d\end{bmatrix}=\begin{bmatrix}2 & -3\\4 & 0\end{bmatrix}-\begin{bmatrix}1 & 4\\2 & -5\end{bmatrix}=\begin{bmatrix}1 & -7\\2 & 5\end{bmatrix}$

$(a, b, c, d)=(1,\ 2,\ -7,\ 5)$.

24. (b) $A^2=\begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=I$.

25. (b) In the product AB, the required element $C_{33}=(-2)3+2.5+0.0=-6+10=4$.

26. (c) $A=\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}\Rightarrow A^5=\begin{bmatrix}2^5 & 0 & 0\\0 & 2^5 & 0\\0 & 0 & 2^5\end{bmatrix}=2^4\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$

$=16A$.

27. (d) Since $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=O=AB$

$B=\begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix}$.

28. (b) It is obvious.

29. (c) Since a square matrix A whose elements $a_{ij}=0$ for $i<j$. Then A is the lower triangular matrix.

30. (b) $[a_{ij}]_{n\times n}$ square matrix is a upper triangular matrix for $a_{ij}=0, i>j$.

31. (d) The matrix $A=\begin{bmatrix}0 & 1 & -2\\-1 & 0 & 3\\\lambda & -3 & 0\end{bmatrix}$ is singular

$|A|=0 \quad 0-1(-3\lambda)+(-2)(3)=0$

$\Rightarrow 3\lambda-6=0\Rightarrow\lambda=2$.

32. (b) It is based on fundamental concept.

33. (c) $|A|=k^n|B|$, by fundamental concept.

34. (b) $A^2B=(A.A)B=\begin{bmatrix}2 & 0 & 0\\0 & -1 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}2 & 0 & 0\\0 & -1 & 0\\0 & 0 & 3\end{bmatrix}.\begin{bmatrix}-1 & 0 & 0\\0 & 3 & 0\\0 & 0 & 2\end{bmatrix}$

$=\begin{bmatrix}4 & 0 & 0\\0 & 1 & 0\\0 & 0 & 9\end{bmatrix}\begin{bmatrix}-1 & 0 & 0\\0 & 3 & 0\\0 & 0 & 2\end{bmatrix}=\begin{bmatrix}-4 & 0 & 0\\0 & 3 & 0\\0 & 0 & 18\end{bmatrix}$.

35. (d) $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & 0\\0 & \cos^2\theta+\sin^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

36. (c) It is based on fundamental concept.

37. (c) $AC=[a\ b]\begin{bmatrix}a\\-a\end{bmatrix}=[a^2-ab]$

$BC=[-b\ -a]\begin{bmatrix}a\\-a\end{bmatrix}=[a^2-ab]$

$AC=BC$.

38. (d) $A^2=A.A=\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2a\\0 & 1\end{bmatrix}$

$A^3=A.A^2=\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2a\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3a\\0 & 1\end{bmatrix}$

$A^4=A.A^3=\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3a\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 4a\\0 & 1\end{bmatrix}$.

39. (a) $\begin{bmatrix}3 & 1\\4 & 1\end{bmatrix}X=\begin{bmatrix}5 & -1\\2 & 3\end{bmatrix}\Rightarrow X=\begin{bmatrix}-3 & 4\\14 & -13\end{bmatrix}$

Since $\begin{bmatrix}3 & 1\\4 & 1\end{bmatrix}\begin{bmatrix}-3 & 4\\14 & -13\end{bmatrix}=\begin{bmatrix}5 & -1\\2 & 3\end{bmatrix}$.

40. (a) We have $(A+B)(A-B)=A^2-AB+BA-B^2$

Option (a) is not true.

41. (b) Since $|B|\neq 0\Rightarrow B^{-1}$ exists, $AB=0$

$(AB)B^{-1}=OB^{-1}\Rightarrow A(BB^{-1})=O$

$AI=O\Rightarrow A=O$.

42. (b) Order will be $(1\times 3)(3\times 3)(3\times 1)=(1\times 1)$.

43. (c) We have $AB=B$ and $BA=A$.

Therefore $A^2+B^2=AA+BB=A(BA)+B(AB)$

$=(AB)A+(BA)B=BA+AB=A+B$,

$(\because AB=B$ and $BA=A)$.

44. (a) Since $(A+B)(A-B)=A^2-B^2$

By matrix distribution law,

$A^2-AB+BA-B^2=A^2-B^2$

$BA-AB=0\Rightarrow BA=AB$.

45. (b) $A=\begin{bmatrix}5 & -3\\2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}6 & -4\\3 & 6\end{bmatrix}$,

$A-B=\begin{bmatrix}-1 & 1\\-1 & -2\end{bmatrix}$.

46. (d) $X=\begin{bmatrix}3 & -4\\1 & -1\end{bmatrix}\Rightarrow X^2=\begin{bmatrix}5 & -8\\2 & -3\end{bmatrix}$.

Clearly for $n=2$, the matrices in (a), (b), (c) do not tally with $\begin{bmatrix}5 & -8\\2 & -3\end{bmatrix}$.

47. (b) $A=\begin{bmatrix}i & 0\\0 & i\end{bmatrix}$; $A^2=A.A=\begin{bmatrix}i & 0\\0 & i\end{bmatrix}\begin{bmatrix}i & 0\\0 & i\end{bmatrix}$

$A^2=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$, $[\because i^2=-1]$.

48. (a) It is obvious.

49. (a) We have $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

So $A^2=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I_2$

$A^4=A^2.A^2=I_2.I_2=I_2=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

50. (d) $A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

$A^2=A.A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

$A^2=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$.

51. (b) $A+B$ is defined A and B are of same order
Also AB is defined Number of columns in A = Number of rows in B
Obviously, both simultaneously mean that the matrices A and B are square matrices of same order.

52. (b) $AB=\begin{bmatrix}5 & 9 & 13\\-1 & 2 & 4\\-2 & 2 & 4\end{bmatrix}$.

53. (b) $A=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}, B=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\Rightarrow AB=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$.

54. (a) Here $AB=\begin{bmatrix}i & 0\\0 & i\end{bmatrix}\begin{bmatrix}0 & -i\\-i & 0\end{bmatrix}=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

and $BA=\begin{bmatrix}0 & -i\\-i & 0\end{bmatrix}\begin{bmatrix}i & 0\\0 & i\end{bmatrix}=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Since $AB=BA$, therefore $(A+B)(A-B)=A^2-B^2$.

55. (b) $5A-3B+2C=\begin{bmatrix}5 & -10\\15 & 0\end{bmatrix}-\begin{bmatrix}-3 & 12\\6 & 9\end{bmatrix}-\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$

$=\begin{bmatrix}5 & -10\\15 & 0\end{bmatrix}-\begin{bmatrix}-3 & 10\\8 & 9\end{bmatrix}=\begin{bmatrix}8 & -20\\7 & -9\end{bmatrix}$.

56. (b) Since $x-2=3-2\Rightarrow x=3$
and $y+4=3-1\Rightarrow y=-2$.

57. (d) $A=\begin{bmatrix}x & 1\\1 & 0\end{bmatrix}, \therefore A^2=I\Rightarrow\begin{bmatrix}x & 1\\1 & 0\end{bmatrix}\begin{bmatrix}x & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\begin{bmatrix}x^2+1 & x\\x & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\Rightarrow x^2+1=1\Rightarrow x=0$.

58. (a) $AB=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}=\begin{bmatrix}i & 0\\0 & -i\end{bmatrix}$

and $BA=\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}-i & 0\\0 & i\end{bmatrix}=-AB$

$\therefore AB+BA=O$
Hence, $(A+B)^2=A^2+B^2$.

59. (b) $(aI+bA)^2=\begin{bmatrix}a & b\\0 & a\end{bmatrix}\begin{bmatrix}a & b\\0 & a\end{bmatrix}=\begin{bmatrix}a^2 & 2ab\\0 & a^2\end{bmatrix}=a^2I+2abA$.

60. (b) Students should remember it.

61. (c) Since $A^2=\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}=\begin{bmatrix}-4 & 2\\-3 & -6\end{bmatrix}\neq A$

$B^2=\begin{bmatrix}-1 & 0\\2 & 3\end{bmatrix}\begin{bmatrix}-1 & 0\\2 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\4 & 9\end{bmatrix}\neq B$

Now $AB=\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\2 & 3\end{bmatrix}=\begin{bmatrix}3 & 6\\3 & 0\end{bmatrix}$

and $BA=\begin{bmatrix}-1 & 0\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}=\begin{bmatrix}-1 & -2\\-7 & 4\end{bmatrix}$

Obviously, $AB\neq BA$.

62. (c) It is a property of matrix multiplication.

63. (d) Matrix $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & \lambda & 5\end{bmatrix}$ be non singular,

only if $\begin{vmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & \lambda & 5\end{vmatrix}\neq 0$

$1(25-6\lambda)-2(20-18)+3(4\lambda-15)\neq 0$

$25-6\lambda-4+12\lambda-45\neq 0$

$6\lambda-24\neq 0 \quad \lambda\neq 4$.

64. (d) $UV=[4]$ and $XY=[16]; \therefore UV+XY=[20]$.

65. (b) $A=\begin{bmatrix}0 & i\\-i & 0\end{bmatrix}\Rightarrow A^2=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$

$(A^2)^{20}=A^{40}=(I)^{20}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

66. (b) $AB=\begin{bmatrix}1 & -2 & 1\\2 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 2\\1 & 1\end{bmatrix}$

$AB=\begin{bmatrix}-3 & -2\\10 & 7\end{bmatrix}\Rightarrow(AB)^T=\begin{bmatrix}-3 & 10\\-2 & 7\end{bmatrix}$.