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MODI Method (Modified Distribution Method)

This method is used to test whether an initial feasible solution is optimal or not.

Step 1

When the initial basic feasible solution is obtained some cells are occupied (ie, allocation made) and others unoccupied, the number of occupied cells is $m+n-1$. Let C_{ij} be the cost of cell (i, j). Then determine $m+n$ numbers called u_i and v_j values by forming $m+n-1$ equations of the form $u_i + v_j = C_{ij}$, corresponding to each occupied cell. For solving the equations, we take one of u_i or v_j values as zero.

Step 2

Calculate the cell evaluations known as d_{ij} values for unoccupied cells by the formula $d_{ij} = C_{ij} - (u_i + v_j)$

Step 3

If all d_{ij} values are positive, the solution is optimal and unique. If atleast one of them is zero and the rest are positive then the solution is optimal but alternative solution exists. If atleast one d_{ij} is negative then the solution is not optimal.

Step 4

If the solution is not optimal, make reallocations. Give maximum allocation to the cell for which d_{ij} is negative making one of the occupied cells empty.

Then repeat the steps 1 to 4 until the solution becomes optimal.

Qn 1. Solve the following transportation problem.

	W_1	W_2	W_3	Supply
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18	

Soln.

By Vogel's method the initial solution is

	W_1	W_2	W_3	Supply
F_1	$\boxed{5}$ 2	7	4	5
F_2	3	3	$\boxed{8}$ 1	8
F_3	5	$\boxed{7}$ 4	7	7
F_4	$\boxed{2}$ 1	$\boxed{2}$ 6	$\boxed{10}$ 2	14
Demand	7	9	18	

Select the numbers u_1, u_2, u_3, u_4 and v_1, v_2, v_3 and solve the equations

$$u_1 + v_1 = 2 \qquad u_4 + v_1 = 1$$
$$u_2 + v_3 = 1 \qquad u_4 + v_2 = 6$$
$$u_3 + v_2 = 4 \qquad u_4 + v_3 = 2$$

Take $u_4 = 0$, then the solution is

$u_1 = 1$, $u_2 = -1$, $u_3 = -2$, $v_1 = 1$, $v_2 = 6$, $v_3 = 2$

c_{ij}			u_i	$u_i + v_j$			$d_{ij} = c_{ij} - (u_i + v_j)$		
×	7	4	1	×	7	4	×	0	1
3	3	×	-1	3	3	×	3	-2	×
5	×	7	-2	5	×	7	6	×	7
×	×	×	0	×	×	×	×	×	×

$v_j \rightarrow$ 1 6 2

Since one of the d_{ij} is negative the solution is not optimal. Make reallocations by giving maximum possible allocation to the cell (2,2) where the negative d_{ij} value occurs.

The revised allocation gives

original

3	$\boxed{8}$ 1
$\boxed{2}$ 6	$\boxed{10}$ 2

changed

$\boxed{2}$ 3	$\boxed{6}$ 1
6	$\boxed{12}$ 2

$\boxed{5}$ 2	7	4
3	$\boxed{2}$ 3	$\boxed{6}$ 1
5	$\boxed{7}$ 4	7
$\boxed{2}$ 1	6	$\boxed{12}$ 2

The equations of u_i and v_j values of occupied cells are

$u_1 + v_1 = 2$ $\quad$ $u_3 + v_2 = 4$
$u_2 + v_2 = 3$ $\quad$ $u_4 + v_1 = 1$
$u_2 + v_3 = 1$ $\quad$ $u_4 + v_3 = 2$

Take $u_4 = 0$, then

$u_1 = 1,\ u_2 = -1,\ u_3 = 0,\ v_1 = 1,\ v_2 = 4,\ v_3 = 2$

c_{ij}			$u_i \downarrow$	$u_i + v_j$			d_{ij}		
x	7	4	1	x	5	3	x	2	1
3	x	x	-1	0	x	x	3	x	x
5	x	7	0	1	x	2	4	x	5
x	6	x	0	x	4	x	x	2	x
			$v_j \rightarrow$	1	4	2			

No d_{ij} value is negative. Thus the solution is optimum.

$\therefore$ Total cost $= (5\times2) + (2\times3) + (6\times1) + (7\times4) + (2\times1) + (12\times2)$
$= \underline{\underline{76}}$