16. (d) A, B are independent as $P(A) = P\left(\frac{A}{B}\right)$

$P\left(\frac{A'}{B}\right) = 1 - \frac{1}{4} = \frac{3}{4}$ as A, B are independent

$\Rightarrow A', B$ are independent.

$P\left(\frac{B'}{A'}\right) = P(B') = 1 - \frac{1}{2} = \frac{1}{2}.$

17. (c) Let A be the event that even face turns up and B be the event that it is 2 or 4.

Then

$P(A) = P(2) + P(4) + P(6) = 0.24 + 0.18 + 0.14 = 0.56$

$P(B) = P(2) + P(4) = 0.24 + 0.18 = 0.42$

So, $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.42}{0.56} = 0.75.$

18. (c) $P[B/(A \cup B^c)] = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$

$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$

$= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)} = \frac{0.7 - 0.5}{0.8} = \frac{1}{4}.$

19. (b) We define the following events :

A_1 : Selecting a pair of consecutive letter from the word LONDON.

A_2 : Selecting a pair of consecutive letters from the word CLIFTON.

E : Selecting a pair of letters 'ON'.

Then $P(A_1 \cap E) = \frac{2}{5}$; as there are 5 pairs of consecutive letters out of which 2 are ON.

$P(A_2 \cap E) = \frac{1}{6}$; as there are 6 pairs of consecutive letters of which one is ON.

The required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}.$$

20. (c, d) Since $P(A \cap B) = P(A)P(B)$

It means A and B are independent events so A^c and B^c will also be independent. Hence

$P(A \cup B)^c = P(A^c \cap B^c) = P(A^c)P(B^c)$ (Demorgan's law)

As A is independent of B, hence

$P(A/B) = P(A)$, $\{\because P(A \cap B) = P(B)P(A/B)\}$.

21. (a) Required probability $= \frac{0.1}{0.1 + 0.32} = \frac{0.1}{0.42} = \frac{5}{21}.$

22. (b) $P(A) = \frac{40}{100}$, $P(B) = \frac{25}{100}$ and $P(A \cap B) = \frac{15}{100}$

So $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{15/100}{40/100} = \frac{3}{8}.$

23. (a) Consider the following events :

$A \rightarrow$ Ball drawn is black; $E_1 \rightarrow$ Bag I is chosen;

$E_2 \rightarrow$ Bag II is chosen and $E_3 \rightarrow$ Bag III is chosen.

Then $P(E_1) = (E_2) = P(E_3) = \frac{1}{3}$, $P\left(\frac{A}{E_1}\right) = \frac{3}{5}.$

$P\left(\frac{A}{E_2}\right) = \frac{1}{5}$, $P\left(\frac{A}{E_3}\right) = \frac{7}{10}$

Required probability $= P\left(\frac{E_3}{A}\right)$

$$= \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} = \frac{7}{15}.$$

24. (b) We define the following events :

A_1 : He knows the answer.

A_2 : He does not know the answer.

E: He gets the correct answer.

Then $P(A_1) = \frac{9}{10}$, $P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}$, $P\left(\frac{E}{A_1}\right) = 1$,

$P\left(\frac{E}{A_2}\right) = \frac{1}{4}$

$\therefore$ Required probability

$$= P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)} = \frac{1}{37}.$$

25. (a) $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$

$n(E) = 4$, $n(F) = 4$ and $n(E \cap F) = 3$

$\therefore P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{4/8} = \frac{3}{4}.$

26. (a) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.6} = \frac{5}{6}.$

27. (a, d) $P(E/F) + P(\overline{E}/F) = \frac{P(E \cap F) + P(\overline{E} \cap F)}{P(F)}$

$= \frac{P\{(E \cap F) \cup (\overline{E} \cap F)\}}{P(F)}$

[$\because E \cap F$ and $\overline{E} \cap F$ are disjoint]

$= \frac{P\{(E \cup \overline{E}) \cap F\}}{P(F)} = \frac{P(F)}{P(F)} = 1$

Similarly we can show that (b) and (c) are not true while (d) is true.

$$P\left(\frac{E}{\overline{F}}\right) + P\left(\frac{\overline{E}}{\overline{F}}\right) = \frac{P(E \cap \overline{F})}{P(F)} + \frac{P(\overline{E} \cap \overline{F})}{P(F)} = \frac{P(\overline{F})}{P(\overline{F})} = 1$$

28. (d) $P\left(\frac{B}{A}\right)=\frac{1}{2}\Rightarrow\frac{P(B\cap A)}{P(A)}=\frac{1}{2}\Rightarrow P(B\cap A)=\frac{1}{8}$

$P\left(\frac{A}{B}\right)=\frac{1}{4}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{1}{4}\Rightarrow P(B)=\frac{1}{2}$

$P(A\cap B)=\frac{1}{8}=P(A).P(B)$

$\therefore$ Events A and B are independent.

Now, $P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}=\frac{P(A')P(B)}{P(B)}=\frac{3}{4}$

and $P\left(\frac{B'}{A'}\right)=\frac{P(B'\cap A')}{P(A')}=\frac{P(B')P(A')}{P(A')}=\frac{1}{2}$.

29. (c) $P(E_1)=\frac{4}{52}=\frac{1}{13}$, $P\left(\frac{E_2}{E_1}\right)=\frac{15}{51}=\frac{5}{17}$

$P(E_1\cap E_2)=P(E_1).P\left(\frac{E_2}{E_1}\right)=\frac{1}{13}.\frac{5}{17}=\frac{5}{221}$.

30. (a) We have to find the bounded probability to get sum 15 when 4 appears first. Let the event of getting sum 15 of three thrown number is A and the event of apearing 4 is B. So we have to find $P\left(\frac{A}{B}\right)$.

But $P\left(\frac{A}{B}\right)=\frac{n(A\cap B)}{n(B)}$

When $n(A\cap B)$ and $n(B)$ respectively denote the number of digits in $A\cap B$ and B.

Now $n(B)=36$, because first throw is of 4. So another two throws stop by $6\times 6=36$ types. Three dices have only two throws, which starts from 4 and give sum 15 *i.e.,* (4, 5, 6) and (4, 6, 5).

So, $n(A\cap B)=2$, $n(B)=36$; $\therefore P\left(\frac{A}{B}\right)=\frac{2}{36}=\frac{1}{18}$.

31. (b) Event $(Y=0)$ is $\{00, 01, 09, 10, 20, ...90\}$

Also $(X=9)\cap(Y=0)=09, 90$, we have

$P(Y=0)=\frac{19}{100}$ and $P(X=9)\cap(Y=0)=\frac{2}{100}$

Hence required probability

$=P\{(X=9)/(Y=0)\}=\frac{\{P(X=9)\cap(Y=0)\}}{P(Y=0)}=\frac{2}{19}$.

32. (a) Let E denote the event that a six occurs and A the event that the man reports that it is a '6', we have

$P(E)=\frac{1}{6}$, $P(E')=\frac{5}{6}$, $P(A/E)=\frac{3}{4}$ and $P(A/E')=\frac{1}{4}$

From Baye's theorem,

$$P(E/A)=\frac{P(E).P(A/E)}{P(E).P(A/E)+P(E').P(A/E')}$$

$$=\frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}=\frac{3}{8}.$$

33. (d) Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag B and E that the ball is red.We have to find $P(E_2/E)$.

Since both the bags are equally likely to be selected, we have $P(E_1)=P(E_2)=\frac{1}{2}$

Also $P(E/E_1)=3/5$ and $P(E/E_2)=5/9$.

Hence by Bay's theorem, we have

$$P(E_2/E)=\frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{2}.\frac{5}{9}}{\frac{1}{2}.\frac{3}{5}+\frac{1}{2}.\frac{5}{9}}=\frac{25}{52}.$$

34. (c) Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, then $P(A)=1/2, P(B)=1/2$, $P(E/A)=2/5$ $P(E/B)=4/6=2/3$.

$P(E)=P(A)P(E/A)+P(B)P(E/B)=\frac{1}{2}\cdot\frac{2}{5}+\frac{1}{2}\cdot\frac{2}{3}=\frac{8}{15}$.

35. (c) It is based on Baye's theorem.

Probability of picked bag A $P(A)=\frac{1}{2}$

Probability of picked bag B $P(B)=\frac{1}{2}$

Probability of green ball picked from bag A

$=P(A).P\left(\frac{G}{A}\right)=\frac{1}{2}\times\frac{4}{7}=\frac{2}{7}$

Probability of green ball picked from bag B

$=P(B).P\left(\frac{G}{B}\right)=\frac{1}{2}\times\frac{3}{7}=\frac{3}{14}$

Total probability of green ball $=\frac{2}{7}+\frac{3}{14}=\frac{1}{2}$

Probability of fact that green ball is drawn from bag B

$$P\left(\frac{G}{B}\right)=\frac{P(B)P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right)+P(B)P\left(\frac{G}{B}\right)}=\frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{7}}=\frac{3}{7}.$$

Binomial distribution

1. (d) The required probability

$$= {}^8C_6\left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8 = \frac{37}{256}.$$

2. (d) Let $P(\text{fresh egg}) = \frac{90}{100} = \frac{9}{10} = p$

$P(\text{rotten egg}) = \frac{10}{100} = \frac{1}{10} = q; \quad n = 5, \quad r = 5$

So the probability that none egg is rotten

$$= {}^5C_5\left(\frac{9}{10}\right)^5 \cdot \left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5.$$

3. (b) Required probability $= {}^5C_1\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^4$

{Here strictly one is swimmer}

4. (b) Here P (without defected) $= \frac{8}{10} = \frac{4}{5} = p$

P (defected) $= \frac{2}{10} = \frac{1}{5} = q$ and $n = 2, \quad r = 2$

Hence required probability $= {}^nC_r p^r . q^{n-r}$

$$= {}^2C_2\left(\frac{4}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^0 = \frac{16}{25}.$$

5. (b) Probability that head occurs 6 times $= {}^nC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6}$ and probability that head occurs 8 times $= {}^nC_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{n-8}$

$$\therefore \; {}^nC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6} = {}^nC_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{n-8}$$

${}^nC_6 = {}^nC_8 \qquad (n-6)(n-7) = 56 \Rightarrow n = 14.$

6. (d) Required probability

$$= {}^3C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2 + {}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) + {}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{91}{216}.$$

7. (a) Required probability $= {}^7C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^3.$

8. (a) Probability of coming 'six' in one throw is $\frac{1}{6}$

Hence required probability is given by

$$= {}^4C_4\left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^0 = \frac{1}{1296}.$$

9. (a) Probability of success $(p) = \frac{3}{5} \Rightarrow q = 1 - p = \frac{2}{5}$

Hence the probability of 2 hits in 5 attempts

$$= {}^5C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^3 = \frac{144}{625}.$$

10. (b) Required probability $= {}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = \frac{125}{3888}.$

11. (d) $np = 6$

$npq = 2 \Rightarrow q = \frac{1}{3}, \; p = \frac{2}{3}$ and $n = 9.$

Hence the binomial distribution is $\left(\frac{1}{3} + \frac{2}{3}\right)^9.$

12. (d) Required probability

$$= {}^{10}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = {}^{10}C_4\left(\frac{1}{2}\right)^{10} = {}^{10}C_6\left(\frac{1}{2}\right)^{10}.$$

13. (d) We have mean $(X) = np = 2$

and variance $(X) = npq = 1 \qquad q = \frac{1}{2}$ or $p = \frac{1}{2}$

and $n = 4$

Thus $p(X \ge 1) = 1 - p(X = 0) = 1 - {}^4C_0\left(\frac{1}{2}\right)^4 = \frac{15}{16}.$

14. (d) Suppose the coin tossed n times. Let X be the number of heads obtained. Then X follows a binomial distribution with parameters n and $p = \frac{1}{2}.$

We have, $P(X \ge 1) \ge 0.8 \Rightarrow 1 - P(X = 0) \ge 0.8$

$1 - {}^nC_0 P^0 (1 - P)^n \ge 0.8$

$\Rightarrow \left(\frac{1}{2}\right)^n \le 0.2 \Rightarrow \left(\frac{1}{2}\right)^n = \frac{1}{5} \Rightarrow 2^n \ge 5$

This shows that the least value of n is 3.

15. (b) Let X denotes the number of tosses required. Then $P(X = r) = (1 - p)^{r-1}. \; p$, for $r = 1, 2, 3 \ldots\ldots$

Let E denote the event that the number of tosses required is even.

Then $P(E) = P[(X = 2) \cup (X = 4) \cup (X = 6) \cup \ldots\ldots..]$

$P(E) = P(X = 2) + P(X = 4) + P(X = 6) + \ldots\ldots$

$P(E) = (1 - p)p + (1 - p)^3 p + (1 - p)^5 p + \ldots\ldots. = \frac{1 - p}{2 - p}$

But we are given that $P(E) = \frac{2}{5}$, then we get $p = \frac{1}{3}.$

16. (a) Let n be the least number of bombs required and X the number of bombs that hit the bridge. Then X follows a binomial distribution with parameter n and $p = \frac{1}{2}.$

Now $P(X \ge 2) > 0.9 \Rightarrow 1 - P(X < 2) > 0.9$

$\Rightarrow P(X = 0) + P(X = 1) < 0.1$

$\Rightarrow {}^nC_0\left(\frac{1}{2}\right)^n + {}^nC_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right) < 0.1 \Rightarrow 10(n + 1) < 2^n$

This gives $n \ge 8.$

17. (c) $9.{}^6C_4p^4q^2 = {}^6C_2p^2q^4$

Putting $q = 1-p$, we get required result.

18. (d) We are given that $n = 3$, $p = \frac{1}{6}$, $q = \frac{5}{6}$

Mean $= np = 3 \times \frac{1}{6} = \frac{1}{2}$

Variance $= nqp = 3 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{12}$.

19. (b) Obviously, $p = \frac{2}{6} = \frac{1}{3} \Rightarrow q = 1 - \frac{1}{3} = \frac{2}{3}$,

also $n = 2$. Therefore, variance $= npq = 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$.

20. (b) Required probability $= P$ (exactly two) $+ P$ (exactly three)

$= {}^3C_2 \left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right) + {}^3C_3 \left(\frac{2}{6}\right)^3 = \frac{2}{9} + \frac{1}{27} = \frac{7}{27}$.

21. (c) Required probability $= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) = \frac{1}{4}$.

22. (c) Required probability

$= {}^3C_1 \left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{2}\right)^2 + {}^3C_2 \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^1 = \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$.

23. (a) Required probability

$= {}^8C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^7 + {}^8C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^8 = \frac{27}{20}\left(\frac{19}{20}\right)^7$

24. (d) We have $p = \frac{3}{4} \Rightarrow q = \frac{1}{4}$ and $n = 5$

Therefore required probability

$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2 + {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) + {}^5C_5 \left(\frac{3}{4}\right)^5$

$= \frac{10.27}{4^5} + \frac{5.81}{4^5} + \frac{243}{4^5} = \frac{270+405+243}{1024} = \frac{459}{512}$.

25. (a) Let the coin be tossed n times

P(7 heads) $= {}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{n-7} = {}^nC_7 \left(\frac{1}{2}\right)^n$

and P(9 heads) $= {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9} = {}^nC_9 \left(\frac{1}{2}\right)^n$

P(7 heads) $= P$(9 heads) $\Rightarrow {}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$\therefore P$ (3 heads) $= {}^{16}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{16-3} = {}^{16}C_3 \left(\frac{1}{2}\right)^{16} = \frac{35}{2^{12}}$.

26. (c) P (correct prediction) $= \frac{1}{3}$; P (wrong prediction) $= \frac{2}{3}$

For exactly 4 right predictions

Probability $= {}^7C_4 \left(\frac{1}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^3 = \frac{280}{3^7}$.

27. (c) It is a fundamental concept.

28. (a) For binomial distribution, mean $= np$ and variance $= npq$

$n = 3$, $p = \frac{2}{6} = \frac{1}{3}$, $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

So, mean $(\mu) = 3 \times \frac{1}{3} = 1$

Variance $(\sigma^2) = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3}$.

29. (d) $4P(X=4) = P(X=2) \Rightarrow 4.{}^6C_4p^4q^2 = {}^6C_2p^2q^4$

$\Rightarrow 4p^2 = q^2 \Rightarrow 4p^2 = (1-p)^2$

$\Rightarrow 3p^2 + 2p - 1 = 0 \Rightarrow p = \frac{1}{3}$.

30. (a) $\sum_{k=0}^{4} P(X=k) = 1 \Rightarrow \sum_{k=0}^{4} Ck^2 = 1$

$\Rightarrow C(1^2 + 2^2 + 3^2 + 4^2) = 1 \Rightarrow C = \frac{1}{30}$.

31. (a) The total number of ways of selecting 4 tickets $= 3^4 = 81$.

The favourable number of ways

= sum of coefficients of $x^2, x^4, \ldots\ldots.$ in $(x + x^2 + x^3)^4$

= sum of coefficients of $x^2, x^4, \ldots\ldots$ in $x^4(1 + x + x^2)^4$.

Let $(1 + x + x^2)^4 = 1 + a_1x + a_2x^2 + \ldots.. + a_8x^8$.

Then $3^4 = 1 + a_1 + a_2 + a_3 + \ldots. + a_8$, (On putting $x = 1$)

and $1 = 1 - a_1 + a_2 - a_3 + \ldots.. + a_8$, (On putting $x = -1$)

$\therefore 3^4 + 1 = 2(1 + a_2 + a_4 + a_6 + a_8)$

$\Rightarrow a_2 + a_4 + a_6 + a_8 = 41$

Thus sum of the coefficients of $x^2, x^4, \ldots\ldots = 41$

Hence the required probaility $= \frac{41}{81}$.

32. (b) Required probability $= {}^{10}C_5 \times \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5 = \frac{63}{256}$.

33. (b) Here $p = \frac{19}{20}$, $q = \frac{1}{20}$, $n = 5$, $r = 5$

The required probability $= {}^5C_5 \left(\frac{19}{20}\right)^5 \cdot \left(\frac{1}{20}\right)^0 = \left(\frac{19}{20}\right)^5$.

34. (a) By Binomial theorem,

$P = {}^5C_3 \times \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 \Rightarrow P = \frac{5}{16}$.

35. (a) Probability of disease is fatal = $p = 10\%$

$p = \frac{10}{100} = \frac{1}{10},\ q = \frac{9}{10}$

Number of patients = 6, Number of die cases = 3

$\therefore$ Probability that 3 will die

$$= {}^6C_3\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^3 = 1458 \times 10^{-5}.$$

36. (c) The chance of being a boy or girl $p = \frac{1}{2},\ q = \frac{1}{2}$

Total child = 2, Number of boys = 1

P(1 boys, 1 girl) $= {}^2C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{2-1} = \frac{1}{2}.$

37. (a) The probability that student is not swimmer $p = \frac{1}{5}$ and probability that student is swimmer $q = \frac{4}{5}$.

$\therefore$ Probability that out of 5 students 4 are swimmer

$$= {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^{5-4} = {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right).$$

38. (c) Probability of failure = $\frac{1}{3}$

Probability for getting success $= \frac{2}{3}$

Required probability

$$= {}^4C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 + {}^4C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)$$

$$= \left(\frac{2}{3}\right)^4 + 4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right) = \frac{16}{27}.$$

39. (a) Given $np = 6, npq = 4$

$\frac{npq}{np} = \frac{4}{6} \Rightarrow q = \frac{2}{3}$ and $p = \frac{1}{3}$

$\because np = 6 \Rightarrow n \times \frac{1}{3} = 6 \Rightarrow n = 18.$

40. (d) Condition for sum of 12 is 2, 2, 2, 3, 3

$\therefore$ Required probability

$= {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10.\frac{1}{2^5} = \frac{5}{16}.$

41. (c) Probability for white ball $= \frac{2}{6} = \frac{1}{3}$

Probability for black ball $= \frac{4}{6} = \frac{2}{3}$

$\therefore$ Required probability

$$= {}^5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$$

$$= \left(\frac{1}{3}\right)^4\left[\frac{1}{3} + 5.\frac{2}{3}\right] = \frac{11}{3^5} = \frac{11}{243}.$$

42. (c) The required probability

= 1 – Probability of equal number of heads and tails

$$= 1 - {}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{2n-n} = 1 - \frac{(2n)!}{n!n!}\left(\frac{1}{4}\right)^n = 1 - \frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}.$$

43. (b) In Binomial distribution, Variance = npq and Mean = np, Variance $= 3 = npq$ Mean $= 4 = np$

Now, $q = \frac{3}{4},\ p = \frac{1}{4}$ and $n = 16$

Probability of success = ${}^{16}C_6\left(\frac{3}{4}\right)^{10}\left(\frac{1}{4}\right)^6.$

44. (d) Probability of getting odd $p = \frac{3}{6} = \frac{1}{2}.$

Probability of getting others $q = \frac{3}{6} = \frac{1}{2}$

Variance $= npq = 5.\frac{1}{2}.\frac{1}{2} = \frac{5}{4}.$

45. (a) Probability for a head $= \frac{1}{2}$ *i.e.*, $p = \frac{1}{2}$

$\therefore q = \frac{1}{2}$ in a toss.

Required probability $= {}^{10}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{63}{256}.$

46. (c) Probability of success $p = \frac{1}{4}$

Probability of unsuccess $q = \frac{3}{4}$

Mean = np

Standard deviation = $\sqrt{\text{Variance}} \Rightarrow$ Variance = 9

$npq = 9 \qquad n.\frac{1}{4}.\frac{3}{4} = 9 \qquad n = 48$

Mean $= np = \frac{1}{4} \times 48 = 12.$

47. (b) $p = P$(getting a head) $= \frac{1}{2}, q = \frac{1}{2}.$

Required probability = P(six successes)

$$= {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \frac{10!}{6!4!}.\frac{1}{2^{10}} = \frac{105}{512}.$$

48. (a) Probability of occurrence of '4' $= \frac{1}{6}$

Probability of inoccurrence of '4' $= \frac{5}{6}$

$\therefore$ Required probability

$$= {}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) + {}^2C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0 = \frac{11}{36}.$$

49. (a) $\left.\begin{array}{l} np = 4 \\ npq = 2 \end{array}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}.$$

50. (b) Let X be the number of heads getting. X follows binomial distribution with parameters $n, p = 1/2$ Given that $P(X \geq 1) \geq 0.8$

$$1 - P(X=0) \geq 0.8 \Rightarrow P(X=0) \leq 0.2$$

$$^nC_0 (1/2)^n \leq 0.2 \;\Rightarrow\; \frac{1}{2^n} \leq \frac{1}{5} \Rightarrow 2^n = 5.$$

$\therefore$ The least value of n is 3.

51. (a) Required probability = $^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{3}{8}$.

52. (a) Let p = Probability of getting tail = $\frac{1}{2}$

q = Probability of getting head = $\frac{1}{2}$

Also, $p + q = 1$ and $n = 100$

$\therefore$ Required probability

$= P(X=1) + P(X=3) + ... + P(X=99)$

$= {}^{100}C_1 p.q^{99} + {}^{100}C_3 p^3 q^{97} + ... + {}^{100}C_{99} p^{99} q^1$

$= \frac{(p+q)^{100} - (p-q)^{100}}{2} = \frac{1}{2}$.

53. (c) Required probability $= \left(\frac{1}{2}\right)^3 . {}^3C_2 + \left(\frac{1}{2}\right)^3 . {}^3C_3 = \frac{4}{8} = \frac{1}{2}$.

{Here at least two heads means two heads or three heads}.

54. (d) Probability of getting odd number $= \frac{3}{6} = \frac{1}{2}$

Hence required probability $= {}^2C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^0 = \frac{1}{4}$.

55. (a) $\left.\begin{array}{l} np = 4 \\ npq = 2 \end{array}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X=2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = 28.\frac{1}{2^8} = \frac{28}{256}.$$

56. (b) Here mean = np and variance = npq

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{{}^nC_k (p)^k (q)^{n-k}}{{}^nC_{k-1} (p)^{k-1} (q)^{n-k+1}} = \frac{{}^nC_k}{{}^nC_{k-1}} \cdot \frac{p}{q}$$

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{n-k+1}{k} \cdot \frac{p}{q}.$$

57. (c) Let X denote a random variable which is the number of aces. Clearly, X takes values, 1, 2.

$$p = \frac{4}{52} = \frac{1}{13}, \; q = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X=1) = 2 \times \left(\frac{1}{13}\right) \times \left(\frac{12}{13}\right) = \frac{24}{169}$$

$$P(X=2) = 2.\left(\frac{1}{13}\right)^2 \left(\frac{12}{13}\right)^0 = \frac{1}{169}$$

Mean = $\sum P_i X_i = \frac{24}{169} + \frac{2}{169} = \frac{26}{169} = \frac{2}{13}$.

58. (b) Since the item are choosen without replacement.

$$P(X=x) = \frac{{}^3C_x + {}^7C_{4-x}}{{}^{10}C_4}$$

Putting $x = 1, 2$ we have

$$P(0 < x < 3) = \frac{{}^3C_1 \times {}^7C_3}{210} + \frac{{}^3C_2 \times {}^7C_2}{210}$$

$$= \frac{3 \times 35 + 3 \times 21}{210} = \frac{105 + 63}{210} = \frac{168}{210} = \frac{4}{5}.$$

Critical Thinking Questions

1. (d) $P(E) \leq P(F) \Rightarrow n(E) \leq n(F)$

$P(E \cap F) > 0 \Rightarrow E \cap F \neq \phi$

These do not mean that E is a sub-set of F or F is a sub-set of E. *i.e.,* $E \subseteq F$ or $F \subseteq E$ or $\overline{E} \subseteq \overline{F}$.

2. (b) Here $P(H) = P(T) = \frac{1}{2}$ and $P(X) = 1$, where X denotes head or tail.

If the sequence of m consecutive heads starts from the first throw, we have $(HH....m \text{ times})(XX.....n \text{ times}$.

$\therefore$ Chance of this event $= \frac{1}{2}.\frac{1}{2}.\frac{1}{2}........m \text{ times} = \frac{1}{2^m}$

$\because m+1$ and subsequent throws may be head or tail since we are considering at least m consecutive heads. If the sequence of m consecutive heads starts from the second throw, the first must be a tail and we have, the chance of this event $= \frac{1}{2}.\frac{1}{2^m} = \frac{1}{2^{m+1}}$.

If the sequence of heads starts from $(r+1)^{th}$ throw then the first $(r-1)$ throws may be head or tail but r^{th} throw must be a tail and we have,

$(XX........(r-1) \text{ times}) T (HH........m \text{ times})$

$(XX......\overline{n-m-r} \text{ times})$

The chance of this event also $\frac{1}{2} \times \frac{1}{2^m} = \frac{1}{2^{m+1}}$

Since all the above events are mutually exclusive, so the required probability $= \frac{1}{2^m} + \left(\frac{1}{2^{m+1}} + \frac{1}{2^{m+1}} + ...n \text{ times}\right)$

$$= \frac{1}{2^m} + \frac{n}{2^{m+1}} = \frac{n+2}{2^{m+1}}.$$

Note : Students should remember this question as a formula.

3. (d) Let $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$ and $p_4 = 0.1$

P (the gun hits the plane) $= P$ (the plane is hit in once)

$= 1 - P$ (the plane is hit in none of the shots)

$= 1-(1-p_1)(1-p_2)(1-p_3)(1-p_4) = 0.6976.$

4. (c) Let W denote the event of drawing a white ball at any draw and B that for a black ball.

Then $P(W) = \frac{a}{a+b}$, $P(B) = \frac{b}{a+b}$

$P(A$ wins the game$) = P(W$ or BBW or $BBBBW$ or .....)

$= P(W) + P(B)P(B)P(W) + P(B)P(B)P(B)P(B)P(W) + ...$

$$= \frac{P(W)}{1-P(B)^2} = \frac{\frac{a}{a+b}}{1-\frac{b^2}{(a+b)^2}} = \frac{a(a+b)}{a^2+2ab} = \frac{(a+b)}{a+2b}$$

Also $P(B$ wins the game$) = 1 - \frac{a+b}{a+2b} = \frac{b}{a+2b}$

According to the given condition,

$\frac{a+b}{a+2b} = 3.\frac{b}{a+2b} \Rightarrow a = 2b \Rightarrow a:b = 2:1.$

5. (a) Since $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ and $\left(\frac{1-2p}{2}\right)$ are the probabilities of the three events, we must have

$0 \le \frac{1+3p}{3} \le 1, 0 \le \frac{1-p}{4} \le 1$ and $0 \le \frac{1-2p}{2} \le 1$

$\Rightarrow -1 \le 3p \le 2, -3 \le p \le 1$ and $-1 \le 2p \le 1$

$\Rightarrow -\frac{1}{3} \le p \le \frac{2}{3}, -3 \le p \le 1$ and $-\frac{1}{2} \le p \le \frac{1}{2}$

Also as $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events

$0 \le \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$

$\Rightarrow 0 \le 4+12p+3-3p+6-12p \le 12 \Rightarrow \frac{1}{3} \le p \le \frac{13}{3}$

Thus the required value of p are such that

Max. $\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le p$ min. $\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$

$\Rightarrow \frac{1}{3} \le p \le \frac{1}{2}.$

6. (c) The last digit of the product will be $1, 2, 3, 4, 6, 7, 8$ or 9 if and only if each of the n positive integers ends in any of these digits. Now the probability of an integer ending in $1, 2, 3, 4, 6, 7, 8$ or 9 is $\frac{8}{10}$. Therefore the probability that the last digit of the product of n integers in $1, 2, 3, 4, 6, 7, 8$ or 9 is $\left(\frac{4}{5}\right)^n$. The probability for an integer to end in $1, 3, 7$ or 9 is $\frac{4}{10} = \frac{2}{5}$.

Therefore the probability for the product of n positive integers to end in $1, 3, 7$ or 9 is $\left(\frac{2}{5}\right)^n$.

Hence the required probability $= \left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n = \frac{4^n - 2^n}{5^n}$.

7. (b) Required probability = probability that either the number is 7 or the number is 8.

i.e., Required Probability $= P_7 + P_8$

Now $P_7 = \frac{1}{2}.\frac{1}{11} + \frac{1}{2}.\frac{6}{36} = \frac{1}{2}\left(\frac{1}{11} + \frac{1}{6}\right)$

$P_8 = \frac{1}{2}.\frac{1}{11} + \frac{1}{2}.\frac{5}{36} = \frac{1}{2}\left(\frac{1}{11} + \frac{5}{36}\right)$

$\therefore P = \frac{1}{2}\left(\frac{2}{11} + \frac{11}{36}\right) = 0.244$

8. (d) There will be no x because $P(AB)$ can never be less than $P(ABC)$.

9. (b) Probability [Person A will die in 30 years] $= \frac{8}{8+5}$

$P(A) = \frac{8}{13} \Rightarrow P(\bar{A}) = \frac{5}{13}$

Similarly, $P(B) = \frac{4}{7} \Rightarrow P(\bar{B}) = \frac{3}{7}$

There are two ways in which one person is alive after 30 years. $\bar{A}B$ and $A\bar{B}$ and event are independent.

So, required probability

$= P(\bar{A}).P(B) + P(A).P(\bar{B}) = \frac{5}{13} \times \frac{4}{7} + \frac{8}{13} \times \frac{3}{7} = \frac{44}{91}.$

10. (a) The probability of hitting in one shot $= \frac{10}{100} = \frac{1}{10}$

If he fires n shots, the probability of hitting at least once

$$= 1 - \left(1 - \frac{1}{10}\right)^n = 1 - \left(\frac{9}{10}\right)^n = \frac{1}{2} \text{ (from the question)}$$

$$\therefore \left(\frac{9}{10}\right)^n = \frac{1}{2}, \ \therefore n\{2\log_{10} 3 - 1\} = -\log_{10} 2$$

$$\therefore n = \left\{\frac{\log_{10} 2}{1 - 12\log_{10} 3} = \frac{0.3010}{1 - 2 \times 0.4771} = 6.5 \text{ (nearly)}\right.$$

$\therefore$ For 6 shots, the probabilty is about 53% while for 7 shots it is nearly 48%.

11. (a) Since m and n are selected between 1 and 100, hence sample space $= 100 \times 100$

Also $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$ etc. Hence $1, 3, 7$ and 9 will be the last digits in the powers of 7. Hence for favourable cases

$n \; m \rightarrow$

$\downarrow$

$1, 1 \quad 1, 2 \quad 1, 3 \ldots\ldots \quad 1, \ 100$

$2, 1 \quad 2, 2 \quad 2, 3 \ldots\ldots \quad 2, \ 100$

....

$100, 1 \quad 100, 2 \quad 100, 3 \ldots\ldots 100, \ 100$

For $m = 1$; $n = 3, 7, 11 \ldots 97$

$\therefore$ Favourable cases = 25

For $m = 2$; $n = 4, 8, 12 \ldots 100$

$\therefore$ Favourable cases = 25

Similarly for every m, favourable n are 25.

$\therefore$ Total favourable cases $= 100 \times 25$

Hence required probability $= \frac{100 \times 25}{100 \times 100} = \frac{1}{4}$.

12. (b) This is a problem of without replacement.

$$P = \frac{\text{one def. from 2 def.}}{\text{any one from 4}} \times \frac{\text{1 def. from remaining 1 def.}}{\text{any one from remaining 3}}$$

Hence required probability $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

Aliter : Number of ways in which two faulty machines may be detected (depending upon the test done to indentify the faulty machines) $= {}^4C_2 = 6$

Number of favourable cases = 1

[When faulty machines are identified in the first and the second test]

Hence required probability $= \frac{1}{6}$.

13. (b) The probability of throwing 9 with two dice $= \frac{4}{36} = \frac{1}{9}$

$\therefore$ The probability of not throwing 9 with two dice $= \frac{8}{9}$

If A is to win he should throw 9 in 1st or 3rd or 5th attempt

If B is to win, he should throw, 9 in 2nd, 4th attempt

B s chances

$$= \left(\frac{8}{9}\right) \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^3 \cdot \frac{1}{9} + \ldots = \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \left(\frac{8}{9}\right)^2} = \frac{8}{17}.$$

14. (b) Favorable number of cases = ${}^{20}C_1 = 20$

Sample space = ${}^{62}C_1 = 62$

$\therefore$ Required probability = $\frac{20}{62} = \frac{10}{31}$.

15. (d) Let A denotes the event that the student is selected in IIT entrance test and B denotes the event that he is selected in Roorkee entrance test. Then

$P(A) = 0.2, \ P(B) = 0.5$ and $P(A \cap B) = 0.3$.

Required probability $= P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.2 + 0.5 - 0.3) = 0.6$.

16. (a) Let $n =$ total number of ways $= 12!$

and $m =$ favourable number of ways $= 2 \times 6! \,.\, 6!$

Since the boys and girls can sit alternately in $6! \,.\, 6!$ ways if we begin with a boy and similarly they can sit alternately in $6! \,.\, 6!$ ways if we begin with a girl

Hence required probability

$$= \frac{m}{n} = \frac{2 \times 6! \,.\, 6!}{12!} = \frac{1}{462}.$$

17. (a) Here the least number of draws to obtain 2 aces are 2 and the maximum number is 50 thus n can take value from 2 to 50.

Since we have to make n draws for getting two aces, in $(n-1)$ draws, we get any one of the 4 aces and in the n^{th} draw we get one ace. Hence the required probability

$$= \frac{{}^4C_1 \times {}^{48}C_{n-2}}{{}^{52}C_{n-1}} \times \frac{3}{52 - (n-1)}$$

$$= \frac{4 \times (48)!}{(n-2)!(48 - n + 2)!} \times \frac{(n-1)!(52 - n + 1)!}{(52)!} \times \frac{3}{52 - n + 1}$$

$= \frac{(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$ (on simplification).

18. (a) We know that the number of sub-sets of a set containing n elements is 2^n. Therefore the number of ways of choosing A and B is 2^n. $2^n = 2^{2n}$

We also know that the number of sub-sets (of X) which contain exactly r elements is ${}^nC_r$. Therefore the number of ways of choosing A and B, so that they have the same number elements is

$({}^nC_0)^2+({}^nC_1)^2+({}^nC_2)^2+......+({}^nC_n)^2={}^{2n}C_n$

Thus the required probability $=\frac{{}^{2n}C_n}{2^{2n}}$.

19. (c) $n(S)=6\times 6\times 6$

$n(E)=$ The number of solutions of $x+y+z=7$, where $1\le x\le 6,\ 1\le y\le 6,\ 1\le z\le 6$

= coefficient of x^7 in $(x+x^2+.......+x^6)^3$

= coefficient of x^4 in $(1+x+......+x^5)^3$

= coefficient of x^4 in $\left(\frac{1-x^6}{1-x}\right)^3$

= coefficient of x^4 in $(1-3.x^6+3.x^{12}-x^{18})(1-x)^{-3}$

= coefficient of x^4 in $(1-3x^6+3x^{12}-x^{18})$

$({}^2C_0+{}^3C_1x+{}^4C_2x^2+{}^5C_3x^3+{}^6C_4x^4+.......)$

$= {}^6C_4=\frac{6\,!}{4\,!.2\,!}=\frac{6\times 5}{2}=15$

$p(E)=\frac{n(E)}{n(S)}=\frac{15}{6\times 6\times 6}=\frac{5}{72}$.

20. (b) The number of ways to arrange 7 white an 3 black balls in a row $=\frac{10!}{7\,!.3\,!}=\frac{10.9.8}{1.2.3}=120$

Numbers of blank places between 7 balls are 6. There is 1 place before first ball and 1 place after last ball. Hence total number of places are 8.

Hence 3 black balls are arranged on these 8 places so that no two black balls are together in number of ways

$={}^8C_3=\frac{8\times 7\times 6}{1\times 2\times 3}=56$

So required probability $=\frac{56}{120}=\frac{7}{15}$.

21. (d) Let p be the probability of the other event, then the probability of the first event is $\frac{2}{3}p$.

Since two events are toally exclusive, we have $p+\left(\frac{2}{3}\right)p=1\Rightarrow p=\frac{3}{5}$

Hence odds in favour of the other are $3:5-3$, *i.e.* $3:2$.

22. (c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cap B)$

$\{\because\ P(A\cap B)=P(A\cup B)\}$

$\Rightarrow 2P(A\cap B)=P(A)+P(B)$

$\Rightarrow 2P(A).\frac{P(A\cap B)}{P(A)}=P(A)+P(B)$

$\Rightarrow 2P(A).P\left(\frac{B}{A}\right)=P(A)+P(B)$.

23. (b) $P(\overline{A}\cap\overline{B})=P(\overline{A\cup B})=1-P(A\cup B)$

Since A and B are mutually exclusive, so $P(A\cup B)=P(A)+P(B)$

Hence required probability $=1-(0.5+0.3)=0.2$.

24. (d) It is obvious.

25. (a) We know that $P(A/B)=\frac{P(A\cap B)}{P(B)}$

Also we know that $P(A\cup B)\le 1$

$\Rightarrow P(A)+P(B)-P(A\cap B)\le 1$

$\Rightarrow P(A\cap B)\ge P(A)+P(B)-1$

$\Rightarrow \frac{P(A\cap B)}{P(B)}\ge\frac{P(A)+P(B)-1}{P(B)}$

$\Rightarrow P(A/B)\ge\frac{P(A)+P(B)-1}{P(B)}$

26. (a) Let the events are

$R_1=A$ red ball is drawn from urn A and placed in B

$B_1=A$ black ball is drawn from urn A and placed in B

$R_2=A$ red ball is drawn from urn B and placed in A

$B_2=A$ black ball is drawn from urn B and placed in A

$R=A$ red ball is drawn in the second attempt from A

Then the required probability

$=P(R_1R_2R)+(R_1B_2R)+P(B_1R_2R)+P(B_1B_2R)$

$=P(R_1)P(R_2)P(R)+P(R_1)P(B_2)P(R)+P(B_1)P(R_2)P(R)+P(B_1)P(B_2)P(R)$

$=\frac{6}{10}\times\frac{5}{11}\times\frac{6}{10}+\frac{6}{10}\times\frac{6}{11}\times\frac{5}{10}+\frac{4}{10}\times\frac{4}{11}\times\frac{7}{10}+\frac{4}{10}\times\frac{7}{11}\times\frac{6}{10}$

$= \frac{32}{55}$.

27. (d) We have $P(A \cup B) \ge \max. \{P(A), P(B)\} = \frac{2}{3}$

$P(A \cap B) \le \min. \{P(A), P(B)\} = \frac{1}{2}$

and

$P(A \cap B) = P(A) + P(B) - P(A \cup B) \ge P(A) - P(B) - 1 = \frac{1}{6}$

$\Rightarrow \frac{1}{6} \le P(A \cap B) \le \frac{1}{2}$

$P(A' \cap B) = P(B) - P(A \cap B)$

Hence $\frac{2}{3} - \frac{1}{2} \le P(A' \cap B) \le \frac{2}{3} - \frac{1}{6}$

$\Rightarrow \frac{1}{6} \le P(A' \cap B) \le \frac{1}{2}$.

28. (c) A leap year consists of 366 days comprising of 52 weeks and 2 days. There are 7 possibilities for these 2 extra days *viz.*

(i) Sunday, Monday, (ii) Monday, Tuesday,

(iii) Tuesday, Wednesday, (iv) Wednesday, Thursday,

(v) Thursday, Friday, (vi) Friday, Saturday and

(vii) Saturday, Sunday.

Let us consider two events :

A: the leap year contains 53 Sundays

B: the leap year contains 53 Mondays.

Then we have $P(A) = \frac{2}{7}$, $P(B) = \frac{2}{7}$, $P(A \cap B) = \frac{1}{7}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$.

29. (b, c) Let M, P and C be the events of passing in mathematics, physics and chemistry respectively.

$P(M \cup P \cup C) = \frac{75}{100} = \frac{3}{4}$

$P(M \cap P) + P(P \cap C) + P(M \cap C) - 2P(M \cap P \cap C) = \frac{50}{100} = \frac{1}{2}$

$P(M \cap P) + P(P \cap C) + P(M \cap C) - 2P(M \cap P \cap C) = \frac{40}{100} = \frac{2}{5}$

$m(1-p)(1-c) + p(1-m)(1-c) + c(1-m)(1-p)$

$+ mp(1-c) + mc(1-p) + pc(1-m) + mpc = \frac{3}{4}$

$\Rightarrow m + p + c - mc - mp - pc + mpc = \frac{3}{4}$ .....(i)

Similarly, $mp(1-c) + pc(1-m) + mc(1-p) + mpc = \frac{1}{2}$

$\Rightarrow mp + pc + mc - 2mpc = \frac{1}{2}$ .....(ii)

$mp(1-c) + pc(1-m) + mc(1-p) = \frac{2}{5}$

$\Rightarrow mp + pc + mc - 3mpc = \frac{2}{5}$ .....(iii)

From (ii) to (iii), $mpc = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

From (i) and (ii), $m + p + c - mpc = \frac{3}{4} + \frac{1}{2}$

$\therefore m + p + c = \frac{3}{4} + \frac{1}{2} + \frac{1}{10} = \frac{15+10+2}{20} = \frac{27}{20}$.

30. (d) Let a white ball be transferred from the first bag to the second. The Probability of selecting a white ball from the first bag $= \frac{5}{9}$.

Now the second bag has 8 white and 9 black. The probability of selecting white ball from the second bag $= \frac{8}{17}$.

Hence required probability $= \frac{5}{9} \times \frac{8}{17} = \frac{40}{153}$

If a black ball be transferred from the first bag to the second, then the probability $= \frac{4}{9} \times \frac{7}{17} = \frac{28}{153}$

Therefore required probability $= \frac{40}{153} + \frac{28}{153} = \frac{4}{9}$.

31. (d) Let the first number be x and second is y.

Let A denotes the event that the difference between the first and second number is at least m. Let E_x denote the event that the first number chosen is x, we must have $x - y \ge m$ or $y \le x - m$. Therefore $x > m$ and $y < n - m$

Thus $P(E_x) = 0$ for $0 < x \le m$ and $P(E_x) = \frac{1}{n}$ for $m < x \le n$. Also $P(A / E_x) = \frac{(x-m)}{(n-1)}$

Therefore, $P(A) = \sum_{x=1}^{n} P(E_x)\, P(A / E_x)$

$= \sum_{x=m+1}^{n} P(E_x)\, P(A / E_x) = \sum_{x=m+1}^{n} \frac{1}{n} \cdot \frac{x-m}{n-1}$

$= \frac{1}{n(n-1)} [1 + 2 + 3 + ... + (n-m)]$

$= \frac{(n-m)(n-m+1)}{2n(n-1)}$.

32. (d) Let E be the event that a new product is introduced.

Then $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

and $P(E / A) = 0.7$, $P(E / B) = 0.6$, $P(E / C) = 0.5$.

$\because A, B$ and C are mutually exclusive and exhaustive events.

$P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C)$

$= 0.5\times 0.7 + 0.3\times 0.6 + 0.2\times 0.5$

$= 0.35 + 0.18 + 0.10 = 0.63.$

33. (a) $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} \Rightarrow \frac{1}{2} = \frac{P(A\cap B)}{1/4}$

$\Rightarrow P(A\cap B) = \frac{1}{8}$

Hence events A and B are not mutually exclusive.

Statement II is incorrect.

$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(B) = \frac{1}{2}$

$\because\ P(A\cap B) = \frac{1}{8} = P(A).P(B)$

events A and B are independent events.

$P\left(\frac{A^c}{B^c}\right) = \frac{P(A^c\cap B^c)}{P(B^c)} = \frac{P(A^c)P(B^c)}{P(B^c)} = \frac{3}{4}.\frac{1}{2}.\frac{2}{1} = \frac{3}{4}$

Hence statement I is correct.

Again $P\left(\frac{A}{B}\right) + P\left(\frac{A}{B^c}\right) = \frac{1}{4} + \frac{P(A\cap B^c)}{P(B^c)}$

$= \frac{1}{4} + \frac{P(A) - P(A\cap B)}{P(B^c)} = \frac{1}{4} + \frac{\frac{1}{4} - \frac{1}{8}}{\frac{1}{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Hence statement III is incorrect.

34. (c) Required probability $= \frac{1}{2}.\frac{4}{7} + \frac{1}{2}.\frac{6}{8} = \frac{37}{56}.$

35. (a) $P($ minimum face value not less than 2 and maximum face value is not greater than 5)

$= P(2\text{ or }3\text{ or }4\text{ or }5) = \frac{4}{6} = \frac{2}{3}$

Hence required probability $= {}^4C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = \frac{16}{81}.$

36. (b) Matches played by India are four. Maximum points in any match are 2.

$\therefore$ Maximum points in four matches can be 8 only.

Therefore probability $(P) = p(7) + p(8)$

$p(7) = {}^4C_1(0.05)(0.5)^3 = 0.0250$

$p(8) = (0.5)^4 = 0.0625$

$\Rightarrow P = 0.0875.$

37. (a) Mean $= np = 3,\ S.D. = \sqrt{npq} = \frac{3}{2}$

$\Rightarrow q = \frac{npq}{np} = \frac{9}{4\times 3} = \frac{3}{4}$

$\Rightarrow p = 1 - \frac{3}{4} = \frac{1}{4}$

Hence binomial distribution is $(q+p)^n = \left(\frac{3}{4} + \frac{1}{4}\right)^{12}.$

38. (a) Let X be the number of times 1, 3 or 4 occur on the die. Then X follows a binomial distribution with parameter and $p = \frac{3}{6} = \frac{1}{2}.$

We have $P(1,\ 3$ or 4 occur at most n times on the die)

$= P(0 \le X \le n) = P(X=0) + P(X=1) + \ldots.. + P(X=n)$

$= {}^{2n+1}C_0\left(\frac{1}{2}\right)^{2n+1} + {}^{2n+1}C_1\left(\frac{1}{2}\right)^{2n+1} + \ldots.. + {}^{2n+1}C_n\left(\frac{1}{2}\right)^{2n+1}$

$= \left[{}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots.. + {}^{2n+1}C_n\right]\left(\frac{1}{2}\right)^{2n+1}$

Let $S = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots.. + {}^{2n+1}C_n$

$\Rightarrow 2S = 2.{}^{2n+1}C_0 + 2.{}^{2n+1}C_1 + \ldots\ldots. + 2^{2n+1}C_n$

$= \left({}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}\right) + \left({}^{2n+1}C_1 + {}^{2n+1}C_{2n}\right) + \ldots\ldots..$

$+ \left({}^{2n+1}C_n + {}^{2n+1}C_{n+1}\right)$

$\Rightarrow S = 2^{2n}.$

Hence required probability $= 2^{2n}\left(\frac{1}{2}\right)^{2n+1} = \frac{1}{2}.$

39. (c) To get 3 white balls in first 6 draw and then a white again in 7th draws.

$P = {}^6C_3 \times \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \Rightarrow P = \frac{5}{32}.$

40. (a) Let p_1, p_2 be the chances of happenig of the first and second event respectively, then according to the given conditions, we have

$p_1 = p_2^2$ and $\frac{1-p_1}{p_1} = \left(\frac{1-p_2}{p_2}\right)^3$

$\Rightarrow p_2 = \frac{1}{3}$ and so $p_1 = \frac{1}{9}.$