7. c) Since there are one A, two I and one O, hence the required probability $=\frac{1+2+1}{11}=\frac{4}{11}$.

8. (b) Required probability is $1-P$ (All letters in right envelope) $=1-\frac{1}{n!}$

{As there are total number of $n!$ ways in which letters can take envelopes and just one way in which they have corresponding envelopes}.

9. (a) Favourable ways $\{29, 92, 38, 83, 47, 74, 56, 65\}$

Hence required probability $=\frac{8}{100}=\frac{2}{25}$.

10. (c) To be both boys the probability $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$.

11. (c) Probability of getting 1 in first throw $=\frac{1}{6}$

Probability of not getting 1 in second throws $=\frac{5}{6}$

Both are independent events, so the required probability $=\frac{1}{6}\times\frac{5}{6}=\frac{5}{36}$.

12. (c) Probability of first card to be a king $=\frac{4}{52}$

and probability of also second to be a king $=\frac{3}{51}$

Hence required probability $=\frac{4}{52}\times\frac{3}{51}=\frac{1}{221}$.

13. (b) Required probability $=\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)=\frac{1}{12}$.

14. (b) Required probability $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$.

15. (c) Required probability $=P(\text{Diamond}).P(\text{king})$

$=\frac{13}{52}.\frac{4}{52}=\frac{1}{52}$.

16. (b) The sum 2 can be found in one way *i.e.* $(1, 1)$

The sum 8 can be found in five ways *i.e.* $(6, 2)$, $(5, 3)$, $(4, 4), (3, 5), (2, 6)$. Similarly the sum twelve can be found in one way *i.e.,* $(6, 6)$.

Hence required probability $=\frac{7}{36}$.

17. (b) Let $P(A)$ and $P(B)$ be the probability of the events then $P(A \text{ and } B)=P(A).P(B)=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$.

18. (a) Required probability is $1-P$ (no ace)

$$=1-\frac{48}{52}.\frac{47}{51}=\frac{663-564}{13.51}=\frac{99}{13.51}=\frac{33}{221}.$$

19. (a) Required probability is $1-P$ (no ace of heart)

$$=1-\frac{51}{52}.\frac{51}{52}=\frac{(52+51)}{52.52}=\frac{103}{2704}.$$

20. (c) Total rusted items $=3+5=8$; unrusted nails $=3$.

$\therefore$ Required probability $=\frac{3+8}{6+10}=\frac{11}{16}$.

21. (a) Required probability $=1-\left(\frac{1}{2}\right)^4=\frac{15}{16}$.

22. (d) Total no. of ways placing 3 letters in three envelops $=3!$, out of these ways only one way is correct.

Hence the required probability $=\frac{1}{3!}=\frac{1}{6}$.

23. (b) Required probability $=\frac{2+1}{36}=\frac{1}{12}$.

24. (b) Total number of ways $=36$

Favourable numbers of cases are $(1\ 4), (2,\ 3), (3,\ 2)$,

$(4,\ 1), (1,\ 5), (2,\ 4), (3,\ 3), (4,\ 2), (5,\ 1)=9$

Hence the required probability $=\frac{9}{36}=\frac{1}{4}$.

25. (b) It is obvious.

26. (b) Here $P(A)=0.4$ and $P(\overline{A})=0.6$

Probability that A does not happen at all $=(0.6)^3$

Thus required Probability $=1-(0.6)^3=0.784$.

27. (b) Required probability $=\frac{1}{2}$.

28. (c) Number of tickets numbered such that it is divisible by 20 are $\frac{10000}{20}=500$

Hence required probability $=\frac{500}{10000}=\frac{1}{20}$.

29. (b) Favourable cases for one are three *i.e.* 2, 4 and 6 and for other are two *i.e.* 3, 6.

Hence required probability $=\left[\left(\frac{3\times 2}{36}\right)2-\frac{1}{36}\right]=\frac{11}{36}$

{As same way happen when dice changes numbers among themselves}

30. (d) The probability of students not solving the problem are $1-\frac{1}{3}=\frac{2}{3}$, $1-\frac{1}{4}=\frac{3}{4}$ and $1-\frac{1}{5}=\frac{4}{5}$

Therefore the probability that the problem is not solved by any one of them $=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$

Hence the probability that problem is solved $=1-\frac{2}{5}=\frac{3}{5}$.

31. (c) Required probability $=\frac{1}{6}$.

32. (b) The probability of card to be queen of club is $\frac{1}{52}$ and also probability of card to be a king of heart is $\frac{1}{52}$.

Both are mutually exclusive events, hence the required probability $= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}$.

33. (c) Total ways are 8 and favourable ways are 4 $S = \{HHH, HHT .... TTT\}$

Hence probability $= \frac{4}{8} = \frac{1}{2}$.

34. (b) It is obvious.

35. (c) Favourable cases to get the sum not less than 11 are $\{(5, 6), (6, 6), (6, 5)\} = 3$

Hence favourable cases to get the sum less than 11 are $(36 - 3) = 33$. So required probability $= \frac{33}{36} = \frac{11}{12}$.

36. (b) In a non-leap year, we have 365 days *i.e.,* 52 weeks and one day. So, we may have any day of seven days. Therefore, 53 Sunday, required probability $= \frac{1}{7}$.

37. (b) Court cards are king, queen and jack

Hence required probability $= \frac{12}{52} = \frac{3}{13}$.

38. (c) $S = \{(3,1), (2,2), (1,3), (6,2), (5,3), (4,4), (3,5), (2,6), (6,6)\}$

Hence required probability $= \frac{9}{36} = \frac{1}{4}$.

39. (a) Required probability $= \frac{5}{25} = \frac{1}{5}$.

40. (b) Required probability $= \frac{4}{6} = \frac{2}{3}$.

41. (d) Required probability $= \frac{4 \,.\, 4}{52 \,.\, 51} \times 2 = \frac{8}{663}$.

42. (c) Obviously numbers will be $\begin{pmatrix} \text{I} & \text{II} \\ 5, & 1 \\ 4, & 2 \\ 2, & 4 \\ 1, & 5 \end{pmatrix}$. Hence required probability $= \frac{4}{6 \,.\, 5} = \frac{2}{15}$.

43. (a) Let E_1 be the event that man will be selected and E_2 the event that woman will be selected. Then

$P(E_1) = \frac{1}{4}$ so $P(\overline{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$ and $P(E_2) = \frac{1}{3}$

So $P(\overline{E}_2) = \frac{2}{3}$

Clearly E_1 and E_2 are independent events.

So, $P(\overline{E}_1 \cap \overline{E}_2) = P(\overline{E}_1) \times P(\overline{E}_2) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.

44. (c) Since both heads and tails appears, so

$n(S) = \{HHT, HTH, THH, HTT, THT, TTH\}$

$n(E) = \{HTT, THT, TTH\}$

Hence required probability $= \frac{3}{6} = \frac{1}{2}$.

45. (d) Probability for white ball $P(W) = \frac{4}{15}$

Probability for red ball $P(R) = \frac{6}{15}$

Probability (white or red ball) $= P(W) + P(R)$

$= \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$.

46. (b) Required probability $= \frac{52 - 16}{52} = \frac{36}{52} = \frac{9}{13}$.

47. (a) Since the total '13' can't be found.

48. (c) Three dice can be thrown in $6 \times 6 \times 6 = 216$ ways. A total 17 can be obtained as $(5,6,6)$, $(6,5,6)$, $(6,6,5)$. A total 18 can be obtained as $(6,6,6)$.

Hence the required probability $= \frac{4}{216} = \frac{1}{54}$.

49. (d) Required probability $= \frac{64}{64}$.

50. (b) It is obvious.

51. (c) P (tail in 3rd) . P (tail in 4th) $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

52. (b) Prime numbers are $\{2, 3, 5, 7, 11\}$.

Hence required probability

$$= \frac{1 + 2 + 4 + 6 + 2}{36} = \frac{15}{36} = \frac{5}{12}.$$

53. (a) Required probability

$$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}.$$

54. (b) Since favourable ways are 6. Total ways are 36.

Hence probability $= \frac{6}{36}$.

55. (d) We have $P(\overline{A}) = 0.05 \Rightarrow P(A) = 0.95$

Hence the probability that the event will take place in 4 consecutive occasions

$$= \{P(A)\}^4 = (0.95)^4 = 0.8145062!.$$

56. (b) Ways $\begin{array}{cccc} 9 & 10 & 11 & 12 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 \end{array}$

Hence the required probability $= \frac{10}{36} = \frac{5}{18}$.

57. (b) It is obvious.

58. (b) Same number can appear in 6 ways.

Hence required probability $= \frac{6}{216} = \frac{1}{36}$.

59. (b) $P(\overline{A}) = 1 - 0.38 = 0.62$.

60. (a) $P(A' \cap B') = \frac{4}{5}.\frac{3}{4} = \frac{3}{5}$.

61. (b) Total probable ways $= 8$
Favourable number of ways $= [HTH, THT]$
Hence required probability $= \frac{2}{8} = \frac{1}{4}$.

62. (c) It is obvious.

63. (a) It is obvious.

64. (d) Odd and prefect square (< 10) are $1, 9$.
Hence required probability $= \frac{2}{10} = \frac{1}{5}$.

65. (d) $2, 4, 6, 8, 10 ....$ *i.e.,* fifty even and ten divisible by 5 like $5, 15, 25 ....$ as $(10, 20, 30 ....)$ have been considered. Hence required probability $= \frac{50+10}{100} = \frac{3}{5}$.

66. (d) Required probability $= \frac{4}{6} = \frac{2}{3}$.

67. (a) Required probability $= \frac{44}{52} = \frac{11}{13}$.

68. (c) Required probability is $1 - P$ (they go in concerned envelopes) $= 1 - \frac{1}{4!} = \frac{23}{24}$.

69. (a) It is a fundamental concept.

70. (b) Required probability is
$1 - P(\text{no girl}) = 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$.

71. (b) A determinant of order 2 is of the form $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$
It is equal to $ad - bc$ The total number of ways of choosing a, b, c and d is $2 \times 2 \times 2 \times 2 = 16$. Now $\Delta \neq 0$ if and only if either $ad = 1,\ bc = 0$ or $ad = 0,\ bc = 1$. But $ad = 1,\ bc = 0$ iff $a = d = 1$ and one of b, c is zero. Therefore $ad = 1,\ bc = 0$ in three cases, similarly $ad = 0,\ bc = 1$ in three cases. Therefore the required probability $= \frac{6}{16} = \frac{3}{8}$.

72. (c) A is independent of itself, if
$P(A \cap A) = P(A).P(A) \Rightarrow P(A) = P(A)^2 \Rightarrow P(A) = 0, 1$.

73. (b) Let A denote the event that the stranger succeeds at the k^{th} trial. Then
$P(A') = \frac{999}{1000} \times \frac{998}{999} \times ..... \times \frac{1000-k+1}{1000-k+2} \times \frac{1000-k}{1000-k+1}$
$\Rightarrow P(A') = \frac{1000-k}{1000}$ $\quad P(A) = 1 - \frac{1000-k}{1000} = \frac{k}{1000}$.

74. (b) Required probability is $1 - P$(no die show up 1)
$= 1 - \left(\frac{5}{6}\right)^3 = \frac{216-125}{216} = \frac{91}{216}$.

75. (a) Required probability $= \frac{2}{52} = \frac{1}{26}$.

76. (b) The probability of husband is not selected $= 1 - \frac{1}{7} = \frac{6}{7}$
The probability that wife is not selected $= 1 - \frac{1}{5} = \frac{4}{5}$
The probability that only husband selected $= \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$
The probability that only wife selected $= \frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$
Hence required probability $= \frac{6}{35} + \frac{4}{35} = \frac{10}{35} = \frac{2}{7}$.

77. (a) Required probability $= \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} \times \frac{2}{17} = \frac{1}{969}$.

78. (b) Here $p_1 = \frac{1}{3},\ p_2 = \frac{2}{7}$ and $p_3 = \frac{3}{8}$
$\Rightarrow q_1 = \frac{2}{3},\ q_2 = \frac{5}{7}$ and $q_3 = \frac{5}{8}$
Required probability $= p_1 q_2 q_3 + q_1 p_2 q_3 + q_1 q_2 p_3$.

79. (a) $7 \rightarrow \{(6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6)\}$
$9 \rightarrow \{(6, 3), (5, 4), (4, 5), (3, 6)\}$
Required probability $= \frac{6+4}{36} = \frac{5}{18}$.

80. (d) The probability of getting an even number in first draw $= \frac{9}{19}$. The probability of getting an even number in second draw $= \frac{8}{18}$. Both are independent event and so required probability $= \frac{9}{19} \times \frac{8}{18} = \frac{4}{19}$.

81. (a) Here $P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$
Hence required probability
$= P(A)P(\overline{B})P(\overline{C}) + P(\overline{A})P(B)P(\overline{C}) + P(\overline{A})P(\overline{B})P(C)$.

82. (a) $n =$ total number of ways $= 2^4 = 16$
$m =$ Favourable number of ways $= 3$
Since the value of determinant is positive when it is
$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$. Hence required probability $= \frac{3}{16}$.

83. (c) Required probability $= \frac{16}{52} = \frac{4}{13}$

(Since diamond has 13 cards including a king and there are another 3 kings).

84. (d) Let R stand for drawing red ball B for drawing black ball and W for drawing white ball.

Then required probability

$= P(WWR) + P(BBR) + P(WBR) + P(BWR) + P(WRR) + P(BRR) + P(RWR) + P(RBR).$

$= \frac{3.2.2}{8.7.6} + \frac{3.2.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.2.1}{8.7.6} + \frac{3.2.1}{8.7.6} + \frac{2.3.1}{8.7.6} + \frac{2.3.1}{8.7.6}$

$= \frac{2}{56} + \frac{2}{56} + \frac{3}{56} + \frac{3}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} = \frac{1}{4}.$

85. (b) Favourable ways are $(2, 6), (3, 5), (4, 4), (5, 3)$ and $(6, 2)$. Hence required probability $= \frac{5}{36}$.

86. (b) It is obvious.

87. (b) The second ball can be red in two different ways

(i) First is white and second red

$P(A) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$

(ii) First is red and second is also red

$P(B) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$

Both are mutually exclusive events, hence required probability is $\frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$.

88. (c) The sample space is $[LWW, WLW]$

$\therefore\ P(LWW) + P(WLW)$

= Probability that in 5 match series, it is India's second win

$= P(L)P(W)P(W) + P(W)P(L)P(W) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}.$

89. (c) Total number of ways $= {}^{100}C_1 = 100$

Favourable numbers are $1^2, 2^2, ...10^2$.

Therefore, favourable ways =10.

$\therefore$ Probability $= \frac{10}{100} = \frac{1}{10}$.

90. (c) $P(5 \text{ or } 6 \text{ or } 7)$ in one draw $= \frac{3}{7}$

$\therefore$ Probability that in each of 3 draws, the chits bear 5 or 6 or $7 = \left(\frac{3}{7}\right)^3$.

91. (b) $P(\overline{A}) + P(\overline{B}) = 0.35 + 0.85 = 1.2$.

92. (a) Since $\overline{E_1} \cap \overline{E_2} = \overline{E_1 \cup E_2}$

and $(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2}) = \phi$

$P\{(E_1 \cup E_2) \cap (\overline{E_1} \cap \overline{E_2})\} = P(\phi) = 0 < \frac{1}{4}$.

93. (c) $P(\text{non-occurrence of } A_i) = 1 - \frac{1}{i+1} = \frac{i}{i+1}$

$\therefore\ P(\text{non-occurrence of any of events})$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{2}{3}\right) .... \left\{\frac{n}{n+1}\right\} = \frac{1}{n+1}.$

94. (b) Probability of getting at least one head in n tosses

$= 1 - \left(\frac{1}{2}\right)^n \ge 0.9 \Rightarrow \left(\frac{1}{2}\right)^n \le 0.1 \Rightarrow 2^n \ge 10 \Rightarrow n \ge 3$

Hence least value of $n = 4$.

95. (c) Second white ball can draw in two ways.

(i) First is white and second is white

Probability $= \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

(ii) First is black and second is white

Probability $= \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$

Hence required probability $= \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$.

96. (c) The probaility of drawing two cards of same suit $= \frac{13}{52} \times \frac{13}{52}$ and it can be of any suit out of 4.

So $P(A) = \frac{4 \times 13 \times 13}{52 \times 52} = \frac{1}{4}$ and $P(B) = \frac{5}{36}$

Thus $P(A \cap B) = P(A).P(B) = \frac{1}{4} \times \frac{5}{36} = \frac{5}{144}$.

97. (a) The required probability

$= \frac{2}{9} \times \frac{1}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 1 \times 1 \times 1 = \frac{1}{1260}.$

98. (b) Exhaustive number of cases $= 6^3 = 216$

Obviously, the second number has to be greater than unity. If the second number is $i\ (i > 1)$, then the first can be chosen in $i-1$ ways and the third in $6-i$ ways and hence three numbers can be chosen in $(i-1) \times 1 \times (6-i)$ ways. But the second number can be $2, 3, 4, 5$. Thus the favourable number of cases

$= \sum_{i=2}^{5} (i-1)(6-i) = 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1 = 20$

Hence the required probability $= \frac{20}{216} = \frac{5}{54}$.

99. (b) There are four aces and 48 other cards. Therefore the required probability $= \frac{48 \cdot 47 \cdot .... \cdot 39}{52 \cdot 51 \cdot .... \cdot 43} \cdot \frac{4}{42} = \frac{164}{4165}$.

100. (d) The probability that one test is held $= 2 \times \frac{1}{5} \times \frac{4}{5} = \frac{8}{25}$

Probability that one test is held on both days

$=\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$

Thus the probability that the student misses at least one test $=\frac{8}{25}+\frac{1}{25}=\frac{9}{25}$.

101. (c) Total cases $=36$. Favourable cases $=2+4+2=8$

$\therefore$ The required probability $=\frac{8}{36}=\frac{2}{9}$.

102. (d) **Trick :** As we know, the sum will be either even or odd but even is more likely to occur than odd (given). Therefore, the probability is greater than $\frac{1}{2}$ which is given in only one option.

103. (d) There are two mutually exclusive cases for the event.

$A=$ India wins the toss and wins the match.

$B=$ India losses the toss and wins the match

Required probability

$=P(A)+P(B)=\frac{3}{4}\times\frac{4}{5}+\frac{1}{4}\times\frac{1}{2}=\frac{29}{40}$.

104. (b) Probability of getting king $=\frac{4}{52}=\frac{1}{13}$

Probability of getting queen $=\frac{4}{52}=\frac{1}{13}$

P (king or queen) $=\frac{1}{13}+\frac{1}{13}=\frac{2}{13}$.

105. (c) Required probability $=\frac{4}{52}\times\frac{4}{51}=\frac{4}{663}$.

106. (d) A denote the event getting I;

B denote the event getting II;

C denote the event getting III;

and D denote the event getting fail.

Obviously, these four event are mutually exclusive and exhaustive, therefore

$P(A)+P(B)+P(C)+P(D)=1\Rightarrow P(D)=1-0.95=0.05$.

107. (b) Probability of success $=\frac{2}{6}=\frac{1}{3}=p$

Probability of failure $=1-\frac{1}{3}=\frac{2}{3}=q$

Probability that success occurs in even number of tosses

$=P(FS)+P(FFFS)+P(FFFFFS)+\dots\dots\dots$

$=pq+q^3p+q^5p+\dots\dots=\frac{pq}{1-q^2}=\frac{2}{5}$.

108. (a) Number of ways $=6\times 6=36$

Sample space $=\left\{\begin{matrix}(6,1)\ (6,2)\ (6,3)\ (6,4)\\(6,5)\ (1,6)\ (2,6)\ (3,6)\\(4,6)\ (5,6)\ (6,6)\end{matrix}\right\}$

Probability of at least one 6

$=P$ (one 6) $+P=\frac{10}{36}+\frac{1}{36}=\frac{11}{36}$. (Both 6)

109. (b) Numbers multiple of $5=(5,10,15,20,25,30)$

Numbers multiple of $7=(7,14,21,28)$

Multiple of both $=(0)$; $P(5\text{ or }7)=\frac{10}{30}=\frac{1}{3}$.

110. (c) Person has to miss all times probability of it will be $\left(\frac{1}{4}\right)^5=\frac{1}{1024}$.

111. (a) Appearance of head on fifth toss does not depend on the outcomes of first four tosses. Hence

P(head on 5th toss) $=\frac{1}{2}$.

112. (c) The condition will be satisfied, if both get $0,1,2$ or 3 heads.

$\therefore$ Either 0 head by A and 0 head by B

or 1 head by A and 1 head by B

or 2 head by A and 2 head by B

or 3 head by A and 3 head by B

$\therefore$ Required probability

$=\left[\frac{1}{8}\times\frac{1}{8}+\frac{3}{8}\times\frac{3}{8}+\frac{3}{8}\times\frac{3}{8}+\frac{1}{8}\times\frac{1}{8}\right]=\frac{5}{16}$.

113. (a) Required pair $=(12,88),(13,87)\dots\dots\dots(88,12)$

Total number of such pairs are 77 and total number of pairs (for which sum is 100) = 99

Required probability $=\frac{7}{9}$

114. (c) Required combinations are (2, 2, 1), (1, 2, 2), (2, 1, 2), (1, 3, 1,), (3, 1, 1) and (1, 1, 3)

Required probability $=\frac{6}{4^3}=\frac{6}{64}=\frac{3}{32}$.

115. (b) The last digit of square will be 1 or 5 only when the integer is 1, 5 or 9.

Required probability $=\frac{3}{10}$.

116. (b) If both integers are even then product is even.
If both integers are odd then product is odd.
If one integer is odd and other is even, then product is even. Required probability $=\frac{2}{3}$.

117. (d) Probability of correct bit appearing is $(1-p)$

Probability of correct number $=(1-p)^{16}$

and hence probability of incorrect number $=1-(1-p)^{16}$.

118. (d) P(at least $1H$)) $=1-P$ (No head)

$=1-P$ (four tail) $=1-\frac{1}{16}=\frac{15}{16}$.

119. (d) We know a leap year is fallen within 4 years, so its probability is $\frac{25}{100}=\frac{1}{4}$

53rd Sunday is a leap year $=\frac{1}{4}\times\frac{2}{7}=\frac{2}{28}$

Similarly probability of 53rd Sunday in a non-leap year $= \frac{75}{100} \times \frac{1}{7} = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$

Required probability $= \frac{2}{28} + \frac{3}{28} = \frac{5}{28}$.

120. (a) Required probability $= \frac{3}{6} = \frac{1}{2}$.

121. (c) Required probability $= \frac{3}{6} = \frac{1}{2}$.

122. (c) Required probability $= \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$.

123. (d) P (at least one head) = $1 - P$ (0 head)

$= 1 - P$ (All tail) $= 1 - \frac{1}{8} = \frac{7}{8}$.

124. (d) Required situation = {(5, 6), (6, 5), (5, 5)}

$\therefore$ Required probability $= \frac{3}{36} = \frac{1}{12}$.

125. (b) Let 100 students studying in which 60 % girls and 40 % boys.
Boys = 40, Girls = 60

25% of boys offer Maths = $\frac{25}{100} \times 40 = 10$ Boys

10% of girls offer Maths = $\frac{10}{100} \times 60 = 6$ Girls

It means, 16 students offer Maths.

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$.

126. (c) Total number of ways $= 6 \times 6 = 36$
Favourable number of ways
= {(1,1) (1,2) (1,3) (1,4) (1, 5) (1,6)}

Required probability = $\frac{6}{36} = \frac{1}{6}$.

127. (c) Total number of digits in any number at the unit place is 10.
$\therefore$ $n(S)$ = 10
To get the last digit in product is 1, 3, 5, or 7, it is necessary the last digit in each number must be 1, 3, 5 or 7.

$n(A) = 4$, $\therefore P(A) = \frac{4}{10} = \frac{2}{5}$

Required probability $= \left(\frac{2}{5}\right)^4 = \frac{16}{625}$.

128. (a) Total number of ways $= 2^n$

If head comes odd times, then favourable ways $= 2^{n-1}$.

Required probability $= \frac{2^{n-1}}{2^n} = \frac{1}{2}$.

129. (b) There are 366 days in a leap year, in which 52 weeks and two days, The combination of 2 days –
Sunday – Monday, Monday – Tuesday, Tuesday – Wednesday, Wednesday – Thursday, Thursday – Friday, Friday – Saturday, Saturday – Sunday

P(53 Fridays) = $\frac{2}{7}$; P(53 Saturdays) $= \frac{2}{7}$

P(53 Fridays and 53 Saturdays) $= \frac{1}{7}$

P(53 Fridays or Saturdays) = P(53 Fridays) + P(53 Saturdays) – P(53 Fridays and Saturdays)

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$.

130. (d) Total number of numbers $= (5)^2$
Favourable cases = [12, 24, 32, 44, 52]

Required probability $= \frac{5}{25} = \frac{1}{5}$.

131. (b) There are two conditions.
(i) When first is an ace of heart and second one is non-ace of heart $= \frac{1}{52} \times \frac{51}{52} \Rightarrow \frac{1}{52}$

(ii) When first is non-ace of heart and second one is an ace of heart $= \frac{51}{52} \times \frac{1}{51} = \frac{1}{52}$

Required probability $= \frac{1}{52} + \frac{1}{52} = \frac{1}{26}$.

132. (a) Probability problem is not solved by $A = 1 - \frac{1}{2} = \frac{1}{2}$

Probability problem is not solved by $B = 1 - \frac{1}{3} = \frac{2}{3}$

Probability problem is not solved by $C = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of solving the problem $= 1 - P$ (not solved by any body)

$P = 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$.

133. (b) Total number of outcomes $= 36$
Favourable number of outcomes = 6
i.e., (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6),

Required probability $= \frac{6}{36} = \frac{1}{6}$.

134. (d) Total number of outcomes $= 36$,
For sum = 7, favourable outcomes are 6 *i.e.*,
(6, 1); (5, 2); (4, 3); (3, 4); (2, 5); (1, 6)
For sum = 12, favourable outcome is only 1 *i.e.*, (6, 6)

Probability $= \frac{6}{36} + \frac{1}{36} = \frac{7}{36}$.

135. (a) Total number of ways $= 20 \times 19 \times 18 \times 17$
The number of ways in which no pair is selected
$= 20 \times 18 \times 16 \times 14$
Hence the required probability

$= 1 - \frac{20\times18\times16\times14}{20\times19\times18\times17} = \frac{99}{323}$.

136. (b) We have total number of balls = 10

$\therefore$ Number of red balls = 3

and number of black balls = 7

and number of balls in the bag = 3 + 7 = 10

$\therefore$ The probability for taking out one red ball out of 10 balls = $\frac{3}{10}$ and the probability for taking out one red ball out of remaining 9 balls = $\frac{2}{9}$

$\therefore$ Probability for both balls to be red

i.e., $p = \frac{3}{10}\times\frac{2}{9} = \frac{1}{15}$.

137. (b) A leap year contain 366 days *i.e.* 52 weeks and 2 days, clearly there are 52 Sundays in 52 weeks

For the remaining two days, we may have any of the following two days.

(i) Sunday and Monday, (ii) Monday and Tuesday,

(iii) Tuesday and Wednesday, (iv) Wednesday and

Thursday, (v) Thursday and Friday, (vi) Friday and

Saturday and (vii) Saturday and Sunday.

Now for 53 Sundays, one of the two days must be Sunday Hence required probability $= \frac{2}{7}$.

138. (b) Required probability $= \frac{1}{2}\times\frac{2}{5} + \frac{1}{2}\times\frac{4}{6} = \frac{8}{15}$.

139. (b) Required probability $= \frac{1}{2}\left(\frac{3}{5}+\frac{2}{6}\right) = \frac{9+5}{30} = \frac{7}{15}$.

140. (c) Total number of pens in first bag = 4 + 2 = 6

and total number of pens in second bag = 3 + 5 = 8.

The probability of selecting a white pen from first bag = $\frac{4}{6} = \frac{2}{3}$ and probability of selecting a white pen from second bag = $\frac{3}{8}$.

$\therefore$ Required probability that both the pens are white $= \frac{2}{3}\times\frac{3}{8} = \frac{1}{4}$.

141. (b) From bag A, $P\,(\text{red ball}) = p_1 = \frac{3}{8}$

$P(\text{ black ball}) = p_2 = \frac{5}{8}$

From bag B, $P\,(\text{red ball}) = p_3 = \frac{6}{10}$

$P\,(\text{black ball}) = p_4 = \frac{4}{10}$

Required probability

$= P$ [(red ball from bag A and black from B) or (red from bag B and black from A)]

$= p_1 \times p_4 + p_2 \times p_3 = \frac{3}{8}\times\frac{4}{10} + \frac{5}{8}\times\frac{6}{10} = \frac{21}{40}$.

142. (a) It is obvious.

143. (c) Required probability is $1 - p$ (no head)

$= 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$.

144. (d) Out of 5 horses only one is the winning horse. The probability that Mr. A selected the losing horse $= \frac{4}{5}\times\frac{3}{4}$

The probability that Mr. A selected the winning horse $= 1 - \frac{4}{5}\times\frac{3}{4} = \frac{2}{5}$.

145. (c) Here $P(X) = \frac{3}{5}$, $P(Y) = \frac{1}{2}$

$\therefore$ Required probability = $P(X).P(\overline{Y}) + P(\overline{X})P(Y)$

$= \left(\frac{3}{5}\right)\left(1-\frac{1}{2}\right) + \left(1-\frac{2}{5}\right)\left(\frac{1}{2}\right) = \frac{3}{5}.\frac{1}{2} + \frac{2}{5}.\frac{1}{2} = \frac{1}{2}$.

146. (c) Here $P(A) = \frac{3}{4}, P(B) = \frac{4}{5}$

$\therefore$ Required probability $= P(A).P(\overline{B}) + P(\overline{A}).P(B) = \frac{7}{20}$.

147. (a) Total number of favorable ways

$n(E) = \{(6,6,4)\},(6,4,6),(4,6,6),(5,5,6),(5,6,5)(6,5,5)\} = 6$

Total number of ways = $n(S) = 6\times6\times6 = 216$

$\therefore$ Required probability = $\frac{n(E)}{n(S)} = \frac{6}{216} = \frac{1}{36}$.

148. (d) We have, the number of divisible by 6 in the series form 1 to 90 is 15.

$\therefore$ Number divisible by 8 in the series from 1 to 90 is equal to 11 and the number divisible by both 6 and 8 in series from 1 to 90 = 3.

$\therefore$ Probability of choosing the number divisible by 6 or 8 is $P = \frac{15+11-3}{90}$ or $P = \frac{23}{90}$.

149. (a) The probability that the problem is not being solved by any of two students = $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$ = $\frac{1}{3}$ and probability that the problem is solved = $1 - \frac{1}{3} = \frac{2}{3}$.

150. (d) For a particular house being selected , Probability = $\frac{1}{3}$

Probability (all the persons apply for the same house)

$= \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3 = \frac{1}{9}$.

151. (b) Required probability

$$=\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)+\dots$$

$$=\frac{\frac{5}{6}.\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2}=\frac{5}{36-25}=\frac{5}{11}.$$

152. (d) P (neither A nor B) = $P(\overline{A}\cap\overline{B})=P(\overline{A}).P(\overline{B})$

$P(\overline{A})=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$

$P(\overline{B})=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$

$\therefore\ P(\overline{A}).P(\overline{B})=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}.$

153. (a) Current in the upper part will flow only if both the switches a and b are closed

$\therefore$ Their probability = p.p. = p^2

Now current will flow in lower part of c, if c is closed, its probability is p. Thus current will flow from A to B if current flows either in upper part or flow in lower part.

Required probability = p^2+p.

Use of permutations and combinations in probability

1. (c) Required probability $=\frac{^{13}C_2}{^{52}C_2}=\frac{13.12}{52.51}=\frac{1}{17}.$

2. (c) Required probability $=\frac{^{26}C_3\,.\,^{26}C_3}{^{52}C_6}.$

3. (c) The required probability is given by

$\frac{^{39}C_1}{^{52}C_1}\times\frac{^{39}C_1}{^{52}C_1}\times\frac{^{13}C_1}{^{52}C_1}=\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{9}{64}.$

4. (b) The total number of ways in which 2 integers can be chosen from the given 20 integers $^{20}C_2$.

The sum of the selected numbers is odd if exactly one of them is given and one is odd.

$\therefore$ Favourable number of outcomes $={}^{10}C_1\times{}^{10}C_1$

$\therefore$ Required probability $=\frac{^{10}C_1\times{}^{10}C_1}{^{20}C_2}=\frac{10}{19}.$

5. (c) Required probability $=\frac{^{3}C_3+{}^{7}C_3+{}^{4}C_3}{^{14}C_3}$

$=\frac{1+35+4}{14.13.2}=\frac{40}{14.26}=\frac{10}{91}.$

6. (a) Total number of ways $={}^{9}C_4$, 2 children are chosen in $^{4}C_2$ ways and other 2 persons are chosen in $^{5}C_2$ ways.

Hence required probability $=\frac{^{4}C_2\times{}^{5}C_2}{^{9}C_4}=\frac{10}{21}.$

7. (c) Required probability is $1-P$

(Both odd numbers are chosen)

$=1-\frac{^{13}C_2}{^{25}C_2}=1-\frac{13.12}{25.24}=\frac{37}{50}.$

8. (a) Required probability $=\frac{^{12}C_2}{^{30}C_2}=\frac{12\times11}{30\times29}=\frac{22}{145}.$

9. (b) Required probability $=\frac{^{7}C_2}{^{11}C_2}=\frac{7\,.\,6}{11\,.\,10}=\frac{21}{55}.$

10. (a) $7, 11$ have always to be in that group of three, therefore 3rd ticket may be chosen in 18 ways. Hence required probability is $\frac{18}{^{20}C_3}=\frac{18.3.2}{20.19.18}=\frac{3}{190}.$

11. (a) Mohan can gets one prize, 2 prizes or 3 prizes and his chance of failure means he get no prize.

Number of total ways $={}^{12}C_3=220$

Favourable number of ways to be failure $={}^{9}C_3=84$

Hence required probability $=1-\frac{84}{220}=\frac{34}{55}.$

12. (d) Required probability $=\frac{^{3}C_1+{}^{7}C_1}{^{10}C_1}=\frac{10}{10}.$

13. (a) Required probability $=\frac{^{4}C_1.11}{^{15}C_2}=\frac{4.11.2}{15.14}=\frac{44}{105}.$

14. (c) Required probability $=\frac{^{4}C_2\times{}^{6}C_1}{^{18}C_3}=\frac{3}{68}.$

15. (c) Required probability $=\frac{^{7}C_3}{^{9}C_5}+\frac{^{7}C_5}{^{9}C_5}=\frac{56}{126}=\frac{4}{9}.$

16. (b) Total ways of arrangements $=\frac{8!}{2!\,.\,4!}$

$\bullet\, w \bullet\, x \bullet\, y \bullet\, z \bullet$

Now 'S' can have places at dot's and in places of w, x, y, z we have to put $2A's$, one I and one N.

Therefore favourable ways $=5\left(\frac{4!}{2!}\right)$

Hence required probability $=\frac{5.4!\,2!\,4!}{2!\,8!}=\frac{1}{14}.$

17. (d) Required probability $=\frac{^{8}C_4}{2^8}.$

18. (a) In 50 tickets 14 are of prize and 36 are blank. Number of ways both the tickets are blank $={}^{36}C_2$

Thus the probability of not winning the prize $=\frac{^{36}C_2}{^{50}C_2}=\frac{18}{35}.$

Hence probability of winning the prize $=1-\frac{18}{35}=\frac{17}{35}.$

19. (b) Probability that both balls are white $= \frac{^7C_2}{^{15}C_2} = \frac{1}{5}$

Probability that both balls are black $= \frac{^8C_2}{^{15}C_2} = \frac{4}{15}$

Probability that one ball is white and one is black

$$= \frac{^7C_1 \times ^8C_1}{^{15}C_2} = \frac{8}{15}.$$

20. (b) Total number of ways

$= ^4C_1 \times ^6C_4 + ^4C_2 \times ^6C_3 + ^4C_3 \times ^6C_2 + ^4C_4 \times ^6C_1 + ^6C_5$

$= 60 + 120 + 60 + 6 + 6 = 252$

No. of ways in which at least one woman exist are

$= ^4C_1 \times ^6C_4 + ^4C_2 \times ^6C_3 + ^4C_3 \times ^6C_2 + ^4C_4 \times ^6C_1 = 246$

Hence required probability $= \frac{246}{252} = \frac{41}{42}$.

21. (c) Total number of ways to form the numbers of three digit with 1, 2, 3 and 4 are $^4P_3 = 4! = 24$

If the numbers are divisible by three then their sum of digits must be 3, 6 or 9

But sum 3 is impossible. Then for sum 6, digits are 1, 2, 3

Number of ways $= 3!$

Similarly for sum 9, digits are 2, 3, 4. Number of ways =3!

Thus number of favourable ways $= 3! + 3!$

Hence required probability $= \frac{3! + 3!}{4!} = \frac{12}{24} = \frac{1}{2}$.

22. (b) Required probability $= \frac{^4C_1 \times ^4C_1 \times ^4C_1}{^{52}C_3}$

23. (c) Total number of ways $= \frac{10!}{2!}$

Favourable number of ways for 'I' come together is $9!$

Thus probability that 'I' come together

$= \frac{9! \times 2!}{10!} = \frac{2}{10} = \frac{1}{5}$.

Hence required probability $= 1 - \frac{1}{5} = \frac{4}{5}$.

24. (a) Let E_i denote the event that the i^{th} object goes to the i^{th} place, we have $P(E_i) = \frac{(n-1)!}{n!} = \frac{1}{n}, \forall\ i$

and $P(E_i \cap E_j \cap E_k) = \frac{(n-3)!}{n!}$ for $i < j < k$

Since we can choose 3 places out of *n* in $^nC_3$ ways.

The probability of the required event is

$^nC_3 . \frac{(n-3)!}{n!} = \frac{1}{6}$.

25. (c) $n =$ Total number of ways $= 6^5$

A total of 12 in 5 throw can be obtained in following two ways –

(i) One blank and four $3's = ^5C_1 = 5$

or (ii) Three $2's$ and two $3's = ^5C_2 = 10$

Hence, the required probability $= \frac{15}{6^5} = \frac{5}{2592}$.

26. (c) $p_1 = \frac{6}{36} = \frac{1}{6}$

To find p_2, the total number of ways $= 6^4$ and since two numbers out of 6 can be selected in $^6C_2$ ways *i.e.* 15 ways and corresponding to each of these ways, there are 8 ways *e.g.*, $(1, 1, 1, 2)\ (1, 1, 2, 1)$.....

Thus favourable ways $= 15 \times 8 = 120$

Hence $p_2 = \frac{120}{6^4} = \frac{5}{54}$. Hence $p_1 > p_2$.

27. (a) Total number of ways $= n!$.

Favourable cases $= 2(n-1)!$

Hence required probability $= \frac{2(n-1)!}{n!} = \frac{2}{n}$.

28. (a) There are 8 prime numbers $2, 3, 5, 7, 11, 13, 17, 19$.

Hence required probability

$= \frac{^8C_2}{^{20}C_2} = \frac{8.7}{20.19} = \frac{14}{95}$.

29. (a) Required probability $= \frac{^9C_2}{^{15}C_2} = \frac{9 \times 8}{15 \times 14} = \frac{12}{35}$.

30. (b) Required probability $= \frac{^5C_1}{^8C_1} = \frac{5}{8}$.

31. (c) Total number of triangles which can be formed is equal to $^6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

Number of equilateral triangles = 2

$\therefore$ Required probability $= \frac{2}{20} = \frac{1}{10}$.

32. (b) Required probability $= \frac{^3C_1 \times ^3C_1}{^6C_2} = \frac{3 \times 3}{15} = \frac{3}{5}$.

33. (a) Five places for 5 volumes of Mathematics from 25 places of books can be select in $^{25}C_5$ ways.

In these places, we are not to permute the 5 volumes since order of these 5 volumes is fixed *i.e.*, V_1, V_2, V_3, V_4, V_5.

Remaining 20 books can be arranged in $20!$ ways.

$\therefore$ Favourable ways $= {}^{25}C_5 . 20! = \frac{25!.20!}{5!\,20!} = \frac{25!}{5!}$

Also total number of ways $= 25!$

$\therefore$ Probability $= \frac{25!}{5!\,25!} = \frac{1}{5!}$.

34. (c) Required probability

$$= \frac{{}^5C_3 \times {}^{10}C_8}{{}^{15}C_{11}} + \frac{{}^5C_4 \times {}^{10}C_7}{{}^{15}C_{11}} + \frac{{}^5C_5 \times {}^{10}C_6}{{}^{15}C_{11}} = \frac{12}{13}.$$

35. (b) $P_1 = \frac{{}^{15}C_2 \times {}^{27}C_2}{{}^{42}C_4} = \frac{27}{82}$

and $P_2 = \frac{{}^{30}C_4 \times {}^{54}C_4}{{}^{84}C_8} = \frac{17.29.45.53}{11.79.82.83}$

(After simplification)

Hence $P_1 > P_2$.

36. (b) m rupee coins and n ten paise coins can be placed in a line in $\frac{(m+n)!}{m!\,n!}$ ways.

If the extreme coins are ten paise coins, then the remaining $n-2$ ten paise coins and m one rupee coins can be arragned in a line in $\frac{(m+n-2)!}{m!(n-2)!}$ ways.

Hence the required probability

$$= \frac{\frac{(m+n-2)!}{m!(n-2)!}}{\frac{(m+n)!}{m!\,n!}} = \frac{n(n-1)}{(m+n)(m+n-1)}.$$

37. (c) The total number of functions from A to itself is n^n and the total number of bijections from A to itself is $n!$. {Since A is a finite set, therefore every injective map from A to itself is bijective also}.

$\therefore$ The required probability $= \frac{n!}{n^n} = \frac{(n-1)!}{n^{n-1}}$.

38. (a) Required probability $= \frac{{}^{12}C_1}{{}^{20}C_1} = \frac{3}{5}$.

39. (c) Number of ways of selecting two good mangoes $= {}^6C_2 = 15$. Number of ways that at least one of the two selected mangoes is to be good $= {}^6C_1 \times {}^9C_1 = 54$

$\therefore$ Required probability $= \frac{15}{64} = \frac{5}{18}$.

40. (a) Required probability $= \frac{{}^5C_1 \times {}^8C_1}{{}^{13}C_2} + \frac{{}^5C_2}{{}^{13}C_2} = \frac{25}{39}$.

41. (d) The total number of ways of choosing two numbers out of $1, 2, 3, ...30$ is ${}^{30}C_2 = 435$.

Since $a^2 - b^2$ is divisible by 3 if either a and b both are divisible by 3 or none of a and b is divisible by 3. Thus the favourable number of cases $= {}^{10}C_2 + {}^{20}C_2 = 235$.

Hence the required probability $= \frac{235}{435} = \frac{47}{87}$.

42. (d) Let each of the friend have x daughters. Then the probability that all the tickets go to the daughters of A is $\frac{{}^xC_3}{{}^{2x}C_3}$. Therefore

$\frac{{}^xC_3}{{}^{2x}C_3} = \frac{1}{20} \Rightarrow x = 3$.

43. (b) There are 10 digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. The last two digits can be dialled in ${}^{10}P_2 = 90$ ways, out of which only one way is favourable, thus the required probability $= \frac{1}{90}$.

44. (c) Total number of ways in which 3 balls can be drawn are ${}^9C_3 = 84$.

Since the drawn balls can't be of red colour, as these are 2 in numbers.

Therefore favourable number of ways $= {}^3C_3 + {}^4C_3$.

Hence the required probability $= \frac{5}{84}$.

45. (b) 6 boys and 6 girls can be arranged in a row in $12!$ ways. If all the 6 girls are together, then the number of arrangement are $7! \times 6!$.

Hence required probability $= \frac{7!.6!}{12!}$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2}{12 \times 11 \times 10 \times 9 \times 8} = \frac{1}{132}.$$

46. (b) Total number of ways to select 6 persons out of $11 = {}^{11}C_6$

Ways to select two ladies $= {}^4C_2$

Ways to select four men $= {}^7C_4$

$\therefore$ Probability that committee contain 2 ladies

$$= \frac{{}^4C_2 . {}^7C_4}{{}^{11}C_6} = \frac{5}{11}.$$

47. (a) Required probability $= \frac{{}^3C_1}{{}^7C_1} \times \frac{{}^2C_1}{{}^6C_1} = \frac{1}{7}$.

48. (d) Required probability $= \frac{{}^5C_3}{{}^{16}C_3} = \frac{5 \times 4 \times 3}{16 \times 15 \times 14} = \frac{1}{56}$.

49. (b) Required probability

$$= \frac{^{20}C_1 \times {}^{20}C_1}{^{40}C_2} = \frac{20 \times 20 \times 2}{40 \times 39} = \frac{20}{39}.$$

50. (d) 3 cards are drawn out of 26 red cards (favourable)

$$= \frac{^{26}C_3}{^{52}C_3} = \frac{26\,!}{3\,!\,23\,!} \times \frac{3\,!\,49\,!}{52\,!} = \frac{2}{17}.$$

51. (d) Required probability

$$= \frac{^{2}C_1 \times {}^{3}C_1 \times {}^{4}C_1}{^{9}C_3} = \frac{2 \times 3 \times 4}{\left(\frac{9 \times 8 \times 7}{3 \times 2}\right)} = \frac{2}{7}.$$

52. (a) Total number of cases obtained by taking multiplication of only two numbers out of $100 = {}^{100}C_2$. Out of hundred $(1, 2, ...100)$ given numbers, there are the numbers $3, 6, 9, 12, ...99$, which are 33 in number such that when any one of these is multiplied with any one of remaining 67 numbers or any two of these 33 are multiplied, then the resulting products is divisible by 3. Then the number of numbers which are the products of two of the given number are divisible by $3 = {}^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2$. Hence the required probability

$$= \frac{^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2}{^{100}C_2} = \frac{2739}{4950} = 0.55.$$

53. (a) By using digits 1, 2, 3, 4, 5, 6 and 8, total 5 digits numbers = $^{7}P_5$

And number of ways to form the numbers, they have even digit at both ends $= 4 \times 3 \times {}^{5}P_3$

$\therefore$ Probability $= \dfrac{4 \times 3 \times {}^{5}P_3}{^{7}P_5} = \dfrac{2}{7}$.

54. (a) Probability $= \dfrac{^{3}C_1 . {}^{4}C_1 . {}^{5}C_1}{^{12}C_3} = \dfrac{3}{11}$.

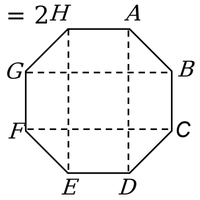
55. (d) Here only 2 rectangles are formed *ADEH, GFCB*.

Number of favourable cases = 2

and total number of cases $= {}^{8}C_4$

Required probability

$$= \frac{2}{^{8}C_4} = \frac{1}{35}.$$



56. (d) Total socks $= 5 + 4 = 9$

The number of ways to select 2 socks out of 9 = $^{9}C_2$

If number of ways to select both brown socks $= {}^{5}C_2$

And number of ways to select both white socks $= {}^{4}C_2$

$\therefore P$ (Both brown or white)

$$= \frac{^{5}C_2 + {}^{4}C_2}{^{9}C_2} = \frac{16}{36} = \frac{48}{108}.$$

57. (c) Required probability $= \dfrac{^{37}C_2}{^{38}C_3} = \dfrac{\binom{37}{2}}{\binom{38}{3}}$.

58. (b) Four boys can be arranged in 4! ways and three girls can be arranged in 3! ways.

$\therefore$ The favourable cases $= 4! \times 3!$

Hence the required probability

$$= \frac{4! \times 3!}{7!} = \frac{6}{7 \times 6 \times 5} = \frac{1}{35}.$$

59. (a) Required probability $= \dfrac{^{88}C_3}{^{90}C_5} = \dfrac{2}{801}$.

60. (a) Total number of ways $= {}^{15}C_{11}$

Favourable cases $= {}^{8}C_6 \times {}^{7}C_5$

Required probability $= \dfrac{^{8}C_6 \times {}^{7}C_5}{^{15}C_{11}}$.

61. (d) P(Black or Red) $= \dfrac{^{5}C_1 + {}^{3}C_1}{^{12}C_1} = \dfrac{2}{3}$.

62. (c) The total number of ways in which 2 integers can be chosen from the given 30 integers is $^{30}C_2$. The sum of the selected numbers is odd if exactly one of them is even and one is odd.

Favourable number of outcomes $= {}^{15}C_1 . {}^{15}C_1$

$\therefore$ Required probability $= \dfrac{^{15}C_1 . {}^{15}C_1}{^{30}C_2} = \dfrac{15}{29}$.

63. (c) The total number of ways in which 3 integers can be chosen from first 20 integers is $^{20}C_3$. The product of three integers will be even if at least one of them is even.

$\therefore$ Required probability = 1 – Probability that none is even

$$= 1 - \frac{^{10}C_3}{^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}.$$

64. (d) Total ways = 2! $^{6}C_2 = 30$

Favourable cases $= 30 - 6 = 24$

Required probability $= \dfrac{24}{30} = \dfrac{4}{5}$.

65. (b) 3 ball can be drawn in $^{18}C_3$ ways

Favourable cases $= {}^{6}C_3$

$\therefore$ Required probability

$$= \frac{^{6}C_3}{^{18}C_3} = \frac{6 \times 5 \times 4}{18 \times 17 \times 16} = \frac{5}{204}.$$

66. (d) Required probability $= \dfrac{4.5.6}{^{15}C_3} = \dfrac{24}{91}$.

67. (b) Required probability $=\frac{(10.15)}{^{25}C_2}=\left(\frac{2.10.15}{25.24}\right)=\frac{1}{2}$.

68. (c) Required probability is $\frac{^4C_1}{^{52}C_3}=\frac{1}{5525}$.

69. (a) Required probability is $1-P$ (No Ace)

$$=1-\frac{^{12}C_2}{^{16}C_2}=1-\frac{12.11}{16.15}=\frac{9}{20}.$$

70. (a) The total number of cases are 2^{100}

The number of favourable ways

$$^{100}C_1+{}^{100}C_3+......+{}^{100}C_{99}=2^{100-1}=2^{99}$$

Hence required probability $=\frac{2^{99}}{2^{100}}=\frac{1}{2}$.

71. (a) We have the following three pattern :

(i) Red, white $P(A)=\frac{3\times 4}{^{12}C_2}$

(ii) Red, blue $P(B)=\frac{3\times 5}{^{12}C_2}$

(iii) Blue, white $P(C)=\frac{4\times 5}{^{12}C_2}$

Since all these cases are exclusive, so the required probability

$$=\frac{(12+15+20)}{^{12}C_2}=\frac{(47\times 2)}{(12\times 11)}=\frac{47}{66}.$$

72. (a) Total ways $=10!$

Two boys can sit side by side in $2\times 9!$ ways.

So probaibility $=\frac{2\times 9!}{10!}=\frac{1}{5}$

Thus the probability that they are not seated together is $1-\frac{1}{5}=\frac{4}{5}$.

73. (a) Out of 9 socks, 2 can be drawn in $^9C_2$ ways.

Two socks drawn from the drawer will match if either both are brown of both are blue. Therefore favourable number of cases is $^5C_2+{}^4C_2$.

Hence the required probability $=\frac{^5C_2+{}^4C_2}{^9C_2}=\frac{4}{9}$.

74. (c) Total number of ways $=5!$

Favourable number of ways $2.4!$

Hence required probability $=\frac{2.4!}{5!}=\frac{2}{5}$.

75. (c) Required probability

= Either the balls are red or the balls are black

$$=\frac{^8C_2}{^{15}C_2}+\frac{^7C_2}{^{15}C_2}=\frac{28+21}{105}=\frac{49}{105}=\frac{7}{15}.$$

76. (b) Total number of ways $={}^{80}C_2$

and favorable ways $={}^{20}C_2$

Required probability $P=\frac{^{20}C_2}{^{80}C_2}=\frac{19}{316}$.

77. (d) Required probability

$$=\frac{^5C_1\,{}^4C_1}{^{12}C_1\,{}^{12}C_1}+\frac{^7C_1\,{}^8C_1}{^{12}C_1\,{}^{12}C_1}=\frac{20+56}{144}=\frac{76}{144}.$$

78. (b) As we know the total number of mappings is n^m and number of injective mappings is $\frac{n!}{(n-m)!n^m}$.

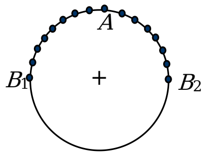
79. (a) Let there be n persons and $(n-2)$ persons not selected are arranged in places stated above by stars and the selected 2 persons can be arranged at places stated by dots (dots are $n-1$ in number) So the favourable ways are $^{n-1}C_2$ and the total ways are $^nC_2$, so

$$\times\bullet\times\bullet\times\bullet\times\bullet\times\bullet\times$$

$$P=\frac{^{n-1}C_2}{^nC_2}=\frac{(n-1)!2!(n-2)!}{(n-3)!2!n!}=\frac{n-2}{n}=1-\frac{2}{n}.$$

80. (d) Let A occupy any seat at the round table. Then there are 14 seats available for B.

If there are to be four persons between A and B.



Then B has only two ways to sit, as show in the fig.

Hence required probability $=\frac{2}{14}=\frac{1}{7}$.

81. (b) Let n = total no. of ways = 10!

m = favourable no. of ways = 2 × 5! . 5!

Since the boys and girls can sit alternately in 5 ! . 5! ways if we begin with a boy and similarly they can sit alternately in 5! . 5! ways if we begin with a girl

Hence, required probability = $\frac{m}{n}$

$$=\frac{2\times 5!.5!}{10!}=\frac{2\times 5!}{10\times 9\times 8\times 7\times 6}=\frac{1}{126}.$$

Odds in favour and odds against, Addition theorem on probability

1. (c) Required probability $=\frac{3}{5}$.

2. (d) Required probability $=1-\frac{3}{8}=\frac{5}{8}$.

3. (c) Probability of the card being a spade or an ace $=\frac{16}{52}=\frac{4}{13}$. Hence odds in favour is $4:9$.

So the odds against his winning is $9:4$.

4. (c) Required probability $=\frac{4}{4+5}=\frac{4}{9}$.

5. (b) Required probability $= \frac{6}{6+5} = \frac{6}{11}$.

6. (b) Probabilities of winning the race by three horses are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$.

Hence required probability $= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$.

7. (b) Let A and B be two given events. The odds against A are $5:2$, therefore $P(A) = \frac{2}{7}$.

The odds in favour of B are $6:5$,

therefore $P(B) = \frac{6}{11}$.

The required probability $= 1 - P(\overline{A})P(\overline{B})$

$$= 1 - \left(1 - \frac{2}{7}\right)\left(1 - \frac{6}{11}\right) = \frac{52}{77}.$$

8. (c) The probability of solving the question by these three students are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively.

$P(A) = \frac{1}{3}$; $P(B) = \frac{2}{7}$; $P(C) = \frac{3}{8}$

Then probability of question solved by only one student

$= P(A\overline{B}\,\overline{C}$ or $\overline{A}B\overline{C}$ or $\overline{A}\,\overline{B}C)$

$= P(A)\,P(\overline{B})\,P(\overline{C}) + P(\overline{A})\,P(B)P(\overline{C}) + P(\overline{A})P(\overline{B})P(C)$

$= \frac{1}{3}.\frac{5}{7}.\frac{5}{8} + \frac{2}{3}.\frac{2}{7}.\frac{5}{8} + \frac{2}{3}.\frac{5}{7}.\frac{3}{8} = \frac{25+20+30}{168} = \frac{25}{56}$.

9. (a) We have ratio of the ships A, B and C for arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively.

The probability of ship A for arriving safely $= \frac{2}{2+5} = \frac{2}{7}$

Similarly, for $B = \frac{3}{3+7} = \frac{3}{10}$ and for C = $\frac{6}{6+11} = \frac{6}{17}$

$\therefore$ Probability of all the ships for arriving safely

$$= \frac{2}{7} \times \frac{3}{10} \times \frac{6}{17} = \frac{18}{595}.$$

10. (a) Required probability $= \frac{(21)!2!}{(22)!} = \frac{1}{11} = \frac{1}{1+10}$.

Odds against = 10 : 1.

11. (a) We have $P(A+B) = P(A) + P(B) - P(AB)$

$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3} \Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

Thus, $P(A).P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} = P(AB)$

Hence events A and B are independent.

12. (b) Since here $P(A+B+C) = P(A) + P(B) + P(C)$

$= \frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{12}$, which is greater than 1.

Hence the statement is wrong.

13. (c) Since we have $P(A+B) = P(A) + P(B) - P(AB)$

$\Rightarrow 0.7 = 0.4 + P(B) - 0.2 \Rightarrow P(B) = 0.5$.

14. (d) $P(A+B) = P(A) + P(B) - P(AB) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$.

15. (d) Required probability is

$P(\text{Red} + \text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= P(\text{Red}) + P(\text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$.

16. (b) Here $P(X) = 0.3$; $P(Y) = 0.2$

Now $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

Since these are independent events, so $P(X \cap Y) = P(X).P(Y)$

Thus required probability $= 0.3 + 0.2 - 0.06 = 0.44$.

17. (b) $0.7 = 0.4 + x - 0.4x \Rightarrow x = \frac{1}{2}$.

18. (a) Since $P(A \cap \overline{B}) + P(A \cap B) = P(A)$

$\Rightarrow P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.25 - 0.15 = 0.1$

19. (a) Since events are mutually exclusive, therefore $P(AB) = 0$ *i.e.*, $P(A \cup B) = P(A) + P(B)$

$\Rightarrow 0.7 = 0.4 + x \Rightarrow x = \frac{3}{10}$.

20. (b) Required probability = A occurs and B does not occur or B occurs and A does not occur

$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$.

21. (d) Total number of ways $= (HH, HT, TH, TT)$

P (head on first toss) $= \frac{2}{4} = \frac{1}{2} = P(A)$

P (head on second toss) $= \frac{2}{4} = \frac{1}{2} = P(B)$

P (head on both toss) $= \frac{1}{4} = P(A \cap B)$

Hence required probability is,

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$.

22. (c) $P(A) + P(B)$ (Fundamental concept).

23. (c) We are given that $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$.

We know that if A and B are any two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 1 - P(\overline{A}) + 1 - P(\overline{B}) - 0.2$

$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - 0.8 = 1.2$.

24. (a) Let $P(A) = \frac{20}{100} = \frac{1}{5}$, $P(B) = \frac{10}{100} = \frac{1}{10}$

Since events are independent and we have to find

$P(A\cup B)=P(A)+P(B)-P(A).P(B)$

$=\frac{1}{5}+\frac{1}{10}-\frac{1}{5}\times\frac{1}{10}=\frac{3}{10}-\frac{1}{50}=\frac{14}{50}=\frac{14}{50}\times 100=28\%.$

25. (a) $P(A\cap B)=\frac{2}{8}+\frac{5}{8}-\frac{6}{8}=\frac{1}{8}.$

26. (d) $P(\text{neither } A \text{ nor } B)=P(\overline{A}\cap\overline{B})$

$=P(\overline{A}).P(\overline{B})=0.6\times 0.5=0.30.$

27. (a) It is obvious.

28. (b) Let A be the event to be multiple of 4 and B be the event to be multiple of 6

So, $P(A)=\frac{25}{100}$, $P(B)=\frac{16}{100}$ and $P(A\cap B)=\frac{8}{100}$

Thus required probability is

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{25}{100}+\frac{16}{100}-\frac{8}{100}=\frac{33}{100}.$

29. (b) $P(A\cup B)=P(A)+P(B)=\frac{1}{4}+\frac{1}{5}=\frac{9}{20}.$

{Since events are mutually exclusive, so $P(A\cap B)=0$}

30. (c) $P(A)=P(A\cap B)+P(A\cup B)-P(B)$

$=\frac{1}{3}+\frac{5}{6}-\frac{2}{3}=\frac{3}{6}=\frac{1}{2}.$

31. (a) Since we have

$P(A\cup B)+P(A\cap B)=P(A)+P(B)=P(A)+\frac{P(A)}{2}$

$\Rightarrow \frac{7}{8}=\frac{3P(A)}{2}\Rightarrow P(A)=\frac{7}{12}.$

32. (b) Required probability is

P[(A will die and B alive) or (B will die and A alive)]

$=P[(A\cap B')\cup(B\cap A')]$

Since events are independent, so

Required probability $=P(A).P(B')+P(B).P(A')$

$=p.(1-q)+q(1-p)=p+q-2pq$

33. (a) $P(A\cap B)=P(A).P(B)=\frac{1}{6}$

$P(\overline{A}\cap\overline{B})=\frac{1}{3}=1-P(A\cup B)$

$\Rightarrow \frac{1}{3}=1-[P(A)+P(B)]+\frac{1}{6}\Rightarrow P(A)+P(B)=\frac{5}{6}.$

Hence $P(A)$ and $P(B)$ are $\frac{1}{2}$ and $\frac{1}{3}$.

34. (a) Since events are independent.

So, $P(A\cap B')=P(A)\times P(B')=\frac{3}{25}$

$\Rightarrow P(A)\times\{1-2P(B)\}=\frac{3}{25}$ .....(i)

Similarly, $P(B)\times\{1-P(A)\}=\frac{8}{25}$ .....(ii)

On solving (i) and (ii), we get $P(A)=\frac{1}{5}$ and $\frac{3}{5}$.

35. (b) $0.8=0.3+x-0.3x\Rightarrow x=5/7.$

36. (d) It is obvious.

37. (c) $P(A)=P(B)$

As this gives $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

or $P(A)=2P(A)-P(A)\Rightarrow P(A\cup B)=P(A\cap B).$

38. (a) $P(A^c\cap B^c)=1-P(A\cup B)$

$=1-[0.25+0.5-0.14]=0.39.$

39. (a) $P(2\cup 3)=P(2)+P(3)-P(2\cap 3)$

$=\frac{6}{12}+\frac{4}{12}-\frac{2}{12}=\frac{2}{3}.$

40. (c) $P(A)=P(B)=2P(C)$,

$P(A)+P(B)+P(C)=1 \quad\Rightarrow P(C)=\frac{1}{5}$ and $P(A)=P(B)=\frac{2}{5}$

Hence $P(A\cup B)=P(A)+P(B)=\frac{2}{5}+\frac{2}{5}=\frac{4}{5}.$

41. (b) $P(A'\cap B')=1-P(A\cup B)$

$=1-\left(\frac{1}{2}+\frac{1}{3}-\frac{7}{12}\right)=1-\frac{1}{4}=\frac{3}{4}.$

42. (d) $P(A)=\frac{1}{5}$, $P(B)=\frac{2}{5}$ and $P(A\cap B)=\frac{1}{20}$

Then $P(\overline{A}\cap\overline{B})=1-P(A\cup B)$

$=1-\left[\frac{1}{5}+\frac{2}{5}-\frac{1}{20}\right]=\frac{9}{20}$ *i.e.,* 45%.

43. (c) $1-P(A'\cap B')=0.6$, $P(A\cap B)=0.3$, then

$P(A'\cup B')=P(A')+P(B')-P(A'\cap B')$

$1-P(A\cap B)=P(A')+P(B')-0.4$

$P(A')+P(B')=0.7+0.4=1.1.$

44. (a) Let A be the event that the husband will be alive 20 years. B be the event that the wife will be alive 20 years. Clearly A and B are independent events

$P(A\cap B)=P(A).P(B)$

Given $P(A)=\frac{3}{5}$, $P(B)=\frac{2}{3}$

The probability that at least of them will be alive 20 years is $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)P(B)=\frac{3}{5}+\frac{2}{3}-\frac{3}{5}.\frac{2}{3}=\frac{9+10-6}{15}=\frac{13}{15}$

Aliter : Required probability is $1-P(A$ and B both will die) $=1-\frac{2}{5}\times\frac{1}{3}=1-\frac{2}{15}=\frac{13}{15}.$

45. (d) $P(A\cup B)=P(A)+P(B)=0.45+0.35=0.8.$

46. (c) It is a fundamental concept.

47. (c) For any two events A and B, we have

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ P(A\cup B)\le P(A)+P(B)$.

Using principle of mathematical induction, it can be easily established that

$P\left(\bigcup_{i=1}^{n} A_i\right)\le \sum_{i=1}^{n} P(A_i)$.

48. (a) Consider the following events :

$A=$ A student is passed in Mathematics,

$B=$ A student is passed in Statistics.

Then $P(A)=\frac{70}{125},\ P(B)=\frac{55}{125},\ P(A\cap B)=\frac{30}{125}$.

Required probability is

$P(A\cap\overline{B})+P(\overline{A}\cap B)=P(A)+P(B)-2P(A\cap B)$

$=\frac{70}{125}+\frac{55}{125}-\frac{60}{125}=\frac{65}{125}=\frac{13}{25}$.

49. (c) $P[A\cap(B\cup C)]=P[(A\cap B)\cup(A\cap C)]$

$=P(A\cap B)+P(A\cap C)-P[(A\cap B)\cap(A\cap C)]$

$=P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)$.

50. (c) P (only one of them occurs)

$=P(E_1\overline{E}_2\overline{E}_3+\overline{E}_1E_2\overline{E}_3+\overline{E}_1\overline{E}_2E_3)$

$\ne P(\overline{E}_1E_2E_3+E_1\overline{E}_2E_3+E_1E_2\overline{E}_3)$

(a) is incorrect.

P (none of them occurs)

$=P(\overline{E}_1\cap\overline{E}_2\cap\overline{E}_3)\ne P(\overline{E}_1+\overline{E}_2+\overline{E}_3)$

(b) is not correct.

P (atleast one of them occurs)

$=P(E_1\cup E_2\cup E_3)=P(E_1+E_2+E_3)$

(c) is correct.

P (all the three occurs)

$=P(E_1\cap E_2\cap E_3)\ne P(E_1+E_2+E_3)$

(d) is not correct.

51. (c) Probability that it is queen $P(A)=\frac{4}{52}=\frac{1}{13}$

Probability that it is heart $P(B)=\frac{13}{52}=\frac{1}{4}$

Probability that it is queen and heart

$P(A\cap B)=\frac{1}{52}$

$\therefore\ P$ (Queen or heart) $=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{13}+\frac{1}{4}-\frac{1}{52}=\frac{4+13-1}{52}=\frac{16}{52}=\frac{4}{13}$.

52. (b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.21+0.49-0.16\Rightarrow P(A\cup B)=0.54$

Probability of none of two occurs

$=1-P(A\cup B)=1-0.54=0.46$.

53. (c) $P(\overline{A}\cap B)=P(B)-P(A\cap B)=y-z$.

54. (a) (i) This question can also be solved by one student

(ii) This question can be solved by two students simultaneously

(ii) This question can be solved by three students all together.

$P(A)=\frac{1}{2},\ P(B)=\frac{1}{4},\ P(C)=\frac{1}{6}$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)$

$-[P(A).P(B)+P(B).P(C)+P(C).P(A)]+[P(A).P(B).P(C)]$

$=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}-\left[\frac{1}{2}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}\right]+\left[\frac{1}{2}\times\frac{1}{4}\times\frac{1}{6}\right]=\frac{33}{48}$.

55. (b) $P(A\cap B)=\frac{1}{6}$ and $P(A^c\cap B^c)=\frac{1}{3}$

Now $P(A\cup B)^c=P(A^c\cap B^c)=\frac{1}{3}$

$1-P(A\cup B)=\frac{1}{3}\Rightarrow P(A\cup B)=\frac{2}{3}$

But $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A)+P(B)=\frac{5}{6}$ .....(i)

$\because$ A and B are independent events

$P(A\cap B)=P(A)P(B)$ $\quad P(A)P(B)=\frac{1}{6}$

$[P(A)-P(B)]^2=[P(A)+P(B)]^2-4P(A)P(B)$

$=\frac{25}{36}-\frac{4}{6}=\frac{1}{36}$

$\Rightarrow P(A)-P(B)=\pm\frac{1}{6}$ ......(ii)

Solving (i) and (ii), we get $P(A)=\frac{1}{2}$ or $\frac{1}{3}$.

56. (c) Probability of king $P(A)=\frac{4}{52}=\frac{1}{13}$

Probability of spade $=\frac{13}{52}=\frac{1}{4}$

P (King or spade)

$=P$(king) + P (spade) $-P$ (King and spade)

$=\frac{1}{13}+\frac{1}{4}-\frac{1}{52}=\frac{4+13-1}{52}=\frac{16}{52}=\frac{4}{13}$.

57. (d) $P(A\cap\overline{B})=P(A)-P(A\cap B)=0.25-0.14=0.11$.

58. (d) It is a fundamental concept.

59. (b) $P(A^c)=1-P(A)\Rightarrow P(A)=\frac{1}{6}$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{5}{6}=\frac{1}{6}+\frac{2}{3}-P(A\cap B)$ $\quad P(A\cap B)=0$

Hence, events A and B are mutually exclusive.

60. (c) Given $P(A\cup B)=\frac{3}{5}$ and $P(A\cap B)=\frac{1}{5}$

We know $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{3}{5}=1-P(\overline{A})+1-P(\overline{B})-\frac{1}{5}$

$2-\frac{4}{5}=P(\overline{A})+P(\overline{B})\Rightarrow P(\overline{A})+P(\overline{B})=\frac{6}{5}$.

61. (b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)\le P(A)+P(B)$,

$(\because P(A\cap B)\geq 0)$.

62. (a) $P(A\cup B)=\frac{3}{4},\ P(A\cap B)=\frac{1}{4}$

$P(\overline{A})=\frac{2}{3}\Rightarrow P(A)=\frac{1}{3}$

$\therefore P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$\frac{1}{4}=\frac{1}{3}+P(B)-\frac{3}{4}\qquad P(B)=\frac{2}{3}$

$P(\overline{A}\cap B)=P(B)-P(A\cap B)=\frac{2}{3}-\frac{1}{4}=\frac{8-3}{12}=\frac{5}{12}$.

63. (b) $E=\{x$ is a prime number$\}$

$P(E)=P(2)+P(3)+P(5)+P(7)=0.62$,

$F=\{x<4\}$, $P(F)=P(1)+P(2)+P(3)=0.50$

and $P(E\cap F)=P(2)+P(3)=0.35$

$\therefore P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$=0.62+0.50-0.35=0.77$.

64. (a) $P(A'\cap B')=1-P(A\cup B)\Rightarrow P(A\cup B)=\frac{2}{3}$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{2}{3}=x+x-\frac{1}{3}\Rightarrow x=\frac{1}{2}$.

65. (d) $P(A'\cap B')=1-P(A\cup B)=1-0.8=0.2$

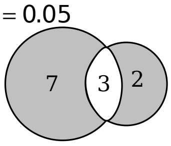
$P(A'\cup B')=1-P(A\cap B)=1-0.3=0.7$

$P(A'\cup B')=P(A')+P(B')-P(A'\cap B')$

$0.7=P(A')+P(B')-0.2\qquad P(A')+P(B')=0.9$.

66. (c) Here, $P(R)=\frac{10}{100}=0.1,\ P(F)=\frac{5}{100}=0.05$

$P(F\cap R)=\frac{3}{100}=0.03$



Required probability $=P(R)+P(F)-2P(F\cap R)$

$=0.1+0.05-2(0.03)=0.09$.

67. (c) P(queen or heart) $=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{4}{13}$.

68. (a) $P(\overline{A\cup B})=\frac{1}{6};\ P(A\cap B)=\frac{1}{4}$,

$P(\overline{A})=\frac{1}{4}\Rightarrow P(A)=\frac{3}{4}$,

$P(\overline{A\cup B})=1-P(A\cup B)=1-P(A)-P(B)+P(A\cap B)$

$\frac{1}{6}=\frac{1}{4}-P(B)+\frac{1}{4}\qquad P(B)=\frac{1}{3}$.

Since $P(A\cap B)=P(A)P(B)$ and $P(A)\neq P(B)$

A and B are independent but not equally likely.

69. (d) Ways of selection two subset of $A=(2^n)^2$

Ways of selection $A\cup B$ and $A\cap B$ are 2^n

Required probability $=\frac{\text{favourable cases}}{\text{total cases}}=\frac{2^n}{(2^n)^2}=\frac{1}{2^n}$.

70. (b) $P(A')=0.3,\qquad P(A)=0.7$

$P(B')=0.6$, $\therefore P(B)=0.4$ and $P(A\cap B')=0.5$

$\therefore P(A\cup B')=P(A)+P(B')-P(A\cap B')$

$=0.7+0.6-0.5=0.8$.

Conditional probability, Baye's theorem

1. (b) Since we are given that 5 appears on first die so to get sum 11, six must be on the second and hence, the required probability $=\frac{1}{6}$.

2. (c) $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{1/4}{1/2}=\frac{1}{2}$.

3. (c) $P\left(\frac{\overline{A}}{\overline{B}}\right)=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{P(\overline{A\cup B})}{P(\overline{B})}=\frac{1-P(A\cup B)}{P(\overline{B})}$.

4. (b) Since 4 has appeared on the first, so we are required 4 or 5 or 6 on second dice.

Hence required probability $=\frac{3}{6}=\frac{1}{2}$.

5. (a) $P\left(\frac{\overline{B}}{\overline{A}}\right)=\frac{1-P(A\cup B)}{P(\overline{A})}=\frac{1-\frac{23}{60}}{1-\frac{1}{3}}=\frac{37}{60}\times\frac{3}{2}=\frac{37}{40}$.

6. (a) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{(3/8)+(5/8)-(3/4)}{(5/8)}=\frac{2}{5}$.

7. (a) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

Since A and B are mutually exclusive.

So, $P(A\cap B)=0$. Hence $P\left(\frac{A}{B}\right)=\frac{0}{P(B)}=0$.

8. (b) Since $A\subseteq B\Rightarrow A\cap B=B\cap A=A$

Hence $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1$.

9. (c) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{P(A).P(B)}{P(B)}=P(A)$.

10. (d) $P(E\cap F)=P(E).P(F)$

Now,

$P(E\cap F^c)=P(E)-P(E\cap F)=P(E)[1-P(F)]=P(E).P(F^c)$

and

$P(E^c\cap F^c)=1-P(E\cup F)=1-[P(E)+P(F)-P(E\cap F)$

$=[1-P(E)][1-P(F)]=P(E^c)P(F^c)$

Also $P(E/F)=P(E)$ and $P(E^c/F^c)=P(E^c)$

$\Rightarrow P(E/F)+P(E^c/F^c)=1$.

11. (a) $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{(1/10)}{(1/4)}=\frac{2}{5}$.

12. (c) Let A be the event that face 4 turns up and B be the event that face 5 turns up then

$P(A)=0.25,\quad P(B)=0.05$. Since A and B are mutually exclusive, so $P(A\cup B)=P(A)+P(B)=0.25+0.05=0.30$.

We have to find $P\left(\frac{A}{A\cup B}\right)$, which is equal to

$$P\frac{[A\cap(A\cup B)]}{P(A\cup B)}=\frac{P(A)}{P(A\cup B)}=\frac{0.25}{0.30}=\frac{5}{6}.$$

13. (c) Let $P(A)=$ probability of a boy in two children $=\frac{3}{4}$

Because cases are $BB, BG, GB, GG=4$

Favourable cases are $BB, BG, GB=3$

The probability that the second child is also boy is

$$P(A\cap B)=\frac{1}{4}$$

We have to find $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{1/4}{3/4}=\frac{1}{3}$.

14. (a) Let E be the event in which all three coins shows tail and F be the event in which a coin shows tail.

$$F=\{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

and $E=\{TTT\}$.

Required probability $=P(E/F)=\frac{P(E\cap F)}{P(E)}=\frac{1}{7}$.

15. (d) $P(A/B)=P(A)$ as independent event $=\frac{1}{2}$.

$$P\{A/(A\cup B)\}=\frac{P[A\cap(A\cup B)]}{P(A\cup B)}$$

{Since $A\cap(A\cup B)=A\cap[A-B-A\cap B]$

$= A-A\cap B-A\cap B=a\}$

$$\Rightarrow P\left(\frac{A}{A\cup B}\right)=\frac{P(A)}{P(A\cup B)}=\frac{\frac{1}{2}}{\frac{1}{2}-\frac{1}{5}-\frac{1}{10}}=\frac{\frac{1}{2}}{\frac{6}{10}}=\frac{5}{6}$$

and similarly $P\left(\frac{A\cap B}{A'\cup B'}\right)$.