11. (b) Let γ be the angle made by $\mathbf{n}$ with z-axis.
Then direction cosines of $\mathbf{n}$ are
$l = \cos 45^o = \frac{1}{\sqrt{2}}$, $m = \cos 60^o = \frac{1}{2}$ and $n = \cos\gamma$.

$$l^2 + m^2 + n^2 = 1 \Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$$

$$n^2 = \frac{1}{4} \Rightarrow n = \frac{1}{2}, \quad [\because \gamma \text{ is acute, }$$
$n = \cos\gamma > 0$]

We have $|\mathbf{n}| = 8$, $\mathbf{n} = |\mathbf{n}|(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$

$$\Rightarrow \mathbf{n} = 8\left(\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}\right) = 4\sqrt{2}\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$$

The required plane passes through the point $(\sqrt{2}, -1, 1)$ having position vector $\mathbf{a} = \sqrt{2}\mathbf{i} - \mathbf{j} + \mathbf{k}$.

So, its vector equation is $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ or
$\mathbf{r}.\mathbf{n} = \mathbf{a}.\mathbf{n}$

$$\mathbf{r}.(4\sqrt{2}\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) = (\sqrt{2}\mathbf{i} - \mathbf{j} + \mathbf{k}).(4\sqrt{2}\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$$

$$\mathbf{r}.(\sqrt{2}\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2.$$

12. (b) Here $d = 8$ and $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{2\mathbf{i} + \mathbf{j} + 2\mathbf{k}}{\sqrt{4+1+4}} = \frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Hence, the required equation of the plane is

$$\mathbf{r}.\left(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = 8 \text{ or } \mathbf{r}.(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 24.$$

13. (a) We know that the perpendicular distance of a point P with position vector $\mathbf{a}$ from the plane $\mathbf{r}.\mathbf{n} = d$ is given by $\frac{|\mathbf{a}.\mathbf{n} - d|}{|\mathbf{n}|}$.

Here $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \mathbf{n} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $d = 9$.

So, required distance

$$= \frac{|(2\mathbf{i} + \mathbf{j} - \mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) - 9|}{\sqrt{1+4+16}}$$

$$= \frac{|2 - 2 - 4 - 9|}{\sqrt{21}} = \frac{13}{\sqrt{21}}.$$

14. (b) The equation of a line through the centre $\mathbf{j} + 2\mathbf{k}$ and normal to the given plane is

$$\mathbf{r} = \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \quad .....(i)$$

This meets the plane at a point for which we must have $((\mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})).(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 15$

$$6 + \lambda(9) = 15 \Rightarrow \lambda = 1.$$

Putting $\lambda = 1$ in (i), we obtain the position vector of the centre as $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$. Hence, the coordinates of the centre of the circle are (1, 3, 4).

15. (d) Let l, m, n be the d.c's of $\mathbf{r}$. Then $l = m = n$, (given)

$$l^2 + m^2 + n^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}} = m = n$$

Now, $\mathbf{r} = |\mathbf{r}|(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) = 6\left(\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}\right)$

Hence, $\mathbf{r} = 2\sqrt{3}(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

16. (c) The required plane is

$$\{\mathbf{r} - (\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})\}\overrightarrow{PQ} = 0.$$

17. (b) The equation of a plane through the line of intersection of the planes $\mathbf{r}.\mathbf{a} = \lambda$ and $\mathbf{r}.\mathbf{b} = \mu$ can be written as

$(\mathbf{r}.\mathbf{a} - \lambda) + k(\mathbf{r}.\mathbf{b} - \mu) = 0$ or $\mathbf{r}.(\mathbf{a} + k\mathbf{b}) = \lambda + k\mu$ .....(i)

This passes through the origin, therefore

$$\mathbf{0}.(\mathbf{a} + k\mathbf{b}) = \lambda + \mu k \Rightarrow k = \frac{-\lambda}{\mu}$$

Putting the value of k in (i), we get the equation of the required plane as $\mathbf{r}.(\mu\mathbf{a} - \lambda\mathbf{b}) = 0 \Rightarrow \mathbf{r}.(\lambda\mathbf{b} - \mu\mathbf{a}) = 0$.

18. (c) The position vectors of two given points are $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ the equation of the given plane is $\mathbf{r}.(5\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) + 9 = 0$ or $\mathbf{r}.\mathbf{n} + d = 0$.

We have, $\mathbf{a}.\mathbf{n} + d = (\mathbf{i} - \mathbf{j} + 3\mathbf{k}).(5\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) + 9$

$$= 5 - 2 - 21 + 9 < 0$$

and, $\mathbf{b}.\mathbf{n} + d = (3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}).(5\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) + 9$

$$= 15 + 6 - 21 + 9 > 0$$

So, the points $\mathbf{a}$ and $\mathbf{b}$ are on the opposite sides of the plane.

19. (b) The equation of a plane parallel to the plane $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) - 7 = 0$ is $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) + \lambda = 0$.

This passes through $2\mathbf{i} - \mathbf{j} - 4\mathbf{k}$.

Therefore, $(2\mathbf{i} - \mathbf{j} - 4\mathbf{k}).(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) + \lambda = 0$

$$8 + 12 + 12 + \lambda = 0 \Rightarrow \lambda = -32$$

So, the required plane is $\mathbf{r}.(4\mathbf{i} - 12\mathbf{j} - 3\mathbf{k}) - 32 = 0$.

20. (a) The vector equation of a plane through the line of intersection of the planes $\mathbf{r}.(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 0$ and $\mathbf{r}.(\mathbf{j} + 2\mathbf{k}) = 0$ can be written as

$(\mathbf{r}.(\mathbf{i} + 3\mathbf{j} - \mathbf{k})) + \lambda(\mathbf{r}.(\mathbf{j} + 2\mathbf{k})) = 0$ .....(i)

This passes through $2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$$(2\mathbf{i} + \mathbf{j} - \mathbf{k}).(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}).(\mathbf{j} + 2\mathbf{k}) = 0$$

or $(2 + 3 + 1) + \lambda(0 + 1 - 2) = 0 \Rightarrow \lambda = 6$

Put the value of λ in (i) we get

$\mathbf{r}.(\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}) = 0$, which is the required plane.

21. (b) The line of intersection of the planes $\mathbf{r}.(3\mathbf{i} - \mathbf{j} + \mathbf{k}) = 1$ and $\mathbf{r}.(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}) = 2$ is common to both the planes. Therefore, it is perpendicular to normals to the two planes *i.e.*, $\mathbf{n}_1 = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$. Hence it is parallel to the vector $\mathbf{n}_1 \times \mathbf{n}_2 = -2\mathbf{i} + 7\mathbf{j} + 13\mathbf{k}$. Thus, we have to find the equation of the plane passing through $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and normal to the vector $\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$. The equation of the required plane is $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ or $\mathbf{r}.\mathbf{n} = \mathbf{a}.\mathbf{n}$

or $\mathbf{r}.(-2\mathbf{i} + 7\mathbf{j} + 13\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - \mathbf{k}).(-2\mathbf{i} + 7\mathbf{j} + 13\mathbf{k})$

or $\mathbf{r}.(2\mathbf{i}-7\mathbf{j}-13\mathbf{k})=1$.

22. (a) The required plane passes through a point having position vector $\mathbf{a}_1$ and is parallel to the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. If $\mathbf{r}$ is the position vector of any point on the plane, then $\mathbf{r}-\mathbf{a}_1, \mathbf{a}_1, \mathbf{a}_2$ are coplanar.
Therefore, $(\mathbf{r}-\mathbf{a}_1).(\mathbf{a}_1\times\mathbf{a}_2)=0$

$[\mathbf{r}\ \mathbf{a}_1\ \mathbf{a}_2]=[\mathbf{a}_1\ \mathbf{a}_1\ \mathbf{a}_2] \Rightarrow [\mathbf{r}\mathbf{a}_1\ \mathbf{a}_2]=0$

Hence, the required plane is $[\mathbf{r}\ \mathbf{a}_1\ \mathbf{a}_2]=0$.

23. (b) Given two lines $\mathbf{r}=(\mathbf{i}+\mathbf{j})+\lambda(\mathbf{i}+2\mathbf{j}-\mathbf{k})$ and $\mathbf{r}=(\mathbf{i}+\mathbf{j})+\mu(-\mathbf{i}+\mathbf{j}-2\mathbf{k})$ pass through $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and are parallel to the vectors $\mathbf{b}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{c}=-\mathbf{i}+\mathbf{j}-2\mathbf{k}$ respectively. Therefore the plane containing them passes through $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and is perpendicular to
$\mathbf{n}=\mathbf{b}\times\mathbf{c}=(\mathbf{i}+2\mathbf{j}-\mathbf{k})\times(-\mathbf{i}+\mathbf{j}-2\mathbf{k})=-3\mathbf{i}+3\mathbf{j}+3\mathbf{k}$.
Hence, the equation of the plane is
$(\mathbf{r}-\mathbf{a}).\mathbf{n}=0 \Rightarrow \mathbf{r}.\mathbf{n}=\mathbf{a}.\mathbf{n} \Rightarrow \mathbf{r}.(\mathbf{i}-\mathbf{j}-\mathbf{k})=0$.

24. (c) We have $\mathbf{r}=(1+\lambda-\mu)\mathbf{i}+(2-\lambda)\mathbf{j}+(3-2\lambda+2\mu)\mathbf{k}$

$\mathbf{r}=(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\lambda(\mathbf{i}-\mathbf{j}-2\mathbf{k})+\mu(-\mathbf{i}+2\mathbf{k})$,

which is a plane passing through $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and parallel to the vectors $b=\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\mathbf{c}=-\mathbf{i}+2\mathbf{k}$
Therefore, it is perpendicular to the vector
$\mathbf{n}=\mathbf{b}\times\mathbf{c}=-2\mathbf{i}-\mathbf{k}$
Hence, its vector equation is $(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$

$\mathbf{r}.\mathbf{n}=\mathbf{a}.\mathbf{n} \Rightarrow \mathbf{r}.(-2\mathbf{i}-\mathbf{k})=-2-3 \Rightarrow \mathbf{r}.(2\mathbf{i}+\mathbf{k})=5$

So, the cartesian equation is $(x\mathbf{i}+y\mathbf{j}+z\mathbf{k}).(2\mathbf{i}+\mathbf{k})=5$
or $2x+z=5$.

25. (a) The vector equation of the plane passing through points $\mathbf{a},\mathbf{b},\mathbf{c}$ is
$\mathbf{r}.(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$
Therefore, the length of the perpendicular from the origin to this plane is given by $\dfrac{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}$.

26. (c) The given plane passes through $\mathbf{a}$ and is parallel to the vectors $\mathbf{b}-\mathbf{a}$ and $\mathbf{c}$. So it is normal to $(\mathbf{b}-\mathbf{a})\times\mathbf{c}$. Hence, its equation is
$(\mathbf{r}-\mathbf{a}).((\mathbf{b}-\mathbf{a})\times\mathbf{c})=0$
or $\mathbf{r}.(\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$
The length of the perpendicular from the origin to this plane is $\dfrac{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}{|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}$.

27. (b) The equation of a line passing through the points $A(\mathbf{i}-\mathbf{j}+2\mathbf{k})$ and $B(3\mathbf{i}+\mathbf{j}+\mathbf{k})$ is
$\mathbf{r}=(\mathbf{i}-\mathbf{j}+2\mathbf{k})+\lambda(3\mathbf{i}+\mathbf{j}+\mathbf{k})$
The position vector of any point P which is a variable point on the line, is $(\mathbf{i}-\mathbf{j}+2\mathbf{k})+\lambda(3\mathbf{i}+\mathbf{j}+\mathbf{k})$

$\overrightarrow{AP}=\lambda(3\mathbf{i}+\mathbf{j}+\mathbf{k}) \Rightarrow |\overrightarrow{AP}|=\lambda\sqrt{11}$

Now, if $\lambda\ \sqrt{11}=3\sqrt{11}$ *i.e.,* $\lambda=3$ then the position vector of the point P is $10\mathbf{i}+2\mathbf{j}+5\mathbf{k}$.

If $\lambda\sqrt{11}=-3\sqrt{11}$, *i.e.,* $\lambda=-3$ then the position vector of the point P is $-8\mathbf{i}-4\mathbf{j}-\mathbf{k}$.

28. (d) The equations of the lines joining $6\mathbf{a}-4\mathbf{b}+4\mathbf{c}, -4\mathbf{c}$ and $-\mathbf{a}-2\mathbf{b}-3\mathbf{c}, \mathbf{a}+2\mathbf{b}-5\mathbf{c}$ are respectively.

$\mathbf{r}=6\mathbf{a}-4\mathbf{b}+4\mathbf{c}+m(-6\mathbf{a}-4\mathbf{b}-8\mathbf{c})$ .....(i)

and $\mathbf{r}=-\mathbf{a}-2\mathbf{b}-3\mathbf{c}+n(2\mathbf{a}+4\mathbf{b}-2\mathbf{c})$ .....(ii)

For the point of intersection, the equations (i)and (ii) should give the same value of $\mathbf{r}$. Hence, equating the coefficients of vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ in the two expressions for $\mathbf{r}$, we get $6m+2n=7$, $2m-2n=1$ and $8m-2n=7$.
Solving first two equations, we get $m=1$, $n=\dfrac{1}{2}$. These values of m and n also satisfy the third equation. Hence, the lines intersect. Putting the value of m in (i), we get the position vector of the point of intersection as $-4\mathbf{c}$.

29. (d) Use the formula, $\sin\theta=\dfrac{\mathbf{n}.\mathbf{b}}{|\mathbf{n}||\mathbf{b}|}$.

30. (d) The required line passes through the point $\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and is perpendicular to the lines

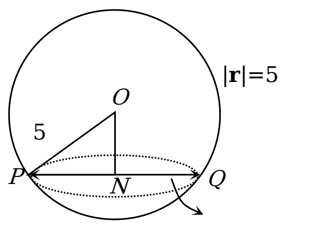
$\mathbf{r}=(\mathbf{i}+2\mathbf{j}-\mathbf{k})+\lambda(2\mathbf{i}+\mathbf{j}+\mathbf{k})$

and $\mathbf{r}=(2\mathbf{i}+6\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$, therefore it is parallel to the vector
$\mathbf{b}=(2\mathbf{i}+\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+3\mathbf{k})=(\mathbf{i}-5\mathbf{j}+3\mathbf{k})$
Hence, the equation of the required line is
$\mathbf{r}=(\mathbf{i}+3\mathbf{j}+2\mathbf{k})+\lambda'(\mathbf{i}-5\mathbf{j}+3\mathbf{k})$

$\mathbf{r}=(\mathbf{i}+3\mathbf{j}+2\mathbf{k})+\lambda(-\mathbf{i}+5\mathbf{j}-3\mathbf{k})$, where $\lambda=-\lambda'$.

31. (a) We have $\overrightarrow{AP}=-3\mathbf{i}-\mathbf{j}+10\mathbf{k}$

$|\overrightarrow{AP}|=\sqrt{9+1+100}=\sqrt{110}$



$\mathbf{r}=(\mathbf{i}+\mathbf{j}+\mathbf{k})=3\sqrt{3}$

$AN=$ Projection of $\overrightarrow{AP}$ on $6\mathbf{i}+3\mathbf{j}-4\mathbf{k}$

$$=\left|\frac{\overrightarrow{AP}.(6\mathbf{i}+3\mathbf{j}-4\mathbf{k})}{|6\mathbf{i}+3\mathbf{j}-4\mathbf{k}|}\right|=\left|\frac{-18-3-40}{\sqrt{61}}\right|=\sqrt{61}$$

$PN=\sqrt{AP^2-AN^2}=\sqrt{110-61}=7$.

32. (b) The vector equation of the line joining the points $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $-2\mathbf{j}+3\mathbf{k}$ is
$\mathbf{r}=(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\lambda(-\mathbf{i}+2\mathbf{k})$

.....(i)

The vector equation of the plane through the origin, $4\mathbf{j}$ and $2\mathbf{i}+\mathbf{k}$ is $\mathbf{r}.(4\mathbf{i}-8\mathbf{k})=0$ .....(ii)

(Using $\mathbf{r}.(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})=[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$)

The position vector of any point on (i) is $(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\lambda(-\mathbf{i}+2\mathbf{k})$.

If it lies on (ii), then

$$((\mathbf{i}-2\mathbf{j}+\mathbf{k})+\lambda(-\mathbf{i}+2\mathbf{k})).(4\mathbf{i}-8\mathbf{k})=0$$

$$-4-20\lambda=0\Rightarrow\lambda=-\frac{1}{5}$$

Putting the value of λ in $(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\lambda(-\mathbf{i}+2\mathbf{k})$, we get the position vector of the required point as $\frac{1}{5}(6\mathbf{i}-10\mathbf{j}+3\mathbf{k})$.

33. (b) The two planes are on the opposite side of the origin. Therefore, if p_1 and p_2 are the lengths of the perpendicular from the origin to the planes $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-2\mathbf{k})+5=0$ and $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-2\mathbf{k})-8=0$ respectively, then the required distance is given by $p_1+p_2=\frac{5}{3}+\frac{8}{3}=\frac{13}{3}$ *unit.*

34. (a) The position vector of any point on the given line is $\mathbf{i}+\mathbf{j}+\lambda(2\mathbf{i}+\mathbf{j}+4\mathbf{k})$ or $(2\lambda+1)\mathbf{i}+(\lambda+1)\mathbf{j}+4\lambda\mathbf{k}$ which lies on $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-\mathbf{k})=3$.

Hence, the plane $\mathbf{r}.(\mathbf{i}+2\mathbf{j}-\mathbf{k})=3$ contains the given line.

35. (c) Since the equation $|\mathbf{r}|^2-2(\mathbf{r}.\mathbf{a})+\lambda=0$ represents a sphere of radius $\sqrt{|\mathbf{a}|^2-\lambda}$, therefore $|\mathbf{r}|^2-\mathbf{r}.(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})-10=0$ represents a sphere of radius

$$=\sqrt{|\mathbf{i}+2\mathbf{j}-\mathbf{k}|^2+10}=\sqrt{6+10}=4.$$

36. (d) It is obvious.

37. (c) The given lines are $\mathbf{r}=\mathbf{a}_1+\lambda\mathbf{b}_1, \mathbf{r}=\mathbf{a}_2+\mu\mathbf{b}_2$,

where $\mathbf{a}_1=3\mathbf{i}-2\mathbf{j}-2\mathbf{k},\ \mathbf{b}_1=\mathbf{i}$

$\mathbf{a}_2=\mathbf{i}-\mathbf{j}+2\mathbf{k},\ \mathbf{b}_2=\mathbf{j}$

$|\mathbf{b}_1\times\mathbf{b}_2|=|\mathbf{i}\times\mathbf{j}|=|\mathbf{k}|=1$

Now, $[(\mathbf{a}_2-\mathbf{a}_1)\,\mathbf{b}_1\,\mathbf{b}_2]=(\mathbf{a}_2-\mathbf{a}_1).(\mathbf{b}_1\times\mathbf{b}_2)$

$=(-2\mathbf{i}+\mathbf{j}+4\mathbf{k})(\mathbf{k})=4$

Shortest distance

$$=\frac{[(\mathbf{a}_2-\mathbf{a}_1)(\mathbf{b}_1-\mathbf{b}_2)]}{|\mathbf{b}_1\times\mathbf{b}_2|}=\frac{4}{1}=4.$$

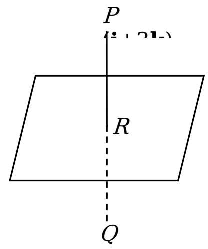
38. (c) It is obvious.

39. (d) Required distance

$$=\left|\frac{d-\mathbf{a}.\mathbf{n}}{|\mathbf{n}|}\right|=\left|\frac{5-(2\mathbf{i}-2\mathbf{j}+3\mathbf{k}).(\mathbf{i}+5\mathbf{j}+\mathbf{k})}{\sqrt{1+25+1}}\right|$$

$$=\left|\frac{5-(2-10+3)}{\sqrt{27}}\right|=\frac{10}{3\sqrt{3}}.$$

40. (c) Let Q be the image of the point $P(\mathbf{i}+3\mathbf{k})$ in the plane $\mathbf{r}.(\mathbf{i}+\mathbf{j}+\mathbf{k})=1$. Then PQ is normal to the plane. Since PQ passes through P and in normal to the given plane, therefore equation of PQ is $\mathbf{r}=(\mathbf{i}+3\mathbf{k})+\lambda(\mathbf{i}+\mathbf{j}+\mathbf{k})$



Since, Q lies on the line PQ, so, let the position vector of Q be $(\mathbf{i}+3\mathbf{k})+\lambda(\mathbf{i}+\mathbf{j}+\mathbf{k})$

$(1+\lambda)\mathbf{i}+\lambda\mathbf{j}+(3+\lambda)\mathbf{k}$.

Since R is the mid point of PQ, therefore position vector of R is

$$\frac{(1+\lambda)\mathbf{i}+\lambda\mathbf{j}+(3+\lambda)\mathbf{k}+\mathbf{i}+3\mathbf{k}}{2}$$

or $\left(\frac{\lambda+2}{2}\right)\mathbf{i}+\left(\frac{\lambda}{2}\right)\mathbf{j}+\left(\frac{6+\lambda}{2}\right)\mathbf{k}$

or $\left(\frac{\lambda}{2}+1\right)\mathbf{i}+\left(\frac{\lambda}{2}\right)\mathbf{j}+\left(3+\frac{\lambda}{2}\right)\mathbf{k}$

Since R lies on the plane $\mathbf{r}.(\mathbf{i}+\mathbf{j}+\mathbf{k})=1$

Therefore,

$$\left[\left(\frac{\lambda}{2}+1\right)\mathbf{i}+\left(\frac{\lambda}{2}\right)\mathbf{j}+\left(3+\frac{\lambda}{2}\right)\mathbf{k}\right].[\mathbf{i}+\mathbf{j}+\mathbf{k}]=1$$

$$\left[\frac{\lambda}{2}+1+\frac{\lambda}{2}+3+\frac{\lambda}{2}\right]=1 \qquad \lambda=-2$$

So, the position vector of Q is

$(\mathbf{i}+3\mathbf{k})-2(\mathbf{i}+\mathbf{j}+\mathbf{k})=-\mathbf{i}-2\mathbf{j}+\mathbf{k}$.

41. (a) Let the equation of plane is

$a(x+1)+b(y+2)+c(z-0)=0$ .....(i)

As it passes through (2, 3, 5)

so, $3a+5b+5c=0$ .....(ii)

also, $2a+5b-c=0$ .....(iii)

$$\therefore \frac{a}{-5-25}=\frac{b}{10+3}=\frac{c}{15-10}$$

$$\therefore \frac{a}{-30}=\frac{b}{13}=\frac{c}{5}$$

Hence equation of plane is, $-30x+13y+5z=4$

or $\mathbf{r}.(-30\mathbf{i}+13\mathbf{j}+5\mathbf{k})=4$.

42. (d) The Given lines are $\mathbf{r}_1=\mathbf{a}_1+\lambda\mathbf{b}_1,\ \mathbf{r}_2=\mathbf{a}_2+\mu\mathbf{b}_2$

Where $\mathbf{a}_1=4\mathbf{i}-3\mathbf{j}-\mathbf{k};\ \ \mathbf{b}_1=\mathbf{i}-4\mathbf{j}+7\mathbf{k}$

$\mathbf{a}_2=\mathbf{i}-\mathbf{j}-10\mathbf{k};\ \ \mathbf{b}_2=2\mathbf{i}-3\mathbf{j}+8\mathbf{k}$

$$|\mathbf{b}_1\times\mathbf{b}_2|=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-4&7\\2&-3&8\end{vmatrix}=-11\mathbf{i}+6\mathbf{j}+5\mathbf{k}$$

Now $[(\mathbf{a}_2-\mathbf{a}_1)\,\mathbf{b}_1\,\mathbf{b}_2]=(\mathbf{a}_2-\mathbf{a}_1).(\mathbf{b}_1\times\mathbf{b}_2)$

$= (-3\mathbf{i} + 2\mathbf{j} - 9\mathbf{k})(-11\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}) = 0$

Therefore, shortest distance $= \frac{[(\mathbf{a}_2 - \mathbf{a}_1)\ \mathbf{b}_1\ \mathbf{b}_2]}{|\mathbf{b}_1 \times \mathbf{b}_2|} = 0$.

43. (b) The point of the given line is $(1+t, -1+t, 1-t)$

Equation of plane is, $x + y + z = 5$

The point of the given line satisfies the equation of plane

$\therefore (1+t) + (-1+t) + (1-t) = 5 \quad 1 + t = 5 \quad t = 4$

Points are $(5, 3, -3)$

Hence, position vector of point is, $5\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$.

44. (a) Centroid of ΔPQR is $2\mathbf{i} - 5\mathbf{j} + 8\mathbf{k}$

$\therefore$ Intercepts on **x**, **y** and **z** axis are $6\mathbf{i}, -15\mathbf{j}$ and $24\mathbf{k}$ respectively.

Hence equation of plane is,

$[\mathbf{r} - 15\mathbf{j}\ \ 24\mathbf{k}] + [\mathbf{r}\ \ 24\mathbf{k}\ \ 6\mathbf{i}] + [\mathbf{r}\ \ 6\mathbf{i} - 15\mathbf{j}] = [6\mathbf{i}\ \ -15\mathbf{j}\ \ 24\mathbf{k}]$

$\therefore\ -\mathbf{r}.(20\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}) = -120$

$\therefore\ \mathbf{r}.(20\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}) = 120.$

45. (b) The line of intersection of the planes $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = 1$ and $\mathbf{r}.(2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) = 2$ is perpendicular to each of the normal vectors $\mathbf{n}_1 = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{n}_2 = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$

$\therefore$ It is parallel to the vector

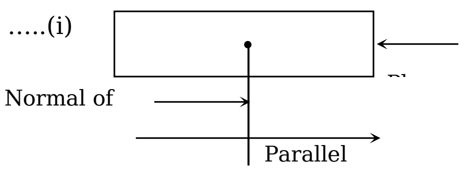
$\mathbf{n}_1 \times \mathbf{n}_2 = (\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 1 \\ 2 & 5 & -3 \end{vmatrix} = 4\mathbf{i} + 5\mathbf{j} + 11\mathbf{k}.$$

46. (a) As plane is parallel to a given vector Normal of plane must perpendicular to the given vectors. Given point to which plane passes through is (2, –1,3).

Let A, B, C are direction ratios of its normal.

Equation of plane is, $A(x-2) + B(y+1) + C(z-3) = 0$ .....(i)



Now normal to plane $A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ is perpendicular to the given vectors $\mathbf{a} = 3\mathbf{i} + 0\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$3A + 0B - C = 0$ .....(i)

$-3A + 2B + 2C = 0$ .....(ii)

Solving (i) and (ii) we get, $\frac{A}{2} = \frac{B}{-3} = \frac{C}{6}$

Equation of plane be $2(x-2) - 3(y+1) + 6(z-3) = 0$

i.e., $2x - 3y + 6z - 25 = 0$.

47. (d) Required ratio $= -\left(\frac{x_1}{x_2}\right) = \frac{-9}{1}$ *i.e.*, – 9 : 1.

Critical Thinking Questions

1. (c) Three vectors meeting at a point are $\mathbf{i} + \mathbf{j}, \mathbf{j} + \mathbf{k}, \mathbf{k} + \mathbf{i}$. Forces of 1, 2, 3 dynes are acting along these directions respectively, therefore resultant force

$= \frac{\mathbf{i} + \mathbf{j}}{\sqrt{2}} + \frac{2(\mathbf{j} + \mathbf{k})}{\sqrt{2}} + \frac{3(\mathbf{k} + \mathbf{i})}{\sqrt{2}} = \frac{4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}}{\sqrt{2}}$,

$\therefore$ Magnitute $= \frac{5\sqrt{2}}{\sqrt{2}} = 5$ dyne.

2. (b) Clearly, $\mathbf{b} \perp \mathbf{c}$, $\therefore\ \mathbf{b}.\mathbf{c} = 0$

Now, $\mathbf{d} = \mathbf{c} - \mathbf{b} \Rightarrow |\mathbf{d}|^2 = |\mathbf{c} - \mathbf{b}|^2$

$= |\mathbf{c}|^2 + |\mathbf{b}|^2 - 2\mathbf{b}.\mathbf{c} = 16 + 16 - 0$

$|\mathbf{d}| = \sqrt{32} = 4\sqrt{2}$ and direction of **d** is west.

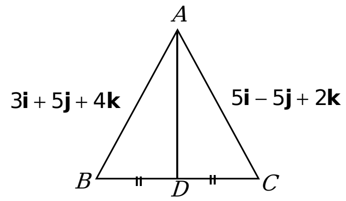
3. (b) $|\mathbf{a} - \mathbf{b}|^2 + |\mathbf{b} - \mathbf{c}|^2 + |\mathbf{c} - \mathbf{a}|^2$

$= 2(\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2) - 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a})$

$= 2 \times 3 - 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a})$

$= 6 - \{(\mathbf{a} + \mathbf{b} + \mathbf{c})^2 - \mathbf{a}^2 - \mathbf{b}^2 - \mathbf{c}^2\} = 9 - |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 \le 9$.

4. (c)



$P.V.$ of $\overrightarrow{AD} = \frac{(3+5)\mathbf{i} + (5-5)\mathbf{j} + (4+2)\mathbf{k}}{2}$

$\overrightarrow{AD} = \frac{8\mathbf{i} + 6\mathbf{k}}{2} = 4\mathbf{i} + 3\mathbf{k}$

Length of median $= |\overrightarrow{AD}| = \sqrt{16+9} = 5$ *unit.*

5. (b) Here, $3\mathbf{p} = (3x + 12y)\mathbf{a} + (6x + 3y + 3)\mathbf{b}$

$2\mathbf{q} = (2y - 4x + 4)\mathbf{a} + (4x - 6y - 2)\mathbf{b}$

On comparing, we get $3x + 12y = 2y - 4x + 4$

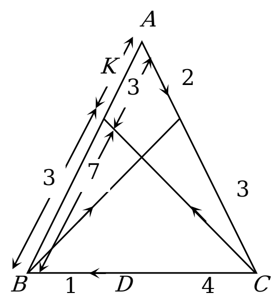
$7x + 10y = 4$ .....(i)

and $2x + 9y = -5$ .....(ii)

On solving equations, we get $x = 2, -1$.

6. (b) Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AC} = \mathbf{b}$

So, $\overrightarrow{AD} = \frac{4\mathbf{a} + \mathbf{b}}{5}$, $\overrightarrow{AE} = \frac{2\mathbf{b}}{5}$, $\overrightarrow{AF} = \frac{3\mathbf{a}}{10}$, and $\overrightarrow{AK} = \frac{\mathbf{a}}{4}$



$$\frac{\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}}{\overrightarrow{CK}}=\frac{\frac{\mathbf{b}+4\mathbf{a}}{5}+\frac{2\mathbf{b}-5\mathbf{a}}{5}+\frac{3\mathbf{a}-10\mathbf{b}}{10}}{\frac{\mathbf{a}-4\mathbf{b}}{4}}$$

$$=\frac{6\mathbf{b}-2\mathbf{a}+3\mathbf{a}-10\mathbf{b}}{10(\mathbf{a}-4\mathbf{b})}\times 4=\frac{2}{5}.$$

7. (e) Since, no vector given in options is collinear with the given vectors. Therefore all vectors can be third vertex of the triangle.

8. (b) Let $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

$$|\mathbf{a}|=\sqrt{x^2+y^2+z^2}=50,\quad \mathbf{b}=6\mathbf{i}-8\mathbf{j}-\frac{15}{2}\mathbf{k}$$

Since $\mathbf{a}$ and $\mathbf{b}$ are collinear, so $\mathbf{a}=k\mathbf{b}$ and

$$\frac{x}{6}=\frac{y}{-8}=\frac{2z}{-15}=k,\ \text{(constant)}$$

$$2500=k^2\left[\frac{144+256+225}{4}\right]$$

$$k=\pm\sqrt{\frac{2500\times 4}{625}}=\pm 4$$

Since $\mathbf{a}$ makes an acute angle with the direction of z-axis, Hence, its z-component must be positive. This is possible only when $k=-4$.

$$\mathbf{a}=k\left[6\mathbf{i}-8\mathbf{j}-\frac{15}{2}\mathbf{k}\right],\quad [\because \mathbf{a}=k\mathbf{b}]$$

Hence, $\mathbf{a}=-24\mathbf{i}+32\mathbf{j}+30\mathbf{k}$.

9. (d) $|\mathbf{c}|=1$, we have $|\mathbf{c}|^2=1$ or $c_1^2+c_2^2+c_3^2=1$ .....(i)

Again, since $\mathbf{c}\perp\mathbf{a}$ and $\mathbf{c}\perp\mathbf{b}$, we have $\mathbf{c}.\mathbf{a}=0$

$\Rightarrow a_1c_1+a_2c_2+a_3c_3=0$ .....(ii)

and $\mathbf{c}.\mathbf{b}=0\Rightarrow b_1c_1+b_2c_2+b_3c_3=0$ .....(iii)

Also since angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, we have $\mathbf{a}.\mathbf{b}=a_1b_1+a_2b_2+a_3b_3$

$$|\mathbf{a}||\mathbf{b}|\cos\frac{\pi}{6}=a_1b_1+a_2b_2+a_3b_3$$

$$\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)=(a_1b_1+a_2b_2+a_3b_3)^2 \quad \text{.....(iv)}$$

Now, $\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}^2=\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$$=\begin{vmatrix} a_1^2+a_2^2+a_3^2 & a_1b_1+a_2b_2+a_3b_3 & 0\\ b_1a_1+b_2a_2+b_3a_3 & b_1^2+b_2^2+b_3^2 & 0\\ 0 & 0 & 1\end{vmatrix}$$

{Using (i), (ii) and (iii)}

$$=\frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2),\ \text{\{Using (iv)\}}$$

$$=\frac{(\Sigma a_1^2)(\Sigma b_1^2)}{4},$$

where $\Sigma a_1^2=a_1^2+a_2^2+a_3^2$ and $\Sigma b_1^2=b_1^2+b_2^2+b_3^2$.

10. (b) $\mathbf{c}=\alpha\mathbf{a}+\beta\mathbf{b}+\gamma(\mathbf{a}\times\mathbf{b})\Rightarrow \mathbf{c}.\mathbf{a}=\alpha$ and $\mathbf{c}.\mathbf{b}=\beta$

$\Rightarrow \alpha=\beta=\cos\theta$

Also, $1=\mathbf{c}.\mathbf{c}$,

$\therefore [\alpha\mathbf{a}+\beta\mathbf{b}+\gamma(\mathbf{a}\times\mathbf{b})].[(\alpha\mathbf{a}+\beta\mathbf{b})+\gamma(\mathbf{a}\times\mathbf{b})]=1$

$\Rightarrow 2\alpha^2+\gamma^2(\mathbf{a}\times\mathbf{b})^2=1,\quad \{\because \alpha=\beta\}$

$\Rightarrow 2\alpha^2+\gamma^2[\mathbf{a}^2\mathbf{b}^2-(\mathbf{a}.\mathbf{b})^2]=1\Rightarrow 2\alpha^2+\gamma^2=1$

Hence, $\gamma^2=1-2\alpha^2=1-2\cos^2\theta=-\cos 2\theta$.

11. (b) Since the angle between $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}$ and the angle between $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}$ are the same, so we have

$$\frac{(\mathbf{a}+\mathbf{b}).\mathbf{a}}{|\mathbf{a}+\mathbf{b}||\mathbf{a}|}=\frac{(\mathbf{a}+\mathbf{b}).\mathbf{b}}{|\mathbf{a}+\mathbf{b}||\mathbf{b}|}$$

$$\Rightarrow \frac{|\mathbf{a}|^2}{|\mathbf{a}+\mathbf{b}||\mathbf{a}|}+\frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}+\mathbf{b}||\mathbf{a}|}=\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}+\mathbf{b}||\mathbf{b}|}+\frac{|\mathbf{b}|^2}{|\mathbf{a}+\mathbf{b}||\mathbf{b}|}$$

$$\Rightarrow \frac{|\mathbf{a}|-|\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|}\left(1-\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)=0$$

Hence $|\mathbf{a}|=|\mathbf{b}|$ or angle between $\mathbf{a}$ and $\mathbf{b}$ is 0.

12. (d) We have $\overrightarrow{BD}=\overrightarrow{OD}-\overrightarrow{OB}=\mathbf{a}-2\mathbf{b}-\mathbf{b}=\mathbf{a}-3\mathbf{b}$ and $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=2\mathbf{a}+3\mathbf{b}-\mathbf{a}=\mathbf{a}+3\mathbf{b}$.

Let θ be the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$.

Then $\cos\theta=\frac{\overrightarrow{BD}.\overrightarrow{AC}}{|\overrightarrow{BD}||\overrightarrow{AC}|}=\frac{|\mathbf{a}|^2-9|\mathbf{b}|^2}{|\overrightarrow{BD}||\overrightarrow{AC}|}$

$$=\frac{9|\mathbf{b}|^2-9|\mathbf{b}|^2}{|\overrightarrow{BD}||\overrightarrow{AC}|},\quad (\because |\mathbf{a}|=3|\mathbf{b}|)$$

$$\Rightarrow \cos\theta=0^\circ\Rightarrow\theta=\frac{\pi}{2}.$$

13. (c) $\vec{A}+t\vec{B}=(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+t(-\mathbf{i}+2\mathbf{j}+\mathbf{k})$

$$=\mathbf{i}(1-t)+\mathbf{j}(2+2t)+\mathbf{k}(3+t)$$

But it is perpendicular to $\vec{C}=3\mathbf{i}+\mathbf{j}$,

So, $\vec{C}.(\vec{A}+t\vec{B})=0\Rightarrow 3(1-t)+2+2t=0\Rightarrow t=5$.

14. (d) Let $\mathbf{r}=\lambda\mathbf{b}+\mu\mathbf{c}$ and $\mathbf{c}=\pm(x\mathbf{i}+y\mathbf{j})$. Since $\mathbf{c}$ and $\mathbf{b}$ are perpendicular, we have $4x+3y=0$

$\Rightarrow \mathbf{c} = \pm x\left(\mathbf{i} - \frac{4}{3}\mathbf{j}\right)$, $\{\because y = -\frac{4}{3}x\}$

Now, projection of $\mathbf{r}$ on $\mathbf{b} = \frac{\mathbf{r}.\mathbf{b}}{|\mathbf{b}|} = 1$

$\Rightarrow \frac{(\lambda\mathbf{b} + \mu\mathbf{c}).\mathbf{b}}{|\mathbf{b}|} = \frac{\lambda\mathbf{b}.\mathbf{b}}{|\mathbf{b}|} = 1 \Rightarrow \lambda = \frac{1}{5}$

Again, projection of $\mathbf{r}$ on $\mathbf{c} = \frac{\mathbf{r}.\mathbf{c}}{|\mathbf{c}|} = 2$

This gives $\mu x = \frac{6}{5} \Rightarrow \mathbf{r} = \frac{1}{5}(4\mathbf{i} + 3\mathbf{j}) + \frac{6}{5}\left(\mathbf{i} - \frac{4}{3}\mathbf{j}\right) = 2\mathbf{i} - \mathbf{j}$

or $\mathbf{r} = \frac{1}{5}(4\mathbf{i} + 3\mathbf{j}) - \frac{6}{5}\left(\mathbf{i} - \frac{4}{3}\mathbf{j}\right) = -\frac{2}{5}\mathbf{i} + \frac{11}{5}\mathbf{j}$.

15. (a,c) Any vector $\mathbf{r}$ in the plane of $\mathbf{b}$ and $\mathbf{c}$ is $\mathbf{r} = \mathbf{b} + t\mathbf{c}$ or $\mathbf{r} = (1+t)\mathbf{i} + (2+t)\mathbf{j} - (1+2t)\mathbf{k}$ ......(i)

Projection of $\mathbf{r}$ on $\mathbf{a}$ is $\sqrt{\left(\frac{2}{3}\right)} \Rightarrow \frac{\mathbf{r}.\mathbf{a}}{|\mathbf{a}|} = \sqrt{\left(\frac{2}{3}\right)}$

or $\frac{2(1+t) - (2+t) - (1+2t)}{\sqrt{6}} = \pm\sqrt{\left(\frac{2}{3}\right)}$

$-t - 1 = \pm 2 \Rightarrow t = -3, 1$

Projection in (i),we get

$\therefore \mathbf{r} = -2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ or $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$.

16. (b) If x, y are the original components; X, Y the new components and α is the angle of rotation, then $x = X\cos\alpha - Y\sin\alpha$ and $y = X\sin\alpha + Y\cos\alpha$

$\therefore 2p = (p+1)\cos\alpha - \sin\alpha$ and $1 = (p+1)\sin\alpha + \cos\alpha$

Squaring and adding, we get $4p^2 + 1 = (p+1)^2 + 1$

$\Rightarrow p + 1 = \pm 2p \Rightarrow p = 1$ or $-\frac{1}{3}$.

17. (b) $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Let vector $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{c}.\mathbf{u} = 0$

$\Rightarrow 2c_1 + 2c_2 - c_3 = 0$ .....(i)

and $\mathbf{c}.\mathbf{v} = 0 \Rightarrow 6c_1 - 3c_2 + 2c_3 = 0$ .....(ii)

Solving equation (i) and (ii) by cross multiplication

$\frac{c_1}{4-3} = \frac{c_2}{-6-4} = \frac{c_3}{-6-12} = \lambda$, (say)

$\Rightarrow \frac{c_1}{1} = \frac{c_2}{-10} = \frac{c_3}{-18} = \lambda$

$\Rightarrow c_1 = \lambda$, $c_2 = -10\lambda$ and $c_3 = -18\lambda$

Thus $\mathbf{c} = \lambda(\mathbf{i} - 10\mathbf{j} - 18\mathbf{k})$

$|\mathbf{c}| = \lambda\sqrt{1 + 100 + 324} = \lambda\sqrt{425}$

Hence required unit vector is, $\frac{\mathbf{c}}{|\mathbf{c}|}$

$= \frac{\lambda(\mathbf{i} - 10\mathbf{j} - 18\mathbf{k})}{\lambda\sqrt{425}} = \frac{1}{\sqrt{425}}(\mathbf{i} - 10\mathbf{j} - 18\mathbf{k})$

$= \frac{1}{5\sqrt{17}}(\mathbf{i} - 10\mathbf{j} - 18\mathbf{k}) = \frac{1}{\sqrt{17}}\left(\frac{1}{5}\mathbf{i} - 2\mathbf{j} - \frac{18}{5}\mathbf{k}\right)$

Aliter : Required vector is $\frac{\mathbf{u} \times \mathbf{v}}{|\mathbf{u} \times \mathbf{v}|} = \frac{\mathbf{i} - 10\mathbf{j} - 18\mathbf{k}}{\sqrt{425}}$.

18. (d) $\mathbf{d} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ gives $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$,

where $\mathbf{d} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ (say)

On solving, $x = -1$, $y = -8$, $z = 2$

Hence $\mathbf{d} = -\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$.

19. (b) Given, $\mathbf{a} \times \mathbf{r} = \mathbf{b} + \lambda\mathbf{a} \Rightarrow (\mathbf{a} \times \mathbf{r}).\mathbf{a} = \mathbf{b}.\mathbf{a} + \lambda\mathbf{a}.\mathbf{a}$

$\Rightarrow 0 = \mathbf{b}.\mathbf{a} + \lambda|\mathbf{a}|^2 \Rightarrow \lambda = -\frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}|^2} = \frac{5}{6}$

Also, $(\mathbf{a} \times \mathbf{r}) \times \mathbf{a} = \mathbf{b} \times \mathbf{a} + \lambda\mathbf{a} \times \mathbf{a} \Rightarrow \mathbf{r} = \frac{7}{6}\mathbf{i} + \frac{2}{3}\mathbf{j}$.

20. (a) A vector perpendicular to the plane P_1 of $\mathbf{a}$, $\mathbf{b}$ is $\mathbf{a} \times \mathbf{b}$

A vector perpendicular to the plane P_2 of $\mathbf{c}$, $\mathbf{d}$ is $\mathbf{c} \times \mathbf{d}$.

$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = 0$ $(\mathbf{a} \times \mathbf{b}) \parallel (\mathbf{c} \times \mathbf{d})$

The angle between the planes is 0^o.

21. (a) Let $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$

Now, $\mathbf{j} - \mathbf{k} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$b_3 - b_2 = 0, b_1 - b_3 = 1, b_2 - b_1 = -1$

$b_3 = b_2, b_1 = b_2 + 1$

Now, $\mathbf{a}.\mathbf{b} = 1 \Rightarrow b_1 + b_2 + b_3 = 1$

$3b_2 + 1 = 1 \Rightarrow b_2 = 0$ $b_1 = 1, b_3 = 0$.

Thus $\mathbf{b} = \mathbf{i}$.

22. (c) It is given that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are the position vectors of vertices of a quadrilateral *ABCD* respectively.

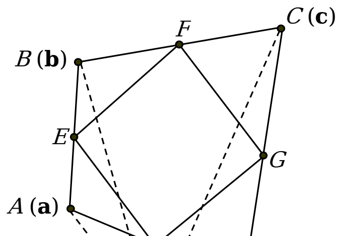
Let *E, F, G* and *H* are the middle points of sides *AB, BC, CD* and *DA* respectively.

The position vectors of these points will be $\overrightarrow{OE} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$, $\overrightarrow{OF} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$,

$\overrightarrow{OG} = \frac{1}{2}(\mathbf{c} + \mathbf{d})$, $\overrightarrow{OH} = \frac{1}{2}(\mathbf{a} + \mathbf{d})$

Then $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE} = \left(\frac{\mathbf{c} - \mathbf{a}}{2}\right)$

and $\overrightarrow{FG} = \frac{1}{2}(\mathbf{d} - \mathbf{b}), \overrightarrow{GH} = \frac{1}{2}(\mathbf{a} - \mathbf{c}), \overrightarrow{GH} = \frac{1}{2}(\mathbf{b} - \mathbf{d})$



It is clear that $\overrightarrow{EF}$ is parallel to $\overrightarrow{GH}$ and $\overrightarrow{FG}$ is parallel to $\overrightarrow{HE}$. Thus $EFGH$ is a parallelogram.

$\therefore \quad \overrightarrow{EF}\times\overrightarrow{FG}=\frac{1}{4}\{(\mathbf{c}-\mathbf{a})\times(\mathbf{d}-\mathbf{b})\}$

$$=\frac{1}{4}(\mathbf{c}\times\mathbf{d}-\mathbf{c}\times\mathbf{b}-\mathbf{a}\times\mathbf{d}+\mathbf{a}\times\mathbf{b})$$

$$=\frac{1}{4}(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{a})$$

$\therefore$ Area of parallelogram $EFGH$ is,

$A=|\overrightarrow{EF}\times\overrightarrow{FG}|=\frac{1}{4}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{d}+\mathbf{d}\times\mathbf{a}|$.

23. (a) Force $\mathbf{F}=\overrightarrow{AB}=(3-1)\mathbf{i}+(-4-2)\mathbf{j}+(2+3)\mathbf{k}$

$$=2\mathbf{i}-6\mathbf{j}+5\mathbf{k}$$

Moment of Force $\vec{F}$ *w.r.t* $M=\overrightarrow{MA}\times\vec{F}$

$\because \overrightarrow{MA}=(1+2)\mathbf{i}+(2-4)\mathbf{j}+(-3+6)\mathbf{k}=3\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

Now $\overrightarrow{MA}\times\vec{F}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & -2 & 3\\ 2 & -6 & 5\end{vmatrix}$

$=\mathbf{i}(-10+18)+\mathbf{j}(6-15)+\mathbf{k}(-18+4)=8\mathbf{i}-9\mathbf{j}-14\mathbf{k}$.

24. (d) Since $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}=0$

Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$,

we get $\begin{vmatrix}a & 1 & 1\\ 1-a & b-1 & 0\\ 1-a & 0 & c-1\end{vmatrix}=0$

On expanding, we get

$a(b-1)(c-1)-(1-a)(c-1)-(1-a)(b-1)=0$

On dividing by $(1-a)(1-b)(1-c)$, we get

$$\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=\frac{1}{1-a}-\frac{a}{1-a}=1.$$

25. (c) Let $\alpha\neq 0$, then $\alpha(\mathbf{a}\times\mathbf{b}).\mathbf{c}+\beta(\mathbf{b}\times\mathbf{c}).\mathbf{c}+\gamma(\mathbf{c}\times\mathbf{a}).\mathbf{c}=0$

$\Rightarrow \alpha[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=0\Rightarrow[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=0$, $\{\because \alpha\neq 0\}$

Hence $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are coplanar.

26. (b) Volume of tetrahedron $ABCD$ is, $\frac{1}{6}|\overrightarrow{AB}\times\overrightarrow{AC}.\overrightarrow{AD}|$, where $A(-1,1,1)$, $B(1,-1,1)$, $C(1,1,-1)$ and $D(0,0,0)$.

$$=\frac{1}{6}|(2\mathbf{i}-2\mathbf{j})\times(2\mathbf{i}-2\mathbf{k}).(\mathbf{i}-\mathbf{j}-\mathbf{k})|$$

$$=\frac{1}{6}\begin{vmatrix}2 & -2 & 0\\ 2 & 0 & -2\\ 1 & -1 & -1\end{vmatrix}=\frac{1}{6}(-4)=-\frac{2}{3}=\frac{2}{3} \text{ cubic unit.}$$

27. (c) Let $\mathbf{a}=\mathbf{i}-\mathbf{j}$, $\mathbf{b}=\mathbf{j}-\mathbf{k}$ and $\mathbf{c}=\mathbf{k}-\mathbf{i}$

Let $\hat{\mathbf{d}}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $|\hat{\mathbf{d}}|=\sqrt{a_1^2+a_2^2+a_3^2}=1$

$\Rightarrow a_1^2+a_2^2+a_3^2=1$ ......(i)

$\mathbf{a}.\hat{\mathbf{d}}=0\Rightarrow a_1-a_2=0$ .....(ii)

$[\mathbf{b}\,\mathbf{c}\,\hat{\mathbf{d}}]=0\Rightarrow \mathbf{b}.(\mathbf{c}\times\hat{\mathbf{d}})=0$

$$\Rightarrow \begin{vmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ a_1 & a_2 & a_3\end{vmatrix}=-1(-a_3-a_1)-1(-a_2)$$

$\therefore a_1+a_2+a_3=0\Rightarrow a_1-a_2+0a_3=0$, {from (ii)}

$\therefore \frac{a_1}{0+1}=\frac{a_2}{1-0}=\frac{a_3}{-1-1}\Rightarrow\frac{a_1}{1}=\frac{a_2}{1}=\frac{a_3}{-2}=\lambda$, (say)

$\Rightarrow a_1=\lambda,\ a_2=\lambda,\ a_3=-2\lambda$

$\therefore \lambda^2+\lambda^2+4\lambda^2=1$, {from (i)}

$\Rightarrow 6\lambda^2=1\Rightarrow\lambda=\pm\frac{1}{\sqrt{6}}$; $\therefore \hat{\mathbf{d}}=\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$.

28. (c) $V=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a\Rightarrow\frac{dV}{da}=3a^2-1$

$$=3\left(a+\frac{1}{\sqrt{3}}\right)\left(a-\frac{1}{\sqrt{3}}\right)$$

$\therefore$ Minimum at $\frac{1}{\sqrt{3}}$.

29. (a) Let $\mathbf{i}$ be a unit vector in the direction of $\mathbf{b}$, $\mathbf{j}$ in the direction of $\mathbf{c}$. Note that $\mathbf{b}=\mathbf{i}$ and $\mathbf{c}=\mathbf{j}$

We have $\mathbf{b}\times\mathbf{c}=|\mathbf{b}||\mathbf{c}|\sin\alpha\,\mathbf{k}=\sin\alpha\,\mathbf{k}$, where $\mathbf{k}$ is a unit vector perpendicular to $\mathbf{b}$ and $\mathbf{c}$.

$\Rightarrow |\mathbf{b}\times\mathbf{c}|=\sin\alpha\Rightarrow\mathbf{k}=\frac{\mathbf{b}\times\mathbf{c}}{|\mathbf{b}\times\mathbf{c}|}$

Any vector $\mathbf{a}$ can be written as a linear combination of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

Let $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$.

Now $\mathbf{a}.\mathbf{b}=\mathbf{a}.\mathbf{i}=a_1$, $\mathbf{a}.\mathbf{c}=\mathbf{a}.\mathbf{j}=a_2$

and $\mathbf{a}.\frac{\mathbf{b}\times\mathbf{c}}{|\mathbf{b}\times\mathbf{c}|}=\mathbf{a}.\mathbf{k}=a_3$

Thus $(\mathbf{a}.\mathbf{b})\mathbf{b}+(\mathbf{a}.\mathbf{c})\mathbf{c}+\frac{\mathbf{a}.(\mathbf{b}\times\mathbf{c})}{|\mathbf{b}\times\mathbf{c}|}(\mathbf{b}\times\mathbf{c}|$

$= a_1\mathbf{b} + a_2\mathbf{c} + a_3\dfrac{(\mathbf{b}\times\mathbf{c})}{|\mathbf{b}\times\mathbf{c}|} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = \mathbf{a}$.

30. (c) $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \dfrac{\mathbf{b}+\mathbf{c}}{\sqrt{2}} \Rightarrow (\mathbf{a}.\mathbf{c})\mathbf{b} - (\mathbf{a}.\mathbf{b})\mathbf{c} = \dfrac{\mathbf{b}+\mathbf{c}}{\sqrt{2}}$

$\Rightarrow \left[(\mathbf{a}.\mathbf{c}) - \dfrac{1}{\sqrt{2}}\right]\mathbf{b} - \left[(\mathbf{a}.\mathbf{b}) + \dfrac{1}{\sqrt{2}}\right]\mathbf{c} = 0$

$\Rightarrow \mathbf{a}.\mathbf{c} = \dfrac{1}{\sqrt{2}},\ \ \mathbf{a}.\mathbf{b} = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow |\mathbf{a}||\mathbf{c}|\cos\theta = \dfrac{1}{\sqrt{2}},\ |\mathbf{a}||\mathbf{b}|\cos\phi = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}},\ \ \cos\phi = -\dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4},\ \ \phi = \dfrac{3\pi}{4}$.

31. (c) We have $(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})$

$= ((\mathbf{a}\times\mathbf{b}).\mathbf{c})\mathbf{b} - ((\mathbf{a}\times\mathbf{b}).\mathbf{b})\mathbf{c} = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{b}$

$(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a}) = ((\mathbf{b}\times\mathbf{c}).\mathbf{a})\mathbf{c} - ((\mathbf{b}\times\mathbf{c}).\mathbf{c})\mathbf{a} = [\mathbf{b}\,\mathbf{c}\,\mathbf{a}]\mathbf{c}$

$(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b}) = ((\mathbf{c}\times\mathbf{a}).\mathbf{b})\mathbf{a} - ((\mathbf{c}\times\mathbf{a}).\mathbf{a})\mathbf{b} = [\mathbf{c}\,\mathbf{a}\,\mathbf{b}]\mathbf{a}$

$\therefore\ [(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]$

$= [[\mathbf{a}\mathbf{b}\mathbf{c}]\mathbf{a}[\mathbf{a}\mathbf{b}\mathbf{c}]\mathbf{b}[\mathbf{a}\mathbf{b}\mathbf{c}]\mathbf{c}] = [\mathbf{a}\mathbf{b}\mathbf{c}]^3[\mathbf{a}\mathbf{b}\mathbf{c}] = [\mathbf{a}\mathbf{b}\mathbf{c}]^4$.

32. (a,c) Since $\mathbf{a},\mathbf{b},\mathbf{c}$ are coplanar, hence $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$

Given $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = \dfrac{1}{6}\mathbf{i} - \dfrac{1}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$

$\Rightarrow [(\mathbf{a}\times\mathbf{b}).\mathbf{d}]\mathbf{c} - [(\mathbf{a}\times\mathbf{b}).\mathbf{c}]\mathbf{d} = \dfrac{1}{6}\mathbf{i} - \dfrac{1}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$

$\Rightarrow [(|\mathbf{a}||\mathbf{b}|\sin 30^\circ)\hat{\mathbf{n}}.\mathbf{d}]\mathbf{c} - 0 = \dfrac{1}{6}\mathbf{i} - \dfrac{1}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$

$\Rightarrow \left[(1)(1)\left(\dfrac{1}{2}\right)\right][|\hat{\mathbf{n}}||\mathbf{d}|\cos\theta]\mathbf{c} = \dfrac{1}{6}\mathbf{i} - \dfrac{1}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$

$\Rightarrow \dfrac{1}{2}\cos\theta(\mathbf{c}) = \dfrac{1}{6}\mathbf{i} - \dfrac{1}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$,

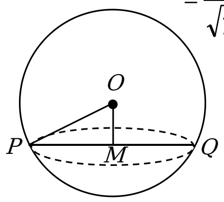
Where $\hat{\mathbf{n}}$ and $\mathbf{d}$ are unit perpendicular vector and angle between $\hat{\mathbf{n}}$ and $\mathbf{d}$ may be 0 or π.

When $\theta = 0^\circ$, $\mathbf{c} = \dfrac{1}{3}[\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}]$

When $\theta = \pi$, $\mathbf{c} = \dfrac{1}{3}[-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}]$.

33. (d) The centre of the sphere $|\mathbf{r}| = 5$ is at the origin and radius $= 5$. Let M be the foot of perpendicular from O to the given plane. Then OM = length of perpendicular from O to the given plane $= \dfrac{|\overrightarrow{OM}.(\mathbf{i}+\mathbf{j}+\mathbf{k}) - 3\sqrt{3}|}{|\mathbf{i}+\mathbf{j}+\mathbf{k}|}$

$= \dfrac{3\sqrt{3}}{\sqrt{1^2+1^2+1^2}} = 3$



Let P be any position of circle, then P lies on plane as well as on sphere.

$\therefore\ OP$ = radius of sphere = 5

In ΔOPM, we have $OP^2 = OM^2 + PM^2$

$\Rightarrow PM = \sqrt{5^2 - 3^2} = 4$.

34. (c) Given $\mathbf{x}$ is parallel to $\mathbf{y}$ and $\mathbf{z}$

$\therefore\ \mathbf{x}.(\mathbf{y}\times\mathbf{z}) = 0 \Rightarrow [\mathbf{x}\ \mathbf{y}\ \mathbf{z}] = 0$

$$\begin{vmatrix} 2 & 1 & \alpha \\ \alpha & 0 & 1 \\ 5 & -1 & 0 \end{vmatrix} = 0 \Rightarrow \alpha = \pm\sqrt{7}.$$

35. (a) The required vector $\mathbf{c}$ is given by $\lambda\left(\dfrac{\mathbf{a}}{|\mathbf{a}|} + \dfrac{\mathbf{b}}{|\mathbf{b}|}\right)$

Now, $\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{1}{9}(7\mathbf{i} - 4\mathbf{j} - 4\mathbf{k})$

and $\dfrac{\mathbf{b}}{|\mathbf{b}|} = \dfrac{1}{3}(-2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

$\Rightarrow \mathbf{c} = \lambda\left(\dfrac{1}{9}\mathbf{i} - \dfrac{7}{9}\mathbf{j} + \dfrac{2}{9}\mathbf{k}\right)$

$\Rightarrow |\mathbf{c}|^2 = \lambda^2.\dfrac{54}{81}$

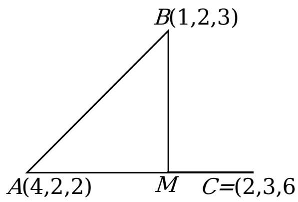
$\Rightarrow \lambda^2 = 225$ or $\lambda = \pm 15$.

Therefore, $\mathbf{c} = \pm\dfrac{5}{3}(\mathbf{i} - 7\mathbf{j} + 2\mathbf{k})$.

36. (b) $BM^2 = AB^2 - AM^2$ .....(i)

$\overrightarrow{AB} = -3\mathbf{i} + 0\mathbf{j} + \mathbf{k}$

$AB^2 = \overrightarrow{AB}^2 = 9 + 1 = 10$



AM = Projection of $\overrightarrow{AB}$ in direction of $\vec{C}$

$= 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

$AM = \dfrac{\overrightarrow{AB}.\vec{C}}{|\vec{C}|} = \dfrac{(-3\mathbf{i} + 0\mathbf{j} + \mathbf{k}).(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})}{7} = 0$

$BM^2 = 10 - 0 = 10$

$\Rightarrow BM = \sqrt{(10)}$, {by (i)}.

37. (b,c) We have $\mathbf{r} = \lambda_1\mathbf{r}_1 + \lambda_2\mathbf{r}_2 + \lambda_3\mathbf{r}_3$

$$\Rightarrow 2\mathbf{a} - 3\mathbf{b} + 4\mathbf{c} = (\lambda_1 - \lambda_2 + \lambda_3)\mathbf{a}$$

$$+(-\lambda_1 + \lambda_2 + \lambda_3)\mathbf{b} + (\lambda_1 + \lambda_2 + \lambda_3)\mathbf{c}$$

$$\Rightarrow \lambda_1 - \lambda_2 + \lambda_3 = 2, -\lambda_1 + \lambda_2 + \lambda_3 = -3, \lambda_1 + \lambda_2 + \lambda_3 = 4$$

($\because$ $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar)

$$\Rightarrow \lambda_1 = \frac{7}{2}, \quad \lambda_2 = 1, \quad \lambda_3 = -\frac{1}{2}$$

Therefore, $\lambda_1 + \lambda_3 = 3$ and $\lambda_1 + \lambda_2 + \lambda_3 = 4$.

38. (a) $\mathbf{c}$ is coplanar with $\mathbf{a}, \mathbf{b}$

$\therefore$ $\mathbf{c} = x\mathbf{a} + y\mathbf{b}$

$\Rightarrow \mathbf{c} = x(2\mathbf{i} + \mathbf{j} + \mathbf{k}) + y(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

$\Rightarrow \mathbf{c} = (2x + y)\mathbf{i} + (x + 2y)\mathbf{j} + (x - y)\mathbf{k}$

$\because$ $\mathbf{a}.\mathbf{c} = 0$

$\therefore$ $2(2x + y) + x + 2y + x - y = 0$

$\Rightarrow y = -2x$

$\mathbf{c} = -3x\mathbf{j} + 3x\mathbf{k} = 3x(-\mathbf{j} + \mathbf{k})$

$\because$ $|\mathbf{c}| = 1$

$\therefore$ $9x^2 + 9x^2 = 1$

$$\Rightarrow x = \pm\frac{1}{3\sqrt{2}} \Rightarrow \mathbf{c} = \frac{1}{\sqrt{2}}(-\mathbf{j} + \mathbf{k}).$$

39. (b) $|\mathbf{p}| = |\mathbf{q}| = |\mathbf{r}| = c$, (say)

and $\mathbf{p}.\mathbf{q} = 0 = \mathbf{p}.\mathbf{r} = \mathbf{q}.\mathbf{r}$

$\mathbf{p}\times|(\mathbf{x} - \mathbf{q})\times\mathbf{p}| + \mathbf{q}\times|(\mathbf{x} - \mathbf{r})\times\mathbf{q}| + \mathbf{r}\times|(\mathbf{x} - \mathbf{p})\times\mathbf{r}| = 0$

$\Rightarrow (\mathbf{p}.\mathbf{p})(\mathbf{x} - \mathbf{q}) - \{\mathbf{p}.(\mathbf{x} - \mathbf{q})\}\mathbf{p} + \ldots\ldots\ldots = 0$

$\Rightarrow c^2(\mathbf{x} - \mathbf{q} + \mathbf{x} - \mathbf{r} + \mathbf{x} - \mathbf{p}) - (\mathbf{p}.\mathbf{x})\mathbf{p} - (\mathbf{q}.\mathbf{x})\mathbf{q} - (\mathbf{r}.\mathbf{x})\mathbf{r} = 0$

$\Rightarrow c^2\{3\mathbf{x} - (\mathbf{p} + \mathbf{q} + \mathbf{r})\} - [(\mathbf{p}.\mathbf{x})\mathbf{p} + (\mathbf{q}.\mathbf{x})\mathbf{q} + (\mathbf{r}.\mathbf{x})\mathbf{r}] = 0$

which is satisfied by $\mathbf{x} = \frac{1}{2}(\mathbf{p} + \mathbf{q} + \mathbf{r})$.

40. (a) We have $\mathbf{r}\times\mathbf{b} = \mathbf{a}\times\mathbf{b}$ and $\mathbf{r}\times\mathbf{a} = \mathbf{b}\times\mathbf{a}$

Adding $\mathbf{r}\times(\mathbf{a} + \mathbf{b}) = 0$ *i.e.*, $\mathbf{r}$ is parallel to $\mathbf{a} + \mathbf{b}$

or $\mathbf{r} = \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{i} - \mathbf{k})$

$\mathbf{r} = \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ for $\lambda = 1 \Rightarrow \mathbf{r} = (3\mathbf{i} + \mathbf{j} - \mathbf{k})$.