Permutations and Combinations

SET Self Evaluation Test - 6

1. The number of ways in which five identical balls can be distributed among ten identical boxes such that no box contains more than one ball, is

(a) 10 ! (b) $\frac{10!}{5!}$

(c) $\frac{10!}{(5!)^2}$ (d) None of these

2. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 and then men select the chairs from amongst the remaining. The number of possible arrangements is **[IIT 1982, 89; Pb. CET 2000]**

(a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4C_2 \times {}^4P_3$

(c) ${}^4P_2 \times {}^4P_3$ (d) None of these

3. The sides *AB, BC, CA* of a triangle *ABC* have respectively 3, 4 and 5 points lying on them. The number of triangles that can be constructed using these points as vertices is **[IIT 1984]**

(a) 205 (b) 220

(c) 210 (d) None of these

4. P, Q, R and S have to give lectures to an audience. The organiser can arrange the order of their presentation in **[BIT Ranchi 1991; Pb. CET 1991]**

(a) 4 ways (b) 12 ways

(c) 256 ways (d) 24 ways

5. Let A be a set containing 10 distinct elements. Then the total number of distinct functions from A to A, is **[MNR 1992]**

(a) $10!$ (b) 10^{10}

(c) 2^{10} (d) $2^{10}-1$

6. How many numbers can be made with the help of the digits 0, 1, 2, 3, 4, 5 which are greater than 3000 (repetition is not allowed)

(a) 180 (b) 360

(c) 1380 (d) 1500

7. How many words can be made from the letters of the word INSURANCE, if all vowels come together **[Dhanbad Engg. 1971]**

(a) 18270 (b) 17280

(c) 12780 (d) None of these

8. In a certain test a_i students gave wrong answers to at least i questions where $i = 1, 2, 3, ...k$. No student gave more than k wrong answers. The total numbers of wrong answers given is **[IIT 1982]**

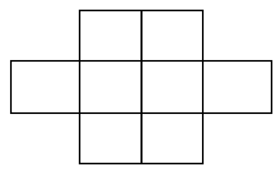
(a) $a_1 + 2a_2 + 3a_3 + ...ka_k$

(b) $a_1 + a_2 + a_3 + ...+ a_k$

(c) Zero

(d) None of these

9. Six '*X*'s have to be placed in the square of the figure such that each row contains at least one *X*. In how many different ways can this be done **[IIT 1978, 89]**



(a) 28 (b) 27

(c) 26 (d) None of these

10. A committee of 12 is to be formed from 9 women and 8 men in which at least 5 women have to be included in a committee. Then the number of committees in which the women are in majority and men are in majority are respectively **[IIT 1994]**

(a) 4784, 1008 (b) 2702, 3360

(c) 6062, 2702 (d) 2702, 1008

11. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is **[Roorkee 1990]**

(a) 10^2 (b) 1023

(c) 2^{10} (d) 10 !

12. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is

(a) 324 (b) 341

(c) 359 (d) None of these

13. The total number of seven digit numbers the sum of whose digits is even is

(a) 9000000 (b) 4500000

(c) 8100000 (d) None of these

14. The number of ways in which the following prizes be given to a class of 20 boys, first and second

Mathematics, first and second Physics, first Chemistry and first English is

(a) $20^4 \times 19^2$ (b) $20^3 \times 19^3$

(c) $20^2 \times 19^4$ (d) None of these

15. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals **[IIT 1998]**

(a) $(n-1)\, a_n$

(b) na_n

(c) $\frac{1}{2} na_n$

(d) None of these

16. ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_{3,}$ then the value of n is **[RPET 2000]**

(a) 7 (b) < 7

(c) > 7 (d) None of these

17. We are to form different words with the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1 / m_2 is equal to **[AMU 2000]**

(a) 30 (b) 60

(c) 90 (d) 180

18. A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends, if two of the friends will not attend the party together is **[DCE 2001]**

(a) 112

(b) 140

(c) 164

(d) None of these

19. In how many ways a team of 10 players out of 22 players can be made if 6 particular players are always to be included and 4 particular players are always excluded **[RPET 2002]**

(a) ${}^{22}C_{10}$ (b) ${}^{18}C_3$

(c) ${}^{12}C_4$ (d) ${}^{18}C_4$

20. There are n distinct points on the circumference of a circle. The number of pentagons that can be formed with these points as vertices is equal to the number of possible triangles. Then the value of n is **[AMU 2002]**

(a) 7 (b) 8

(c) 15 (d) 30

Answers and Solutions

(SET - 6)

1. (c) Out of 10 boxes we have to choose only 5 boxes because the balls are identical and the boxes are also identical (but they can occupy different places), the required number of ways $={}^{10}C_5=\frac{10!}{(5!)^2}$.

2. (d) Required number of ways are $={}^4P_2\times{}^6P_3$.

{After selecting by women there are 6 chairs remaining}.

3. (a) In all there are 3 + 4+ 5 = 12 points in a plane.

The number of required triangles = (The number of triangles formed by these 12 points) – (The number of triangles formed by the collinear points)

$={}^{12}C_3-({}^3C_3+{}^4C_3+{}^5C_3)=220-(1+4+10)=205.$

4. (d) The arrangement can be done in ${}^4P_4=24$ ways.

5. (b) Total number of distinct functions from A to A are n^r *i.e.* 10^{10}.

6. (c) All the 5 digit numbers and 6 digit numbers are greater than 3000. Therefore number of 5 digit numbers $={}^6P_5-{}^5P_5=600$.

{Since the case that 0 will be at ten thousand place should be omit}.

Similarly number of 6 digit numbers $6!-5!=600$.

Now the numbers of 4 digit numbers which are greater than 3000, having 3, 4 or 5 at first place, this can be done in 3 ways and remaining 3 digit may be filled from remaining 5 digits *i.e.* required number of 4 digit numbers are ${}^5P_3\times3=180$. Hence total required number of numbers $=600+600+180=1380$.

7. (d) IUAENSRNC

Obviously required number of words are

$\frac{6!}{2!}\times4!=8640.$

8. (b) Total number of wrong answers

$=1(a_1-a_2)+2(a_2-a_3)+......+(k-1)(a_{k-1}-a_k)+ka_k$

$=a_1+a_2+......+a_k$.

9. (c) In all, we have 8 squares in which $6\ X's$ have to be placed and it can be done in ${}^8C_6=28$ ways.

But this includes the possibility that either the top horizontal row does not have any X or the bottom horizontal has no X. Since we want each row must have at least one X, these two possibilities are to be excluded. Hence required number of ways are $28-2=26$.

10. (d) The number of ways in which at least 5 women can be included in a committee is

${}^9C_5\times{}^8C_7+{}^9C_6\times{}^8C_6+{}^9C_7\times{}^8C_5+{}^9C_8\times{}^8C_4+{}^9C_9\times{}^8C_3$

$=1008+2352+2016+630+56=6062$ ways

(i) The women are in majority in

$(2016+630+56)=2702$ cases.

(ii) Men are in majority in 1008 cases.

11. (b) $2^{10}-1=1023; -1$ corresponds to none of the lamps is being switched on.

12. (a) Words starting from A are $5!=120$

Words starting from I are 5 ! = 120

Words starting from KA are 4 ! = 24

Words starting from KI are 4 ! = 24

Words starting from KN are 4 ! = 24

Words starting from KRA are 3 ! = 6

Words starting from KRIA are 2 ! = 2

Words starting from KRIN are 2 ! = 2

Words starting from KRISA are 1 ! = 1

Words starting from KRISNA are 1 ! = 1

Hence rank of the word KRISNA is 324.

13. (b) Suppose $x_1x_2x_3x_4x_5x_6x_7$ represents a seven digit number. Then x_1 takes the value

$1, 2, 3, \ldots\ldots 9$ and $x_2, x_3, \ldots x_7$ all take values 0, 1, 2, 3, ..., 9.

If we keep $x_1, x_2, \ldots\ldots, x_6$ fixed, then the sum $x_1 + x_2 + \ldots\ldots + x_6$ is either even or odd. Since x_7 takes 10 values 0, 1, 2, ..., 9, five of the numbers so formed will be even and 5 odd.

Hence the required number of numbers

$= 9 \,.\, 10 \,.\, 10 \,.\, 10 \,.\, 10 \,.\, 10 \,.\, 5 = 4500000$.

14. (a) Four first prizes can be given in 20^4 ways since first prize of Mathematics can be given in 20 ways, first prize of Physics also in 20 ways, similarly first prizes of Chemistry and English can be given in 20 ways each. (Note that a boy can stand first in all the four subjects). Then two second prizes can be given in 19^2 ways since a boy cannot get both the first and second prizes.

Hence the required number of ways $= 20^4 \times 19^2$.

15. (c) Given $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

Let $b_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

Then $b_n = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \ldots\ldots + \frac{n}{{}^nC_n}$

and $b_n = \frac{n}{{}^nC_0} + \frac{n-1}{{}^nC_1} + \frac{n-2}{{}^nC_2} + \ldots + \frac{0}{{}^nC_n}$

$[\because {}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1} \ldots \text{as } {}^nC_r = {}^nC_{n-r}]$

By adding $2b_n = \frac{n}{{}^nC_0} + \frac{n}{{}^nC_1} + \ldots\ldots + \frac{n}{{}^nC_n}$

$= n\left[\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + \ldots\ldots + \frac{1}{{}^nC_n}\right] \Rightarrow 2b_n = na_n$

$\therefore b_n = \frac{1}{2} na_n$

16. (c) ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3 \qquad {}^nC_4 > {}^nC_3$

$$\frac{{}^nC_4}{{}^nC_3} > 1 \Rightarrow \frac{n-3}{4} > 1 \Rightarrow n > 7.$$

17. (a) We have 5 letters other than ' I ' and 'N ' of which two are identical (E's). We can arrange these letters in a line in $\frac{5!}{2!}$ ways. In any such arrangement, 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways, so required number $= \frac{5!}{2!} \cdot {}^6P_2 = m_1$. Now, if word start with ' I ' and end with 'R' then the remaining letters are 5. So total no. of ways $= \frac{5!}{2!} = m_2$

$$\frac{m_1}{m_2} = \frac{5!}{2!} \cdot \frac{6!}{4!} \cdot \frac{2!}{5!} = 30.$$

18. (b) Either 6 selected out of 8 or one from 2 and 5 from 8 $= {}^8C_6 + {}^2C_1 + {}^8C_5 = 140$.

19. (c) 6 particular players are always to be included and 4 are always excluded so total no. of selection, now,4 players out of 12, hence number of ways $= {}^{12}C_4$.

20. (b) ${}^nC_5 = {}^nC_3 \Rightarrow n = 8$.

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