Vector Algebra

SET Self Evaluation Test - 19

1. If the moduli of the vectors **a, b, c** are 3, 4, 5 respectively and **a** and $\mathbf{b}+\mathbf{c}$, **b** and $\mathbf{c}+\mathbf{a}$, **c** and $\mathbf{a}+\mathbf{b}$ are mutually perpendicular, then the modulus of $\mathbf{a}+\mathbf{b}+\mathbf{c}$ is

[IIT 1981; Kerala (Engg.) 2005]

(a) $\sqrt{12}$ (b) 12

(c) $5\sqrt{2}$ (d) 50

2. What will be the length of longer diagonal of the parallelogram constructed on $5\mathbf{a}+2\mathbf{b}$ and $\mathbf{a}-3\mathbf{b}$. If it is given that $|\mathbf{a}|=2\sqrt{2}, |\mathbf{b}|=3$ and angle between **a** and **b** is $\frac{\pi}{4}$ **[UPSEAT 2002]**

(a) 15 (b) $\sqrt{113}$

(c) $\sqrt{593}$ (d) $\sqrt{369}$

3. If $|\mathbf{a}|=a$ and $|\mathbf{b}|=b$, then $\left(\frac{\mathbf{a}}{a^2}-\frac{\mathbf{b}}{b^2}\right)^2=$

(a) $\left(\frac{\mathbf{a}+\mathbf{b}}{ab}\right)^2$ (b) $\frac{(\mathbf{a}-\mathbf{b})^2}{ab}$

(c) $\left(\frac{\mathbf{a}-\mathbf{b}}{ab}\right)^2$ (d) $\frac{(\mathbf{a}+\mathbf{b})^2}{ab}$

4. The point B divides the arc AC of a quadrant of a circle in the ratio 1 : 2. If O is the centre and $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$, then the vector $\overrightarrow{OC}$ is

(a) $\mathbf{b}-2\mathbf{a}$ (b) $2\mathbf{a}-\mathbf{b}$

(c) $3\mathbf{b}-2\mathbf{a}$ (d) None of these

5. If S is the circumcentre, G the centroid, O the orthocentre of a triangle ABC, then $\overrightarrow{SA}+\overrightarrow{SB}+\overrightarrow{SC}=$

[MNR 1987; EAMCET 1994]

(a) $\overrightarrow{SG}$ (b) $\overrightarrow{OS}$

(c) $\overrightarrow{SO}$ (d) $\overrightarrow{OG}$

6. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$ are linearly dependent vectors and $|\mathbf{c}|=\sqrt{3}$, then

[IIT 1998]

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm 1$

(c) $\alpha=-1, \beta=\pm 1$ (d) $\alpha=\pm 1, \beta=1$

7. Vectors **a, b, c** are inclined to each other at an angle of 60^o and $|\mathbf{a}|=|\mathbf{b}|=2$ and $|\mathbf{c}|=2$, then $(2\mathbf{a}+3\mathbf{b}-5\mathbf{c}).(4\mathbf{a}-6\mathbf{b}+10\mathbf{c})=$

(a) 167 (b) – 167

(c) 120 (d) – 120

8. The vectors **a, b** and **c** are of the same length and taken pairwise, they form equal angles. If $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=\mathbf{j}+\mathbf{k}$, then the co-ordinates of **c** are

(a) (1, 0, 1) (b) (1, 2, 3)

(c) $(-1, 1, 2)$ (d) None of these

9. The position vector of coplanar points A, B, C, D are **a, b, c** and **d** respectively, in such a way that $(\mathbf{a}-\mathbf{d}).(\mathbf{b}-\mathbf{c})=(\mathbf{b}-\mathbf{d}).(\mathbf{c}-\mathbf{a})=0$, then the point D of the triangle ABC is **[IIT 1984]**

(a) Incentre

(b) Circumcentre

(c) Orthocentre

(d) None of these

10. Let **p** and **q** be the position vectors of P and Q respectively with respect to O and $|\mathbf{p}|=p, |\mathbf{q}|=q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If $\overrightarrow{OR}$ and $\overrightarrow{OS}$ are perpendicular, then

[IIT Screening 1994]

(a) $9p^2=4q^2$ (b) $4p^2=9q^2$

(c) $9p=4q$ (d) $4p=9q$

11. A unit vector in xy-plane that makes an angle 45^o with the vector $(\mathbf{i}+\mathbf{j})$ and an angle of 60^o with the vector $(3\mathbf{i}-4\mathbf{j})$ is **[Kurukshetra CEE 2002]**

(a) **i** (b) $\frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$

(c) $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ (d) None of these

12. Let **u** = **i** + **j**, **v** = **i** – **j** and **w** = **i** + 2**j** + 3**k**. If **n** is a unit vector such that **u.n** = 0 and **v.n** = 0 then $|\mathbf{w}.\mathbf{n}|$ is equal to

(a) 0 (b) 1

(c) 2 (d) 3

13. The value of c so that for all real x, the vectors $cx\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$, $x\mathbf{i} + 2\mathbf{j} + 2cx\mathbf{k}$ make an obtuse angle are

[EAMCET 1994]

(a) $c < 0$ (b) $0 < c < \frac{4}{3}$

(c) $-\frac{4}{3} < c < 0$ (d) $c > 0$

14. The vector $\mathbf{b} = 3\mathbf{j} + 4\mathbf{k}$ is to be written as the sum of a vector $\mathbf{b}_1$ parallel to $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and a vector $\mathbf{b}_2$ perpendicular to $\mathbf{a}$. Then $\mathbf{b}_1 =$ **[MNR 1993; UPSEAT 2000]**

(a) $\frac{3}{2}(\mathbf{i} + \mathbf{j})$ (b) $\frac{2}{3}(\mathbf{i} + \mathbf{j})$

(c) $\frac{1}{2}(\mathbf{i} + \mathbf{j})$ (d) $\frac{1}{3}(\mathbf{i} + \mathbf{j})$

15. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be such that $|\mathbf{u}| = 1, |\mathbf{v}| = 2, |\mathbf{w}| = 3$. If the projection $\mathbf{v}$ along $\mathbf{u}$ is equal to that of $\mathbf{w}$ along $\mathbf{u}$ and $\mathbf{v}, \mathbf{w}$ are perpendicular to each other then $|\mathbf{u} - \mathbf{v} + \mathbf{w}|$ equals **[AIEEE 2004]**

(a) 14 (b) $\sqrt{7}$

(c) $\sqrt{14}$ (d) 2

16. Forces of magnitudes 3 and 2 units acting in the directions $5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ respectively act on a particle which is displaced from the points (1, –1, –1) to (3, 3, 1). The work done by the forces is equal to

(a) $50\sqrt{2}$ *unit* (b) $40\sqrt{2}$ *unit*

(c) $\frac{57}{5}\sqrt{2}$ *unit* (d) $8\sqrt{2}$ *unit*

17. If $\mathbf{a} = (1, 1, 1), \mathbf{c} = (0, 1, -1)$ are two vectors and $\mathbf{b}$ is a vector such that $\mathbf{a} \times \mathbf{b} = \mathbf{c}$ and $\mathbf{a}.\mathbf{b} = 3$, then $\mathbf{b}$ is equal to

[IIT 1985, 91]

(a) $\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{2}{3}, \frac{5}{3}, \frac{2}{3}\right)$

(c) $(5, 2, 2)$ (d) $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$

18. $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = 6\mathbf{i} + 3\mathbf{j}$ are two vectors and $\mathbf{c}$ is a vector such that $\mathbf{c} = \mathbf{a} \times \mathbf{b}$, then $|\mathbf{a}| : |\mathbf{b}| : |\mathbf{c}|$ is

[AIEEE 2002]

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$

(b) $\sqrt{34} : \sqrt{45} : 39$

(c) 34 : 39 : 45

(d) 39 : 35 : 34

19. If $|\mathbf{a} \times \mathbf{b}| = 4$ and $|\mathbf{a}.\mathbf{b}| = 2$, then $|\mathbf{a}|^2 |\mathbf{b}|^2 =$

[Karnataka CET 2003]

(a) 2 (b) 6

(c) 8 (d) 20

20. **a, b, c** are three non-zero, non-coplanar vectors and **p, q, r** are three other vectors such that $\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{\mathbf{a}.\mathbf{b} \times \mathbf{c}}$, $\mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{\mathbf{a}.\mathbf{b} \times \mathbf{c}}$, $\mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{\mathbf{a}.\mathbf{b} \times \mathbf{c}}$. Then $[\mathbf{pqr}]$ equals **[CEE 1993]**

(a) $\mathbf{a} . \mathbf{b} \times \mathbf{c}$ (b) $\frac{1}{\mathbf{a}.\mathbf{b} \times \mathbf{c}}$

(c) 0 (d) None of these

21. If **a, b** and **c** are the three non-coplanar vectors, then $(\mathbf{a} + \mathbf{b} + \mathbf{c}).[(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{c})]$ is equal to

[IIT 1995; UPSEAT 2004]

(a) [**a b c**] (b) 2 [**a b c**]

(c) – [**a b c**] (d) 0

22. Let $\mathbf{V} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{W} = \mathbf{i} + 3\mathbf{k}$. If **U** is a unit vector, then the maximum value of the scalar triple product [**U V W**] is

(a) – 1 (b) $\sqrt{10} + \sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

23. If $\begin{vmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{vmatrix} = 0$ and $\mathbf{a} = (1, a, a^2), \mathbf{b} = (1, b, b^2)$, and $\mathbf{c} = (1, c, c^2)$ are non-coplanar vectors, then abc is equal to

[IIT 1985; AIEEE 2003; Pb. CET 2003]

(a) – 1 (b) 0

(c) 1 (d) 4

24. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be non-zero vectors such that $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$. If is the acute angle between the vectors $\mathbf{b}$ and $\mathbf{c}$, then $\sin\theta$ equals**[AIEEE 20**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

25. A non-zero vector **a** is parallel to the line of intersection of the plane determined by the vectors $\mathbf{i}, \mathbf{i} + \mathbf{j}$ and the plane determined by the vectors $\mathbf{i}-\mathbf{j}, \mathbf{i}+\mathbf{k}$. The angle between $\mathbf{a}$ and the vector $\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ is **[IIT 1996]**

(a) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (b) $\frac{2\pi}{4}$ or $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ (d) None of these

Answers and Solutions

(SET - 19)

1. (c) According to the given condition,

$\mathbf{a}.(\mathbf{b}+\mathbf{c})=0$ .....(i)

$\mathbf{b}.(\mathbf{c}+\mathbf{a})=0$ .....(ii)

$\mathbf{c}.(\mathbf{a}+\mathbf{b})=0$ .....(iii)

Now adding (i), (ii) and (iii), we get

$2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=0$, $\{\because \mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{a} \text{ etc}\}$

Hence,

$$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=a^2+b^2+c^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})$$

$$=3^2+4^2+5^2$$

$$|\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{50}=5\sqrt{2}.$$

2. (c) Length of two diagonals will be

$$\mathbf{d}_1=|(5\mathbf{a}+2\mathbf{b})+(\mathbf{a}-3\mathbf{b})|$$

and $\mathbf{d}_2=|(5\mathbf{a}+2\mathbf{b})-(\mathbf{a}-3\mathbf{b})|$ $\mathbf{d}_2=|4\mathbf{a}+5\mathbf{b}|$

Thus,

$$\mathbf{d}_1=\sqrt{|6\mathbf{a}|^2+|-\mathbf{b}|^2+2|6\mathbf{a}||-\mathbf{b}|\cos\left(\pi-\frac{\pi}{4}\right)}$$

$$=\sqrt{36(2\sqrt{2})^2+9+12.2\sqrt{2}.3\left(\frac{-1}{\sqrt{2}}\right)}=15$$

$$\mathbf{d}_2=\sqrt{|4\mathbf{a}|^2+|5\mathbf{b}|^2+2|4\mathbf{a}||5\mathbf{b}|\cos\left(\frac{\pi}{4}\right)}$$

$$=\sqrt{16\times 8+25\times 9+40\times 2\sqrt{2}\times 3\times\frac{1}{\sqrt{2}}}$$

$$=\sqrt{593}.$$

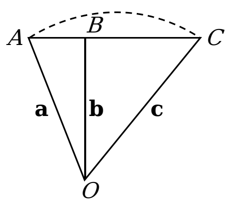
$\therefore$ Length of the longer diagonal $=\sqrt{593}$.

3. (c) $\left(\frac{\mathbf{a}}{a^2}-\frac{\mathbf{b}}{b^2}\right)^2=\frac{a^2}{a^4}+\frac{b^2}{b^4}-\frac{2\mathbf{a}.\mathbf{b}}{a^2b^2}$, $\{\because \mathbf{a}^2=a^2 \text{ etc}\}$

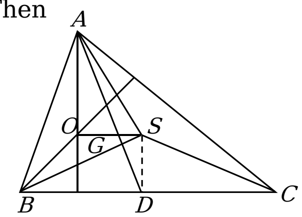
$$=\frac{a^2+b^2-2\mathbf{a}.\mathbf{b}}{a^2b^2}=\left(\frac{\mathbf{a}-\mathbf{b}}{ab}\right)^2.$$

4. (c) $\overrightarrow{OC}=\frac{3\mathbf{b}-2\mathbf{a}}{3-2}=3\mathbf{b}-2\mathbf{a}$

$\{\because AC:BC=3:2 \text{(externally)}\}$



5. (c) Let P be any point in the plane of the triangle ABC. Then



$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\overrightarrow{PA}+(\overrightarrow{PB}+\overrightarrow{PC})=\overrightarrow{PA}+2\overrightarrow{PD}$$

$$=(1+2)\overrightarrow{PG}=3\overrightarrow{PG}$$

Since G divides AD in the ratio $2:1$.

$\therefore$ $\overrightarrow{SA}+\overrightarrow{SB}+\overrightarrow{SG}=3\overrightarrow{SG}=2\overrightarrow{SG}+\overrightarrow{SG}$

$=\overrightarrow{GO}+\overrightarrow{SG}=\overrightarrow{SO}$, $(\because OG=2SG)$.

6. (d) $|\mathbf{c}|=1+\alpha^2+\beta^2=3\Rightarrow\alpha^2+\beta^2=2$

Since $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly dependent.

Hence, $\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=1-\beta=0\Rightarrow\beta=1$

$\therefore$ $\alpha^2=1\Rightarrow\alpha=\pm 1$.

7. (d) $(2\mathbf{a}+3\mathbf{b}-5\mathbf{c}).(4\mathbf{a}-6\mathbf{b}+10\mathbf{c})$

$=8\mathbf{a}.\mathbf{a}-18\mathbf{b}.\mathbf{b}-50\mathbf{c}.\mathbf{c}-12\mathbf{a}.\mathbf{b}+20\mathbf{a}.\mathbf{c}$

$+12\mathbf{b}.\mathbf{a}+30\mathbf{b}.\mathbf{c}-20\mathbf{c}.\mathbf{a}+30\mathbf{c}.\mathbf{b}$

$=8a^2-18b^2-50c^2+60\mathbf{c}.\mathbf{b}$

$=32-72-200+60.|\mathbf{b}|.|\mathbf{c}|\cos 60^\circ=-120$

8. (a) Let $\mathbf{c}=(c_1, c_2, c_3)$, then

$|\mathbf{c}|=|\mathbf{a}|=|\mathbf{b}|=\sqrt{2}=\sqrt{c_1^2+c_2^2+c_3^2}$

It is given that the angles between the vectors are identical and equal to ϕ (say), then

$$\cos\phi=\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}=\frac{0+1+0}{\sqrt{2}\sqrt{2}}=\frac{1}{2}$$

$$\frac{\mathbf{a}.\mathbf{c}}{|\mathbf{a}||\mathbf{c}|}=\frac{c_1+c_2}{2}=\frac{1}{2} \text{ and } \frac{\mathbf{b}.\mathbf{c}}{|\mathbf{b}||\mathbf{c}|}=\frac{c_2+c_3}{2}=\frac{1}{2}$$

Hence $c_1+c_2=1, c_2+c_3=1$ and $c_1^2+c_2^2+c_3^2=2$

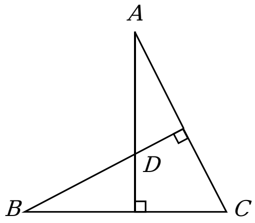
On solving the equation, we get

$c_1 = 1, -\frac{1}{3}; c_2 = 0, \frac{4}{3}; c_3 = 1, -\frac{1}{3}$

Hence co-ordinates of **c** are $(1, 0, 1)$ or $\left(-\frac{1}{3}, \frac{4}{3}, \frac{-1}{3}\right)$.

Trick : Obviously, length of the vector $(1, 0, 1)$ *i.e.*, $\mathbf{i}+\mathbf{k}$ is equal to length of **a** and **b**. Also it makes equal angle with **a** and **b** and equal to that of between **a** and **b** *i.e.*, $\frac{\pi}{3}$.

9. (c) $\overrightarrow{DA}.\overrightarrow{CB} = \overrightarrow{DB}.\overrightarrow{AC} = 0 \Rightarrow \overrightarrow{DA} \perp \overrightarrow{CB}$ and $\overrightarrow{DB} \perp \overrightarrow{AC}$.



Hence the point D is orthocentre of ΔABC.

10. (a) The position vectors of R and S are $\frac{3\mathbf{p}+2\mathbf{q}}{5}$ and $3\mathbf{p}-2\mathbf{q}$ respectively. Therefore $\overrightarrow{OR} = \frac{3\mathbf{p}+2\mathbf{q}}{5}$ and $\overrightarrow{OS} = 3\mathbf{p}-2\mathbf{q}$. Since $\overrightarrow{OR} \perp \overrightarrow{OS}$, therefore $\overrightarrow{OR}.\overrightarrow{OS} = 0 \Rightarrow \left(\frac{3\mathbf{p}+2\mathbf{q}}{5}\right).(3\mathbf{p}-2\mathbf{q}) = 0$

$\Rightarrow 9|\mathbf{p}|^2 - 4|\mathbf{q}|^2 = 0$

$\Rightarrow 9|\mathbf{p}|^2 = 4|\mathbf{q}|^2 \Rightarrow 9p^2 = 4q^2$.

11. (d) Let the vector be $x\mathbf{i}+y\mathbf{j}$

$\therefore \cos 45^o = \frac{x+y}{\sqrt{2}\sqrt{x^2+y^2}} \Rightarrow 1 = \frac{x+y}{\sqrt{x^2+y^2}}$

$\Rightarrow x+y = \sqrt{x^2+y^2}$ also $\sqrt{x^2+y^2} = 1 \Rightarrow x+y = 1$

Again $\cos 60^o = \frac{3x-4y}{5} \Rightarrow \frac{5}{2} = 3x-4y$

$5 = 6x - 8y$ .....(i)

$1 = x+y$ .....(ii)

No value in the given options set satisfies the above relations. Thus (d) is correct.

12. (d) $\because$ **n** is perpendicular to **u** and **v**

$\mathbf{n} = \mathbf{u} \times \mathbf{v}$

$$\mathbf{n} = \frac{\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2}\times\sqrt{2}} = \frac{-2\mathbf{k}}{2} = -\mathbf{k}$$

$|\mathbf{w}.\mathbf{n}| = |(\mathbf{i}+2\mathbf{j}+3\mathbf{k}).(-\mathbf{k})| = |-3| = 3$.

13. (c) Since the vectors $\mathbf{a} = cx\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = x\mathbf{i} + 2\mathbf{j} + 2cx\mathbf{k}$ make an obtuse angle, therefore

$\mathbf{a}.\mathbf{b} < 0 \Rightarrow cx^2 - 12 + 6cx < 0$ for all x

$\Rightarrow c < 0$ and Discriminant < 0

$\Rightarrow c < 0$ and $36c^2 + 48c < 0$

$\Rightarrow c < 0$ and $3c + 4 > 0 \Rightarrow c < 0$ and $3c + 4 > 0$

$\Rightarrow -\frac{4}{3} < c < 0$.

14. (a) Since $\mathbf{b}_1 || \mathbf{a}$, therefore $\mathbf{b}_1 = a(\mathbf{i}+\mathbf{j})$

$\mathbf{b}_2 = \mathbf{b} - \mathbf{b}_1 = (3-a)\mathbf{i} - a\mathbf{j} + 4\mathbf{k}$

Also $\mathbf{b}_2.\mathbf{a} = 0 \Rightarrow (3-a) - a \Rightarrow a = \frac{3}{2}$

Hence $\mathbf{b}_1 = \frac{3}{2}(\mathbf{i}+\mathbf{j})$.

15. (c) Without loss of generality, we can assume $\mathbf{v} = 2\mathbf{i}$ and $\mathbf{w} = 3\mathbf{j}$, $(\because \mathbf{v} \perp \mathbf{w})$.

Let $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$|\mathbf{u}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$

and projection of **v** along **u** = projection of **w** along **u**

$\mathbf{v}.\mathbf{u} = \mathbf{w}.\mathbf{u}$ $\quad 2\mathbf{i}(x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) = 3\mathbf{j}(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})$

$2x = 3y \quad 3y - 2x = 0$

Now, $|\mathbf{u} - \mathbf{v} + \mathbf{w}| = |x\mathbf{i} + y\mathbf{j} + z\mathbf{k} - 2\mathbf{i} + 3\mathbf{j}|$

$= |(x-2)\mathbf{i} + (y+3)\mathbf{j} + z\mathbf{k}| = \sqrt{(x-2)^2 + (y+3)^2 + z^2}$

$= \sqrt{(x^2+y^2+z^2+2(3y-2x)+13}$

$= \sqrt{1+2(0)+1(3)} = \sqrt{14}$.

16. (c) Unit vector in the direction of $5\mathbf{i}+3\mathbf{j}+4\mathbf{k}$

$= \frac{5\mathbf{i}+3\mathbf{j}+4\mathbf{k}}{\sqrt{25+9+16}}$

Unit vector in the direction of

$3\mathbf{i}+4\mathbf{j}-5\mathbf{k} = \frac{3\mathbf{i}+4\mathbf{j}-5\mathbf{k}}{\sqrt{9+16+25}}$

Force of magnitude 3 *i.e.*, $\mathbf{F}_1 = \frac{3(5\mathbf{i}+3\mathbf{j}+4\mathbf{k})}{5\sqrt{2}}$

Force of magnitude 2 *i.e.*, $\mathbf{F}_2 = \frac{2(3\mathbf{i}+4\mathbf{j}-5\mathbf{k})}{5\sqrt{2}}$

Net force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \Rightarrow \mathbf{F} = \frac{1}{5\sqrt{2}}(21\mathbf{i}+17\mathbf{j}+2\mathbf{k})$

Displacement $\mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1 = (3\mathbf{i}+3\mathbf{j}+\mathbf{k}) - (\mathbf{i}-\mathbf{j}-\mathbf{k})$

$\mathbf{d} = (2\mathbf{i}+4\mathbf{j}+2\mathbf{k})$

Work done

$= \mathbf{F}.\mathbf{d} = \frac{1}{5\sqrt{2}}(21\mathbf{i}+17\mathbf{j}+2\mathbf{k}).(2\mathbf{i}+4\mathbf{j}+2\mathbf{k})$

$= \frac{57\sqrt{2}}{5}$ *unit.*

17. (d) Let $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$

$\mathbf{a}.\mathbf{b} = b_1 + b_2 + b_3 = 3$ ......(i)

$\mathbf{a}\times\mathbf{b}=(b_3-b_2)\mathbf{i}+(b_1-b_3)\mathbf{j}+(b_2-b_1)\mathbf{k}=\mathbf{c}$

Comparing the coefficients of $\mathbf{i},\mathbf{j},\mathbf{k}$ of $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}$.

we get,

$b_3-b_2=0$ ......(ii)

$b_1-b_3=1$ .....(iii)

$b_2-b_1=-1$ .....(iv)

On solving equations, we get

$b_1=\frac{5}{3},\ b_2=\frac{2}{3},\ b_3=\frac{2}{3},$

Hence $\mathbf{b}=\left(\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$.

18. (b) $\mathbf{c}=\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&-5&0\\6&3&0\end{vmatrix}=\mathbf{i}(0)-\mathbf{j}(0)+\mathbf{k}(9+30)=39\mathbf{k}$

Now, $|\mathbf{a}|=\sqrt{9+25}=\sqrt{34}$

$|\mathbf{b}|=\sqrt{36+9}=\sqrt{45}$

$|\mathbf{c}|=\sqrt{(39)^2}=39.$

19. (d) Given $|\mathbf{a}\times\mathbf{b}|=4\Rightarrow\left||\mathbf{a}||\mathbf{b}|\sin\theta\ \hat{\mathbf{n}}\right|=4$

$\Rightarrow\left||\mathbf{a}||\mathbf{b}|\sin\theta\right|=4$ .....(i)

Also $|\mathbf{a}.\mathbf{b}|=2\Rightarrow\left||\mathbf{a}||\mathbf{b}|\cos\theta\right|=2$

$\Rightarrow\left||\mathbf{a}||\mathbf{b}|\cos\theta\right|=2$ .....(ii)

Now squaring and adding equation (i) and (ii), we get

$|\mathbf{a}|^2.|\mathbf{b}|^2\sin^2\theta+|\mathbf{a}|^2.|\mathbf{b}|^2\cos^2\theta=4^2+2^2$

$\therefore|\mathbf{a}|^2.|\mathbf{b}|^2(\sin^2\theta+\cos^2\theta)=16+4$

$\Rightarrow|\mathbf{a}|^2.|\mathbf{b}|^2\times1=20.$

20. (b) $[\mathbf{pqr}]=\mathbf{p}.(\mathbf{q}\times\mathbf{r})$

$=\frac{\mathbf{b}\times\mathbf{c}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}.\left[\frac{\mathbf{c}\times\mathbf{a}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}\times\frac{\mathbf{a}\times\mathbf{b}}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}\right]$

$=\frac{(\mathbf{b}\times\mathbf{c}).[(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$\frac{(\mathbf{b}\times\mathbf{c}).[\{(\mathbf{c}\times\mathbf{a}).\mathbf{b}\}\mathbf{a}-\{(\mathbf{c}\times\mathbf{a}).\mathbf{a}\}\mathbf{b}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$=\frac{(\mathbf{b}\times\mathbf{c})[\{(\mathbf{c}\times\mathbf{a}).\mathbf{b}\}\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$, $\{\because(\mathbf{c}\times\mathbf{a}).\mathbf{a}=0\}$

$=\frac{(\mathbf{b}\times\mathbf{c})[\{(\mathbf{a}.(\mathbf{b}\times\mathbf{c})\}\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}=\frac{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})][(\mathbf{b}\times\mathbf{c}).\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$=\frac{1}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}.$

21. (c) $(\mathbf{a}+\mathbf{b}+\mathbf{c}).[(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{c})]$

$=(\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}\times\mathbf{a}+\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c})$

$=(\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c})$

$=[\mathbf{aac}]+[\mathbf{aba}]+[\mathbf{abc}]+[\mathbf{bac}]+[\mathbf{bba}]$

$+[\mathbf{bbc}]+[\mathbf{cac}]+[\mathbf{cba}]+[\mathbf{cbc}]$

$=-[\mathbf{abc}]$.

22. (c) $\mathbf{V}\times\mathbf{W}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&1&-1\\1&0&3\end{vmatrix}=3\mathbf{i}-7\mathbf{j}-\mathbf{k}$

But $\mathbf{U}$ is a unit vector, $\therefore$ $\mathbf{U}=\frac{3\mathbf{i}-7\mathbf{j}-\mathbf{k}}{\sqrt{59}}$

Hence, $[\mathbf{U}\,\mathbf{V}\,\mathbf{W}]=\frac{3^2+7^2+1^2}{\sqrt{59}}=\sqrt{59}$.

* * *

23. (a) Since $(1,a,a^2),(1,b,b^2)$ and $(1,c,c^2)$ are non-coplanar, therefore $\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}\neq0=\Delta\text{(say)}$

and $\begin{vmatrix}a&a^2&1+a^3\\b&b^2&1+b^3\\c&c^2&1+c^3\end{vmatrix}=\begin{vmatrix}a&a^2&1\\b&b^2&1\\c&c^2&1\end{vmatrix}+\begin{vmatrix}a&a^2&a^3\\b&b^2&b^3\\c&c^2&c^3\end{vmatrix}=0$

$\Rightarrow\Delta+abc\Delta=0$

$\Rightarrow\Delta(abc+1)=0\Rightarrow abc=-1.$

24. (a) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$

$(\mathbf{a}.\mathbf{c})\mathbf{b}-(\mathbf{b}.\mathbf{c})\mathbf{a}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$

$(\mathbf{a}.\mathbf{c})\mathbf{b}=\left\{(\mathbf{b}.\mathbf{c})+\frac{1}{3}|\mathbf{b}||\mathbf{c}|\right\}\mathbf{a}$

$(\mathbf{a}.\mathbf{c})\mathbf{b}=|\mathbf{b}||\mathbf{c}|\left\{\cos\theta+\frac{1}{3}\right\}\mathbf{a}$

As $\mathbf{a}$ and $\mathbf{b}$ are not parallel, $\mathbf{a}.\mathbf{c}=0$ and $\cos\theta+\frac{1}{3}=0$

$\cos\theta=-\frac{1}{3}$. Hence $\sin\theta=\frac{2\sqrt{2}}{3}$.

25. (a) Equation of the plane containing $\mathbf{i}$ and $\mathbf{i}+\mathbf{j}$ is $|\mathbf{r}-\mathbf{i}\ \ \mathbf{i}\ \ \mathbf{i}+\mathbf{j}|=0$

$\Rightarrow(\mathbf{r}-\mathbf{i}).[\mathbf{i}\times(\mathbf{i}+\mathbf{j})]=0$

$\Rightarrow[(x-1)\mathbf{i}+y\mathbf{j}+z\mathbf{k}].\mathbf{k}=0\Rightarrow z=0$ .....(i)

Equation of the plane containing $\mathbf{i}-\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$ is

$\Rightarrow[\mathbf{r}-(\mathbf{i}-\mathbf{j})\ \ \mathbf{i}-\mathbf{j}\ \ \mathbf{i}+\mathbf{k}]=0$

$\Rightarrow (\mathbf{r}-\mathbf{i}+\mathbf{j}).[(\mathbf{i}-\mathbf{j})\times(\mathbf{i}+\mathbf{k})]=0$

$\Rightarrow [(x-1)\mathbf{i}+(y+1)\mathbf{j}+z\mathbf{k}].(-\mathbf{i}-\mathbf{j}+\mathbf{k})=0$

$\Rightarrow x+y-z=0$ .....(ii)

Let $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$. Since $\mathbf{a}$ is parallel to (i) and (ii) $a_3=0$ and $a_1+a_2-a_3=0 \Rightarrow a_1=-a_2$

Thus a vector in the direction of $\mathbf{a}$ is $\mathbf{u}=\mathbf{i}-\mathbf{j}$.

If θ is the angle between $\mathbf{a}$ and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, then

$$\cos\theta = \pm\frac{(1)(1)+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}} = \pm\frac{3}{(\sqrt{2})(3)}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}.$$