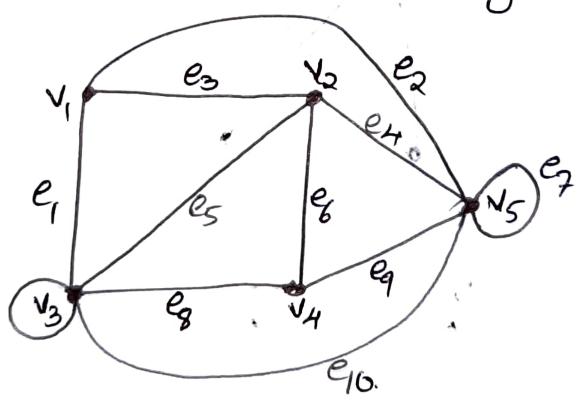
Adjacency Matrix

Let $V = \{v_1, v_2, \dots, v_n\}$. The adjacency matrix A of the graph is $A = [a_{ij}]_{n \times n}$ where $a_{ij} = 1$ if $\{v_i, v_j\} \in E$, otherwise $a_{ij} = 0$

If $E = \{e_1, e_2, \dots, e_k\}$ the incidence matrix. I is the $n \times k$ matrix $[b_{ij}]_{n \times k}$ where $b_{ij} = 1$ if v_i is the vertex on the edge e_j, otherwise. $e_{ij} = 0$

(1) Find the adjacency matrix and incidence matrix associated with the given graph.



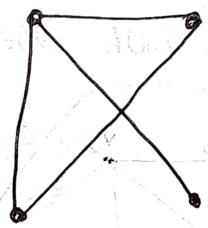
Adjacency matrix

$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$

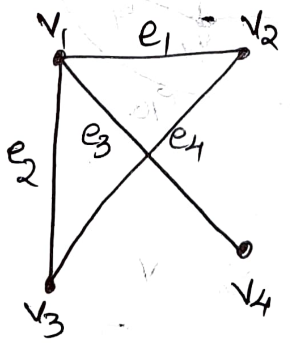
Incidence matrix

$$
\begin{array}{c}
 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

2. Find the adjacency matrix and incidence matrix of the graph.



Solution



Adjacency matrix.

$$
\begin{array}{c|cccc}
 & V_1 & V_2 & V_3 & V_4 \\
\hline
V_1 & 0 & 1 & 1 & 1 \\
V_2 & 1 & 0 & 1 & 0 \\
V_3 & 1 & 1 & 0 & 0 \\
V_4 & 1 & 0 & 0 & 0
\end{array}
$$

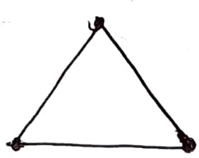
Incidence matrix

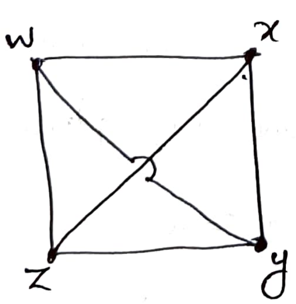
$$
\begin{array}{c|cccc}
 & e_1 & e_2 & e_3 & e_4 \\
\hline
v_1 & & & & \\
v_2 & & & & \\
v_3 & & & & \\
v_4 & & & &
\end{array}
\quad
\begin{bmatrix}
1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

Planar Graph.

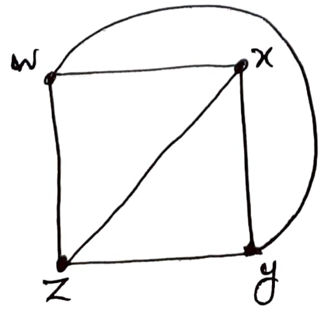
A graph G is called planar if G can be drawn in the plane with its edges intersecting only at vertices of G. Such a drawing of G is called an embedding of G in the plane.

K_3 is planar





The graph is non planar the edges $\{x, z\}$ and $\{w, y\}$ overlap at a point other than a vertex.



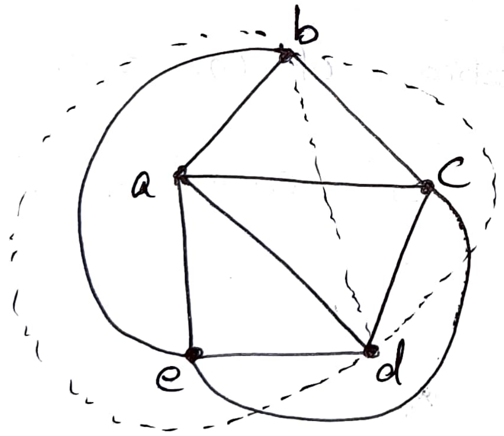
But K_4 is planar.

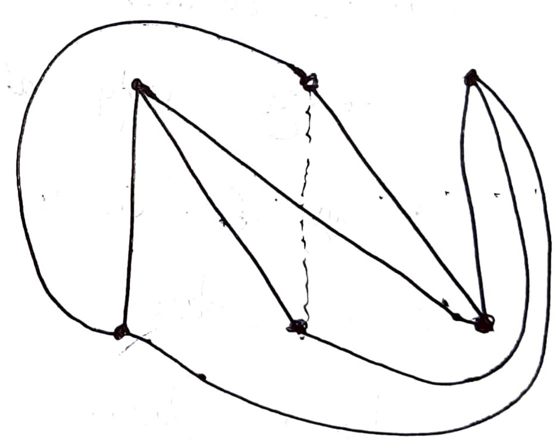
Bipartite graph

A graph $G = (V, E)$ is called bipartite if $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \phi$, and every edge of G is of the from $\{a, b\}$ with $a \in V_1$ and $b \in V_2$.

If each vertex in V_1 is joined with every vertex in V_2, the graph is called Complete bipartite graph. In this case if $|V_1| = m$, $|V_2| = n$, the graph is denoted by $K_{m,n}$.

The graphs K_5 and $K_{3,3}$ are non planar





Let G be a Connected planar graph with $|V| = v$ and $|E| = e$. Let r be the number of regions in the plane determined by planar embedding of G, then $v - e + r = 2$.

(If f is the number of faces $v - e + f = 2$)

Thm

Let $G = (V,E)$ be a loop free connected planar graph with $|V| = v$, $|E| = e > 2$, and r regions. Then $3r \le 2e$ and $e \le 3v - 6$.

Solution

Since G is a ~~region~~ loop free and is not a multigraph, the boundary of each region contains atleast 3 edges. Hence each region has degree ≥ 3. Consequently $2e = 2|E| =$ the sum of the degrees of r regions determined by G and $2e \ge 3r$. We have $v - e + r = 2$.

$$2 = v - e + r \le v - e + \left(\tfrac{2}{3}\right)e.$$

$$= v - \left(\tfrac{1}{3}\right)e$$

$$2 \le v - \frac{1}{3}e.$$

$$2 \le \frac{3v - e}{3}$$

$$6 \le 3v - e.$$

$$e \le 3v - 6.$$