Probability

SET Self Evaluation Test - 28

1. A box contains 15 tickets numbered 1, 2, ... 15. Seven tickets are drawn at random one after the other with replacement. The probability that the greatest number on a drawn ticket is 9, is **[CEE 1993;**

(a) $\left(\frac{9}{10}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) None of these

2. Two squares are chosen at random on a chess-board. The probability that they have a side in common, is

(a) 1/9 (b) 2/7

(c) 1/18 (d) None of these

3. The probability that a certain beginner at golf gets a good shot if he uses the correct club is $\frac{1}{3}$ and the probability of a good shot with an incorrect club is $\frac{1}{4}$. In his bag are 5 different clubs, only one of which is correct for the shot in question. If he chooses a club at random and takes a stroke, then the probability that he gets a good shot, is

(a) 1/3 (b) 1/12

(c) 4/15 (d) 7/12

4. If $\frac{1-3p}{2}, \frac{1+4p}{3}$ and $\frac{1+p}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is **[MNR 1992; RPET 20**

(a) $[0, 1]$ (b) $\left[-\frac{1}{4}, \frac{1}{3}\right]$

(c) $\left[0, \frac{1}{3}\right]$ (d) $(0, \infty)$

5. A man alternately tosses a coin and throws a dice beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is **[Roorkee 1988]**

(a) 3/4 (b) 1/2

(c) 1/3 (d) None of these

6. Two persons 'A' and 'B' have respectively $n+1$ and n coins which they toss simultaneously. Then the probability that A will have more heads than B is

(a) $\frac{1}{2}$ (b) $>\frac{1}{2}$

(c) $<\frac{1}{2}$ (d) None of these

7. Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is

(a) 1/2 (b) 7/18

(c) 3/4 (d) 5/12

8. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is

(a) $\left(\frac{3}{4}\right)^{50}$ (b) $\left(\frac{2}{7}\right)^{50}$

(c) $\left(\frac{1}{8}\right)^{50}$ (d) $\left(\frac{7}{8}\right)^{50}$

9. Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

[IIT Screening 2004; Kerala Engg. 2005]

(a) 4/25 (b) 4/35

(c) 4/55 (d) 4/1155

10. Three six faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $k (3 \le k \le 8)$, is

(a) $\frac{(k-1)(k-2)}{432}$ (b) $\frac{k(k-1)}{432}$

(c) $\frac{k^2}{432}$ (d) None of these

11. Out of 21 tickets marked with numbers from 1 to 21, three are drawn at random. The chance that the numbers on them are in A.P., is **[Roorkee 1988; DCE 1**

(a) 10/133 (b) 9/133

(c) 9/1330 (d) None of these

12. Three squares of a chess board are chosen at random, the probability that two are of one colour and one of another is

(a) 16/21 (b) 8/21

(c) 32/12 (d) None of these

13. A five digit number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetitions. Then the probability that the number is divisible by 4 is

[Orissa JEE 2003]

(a) 3/5 (b) 18/5

(c) 1/5 (d) 6/5

14. If a party of n persons sit at a round table, then the odds against two specified individuals sitting next to each other are

(a) $2:(n-3)$ (b) $(n-3):2$

(c) $(n-2):2$ (d) $2:(n-2)$

15. Let E and F be two independent events. The probability that both E and F happens is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then **[IIT 1993]**

(a) $P(E)=\frac{1}{3},\ P(F)=\frac{1}{4}$ (b) $P(E)=\frac{1}{2},\ P(F)=\frac{1}{6}$

(c) $P(E)=\frac{1}{6},\ P(F)=\frac{1}{2}$ (d) None of these

16. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is **[IIT 1989]**

(a) 1/5 (b) 2/5
(c) 4/5 (d) None of theses

17. Two dice are thrown simultaneously. The probability that sum is odd or less than 7 or both, is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

18. A student appears for test I, II and III. The student is successful if he passes either in tests I and II or test I and III. The probabilities of the student passing in tests I, II, III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then **[IIT 1986]**

(a) $p=1,\ q=0$

(b) $p=\frac{2}{3},\ q=\frac{1}{2}$

(c) There are infinitely many values of p and q
(d) All of the above

19. For the three events A, B and C, P (exactly one of the events A or B occurs)= P (exactly one of the events B or C occurs)= P (exactly one of the events C or A occurs)= p and P (all the three events occur simultaneously) $=p^2$, where $0<p<1/2$. Then the probability of at least one of the three events A, B and C occurring is **[IIT 1996]**

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

20. If $P(B)=\frac{3}{4}$, $P(A\cap B\cap \overline{C})=\frac{1}{3}$ and $P(\overline{A}\cap B\cap \overline{C})=\frac{1}{3}$, then $P(B\cap C)$ is **[IIT Screening 2003]**

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

21. A box contains 100 tickets numbered 1, 2 ...... 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability **[IIT 1985]**

(a) $\frac{1}{8}$ (b) $\frac{13}{15}$

(c) $\frac{1}{7}$ (d) None of these

22. Let p denotes the probability that a man aged x years will die in a year. The probability that out of n men $A_1, A_2, A_3.....A_n$ each aged x, A_1 will die in a year and will be the first to die, is **[MNR 1987; UPSEAT 20**

(a) $\frac{1}{n}[1-(1-p)^n]$ (b) $[1-(1-p)^n]$

(c) $\frac{1}{n-1}[1-(1-p)^n]$ (d) None of these

23. A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing cards is black, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{^{25}C_{13}}{^{51}C_{13}}$

24. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is **[IIT 1998]**

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$

(c) $\frac{1}{32}$ (d) $\frac{3}{16}$

25. One hundred identical coins each with probability p of showing up heads are tossed once. If $0<p<1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is **[IIT 1988, CEE 1993; MP PET 2001]**

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$

(c) $\frac{50}{101}$ (d) $\frac{51}{101}$

26. A fair coin is tossed n times. Let X be the number of times head is observed. If $P(X=4)$, $P(X=5)$ and $P(X=6)$ are in H.P., then n is equal to

(a) 7 (b) 10
(c) 14 (d) None of these

27. Suppose X follows a binomial distribution with parameters n and p, where $0<p<1$. If $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then

(a) $p=\frac{1}{2}$ (b) $p=\frac{1}{3}$

(c) $p=\frac{1}{4}$ (d) None of these

28. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is

(a) ${}^{11}C_6\,(0.24)^5$ (b) ${}^{11}C_6(0.4)^6(0.6)^5$

(c) ${}^{11}C_6(0.6)^6\,(0.4)^5$ (d) None of these

29. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is

(a) At least 30 (b) At most 20

(c) Exactly 25 (d) None of the above

30. A random variable X has Poisson's distribution with mean 2. Then $P(X > 1.5)$ equals

(a) $1-\frac{3}{e^2}$ (b) $\frac{3}{e^2}$

(c) $\frac{2}{e^2}$ (d) 0

Answers and Solutions

(SET - 28)

1. (c) On trial, $n = 15$ since any of the 15 numbers can be on the selected coin and $m = 9$ since the largest number is 9 and so it can be 1 or 2 or 3........or 9.

We have required probability $=\left(\frac{9}{15}\right)^7=\left(\frac{3}{5}\right)^7$.

2. (c) The number of ways of chosing the first square is 64 and that for the second square is 63. Therefore the number of ways of chosing the first and second square is $64\times 63 = 4032$ Now we proceed to find the number of favourable ways. If the first happens to be any of the four squares in the corner, the second square can be chosen in two ways. If the first square happens to be any of the 24 square on either side of the chess board, the second square can be chosen in 3 ways. If the first square happens to be any of the 36 remaining squares, the second square can be chosen in 4 ways.

Therefore the number of favourable ways is

$(4)(2)+(24)(3)+(36)(4) = 224$

Hence the required probability $=\frac{224}{4032}=\frac{1}{18}$.

3. (c) Required probability = probability of right club and good shot or probability of wrong club and good shot $=\frac{1}{5}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}=\frac{4}{15}$.

4. (b) Since events are mutually exclusive and exhaustive

$$\Rightarrow \frac{1-3p}{2}+\frac{1+4p}{3}+\frac{1+p}{6}=1$$

Also $0\le\frac{1-3p}{2}\le 1$ and $0\le\frac{1+4p}{3}\le 1$

and $0\le\frac{1+p}{6}\le 1.$

$$0\le\frac{1+3p}{2}\le 1\Rightarrow p\in\left[-\frac{1}{3},\frac{1}{3}\right],$$

$$0\le\frac{1+4p}{3}\le 1\Rightarrow p\in\left[-\frac{1}{4},\frac{1}{2}\right],$$

$$0\le\frac{1+p}{6}\le 1\Rightarrow p\in[-1,5]$$

Hence the set of value satisfying all the above inequalities are $\left[-\frac{1}{4},\frac{1}{3}\right]$.

5. (a) Probability of getting head $=\frac{1}{2}$ and probability of throwing 5 or 6 with a dice $=\frac{2}{6}=\frac{1}{3}$. He starts with a coin and alternately tosses the coin and throws the dice and he will win if he get a head before he get 5 or 6.

Probability

$$=\frac{1}{2}+\left(\frac{1}{2}\cdot\frac{2}{3}\right)\cdot\frac{1}{2}+\left(\frac{1}{2}\cdot\frac{2}{3}\right)\cdot\left(\frac{1}{2}\cdot\frac{2}{3}\right)\times\frac{1}{2}+..$$

$$=\frac{1}{2}\left[1+\frac{1}{3}+\left(\frac{1}{3}\right)^2+.......\right]=\frac{1}{2}.\frac{1}{1-(1/3)}=\frac{3}{4}.$$

6. (a) Let λ,μ and λ',μ' be the numbers of heads and tails thrown by A and B respectively, so that

$\lambda+\lambda'=n+1$ and $\mu+\mu'=n.$

The required probability P is the probability of the inequality $\lambda>\mu.$. The probability $1-P$ of the opposite event $\lambda\le\mu$ is at the same time the probability of the inequality $\lambda'>\mu'$ *i.e.*, $1-P$ is the probability that A will throw more tails than B.

[**Reason :** $\lambda\le\mu\Rightarrow n+1-\lambda'\le n-\mu'$

$\Rightarrow 1-\lambda'\le-\mu'\Rightarrow\lambda'-1\ge\mu'\Rightarrow\lambda'\ge\mu'+1>\mu'$]

By reason of symmetry $1-P=P$ or $P=\frac{1}{2}$.

7. (d) Consider two events :

$A_i\rightarrow$ getting a number i on first die.

$B_i\rightarrow$ getting a number more than i on second die.

The required probability

$= P(A_1\cap B_1)+P(A_2\cap B_2)+P(A_3\cap B_3)+P(A_4\cap B_4)$

$+P(A_5 \cap B_5)$

$$= \sum_{i=1}^{5} P(A_i \cap B_i) = \sum_{i=1}^{5} P(A_i)P(B_i)$$

($\because$ A_i, B_i are independent)

$$= \frac{1}{6}(P(B_1) + P(B_2) + ... + P(B_5))$$

$$= \frac{1}{6}\left(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}\right) = \frac{5}{12}.$$

8. (a) For each toss there are four choices :

(i) A gets head, B gets head, (ii) A gets tail, B gets head,

(iii) A gets head, B gets tail (iv) A gets tail, B gets tail.

Thus exhaustive number of ways $= 4^{50}$.

Out of the four choices listed above (iv) is not favourable to the required event in a toss.

Therefore favourable number of cases is 3^{50}.

Hence the required probability $= \left(\frac{3}{4}\right)^{50}$.

9. (d) The numbers should be divisible by 6. Thus, the number of favourable ways is ${}^{16}C_3$ (as there are 16 numbers in first 100 natural numbers, divisible by 6).

Required probability is $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$.

10. (a) The total number of cases $= 6 \times 6 \times 6 = 216$

The number of favourable ways

= Coefficient of x^k in $(x + x^2 + ... + x^6)^3$

= Coefficient of x^{k-3} in $(1 - x^6)(1 - x)^{-3}$

= Coefficient of x^{k-3} in $(1 - x)^{-3}$, $\{0 \le k - 3 \le 5\}$

= Coefficient of x^{k-3} in $(1 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + ...)$

$= {}^{k-1}C_2 = \frac{(k-1)(k-2)}{2}$

Thus the probability of the required event is $\frac{(k-1)(k-2)}{432}$.

11. (a) Total number of ways $= {}^{21}C_3 = 1330$ If common difference of the AP is to be 1 then the possible groups are $1, 2, 3;$ $2, 3, 4; .... 19, 20, 21.$

If the common difference is 2, then possible groups are $1, 3, 5;$ $2, 4, 6; .... 17, 19, 21.$

Proceeding in the same way if the common difference is 10 then the possible group is 1, 10, 21. Thus if the common difference of the $A.P.$ is to be ≥ 11, obviously there is no favourable case.

Hence total number of favourable cases are $= 19 + 17 + 15 + ... + 3 + 1 = 100$

Hence required probability $= \frac{100}{1330} = \frac{10}{133}$.

12. (a) Three squares can be chosen out of 64 squares in ${}^{64}C_3$ ways. Two squares of one colour and one another colour can be chosen in two mutually exclusive ways :

(i) Two white and one black and (ii) Two black and one white. Thus the favourable number of cases

$= {}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2$.

Hence the required probability $= \frac{2({}^{32}C_1 . {}^{32}C_2)}{{}^{64}C_3} = \frac{16}{21}$.

13. (c) The number is divisible by 4 if last two digits are 12, 24, 32 and 52. Remaining three places can be filled by 3! ways.

$\therefore$ Favouable cases = $3! \times 4$

Required probability = $\frac{3! \times 4}{5!} = \frac{1}{5}$.

14. (b) The total number of ways in which n persons can sit at a round table $= (n-1)!$.

$\therefore$ Favourable number of cases $= 2!(n-2)!$

Thus the required probability $= \frac{2!(n-2)!}{(n-1)!} = \frac{2}{n-1}$

Hence the odds against are $(1-p) : p$ or $(n-3) : 2$.

15. (a) We are given $P(E \cap F) = \frac{1}{12}$ and $P(\bar{E} \cap \bar{F}) = \frac{1}{2}$

$\Rightarrow P(E).P(F) = \frac{1}{12}$ .....(i)

and $P(\bar{E}).P(\bar{F}) = \frac{1}{2}$ .....(ii)

$\Rightarrow \{(1 - P(E)\}\{(1 - P)(F)\} = \frac{1}{2}$

$\Rightarrow 1 + P(E)P(F) - P(E) - P(F) = \frac{1}{2}$

$\Rightarrow 1 + \frac{1}{12} - [P(E) + P(F)] = \frac{1}{2}$

$\Rightarrow P(E) + P(F) = \frac{7}{12}$ .....(iii)

On solving (i) and (iii), we get

$P(E) = \frac{1}{3}, \frac{1}{4}$ and $P(F) = \frac{1}{4}, \frac{1}{3}$.

16. (d) Let A denotes the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs, we have $P(A)=\frac{4}{36}, P(B)=\frac{6}{36}$

and $P(C)=\frac{26}{36}=\frac{13}{18}$

Thus $P(A$ occurs before $B)$

$= P[A$ or $(C\cap A)$ or $(C\cap C\cap A)$ or ....]

$= P(A)+P(C\cap A)+P(C\cap C\cap A)+........$

$= P(A)+P(C).P(A)+P(C)^2P(A)+.........$

$= \frac{P(A)}{1-P(C)}, [\text{by G.P.}] = \frac{\frac{1}{9}}{1-\frac{13}{18}} = \frac{2}{3}.$

17. (c) Required probability

$= P(\text{less than } 7)+P(\text{odd})+P(\text{both})-P(7\cap \text{odd})$

$-P(7\cap \text{both})-P(\text{odd}\cap \text{both})+P(\text{odd}\cap 7\cap \text{both})$

But

$P(\text{both}) = P(7\cap \text{odd}) = P(7\cap \text{both}) = P(\text{odd}\cap \text{both})$

$= P(\text{odd}\cap 7\cap \text{both})$

Therefore required probability

$= P(\text{Less than } 7)+P(\text{odd})-P(7\cap \text{odd})$

$P(\text{odd}) = \frac{18}{36}=\frac{1}{2}$

$P(\text{less than } 7)=\frac{15}{36}=\frac{5}{12},\ P(\text{both})=\frac{6}{36}=\frac{1}{6}$

Hence required probability $=\frac{5}{12}+\frac{1}{2}-\frac{2}{12}=\frac{9}{12}=\frac{3}{4}.$

18. (d) Let A, B and C be the events that the student is successful in test I, II and III respectively, then $P($ the student is successful)

$P[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A\cap B\cap C)]$

$= P(A\cap B\cap C')+P(A\cap B'\cap C)+P(A\cap B\cap C)$

$P(A).P(B).P(C')+P(A)P(B')P(C)+P(A)P(B)P(C)$

$\{\because A, B, C \text{ are independent}\}$

$= pq\left(1-\frac{1}{2}\right)+p(1-q)\left(\frac{1}{2}\right)+pq\left(\frac{1}{2}\right)=\frac{1}{2}p(1+q)$

$\Rightarrow \frac{1}{2}=\frac{1}{2}p(1+q)\Rightarrow p(1+q)=1$

This equation has infinitely many values of p and q.

19. (a) We know that P (exactly one of A or B occurs)

$= P(A)+P(B)-2P(A\cap B)$

Therefore, $P(A)+P(B)-2P(A\cap B)=p$ .....(i)

Similarly, $P(B)+P(C)-2P(B\cap C)=p$ .....(ii)

and $P(C)+P(A)-2P(C\cap A)=p$ .....(iii)

Adding (i), (ii) and (iii), we get

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=\frac{3p}{2}$ .....(iv)

We are also given that $P(A\cap B\cap C)=p^2$ .....(v)

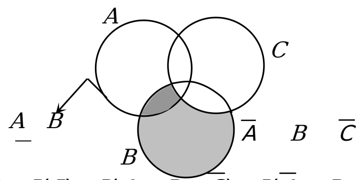
Now, P (at least one of A, B and C)

$= P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)$

$-P(C\cap A)+P(A\cap B\cap C)$

$=\frac{3p}{2}+p^2$, [By (iv) and (v)].

20. (a) From Venn diagram, we can see that



$P(B\cap C)=P(B)-P(A\cap B\cap \overline{C})-P(\overline{A}\cap B\cap \overline{C})$

$=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{1}{12}.$

21. (b) Let A be the event that the maximum number on the two chosen tickets is not more than 10 *i.e.*, the number on them ≤ 10 and B be the event that the maximum number on them is 5, *i.e.*, the number on them is ≥ 5 we have to find $P(B/A)$.

Now $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{n(A\cap B)}{n(A)}$

Now the number of ways of getting a number r on the two tickets is the coefficient of x^r in the expansion of

$(x^1+x^2+x^3+.......+x^{100})^2 = x^2(1+x+.......+x^{99})^2$

$= x^2\left(\frac{1-x^{100}}{1-x}\right)^2 = x^2(1-2x^{100}+x^{200})(1-x)^{-2}$

$= x^2(1-2x^{100}+x^{200})(1+2x+3x^2+....+(r+1)x^r+.....)$

Thus coefficient of $x^2=1$, of $x^3=2$, of $x^4=3$...... of x^{10} is 9.

Hence $n(A)=1+2+3+4+5+6+7+8+9=45$

and $n(A\cap B)=4+5+6+7+8+9=39$

[Note that in finding $n(A)$ we have to add the coefficients of $x^2, x^3, ...x^{10}$ and in $n(A\cap B)$ we add the coefficient of $x^5, x^6, ...x^{10}$]

Hence required probability $=\frac{39}{45}=\frac{13}{15}.$

22. (a) Let E_i denote the event that A_i dies in a year.

Then $P(E_i)=p$ and $P(E_i')=1-p$ for $i=1, 2, ...n$

$P(\text{none } A_1, A_2, ...A_n \text{ dies in a year})$

$= P(E_1'\cap E_2'\cap ......E_n') = P(E_1')P(E_2')......P(E_n')=(1-p)^n$

Because $E_1, E_2, ...E_n$ are independent.

Let E denote the event that at least one of $A_1, A_2, \ldots\ldots A_n$ dies in a year. Then

$P(E) = 1 - P(E_1' \cap E_2' \cap \ldots\ldots \cap E_n') = 1 - (1-p)^n$

Let F denote the event that A_1 is the first to die.

Then $P(F/E) = \frac{1}{n}$.

Also, $P(F) = P(E).P(F/E) = \frac{1}{n}[1-(1-p)^n]$.

23. (b) Let A_1 be the event that the black card is lost, A_2 be the event that red card is lost and let E be the event that first 13 cards examined are red.

Then the required probability $= P\left(\frac{A_1}{E}\right)$.

We have $P(A_1) = P(A_2) = \frac{1}{2}$; as black and red cards were intially equal in number.

Also, $P\left(\frac{E}{A_1}\right) = \frac{^{26}C_{13}}{^{51}C_{13}}$ and $P\left(\frac{E}{A_2}\right) = \frac{^{25}C_{13}}{^{51}C_{13}}$

The required probability $= P\left(\frac{A_1}{E}\right)$

$$= \frac{P(E/A_1)P(A_1)}{P(E/A_1)P(A_1) + P(E/A_2)P(A_2)}$$

$$= \frac{\frac{1}{2}.\frac{^{26}C_{13}}{^{51}C_{13}}}{\frac{1}{2}.\frac{^{26}C_{13}}{^{51}C_{13}} + \frac{1}{2}.\frac{^{25}C_{13}}{^{51}C_{13}}} = \frac{2}{3}.$$

24. (a) Let $P(W_i)$ and $P(B_i)$ be the probabilities of drawing one white and one black ball from the i^{th} box where $i = 1, 2, 3$ respectively. Hence

$P(W_1) = \frac{3}{4}$, $P(B_1) = \frac{1}{4}$

$P(W_2) = \frac{2}{4} = \frac{1}{2}$, $P(B_2) = \frac{2}{4} = \frac{1}{2}$

$P(W_3) = \frac{1}{4}$, $P(B_3) = \frac{3}{4}$

Two white and one black ball may be drawn from 3 boxes in the following three ways –

	Box1	Box2	Box3
Way1	W	W	B
Way2	W	B	W
Way3	B	W	W

$\therefore$ Required probability

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$= \frac{3}{4}.\frac{2}{4}.\frac{3}{4} + \frac{3}{4}.\frac{2}{4}.\frac{1}{4} + \frac{1}{4}.\frac{2}{4}.\frac{1}{4} = \frac{18+6+2}{64} = \frac{13}{32}$.

25. (d) We have $^{100}C_{50}\, p^{50}(1-p)^{50} = {}^{100}C_{51}(1-p)^{49}$

or $\frac{1-p}{p} = \frac{100!}{51!\,.\,49!} \times \frac{50!\,.\,50!}{100!} = \frac{50}{51}$

or $51 - 51p = 50p$

$\Rightarrow p = \frac{51}{101}$.

26. (d) No value of n can satisfy it.

27. (a) We have,

$$\frac{P(X=r)}{P(X=n-r)} = \frac{^nC_r p^r (1-p)^{n-r}}{^nC_{n-r} p^{n-r}(1-p)^r} = \left(\frac{1}{p} - 1\right)^{n-2r}$$

Note that $\frac{1}{p} - 1 > 0$. Therefore the ratio will be independent of n and r, if $\frac{1}{p} - 1 = 1$ or $p = \frac{1}{2}$.

* * *

28. (a) The man will be one step away from the starting point if (i) either he is one step ahead or (ii) one step behind the starting point.

$\therefore$ The required probability $= P(i) + P(ii)$

The man will be one step ahead at the end of eleven steps if he moves six steps forward and five steps backward. The probability of this event $= {}^{11}C_6 (0.4)^6 (0.6)^5$.

The man will be one step behind at the end of eleven steps if he moves six steps backward and five steps forward.

The probability of this event $= {}^{11}C_6(0.6)^6(0.4)^5$.

Hence the required probability

$= {}^{11}C_6(0.4)^6(0.6)^5 + {}^{11}C_6(0.6)^6(0.4)^5$

$= {}^{11}C_6(0.4)^5(0.6)^5(0.4+0.6) = {}^{11}C_6(0.24)^5$.

29. (c) Let number of newspaper be n, then

$60n = 300 \times 5 \Rightarrow n = 25$.

30. (a) $P(x > 1.5) = 1 - P(x = 0) - P(x = 1)$

$$P(x = k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

$$P(x > 1.5) = 1 - \frac{1}{e^2} - \frac{2}{e^2} = 1 - \frac{3}{e^2}.$$