? suppose that the life time of bulbs produced by a company are normally distributed with mean 1000 hours and std deviation 100 hours. Is this company correct when it claims that 95% of its bulbs last atleast 900 hours?

A). Let x denote the lifetime of a bulb.

The company claims that $p(x \geq 900) = 95\%$

Here $x \sim N(\mu, \sigma^2)$ with $\mu = 1000$ & $\sigma = 100$

~~$p(x \geq 900)$~~-

we can change to std normal distribution

by $z = \frac{x-\mu}{\sigma} = \frac{x-1000}{100} \sim N(0,1)$

$p(x \geq 900) = p\left(\frac{x-1000}{100} \geq \frac{900-1000}{100}\right)$

$= p(z \geq -1) = 0.8413$

Then 84.13 % of the bulbs would last more than 900 hours & the companys claim is false.

2). If x is a normal random variable with mean 50 and std deviation 10. find α and β such that $p(x < \alpha) = 0.1$ & $p(x > \beta) = 0.05$

A) suppose that $x \sim N(\mu, \sigma^2)$. where $\mu = 50, \sigma = 10$

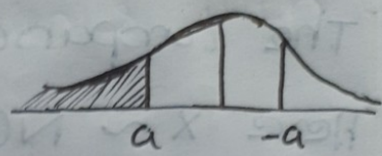
we can change to std normal variable using

$z = \frac{x-\mu}{\sigma} = \frac{x-50}{10}$ then $z \sim N(0,1)$

$p(x < \alpha) = 0.1 \Rightarrow p\left(z < \frac{\alpha - 50}{10}\right) = 0.1$

$\Rightarrow p(z < a) = 0.1$, where, $a = \frac{\alpha - 50}{10}$

Since $p(z<a) = 0.1 < 0.5$, a less lies to the left side of the origen.



$\therefore p(z<a) + p(a<z<0) = 0.5$

$0.1 + p(0<z<-a) = 0.5$

$p(0<z<-a) = 0.4$

From table, $-a = 1.28$ → 0.4 ന് അടുത്ത് കിടക്കുന്ന value ഉം equal difference ആണെങ്കിൽ, take bigger one

$a = 1.28$

$\frac{\alpha - 50}{10} = 1.28$

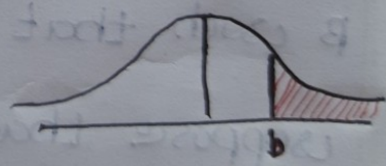
$\alpha - 50 = 12.8$

$\alpha = 37.2$

$P(X > \beta) = 0.05 \Rightarrow P\left(z > \frac{\beta - 50}{10}\right) = 0.05$

$\Rightarrow P(z > b) = 0.05$, where $b = \frac{\beta - 50}{10}$

which shows that b lies right side of the origen



$P(0<z<b) = 0.5 - P(z>b)$

$= 0.45$

$\therefore$ From table, $b = 1.65$

ie, $\frac{\beta - 50}{10} = 1.65$

$\beta - 50 = 16.5$

$\beta = 66.5$

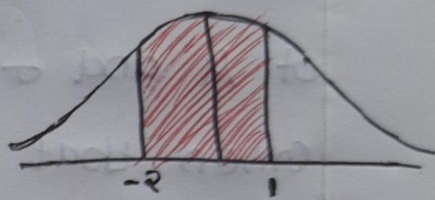
3. If X is normally distributed with mean 1 and variance 4

(1) find $P(-3<X<3)$ and

(2) obtain K if $P(X\leq K)=0.9$

A) $X \sim N(\mu,\sigma^2)$ where $\mu=1$ & $\sigma=2$

$\therefore Z=\frac{X-1}{2} \sim N(0,1)$



$P(-3<X<3) = P\left(-\frac{3-1}{2}<Z<\frac{3-1}{2}\right)$

$= P(-2<Z<1)$

$= P(0<Z<1) + P(0<Z<2)$

$= 0.3413 + 0.4772$

$= \underline{\underline{0.8185}}$

(b) $P(X\leq K) = 0.9 \Rightarrow P\left(Z\leq \frac{K-1}{2}\right)=0.9$

$\Rightarrow P(Z\leq K_1) = 0.9$, where $K_1=\frac{K-1}{2}$

$P(0<Z<K_1) = 0.9-0.5$

$=0.4$

$\frac{K-1}{2} = 1.29$

$K = \underline{\underline{3.58}}$

4) The marks obtained by a batch of students in a certain subject are normally distributed. 10% of students got less than

45 marks while 5% got more than 75.
Find the percentage of students with score
b/w 45 & 60.

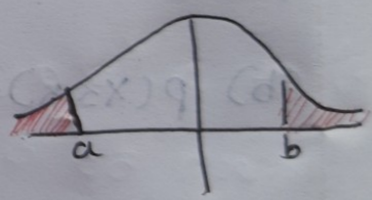
A). Let x denote the marks obtained by a student, and assume that x has mean μ and variance σ^2. we first find the radius of μ and σ.

Given that $p(x<45) = 0.1$ & $p(x>75) = 0.05$

ie, $p(z < \frac{45-\mu}{\sigma}) = 0.1$ $p(z > \frac{75-\mu}{\sigma}) = 0.05$.

Let $a = \frac{45-\mu}{\sigma}$ $b = \frac{75-\mu}{\sigma}$

$\therefore p(z<a) = 0.1$ $p(x>b) = 0.05$



since $p(z<a) = 0.1 < 0.5$

$a<0$

$p(z>b) = 0.05 < 0.5 \therefore b>0$

$p(a<z<0) = 0.4$

$\Rightarrow p(0<z<-a) = 0.4$

$\Rightarrow -a = 1.28$

$a = -1.28$

$\frac{45-\mu}{\sigma} = -1.28$ ——— (2)

From figure, $p(0<z<b) = 0.45$

$\Rightarrow b = 1.65$

$\Rightarrow \frac{75-\mu}{\sigma} = 1.65$ ——— (2)

From (1) & (2)

$$\frac{\frac{45-\mu}{\sigma}}{\frac{75-\mu}{\sigma}} = \frac{-1.28}{1.65}$$

$$\frac{45-\mu}{75-\mu} = -0.7758$$

$$45-\mu = -58.185 + 0.7758\mu$$

$$45 + 58.185 = 1.7758\mu$$

$$\mu = 58.1062$$

$$\frac{45-\mu}{\sigma} = -1.28$$

$$\frac{45 - 58.1062}{\sigma} = -1.28$$

$$\sigma = 10.24$$

$$P(45 < x < 60) = P(-1.28 < z < \underset{\sim 0.19}{0.1855})$$

$$= 0.3997 + 0.07535$$

$$= \underline{\underline{0.475}}$$

$\therefore$ 47.5% students score marks be 45 & 60