Determinants and Matrices — SET Self Evaluation Test - 9

1. If $A=\begin{vmatrix}\sin(\theta+\alpha) & \cos(\theta+\alpha) & 1\\ \sin(\theta+\beta) & \cos(\theta+\beta) & 1\\ \sin(\theta+\gamma) & \cos(\theta+\gamma) & 1\end{vmatrix}$, then **[Orissa JEE 2003]**

(a) $A=0$ for all θ (b) A is an odd Function of
(c) $A=0$ for $\theta=\alpha+\beta+\gamma$ (d) A is independent of θ

2. If $f(x)=\begin{vmatrix}1 & x & x+1\\ 2x & x(x-1) & (x+1)x\\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$ then $f(100)$ is equal to

[IIT 1999; MP PET 2000; Kerala (Engg.) 2005; DCE 2005]

(a) 0 (b) 1
(c) 100 (d) –100

3. The parameter on which the value of the determinant $\begin{vmatrix}1 & a & a^2\\ \cos(p-d)x & \cos px & \cos(p+d)x\\ \sin(p-d)x & \sin px & \sin(p+d)x\end{vmatrix}$ does not depend upon

[IIT (Re-Exam) 1997; Pb. CET 2000]

(a) a (b) p
(c) d (d) x

4. If $1,\omega,\omega^2$ are the cube roots of unity, then

$\Delta=\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^n & \omega^{2n} & 1\\ \omega^{2n} & 1 & \omega^n\end{vmatrix}$ is equal to

(a) 0 (b) 1
(c) ω (d) ω^2

5. The value of the determinant $\begin{vmatrix}10! & 11! & 12!\\ 11! & 12! & 13!\\ 12! & 13! & 14!\end{vmatrix}$ is

[Orissa JEE 2003]

(a) $2(10!\,11!)$ (b) $2(10!\,13!)$
(c) $2(10!\,11!\,12!)$ (d) $2(11!\,12!\,13!)$

6. For all values of A,B,C and P,Q,R, the value of $\begin{vmatrix}\cos(A-P) & \cos(A-Q) & \cos(A-R)\\ \cos(B-P) & \cos(B-Q) & \cos(B-R)\\ \cos(C-P) & \cos(C-Q) & \cos(C-R)\end{vmatrix}$ is **[IIT 1994]**

(a) 0 (b) $\cos A\cos B\cos C$
(c) $\sin A\sin B\sin C$ (d) $\cos P\cos Q\cos R$

7. The number of distinct real roots of $\begin{vmatrix}\sin x & \cos x & \cos x\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix}=0$ in the interval $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$ is

[IIT Screening 2001]

(a) 0 (b) 2
(c) 1 (d) 3

8. Let $\Delta_1=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$ and $\Delta_2=\begin{vmatrix}\alpha_1 & \beta_1 & \gamma_1\\ \alpha_2 & \beta_2 & \gamma_2\\ \alpha_3 & \beta_3 & \gamma_3\end{vmatrix}$, then $\Delta_1\times\Delta_2$ can be expressed as the sum of how many determinants **[Tamilnadu (Engg.)2001]**

(a) 9 (b) 3
(c) 27 (d) 2

9. If $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$, $B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \sin\phi\cos\phi & \sin^2\phi\end{bmatrix}$

and θ and ϕ differs by $\frac{\pi}{2}$, then $AB=$

(a) I (b) O
(c) $-I$ (d) None of these

10. If the multiplicative group of 2×2 matrices of the form $\begin{pmatrix}a & a\\ a & a\end{pmatrix}$, for $a\neq 0$ and $a\in R$, then the inverse of $\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}$ is

[Karnataka CET 1999]

(a) $\begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8}\end{pmatrix}$ (b) $\begin{pmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{4}\end{pmatrix}$

(c) $\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ (d) Does not exist

11. If for $AX=B$, $B=\begin{bmatrix}9\\ 52\\ 0\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}3 & -\frac{1}{2} & -\frac{1}{2}\\ -4 & \frac{3}{4} & \frac{5}{4}\\ 2 & -\frac{1}{4} & -\frac{3}{4}\end{bmatrix}$, then X is equal to

(a) $\begin{bmatrix}1\\ 3\\ 5\end{bmatrix}$ (b) $\begin{bmatrix}-\frac{1}{2}\\ -\frac{1}{2}\\ 2\end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ \frac{3}{4} \\ -\frac{3}{4} \end{bmatrix}$

12. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then for $n \geq 2, A^n =$ `

(a) $nA-(n-1)I$ (b) $nA-I$

(c) $2^{n-1}A-(n-1)I$ (d) $2^{n-1}A-I$

13. Let p a non singular matrix $1+p+p^2+....+p^n = O$ (O denotes the null matrix), then $p^{-1} =$ **[Orissa JEE 2(**

(a) p^n

(b) $-p^n$

(c) $-(1+p+....+p^n)$

(d) None of these

14. If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ then $P_{22} =$

[Orissa JEE 2004]

(a) 40 (b) – 40

(c) – 20 (d) 20

15. If $C = 2\cos\theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is **[Orissa JEE 2002]**

(a) $\frac{\sin 4\theta}{\sin\theta}$

(b) $\frac{2\sin^2 2\theta}{\sin\theta}$

(c) $4\cos^2\theta(2\cos\theta - 1)$

(d) None of these

16. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

[MNR 1986; Kurukshetra CEE 1998; UPSEAT 2000]

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(c) $\frac{d}{dx}(\Delta_1) = 2(\Delta_2)^2$ (d) $\Delta_1 = 3\Delta_2^{3/2}$

17. If $a_1, a_2, a_3....a_n....$ are in G.P. then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is

[AIEEE 2004, 05]

(a) – 2 (b) 1

(c) 2 (d) 0

18. In the determinant $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$, the ratio of the co-factor to its minor of the element – 3 is **[MP PET 1992]**

(a) –1 (b) 0

(c) 1 (d) 2

19. If value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be **[Roorkee 1990; DCE 2000]**

(a) 11 (b) 121

(c) 1331 (d) 14641

20. Consider the system of linear equations $a_1x+b_1y+c_1z+d_1 = 0$, $a_2x+b_2y+c_2z+d_2 = 0$ and $a_3x+b_3y+c_3z+d_3 = 0$. Let us denote by $\Delta(a,b,c)$ the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ if $\Delta(a,b,c) \neq 0$, then the value of x in the unique solution of the above equations is

[Pb. CET 2004]

(a) $\frac{\Delta(bcd)}{\Delta(abc)}$ (b) $\frac{-\Delta(bcd)}{\Delta(abc)}$

(c) $\frac{\Delta(acd)}{\Delta(abc)}$ (d) $-\frac{\Delta(abd)}{\Delta(abc)}$

21. If a matrix A is such that $4A^3+2A^2+7A+I = O$, then A^{-1} equals **[MP PET 2001]**

(a) $(4A^2+2A+7I)$ (b) $-(4A^2+2A+7I)$

(c) $-(4A^2-2A+7I)$ (d) $(4A^2+2A-7I)$

22. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $[F(\alpha)G(\beta)]^{-1} =$

(a) $F(\alpha) - G(\beta)$ (b) $-F(\alpha) - G(\beta)$

(c) $[F(\alpha)]^{-1}[G(\beta)]^{-1}$ (d) $[G(\beta)]^{-1}[F(\alpha)]^{-1}$

23. If the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than

[Orissa JEE 2003]

(a) 3 (b) 2

(c) 1 (d) None of these

24. For how many value (s) of x in the closed interval $[-4, -1]$ is the matrix $\begin{bmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix}$ singular

[Karnataka CET 2002]

(a) 2 (b) 0
(c) 3 (d) 1

25. If 3, – 2 are the Eigen values of a non-singular matrix A and $|A| = 4$, then the Eigen values of $adj(A)$ are

[Kurukshetra CEE 2002]

(a) $\frac{3}{4}, \frac{-1}{2}$ (b) $\frac{4}{3}, -2$
(c) 12, – 8 (d) – 12, 8

Answers and Solutions

(SET - 9)

1. (d) Given $A = \begin{vmatrix} \sin(\theta+\alpha) & \cos(\theta+\alpha) & 1 \\ \sin(\theta+\beta) & \cos(\theta+\beta) & 1 \\ \sin(\theta+\gamma) & \cos(\theta+\gamma) & 1 \end{vmatrix}$

Operate $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\therefore A = \{\cos(\theta+\gamma) - \cos(\theta+\alpha)\}$

$\{\sin(\theta+\beta) - \sin(\theta+\alpha)\} - \{\cos(\theta+\beta)$

$-\cos(\theta+\alpha)\}\{\sin(\theta+\gamma) - \sin(\theta+\alpha)\}$

$= \sin(\beta-\gamma) - \sin(\beta-\alpha) - \sin(\alpha-\gamma)$,

which is independent of θ.

2. (a) Given determinant

$$= \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

$$= x(x+1)\begin{vmatrix} 1 & x & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$$

$$= x(x+1)(x-1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$$

Applying $C_1 - C_3$ and $C_2 - C_3$

$$x(x+1)(x-1)\begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x & -2 & x \end{vmatrix} = x(x+1)(x-1)[-2x+2x] = 0$$

$\therefore f(x) = 0 \Rightarrow f(100) = 0$.

3. (b) $C_1 \to C_1 + C_3 - 2C_2 \cos dx$ gives

$$\Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

$= (1+a^2-2a\cos dx)\sin dx$, (which is independent of p).

4. (a) $\Delta = 1(\omega^{3n}-1) + \omega^n(\omega^{2n}-\omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$\Delta = [(\omega^3)^n - 1] + 0 + \omega^{2n}[\omega^n - (\omega^3)^n.\omega^n]$

$\Delta = 1 - 1 + 0 + \omega^{2n}[\omega^n - \omega^n] = 0$.

5. (c) $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} = 10!\ 11!\ 12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 1 & 12 & 12\times 13 \\ 1 & 13 & 13\times 14 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 10!11!12!\begin{vmatrix} 1 & 11 & 11\times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix} = 2(10!\ 11!\ 12!).$$

6. (a) The determinant can be expanded as

$$\begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos A\cos Q + \sin A\sin Q & \cos A\cos R + \sin A\sin R \\ \cos B\cos P + \sin B\sin P & \cos B\cos Q + \sin B\sin Q & \cos B\cos R + \sin B\sin R \\ \cos C\cos P + \sin C\sin P & \cos C\cos Q + \sin C\sin Q & \cos C\cos R + \sin C\sin R \end{vmatrix}$$

This determinant can be written as 8 determinants and the value of each of these 8 determinants is zero;

e.g., $\cos P\cos Q\cos R\begin{vmatrix} \cos A & \cos A & \cos A \\ \cos B & \cos B & \cos B \\ \cos C & \cos C & \cos C \end{vmatrix} = 0$

Similarly other determinants can be shown zero.

7. (c) Here, $(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

or

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

or $(2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \tan x = -2, 1$. But $\tan x \neq -2$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\therefore \tan x = 1$. So, $x = \frac{\pi}{4}$.

8. (c) Each term in $\Delta_1 \times \Delta_2$ is the sum of three terms. So each entry in C_1 or C_2 or C_3 in $\Delta_1 \times \Delta_2$ is the sum of three terms. Hence, $\Delta_1 \times \Delta_2$ can be written as the sum of $3 \times 3 \times 3 = 27$ determinants.

9. (b) $AB = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi \end{bmatrix}$

$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\theta\sin\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$

$= \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \cos\theta\sin\phi & \sin\theta\sin\phi \end{bmatrix}$

$= O, \left(\because \theta - \phi = \frac{\pi}{2}\right)$.

10. (d) Given, A multiplicative group of 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$. Let $A = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix}$

since $|A| = 0$, therefore inverse of A does not exist.

11. (a) $AX = B \Rightarrow A^{-1}.AX = A^{-1}B \Rightarrow X = A^{-1}B = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$.

12. (a) As we have $A^2 = 2A - I \Rightarrow A^2.A = (2A - I)A$

$\Rightarrow A^3 = 2A^2 - IA = 2(2A - I) - A \Rightarrow A^3 = 3A - 2I$

$\{\because IA = A \text{ and } A^2 = 2A - I\}$

Similarly, $A^4 = 4A - 3I, A^5 = 5A - 4I$..

Hence $A^n = nA - (n-1)I$.

13. (a) $1 + p + p^2 + \dots\dots\dots + p^n = O$

Pre-multiplying both sides by p^{-1}

$p^{-1} + I + Ip + \dots\dots\dots\dots + p^{n-1}I = O.p^{-1}$

$p^{-1} + I(1 + p + p^2 + \dots\dots\dots + p^{n-1}) = O$

$p^{-1} = -(1 + p + p^2 + \dots\dots\dots + p^{n-1})I$

$p^{-1} = -(-p^n) = p^n$.

14. (a) $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}_{3\times3} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix}_{3\times2} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}_{2\times3}$

$P = \begin{bmatrix} -3 & -14 \\ -8 & -20 \\ -11 & -26 \end{bmatrix}_{3\times2} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}_{2\times3}$

$P = \begin{bmatrix} 12 & 15 & 4 \\ 32 & 40 & 28 \\ 44 & 55 & 40 \end{bmatrix}_{3\times3} \Rightarrow P_{22} = 40$.

15. (d) $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C[C^2 - 1] - 1[C - 6]$

$\because C = 2\cos\theta$

$\Delta = 2\cos\theta(4\cos^2\theta - 1) - (2\cos\theta - 6)$

$\Delta = 8\cos^3\theta - 4\cos\theta + 6$.

16. (b) $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx \quad \frac{d}{dx}\Delta_1 = 3(x^2 - ab)$

and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab \quad \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2$.

17. (d) We have $a_1, a_2, a_3 \dots\dots a_n$ in G.P.

then $r = \frac{a_2}{a_1}$ *i.e.,* $r = \frac{a_{n+1}}{a_n} = \frac{a_{n+2}}{a_{n+1}} = \dots\dots\dots$

Hence

$\log r = \log(a_{n+1}) - \log(a_n) = \log(a_{n+2}) - \log(a_{n+1}) = \dots$

Now $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

Operate $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$

$= \begin{vmatrix} \log a_n & (\log a_{n+1} - \log a_n) & (\log a_{n+2} - \log a_{n+1}) \\ \log a_{n+3} & (\log a_{n+4} - \log a_{n+3}) & (\log a_{n+5} - \log a_{n+4}) \\ \log a_{n+6} & (\log a_{n+7} - \log a_{n+6}) & (\log a_{n+8} - \log a_{n+7}) \end{vmatrix}$

$= \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0$.

18. (a) Ratio of cofactor to its minor of the element, which is in the 3rd row and 2nd column $= (-1)^{3+2} = -1$.

19. (d) $\Delta^c = \Delta^{n-1} = \Delta^{3-1} = \Delta^2 = (11)^2 = 121$.

But we have to find the value of the square of the determinant, so required value is $(121)^2 = 14641$.

20. (b) From Cramer's rule, $x = \frac{D_x}{D} = \frac{\begin{bmatrix} -d_1 & b_1 & c_1 \\ -d_2 & b_2 & c_2 \\ -d_3 & b_3 & c_3 \end{bmatrix}}{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}$

$x = \frac{-\Delta(bcd)}{\Delta(abc)}$.

21. (b) Given $4A^3 + 2A^2 + 7A + I = O$

Pre-multiply with A^{-1}

$A^{-1}[4A^3 + 2A^2 + 7A + I] = O$

$4IA^2 + 2IA + 7I + A^{-1}I = O.A^{-1}$

$I(4A^2 + 2A + 7) + A^{-1}I = O$

$A^{-1} = -(4A^2 + 2A + 7I)$.

22. (d) Since $(AB)^{-1} = B^{-1}A^{-1}$.

23. (b) The given matrix is $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}, \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{vmatrix}$

***

Using $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \neq 0$$

$\therefore$ The rank of matrix is always less than 2.

24. (d) For the given matrix to be singular, we must have, $\begin{vmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix} = 0$

$$\begin{vmatrix} 3 & -1+x & 2 \\ 0 & -x & x \\ x & -x & 0 \end{vmatrix} = 0 \quad ,$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$

$$\begin{vmatrix} x+4 & -1+x & 2 \\ 0 & -x & x \\ 0 & -x & 0 \end{vmatrix} = 0, \; [C_1 \to C_1 + C_2 + C_3]$$

$$(x+4)(0+x^2) = 0 \Rightarrow x = -4, 0$$

Note that only $-4 \in [-4, -1]$.

25. (b) Since $A^{-1} = \dfrac{adjA}{|A|}$ and if is eigen value of A, then λ^{-1} is eigen value of A^{-1}.

Thus for $adj(A)X = (A^{-1}X)|A| = |A|\lambda^{-1}I$.

Thus, eigen value corresponding to $\lambda = 3$ is 4/3 and corresponding to $\lambda = -2$ is 4/–2 = – 2.