Trigonometrical Equations and Inequations, Properties of Triangles, Height and Distance

SET Self Evaluation Test - 11

1. The solution of the equation $\cos^2 x - 2\cos x = 4\sin x - \sin 2x,\ (0 \le x \le \pi)$ is **[DCE 2001]**

(a) $\pi - \cot^{-1}\left(\frac{1}{2}\right)$ (b) $\pi - \tan^{-1}(2)$

(c) $\pi + \tan^{-1}\left(-\frac{1}{2}\right)$ (d) None of these

2. If $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} = 3$, then the value of θ and ϕ are

(a) $\theta = n\pi \pm \frac{\pi}{3}, \phi = n\pi \pm \frac{\pi}{6}$ (b) $\theta = n\pi - \frac{\pi}{3}, \phi = n\pi - \frac{\pi}{6}$

(c) $\theta = n\pi \pm \frac{\pi}{2}, \phi = n\pi + \frac{\pi}{3}$ (d) None of these

3. If $2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) 2\cos x = 0$ then **[Karnataka CET 2002]**

(a) $x = \frac{\pi}{6}(4n+1)$ or $x = \frac{\pi}{2}(4n-1)$

(b) $x = \frac{\pi}{6}(4n-1)$ or $x = \frac{\pi}{2}(4n-1)$

(c) $x = \frac{\pi}{6}(4n+1)$ or $x = \frac{\pi}{2}(4n+1)$

(d) None of these

4. The smallest positive values of x and y which satisfy $\tan(x-y) = 1,\ \sec(x+y) = \frac{2}{\sqrt{3}}$ are

(a) $x = \frac{25\pi}{24}, y = \frac{19\pi}{24}$

(b) $x = \frac{37\pi}{24}, y = \frac{7\pi}{24}$

(c) $x = \frac{\pi}{4}, y = \frac{\pi}{2}$

(d) $x = \frac{\pi}{3}, y = \frac{7\pi}{12}$

5. If θ and ϕ are acute satisfying $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then $\theta + \phi \in$ **[IIT Screening 2004]**

(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$

6. The number of solution of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $(0, 2\pi)$ is **[IIT 1993; Kurushetra CEE 1998; AIEEE 2002; MP PET 2000]**

(a) 0 (b) 1

(c) 2 (d) 3

7. The number of integral values of k, for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution, is **[IIT Screening 2002]**

(a) 4 (b) 8

(c) 10 (d) 12

8. Let $f(x) = \cos\sqrt{x}$, then which of the following is true **[Kurukshetra CEE 1998]**

(a) $f(x)$ is periodic with period $\sqrt{2}\pi$

(b) $f(x)$ is periodic with period $\sqrt{\pi}$

(c) $f(x)$ is periodic with period $4\pi^2$

(d) $f(x)$ is not a periodic function

9. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex on the hypotenuse, then the other two angles are **[MNR 1994]**

(a) $\frac{\pi}{3}, \frac{\pi}{6}$ (b) $\frac{\pi}{4}, \frac{\pi}{4}$

(c) $\frac{\pi}{8}, \frac{3\pi}{8}$ (d) $\frac{\pi}{12}, \frac{5\pi}{12}$

10. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1 + \sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is **[AIEEE 2004]**

(a) 150° (b) 90°

(c) 120° (d) 60°

11. In a ΔABC, $\sin A + \sin B + \sin C = 1 + \sqrt{2}$ and $\cos A + \cos B + \cos C = \sqrt{2}$ if the triangle is

(a) Equilateral (b) Isosceles

(c) Right angled (d) Right angled isosceles

12. In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3. Then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to **[IIT 1995; UPSEAT 2001,03]**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{\frac{2}{3}}$

13. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0, 0 < k < 1$, then the measure of angle C is **[IIT 1990; DCE 2001]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

14. If p_1, p_2, p_3 are altitudes of a triangle ABC form the vertices A, B, C, and Δ, the area of the triangle, then $p_1^{-1} + p_2^{-1} - p_3^{-1}$ is equal to

(a) $\frac{s-a}{\Delta}$ (b) $\frac{s-b}{\Delta}$

(c) $\frac{s-c}{\Delta}$ (d) $\frac{s}{\Delta}$

15. Which of the following pieces of data does not uniquely determine an acute angled ΔABC (R = circum-radius) **[IIT Screening 2002]**

(a) $a, \sin A, \sin B$

(b) a, b, c

(c) $a, \sin B, R$

(d) $a, \sin A, R$

16. The upper $\frac{3}{4}th$ portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[AIEEE 2003]**

(a) 20 m (b) 40 m

(c) 60 m (d) 80 m

17. A spherical baloon of radius r subtends an angle α at the eye of an observer. If the angle of elevation of the centre of the baloon be β. The height of the centre of the baloon is **[IIT 1970]**

(a) $r\,\text{cosec}\left(\frac{\alpha}{2}\right)\sin\beta$ (b) $r\,\text{cosec}\,\alpha\sin\left(\frac{\beta}{2}\right)$

(c) $r\sin\left(\frac{\alpha}{2}\right)\text{cosec}\,\beta$ (d) $r\sin\alpha\,\text{cosec}\left(\frac{\beta}{2}\right)$

18. In ambiguous case if a, b and A are given and if there are two possible values of third side, are c_1 and c_2, then **[UPSEAT 1999]**

(a) $c_1 - c_2 = 2\sqrt{(a^2 + b^2\sin^2 A)}$

(b) $c_1 - c_2 = 2\sqrt{(a^2 - b^2\sin^2 A)}$

(c) $c_1 - c_2 = 4\sqrt{(a^2 + b^2\sin^2 A)}$

(d) $c_1 - c_2 = 3\sqrt{(a^2 - b^2\sin^2 A)}$

19. A tower AB leans towards west making an angle α with the vertical. The angular elevation of B, the top most point of the tower is β as observed from a point C due east of A at a distance d from A. If the angular elevation of B from a point D due east of C at a distance $2d$ from C is γ, then $2\tan\alpha$ can be given as **[IIT 1994]**

(a) $3\cot\beta - 2\cot\gamma$ (b) $3\cot\gamma - 2\cot\beta$

(c) $3\cot\beta - \cot\gamma$ (d) $\cot\beta - 3\cot\gamma$

20. There exists a triangle ABC satisfying the conditions **[IIT 1986; Pb. CET 2003]**

(a) $b\sin A = a,\ A < \frac{\pi}{2}$

(b) $b\sin A > a, A > \frac{\pi}{2}$

(c) $b\sin A > a,\ A < \frac{\pi}{2}$

(d) None of these

Answers and Solutions

(SET - 11)

1. (c) Given equation is $\cos^2 x - 2\cos x = 4\sin x - \sin 2x$

$\cos^2 x - 2\cos x = 4\sin x - 2\sin x\cos x$

$\cos x(\cos x - 2) = 2\sin x(2 - \cos x)$

$(\cos x - 2)(\cos x + 2\sin x) = 0$

$\cos x + 2\sin x = 0$, $(\because \cos x \neq 2)$

$\tan x = -\frac{1}{2}$ $\quad x = n\pi + \tan^{-1}(-1/2), n \in I$

As $0 \le x \le \pi$, therefore, $x = \pi + \tan^{-1}(-1/2)$.

2. (a) $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi}$

$\Rightarrow \sin\theta.\cos\theta = \sin\phi\cos\phi$

$\Rightarrow \sin 2\theta = \sin 2\phi$

$2\theta = \pi - 2\phi \Rightarrow \theta = \frac{\pi}{2} - \phi$

But $\frac{\tan\theta}{\tan\phi} = 3 \Rightarrow \frac{\tan\theta}{\cot\theta} = 3 \Rightarrow \tan^2\theta = 3$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$, so that $\phi = n\pi \pm \frac{\pi}{6}$.

Trick : Check with the options for $n = 0, 1$.

3. (a)
$2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) + 2\cos x = 0$

$2\sin x - 2 + 4\sin^2 x - 2\sin x\cos x - 4\sin^2 x\cos x$
$+ 2\cos x = 0$

$4\sin^2 x + 2\sin x - 2 - \cos x[4\sin^2 x + 2\sin x - 2] = 0$

$(1 - \cos x)(\sin x + 1)(4\sin x - 2) = 0$

Hence $\sin x = -1$ or $\cos x = 1$ or $\sin x = \frac{1}{2}$

$x = (4n - 1)\frac{\pi}{2}$ and $x = (4n + 1)\frac{\pi}{6}$.

4. (a,b) $\tan(x - y) = 1 \Rightarrow x - y = \frac{\pi}{4}, \frac{5\pi}{4}$

(Considering values which lie between 0 and 2π)

$\sec(x + y) = \frac{2}{\sqrt{3}} \Rightarrow x + y = \frac{\pi}{6}, \frac{11\pi}{6}$

(Consider values which lie between 0 and 2π)
Since x, y are positive, therefore $x + y > x - y$

Thus we have $x + y = \frac{11\pi}{6}$ and $x - y = \frac{\pi}{4}$

or $x + y = \frac{11\pi}{6}$ and $x - y = \frac{5\pi}{4}$

Solving these two systems of equations, we get

$x = \frac{25\pi}{24}$ and $y = \frac{19\pi}{24}$ or $x = \frac{37\pi}{24}$ and $y = \frac{7\pi}{24}$.

5. (b) $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$\cos\phi = \frac{1}{3} \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2}$. Thus, $\frac{\pi}{2} < (\theta + \phi) < \frac{2\pi}{3}$.

6. (c) Given, $\tan x + \sec x = 2\cos x$ .....(i)

$\Rightarrow (\sin x + 1) = 2\cos^2 x$

$\Rightarrow (\sin x + 1) = 2(1 - \sin x)(1 + \sin x)$

$\Rightarrow$
$(1 + \sin x)[2(1 - \sin x) - 1] = 0 \Rightarrow 2(1 - \sin x) - 1 = 0$

[$\because \sin x \neq -1$ otherwise $\cos x = 0$ and $\tan x, \sec x$ will be undefined]

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ in $(0, 2\pi)$.

7. (b) $-\sqrt{7^2 + 5^2} \le (7\cos x + 5\sin x) \le \sqrt{7^2 + 5^2}$

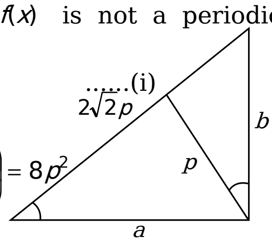
So, for solution $-\sqrt{74} \le (2k + 1) \le \sqrt{74}$

or $-8.6 \le (2k + 1) \le 8.6$ or $-9.6 \le 2k \le 7.6$

or $-4.8 \le k \le 3.8$. So, integral values of k are $-4, -3, -2, -1, 0, 1, 2, 3$ (eight values).

8. (d) If $f(x) = \cos\sqrt{x}$, then $f(x)$ is not a periodic function.

9. (c) Here $a^2 + b^2 = 8p^2$ ......(i)



where $p = a\sin\theta = b\cos\theta$

By (i), $p^2\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right) = 8p^2$

$\Rightarrow 2\sin^2 2\theta = 1$

$\Rightarrow \sin 2\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{8}$.

Here the other angle is $\frac{3\pi}{8}$.

10. (c) $a = \sin\alpha, b = \cos\alpha, c = \sqrt{1 + \sin\alpha\cos\alpha}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{\sin^2\alpha + \cos^2\alpha - (1 + \sin\alpha.\cos\alpha)}{2\sin\alpha.\cos\alpha}$

$= \frac{1 - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$

$\cos C = \frac{-1}{2} = \cos\frac{2\pi}{3}$ $\quad C = \frac{2\pi}{3} = 120^\circ$.

11. (d) **Trick :** If triangle is equilateral, then

$\sin A + \sin B + \sin C = \frac{3\sqrt{3}}{2}$

If triangle is isosceles, let $A = 30^o, B = 30^o, C = 120^o$.

Then $\sin A + \sin B + \sin C = 1 + \frac{\sqrt{3}}{2}$

If triangle is right angled, let $A = 90^o, B = 30^o, C = 60^o$.

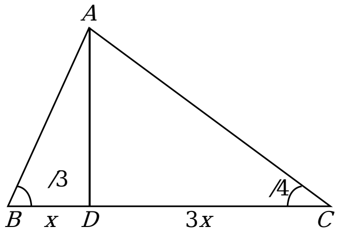
Then $\sin A + \sin B + \sin C = \frac{3+\sqrt{3}}{2}$

If the triangle is right angled isosceles, then one of the angles is 90^o and the remaining two are 45^o each,

so that $\sin A + \sin B + \sin C = 1 + \sqrt{2}$

and $\cos A + \cos B + \cos C = \sqrt{2}$.

12. (c) Let $\angle BAD = \alpha$, $\angle CAD = \beta$



In ΔADB, applying sine formula, we get

$$\frac{x}{\sin\alpha} = \frac{AD}{\sin(\pi/3)} \quad .....(i)$$

In ΔADC, applying sine formula, we get

$$\frac{3x}{\sin\beta} = \frac{AD}{\sin(\pi/4)} \quad .....(ii)$$

Dividing (i) by (ii), we get

$$\frac{x}{\sin\alpha} \times \frac{\sin\beta}{3x} = \frac{AD}{\sin(\pi/3)} \times \frac{\sin(\pi/4)}{AD}$$

$$\frac{\sin\beta}{3\sin\alpha} = \frac{1/\sqrt{2}}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{\sin\beta}{\sin\alpha} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{6}$$

$$\therefore \quad \frac{\sin\angle BAD}{\sin\angle CAD} = \frac{\sin\alpha}{\sin\beta} = \frac{1}{\sqrt{6}}.$$

13. (c) According to the condition $\sin 3A = \sin 3B = k$

$$\Rightarrow \ 2\sin\frac{3A-3B}{2}\cos\frac{3A+3B}{2} = 0$$

Either $A = B$. This ruled out as $A > B$

or $\frac{3A+3B}{2} = \frac{\pi}{2} \ \Rightarrow \ A + B = \frac{\pi}{3} \ \Rightarrow \ C = \frac{2\pi}{3}$.

14. (c) We have, $\frac{1}{2}ap_1 = \Delta, \frac{1}{2}bp_2 = \Delta, \frac{1}{2}cp_3 = \Delta$

$$p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$$

$$\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{a}{2\Delta} + \frac{b}{2\Delta} - \frac{c}{2\Delta} = \frac{a+b-c}{2\Delta}$$

$$= \frac{2(s-c)}{2\Delta} = \frac{s-c}{\Delta}.$$

15. (d) $\frac{a}{\sin A} = R$ and $b = 2R\sin B$. So, two sides and two angles are known. So, $\angle C$ is known. Therefore, two sides and included angle is known. So, Δ is uniquely known in case (a). If *a, b, c* are known the Δ is uniquely known in case (b). $b = 2R\sin B$, $\sin A = \frac{a}{2R}$. So, sides *a, b* and angle *A, B* are known. So, $\angle C$ is known. Therefore two sides and included angle is known. So, Δ is uniquely known in case (c).

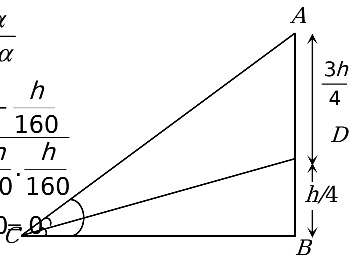
$\frac{a}{\sin A} = R$. So, only a side and an angle is known. So, Δ is not uniquely known in case (d).

16. (b) $\theta = \alpha + \beta$; $\beta = \theta - \alpha$

$$\tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha}$$

$$\frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40}\cdot\frac{h}{160}}$$

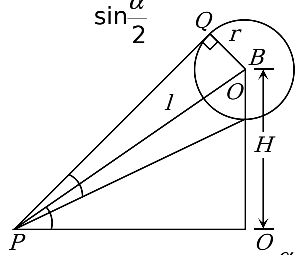
$$h^2 - 200h + 6400 = 0$$



$h = 40$ or 160 *metre*.

$\therefore$ Possible height = 40 *metre*.

17. (a) In $\Delta PQB, l = \frac{r}{\sin\frac{\alpha}{2}}$ and in ΔPOB,



$$H = l\sin\beta \Rightarrow H = r\sin\beta\,\mathrm{cosec}\frac{\alpha}{2}.$$

18. (b) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or $c^2 - (2b\cos A)c + (b^2 - a^2) = 0$, which is quadratic equation in *c*. Let there be two roots c_1 and c_2 of above quadratic equation, then $c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$

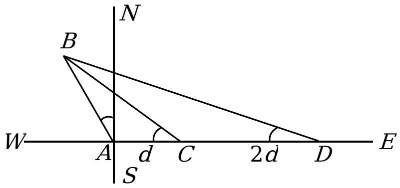
$$\therefore c_1 - c_2 = \sqrt{[(c_1 + c_2)^2 - 4c_1c_2]}$$

$$= \sqrt{[(2b\cos A)^2 - 4(b^2 - a^2)]}$$

$$= \sqrt{[4a^2 - 4b^2(1 - \cos^2 A)]} = 2\sqrt{(a^2 - b^2\sin^2 A)}.$$

19. (c) By $m-n$ theorem at C

$(d+2d)\cot\beta = d\cot\gamma - 2d\cot(90^o+\alpha)$



$3d\ \cot\beta = d\cot\gamma + 2d\tan\alpha$

$\Rightarrow\ \ 3\cot\beta = \cot\gamma + 2\tan\alpha$

$\therefore\ \ 2\tan\alpha = 3\cot\beta - \cot\gamma$.

20. (a) We have, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} \Rightarrow a\sin B = b\sin A$

$\therefore$ From option (a), $b\sin A = a \Rightarrow a\sin B = a$

$\Rightarrow\ \ \sin B = 1 \Rightarrow B = \dfrac{\pi}{2}$

Since $A < \dfrac{\pi}{2}$, the ΔABC is possible.

Now, form option (b), $b\sin A > a \Rightarrow a\sin B > a$

$\Rightarrow\ \ \sin B > 1$, which is impossible.

Similarly option (c) can be shown to be impossible.