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Module - 4

Network Analysis

A project is composed of a number of jobs, activities that are related one to other and all of these should be completed in order to complete the project. An activity of a project can be started only at the completion of many other activities. A network is a combination of activities and events of a project.

Objectives of Network Analysis

1) Minimisation of total cost of a project
2) Minimisation of total time of project
3) Minimisation of cost of a project for a given total time
4) Minimisation of time of a project for a given total cost.
5) Minimisation of idle resources
6) Planning, scheduling and controlling projects.

Network diagram :

⇒ Fulkersons Rule for numbering of events:

The numbering of events is necessary in a network. Event activity has two events known as tail and head events. These two events are identified by the numbers given to them. The following steps may be adopted for numbering of events.

(i) The initial event of the network diagram is numbered 1

(ii) The arrows emerging from the events 1 are then considered. Those arrows end in new events treat them as initial events and number them as 2, 3, 4.. From these new initial arrow emerge which end in new events. They may be treated as new initial events and numbered them as 5, 6, 7 ... Follow step (ii) until last event which has no emerging arrows.

⇒ Rules for constructing network diagram.

(1) Each activity is represented by one and only one arrow in the network.

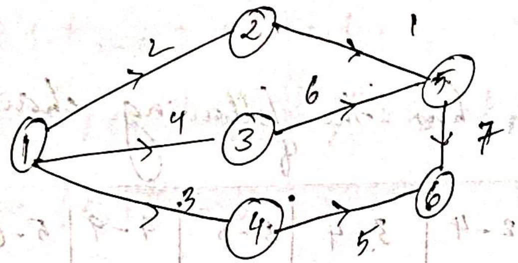
(2) No two activities can be identified by the same head and tail events

(3) Except the beginning and ending nodes. Every node must have atleast one activity proceeding it and atleast one following it.

(4) Only one activity may connect any two nodes.

Q.) Draw the network diagram to the following activities

Activity	Duration
1-2	2
1-3	4
1-4	3
2-5	1
3-5	6
4-6	5
5-6	7



Starting event - 1

Ending event - 6.

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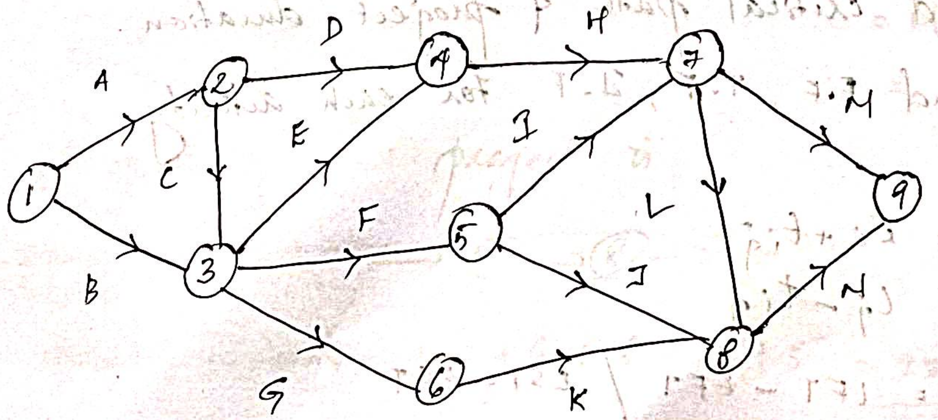
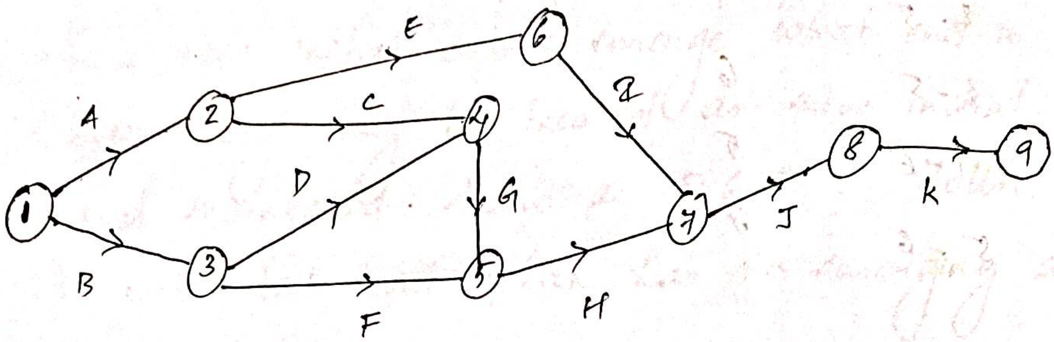
Q.) Draw the network diagram for the following activities :

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Preceding activities	–	–	A	A	B&C	B&C	B&C	D&E	F	F	G	H&I	H&I	J, K&L



Q.) Draw the network diagram:

Activity	A	B	C	D	E	F	G	H	I	J	K
Prerequisties	-	-	A	B	A	B	C, D	G, F	E	H, I	J



Q.) A project schedule has the following characteristics:

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	7

(i) Construct a network diagram

(ii) Compute LST, EST, LFT, EFT of all activities.

(iii) Find critical path & project duration.

(iv) Find F.F, T.F, I.F for each activity.

LST - Latest Starting time

EST - Earliest Starting time

LFT - Latest Finishing time

EFT - Earliest Finishing time.

FF - Free Float

TF - Total Float

IF - Independent Float

$EFT = E_i + t_{ij}$

$LST = L_j - t_{ij}$

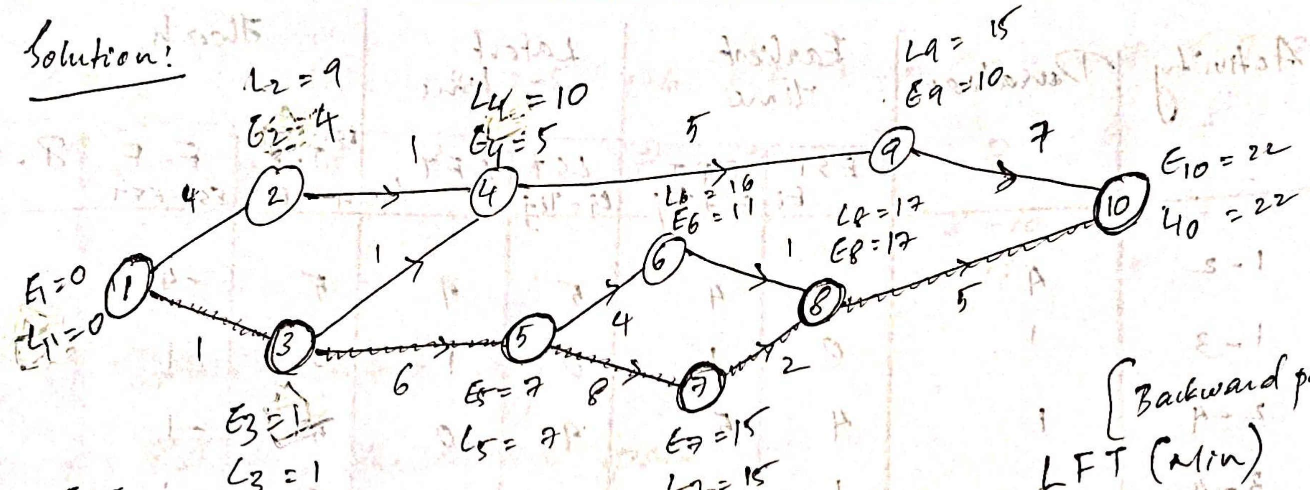
$T.F = LFT - EFT \;/\; LST - EST$

F.F =

Successor activity j | present activity i

Solution:

i)



[Backward pass]

ii) **EST (Max)** [Forward pass]

$ES1 = 0$

$ES2 = ES1 + t_{1-2} = 0+4 = 4$

$ES3 = ES1 + t_{1-3} = 0+1 = 1$

$ES4 = \text{Max}\{ES2 + t_{2-4},\ ES3 + t_{3-4}\}$

$= \text{Max}\{5, 2\} = 5$

$ES5 = ES3 + t_{1-5} = 1+6 = 7$

$ES6 = 7+4 = 11$

$ES7 = 7+8 = 15$

$ES8 = ES7 + 2 = 15+2 = 17$

$ES9 = ES4 + 5 = 5+5 = 10$

$ES10 = ES8 + 5 = 17+5 = 22$

LFT (Min)

$L_{10} = 22$

$L_9 = 22-7 = 15$

$L_8 = 22-5 = 17$

$L_7 = 17-2 = 15$

$L_6 = 17-1 = 16$

$L_5 = \text{Min}\{15-8,\ 16-4\}$

$= \text{Min}\{7, 12\} = 7$

$L_4 = L_9 - 5 = 15-5 = 10$

$L_3 = \text{Min}\{7-6,\ 10-1\}$

$= \text{Min}\{1, 9\}$

$= 1$

$L_2 = 10-1 = 9$

$L_1 = \text{Min}\{9-4,\ 1-1\}$

$= \text{Min}\{5, 0\}$

$= 0$

iii) The critical path of this project is

$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 10$

project duration $= 1+6+8+2+5 = \underline{\underline{22}}$

Activity	Duration	Earliest Time		Latest time		Floats		
		EST E_i	EFT $E_i + t_{ij}$	LST $L_j - T_{ij}$	LFT L_j	T.F (LST−EST, LFT−EFT)	F.F	I.F
1-2	4	0	4	5	9	5		
1-3	1	0	1	0	1	0		
2-4	1	4	5	9	10	5		
3-4	1	1	2	9	10	8		
3-5	6	1	7	1	7	0		
4-9	5	5	10	10	15	5		
5-6	4	7	11	12	16	5		
5-7	8	7	15	7	15	0		
6-8	1	11	12	16	17	5		
7-8	2	15	17	15	17	0		
8-10	5	17	22	17	22	0		
9-10	7	10	17	15	22	5		

Q)

Activity	1-2	1-3	1-4	2-5	4-6	3-7	5-7	6-7	5-8	6-9	7-10	8-10	9-10
Time	10	8	9	8	7	16	7	7	6	5	12	13	15

i) Draw network dig.

ii) Identify the critical path

iii) Find the project duration

iv) Compute EST, EFT, LST, LFT of all ~~activities~~ criteria

v) Compute TF, FF, IntF, IF for all activities

TF - Total Float

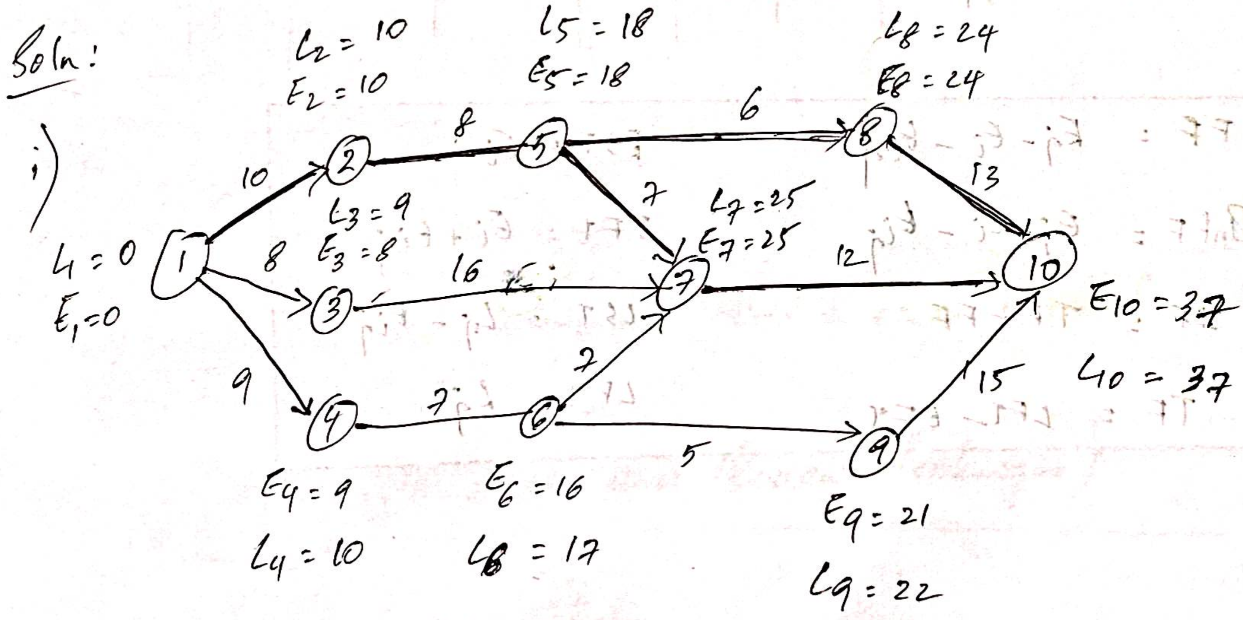
FF - Free Float

IntF - Interfering Float

IF - Independent Float

Soln:

i)



ii) Critical paths are:

$$1 \xrightarrow{10} 2 \xrightarrow{8} 5 \xrightarrow{6} 8 \xrightarrow{13} 10$$

$$1 \xrightarrow[10]{} 2 \xrightarrow[8]{} 5 \xrightarrow[7]{} 7 \xrightarrow[12]{} 10$$

iii) Project duration

$= 10 + 8 + 6 + 13$

$= 37$

Activity	Duration	Earliest Time		Latest Time		Remark	Floats			
		EST E_i	EFT E_i+t_{ij}	LST L_j-t_{ij}	LFT L_j		TF LFT-EFT	FF	IntF	IF
1-2	10	$E_1=0$	10	0	$L_2=10$	C.A	0	0	0	0
1-3	8	$E_1=0$	8	1	9	—	1	0	0	
1-4	9	$E_1=0$	9	1	10	—	1	0		
2-5	8	$E_2=10$	18	10	18	C.A	0	0	0	0
4-6	7	$E_4=9$	16	10	17	—	1	0		
3-7	16	$E_3=8$	24	9	25	—	1	1		
5-7	7	$E_5=18$	25	18	25	C.A	0	0	0	0
6-7	7	$E_6=16$	23	18	25	—	2	2		
5-8	6	$E_5=18$	24	18	24	C.A	0	0	0	0
6-9	5	$E_6=16$	21	17	22	—	1	0		
7-10	12	$E_7=25$	37	25	37	C.A	0	0	0	0
8-10	13	$E_8=24$	37	24	37	C.A	0	0	0	0
9-10	15	$E_9=21$	36	22	37	—	1	1		

$FF = E_j - E_i - t_{ij}$

$IntF = E_j - L_i - t_{ij}$

$IF = TF - FF$

$TF = LFT - EFT$

$EST = E_i$

$EFT = E_i + t_{ij}$

$LST = L_j - t_{ij}$

$LFT = L_j$

1) Define Network analysis. Explain their objectives.

2) Applications of network techniques.

3) Define Fulkerson's rule for numbering of events.

4) Define the following:

> Activity

> Event

> Dummy Activity.

5) Define:

i) Event

ii) Earliest & latest event time

iii) EST, EFT, LST, LFT.

6) Define slack and float. What are the different types of float. Explain it.

7) Define Critical Activity.

8) Define critical path method

9) Explain critical path method (CPM) & steps involved in CPM.

PERT (Project Evaluation and Review Technique)

Q.) For the project given below.

i) Find expected time for each activities

ii) EST, EFT, LST, LFT

iii) Critical path & project duration.

iv) Variance of the project & its standard deviation.

v) Find the probability of completing the project by 39 days.

Task	1-2	1-3	2-3	1-4	3-5	4-5	4-6	5-7	5-6	6-8	7-8
t_o	4	5	8	2	4	7	8	4	3	5	6
t_p	6	9	12	6	10	15	16	8	7	11	12
t_m	5	7	10	4	7	8	12	6	5	8	9
t_e	5	7	10	4	7	9	12	6	5	8	9

$t_o \rightarrow$ optimistic time

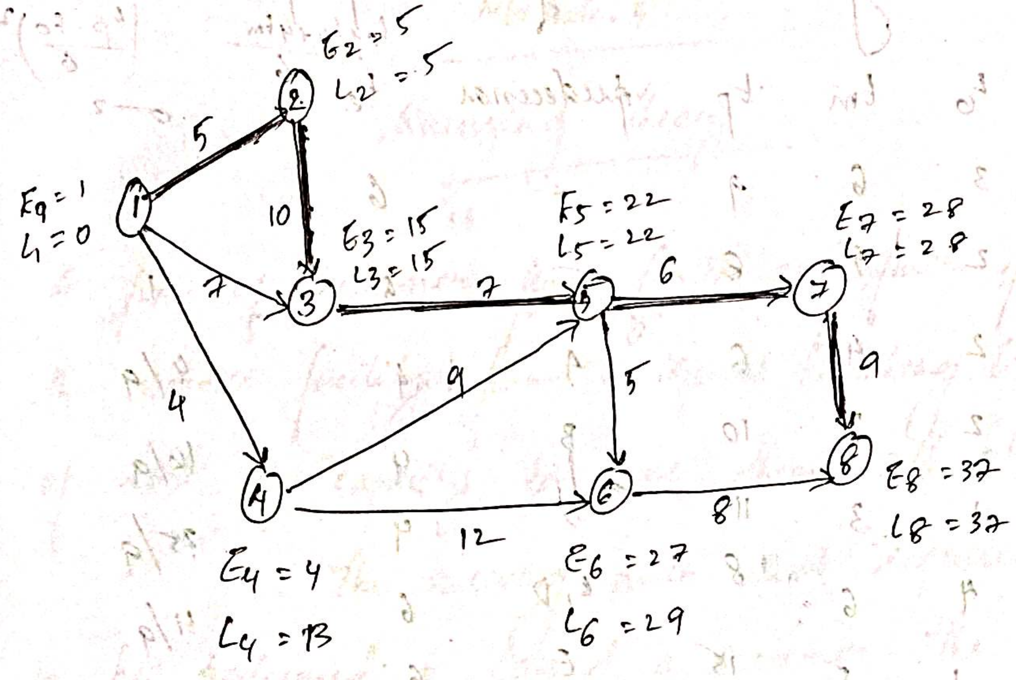
$t_p \rightarrow$ pessimistic time

$t_m \rightarrow$ most likely time

t_e (expected time for each activity) t_e

$$t_e = \frac{t_o + t_p + 4t_m}{6}$$

TASK	Duration	EST (E_i)	EFT	LST	LFT (L_j)
1-2	5	0	5	0	5
1-3	7	0	7	8	15
2-3	10	5	15	5	15
1-4	4	0	4	9	13
3-5	7	15	22	15	22
4-5	9	4	13	13	22
4-6	12	4	16	17	29
5-7	6	22	28	22	28
5-6	5	22	27	24	29
6-8	8	27	35	29	37
7-8	9	28	37	28	37



Critical path:

$$1 \xrightarrow{5} 2 \xrightarrow{10} 3 \xrightarrow{7} 5 \xrightarrow{6} 7 \xrightarrow{9} 8$$

project duration : 37

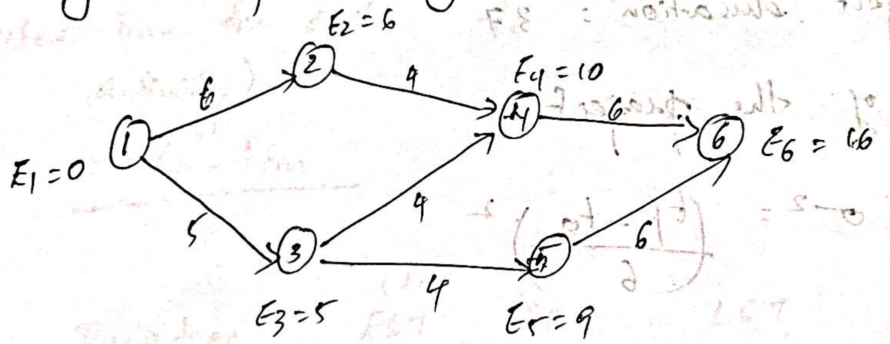
Variance of the project

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Q) Consider the following activities of the project.

Activity		t_o	t_m	t_p	predecessor	$t_e = \frac{t_o + t_p + 4t_m}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	A	3	6	9	—	6	
1-3	B	2	5	8	—	5	1
2-4	C	2	4	6	A	4	4/9
3-4	D	2	3	10	B	4	16/9
3-5	E	1	3	11	B	4	25/9
4-6	F	4	6	8	C, D	6	4/9
5-6	G	1	5	15	E	6	49/9

Find the critical path & SD. Also find probability of completing the project by 18 days.



project duration $m = 16$

$$\sigma^2 = 1 + \frac{4}{9} + \frac{4}{9} = 1.888$$

$$SD = \sigma = \sqrt{1.888} = 1.374$$

$$p(x \le 18) = p\left(z \le \frac{18 - m}{\sigma}\right) = p(z \le 1.45)$$

$$= p(1.45)$$

$$= 0.92647$$

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Module - 3

1) Assignment problem - optimal solution - Hungarian Method of arrangement . Maximization in Assignment problem.
2) Transportation problem - Solution of TP - Finding an IBFS - NWCR - Matrix minima method - vogel's approximation method - Test for optimality - Mod Method.

Unbalanced TP, Maximisation in TP

Assignment Problem

The assignment algorithm (Hungarian technique) - minimisation case :

Step 1 : Check whether the number of rows and columns of the cost matrix are equal. (ie, the problem is balanced)

Step 2 : If the problem is not balanced, balance it by adding suitable number of dummy rows and columns with zero cost

Step 3 : Substract from every row its smallest element. Substract from every column its smallest element.

Step 4 : Draw the least number of horizontal and vertical lines to cover all the zeros. If the number of lines is equal to the number of rows / columns then go to Step 5.

Step 5 : Examine the rows successively to find one with exactly one unmarked zero. Make an assignment thereby encircling the zero and cancel all the zeros in that column to show that they cannot be considered for future assignment

Step 6 : If the no. of lines not equal to no. of rows, identify the smallest uncovered element (not covered by the lines). Substract it from all the uncovered elements and add the same to the element at the intersection of lines. We get a reduced matrix. Then go to step 4.

Q.) Solve:

		Machines				
		I	II	III	IV	V
	1	11	17	8	16	20
	2	9	7	12	6	15
Jobs	3	13	16	15	12	16
	4	21	24	17	28	26
	5	14	10	12	11	15

Solution:

No. of rows = No. of columns

∴ Given AP is balanced

		M				
		I	II	III	IV	V
	1	3	9	0	8	12
	2	3	1	6	0	9
J	3	1	4	3	0	4
	4	4	7	0	11	9
	5	4	0	2	1	5

	I	II	III	IV	V
1	2	9	0	8	8
2	2	1	6	0	5
3	0	4	3	0	0
4	3	7	0	11	5
5	3	0	2	1	1

	I	II	III	IV	V
1	2	9	0	8	8
2	2	1	6	0	5
3	0	4	3	0	0
4	3	7	0	11	5
5	3	9	2	1	1

Here the minimum no. of lines drawn = 4

Here no. of lines is not equal to no. of rows/columns.

	I	II	III	IV	V
1	1	9	0	7	7
2	2	2	7	0	5
3	0	5	4	0	0
4	2	7	0	10	4
5	2	0	2	0	0

Here also, no. of lines is not equal to no. of rows/columns.

	I	II	III	IV	V
1	0	8	0	7	6
2	1	1	7	0	4
3	0	5	5	1	0
4	1	6	0	10	3
5	2	0	3	1	0

Here, no. of lines = No. of rows/columns.

Assignment :

M

J	I	II	III	IV	V
1	0	x	$\cancel{0}$	x	x
2	x	x	x	0	x
3	$\cancel{0}$	x	x	x	0
4	x	x	0	x	x
5	x	0	x	x	$\cancel{0}$

Job 1 assigned to Machine I
Job 2 assigned to Machine IV
Job 3 assigned to Machine V
Job 4 assigned to Machine III
Job 5 assigned to Machine II

The minimum assignment cost = 11 + 6 + 16 + 17 + 10
= 60

Q.) Solve the following assignment problem.

M

J	1	2	3	4
A	10	12	19	1
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Solution:

Clearly, the given assignment problem is balanced.
ie, No. of rows = No. of columns.

	1	2	3	4
A	10	12	19	1
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

⇓ Row reduction.

	1	2	3	4
A	9	11	18	0
B	0	5	2	3
C	1	3	2	0
D	0	7	3	1

⇓ Column reduction.

	1	2	3	4	
A	9	8	16	0	(1)
B	0	2	0	3	(2)
C	1	0	0	0	(3)
D	0	4	1	1	(1)
	(2)	(1)	(2)	(2)	

Here, the minimum no. of lines = no. of rows/columns.

Assignment :

	1	2	3	4
A	×	×	×	0
B	~~0~~	×	0	×
C	×	0	~~0~~	~~0~~
D	0	×	×	×

Job	Machine
A	4
B	3
C	2
D	1

The minimum cost = 1 + 7 + 14 + 8 = 30

2.) Assign the job the profit is minimum

	A	B	C	D
J_1	62	78	50	101
J_2	71	84	61	73
J_3	87	92	111	71
J_4	48	64	87	77

Solution :

This is a maximisation problem. But we can convert it into a minimisation problem by subtracting every element of the given cost matrix from the maximum element 111.

So the initial matrix is :

	A	B	C	D
J_1	49	33	61	10
J_2	40	27	50	38
J_3	24	19	0	40
J_4	63	47	24	34

The problem is balanced. (no. of rows = no. of columns)

	A	B	C	D
J_1	39	23	51	0
J_2	13	0	23	11
J_3	24	19	0	40
J_4	39	23	0	10

	A	B	C	D
J_1	26	23	51	0
J_2	0	0	23	11
J_3	11	19	0	40
J_4	26	23	0	10

Here, minimum no. of lines ≠ no. of rows.

	A	B	C	D
J_1	15	12	51	0
J_2	0	0	34	22
J_3	0	8	0	40
J_4	15	12	0	10

Here, no. of lines = no. of rows.

	A	B	C	D
J_1	x	x	x	0
J_2	~~0~~	0	x	x
J_3	0	x	~~0~~	x
J_4	x	x	0	x

$J_1 \rightarrow D$

$J_2 \rightarrow B$

$J_3 \rightarrow A$

$J_4 \rightarrow C$

Minimum Cost = 101 + 84 + 87 + 87 = 359

Transportation Problem

1) North West Corner Rule [NWCR] - Balanced transportation problem.

Q.) Obtain an Initial basic feasible solution to the following transportation problem using north west corner rule (NWCR)

Demand

		D_1	D_2	D_3	D_4	a_i
Supply	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
	b_j	6	10	15	4	

Solution : Here, total supply = 35 and total demand = 35.
Therefore, the given transportation problem is balanced.

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	~~14~~ ~~8~~ 0
O_2	8	(2) 9	(14) 2	7	~~16~~ ~~14~~ 0
O_3	4	3	(1) 6	(4) 2	~~5~~ 1
b_j	~~6~~ 0	~~10~~ ~~2~~ 0	~~15~~ ~~1~~ 0	~~4~~ 0	

(non-degenerate)

Min trans cost = $6\times6 + 4\times8 + 9\times2 + 2\times14 + 6\times1 + 2\times4$

$= 128$

Q.)

Demand

		D_1	D_2	D_3	D_4	a_i
Supply	O_1	2	4	3	6	20
	O_2	7	3	8	2	10
	O_3	2	2	9	11	15
	B_j	15	15	8	7	

Total supply = 15+15+8+7 = 45
Total demand = 20+10+15 = 45.
Therefore, the given TP is balanced.

	D_1	D_2	D_3	D_4	a_i
O_1	(15) 2	(5) 4	3	6	~~20~~ ~~5~~ 0
O_2	7	(10) 3	8	2	~~10~~ 0
O_3	2	2	(8) 9	(7) 11	~~15~~ ~~7~~ 0
b_j	~~15~~ 0	~~15~~ ~~10~~ 0	~~8~~ 0	~~7~~ 0	

[IBFS is degenerate]

Min trans Cost $= 15\times2 + 5\times4 + 10\times3 + 8\times9 + 7\times11$

$= 30 + 20 + 30 + 72 + 77$

$= 229$

Note:

The number of allocations = no. of rows + no. of columns − 1

Then, the initial Basic Feasible Solution is non-degenerate.

2) Vogel's Approximation Method [VAM]

Q.) Find an IBFS to the following transportation problem using VAM.

	D_1	D_2	D_3	D_4	Supply
O_1	8	4	6	6	34
O_2	6	6	8	7	15
O_3	9	7	7	6	12
O_4	7	2	7	5	19
Demand	21	25	17	17	80

Soln: Total Supply = Total Demand

$\therefore$ The given TP is balanced

Step 1: Calculate the row & column penalty as the difference b/w the smallest and the next smallest cost for each row and column.

	D_1	D_2	D_3	D_4	Supply	
O_1	8	4	6	6	34	(2)
O_2	6	6	8	7	15	(1)
O_3	9	7	7	6	12	(1)
O_4	7	(19) 2	7	5	~~19~~ 0	(3) ⇐
Demand	21	~~25~~ 6	12	17		
	(1)	(2)	(1)	(1)		

Step 2: Select the row/column with the largest penalty. If there is a tie, break it arbitrarily.

Choose the cell with least cost in the selected row/column. Make an allotment there.

	D_1	D_2	D_3	D_4	Supply	
O_1	8	(6) 4	6	6	~~34~~ 28	(2)
O_2	6	6	8	7	15	(1)
O_3	9	7	7	6	12	(1)
O_4	7	(19) 2	7	5	~~19~~ 0	×
	21	~~25~~ ~~6~~	17	12		
	(2)	(2) 0	(1)	(1)		