Answers and Solutions

Modulus of vector, Algebra of vectors

1. (a) $l_1 = \sqrt{25+25} = 5\sqrt{2}$, $l_2 = \sqrt{25+25} = 5\sqrt{2}$, $l_3 = 5\sqrt{2}$.

Hence, $l_1 + l_2 + l_3 = 3\sqrt{50} = \sqrt{450}$.

2. (d) $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \Rightarrow |a| = \sqrt{4+1+4} = \sqrt{9}$

$\mathbf{b} = -\mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow |b| = \sqrt{1+1+16} = \sqrt{18}$

$\mathbf{c} = -\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \Rightarrow |c| = \sqrt{1+4+4} = \sqrt{9}$

$|\mathbf{a}| = |\mathbf{c}|$ and also, $\mathbf{b}^2 = \mathbf{a}^2 + \mathbf{c}^2$

Hence it is isosceles and right angled triangle.

3. (d) $|\mathbf{a}| = \sqrt{9+16+25} = 5\sqrt{2}$

Area $= |\mathbf{a}|^2 = 25 \times 2 = 50$.

4. (a) $|x\mathbf{a}| = |x||\mathbf{a}| \Rightarrow |x|\sqrt{4+4+1} = 1 \Rightarrow x = \pm\frac{1}{3}$.

5. (c) Since $\sin^2 2\theta + \cos^2\theta$ is not equal to one necessarily.

6. (c) Equal in magnitude, as bisector $= \frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{\sqrt{2}}$ if $|\mathbf{a}| = |\mathbf{b}|$.

7. (b) $|\mathbf{b}|\,\hat{\mathbf{a}} = \sqrt{9+36+4}\left(\frac{\mathbf{i}+2\mathbf{j}+2\mathbf{k}}{\sqrt{1+4+4}}\right) = \frac{7}{3}(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$.

8. (d) $\mathbf{p} - 2\mathbf{q} = \mathbf{i} - 4\mathbf{j} - 7\mathbf{k}$

$|\mathbf{p} - 2\mathbf{q}| = \sqrt{49+16+1} = \sqrt{66}$.

9. (c) $\mathbf{b} = \cos 120^\circ\mathbf{i} + \sin 120^\circ\mathbf{j}$ or $\mathbf{b} = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$.

Therefore $\mathbf{a} + \mathbf{b} = \mathbf{i} - \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$.

10. (c) Equilateral, since each side is of length $\sqrt{6}$.

11. (a) $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \Rightarrow |\mathbf{a}| = \sqrt{16+16+4} = 6$

$\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} + 12\mathbf{k} \Rightarrow |\mathbf{b}| = \sqrt{144+4+9} = \sqrt{157}$

$\mathbf{c} = -\mathbf{i} - 4\mathbf{j} - 8\mathbf{k} \Rightarrow |\mathbf{c}| = \sqrt{64+16+1} = 9$

Hence perimeter is $15 + \sqrt{157}$.

12. (b) $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \Rightarrow |\overrightarrow{AB}| = 3$.

13. (b) $R = \sqrt{4+100+121} = 15$.

14. (a) It is a fundamental concept.

15. (d) Resultant vector $= 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

Direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

16. (a) $7 = \sqrt{(5+1)^2 + (4-2)^2 + (a+2)^2} \Rightarrow a + 2 = \pm 3$

or $a = -5, 1$.

17. (a) Direction is not determined.

18. (c) Let $\mathbf{a} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$, where $l^2 + m^2 + n^2 = 1$.

$\mathbf{a}$ makes an angle $\frac{\pi}{4}$ with $z-$axis.

$\therefore n = \frac{1}{\sqrt{2}}$, $l^2 + m^2 = \frac{1}{2}$ .....(i)

$\therefore \mathbf{a} = l\mathbf{i} + m\mathbf{j} + \frac{\mathbf{k}}{\sqrt{2}}$

$\mathbf{a} + \mathbf{i} + \mathbf{j} = (l+1)\mathbf{i} + (m+1)\mathbf{j} + \frac{\mathbf{k}}{\sqrt{2}}$

Its magnitude is 1, hence $(l+1)^2 + (m+1)^2 = \frac{1}{2}$ .....(ii)

From (i) and (ii), $2lm = \frac{1}{2} \Rightarrow l = m = -\frac{1}{2}$

Hence $\mathbf{a} = -\frac{\mathbf{i}}{2} - \frac{\mathbf{j}}{2} + \frac{\mathbf{k}}{\sqrt{2}}$.

19. (b) It is a fundamental concept.

20. (d) Given $\mathbf{a} + \mathbf{c} = \mathbf{b} + \mathbf{d} \Rightarrow \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \frac{1}{2}(\mathbf{b} + \mathbf{d})$

Here, mid points of $\overrightarrow{AC}$ and $\overrightarrow{BD}$ coincide, where $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are diagonals. In addition, we know that diagonals of a parallelogram bisect each other.

Hence quadrilateral is parallelogram.

21. (b) $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$

Direction cosine along $y-$ axis

$= \frac{-5}{\sqrt{16+25+121}} = \frac{-5}{\sqrt{162}}$.

22. (a) $|\overrightarrow{AB}| = |Q| = \sqrt{P^2 + P^2} = P\sqrt{2}$.

23. (c) $\frac{3}{\sqrt{3^2 + 4^2 + 5^2}} = \frac{3}{\sqrt{50}}$.

24. (c) Here, $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$\overrightarrow{OC} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

So, $\overrightarrow{AB} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\overrightarrow{BC} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\overrightarrow{CA} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$

Clearly $|AB| = |BC| = |CA| = \sqrt{6}$

So these points are vertices of an equilateral triangle.

25. (b) Let P, Q and R be points having position vectors $\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$, $\beta\mathbf{i} + \gamma\mathbf{j} + \alpha\mathbf{k}$ and $\gamma\mathbf{i} + \alpha j + \beta\mathbf{k}$ respectively.

Then,

$|\overrightarrow{PQ}| = |\overrightarrow{QR}| = |\overrightarrow{RP}| = \sqrt{(\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2}$

Hence ΔPQR is an equilateral triangle.

26. (b) We have $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$

$\therefore\ 25 + |\mathbf{a}-\mathbf{b}|^2 = 2(9+16) \Rightarrow |\mathbf{a}-\mathbf{b}| = 5$.

27. (b) **Trick :** Here is the only vector $4(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$, whose length is 8.

28. (b) Since $\mathbf{a}$ and $\mathbf{b}$ are non-collinear, so $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ will also be non-collinear. Hence, $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ are linearly independent vectors.

29. (b) $\overrightarrow{AB}$ = Position vector of $\vec{B}$ – Position vector of $\vec{A}$ $= (2\mathbf{i}+3\mathbf{j}-6\mathbf{k})-(6\mathbf{i}-2\mathbf{j}+3\mathbf{k}) = -4\mathbf{i}+5\mathbf{j}-9\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{16+25+81} = \sqrt{122}$, $\overrightarrow{BC} = \mathbf{i}+3\mathbf{j}+4\mathbf{k}$

$|\overrightarrow{BC}| = \sqrt{1+9+16} = \sqrt{26}$ and $\overrightarrow{AC} = -3\mathbf{i}+8\mathbf{j}-5\mathbf{k}$

$|\overrightarrow{AC}| = \sqrt{98}$

Therefore, $AB^2 = 122$, $BC^2 = 26$ and $AC^2 = 98$.

$\Rightarrow AB^2 + BC^2 = 26+122 = 148$

Since $AC^2 < AB^2 + BC^2$, therefore ΔABC is an obtuse-angled triangle.

30. (c) $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$(\sqrt{7}Q)^2 = P^2 + Q^2 + 2PQ\cos 60^\circ$

$7Q^2 = P^2 + Q + PQ \qquad P^2 + PQ - 6Q^2 = 0$

$P^2 + 3PQ - 2PQ - 6Q^2 = 0$

$P(P+3Q) - 2Q(P+3Q) = 0$

$(P-2Q)(P+3Q) = 0$

$P-2Q = 0$ or $P+3Q = 0$

From $P-2Q = 0 \qquad \frac{P}{Q} = 2$.

31. (b) Vector $\vec{A} = 3\mathbf{i}-4\mathbf{j}+5\mathbf{k}$. We know that direction cosines of

$$\vec{A} = \frac{3}{\sqrt{3^2+4^2+5^2}}, \frac{-4}{\sqrt{3^2+4^2+5^2}}, \frac{5}{\sqrt{3^2+4^2+5^2}}$$

$$= \frac{3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}.$$

32. (d) $\overrightarrow{AB} = (6-2)\mathbf{i} + (-3+9)\mathbf{j} + (8+4)\mathbf{k} = 4\mathbf{i}+6\mathbf{j}+12\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{16+36+144} = 14$.

33. (d) $\overline{PQ} = (5-1)\mathbf{i} + (-2-3)\mathbf{j} + (4+7)\mathbf{k} = 4\mathbf{i}-5\mathbf{j}+11\mathbf{k}$

$|\overline{PQ}| = \sqrt{16+25+121} = \sqrt{162}$.

34. (c) $m\mathbf{a}$ is a unit vector if and only if $|m\mathbf{a}| \qquad m$ $|\mathbf{a}| = 1$

$$m = \frac{1}{|\mathbf{a}|}.$$

35. (c) $\overrightarrow{AB} = (3-2)\mathbf{i} + (-2-1)\mathbf{j} + (1+1)\mathbf{k} = \mathbf{i}-3\mathbf{j}+2\mathbf{k}$

$\overrightarrow{BC} = (1-3)\mathbf{i} + (4+2)\mathbf{j} + (-3-1)\mathbf{k} = -2\mathbf{i}+6\mathbf{j}-4\mathbf{k}$

$\overrightarrow{CA} = (2-1)\mathbf{i} + (1-4)\mathbf{j} + (-1+3)\mathbf{k} = \mathbf{i}-3\mathbf{j}-2\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{1+9+4} = \sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{4+36+16} = \sqrt{56} = 2\sqrt{14}$

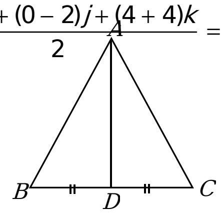
$|\overrightarrow{CA}| = \sqrt{1+9+4} = \sqrt{14}$

So, $|\overrightarrow{AB}| + |\overrightarrow{AC}| = |\overrightarrow{BC}|$ and angle between AB and BC is 180°. Points A, B, C can not form an isosceles triangle.

Hence A, B, C are collinear.

36. (c) P.V. of

$$\overrightarrow{AD} = \frac{(3+5)i + (0-2)j + (4+4)k}{2} = 4\mathbf{i}-\mathbf{j}+4\mathbf{k}$$



$|\overrightarrow{AD}| = \sqrt{16+16+1} = \sqrt{33}$.

37. (d) Given, position vectors of A, B and C are $7\mathbf{j}+10\mathbf{k}$, $-\mathbf{i}+6\mathbf{j}+6\mathbf{k}$ and $-4\mathbf{i}+9\mathbf{j}+6\mathbf{k}$ respectively.

$|\overrightarrow{AB}| = |-\mathbf{i}-\mathbf{j}-4\mathbf{k}| = \sqrt{18}$

$|\overrightarrow{BC}| = |-3\mathbf{i}+3\mathbf{j}| = \sqrt{18}$

$|\overrightarrow{AC}| = |-4\mathbf{i}+2\mathbf{j}-4\mathbf{k}| = \sqrt{36}$

Clearly, $AB = BC$ and $(AC)^2 = (AB)^2 + (BC)^2$

Hence, triangle is right angled isosceles.

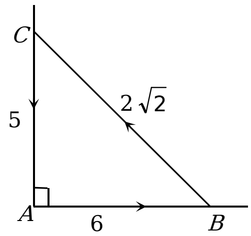
38. (c) Let $A \equiv (1,1,-1), B \equiv (2,3,0), C \equiv (3,5,-2), D \equiv (0,-1,1)$

So,

$\overrightarrow{AB} = (1,2,1), \overrightarrow{BC} = (1,2,-2), \overrightarrow{CD} = (-3,-6,3), \overrightarrow{DA} = (1,2,-2)$

Clearly, $\overrightarrow{BC} \parallel \overrightarrow{DA}$, but $AB \neq CD$ So, it is a trapezium.

39. (b) $R\cos\theta = 6\cos 0^\circ + 2\sqrt{2}\cos(180^\circ - B) + 5\cos 270^\circ$



$R\cos\theta = 6 - 2\sqrt{2}\cos B$ .....(i)

$R\sin\theta = 6\sin 0^\circ + 2\sqrt{2}\sin(180^\circ - B) + 5\sin 270^\circ$

$R\sin\theta = 2\sqrt{2}\sin B - 5$ .....(ii)

From (i) and (ii),

$$R^2 = 36 + 8\cos^2 B - 24\sqrt{2}\cos B + 8\sin^2 B + 25 - 20\sqrt{2}\sin B$$

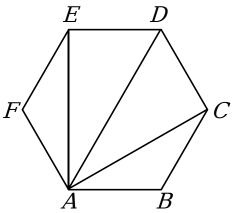
$$= 61 + 8(\cos^2 B + \sin^2 B) - 24\sqrt{2}\cos B - 20\sqrt{2}\sin B$$

$\because$ ABC is a right angled isosceles triangle *i.e.*, $\angle B = \angle C = 45^\circ$

$$\therefore R^2 = 61 + 8(1) - 24\sqrt{2}\cdot\frac{1}{\sqrt{2}} - 20\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 25$$

$R = 5$.

40. (b) By triangle law, $\overrightarrow{AB} = \overrightarrow{AD} - \overrightarrow{BD}$, $\overrightarrow{AC} = \overrightarrow{AD} - \overrightarrow{CD}$

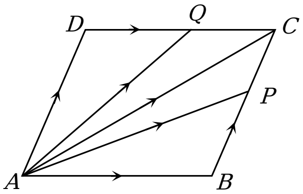


Therefore, $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= 3\overrightarrow{AD} + (\overrightarrow{AE} - \overrightarrow{BD}) + (\overrightarrow{AF} - \overrightarrow{CD}) = 3\overrightarrow{AD}$

Hence $\lambda = 3$, [Since $\overrightarrow{AE} = \overrightarrow{BD}, \overrightarrow{AF} = \overrightarrow{CD}$].

41. (d) $\overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{BP} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$ .....(i)



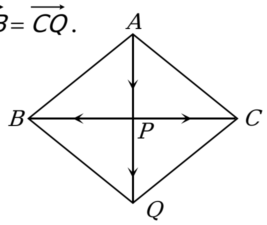
$\overrightarrow{AQ} = \overrightarrow{AD} + \overrightarrow{DQ} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB}$ .....(ii)

By (i) and (ii), we get,

$\overrightarrow{AP} + \overrightarrow{AQ} = \frac{3}{2}(\overrightarrow{AB} + \overrightarrow{AD}) = \frac{3}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{3}{2}\overrightarrow{AC}$.

42. (c) $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PQ}$ or $\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PQ} + \overrightarrow{CP}$

or $\overrightarrow{AB} = \overrightarrow{CQ}$.



Hence it is a parallelogram.

43. (b) $\mathbf{v} = \mathbf{b} + \mathbf{c}$ .....(i)

$\mathbf{w} = \mathbf{b} + \mathbf{a}$ .....(ii)

We have, $\mathbf{x} = \mathbf{v} + \mathbf{w} = \mathbf{a} + 2\mathbf{b} + \mathbf{c}$.

44. (b) It is obvious.

45. (c) Since $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD} \Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

$= (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) - (2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) = -\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}$

Hence unit vector in the direction of $\overrightarrow{BD}$ is

$\frac{-\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}}{|-\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}|} = \frac{-\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}}{\sqrt{69}}$.

46. (d) Let $\mathbf{a} + 2\mathbf{b} = x\mathbf{c}$ and $\mathbf{b} + 3\mathbf{c} = y\mathbf{a}$, then

$\mathbf{a} + 2\mathbf{b} + 6\mathbf{c} = (x + 6)\mathbf{c}$ and $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c} = (1 + 2y)\mathbf{a}$

So, $(x + 6)\mathbf{c} = (1 + 2y)\mathbf{a}$

Since $\mathbf{a}$ and $\mathbf{c}$ are non-zero and non-collinear, we have $x + 6 = 0$ and $1 + 2y = 0$ *i.e.,* $x = -6$ and $y = -\frac{1}{2}$. In either case, we have $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c} = \mathbf{0}$.

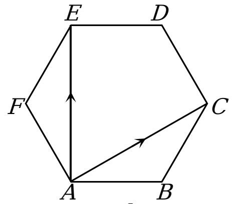
47. (d) $\mathbf{a} + \mathbf{b} = 4\mathbf{i} + 4\mathbf{j}$, therefore unit vector $\frac{4(\mathbf{i} + \mathbf{j})}{\sqrt{32}} = \frac{\mathbf{i} + \mathbf{j}}{\sqrt{2}}$.

48. (a) Let $\mathbf{b}$ should be added, then $\mathbf{a} + \mathbf{b} = \mathbf{i}$

$\Rightarrow \mathbf{b} = \mathbf{i} - \mathbf{a} = \mathbf{i} - (3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}) = -2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

49. (c) $\mathbf{R} = 3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} \Rightarrow \hat{\mathbf{R}} = \frac{3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}}{5\sqrt{2}}$.

50. (b) Obviously, $\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE}$



$= \overrightarrow{AC} + \overrightarrow{AF} - \overrightarrow{AB}$, $\{\because \overrightarrow{CD} = \overrightarrow{AF} \text{ and } \overrightarrow{DE} = -\overrightarrow{AB}\}$.

51. (c) $3\overrightarrow{OD} + \overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC}$

$= \overrightarrow{OD} + \overrightarrow{DA} + \overrightarrow{OD} + \overrightarrow{DB} + \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$.

52. (b) Let $-2\mathbf{a} + 3\mathbf{b} - \mathbf{c} = x\mathbf{p} + y\mathbf{q} + z\mathbf{r}$

$\Rightarrow -2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$

$= (2x + y - 3z)\mathbf{a} + (-3x - 2y + z)\mathbf{b} + (y + 2z)\mathbf{c}$

$\therefore 2x + y - 3z = -2$, $-3x - 2y + z = 3$ and $y + 2z = -1$

Solving these, we get $x = 0$, $y = -\frac{7}{5}$, $z = \frac{1}{5}$

$-2\mathbf{a} + 3\mathbf{b} - \mathbf{c} = \frac{(-7\mathbf{q} + \mathbf{r})}{5}$.

Trick : Check alternates one by one

i.e., (a) $\mathbf{p} - 4\mathbf{q} = -2\mathbf{a} + 5\mathbf{b} - 4\mathbf{c}$

(b) $\frac{-7\mathbf{q} + \mathbf{r}}{5} = -2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$.

53. (a) We have,

$\mathbf{p} = \overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AC} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AC} + \lambda\overrightarrow{AD} + \overrightarrow{CD}$

$= \lambda\overrightarrow{AD} + (\overrightarrow{AC} + \overrightarrow{CD}) = \lambda\overrightarrow{AD} + \overrightarrow{AD} = (\lambda + 1)\overrightarrow{AD}$.

Therefore $\mathbf{p} = \mu\overrightarrow{AD} \Rightarrow \mu = \lambda + 1$.

54. (a) $|\mathbf{a} + \mathbf{b}| = |3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}| = |\sqrt{3^2 + 4^2 + 12^2}| = 13$

$\therefore \mathbf{a} + \mathbf{b} = 13$.

55. (b) A, B, C, D, E are five co-planar points.

$\overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC} + \overrightarrow{AE} + \overrightarrow{BE} + \overrightarrow{CE}$

$= (\overrightarrow{DA} + \overrightarrow{AE}) + (\overrightarrow{DB} + \overrightarrow{BE}) + (\overrightarrow{DC} + \overrightarrow{CE})$

$= \overrightarrow{DE} + \overrightarrow{DE} + \overrightarrow{DE} = 3\overrightarrow{DE}$.

56. (c) $\mathbf{a} + \mathbf{b} + \mathbf{c} = (3 + 2 - 1)\mathbf{i} + (-2 - 4 + 2)\mathbf{j} + (1 - 3 + 2)\mathbf{k}$

$= 4\mathbf{i} - 4\mathbf{j}$.

57. (b) Points *A, B, C, D, E* are in a plane.

Resultant $= (\overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE}) + (\overrightarrow{CB} + \overrightarrow{DB} + \overrightarrow{EB})$

$= (\overrightarrow{AC} + \overrightarrow{CB}) + (\overrightarrow{AD} + \overrightarrow{DB}) + (\overrightarrow{AE} + \overrightarrow{EB})$

$= \overrightarrow{AB} + \overrightarrow{AB} + \overrightarrow{AB} = 3\overrightarrow{AB}$.

58. (a) $P + Q = 18$, $R = 12$, $\theta = 90^o$, (say)

$\tan\theta = \tan 90^o = \infty$

$\Rightarrow \; P+Q\cos\alpha=0\,, \quad \therefore \cos\alpha=\dfrac{-P}{Q}$

Also, $(12)^2 = P^2+Q^2+2PQ\cos\alpha$

or $\quad 144 = P^2+Q^2+(2P)(-P)$

$\Rightarrow 144 = Q^2-P^2=(Q+P)(Q-P)$

or $\quad 144 = 18(Q-P)$ or $\quad Q-P=8$

After solving $Q=13$, $P=5$.

59. (a) Resultant vector

$=(2\mathbf{i}+4\mathbf{j}-5\mathbf{k})+(\mathbf{i}+2\mathbf{j}+3\mathbf{k})=3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$

Unit vector $=\dfrac{3\mathbf{i}+6\mathbf{j}-2\mathbf{k}}{\sqrt{9+36+4}}=\dfrac{1}{7}(3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$.

60. (a) It is obvious.

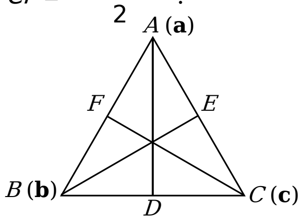
61. (b) It is obvious.

62. (a) $\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}=\dfrac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{2}$,

(where O is the origin for reference)

Similarly, $\quad \overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}=\dfrac{\mathbf{c}+\mathbf{a}}{2}-\mathbf{b}=\dfrac{\mathbf{c}+\mathbf{a}-2\mathbf{b}}{2}$

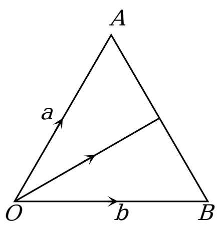
and $\overrightarrow{CF}=\dfrac{\mathbf{a}+\mathbf{b}-2\mathbf{c}}{2}$.



Now, $\quad \overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$

$=\dfrac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{2}+\dfrac{\mathbf{c}+\mathbf{a}-2\mathbf{b}}{2}+\dfrac{\mathbf{a}+\mathbf{b}-2\mathbf{c}}{2}=\mathbf{0}$.

63. (d) Since given that $\overrightarrow{AC}=3\overrightarrow{AB}$. It means that point C divides AB externally. Thus $\overrightarrow{AC}:\overrightarrow{BC}=3:2$



Hence $\overrightarrow{OC}=\dfrac{3.\mathbf{b}-2.\mathbf{a}}{3-2}=3\mathbf{b}-2\mathbf{a}$.

64. (b) $\dfrac{\mathbf{a}+\mathbf{b}}{2}=2\mathbf{i}-\mathbf{j}+\dfrac{5}{2}\mathbf{k}$.

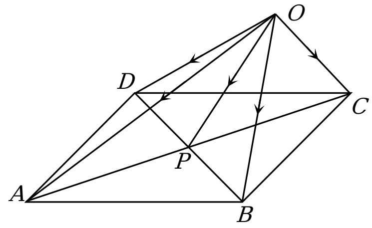
65. (b) Let position vector of D is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then $\overrightarrow{AB}=\overrightarrow{DC}$

$\Rightarrow -2\mathbf{j}-4\mathbf{k}=(7-x)\mathbf{i}+(7-y)\mathbf{j}+(7-z)\mathbf{k}$

$\Rightarrow x=7,\ y=9,\ z=11$.

Hence position vector of D will be $7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$.

66. (d) We know that P will be the mid point of AC and BD



$\therefore \quad \overrightarrow{OA}+\overrightarrow{OC}=2\overrightarrow{OP}$ ......(i)

and $\overrightarrow{OB}+\overrightarrow{OD}=2\overrightarrow{OP}$ .....(ii)

Adding (i) and (ii), we get, $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\overrightarrow{OP}$.

67. (a) Let the position vector of P is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then $\overrightarrow{AB}=\overrightarrow{CP}\Rightarrow \mathbf{j}-\mathbf{i}=x\mathbf{i}+y\mathbf{j}+(z-1)\mathbf{k}$

By comparing the coefficients of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, we get $x=-1$, $\;y=1$ and $z-1=0\Rightarrow z=1$

Hence required position vector is $-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

68. (b) $\overrightarrow{AB}=-\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{CD}=6\mathbf{i}+6\mathbf{j}+12\mathbf{k}$

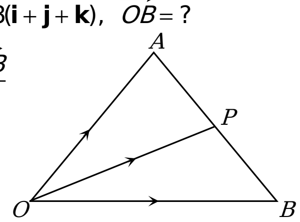
Hence, $\overrightarrow{AB}\,||\,\overrightarrow{CD}$.

69. (c) $\overrightarrow{OA}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $\;\overrightarrow{OP}=3(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $\;\overrightarrow{OB}=?$

we have $\overrightarrow{OP}=\dfrac{\overrightarrow{OA}+\overrightarrow{OB}}{2}$

$\Rightarrow \overrightarrow{OB}=2\overrightarrow{OP}-\overrightarrow{OA}$

$=4\mathbf{i}+3\mathbf{j}+7\mathbf{k}$



Trick : By inspection, middle point of $4\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ and $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ is $3(\mathbf{i}+\mathbf{j}+\mathbf{k})$.

70. (d) $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\mathbf{0}$ and $\overrightarrow{G'A'}+\overrightarrow{G'B'}+\overrightarrow{G'C'}=\mathbf{0}$

$\Rightarrow (\overrightarrow{GA}-\overrightarrow{G'A'})+(\overrightarrow{GB}-\overrightarrow{G'B'})+(\overrightarrow{GC}-\overrightarrow{G'C'})=\mathbf{0}$

$\Rightarrow (\overrightarrow{GA}+\overrightarrow{G'G}-\overrightarrow{G'A'})+(\overrightarrow{GB}+\overrightarrow{G'G}-\overrightarrow{G'B'})$

$+(\overrightarrow{GC}+\overrightarrow{G'G}-\overrightarrow{G'C'})=3\overrightarrow{G'G}$

$\Rightarrow (\overrightarrow{GA}-\overrightarrow{GA'})+(\overrightarrow{GB}-\overrightarrow{GB'})+(\overrightarrow{GC}-\overrightarrow{GC'})=3\overrightarrow{G'G}$

$\Rightarrow \overrightarrow{A'A}+\overrightarrow{B'B}+\overrightarrow{C'C}=3\overrightarrow{G'G}\Rightarrow \overrightarrow{AA'}+\overrightarrow{BB'}+\overrightarrow{CC'}=3\overrightarrow{GG'}$.

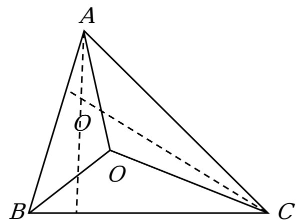
71. (b) $\overrightarrow{O'A}=\overrightarrow{O'O}+\overrightarrow{OA}$

$\overrightarrow{O'B}=\overrightarrow{O'O}+\overrightarrow{OB}$

$\overrightarrow{O'C}=\overrightarrow{O'O}+\overrightarrow{OC}$

$\Rightarrow \overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}$

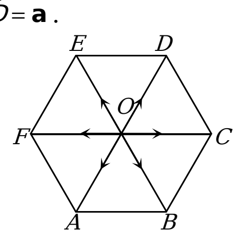
$=3\overrightarrow{O'O}+\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$



Since $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OO'}=-\overrightarrow{O'O}$

$\therefore \quad \overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}=2\overrightarrow{O'O}$.

72. (a) As in figure $\overrightarrow{AB}=\mathbf{a}$, $\;\overrightarrow{BC}=\mathbf{b}$, so $\overrightarrow{AD}=2\mathbf{b}$ and $\overrightarrow{ED}=\mathbf{a}$.



Now, $\overrightarrow{AE}+\overrightarrow{ED}=\overrightarrow{AD} \Rightarrow \overrightarrow{AE}=\overrightarrow{AD}-\overrightarrow{ED}=2\mathbf{b}-\mathbf{a}$.

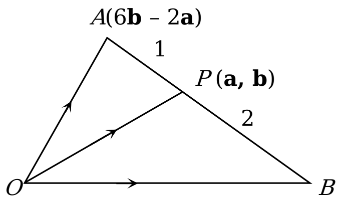
73. (a) Since position vector of a point C with respect to B is

$\overrightarrow{BC}=\mathbf{i}+\mathbf{j}$ .....(i)

Similarly, $\overrightarrow{AB}=\mathbf{i}-\mathbf{j}$ .....(ii)

Now by (i) and (ii), $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}$.

74. (a) $\overrightarrow{OP}=\dfrac{1(\overrightarrow{OB})+2(6\mathbf{b}-2\mathbf{a})}{1+2}$

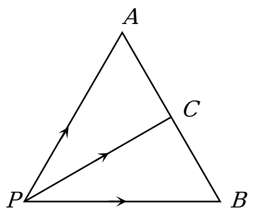


$\Rightarrow 3(\mathbf{a}-\mathbf{b})=\overrightarrow{OB}+12\mathbf{b}-4\mathbf{a} \Rightarrow \overrightarrow{OB}=7\mathbf{a}-15\mathbf{b}$.

75. (b) $\dfrac{3\mathbf{i}-\mathbf{j}-3\mathbf{k}+\mathbf{i}+3\mathbf{j}-\mathbf{k}}{2}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}$.

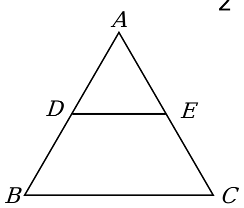
76. (b) $\overrightarrow{PA}+\overrightarrow{PB}=(\overrightarrow{PA}+\overrightarrow{AC})+(\overrightarrow{PB}+\overrightarrow{BC})-(\overrightarrow{AC}+\overrightarrow{BC})$

$=\overrightarrow{PC}+\overrightarrow{PC}-(\overrightarrow{AC}-\overrightarrow{CB})=2\overrightarrow{PC}-0, \; (\because \overrightarrow{AC}=\overrightarrow{CB})$



$\therefore \; \overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$.

77. (d) We know by fundamental theorem of proportionality that $\overrightarrow{DE}=\dfrac{1}{2}\overrightarrow{BC}$



In triangle, $\overrightarrow{BC}=\mathbf{b}-\mathbf{a}$; Hence, $\overrightarrow{DE}=\dfrac{1}{2}(\mathbf{b}-\mathbf{a})$.

78. (b) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0 \Rightarrow \mathbf{a}+\mathbf{b}-\mathbf{c}=0$.

79. (c) $\overrightarrow{AE}+\overrightarrow{ED}+\overrightarrow{DC}+\overrightarrow{AB}=\overrightarrow{AD}+\overrightarrow{DC}+\overrightarrow{AB}=\overrightarrow{AC}+\overrightarrow{AB}$.

Obviously, if $\overrightarrow{BC}$ is added to this system, then it will be $\overrightarrow{AC}+\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}+\overrightarrow{AC}=2\overrightarrow{AC}$.

80. (c) Since $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $2AC=CO$

By section formula $\overrightarrow{OC}=\dfrac{2}{3}\mathbf{a}$.

Therefore, $|\overrightarrow{CD}|=3|\overrightarrow{OB}| \Rightarrow \overrightarrow{CD}=3\mathbf{b}$

$\Rightarrow \overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{CD}=\dfrac{2}{3}\mathbf{a}+3\mathbf{b}$

Hence, $\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=\dfrac{2}{3}\mathbf{a}+3\mathbf{b}-\mathbf{a}=3\mathbf{b}-\dfrac{1}{3}\mathbf{a}$.

81. (a) $2\overrightarrow{OA}+3\overrightarrow{OB}=2(\overrightarrow{OC}+\overrightarrow{CA})+3(\overrightarrow{OC}+\overrightarrow{CB})$

$=5\overrightarrow{OC}+2\overrightarrow{CA}+3\overrightarrow{CB}=5\overrightarrow{OC}, \quad \{\because 2\overrightarrow{CA}=-3\overrightarrow{CB}\}$.

82. (c) $\overrightarrow{AB}=\overrightarrow{BC}$ (as given). Hence it is an isosceles triangle.

83. (a) It should be remembered.

84. (b) Position vectors of the points which divides internally is, $\dfrac{3(2\mathbf{a}-3\mathbf{b})+2(3\mathbf{a}-2\mathbf{b})}{5}=\dfrac{12\mathbf{a}-13\mathbf{b}}{5}$.

85. (a) $\overrightarrow{AB}=\overrightarrow{CX} \Rightarrow \mathbf{j}-\mathbf{i}=\text{P.V.of } X-\mathbf{k} \Rightarrow \text{P.V.of } X=-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

86. (a) It is obvious.

87. (a) Position vectors of vertices A, B and C of the triangle ABC = **a, b** and **c**. We know that position vector of centroid of the triangle $(G)=\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$.

Therefore , $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$

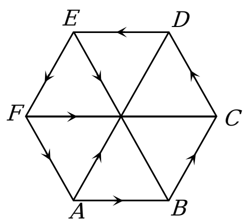
$=\left(\mathbf{a}-\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}\right)+\left(\mathbf{b}-\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}\right)+\left(\mathbf{c}-\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}\right)$

$=\dfrac{1}{3}[2\mathbf{a}-\mathbf{b}-\mathbf{c}+2\mathbf{b}-\mathbf{a}-\mathbf{c}+2\mathbf{c}-\mathbf{a}-\mathbf{b}]=\mathbf{0}$.

88. (c) Co-ordinate of C is $\left(\dfrac{2-4}{2},\dfrac{-1+3}{2}\right)=(-1,1)$

$\therefore \overrightarrow{OC}=-\mathbf{i}+\mathbf{j}$.

89. (d) A regular hexagon $ABCDEF$.



We know from the hexagon that $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$ or $\overrightarrow{AD}=2\overrightarrow{BC}$; $\overrightarrow{EB}$ is parallel to $\overrightarrow{FA}$ or $\overrightarrow{EB}=2\overrightarrow{FA}$, and $\overrightarrow{FC}$ is parallel to $\overrightarrow{AB}$ or $\overrightarrow{FC}=2\overrightarrow{AB}$.

Thus $\overrightarrow{AD}+\overrightarrow{EB}+\overrightarrow{FC}=2\overrightarrow{BC}+2\overrightarrow{FA}+2\overrightarrow{AB}$

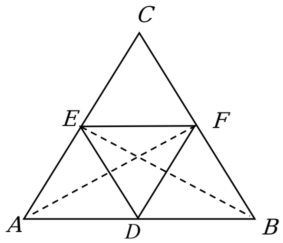
$=2(\overrightarrow{FA}+\overrightarrow{AB}+\overrightarrow{BC})=2(\overrightarrow{FC})=2(2\overrightarrow{AB})=4\overrightarrow{AB}$.

90. (c) If $\mathbf{x}$ be the position vector of B, then $\mathbf{a}$ divides AB in the ratio 2 : 3.

$$\mathbf{a}=\frac{2\mathbf{x}+3(\mathbf{a}+2\mathbf{b})}{2+3}$$

$5\mathbf{a}-3\mathbf{a}-6\mathbf{b}=2\mathbf{x}\Rightarrow \mathbf{x}=\mathbf{a}-3\mathbf{b}.$

91. (a) $\overrightarrow{BE}+\overrightarrow{AF}=\overrightarrow{OE}-\overrightarrow{OB}+\overrightarrow{OF}-\overrightarrow{OA}$



$$=\frac{\overrightarrow{OA}+\overrightarrow{OC}}{2}-\overrightarrow{OB}+\frac{\overrightarrow{OB}+\overrightarrow{OC}}{2}-\overrightarrow{OA}$$

$$=\overrightarrow{OC}-\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}=\overrightarrow{OC}-\overrightarrow{OD}=\overrightarrow{DC}.$$

92. (a) Let the bisector of angle A meets BC at D, then AD divides BC in the ratio AB: AC

Position vectors of D

$$=\frac{|\overrightarrow{AB}|\,(2\mathbf{i}+5\mathbf{j}+7\mathbf{k})+|\overrightarrow{AC}|\,(2\mathbf{i}+3\mathbf{j}+4\mathbf{k})}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$$

Here, $|\overrightarrow{AB}|=|-2\mathbf{i}-4\mathbf{j}-4\mathbf{k}|=6$ and

$|\overrightarrow{AC}|=|-2\mathbf{i}-2\mathbf{j}-\mathbf{k}|=3$

Position vector of

$$D=\frac{6(2\mathbf{i}+5\mathbf{j}+7\mathbf{k})+3(2\mathbf{i}+3\mathbf{j}+4\mathbf{k})}{6+3}$$

$$=\frac{18\mathbf{i}+39\mathbf{j}+54\mathbf{k})}{9}=\frac{1}{3}(6\mathbf{i}+13\mathbf{j}+18\mathbf{k}).$$

93. (d) $\mathbf{c}=\lambda\mathbf{a}+\mu\mathbf{b}=(\lambda+\mu)\mathbf{i}-\lambda\mathbf{j}+\mu\mathbf{k}$

Now, $\mathbf{c}.\mathbf{a}=0\Rightarrow 2\lambda+\mu=0\Rightarrow \mu=-2\lambda$

Therefore, $\mathbf{c}=-\lambda\mathbf{i}-\lambda\mathbf{j}-2\lambda\mathbf{k}=(\sqrt{6})(-\lambda)\left[\frac{\mathbf{i}+\mathbf{j}+2\mathbf{k}}{\sqrt{6}}\right]$

Hence, unit vector $=\frac{(\mathbf{i}+\mathbf{j}+2\mathbf{k})}{\sqrt{6}}$.

94. (d) Here $\overrightarrow{AB}=-2\mathbf{j},\ \overrightarrow{BC}=(a-1)\mathbf{i}+(b+1)\mathbf{j}+c\mathbf{k}$

The points are collinear, then $\overrightarrow{AB}=k(\overrightarrow{BC})$

$-2\mathbf{j}=k\{(a-1)\mathbf{i}+(b+1)\,\mathbf{j}+c\mathbf{k}\}$

On comparing, $k(a-1)=0,\ k(b+1)=-2,\ kc=0$.

Hence $c=0,\ a=1$ and b is arbitrary scalar.

95. (d) $(\mathbf{a}-\mathbf{b})-(\mathbf{a}+\mathbf{b})=[(\mathbf{a}+k\mathbf{b})-(\mathbf{a}-\mathbf{b})]$

$\Rightarrow -2\mathbf{b}=(k+1)\mathbf{b}$. Hence $k\in R$.

96. (a) Here $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$ and $\overrightarrow{AC}=(3\mathbf{a}-2\mathbf{b})-(\mathbf{a})=-2(\mathbf{b}-\mathbf{a})$

Therefore, it is of the form $\overrightarrow{AB}=m\overrightarrow{AC}$.
Hence A, B, C are collinear.

97. (a) It is obvious.

98. (d) $\frac{3}{a}=\frac{1}{b}=\frac{-5}{-15}\Rightarrow a=9,\ b=3.$

99. (a) If the given points be A, B, C, then $\overrightarrow{AB}=k(\overrightarrow{BC})$

$\Rightarrow -20\mathbf{i}-11\mathbf{j}=k[(a-40)\mathbf{i}-44\mathbf{j}]$

On comparing, $-11=-44k\Rightarrow k=\frac{1}{4}$

And $-20=\frac{1}{4}(a-40)\Rightarrow a=-40.$

100. (c) Unit vector prarallel to $\overrightarrow{OA}$

$$=\frac{4\mathbf{i}+5\mathbf{j}}{\sqrt{16+25}}=\frac{1}{\sqrt{41}}(4\mathbf{i}+5\mathbf{j}).$$

101. (c) $\overrightarrow{AB}=-4\mathbf{i}+5\mathbf{k}$, which is a vector lying in a plane parallel to $xz-$ plane.

102. (c) If given points be A, B, C then

$\overrightarrow{AB}=k(\overrightarrow{BC})$ or $2\mathbf{i}-8\mathbf{j}=k[(a-12)\mathbf{i}+16\mathbf{j}]\Rightarrow k=\frac{-1}{2}$

Also, $2=k(a-12)\Rightarrow a=8.$

103. (d) $\overrightarrow{AB}=\lambda\overrightarrow{BC}$, (for collinearity)

Here $\overrightarrow{AB}=-2\mathbf{b},\ \overrightarrow{BC}=(k+1)\mathbf{b}$

Hence $\forall\ k\in R\Rightarrow \overrightarrow{AB}=\lambda\overrightarrow{BC}.$

104. (b) $\overrightarrow{AB}=-\mathbf{i}-4\mathbf{j},\ \overrightarrow{CD}=-2\mathbf{i}+(\lambda-2)\mathbf{j}$

$\because \overrightarrow{AB}\,||\,\overrightarrow{CD}$. So, $\frac{-1}{-2}=\frac{-4}{\lambda-2}$, $\lambda-2=-8$ or $\lambda=-6.$

105. (a) Obviously, $\frac{3}{6}=\frac{2}{-4x}=-\frac{1}{y}\Rightarrow x=-1$ and $y=-2.$

106. (d) Comparing the coefficients of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$, the corresponding equations are

$x+3y-4z=\lambda x$ or $(1-\lambda)x+3y-4z=0$ ......(i)

$x-(\lambda+3)y+5z=0$ ......(ii)

$3x+y-\lambda z=0$ .....(iii)

These equations (i), (ii) and (iii) have a non-trivial solution, if

$$\begin{vmatrix}(1-\lambda) & 3 & -4\\ 1 & -(\lambda+3) & 5\\ 3 & 1 & -\lambda\end{vmatrix}=0\Rightarrow \lambda=0,-1.$$

107. (b) These are coplanar because $1(\mathbf{a})+1(\mathbf{b})=\mathbf{a}+\mathbf{b}$.

108. (d) It is a fundamental concept.

109. (c) $\mathbf{a}+\mathbf{b}=3\mathbf{i}+9\mathbf{j}=3(\mathbf{i}+3\mathbf{j})$. Hence it is parallel to $(1,3)$.

110. (c) Since $\mathbf{c}=(x-2)\mathbf{a}+\mathbf{b}$ and $\mathbf{d}=(2x+1)\mathbf{a}-\mathbf{b}$ are collinear, therefore $\mathbf{c}=\lambda\mathbf{d}$

$\Rightarrow (x-2)\mathbf{a}+\mathbf{b}=\lambda(2x+1)\mathbf{a}-\lambda\mathbf{b}$

or $[(x-2)-\lambda(2x+1)]\mathbf{a}+(\lambda+1)\mathbf{b}=0$

$(x-2)-\lambda(2x+1)=0,\lambda+1=0$

($\because$ $\mathbf{a}, \mathbf{b}$ are linearly independent)

$\Rightarrow x-2+2x+1=0 \Rightarrow x=\frac{1}{3}$.

111. (a) $\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 7 \\ -3 & -2 & -5 \end{vmatrix}=0 \Rightarrow \lambda=3.$

112. (a) $\overrightarrow{PQ}=3\mathbf{a}+3\sqrt{3}\mathbf{b}$ and $\overrightarrow{RS}=2\mathbf{a}+2\sqrt{3}\mathbf{b}$

Hence $\overrightarrow{PQ} \| \overrightarrow{RS}$.

113. (c) Condition for collinearity, $\mathbf{b}=\lambda\mathbf{a}$

$\Rightarrow (-2\mathbf{i}+m\mathbf{j})=\lambda(\mathbf{i}-\mathbf{j})$

Comparison of coefficient, we get

$\Rightarrow \lambda=-2$ and $-\lambda=m$ So, $m=2$.

114. (b) Let the B divide AC in ratio $\lambda:1$, then

$5\mathbf{i}-2\mathbf{k}=\frac{\lambda(11\mathbf{i}+3\mathbf{j}+7\mathbf{k})+\mathbf{i}-2\mathbf{j}-8\mathbf{k}}{\lambda+1}$

$\Rightarrow 3\lambda-2=0 \Rightarrow \lambda=\frac{2}{3}$ *i.e.*, ratio = 2 : 3.

115. (c) If $\mathbf{a}, \mathbf{b}$ are two non-zero, non-collinear vectors and x, y are two scalars such that $x\mathbf{a}+y\mathbf{b}=0$, then $x=0, y=0$. Because otherwise one will be a scalar multiple of the other and hence collinear which is a contradiction.

116. (a) If A, B, C are collinear. Then $\overrightarrow{AB}=\lambda\overrightarrow{BC}$

$2\mathbf{i}+(4-x)\mathbf{j}+4\mathbf{k}=\lambda[(y-3)\mathbf{i}-6\mathbf{j}-12\mathbf{k}]$

$\Rightarrow 2=(y-3)\lambda$ .....(i)

and $4-x=-6\lambda$ .....(ii)

$4=-12\lambda \Rightarrow \lambda=\frac{-1}{3}$

By (i), $y=-3$ and by (ii), $x=2$; $\therefore (x,y)=(2,-3)$.

117. (c) $x\mathbf{a}+y\mathbf{b}$ represents a vector coplanar with $\mathbf{a}$ and $\mathbf{b}$.

118. (a) We have $\mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\mathbf{d}$ and $\mathbf{b}+\mathbf{c}+\mathbf{d}=\beta\mathbf{a}$

$\therefore$ $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=(\alpha+1)\mathbf{d}$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=(\beta+1)\mathbf{a}$.

$\Rightarrow (\alpha+1)\mathbf{d}=(\beta+1)\mathbf{a}$

If $\alpha\neq-1$, then $(\alpha+1)\mathbf{d}=(\beta+1)\mathbf{a} \Rightarrow \mathbf{d}=\frac{\beta+1}{\alpha+1}\mathbf{a}$

$\Rightarrow \mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\mathbf{d} \Rightarrow \mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\left(\frac{\beta+1}{\alpha+1}\right)\mathbf{a}$

$\Rightarrow \left(1-\frac{\alpha(\beta+1)}{\alpha+1}\right)\mathbf{a}+\mathbf{b}+\mathbf{c}=0$

$\Rightarrow \mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar which is contradiction to the given condition, $\therefore \alpha=-1$ and so $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=0$.

119. (a) Since $\mathbf{a}$ and $\mathbf{b}$ are collinear, we have $\mathbf{a}=m\mathbf{b}$ for some scalar m.

$\mathbf{i}-\mathbf{j}=m(-2\mathbf{i}+k\mathbf{j})$ $\quad \mathbf{i}-\mathbf{j}=-2m\mathbf{i}+km\mathbf{j}$

$-2m=1, km=-1$

$m=-\frac{1}{2}$, So $k=2$.

Scalar or Dot product of two vectors and its applications

1. (a) Let $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Then $(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}=\mathbf{a}$.

2. (d) Let $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Since $\mathbf{r}.\mathbf{i}=\mathbf{r}.\mathbf{j}=\mathbf{r}.\mathbf{k}$

$\Rightarrow x=y=z$ .....(i)

Also $|\mathbf{r}|=\sqrt{x^2+y^2+z^2}=3 \Rightarrow x=\pm\sqrt{3}$, {By (i)}

Hence the required vector $\mathbf{r}=\pm\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$.

Trick : As the vector $\pm\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ satisfies both the conditions.

3. (c) $\mathbf{a}.\mathbf{b}=\mathbf{a}.\mathbf{c} \Rightarrow \mathbf{a}.\mathbf{b}-\mathbf{a}.\mathbf{c}=0 \Rightarrow \mathbf{a}.(\mathbf{b}-\mathbf{c})=0$

$\Rightarrow$ Either $\mathbf{b}-\mathbf{c}=\mathbf{0}$ or $\mathbf{a}=\mathbf{0} \Rightarrow \mathbf{b}=\mathbf{c}$ or $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$.

4. (b) $\mathbf{a}.\mathbf{b}=-|\mathbf{a}||\mathbf{b}|$, ($\because \cos\theta=-1$).

5. (c) Squaring $(\mathbf{a}+\mathbf{b}+\mathbf{c})=\mathbf{0}$,

we get $\mathbf{a}^2+\mathbf{b}^2+\mathbf{c}^2+2\mathbf{a}.\mathbf{b}+2\mathbf{b}.\mathbf{c}+2\mathbf{c}.\mathbf{a}=0$

$|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=0$

$2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=-3 \Rightarrow \mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}=-\frac{3}{2}$.

6. (c) Since $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are mutually perpendicular, so $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$

Angle between $\mathbf{a}$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}$ is

$\cos\theta=\frac{\mathbf{a}.(\mathbf{a}+\mathbf{b}+\mathbf{c})}{|\mathbf{a}||\mathbf{a}+\mathbf{b}+\mathbf{c}|}$ .....(i)

Now $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=a$

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=\mathbf{a}^2+\mathbf{b}^2+\mathbf{c}^2+2\mathbf{a}.\mathbf{b}+2\mathbf{b}.\mathbf{c}+2\mathbf{c}.\mathbf{a}$

$=a^2+a^2+a^2+0+0+0$

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=3a^2 \Rightarrow |\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{3}a$

Putting this value in (i), we get $\theta=\cos^{-1}\frac{1}{\sqrt{3}}$.

7. (a) Three mutually perpendicular unit vectors $=\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Therefore $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1$ and $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$.

We know that

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=(\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}+\mathbf{b}+\mathbf{c})=|\mathbf{a}|^2+|\mathbf{b}|^2$

$+|\mathbf{c}|^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=1+1+1+0=3$

or $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{3}$.

8. (c) $\mathbf{a}+\mathbf{b}=\mathbf{c} \Rightarrow |\mathbf{a}|^2+|\mathbf{b}|^2+2\mathbf{a}.\mathbf{b}=|\mathbf{c}|^2$

and $|\mathbf{a}|+|\mathbf{b}|=|\mathbf{c}| \Rightarrow |\mathbf{a}|^2+|\mathbf{b}|^2+2|\mathbf{a}||\mathbf{b}|=|\mathbf{c}|^2$

$\therefore \mathbf{a}.\mathbf{b}=|\mathbf{a}||\mathbf{b}| \Rightarrow \cos\theta=1$ $\quad \theta=0$.

9. (d) Since $\mathbf{a}\perp\mathbf{b} \Rightarrow \mathbf{a}.\mathbf{b}=0$

$|\mathbf{a}-\mathbf{b}|^2=(\mathbf{a}-\mathbf{b})^2=a^2+b^2-2\mathbf{a}.\mathbf{b}=25+25$

$\Rightarrow |\mathbf{a}-\mathbf{b}|=5\sqrt{2}$.

10. (b) $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b}) = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}.\mathbf{b}$

or $|\mathbf{a}+\mathbf{b}|^2 = 2.2\cos^2\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|\mathbf{a}+\mathbf{b}|$.

11. (d) $\mathbf{a}+\mathbf{b} = -\mathbf{c} \Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}||\mathbf{b}|\cos\theta = |\mathbf{c}|^2$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$.

12. (a) $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}-\mathbf{b}|$

Squaring both sides, we get

$a^2 + b^2 + 2\mathbf{a}.\mathbf{b} > a^2 + b^2 - 2\mathbf{a}.\mathbf{b}$

$4\mathbf{a}.\mathbf{b} > 0$ $\quad \cos\theta > 0$. Hence $\theta < 90^\circ$, (acute).

13. (a) Given that $\mathbf{a} = \mathbf{b}+\mathbf{c}$ and angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{\pi}{2}$.

So, $\quad \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 2\mathbf{b}.\mathbf{c}$

or $\quad \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 2|\mathbf{b}||\mathbf{c}|\cos\frac{\pi}{2}$

or $\quad \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 0, \quad \therefore \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2$

i.e., $a^2 = b^2 + c^2$.

14. (d) Obviously $\mathbf{a}, \mathbf{b}$ are unit vectors.

15. (d) Angle between $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}$ is equal to

$\cos^{-1}\left\{\frac{(\mathbf{i}+\mathbf{j}+\mathbf{k}).\mathbf{i}}{|\mathbf{i}+\mathbf{j}+\mathbf{k}||\mathbf{i}|}\right\} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Similarly angle between $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{j}$ is $\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and between $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{k}$ is $\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$. Hence $\alpha = \beta = \gamma$.

16. (b) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow \mathbf{r}.\mathbf{i} = x, \ \mathbf{r}.\mathbf{j} = y, \ \mathbf{r}.\mathbf{k} = z$

$\Rightarrow (\mathbf{r}.\mathbf{i})^2 + (\mathbf{r}.\mathbf{j})^2 + (\mathbf{r}.\mathbf{k})^2 = x^2 + y^2 + z^2 = r^2$.

17. (d) Parallel vector $= (2+b)\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$

Unit vector $= \frac{(2+b)\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}}{\sqrt{b^2 + 4b + 44}}$

According to the condition, $1 = \frac{(2+b)+6-2}{\sqrt{b^2+4b+44}}$

$\Rightarrow b^2 + 4b + 44 = b^2 + 12b + 36 \Rightarrow 8b = 8 \Rightarrow b = 1$.

18. (b) Let unit vector be $y\mathbf{i} + z\mathbf{k}$, then $\sqrt{y^2+z^2} = 1$ .....(i)

Since given that $\cos 30^\circ = \frac{(y\mathbf{j}+z\mathbf{k}).(y\mathbf{j})}{|y\mathbf{j}+z\mathbf{k}||y\mathbf{j}|}$

$\Rightarrow \frac{y^2}{\left(\sqrt{y^2+z^2}\right)y} = \frac{\sqrt{3}}{2} \Rightarrow y = \frac{\sqrt{3}}{2}$,

$(\because \sqrt{y^2+z^2} = 1$ by (i))

Similarly, $\cos 60^\circ = \frac{(y\mathbf{j}+z\mathbf{k}).z\mathbf{k}}{|y\mathbf{j}+z\mathbf{k}||z\mathbf{k}|} \Rightarrow z = \frac{1}{2}$

Hence the components of unit vector are $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$.

Trick : Since the vector lies in $yz-$ plane, so it will be either $0\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$ or $0\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\sqrt{3}}{2}\mathbf{k}$.

But the vector $\frac{\sqrt{3}}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$ makes angle 30° with $y-$axis and that of 60° with z-axis.

19. (c) $\Sigma\mathbf{F} = 2\mathbf{j} - \mathbf{k}$, $\quad \overrightarrow{AB} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$,

$\Sigma\mathbf{F}.\overrightarrow{AB} = 8 + 1 = 9$.

20. (c) $\mathbf{a}.\mathbf{b} = aa\cos 120^\circ$, $\quad \{\because |\mathbf{a}| = |\mathbf{b}| = a(\text{say})\}$

$\Rightarrow -8 = -\frac{a^2}{2} \Rightarrow a = 4$

(Negative sign does not occur in moduli).

21. (b) $|4\mathbf{a}+3\mathbf{b}| = \sqrt{(4\mathbf{a}+3\mathbf{b}).(4\mathbf{a}+3\mathbf{b})}$

$= \sqrt{16|\mathbf{a}|^2 + 9|\mathbf{b}|^2 + 24\mathbf{a}.\mathbf{b}}$

$= \sqrt{144 + 144 + 24 \times 3 \times 4 \times \left(\frac{-1}{2}\right)} = 12$.

22. (d) Let the required vector be $\alpha = d_1\mathbf{i} + d_2\mathbf{j} + d_3\mathbf{k}$, where $d_1^2 + d_2^2 + d_3^2 = 51$, (given) .....(i)

Now, each of the given vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is a unit vector $\cos\theta = \frac{\mathbf{d}.\mathbf{a}}{|\mathbf{d}||\mathbf{a}|} = \frac{\mathbf{d}.\mathbf{b}}{|\mathbf{d}||\mathbf{b}|} = \frac{\mathbf{d}.\mathbf{c}}{|\mathbf{d}||\mathbf{c}|}$

or $\mathbf{d}.\mathbf{a} = \mathbf{d}.\mathbf{b} = \mathbf{d}.\mathbf{c}$

$|\mathbf{d}| = \sqrt{51}$ cancels out and $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

Hence,

$\frac{1}{3}(d_1 - 2d_2 + 2d_3) = \frac{1}{5}(-4d_1 + 0d_2 - 3d_3) = d_2$

$\Rightarrow d_1 - 5d_2 + 2d_3 = 0$ and $4d_1 + 5d_2 + 3d_3 = 0$

On solving, $\frac{d_1}{5} = \frac{d_2}{-1} = \frac{d_3}{-5} = \lambda$ (say)

Putting d_1, d_2 and d_3 in (i), we get $\lambda = \pm 1$

Hence the required vectors are $\pm(5\mathbf{i} - \mathbf{j} - 5\mathbf{k})$.

Trick : Check it with the options.

23. (b) Since $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar, therefore there exists (x, y, z not all zero) such that

$x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = 0$ .....(i)

Multiply be $\mathbf{a}$ scalarly, we get

$x(\mathbf{a}.\mathbf{a}) + (\mathbf{a}.\mathbf{b}) + z(\mathbf{a}.\mathbf{c}) = 0$ ......(ii)

and $x(\mathbf{a}.\mathbf{b}) + y(\mathbf{b}.\mathbf{b}) + z(\mathbf{b}.\mathbf{c}) = 0$ .....(iii)

Eliminating x, y and z from (i), (ii) and (iii), we get $\begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{b} & \mathbf{a}.\mathbf{c} \\ \mathbf{a}.\mathbf{b} & \mathbf{b}.\mathbf{b} & \mathbf{b}.\mathbf{c} \end{vmatrix} = 0$.

Note: Students should remember this question as a formula.

24. (c) It is obvious.

25. (c) $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q} \Rightarrow \mathbf{r}.\mathbf{q} = \mathbf{p}.\mathbf{q} + \lambda\mathbf{q}.\mathbf{q}$

$\Rightarrow 0 = 7 + 14\lambda \Rightarrow \lambda = -\frac{1}{2}$

Therefore, $\mathbf{r} = -\frac{1}{2}(\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$.

26. (a) $\mathbf{d}.\mathbf{c} = \lambda(\mathbf{a}\times\mathbf{b}).\mathbf{c} + \mu(\mathbf{b}\times\mathbf{c}).\mathbf{c} + \nu(\mathbf{c}\times\mathbf{a}).\mathbf{c}$

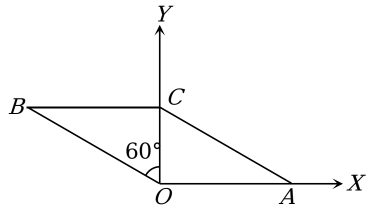
$= \lambda[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] + 0 + 0 = \lambda[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \frac{\lambda}{8}$

Hence $\lambda = 8(\mathbf{d}.\mathbf{c})$, $\mu = 8(\mathbf{d}.\mathbf{a})$ and $\nu = 8(\mathbf{d}.\mathbf{b})$

Therefore, $\lambda + \mu + \nu = 8\mathbf{d}.\mathbf{c} + 8\mathbf{d}.\mathbf{a} + 8\mathbf{d}.\mathbf{b}$

$= 8\mathbf{d}.(\mathbf{a} + \mathbf{b} + \mathbf{c})$.

27. (c) Let $\overrightarrow{OA} = P_1\mathbf{i}$, $\overrightarrow{CB} = -P_1\mathbf{i}$, $\overrightarrow{OB} = -P_1\mathbf{i} + P\mathbf{j}$



$\frac{\overrightarrow{OB}.\mathbf{j}}{OB} = \cos 60^\circ \Rightarrow \frac{(-P_1\mathbf{i} + P\mathbf{j}).\mathbf{j}}{\sqrt{P_1^2 + P^2}} = \frac{1}{2}$

$\Rightarrow 2P = \sqrt{P^2 + P_1^2} \Rightarrow P_1 = P\sqrt{3}$

$|\overrightarrow{OB}| = \sqrt{P^2 + P_1^2} = \sqrt{P^2 + 3P^2} = 2P$.

28. (d) It is obvious, since $\mathbf{a}.\mathbf{b} = 0$.

Hence $(\mathbf{a} + \mathbf{b})^2 = a^2 + b^2 = (\mathbf{a} - \mathbf{b})^2$.

29. (a) It is a fundamental concept.

30. (a) $(\mathbf{a} + \mathbf{b} + \mathbf{c})^2 = 0$

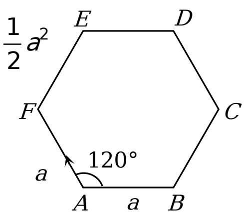
$|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2\mathbf{a}.\mathbf{b} + 2\mathbf{b}.\mathbf{c} + 2\mathbf{c}.\mathbf{a} = 0$

$9 + 1 + 16 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0$

$\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = -\frac{26}{2} = -13$.

31. (d) $\overrightarrow{AB}.\overrightarrow{AF} = |\mathbf{a}||\mathbf{a}|\cos 120^\circ = \frac{-1}{2}a^2$ and

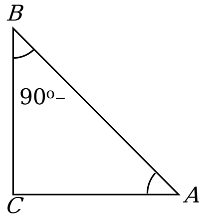
$\frac{1}{2}\overrightarrow{BC}^2 = \frac{1}{2}a^2$



Therefore, $\overrightarrow{AB}.\overrightarrow{AF} + \frac{1}{2}\overrightarrow{BC}^2 = \frac{1}{2}a^2 - \frac{1}{2}a^2 = 0$.

32. (c) We have $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$

$(AB)(AC)\cos\theta + (BC)(BA)\cos(90^o - \theta) + 0$



$= AB(AC\cos\theta + BC\sin\theta) = AB\left(\frac{(AC)^2}{AB} + \frac{(BC)^2}{AB}\right)$

$= AC^2 + BC^2 = AB^2 = p^2$.

33. (d) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$ or $\overrightarrow{CA} = -(\mathbf{a} + \mathbf{b})$

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{b} + \mathbf{c}$

Therefore, $\overrightarrow{AB}.\overrightarrow{CD} + \overrightarrow{BC}.\overrightarrow{AD} + \overrightarrow{CA}.\overrightarrow{BD}$

$= \mathbf{a}.\mathbf{c} + \mathbf{b}.(\mathbf{a} + \mathbf{b} + \mathbf{c}) + (-\mathbf{a} - \mathbf{b}).(\mathbf{b} + \mathbf{c})$

$= \mathbf{a}.\mathbf{c} + \mathbf{b}.\mathbf{a} + \mathbf{b}.\mathbf{b} + \mathbf{b}.\mathbf{c} - \mathbf{a}.\mathbf{b} - \mathbf{a}.\mathbf{c} - \mathbf{b}.\mathbf{b} - \mathbf{b}.\mathbf{c} = 0$.

34. (a) Let $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$, then $\mathbf{a}.\mathbf{b} = 0$

$\Rightarrow 4x - 3y = 0 \Rightarrow \frac{x}{3} = \frac{y}{4} \Rightarrow x = 3\lambda,\ y = 4\lambda,\ \lambda \in R$.

Now $|\mathbf{a}| = |\mathbf{b}| \Rightarrow x^2 + y^2 = 16 + 9 + 25$

$= 9\lambda^2 + 16\lambda^2 = 50$

$\Rightarrow \lambda = \pm\sqrt{2} \Rightarrow x = \pm 3\sqrt{2},\ y = \pm 4\sqrt{2}$

Hence, $\mathbf{a} = \pm\sqrt{2}(3\mathbf{i} + 4\mathbf{j})$.

35. (a) Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, then $\mathbf{a}.\mathbf{i} = a_1$, $\mathbf{a}.\mathbf{j} = a_2$, $\mathbf{a}.\mathbf{k} = a_3$

$\therefore\ \mathbf{a} = (\mathbf{a}.\mathbf{i})\mathbf{i} + (\mathbf{a}.\mathbf{j})\mathbf{j} + (\mathbf{a}.\mathbf{k})\mathbf{k}$.

36. (a) $(\mathbf{b} - \mathbf{a}).\left(\mathbf{c} - \frac{\mathbf{a} + \mathbf{b}}{2}\right) = \mathbf{b}.\mathbf{c} - \mathbf{b}.\left(\frac{\mathbf{a} + \mathbf{b}}{2}\right) - \mathbf{a}.\mathbf{c} + \frac{\mathbf{a}}{2}(\mathbf{a} + \mathbf{b})$

and $|\mathbf{a} - \mathbf{c}| = |\mathbf{b} - \mathbf{c}| \Rightarrow |\mathbf{a} - \mathbf{c}|^2 = |\mathbf{b} - \mathbf{c}|^2$

$\mathbf{a} + \mathbf{b} = 2\mathbf{c}$

Therefore, $(\mathbf{b} - \mathbf{a}).\left(\mathbf{c} - \frac{\mathbf{a} + \mathbf{b}}{2}\right) = 0$.

37. (d) It is obvious.

38. (a) $\mathbf{a} = (1, -1, 2)$, $\mathbf{b} = (-2, 3, 5)$, $\mathbf{c} = (2, -2, 4)$

So, $\mathbf{a} = (1,-1,2) \equiv \mathbf{i} - \mathbf{j} + 2\mathbf{k}$; $\mathbf{b} = (-2,3,5) \equiv -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

and $\mathbf{c} = (2,-2,4) \equiv 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

$\Rightarrow \mathbf{a} - 2\mathbf{b} + 3\mathbf{c} = (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) - 2(-2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$
$+ 3(2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$

$= 11\mathbf{i} - 13\mathbf{j} + 4\mathbf{k}$ and $(\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}).\mathbf{i} = 11$.

39. (a) It is obvious.

40. (d) $\because\ \mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

Squaring both sides, we get

$|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0$

$\Rightarrow 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = -(9 + 16 + 25)$

$\Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = -25$.

41. (c) $\because$ $\mathbf{a}.\mathbf{a} = |\mathbf{a}||\mathbf{a}|\cos\theta = |\mathbf{a}|^2$, $(\because \theta = 0^o)$

and $\mathbf{b}.\mathbf{b} = |\mathbf{b}||\mathbf{b}|\cos\theta = |\mathbf{b}|^2$, (Here $\theta = 0^o$)

Also, since $\mathbf{a}$ and $\mathbf{b}$ are sides of rhombus

$|\mathbf{a}| = |\mathbf{b}|$. Hence $\mathbf{a}.\mathbf{a} = \mathbf{b}.\mathbf{b}$.

42. (c) $|\mathbf{x}-\mathbf{y}|^2 = (\mathbf{x}-\mathbf{y}).(\mathbf{x}-\mathbf{y}) = 1+1-2|\mathbf{x}||\mathbf{y}|\cos\pi$

$= 2 - 2\cos\pi$, $\therefore |\mathbf{x}-\mathbf{y}|^2 = 4$

So, $\frac{1}{2}|\mathbf{x}-\mathbf{y}| = 1$, $[\because |\mathbf{x}|^2 = |\mathbf{y}|^2 = 1, |\mathbf{x}| = |\mathbf{y}| = 1]$.

43. (a) Let $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

Then $\mathbf{a}.\mathbf{i} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}).\mathbf{i} = x$ and $\mathbf{a}.(\mathbf{i}+\mathbf{j}) = x + y$

and $\mathbf{a}.(\mathbf{i}+\mathbf{j}+\mathbf{k}) = x + y + z$

$\because$ Given that $x = x + y = x + y + z$

Now $x = x + y \Rightarrow y = 0$ and $x + y = x + y + z \Rightarrow z = 0$

Hence $x = 1$; $\therefore \mathbf{a} = \mathbf{i}$.

44. (b) It is obvious.

45. (c) $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b}) = \mathbf{a}.\mathbf{a} + \mathbf{b}.\mathbf{a} - \mathbf{b}.\mathbf{a} - \mathbf{b}.\mathbf{b}$

$= \mathbf{a}.\mathbf{a} - \mathbf{b}.\mathbf{b} = |\mathbf{a}|^2 - |\mathbf{b}|^2$

$= 0$, $(\because |\mathbf{a}| = |\mathbf{b}|)$.

46. (b) $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$ $\quad (\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}+\mathbf{b}+\mathbf{c}) = \mathbf{0}$

$|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0$

$\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = \frac{-1-4-9}{2} = -7.$

47. (b) Let the vector be given as $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. For this vector to be coplanar with $\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+\mathbf{k}$, we will have

$a\mathbf{i} + b\mathbf{j} + c\mathbf{k} = p(\mathbf{i}+\mathbf{j}+2\mathbf{k}) + r(\mathbf{i}+2\mathbf{j}+\mathbf{k})$

This gives, $a = p + r$ .....(i)

$b = p + 2r$ .....(ii)

$c = 2p + r$ .....(iii)

For the vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ to be perpendicular to $\mathbf{i}+\mathbf{j}+\mathbf{k}$, we will have

$(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(\mathbf{i}+\mathbf{j}+\mathbf{k}) = 0$

$\Rightarrow a + b + c = 0$ ......(iv)

Adding equation (i) to (iii), we get $4p + 4r = a + b + c$

$\Rightarrow 4(p + r) = 0 \Rightarrow p = -r$

Now with the help of (i), (ii) and (iii), we get

$a = 0$, $b = r$, $c = p = -r$

Hence the required vector is $r(\mathbf{j}-\mathbf{k})$

To be its unit vector $r^2 + r^2 = 1 \Rightarrow r = \pm\frac{1}{\sqrt{2}}$

Hence the required unit vector is, $\pm\frac{1}{\sqrt{2}}(\mathbf{j}-\mathbf{k})$.

Trick : Check for option (a) $\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$ is a unit vector and perpendicular to $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

But $\begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -\frac{4}{\sqrt{2}} \neq 0$.

So it is not coplanar with the given vector.

Check for option (b), $\pm\left(\frac{\mathbf{j}-\mathbf{k}}{\sqrt{2}}\right)$ is a unit vector and also perpendicular to $\mathbf{i}+\mathbf{j}+\mathbf{k}$,

$\begin{vmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$.

So, it is also coplanar with the given vectors.

48. (b) Since $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$, then their product will be zero.

So, $(\mathbf{a}+\lambda\mathbf{b}).(\mathbf{a}-\lambda\mathbf{b}) = 0$ $\quad |\mathbf{a}|^2 - \lambda^2|\mathbf{b}|^2 = 0$

or $\lambda^2 = \frac{|\mathbf{a}|^2}{|\mathbf{b}|^2} \Rightarrow \lambda^2 = \frac{9}{16}$ or $\lambda = \pm\frac{3}{4}$, $[\because |\mathbf{a}| = 3, |\mathbf{b}| = 4]$.

49. (b) Here $|\mathbf{a}| = 4$; $|\mathbf{b}| = 4$; $|\mathbf{c}| = 2$

and $\mathbf{a}.(\mathbf{b}+\mathbf{c}) = 0 \Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{a}.\mathbf{c} = 0$ .....(i)

$\mathbf{b}.(\mathbf{c}+\mathbf{a}) = 0 \Rightarrow \mathbf{b}.\mathbf{c} + \mathbf{b}.\mathbf{a} = 0$ .....(ii)

$\mathbf{c}.(\mathbf{a}+\mathbf{b}) = 0 \Rightarrow \mathbf{c}.\mathbf{a} + \mathbf{c}.\mathbf{b} = 0$ .....(iii)

Adding (i), (ii) and (iii), we get,

$2[\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}] = 0$

$|\mathbf{a}+\mathbf{b}+\mathbf{c}| = \sqrt{|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a})}$

$= \sqrt{|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2} = \sqrt{16+16+4}$

$|\mathbf{a}+\mathbf{b}+\mathbf{c}| = 6$.

50. (b) $\cos\theta = \frac{3(2)+(1)(-2)+2(4)}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{14}\sqrt{24}} = \frac{6}{\sqrt{14}\sqrt{6}}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{\sqrt{7}} \Rightarrow \sin\theta = \frac{2}{\sqrt{7}}$ $\quad \theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

51. (d) $\overrightarrow{AB} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\overrightarrow{CD} = -2\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$

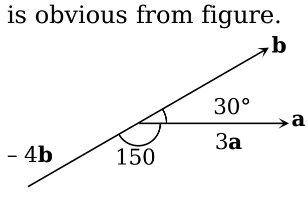
$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}|.|\overrightarrow{CD}|} = \frac{-2-32-2}{\sqrt{18}.\sqrt{72}}$

$= \frac{-2-32-2}{2\times 18} = -1 \Rightarrow \theta = \pi$.

52. (b) $(\mathbf{a}-\sqrt{2}\mathbf{b})^2 = 1 \Rightarrow 1 + 2 - 2\sqrt{2}\mathbf{a}.\mathbf{b} = 1$

$\Rightarrow \mathbf{a}.\mathbf{b} = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$.

53. (a) It is obvious from figure.



54. (d) $(\mathbf{i}-\mathbf{j}+\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})=\sqrt{3}\sqrt{6}\cos\theta$

$\Rightarrow \cos\theta=\frac{0}{\sqrt{3}\sqrt{6}} \Rightarrow \theta=\frac{\pi}{2}$.

55. (d) Here $\overrightarrow{AB}=-3\mathbf{i}+6\mathbf{j}-3\mathbf{k}$, $\overrightarrow{BC}=-2\mathbf{i}+\mathbf{j}+4\mathbf{k}$ and $\overrightarrow{AB}.\overrightarrow{BC}=6+6-12=0 \Rightarrow \angle ABC=90^\circ$.

56. (b) According to the given conditions, $\mathbf{a}.\mathbf{b}>0$ and $\mathbf{b}.\mathbf{c}<0$, where $\mathbf{c}=(0,1,0)$.

$\Rightarrow 2x^2-3x+1>0$ and $x<0$. Hence the result.

57. (b) $(\mathbf{a}-c\mathbf{b})^2=1=2-2\cos \Rightarrow \theta=60^\circ$.

58. (c) As we know $\mathbf{a}.\mathbf{b}=ab\cos\theta$

Obviously, $\cos\theta\ge 0$ for $0\le\theta\le\frac{\pi}{2}$ and $\frac{3\pi}{2}\le\theta\le 2\pi$.

59. (c) We have $\mathbf{a}+\mathbf{b}=4\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{a}-\mathbf{b}=-2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$.

Clearly $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})=0$. Hence $(\mathbf{a}+\mathbf{b})\perp(\mathbf{a}-\mathbf{b})$.

60. (b) For acute angle $\mathbf{a}.\mathbf{b}>0$

i.e., $-3x+2x^2+1>0 \Rightarrow (x-1)(2x-1)>0$

For obtuse angle between $\mathbf{b}$ and x-axis $\mathbf{b}.\mathbf{i}<0$

$x<0$.

61. (c) $\theta=\cos^{-1}\left(\frac{2\times 12+6\times(-4)+3(3)}{\sqrt{2^2+6^2+3^2}\sqrt{12^2+4^2+3^2}}\right)$

$=\cos^{-1}\left(\frac{9}{7\times 13}\right)=\cos^{-1}\left(\frac{9}{91}\right)$.

62. (d) $\cos\frac{\pi}{3}=\frac{1+a}{\sqrt{2}\sqrt{2+a^2}} \Rightarrow a=0$.

63. (c) Given, $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0} \Rightarrow \mathbf{a}+\mathbf{b}=-\mathbf{c}$

Squaring on both sides,

$|\mathbf{a}|^2+|\mathbf{b}|^2+2|\mathbf{a}||\mathbf{b}|\cos\theta=|-\mathbf{c}|^2$

$\Rightarrow 9+25+30\cos\theta=49 \qquad \cos\theta=\frac{1}{2} \Rightarrow \theta=60^\circ$.

64. (d) Given condition is $\mathbf{a}+\mathbf{b}=\mathbf{c}$.

Using dot product, $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\mathbf{c}.\mathbf{c}$

$\Rightarrow \mathbf{a}.\mathbf{a}+\mathbf{b}.\mathbf{b}+2\mathbf{a}.\mathbf{b}=\mathbf{c}.\mathbf{c}$

$\Rightarrow |\mathbf{a}|.|\mathbf{a}|\cos 0^\circ+|\mathbf{b}|.|\mathbf{b}|\cos 0^\circ+2|\mathbf{a}|.|\mathbf{b}|\cos\alpha$

$=|\mathbf{c}|.|\mathbf{c}|\cos 0^\circ$, $(\because |\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1)$

$\Rightarrow 1+1+2\cos\alpha=1 \Rightarrow \cos\alpha=-\frac{1}{2} \Rightarrow \alpha=\frac{2\pi}{3}$.

65. (b) Let $|\mathbf{a}|=1$ and $|\mathbf{b}|=1$

Also $|\mathbf{a}+\mathbf{b}|^2=1^2 \Rightarrow 1+1+2\cos\theta=1 \Rightarrow \theta=120^\circ$

$\therefore |\mathbf{a}-\mathbf{b}|^2=1+1-2\cos\theta=3 \Rightarrow |\mathbf{a}-\mathbf{b}|=\sqrt{3}$.

66. (a) Let $\mathbf{a}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{j}+\mathbf{k}$

Since $\cos\theta=\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

$=\frac{(2\mathbf{i}+3\mathbf{j}+\mathbf{k}).(2\mathbf{i}-\mathbf{j}-\mathbf{k})}{\sqrt{(2)^2+(3)^2+(1)^2}\sqrt{(2)^2+(-1)^2+(-1)^2}}$

$=\frac{4-3-1}{\sqrt{(4+9+1)}\sqrt{(4+1+1)}}=0$

$\therefore \theta=\frac{\pi}{2}$.

67. (a) $\cos\theta=\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}=\frac{12-6-2}{\sqrt{4+4+1}\sqrt{36+9+4}}=\frac{4}{21}$.

68. (b) $(\mathbf{a}+2\mathbf{b}).(5\mathbf{a}-4\mathbf{b})=0$ or $5\mathbf{a}^2+6\mathbf{a}.\mathbf{b}-8\mathbf{b}^2=0$

or $6\mathbf{a}.\mathbf{b}=3$, $(\because \mathbf{a}^2=1, \mathbf{b}^2=1)$

$\therefore \mathbf{a}.\mathbf{b}=\frac{1}{2}$ or $|\mathbf{a}||\mathbf{b}|\cos\theta=\frac{1}{2}$

$\therefore \cos\theta=\frac{1}{2}, \quad \therefore \theta=60^\circ$.

69. (a) $|\mathbf{a}-\mathbf{b}|=\sqrt{1^2+1^2-2.1^2\cos\theta}=\sqrt{2(1-\cos\theta)}$

$=\sqrt{2}\times\sqrt{2}\sin\frac{\theta}{2}=2\sin\frac{\theta}{2} \Rightarrow \sin\frac{\theta}{2}=\frac{|\mathbf{a}-\mathbf{b}|}{2}$.

70. (a) $\mathbf{a}=(1,1,4)=\mathbf{i}+\mathbf{j}+4\mathbf{k}$, $\mathbf{b}=(1,-1,4)=\mathbf{i}-\mathbf{j}+4\mathbf{k}$

$\mathbf{a}+\mathbf{b}=2\mathbf{i}+8\mathbf{k} \qquad \mathbf{a}-\mathbf{b}=2\mathbf{j}$

Since, $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})=0$

$(\mathbf{a}+\mathbf{b})\perp(\mathbf{a}-\mathbf{b})$. Hence $\theta=90^\circ$.

71. (a) Let the vector is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Now according to the conditions,

$\sqrt{x^2+y^2+z^2}=3 \Rightarrow x^2+y^2+z^2=9$ .....(i)

$6x+5y-2z=0$ .....(ii)

and $3x+y-4z=0$ .....(iii)

[$\because$ it is perpendicular to both vectors, hence by $a_1b_1+a_2b_2+a_3b_3=0$]

On solving the equation (i), (ii) and (iii), we get $x=2$, $y=-2$ and $z=1$.

Therefore, the required vector is $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$.

Trick : By inspection, the vector $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ is of length 3 and also perpendicular to the given vectors.

72. (b) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$; Squaring both sides, we get $4\mathbf{a}.\mathbf{b}=0$

$\Rightarrow \mathbf{a}$ is perpendicular to $\mathbf{b}$.

73. (d) $\mathbf{a}.\mathbf{b}=0=4-a-1 \Rightarrow a=3$.

74. (d) $\mathbf{a}+t\mathbf{b}=2\mathbf{i}+2\mathbf{j}+3\mathbf{k}+(-t\mathbf{i}+2t\mathbf{j}+t\mathbf{k})$

$=(2-t)\mathbf{i}+(2+2t)\mathbf{j}+(3+t)\mathbf{k}$

Given that it is perpendicular to $\mathbf{c}=3\mathbf{i}+\mathbf{j}$

Hence $(2-t)3+(2+2t)1+(3+t)0=0$

$\Rightarrow 6-3t+2+2t=0 \Rightarrow t=8$.

75. (c) $\mathbf{a}.\mathbf{b}=(2-4-\lambda)=0 \Rightarrow \lambda=-2$.

76. (b) To be perpendicular, $2a+3b-4c=0$ and option(b) satisfies this equation.

77. (b) $x^2+y^2=1$

Let vector be $x\mathbf{i}+y\mathbf{j}$, then $4x-3y=0$

$\Rightarrow 4x=3y \Rightarrow x=\frac{3}{5}, y=\frac{4}{5}$,

Hence the required vector is $\frac{1}{5}(3\mathbf{i}+4\mathbf{j})$.

78. (c) $l\mathbf{a}+m\mathbf{b}+n\mathbf{c}=\mathbf{0}$

Squaring both sides, we get

$a^2l^2+m^2b^2+n^2c^2+2lm\mathbf{a}.\mathbf{b}+2ln\mathbf{a}.\mathbf{c}+2mn\mathbf{b}.\mathbf{c}=0$

But $\mathbf{a},\mathbf{b},\mathbf{c}$ are mutually perpendicular

So, $\mathbf{a}.\mathbf{b}$, $\mathbf{b}.\mathbf{c}$ and $\mathbf{c}.\mathbf{a}$ are equal to zero.

Therefore, $a^2l^2+m^2b^2+n^2c^2=0$ *i.e.*, l, m, n are equal to zero because a^2, b^2 and c^2 cannot be equal to zero.

79. (b) $\vec{L}=\mathbf{i}+4\mathbf{j}$

Therefore, vector perpendicular to $\vec{L}=\lambda(4\mathbf{i}-\mathbf{j})$

Unit vector is $\frac{4\mathbf{i}-\mathbf{j}}{\sqrt{17}}$.

But it points towards origin

$\therefore$ Required vector $=\frac{-4\mathbf{i}+\mathbf{j}}{\sqrt{17}}$.

80. (a) Obviously, $3a-12-15=0 \Rightarrow a=9$.

81. (a) Since $(2\lambda\mathbf{i}+\mathbf{j}-\mathbf{k}).(2\mathbf{j}+\mathbf{k})=1$ for every λ, therefore the vectors will not be perpendicular for any λ.

82. (c) It is obvious.

83. (c) Clearly, $8\times 2-3\times 4+2\times\lambda=0 \Rightarrow \lambda=-2$.

84. (a, c, d) Check it with the options.

85. (d) Accordingly, $a\times(-1)+2(5)+3(a)=0 \Rightarrow a=-5$.

86. (d) It is obvious.

87. (d) $\frac{1}{2}=\frac{-2}{\lambda} \Rightarrow \lambda=-4$.

88. (a) We know that as the vectors are perpendicular, therefore their dot product is zero

or $(a\mathbf{i}+6\mathbf{j}-\mathbf{k}).(7\mathbf{i}-3\mathbf{j}+17\mathbf{k})=0$

or $7a-18-17=0$ or $7a=35$ or $a=5$.

89. (c) Since angle between the vectors is 90^o, therefore dot product of the vectors will be zero

or $(4\mathbf{i}+\mathbf{j}-\mathbf{k}).(3\mathbf{i}+m\mathbf{j}+2\mathbf{k})=0$

or $12+m-2=0$ or $m=-10$.

90. (c) Let $\mathbf{a}=3\mathbf{i}+\lambda\mathbf{j}+\mathbf{k}$, $\mathbf{b}=2\mathbf{i}-\mathbf{j}+8\mathbf{k}$

$\because \mathbf{a}\perp\mathbf{b}$, $\mathbf{a}.\mathbf{b}=0$

$(3\mathbf{i}+\lambda\mathbf{j}+\mathbf{k}).(2\mathbf{i}-\mathbf{j}+8\mathbf{k})=0$

$6-\lambda+8=0 \Rightarrow \lambda=14$.

91. (d) $(\mathbf{b}.\hat{\mathbf{a}})\hat{\mathbf{a}}=\left\{\frac{(\mathbf{a}.\mathbf{b})}{|\mathbf{a}|}\right\}\hat{\mathbf{a}}=\frac{(\mathbf{a}.\mathbf{b})\mathbf{a}}{\mathbf{a}.\mathbf{a}}$.

92. (a) $14\cos 60^o, 14\sin 60^o$ or $7, \frac{14\sqrt{3}}{2}$ or $7, 7\sqrt{3}$.

93. (b) The component of vector $\mathbf{a}$ along $\mathbf{b}$ is $\frac{(\mathbf{a}.\mathbf{b})\mathbf{b}}{|\mathbf{b}|^2}=\frac{18}{25}(3\mathbf{j}+4\mathbf{k})$.

94. (b) $\mathbf{b}_2=\mathbf{b}-\mathbf{b}_1=-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}$ and obviously $\mathbf{b}_2$ is perpendicular to $\mathbf{a}$.

95. (b) $\left[(\mathbf{i}+\mathbf{j}).\frac{(\mathbf{j}+\mathbf{k})}{\sqrt{2}}\right]\frac{(\mathbf{j}+\mathbf{k})}{\sqrt{2}}=\frac{(\mathbf{j}+\mathbf{k})}{2}$.

96. (b) $(2\mathbf{i}+3\mathbf{j}-2\mathbf{k}).\frac{(\mathbf{i}+2\mathbf{j}+3\mathbf{k})}{\sqrt{14}}=\frac{2}{\sqrt{14}}$.

97. (b) Required value $=\frac{\mathbf{b}.\mathbf{a}}{|\mathbf{b}|}\Big/\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}=\frac{|\mathbf{a}|}{|\mathbf{b}|}=\frac{7}{3}$.

98. (c) It is a fundamental concept.

99. (a) Vectors $\mathbf{a}=2\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{b}=5\mathbf{i}-3\mathbf{j}+\mathbf{k}$.

We know that the projection of $\mathbf{b}$ on

$\mathbf{a}=\frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}|}=\frac{(2\mathbf{i}+\mathbf{j}+2\mathbf{k}).(5\mathbf{i}-3\mathbf{j}+\mathbf{k})}{\sqrt{(2)^2+(1)^2+(2)^2}}=\frac{10-3+2}{\sqrt{9}}=\frac{9}{3}=3.$

100. (b) Projection of $\mathbf{a}$ on $\mathbf{b}$

$=|\mathbf{a}|\cos\theta=|\mathbf{a}|\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}=\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

$=\frac{4+8+7}{\sqrt{16+16+49}}=\frac{19}{\sqrt{81}}=\frac{19}{9}$.

101. (a) Projection of the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ along vector $\mathbf{j}=\frac{(\mathbf{i}+\mathbf{j}+\mathbf{k}).\mathbf{j}}{|\mathbf{j}|}=\frac{1}{1}=1$.

102. (a) $|\mathbf{W}|=(\mathbf{i}-3\mathbf{j}+5\mathbf{k}).(-2\mathbf{i}-4\mathbf{j}+\mathbf{k})=-2+12+5=15$ *unit.*

103. (a) $|\mathbf{W}|=(\mathbf{i}+2\mathbf{j}+3\mathbf{k}).(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=1-4+6=3$.

104. (a) Here $\mathbf{F}=2\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, $\mathbf{d}=-2\mathbf{i}-2\mathbf{j}-2\mathbf{k}$

Work done $=\mathbf{F}.\mathbf{d}=-4+6-4=-2$ or 2 *unit.*

105. (c) Resultant force $\mathbf{F}=5\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and displacement $\mathbf{d}=2\mathbf{i}-2\mathbf{j}+10\mathbf{k}$, then

Work done $\mathbf{W}=\mathbf{F}.\mathbf{d}=10-6+20=24$ *unit*

106. (d) Required work done

$=(3\mathbf{i}+2\mathbf{j}-3\mathbf{k}+2\mathbf{i}+4\mathbf{j}+2\mathbf{k}).(5\mathbf{i}+4\mathbf{j}+2\mathbf{k}-\mathbf{i}-2\mathbf{j}-\mathbf{k})$

$=(5\mathbf{i}+6\mathbf{j}-\mathbf{k}).(4\mathbf{i}+2\mathbf{j}+\mathbf{k})=20+12-1=31.$

107. (c)

$W=\mathbf{F}.\mathbf{d}=(3\mathbf{i}+2\mathbf{j}-5\mathbf{k}).(2\mathbf{i}-\mathbf{j}-\mathbf{k})=6-2+5=9$ *unit.*

108. (b) Required work done = (Force vector).(Displacement vector)

Force vector $=5.\left(\frac{2\mathbf{i}-2\mathbf{j}+\mathbf{k}}{|2\mathbf{i}-2\mathbf{j}+\mathbf{k}|}\right)=\frac{5}{3}(2\mathbf{i}-2\mathbf{j}+\mathbf{k})$

Required work done

$=\frac{5}{3}(2\mathbf{i}-2\mathbf{j}+\mathbf{k}).[(5\mathbf{i}+3\mathbf{j}+7\mathbf{k})-(\mathbf{i}+2\mathbf{j}+3\mathbf{k})]$

$= \frac{5}{3}[(2\mathbf{i}-2\mathbf{j}+\mathbf{k}).(4\mathbf{i}+\mathbf{j}+4\mathbf{k})] = \frac{5}{3}[8-2+4] = \frac{50}{3}$ *unit.*

109. (c) $\vec{F}+\vec{F}_1+\vec{F}_2 = 7\mathbf{i}+2\mathbf{j}-4\mathbf{k}$

$\mathbf{d}$ = P.V. of $\vec{B}$ – P.V. of $\vec{A} = 4\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

$W = \mathbf{F}.\mathbf{d} = 28+4+8 = 40$ *unit.*

110. (a) Projection of $x\mathbf{i}-\mathbf{j}+\mathbf{k}$ on $2\mathbf{i}-\mathbf{j}+5\mathbf{k}$

$$= \frac{(x\mathbf{i}-\mathbf{j}+\mathbf{k})(2\mathbf{i}-\mathbf{j}+5\mathbf{k})}{\sqrt{4+1+25}} = \frac{2x+1+5}{\sqrt{30}}$$

But, given $\frac{2x+6}{\sqrt{30}} = \frac{1}{\sqrt{30}}$ $\quad 2x+6=1 \quad x = \frac{-5}{2}$.

111. (b) $\mathbf{x}+\mathbf{y}+\mathbf{z}=0 \Rightarrow \mathbf{x} = -(\mathbf{y}+\mathbf{z})$

$|\mathbf{x}|^2 = (\mathbf{y}+\mathbf{z}).(\mathbf{y}+\mathbf{z})$

$|\mathbf{x}|^2 = |\mathbf{y}|^2 + |\mathbf{z}|^2 + 2\mathbf{y}.\mathbf{z}$

$|\mathbf{x}|^2 = |\mathbf{y}|^2 + |\mathbf{z}|^2 + 2|\mathbf{y}||\mathbf{z}|\cos\theta$

$4 = 4+4+2\times 2\times 2\cos\theta$

$\cos\theta = \frac{-1}{2} \Rightarrow \theta = 120^o$

$$\text{cosec}^2 120^o + \cot^2 120^o = \left(\frac{2}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}+\frac{1}{3} = \frac{5}{3}.$$

112. (c) $\frac{(2\mathbf{i}+\mathbf{j}-3\mathbf{k}).(\mathbf{i}-2\mathbf{j}+\mathbf{k})}{|\mathbf{i}-2\mathbf{i}+\mathbf{k}|} = \frac{2-2-3}{\sqrt{1+4+1}} = \frac{-3}{\sqrt{6}} = -\sqrt{\frac{3}{2}}$.

113. (b) $(3\mathbf{a}-4\mathbf{b})(2\mathbf{a}+5\mathbf{b}) = 6|\mathbf{a}|^2 - 20|\mathbf{b}|^2 + 7\mathbf{a}.\mathbf{b} = 6-20+7\mathbf{a}.\mathbf{b}$

Given, $|\mathbf{a}+\mathbf{b}|^2 = (\sqrt{3})^2$ $\quad |\mathbf{a}|^2+|\mathbf{b}|^2+2\mathbf{a}.\mathbf{b} = 3$

$2\mathbf{a}.\mathbf{b} = 1 \quad \mathbf{a}.\mathbf{b} = \frac{1}{2}$

Therefore, $(3\mathbf{a}-4\mathbf{b}).(2\mathbf{a}+5\mathbf{b})$

$$= 6-20+7\times\frac{1}{2} = -14+\frac{7}{2} = \frac{-21}{2}.$$

114. (d) Position vectors in the plane of vectors $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+2\mathbf{k}$ are $= (\mathbf{i}+2\mathbf{j}+\mathbf{k})-(\mathbf{i}+\mathbf{j}+2\mathbf{k}) = \mathbf{j}-\mathbf{k}$

Unit vector $= \frac{\mathbf{j}-\mathbf{k}}{\sqrt{2}}$

$$\left(\text{Since}\left(\frac{\mathbf{j}-\mathbf{k}}{\sqrt{2}}\right).(2\mathbf{i}+\mathbf{j}+\mathbf{k}) = \frac{-1+1}{\sqrt{2}} = 0\right).$$

115. (d) $\mathbf{a}.(\mathbf{b}+\mathbf{c}) = 0 \Rightarrow \mathbf{a}.\mathbf{b}+\mathbf{c}.\mathbf{a} = 0$ .....(i)

$\mathbf{b}(\mathbf{c}+\mathbf{a}) = 0 \quad \mathbf{b}.\mathbf{c}+\mathbf{a}.\mathbf{b} = 0$ .....(ii)

$\mathbf{c}.(\mathbf{a}+\mathbf{b}) = 0 \quad \mathbf{c}.\mathbf{a}+\mathbf{b}.\mathbf{c} = 0$ .....(iii)

From (i), (ii) and (iii),

$2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}) = 0$

Now, $|\mathbf{a}+\mathbf{b}|^2 |\mathbf{b}+\mathbf{c}|^2 + |\mathbf{c}+\mathbf{a}|^2 = 6^2+8^2+10^2$

$2[|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2] + 2[\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}] = 200$

$2|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2 = 200 \quad |\mathbf{a}+\mathbf{b}+\mathbf{c}| = 10.$

Vector or Cross product of two vectors and its applications

1. (d) It is obvious.

2. (c) $|\mathbf{a}\times\mathbf{b}| = 1 \Rightarrow |\sin\theta| = 1 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$.

3. (b) $(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{b}) = \mathbf{0}$ or $\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a} = \mathbf{0}$.

4. (a) $(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b}) = \mathbf{a}\times\mathbf{a}-\mathbf{b}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{b}$

$= \mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{a} = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{b} = 2(\mathbf{a}\times\mathbf{b})$.

5. (c) Since $\mathbf{a}+\mathbf{b}+\mathbf{c} = \mathbf{0}$

$\Rightarrow \mathbf{a}\times(\mathbf{a}+\mathbf{b}+\mathbf{c}) = \mathbf{0} \Rightarrow \mathbf{a}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c} = \mathbf{0}$

$\Rightarrow \mathbf{a}\times\mathbf{b} = -\mathbf{a}\times\mathbf{c} = \mathbf{c}\times\mathbf{a}$ .....(i)

Similarly, $\mathbf{b}\times(\mathbf{a}+\mathbf{b}+\mathbf{c}) = \mathbf{0}$ $\Rightarrow \mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c}$ .....(ii)

By (i) and (ii), we get $\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c} = \mathbf{c}\times\mathbf{a}$.

6. (c) $|\mathbf{a}\times\mathbf{b}| = (\mathbf{a}.\mathbf{b}) \Rightarrow ab\sin\theta = ab\cos\theta$

$\Rightarrow \tan\theta = \frac{ab}{ab} = 1 \Rightarrow \theta = \frac{\pi}{4}$.

7. (b) $14(\mathbf{a}\times\mathbf{b})+15(\mathbf{b}\times\mathbf{a}) = \mathbf{b}\times\mathbf{a}$.

8. (c) $\mathbf{a}.\mathbf{b} = 0 \Rightarrow \mathbf{a}\perp\mathbf{b}$ or $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$

and $\mathbf{a}\times\mathbf{b} = \mathbf{0} \Rightarrow \mathbf{a}||\mathbf{b}$ or $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$

Hence, either $\mathbf{a}$ or $\mathbf{b}$ is a null vector.

9. (b) Component of $\mathbf{a}$ along $\mathbf{b} = a\cos\theta = \frac{|\mathbf{a}.\mathbf{b}|}{|\mathbf{b}|}$

Similarly component of $\mathbf{a}$ perpendicular to $\mathbf{b}$ $= a\sin\theta = \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}|}$.