1. Prove that K_5 is nonplanar.

Solution.

$e = 10, \ v = 5.$

$3v - 6 = 15 - 6 = 9 < 10$

$e > 3v - 6$

$\therefore K_5$ is non planar.

2. Show that $K_{3,3}$ is nonplanar

Solution

~~To~~ A connected planar graph with e edges and v vertices with $v \geq 3$ and no circuits of length 3 has $e \leq 2v-4$.

$K_{3,3}$ has six vertices and nine edges.

It has no circuits of length 3. since it is bipartite.

We have $e=9$, $v=6$ so $2v-4 = 8 < 9$.

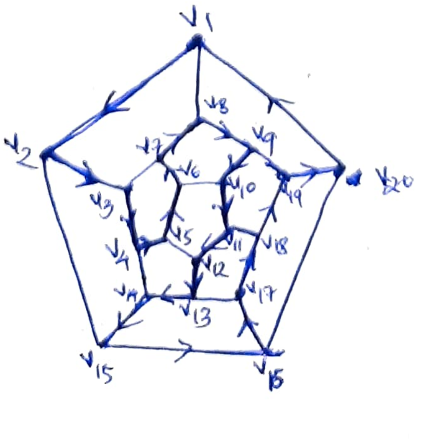
$\therefore e > 2v-4$.

So $K_{3,3}$ is not planar.

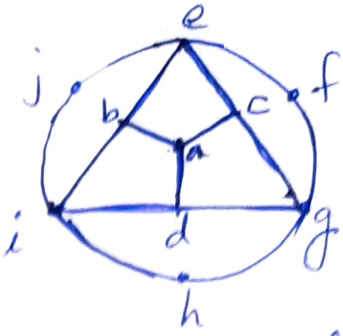
Hamilton Paths and Cycles

Let $G=(V,E)$ is a graph or multigraph with $|V| \geq 3$, G has a Hamilton cycle if there is a cycle in G that contains every vertex in V.

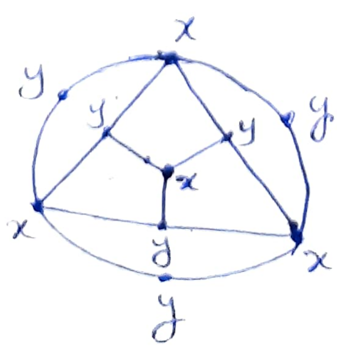
A Hamilton path is a path in G that contains each vertex.



Consider the graph.



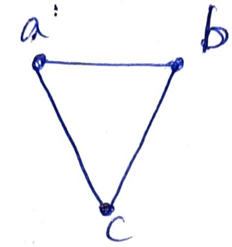
First label the vertex a with the letter x. Those vertex adjacent to a namely b, c and d are then labeled with the letter y. Then labeled the unlabeled vertices adjacent to b, c and d with x. This results in the label x on the vertices e, g and i. Finally, label the unlabeled vertices adjacent to e, g or i with the label y. At this point, all the vertices in G are labeled. Since $|V| = 10$, if G is to have a Hamilton path there must be an alternating sequence of five x's and five y's. Only four vertices are labeled with x, so this is impossible. Hence G has no Hamilton path or cycle



Problem

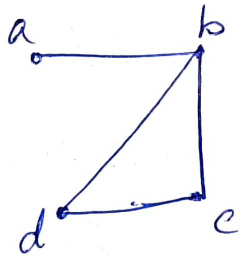
(i) Give an example of a connected graph that has

(a) both a Hamilton cycle and and an Euler circuit



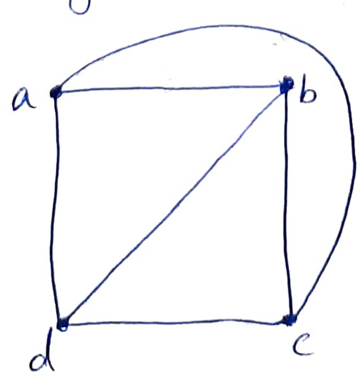
This graph has a Hamilton cycle a, b, c, a. (touching each vertex exactly once) and has a Euler circuit a, b, c, a (traversing each edge exactly once)

(b) Has a Hamilton path but no Hamilton circuit.



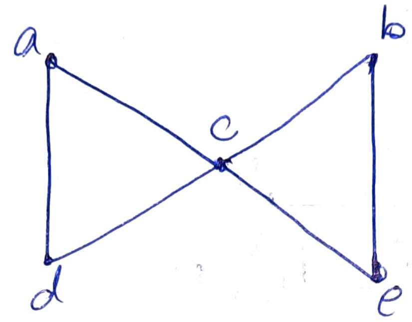
Graph has a Hamilton path d, c, b, a (with every vertex exactly once) but has no Hamilton circuit. since any circuit containing every vertex must contain the edge $\{a, b\}$ twice.

(c) Hamilton cycle but no Euler circuit.



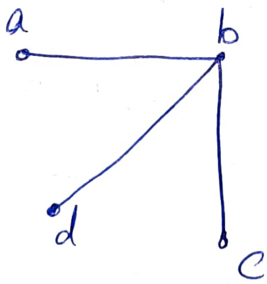
Graph has a Hamilton cycle a, b, c, d, a but has no Euler circuit covering all edges exactly once.

(d). An Euler Circuit but no Hamilton cycle.



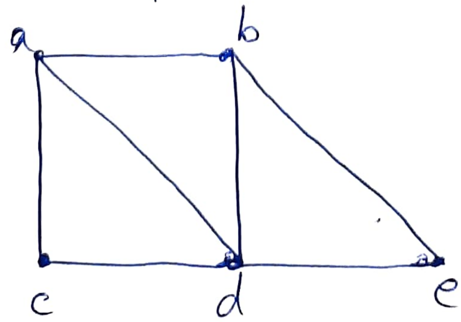
Graph has a Euler Circuit. a, c, e, b, c, d, a. in which every edge is traversed exactly once but has no Hamilton cycle contains every vertex exactly once (Here vertex c will appear twice)

(e) Neither an Euler ~~path~~ Circuit nor a Hamilton cycle.



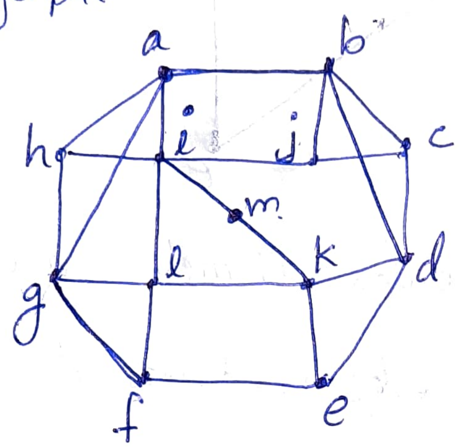
Graph contains no cycle, neither an Euler circuit nor a Hamilton cycle without traversing an edge more than once or traversing a vertex more than once.

(f) An Euler path but no Euler Circuit.



Graph has Euler path a, c, d, e, b, d, a, b but has no Euler circuit (because for a cycle an edge is to be traversed more than once).

2. Find a Hamilton cycle if it exists in the given graph.



A Hamilton cycle contains each vertex exactly once. The cycle a, b, c, d, e, f, g, h, a is not Hamilton since it does not cover all the vertices. So at 'c', traverse j, i, m, k, d. Then at f again go to l, later proceed to g, h, a. Thus a Hamilton cycle in the graph is a, b, c, j, i, m, k, d, e, f, l, g, h, a.