⇒ **Poisson Distribution:-**

A random variable X is said to follow the poisson distrib^n if its pdf is

$$f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & ; x = 0,1,2\ldots\infty \\ & \lambda \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

⇒ Poisson distrib^n as the limiting form of binomial.

Proof:-

Binomial distrib^n tends to the poisson distrib^n when $n \to \infty$ as $p \to 0$ such that $np = \lambda$ (say) is finite.

To prove this result we have to show that the pdf of the binomial distrib^n tends to the poissen distrib^n under these conditions.

Let $f(x)$ be the pdf of binomial

ie; $f(x) = nC_x\, p^x q^{n-x}$; $x = 0,1,2\ldots n$

$$= \frac{n!}{x!\,(n-x)!}\, p^x q^{n-x}$$

$$f(x) = p^x q^{n-x}\, \frac{1\cdot 2\cdots(n-x)(n-x+1)(n-x+2)\cdots(n-1)n}{x!\,(n-x)!}$$

$$= \frac{\cancel{(n-x)!}\,(n-x+1)(n-x+2)\cdots(n-1)\cdot n \times p^x q^{n-x}}{x!\,\cancel{(n-x)!}}$$

$$= \frac{n^x\left[1-\left(\frac{x-1}{n}\right)\right]\left[1-\left(\frac{x-2}{n}\right)\right]\cdots\left(1-\frac{1}{n}\right)\cdot 1}{x!}\, p^x q^{n-x}$$

$$= (np)^x\left[1-\left(\frac{x-1}{n}\right)\right]\left[1-\left(\frac{x-2}{n}\right)\right]\cdots\left(1-\frac{1}{n}\right)\frac{q^{n-x}}{x!}$$

$$= \lambda^x\left[1-\left(\frac{x-1}{n}\right)\right]\left[1-\left(\frac{x-2}{n}\right)\right]\cdots\left(1-\frac{1}{n}\right)\frac{q^{n-x}}{x!}$$

Taking limits as $n \to \infty$, $p \to 0$

$$\lim_{n\to\infty} f(x) = \lim_{n\to\infty}\left[1-\left(\frac{x-1}{n}\right)\ 1-\left(\frac{x-2}{n}\right)\cdots\left(1-\frac{1}{n}\right) \times \frac{\lambda^x q^{n-x}}{x!}\right]$$

as $n \to \infty$, $1-\left(\frac{x-1}{n}\right), 1-\left(\frac{x-2}{n}\right)\cdots\frac{1}{n} \to 1$

$$f(x) = 1\cdot 1\cdots 1 \lim_{n\to\infty} \frac{\lambda^x (1-p)^{n-x}}{x!}$$

$$= \lim_{n\to\infty} \frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^{n-x} \quad \left[\begin{array}{l} np=\lambda \\ \Rightarrow p = \lambda/n \end{array}\right.$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\left[1+\frac{(-\lambda)}{n}\right]^n\cdot 1$$

$$= \frac{\lambda^x}{x!}\, e^{-\lambda} \qquad \boxed{\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x}$$

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$$\therefore \lim_{n\to\infty} f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & ; x = 0,1,2,\ldots\infty \\ 0 & ; \text{elsewhere} \end{cases}$$

Note :-

1. The poisson distrib^n is a discrete distrib^n with single parameter λ.

2. $$\sum_0^\infty f(x) = \sum_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}$$
$$= e^{-\lambda}\sum_0^\infty \frac{\lambda^x}{x!}$$
$$= e^{-\lambda}\left[\frac{1}{0!} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$
$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$
$$= e^{-\lambda}\cdot e^{\lambda}$$
$$= e^{-\lambda+\lambda} = e^0 = 1$$

Mean & Variance :-

Mean μ, $= E(x)$
$$= \sum_0^\infty x\, f(x)$$
$$= \sum_0^\infty x \frac{e^{-\lambda}\lambda^x}{x!}$$

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$$= e^{-\lambda}\sum_1^\infty \frac{\lambda^x}{(x-1)!}$$
$$= e^{-\lambda}\cdot\lambda\sum_1^\infty \frac{\lambda^{x-1}}{(x-1)!}$$
$$= e^{-\lambda}\cdot\lambda\left[\frac{1}{0!} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} \cdots\right]$$
$$= \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} \cdots\right]$$
$$= \lambda e^{-\lambda}\cdot e^{\lambda} = \lambda e^{-\lambda+\lambda} = \lambda e^0$$
$$\mu = \lambda$$

Variance $= E(x^2) - (E(x))^2$ —— ①

$$E(x^2) = \sum_0^\infty x^2 f(x)$$
$$= \sum_0^\infty (x(x-1)+x)\frac{e^{-\lambda}\lambda^x}{x!}$$
$$= \sum_0^\infty x(x-1)\frac{e^{-\lambda}\lambda^x}{x!} + \sum_0^\infty x\frac{e^{-\lambda}\lambda^x}{x!}$$
$$= e^{-\lambda}\sum_2^\infty \frac{\lambda^x}{(x-2)!} + \lambda$$
$$= e^{-\lambda}\lambda^2\sum_2^\infty \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$
$$= \lambda^2 e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} \cdots\right) + \lambda$$

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$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$

$= \lambda^2 e^{-\lambda+\lambda} + \lambda$

$= \lambda^2 e^0 + \lambda$

$= \lambda^2 + \lambda$

$\therefore$ from ①, we get

$Var(x) = \lambda^2 + \lambda - \lambda^2$

$= \lambda$

$\therefore$ for a poisson distribution,

Mean $= Var(x) = \lambda$

and $S.D = \sqrt{Var} = \sqrt{\lambda}$

Q. If a RV has a poisson distrbn such that $P(1) = P(2)$. find mean of the distrbn and $P(4)$.

$\rightarrow$ $P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Since $P(1) = P(2)$

$\Rightarrow \dfrac{\cancel{e^{-\lambda}}\lambda}{1!} = \dfrac{\cancel{e^{-\lambda}}\lambda^2}{2!}$

$\Rightarrow 1 = \dfrac{\lambda}{2} \Rightarrow 2 = \lambda.$

$\therefore$ mean $= \lambda = 2$

Also $P(4) = \dfrac{e^{-2}2^4}{4!} = 0.0902.$

Q. A RV 'x' follows poisson distrbn s.t. $P(x=0) = \dfrac{2}{3}P(x=1)$. find $P(x=3)$ and $P(x>3)$.

$\rightarrow$ $P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Since $P(x=0) = \dfrac{2}{3}P(x=1)$

$\Rightarrow \dfrac{e^{-\lambda}\lambda^0}{0!} = \dfrac{2}{3}\dfrac{e^{-\lambda}\lambda^1}{1!}$

$\Rightarrow \cancel{e^{-\lambda}} = \dfrac{2}{3}\cancel{e^{-\lambda}}\cdot\lambda.$

$\Rightarrow \dfrac{2}{3}\lambda = 1 \Rightarrow \lambda = 3/2$

(i) $P(x=3) = \dfrac{e^{-3/2}(3/2)^3}{3!} = 0.1254$

(ii) $P(x>3) = 1 - P(x \le 3)$

$= 1 - P(x = 0,1,2,3)$

$= 1 - [P(X=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[e^{-3/2} + \dfrac{e^{-3/2}(3/2)}{1!} + \dfrac{e^{-3/2}(3/2)^2}{2!} + \dfrac{e^{-3/2}(3/2)^3}{3!}\right]$

$= 1 - e^{-3/2}(1 + 3/2 + 9/8 + 27/48)$

$= 0.065$

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Q. If 1000 values approximate a poisson distbn with mean 2.5, how many of them would have values 2, 4, 10?

→ Here $\lambda = 2.5$

$\therefore p(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2.5}(2.5)^x}{x!}$; $x = 0, 1, \dots \infty$

i) $P(2) = \frac{e^{-2.5}(2.5)^2}{2!} = 0.2565$

Req. no. of items $= 1000 \times 0.2565$
$= 256.5 \approx 257$

(ii) $P(4) = \frac{e^{-2.5}(2.5)^4}{4!} = 0.1336$

required no. of items $= 1000 \times 0.1336$
$= 133.6 \approx 134$

(iii) $P(10) = \frac{e^{-2.5}(2.5)^{10}}{10!} = 0.0002$

req. no. of items $= 1000 \times 0.0002$
$= 0.2 \approx 0$

Q. If 3% of electric bulbs are manufactured by a company are defective. find probability that in a sample of 100 bulbs, there are 5 defective bulbs.

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→ Here $p = 3\% = \frac{3}{100}$ and $n = 100$.

$\lambda = np = \frac{3}{100} \times 100 = 3$

$\therefore p(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3}3^x}{x!}$; $x = 0, 1, \dots n$

Now, $P(5) = \frac{e^{-3}3^5}{5!} = 0.1008$

Q. If the pb that an individual suffers a bad reaction from a certain injection is 0.002, determine pb that out of 1000 individuals

a) exactly 3

b) more than 2 will suffer a bad reaction

→ $p = 0.002$; $n = 1000$

$\lambda = np = 0.002 \times 1000 = 2$

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2}2^x}{x!}$; $x = 0, 1, \dots \infty$

a) $P(3) = \frac{e^{-2}\lambda^3}{3!} = 0.1804$

b) $P(\text{more than } 2) = P(x > 2)$
$= 1 - P(x = 0, 1, 2)$
$= 1 - [P(x=0) + P(x=1) + P(x=2)]$
$= 1 - \left(e^{-2} + 2 \times e^{-2} + \frac{e^{-2} \times 2^2}{2!}\right)$
$= 0.323$

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Q) 6 coins are tossed 3200 times using poisson distrb^n. Obtain approximate pb of getting 6 heads 'x' times.

→ $\lambda = np = 3200 \times (1/2)^6$
$= 50$

$\therefore p(x) = \dfrac{e^{-50}(50)^x}{x!}$

⇒ Fitting a Poisson Distrb^n:-

For fitting a poisson distrb^n to given data, we have to determine the most appropriate value of the parameter λ.

For the mean of the given data is $\bar{x}$.
Mean of poisson distrb^n is λ.

Equate $\lambda = \bar{x}$.

$\therefore f(x) = \dfrac{e^{-\bar{x}}(\bar{x})^x}{x!}$; $x = 0, 1 \ldots \infty$.

The theoretical frequencies are obtained by putting $x = 0, 1, 2 \ldots$ in $N.f(x)$.

$N.f(0) = Ne^{-\bar{x}}$

$N.f(1) = N.f(0) \cdot \dfrac{\bar{x}}{1}$

$N.f(2) = N.f(1) \cdot \dfrac{\bar{x}}{2}$.

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$N.f(3) = N.f(2) \cdot \dfrac{\bar{x}}{3}$

where $e = 2.7183$

Q. Fit a poisson distrb^n of this given data:

x	f	fx
0	229	0
1	325	325
2	257	514
3	119	357
4	50	200
5	17	85
6	2	12
7	1	7
8	0	0
	1000	1500

→ $\bar{x} = \dfrac{1500}{1000} = 1.5$

$\lambda = \bar{x} = 1.5$

$f(x) = \dfrac{e^{-\bar{x}} \cdot (\bar{x})^x}{x!}$

$= \dfrac{e^{-1.5}(1.5)^x}{x!}$; $x = 0, 1, \ldots$

$N.f(0) = 1000 \times \dfrac{e^{-1.5} \times (1.5)^0}{0!} = 1000 \times 0.223$
$= 223.1$

$N f(1) = N.f(0) \times \dfrac{\bar{x}}{1} = 223.1 \times 1.5$
$= 335$

$N.f(2) = N.f(1) \times \dfrac{\bar{x}}{2} = 335 \times \dfrac{1.5}{2} = 251.1$

$N.f(3) = N \times f(2) \times \dfrac{\bar{x}}{3} = 251.1 \times \dfrac{1.5}{3} = 125$

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$N.f(4) = N \times f(3) \times \frac{\bar{x}}{4} = \frac{125 \times 1.5}{4} = 47$

$N.f(5) = N \times f(4) * \frac{\bar{x}}{5} = \frac{47 \times 1.5}{5} = 14.1$

$N.f(6) = N \times f(5) \cdot \frac{\bar{x}}{6} = \frac{14.1 \times 1.5}{6} = 4$

$N \times f(7) = N \times f(6) \cdot \frac{\bar{x}}{7} = \frac{4 \times 1.5}{7} = 0.851 \approx 1$

$N \times f(8) = N \times f(7) \cdot \frac{\bar{x}}{8} = \frac{0.857 \times 1.5}{8} = 0.16 \approx 0$

x	0	1	2	3	4	5	6	7	8	Total
f	229	325	257	119	50	17	2	1	0	1000
Tf	223	335	251	125	47	14	4	1	0	1000

? Fit a poissen distrbⁿ for the following data.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

→ $\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{400}{400} = 1$

$\therefore \lambda = 1$

x	0	1	2	3	4	5	6	7	8
Tf	147.15	147.15	73.58	24.53	6.13	1.23	0	0	0
≈	147	147	74	25	6	1	0	0	0

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? An insurance company insures 4000 people against loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumption that on the avg 10 persons in 100,000 will have car accident each year that result in this type of injury. What is the pb that more than 3 of the insured will collect on their policy in a given year?

→ $n = 4000$

p = Pb of loss of both eyes in car accide

$= \frac{10}{100000} = 0.0001$

Since p is very small and 'n' is ver large, we may approximate the given ~~poisson~~ distrbⁿ by poisson distrbⁿ.

$\therefore \lambda = np = 4 \times 0.0001$

$= 0.4$

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.4}(0.4)^x}{x!}; x = 0, 1, ...$

$P(x > 3) = 1 - P(x \le 3) = 1 - P(x = 0, 1, 2, 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - e^{-0.4}\left((0.4)^0 + 0.4 + \frac{(0.4)^2}{2!} + \frac{(0.4)^3}{3!}\right)$

$= 1 - 0.6703(1 + 0.4 + 0.08 + 0.0107)$
$= 1 - 0.6703 \times 1.4907$
$= 0.0008$ //

9. A manufacturer, who produces medicine bottles, find that 0.1% of bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the bottle producer. Using Poisson distrbn, find how many boxes will contain
i) no defective
ii) atleast 2 defective.

→ $N = 100$; $n = 500$

$p =$ pb of a defective

$= \frac{0.1}{100} = 0.001$ //

$\lambda = np = 500 \times 0.001 = 0.5$ //

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.5}(0.5)^x}{x!} \; ; x = 0, 1, 2 \ldots$$

i) P(no defective) $= P(X=0)$

$$= \frac{e^{-0.5}(0.5)^0}{0!} = \underline{\underline{0.6065}}$$

Required number $= N \times P(X=0)$
$= 100 \times 0.6065 = 60.65$
≈ 61 //

ii) P(atleast 2 defective) $= P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - P(X = 0, 1)$
$= 1 - \{P(X=0) + P(X=1)\}$
$= 1 - (0.6065 + 0.5 \times 0.6065)$
$= 0.09029$ //

Req. number $= N \times P(X \geq 2)$
$= 100 \times 0.09029$
$= 9$ //

9. Suppose X has a poisson distrbn if $P(X=2) = \frac{2}{3} P(X=1)$. Evaluate
i) $P(X=0)$ ii) $P(X=3)$

4 qns. → $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Since $P(X=2) = \frac{2}{3} P(X=1)$

$$\Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1!}$$

$$\Rightarrow \frac{\lambda}{2} = \frac{2}{3} \Rightarrow 3\lambda = 4$$
$$\Rightarrow \lambda = \underline{\underline{4/3}}$$

(i) $P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!}$

$$= \frac{e^{-4/3}(4/3)^0}{0!} = e^{-4/3} = \underline{\underline{0.26}}$$

(ii) $P(X=3) = \frac{e^{-4/3}(4/3)^3}{3!} = \underline{\underline{0.104}}$

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? If X has a poissons distrbⁿ such that $P(X=1) = P(X=2)$. find $P(X=4)$

→ $P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$P(X=1) = P(X=2)$

$\Rightarrow \frac{\cancel{e^{-\lambda}}\lambda}{1!} = \frac{\cancel{e^{-\lambda}}\lambda^{2}}{2!} \Rightarrow \frac{\lambda}{2} = 1 \Rightarrow \lambda = 2.$

$P(X=4) = \frac{e^{-2}2^{4}}{4!} = 0.09$

? If X has a poissons variate such that $P(X=1) = 2P(X=2)$. find

i) $P(X=0)$ (ii) Mean (iii) Variance

→ $P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

Since $P(X=1) = 2P(X=2)$

$\Rightarrow \frac{\cancel{e^{-\lambda}}\cancel{\lambda}}{1!} = 2\times\frac{\cancel{e^{-\lambda}}\lambda^{2}}{2!}$

$\Rightarrow 1 = \frac{\cancel{2}\lambda}{\cancel{2!}} \Rightarrow \lambda = 1$

(i) $P(X=0) = \frac{e^{-1}1^{0}}{0!} = e^{-1} = 0.37.$

(ii) Mean $= \lambda = 1$ //

(iii) Variance $= \lambda = 1$ //

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? The no: of monthly breakdown of a comput is a RV having a poissons distrbⁿ with mean equal to 1.8. find the pb that this computer will funcⁿ for a month

a) without a breakdown.

b) with only one breakdown

c) with atleast one breakdown.

→ Given, mean = 1.8

$\therefore \lambda = 1.8$

$P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

a) P(without a breakdown) $= P(X=0)$

$= \frac{e^{-1.8}(1.8)^{0}}{0!}$

$= e^{-1.8} = 0.1653$

b) P(with only one breakdown) $= P(X=1)$

$= \frac{e^{-1.8}(1.8)^{1}}{1!}$

$= 0.2975$ //

c) P(with atleast one breakdown) $= P(X \geq 1)$

$= 1 - P(X=0)$

$= 1 - 0.1653$

$= 0.8347$

Q. It is known that the pb of an items produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find no. of packets containing atleast, exactly and atmost 2 defective items in a consignment of 100 packets using poisson distribn.

→ $p = 0.05$; $n = 20$; $N = 100$. $\quad \frac{e^{-\lambda}\lambda^x}{x!}$

$\lambda = np = 20 \times 0.05 = 1$

$q = 1 - p = 1 - 0.05 = 0.95$

(i) P(atleast 2 defective) $= P(x = 2)$

$= \frac{e^{-1} 1^2}{2!}$

$= 0.1839$

Required no. of packets $= N \times P(x = 2)$

$= 100 \times 0.1839$

$= 18.4$

(ii) P(exactly 2 defectives) $= P(x \geq 2)$

$= 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - P(x=0) - P(x=1)$

$= 1 - \frac{e^{-1} 1^0}{0!} - \frac{e^{-1} 1^1}{1!}$

$= 1 - 0.368 - 0.368$

$= 0.2642$

Required no. of packets $= 100 \times 0.2642$

$= 26.4$

(iii) P(atmost 2 defectives) $= P(x \leq 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 0.368 + 0.368 + 0.1839$

$= 0.9199$

Required no. of packets $= 100 \times 0.9199$

$= 919.9$

$= 920$

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Continuous Random Variable :-

If 'x' is a RV which can take all values in an interval, then x is called a continuous random variable.

eg:- ① X represents the service time of the doctor on his next patient

② Life time of the tube.

Probability Density func^n (pdf) :-

If X is a continuous random variable such that

$$P\left(x - \frac{dx}{2} < X < x + \frac{dx}{2}\right) = f(x)\,dx$$

then f(x) is called the pb density func^n.

Definition :-

Let X be a continuous random variable, let F(x) be a func^n defined by $F(x) = P(X \le x)$. Then F(x) is called cumulative distribution func^n (cdf).

If X is continuous,

$$F(x) = P(-\infty \le X \le x) = \int_{-\infty}^{\infty} f(x)\,dx.$$

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Properties :-

1) If 'x' is a continuous random variable, then $\frac{d}{dx}F(x) = f(x)$, at all points where F(x) is differentiable.

2) $F(-\infty) = 0$ and $F(+\infty) = 1 \Rightarrow 0 \le F(x) \le 1$

3) $F(+\infty)$ is a continuous func^n of 'x' on the right

4) $P(a \le x \le b) = \int_a^b f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$

$= P(X \le b) - P(X \le a) = F(b) - F(a)$

Definition :-

Let $f(x) = \frac{dF(x)}{dx}$, then f(x) is called probability density func^n (pdf).

Properties :-

1) $f(x) \ge 0$.

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3) $P(a \le x \le b) = P(a < x < b) = \int_a^b f(x)\,dx$.

Note : Distribution func^n :-

$f(x_1)$ $f(x_2)$ $f(x_3)$

$-\infty$ a b c d ∞

let $f(x) = f(x_1)$ if $a \le x \le b$

$= f(x_2)$ if $b \le x \le c$

$= f(x_3)$ if $c \le x \le d$

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If $x < a$, $F(x) = 0$.

If $a \le x \le b$, $F(x) = \int_{-\infty}^{a} 0\, dx + \int_{a}^{x} f(x_1)\, dx$

If $b \le x \le c$, $F(x) = \int_{-\infty}^{a} 0\, dx + \int_{a}^{b} f(x_1)\, dx + \int_{b}^{x} f(x_2)\, dx$

If $c \le x \le d$,
$F(x) = \int_{-\infty}^{a} 0\, dx + \int_{a}^{b} f(x_1)dx + \int_{b}^{c} f(x_2)dx + \int_{c}^{x} f(x_3)\, dx$

If $d \le x \le \infty$, $F(x) = 1$.

* Note 2:-

Mean, $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$.

Variance, $\sigma^2 = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx - \mu^2$.

Q. Examine whether the following is a density func^n,

$$f(x) = \begin{cases} 2x & ; 0 < x < 1 \\ 4-2x & ; 1 < x < 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

→ If $f(x)$ is a density func^n it has to satisfy the following 2 conditions:

1) $f(x) \ge 0$.

2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

Here $f(x) \ge 0$.

But $\int_{-\infty}^{\infty} f(x)\, dx = \int_0^1 2x\, dx + \int_1^2 (4-2x)\, dx$

$= \left[\cancel{2}\frac{x^2}{\cancel{2}}\right]_0^1 + \left[4x - \frac{\cancel{2}x^2}{\cancel{2}}\right]_1^2$

$= (1-0) + 8-4-(4-1)$

$= 1+4-3 = 2 \ne 1$

$\therefore f(x)$ is not a density func^n.

Q. If $p(x) = \begin{cases} x e^{-x^2/2} & ; x \ge 0 \\ 0 & ; x < 0 \end{cases}$

a) Show that $p(x)$ is a pdf

b) find its distribution func^n $F(x)$

→ a) If $p(x)$ is to be a pdf,
$p(x) \ge 0$ and $\int_{-\infty}^{\infty} p(x)\, dx = 1$.

Clearly, $p(x) = x e^{-x^2/2} \ge 0$, when $x \ge 0$.

Now $\int_0^{\infty} p(x)\, dx = \int_0^{\infty} x \cdot e^{-x^2/2}\, dx$

$= \int_0^{\infty} e^{-t}\, dt = \left[\frac{e^{-t}}{-1}\right]_0^{\infty}$

$= -\left(e^{-\infty} - e^{0}\right)$

$= -(-1) = 1$

$\therefore p(x)$ is a pdf of a random variable x.

put $t = x^2/2$.

$\frac{dt}{dx} = \frac{2x}{2}$

$dt = \frac{\cancel{2}x}{\cancel{2}}\, dx$

$x = 0 \Rightarrow t = 0$

$x = \infty \Rightarrow t = \infty$

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b) $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx + \int_{0}^{x} x \cdot e^{-x^2/2}\,dx$

$= \int_{0}^{x^2/2} e^{-t}\,dt$

put $t = x^2/2$
$\frac{dt}{dx} = x^2$
$dt = x^2 dx$
$x=0 \Rightarrow t=0$
$x=x \Rightarrow t=x^2/2$

$= \left(\frac{e^{-t}}{-1}\right)_0^{x^2/2}$

$= -\left(e^{-x^2/2} - e^{0}\right)$

$= -\left(e^{-x^2/2} - 1\right)$

$= 1 - e^{-x^2/2}$ //

$x<0$; $F(x) = 0$

Q. If the density func^n of a continuous random variable X is given by

$$f(x) = \begin{cases} ax & ; 0 \le x \le 1 \\ a & ; 1 \le x \le 2 \\ 3a - ax & ; 2 \le x \le 3 \\ 0 & ; \text{elsewhere} \end{cases}$$

a) find the value of 'a'.
b) find the cdf of 'X'
c) $P(X > 1.5)$

→ a) Since $f(x)$ is a pdf, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie; $\int_{0}^{3} f(x)\,dx = 1$

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$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$

$\Rightarrow a\left[\frac{x^2}{2}\right]_0^1 + a[x]_1^2 + 3a[x]_2^3 - a\left[\frac{x^2}{2}\right]_2^3 = 1$

$\Rightarrow \frac{a}{2}(1-0) + a(2-1) + 3a(3-2) - \frac{a}{2}(9-4) = 1$

$\Rightarrow \frac{a}{2} + a + 3a - \frac{5a}{2} = 1$

$\Rightarrow 4a - 2a = 1 \Rightarrow 2a = 1 \Rightarrow a = 1/2$

b) $x < 0$; $F(x) = P(X \le x) = 0$.

$0 \le x \le 1$, $F(x) = \int_0^x f(x)\,dx = \int_0^x \frac{x}{2}\,dx = \frac{x^2}{4}$ //

$a = 1/2 \Rightarrow f(x) = \begin{cases} x/2 & ; 0 \le x \le 1 \\ 1/2 & ; 1 \le x \le 2 \\ \frac{3}{2} - \frac{x}{2} & ; 2 \le x \le 3 \\ 0 & ; \text{elsewhere} \end{cases}$

$1 \le x \le 2$, $F(x) = \int_0^x f(x)\,dx$.

$= \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$

$= \int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,dx$

$= \frac{x}{2} - \frac{1}{4}$ //

$\frac{a - 5a}{2} = \frac{-4a}{2} = -2a$.

$2 \le x \le 3$; $F(x) = \int_0^x f(x)\,dx$

$= \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^x f(x)dx$

$= \int_0^1 \frac{x}{2}dx + \int_1^2 \frac{1}{2}dx + \int_2^x \left(\frac{3}{2} - \frac{x}{2}\right)dx$

$= \frac{3}{2}x - \frac{x^2}{4} - \frac{5}{4}$

$x > 3$; $F(x) = 1$

$F(x) = \int_0^1 \frac{x}{2}dx + \int_1^2 \frac{1}{2}dx + \int_2^3 \left[\frac{3}{2} - \frac{x}{2}\right]dx = 1$

c) $P(X > 1.5) = \int_{1.5}^3 f(x)dx = \int_{1.5}^2 \frac{1}{2}dx + \int_2^3 \left(\frac{3}{2} - \frac{x}{2}\right)dx$

$= \frac{1}{2}[x]_{1.5}^2 + \left[\frac{3}{2}x + \frac{x^2}{4}\right]_2^3$

$= \underline{1/2}$

Q. A continuous RV X that can assume any value b/w $x=2$ and $x=5$ has a density funcⁿ given by $f(x) = k(1+x)$. Find $P(x<4)$.

$\rightarrow$ $f(x) = k(1+x)$; $x=2$ & $x=5$

$\int_2^5 k(1+x)dx = k\left[x + \frac{x^2}{2}\right]_2^5$ $\qquad$ $5+12.5-4$ $\quad$ 13.5

$= k\left(5 + \frac{25}{2} - 2 - \frac{4}{2}\right)$

$\therefore \int_2^5 k(1+x)dx = 1$ $\qquad$ $4+8-4$

$\therefore k \times \frac{27}{2} = 1 \Rightarrow k = \frac{2}{27}$

$P(X<4) = \int_2^4 \frac{2}{27}(1+x)dx$

$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4$

$= \frac{2}{27}\left(4 + \frac{16}{2} - 2 - \frac{4}{2}\right) = \frac{2}{27} \times 8 = \underline{\frac{16}{27}}$

Q. A continuous rv X has a pdf $f(x) = kx^2e^{-x}$; $x \ge 0$. Find k, mean & variance.

$\rightarrow$ $\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_0^{\infty} k \cdot x^2 e^{-x}dx = 1 \Rightarrow k\int_0^{\infty} x^2 e^{-x}dx = 1$

$= k\left[x^2\frac{e^{-x}}{-1} - 2x\cdot\frac{e^{-x}}{(-1)^2} \ldots\right]$

$\Rightarrow 2k = 1 \Leftrightarrow k = 1/2$

Mean, $\mu = E(x) = \int_0^{\infty} x f(x)dx$

$= \int_0^{\infty} x \cdot \frac{1}{2}x^2 e^{-x}dx$

$= \frac{1}{2}\int_0^{\infty} x^3 e^{-x}dx$

$= \frac{1}{2}\left[x^3\frac{e^{-x}}{-1} - 3x^2\frac{e^{-x}}{(-1)^2} + 6x\frac{e^{-x}}{(-1)^3} - \frac{6e^{-x}}{(-1)^4}\right]_0^{\infty}$

$t = x$, $\frac{dt}{dx} = 2x$, $\frac{dt}{2} = x\,dx$

$= 2k(-e^{-x})_0^{\infty}$, $= 2k(-e^{-\infty} + e^{-0})$, $= 2k(0+1) = 1$, $= +2k = 1 \Rightarrow k = \frac{1}{2}$

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$$= \frac{1}{2}\left[-x^3 e^{-x} - 3x^2 e^{-x} - 6x e^{-x} - 6e^{-x}\right]_0^\infty$$

$$= \frac{1}{2}\left[0-0-0-6(e^{-\infty}-e^0)\right]$$

$$= \frac{1}{2}\left[-6(0-1)\right] = \frac{6}{2} = 3$$

$Var(x) = E(x^2) - [E(x)]^2$ — ①

$$E(x^2) = \int_0^\infty x^2 \cdot f(x)\,dx$$

$$= \int_0^\infty x^2 \cdot \frac{1}{2} x^2 e^{-x}\,dx = \frac{1}{2}\int_0^\infty x^4 e^{-x}\,dx$$

$$= \frac{1}{2}\left[x^4(-e^{-x}) - 4x^3(e^{-x}) + 12x^2(-e^{-x}) - 24x(e^{-x}) + 24(-e^{-x})\right]_0^\infty$$

$$= \frac{1}{2} \times 24\left(-e^{-\infty} - e^0\right)$$

$$= 12$$

$\therefore$ ① $\Rightarrow Var(x) = 12 - 3^2$

$= 12 - 9 = 3$

Q. A continuous RV has a pdf $f(x) = 3x^2$; $0 \le x \le 1$. Find a & b such that

i) $P(X \le a) = P(X > a)$

ii) $P(X > b) = 0.05$

i) $P(X \le a) = P(X > a)$

ie; $\int_0^a f(x)\,dx = \int_a^1 f(x)\,dx$

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$$\therefore \int_0^a 3x^2\,dx = \int_a^1 3x^2\,dx$$

$$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^a = 3\left[\frac{x^3}{3}\right]_a^1$$

$$a^3 - 0 = 1 - a^3$$

$$2a^3 = 1 \Rightarrow a^3 = 1/2 \Rightarrow a = \sqrt[3]{1/2} = 0.79$$

$= 0.794$

ii) $P(X > b) = 0.05$

$$\Rightarrow \int_b^1 f(x)\,dx = 0.05$$

$$\Rightarrow \int_b^1 3x^2\,dx = 0.05 \Rightarrow 3\left[\frac{x^3}{3}\right]_b^1 = 0.05$$

$$\Rightarrow 1 - b^3 = 0.05 \Rightarrow b^3 = 1 - 0.05$$

$$\Rightarrow b^3 = 0.95 \Rightarrow b = \sqrt[3]{0.95} = 0.983$$

Q. The distrib^n func^n of a rv X is given by $F(x) = 1-(1+x)e^{-x}$, $x \ge 0$. Find the density func^n, mean & variance of X.

Given $F(x) = 1-(1+x)e^{-x}$; $x \ge 0$.

We know that

$$f(x) = \frac{d}{dx}F(x)$$

$$f(x) = -\left[e^{-x} + (1+x)e^{-x}(-1)\right]$$

$$= -\left[\cancel{e^{-x}} - \cancel{e^{-x}} - xe^{-x}\right] = xe^{-x}, x \ge 0$$

$$E(x) = \int_0^\infty x \cdot f(x)\,dx = \int_0^\infty x \cdot xe^{-x}\,dx.$$

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$$= \int_0^\infty x^2 e^{-x}\, dx$$
$$= \left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^\infty$$
$$= 2(-e^{-\infty} + e^0) = 2$$

$Var(X) = E(X^2) - [E(X)]^2$ — (1)

$E(X^2) = \int_0^\infty x^2 \cdot x e^{-x}\, dx = \int_0^\infty x^3 e^{-x}\, dx = 6$

$\therefore (1) \Rightarrow Var(X) = 6 - 2^2 = 6 - 4 = 2$

9. The cdf of a continuous rv X is given by
$$F(x) = \begin{cases} 0 ; & x < 0 \\ x^2 ; & 0 \le x < 1/2 \\ 1 - \frac{3}{25}(3-x)^2 ; & 1/2 \le x < 3 \\ 1 ; & x \ge 3 \end{cases}$$
find pdf of X and evaluate (i) $P(|X| \le 1)$
(ii) $P(|X| \le 1)$ and $P(1/3 \le X < 4)$ using both the pdf and cdf.

→
$$f(x) = \frac{d}{dx}F(x) = \begin{cases} 0 ; & x < 0 \\ 2x ; & 0 \le x < 1/2 \\ \frac{6}{25}(3-x) ; & 1/2 \le x < 3 \\ 0 ; & x \ge 3 \end{cases}$$

(i) $P(|X| \le 1) = P(-1 \le X \le 1)$
$$= \int_{-1}^{1} f(x)\,dx$$

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$$= \int_{-1}^{1/2} 2x\,dx + \int_{1/2}^{1} \frac{6}{25}(3-x)\,dx$$
$$= 2\left[\frac{x^2}{2}\right]_{-1}^{1/2} + \frac{6}{25}\left[3x - \frac{x^2}{2}\right]_{1/2}^{1}$$
$$= \left(\frac{1}{4} - 1\right) + \frac{6}{25}\left(3 - \frac{1}{2} - \frac{3}{2} - \frac{1}{8}\right)$$
$$= \frac{-3}{4} + \frac{6}{25} \times \frac{7}{8} = \frac{13}{25}$$

(ii) $P(1/3 < X < 4) = \int_{1/3}^{1/2} f(x)\,dx + \int_{1/2}^{3} f(x)\,dx + \int_3^4 f(x)\,dx$
$$= \int_{1/3}^{1/2} 2x\,dx + \int_{1/2}^{3} \frac{6}{25}(3-x)\,dx + \int_3^4 0 \cdot dx$$
$$= 2\left[\frac{x^2}{2}\right]_{1/3}^{1/2} + \frac{6}{25}\left[3x - \frac{x^2}{2}\right]_{1/2}^{3} + 0$$
$$= \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{6}{25}\left[3(3 - 1/2) - \left(\frac{3^2}{2} - \frac{1/4}{2}\right)\right]$$
$$= \frac{1}{4} - \frac{1}{9} + \frac{6}{25}\left(\frac{15}{2} - \frac{35}{8}\right)$$
$$= \frac{8}{9}$$

9. The diameter of an electric cable, say X, is assumed to be a continuous random variable with pdf $f(x) = 6x(1-x) ; 0 \le x \le 1$
(i) Check that f(x) is pdf
(ii) Determine a number 'b' such that $P(X < b) = P(X > b)$.

$\frac{3-2}{6} = \frac{1}{6}$

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$f(x) = 6x(1-x)$, $0 \le x \le 1$

→ (i) • $f(x) \ge 0$ is true

• $\int_0^1 f(x)\,dx = 1$

$\Rightarrow \int_0^1 6x(1-x)\,dx = 6\int_0^1 x - x^2\,dx$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 6\left(\frac{1}{2} - \frac{1}{3}\right) = 6 \times \frac{1}{6} = 1$

$\therefore f(x)$ is a pdf

(ii) $P(X < b) = P(X > b)$

$\int_0^b f(x)\,dx = \int_b^1 f(x)\,dx$

$\Rightarrow \int_0^b 6x(1-x)\,dx = \int_b^1 6x(1-x)\,dx$

$\Rightarrow 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_b^1$

$\Rightarrow \frac{b^2}{2} - \frac{b^3}{3} = \frac{1}{2}(1-b^2) - \frac{1}{3}(1-b^3)$

$\Rightarrow \frac{b^2}{2} - \frac{b^3}{3} = \frac{1}{2} - \frac{b^2}{2} - \frac{1}{3} + \frac{b^3}{3}$

$\frac{3-2}{6} = \frac{1}{6}$

$\Rightarrow \frac{b^2}{2} + \frac{b^2}{2} - \frac{b^3}{3} - \frac{b^3}{3} = \frac{1}{2} - \frac{1}{3}$

$\Rightarrow \frac{\cancel{2}b^2}{\cancel{2}} - \frac{2b^3}{3} = \frac{1}{6} \Rightarrow b^2 - \frac{2b^3}{3} = \frac{1}{6}$

$\Rightarrow 6\left[b^2 - \frac{2b^3}{3}\right] = 1 \Rightarrow 6b^2 - 4b^3 - 1 = 0$

$\frac{2 \times 6b^2}{\cancel{3}}$

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$\Rightarrow 4b^3 - 6b^2 + 1 = 0$

$\Rightarrow (2b-1)(2b^2 - 2b - 1) = 0$

$\Rightarrow b = 1/2$; $b = \frac{1 \pm \sqrt{3}}{2}$

$b = 1/2$ is the only real value lying b/w 0 & 1.

Q. Let X be a continuous random variable with pdf $f(x) = \begin{cases} ax & ; 0 \le x \le 1 \\ a & ; 1 \le x \le 2 \\ -ax + 3a & ; 2 \le x \le 3 \\ 0 & ; \text{elsewhere} \end{cases}$

i) Determine the constant 'a'

ii) Compute $P(X \le 1.5)$

→ (i) $\int_0^\infty f(x)\,dx = 1 \Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx + \int_3^\infty 0\,dx$,

$\frac{9}{2} - 2$

$\frac{9-4}{2} = \frac{5}{2}$

$\Rightarrow a\left[\frac{x^2}{2}\right]_0^1 + a[x]_1^2 + -a\left[\frac{x^2}{2}\right]_2^3 + 3a[x]_2^3 = 1$

$\Rightarrow \frac{a}{2} + a - a\left(\frac{9}{2} - 2\right) + 3a(3-2) = 1$

$\frac{a}{2} - \frac{5a}{2}$

$\frac{1}{2}(a - 5a)$

$-\frac{4a}{2} = -2a$

$\Rightarrow \frac{a}{2} + a - \frac{5a}{2} + 3a = 1$

$\Rightarrow 4a - 2a = 1 \Rightarrow 2a = 1 \Rightarrow a = 1/2$

(iii) $P(X \le 1.5) = \int_{-\infty}^{1.5} f(x)\,dx + \int_{-\infty}^{0} f(x)\,dx + \int_0^{1.5} f(x)\,dx + \int_1 f(x)\,dx$

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$$= \int_{-\infty}^{0} 0\, dx + \int_0^1 x/2\, dx + \int_1^{1.5} 1/2\, dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\left[x\right]_1^{1.5}$$

$$= \frac{1}{4} + \frac{1}{2}(1.5-1) = \frac{1}{4} + \frac{1}{2}\times\frac{1}{2} = 1/2$$

$\frac{1}{4}+\frac{1}{4}$, $1/2$

Q) In a continuous distrib^n, the pb density is given by $f(x) = kx(2-x)$; $0<x<2$. Find k, mean, variance and the distrib^n func^n.

→ $f(x) = kx(2-x)$; $0<x<2$.

$$\int_0^2 f(x)\,dx = \int_0^2 kx(2-x)\,dx = 1$$

$$= k\int_0^2 (2x - x^2)\,dx = k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= k\left[(4-0) - \frac{1}{3}(8-0)\right] = 1$$

$$= k\left(4 - \frac{8}{3}\right) = 1$$

$4-\frac{8}{3}$, $\frac{12-8}{3} = \frac{4}{3}$

$$= k\cdot\frac{4}{3} = 1 \Rightarrow k = 3/4$$

$$E(x) = \int_0^2 x\cdot f(x)\,dx.$$

$$= \int_0^2 x\cdot\frac{3}{4}x(2-x)\,dx = \frac{3}{4}\int_0^2 x^2(2-x)\,dx$$

$$= \frac{3}{4}\int_0^2 2x^2 - x^3\,dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = \frac{3}{4}\left[\frac{2}{3}(8-0) - \frac{1}{4}\times 16\right]$$

$8 - \frac{32}{5} = \frac{40-32}{5} = \frac{8}{5}$, $\frac{16}{3}-4$, $\frac{16-12}{3} = \frac{4}{3}$

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$$= \frac{3}{4}\left(\frac{16}{3} - \frac{16}{4}\right) = \frac{3}{4}\left(\frac{16}{3} - 4\right)$$

$$= \frac{3}{4}\times\frac{4}{3} = 1$$

$$V(x) = \int_0^2 x^2 f(x)\,dx = \int_0^2 x^2\cdot\frac{3}{4}x(2-x)\,dx$$

$$= \frac{3}{4}\int_0^2 x^3(2-x)\,dx = \frac{3}{4}\int_0^2 (2x^3 - x^4)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{1}{2}\times 16 - \frac{1}{5}(32)\right] = \frac{3}{4}\left(8 - \frac{32}{5}\right)$$

$$= \frac{3}{4}\times\frac{8}{5} = \frac{6}{5}$$

$$f(x) = \frac{3}{4}x(2-x) ; 0\le x<2.$$

$$F(x) = \begin{cases} 0 & ; x<0 \\ \frac{1}{4}(3x^2 - x^3) & ; 0\le x<2 \\ 1 & ; x\ge 2 \end{cases}$$

Q) The diameter of an electric cable X is a continuous r.v with pdf $f(x) = kx(1-x)$; $0\le x\le 1$

Find (i) value of k.

(ii) cdf of X

(iii) Value of 'a' such that $P(X<a) = 2P(X>a)$

(iv) $P(X\le 1/2 \,/\, 1/3 < X < 2/3)$

$\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$

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(i) $f(x) = kx(1-x)$; $0 \le x \le 1$.

$\int_0^1 f(x)\,dx = 1 \Rightarrow \int_0^1 kx(1-x)\,dx = 1$

$\Rightarrow k\int_0^1 (x-x^2)\,dx = k\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 1$

$\Rightarrow k\left(\frac{1}{2}-\frac{1}{3}\right) = 1 \Rightarrow \frac{1}{6}k = 1 \Rightarrow k = 6$.

$f(x) = 6x(1-x) = 6x-6x^2$

(ii) $F(x) = \begin{cases} 0 & ; x<0 \\ 3x^2-2x^3 & ; 0 \le x < 1 \\ 1 & ; x \ge 1 \end{cases}$

(iii) Given $P(X<a) = 2P(X>a)$

$\Rightarrow \int_0^a f(x)\,dx = 2\int_a^1 f(x)\,dx$

$\Rightarrow \int_0^a (6x-6x^2)\,dx = 2\int_a^1 (6x-6x^2)\,dx$

$\Rightarrow \left[\frac{6x^2}{2}-\frac{6x^3}{3}\right]_0^a = 2\left[\frac{6x^2}{2}-\frac{6x^3}{3}\right]_a^1$

$\Rightarrow \left[3x^2-2x^3\right]_0^a = 2\left[3x^2-2x^3\right]_a^1$

~~$\Rightarrow 3a^2-2a^3 = 6a^2-4a^3$~~

~~$\Rightarrow 4a^3-2a^3 = 6a^2-3a^2$~~

~~$\Rightarrow 2a^3 = 3a^2$~~

$\Rightarrow 3a^2-2a^3 = 2\left[3(1-a^2)-2(1-a^3)\right]$

$\Rightarrow 3a^2-2a^3 = 6-6a^2-4+4a^3$

$\Rightarrow 6a^2+3a^2-2a^3-4a^3 = 2$

$\Rightarrow 9a^2-6a^3-2=0$

$-4(1-a^3)$
$-4a^3$

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$\Rightarrow 6a^3-9a^2+2=0$

$a = 1.3$; $a = 0.4$

that lies b/w 0 & 1.

$\frac{1}{4}-\frac{1}{9}$, $\frac{9-4}{36}=\frac{5}{36\times 2}$, $\frac{5}{72}$, $\frac{27\times 2}{3}$, 81

(iv) $P(X<1/2 \,/\, 1/3 < X < 2/3)$.

$= \int_{1/3}^{1/2} f(x)\,dx = 6\int_{1/3}^{1/2}(x-x^2)\,dx$

$= 6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{1/3}^{1/2} = 6\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{9}\right)-\frac{1}{3}\left(\frac{1}{8}-\frac{1}{27}\right)\right]$

$= 6\left(\frac{1}{8}-\frac{1}{18}-\frac{1}{24}+\frac{1}{81}\right) = 6\times\frac{13}{324} = \frac{13}{54}$

9. X is a continuous r.v with pdf given by

$f(x) = \begin{cases} kx & ; 0 \le x \le 2 \\ 2k & ; 2 \le x \le 4 \\ 6k-kx & ; 4 \le x \le 6 \end{cases}$

$6k-kx$.
$k(6-x)$

find value of k and F(x).

$\rightarrow \int_0^6 f(x)\,dx = 1$

$\Rightarrow \int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (6k-kx)\,dx = 1$

$\Rightarrow k\left[\frac{x^2}{2}\right]_0^2 + 2k\left[x\right]_2^4 + k\left[6x-\frac{x^2}{2}\right]_4^6 = 1$

$\Rightarrow k\times\frac{4}{2} + 2k(4-2) + k\left(36-\frac{36}{2}-24+\frac{16}{2}\right) = 1$

$\Rightarrow 2k+4k+2k = 1 \Rightarrow 8k = 1 \Rightarrow k = 1/8$

$\frac{x}{8}=\frac{1}{4}$

$\frac{6}{8}-\frac{x}{8}$

$\frac{3}{4}$

$$f(x)=\begin{cases} x/8 & ; 0\le x<2 \\ 1/4 & ; 2\le x<4 \\ \frac{3}{4}-\frac{x}{8} & ; 4\le x\le 6. \end{cases}$$

$$F(x)=\begin{cases} 0 & ; x<0 \\ x^2/16 & ; 0\le x<2 \\ \frac{x-1}{4} & ; 2\le x\le 4 \\ -\frac{(20-12x+x^2)}{16} & ; 4\le x<6 \\ 1 & ; x\ge 6. \end{cases}$$

Q. If the distrib^n as a continuous r.v. X is given by
$$F(x)=\begin{cases} 0 & ; x<0 \\ x & ; 0\le x<1 \\ 1 & ; x\ge 1. \end{cases}$$

find pdf of X. Also find $P(1/3<X<1/2)$ and $P(1/2<X<2)$ using cdf of X.

→ $f(x)=1$; $0\le x\le 1$.

(i) $P(1/3<X<1/2)=\int_{1/3}^{1/2} f(x)\,dx=\int_{1/3}^{1/2} 1\cdot dx$

$=[x]_{1/3}^{1/2}=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

(ii) $P(1/2<X<2)=\int_{1/2}^{1} 1\,dx=[x]_{1/2}^{1}$

$=1-\frac{1}{2}=\frac{1}{2}$

Q. If the r.v. K is uniformly distributed over (1, 7), what is the pb that roots of the eq^n $x^2+2kx+(2k+3)=0$ are real.

Page No: $4\times4(K+2)$ $\;4(4k+8)$ $\;16k+32$

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Q. If r.v. k is uniformly distributed over $(0,5)$, what is the pb that the roots of eqn $4x^2+4kx+(k+2)=0$ are real?

→ The rv is $u(0,5)$

b^2-4ac

$\therefore$ pdf of $k = 1/5 \;;\; 0<k<5$

ie, $f(k) = 1/5$ in $(0,5)$

P(roots of $4x^2+4kx+(k+2)$ are real)

$= P(\text{discriminant of eq}^n \geq 0)$

$= P((k^2-k-2)\geq 0)$

$= P((k-2)(k+1)\geq 0)$

$= P((k\geq -1)\text{and}(k\geq 2))$

$= P(k\geq 2 \text{ or } k\leq -1)$

$= P(k\geq 2)$ since k values in $(0,5)$

$= \int_2^5 f(k)\,dk = \int_2^5 \frac{1}{5}\,dk$

$= \frac{1}{5}[k]_2^5 = \frac{1}{5}(5-2) = 3/5$

⇒ Exponential Distribution:-

A continuous rv X is said to follow the exponential distbn if its pdf is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} \;; & x>0 \\ 0 \;; & x\leq 0. \end{cases}$$

Mean & Variance:-

Mean $= E(X) = \int_0^\infty x\cdot f(x)\,dx$

$= \int_0^\infty x\lambda e^{-\lambda x}\,dx$

put $\lambda x = y \Rightarrow x = y/\lambda$ & $dx = \frac{1}{\lambda}dy$.

$\therefore E(x) = \int_0^\infty \frac{y}{\lambda}\cdot \lambda e^{-y}\frac{dy}{\lambda}$

$= \frac{1}{\lambda}\int_0^\infty y e^{-y}\,dy$

$$\boxed{\Gamma n = \int_0^\infty e^{-x}x^{n-1}dx \qquad \Gamma n = (n-1)!}$$

$= \frac{1}{\lambda}\int_0^\infty e^{-y}y^{2-1}\,dy$

$= \frac{1}{\lambda}\Gamma 2 = \frac{1}{\lambda}(2-1)!$

$= \frac{1}{\lambda}\times 1! = \frac{1}{\lambda}$

$Var(x) = E(x^2) - [E(x)]^2$ —— (1).

$E(x^2) = \int_0^\infty x^2\cdot\lambda e^{-\lambda x}\,dx$

$= \int_0^\infty \left(\frac{y}{\lambda}\right)^2 \lambda e^{-y}\frac{dy}{\lambda}$

$= \frac{1}{\lambda^2}\int_0^\infty y^2 e^{-y}\,dy = \frac{1}{\lambda^2}\int_0^\infty e^{-y}y^{3-1}\,dy$

$= \frac{1}{\lambda^2}\Gamma 3 = \frac{(3-1)!}{\lambda^2} = \frac{2!}{\lambda^2} = \frac{2}{\lambda^2}$

(1) ⇒ $Var(x) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$

$= \frac{1}{\lambda^2}$