14. (b) $n(A)$ = 40% of 10,000 = 4,000
$n(B)$ = 20% of 10,000 = 2,000
$n(C)$ = 10% of 10,000 = 1,000
$n(A \quad B)$ = 5% of 10,000 = 500
$n(B \quad C)$ = 3% of 10,000 = 300
$n(C \quad A)$ = 4% of 10,000 = 400
$n(A \quad B \quad C)$ = 2% of 10,000 = 200
We want to find $n(A \quad B^c \quad C^c) = n[A \quad (B \quad C)^c]$
$= n(A) - n[A \quad (B \quad C)] = n(A) - n[(A \quad B) \quad (A \quad C)]$
$= n(A) - [n(A \quad B) + n(A \quad C) - n(A \quad B \quad C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300$.

15. (c) $n(C) = 20$, $n(B) = 50$, $n(C \quad B) = 10$
Now $n(C \quad B) = n(C) + n(B) - n(C \quad B)$
$= 20 + 50 - 10 = 60$.
Hence, required number of persons = 60%.

16. (d) $n(M) = 23$, $n(P) = 24$, $n(C) = 19$
$n(M \quad P) = 12$, $n(M \quad C) = 9$, $n(P \quad C) = 7$
$n(M \quad P \quad C) = 4$
We have to find $n(M \quad P \quad C)$, $n(P \quad M \quad C)$,
$n(C \quad M \quad P)$
Now $n(M \quad P \quad C) = n[M \quad (P \quad C)]$
$= n(M) - n(M \quad (P \quad C))$
$= n(M) - n[(M \cap P) \cup (M \cap C)]$
$= n(M) - n(M \quad P) - n(M \quad C) + n(M \quad P \quad C)$
$= 23 - 12 - 9 + 4 = 27 - 21 = 6$
$n(P \quad M \quad C) = n[P \quad (M \quad C)]$
$= n(P) - n[P \quad (M \quad C)] = n(P) - n[(P \cap M) \cup (P \cap C)]$
$= n(P) - n(P \quad M) - n(P \quad C) + n(P \quad M \quad C)$
$= 24 - 12 - 7 + 4 = 9$
$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$
$= 19 - 7 - 9 + 4 = 23 - 16 = 7$.

17. (a) It is distributive law.

18. (b) It is De' Morgan law.

19. (c) $(A - B) \quad (B - A) = (A \quad B) - (A \quad B)$.

20. (b) $A \times B$ = {(2, 7), (2, 8), (2, 9), (4, 7), (4, 8), (4, 9), (5, 7), (5, 8), (5, 9)}
$n(A \times B) = n(A) \cdot n(B) = 3 \times 3 = 9$.

21. (c) $n(A \times B) = pq$.

22. (c) $B \quad C = \{c, d\} \quad (d, e\} = \{c, d, e\}$
$\therefore A \times (B \quad C) = \{a, b\} \times \{c, d, e\}$
$= \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$.

23. (a,b) $R \times (P^c \cup Q^c)^c = R \times [(P^c)^c \cap (Q^c)^c]$
$= R \times (P \cap Q) = (R \times P) \cap (R \times Q) = (R \times Q) \cap (R \times P)$.

24. (d) It is fundamental concept.

25. (b) It is fundamental concept.

26. (b) Since $\frac{1}{y} \neq 0, \frac{1}{y} \neq 2, \frac{1}{y} \neq \frac{-2}{3}$, $[\because y \in N]$
$\frac{1}{y}$ can be 1, $[\because y$ can be 1].

27. (d) Null set is the subset of all given sets.

28. (b) $S = \{0, 1, 5, 4, 7\}$,
then, total number of subsets of S is 2^n.
Hence, $2^5 = 32$.

29. (a) The number of non- empty subsets = $2^n - 1$
$2^4 - 1 = 16 - 1 = 15$.

30. (b) Given $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$. Hence, $A = \{3, 5, 9\}$.

31. (b) Since $A \cap B = B$, $\therefore B \subset A$.

32. (c) Let $x \in A \Rightarrow x \in A \cup B$, $[\because A \subseteq A \cup B]$
$x \in A \cap B$, $[\because A \cup B = A \cap B]$
$x \in A$ and $x \in B \quad x \in B, \quad A \subseteq B$
Similarly, $x \in B \quad x \in A, \quad B \subseteq A$
Now $A \subseteq B, B \subseteq A \quad A = B$.

33. (b) $A \cap B \subseteq A \subseteq A \cup B, \quad A \cap B \subseteq A \cup B$.

34. (b) $\because y = e^x$, $y = e^{-x}$ will meet, when $e^x = e^{-x}$
$e^{2x} = 1$, $\therefore x = 0, y = 1$
A and B meet on (0, 1), $\quad A \cap B = \phi$.

35. (a) $A \cap B = \{2, 3, 4, 8, 10\} \cap \{3, 4, 5, 10, 12\}$
$= \{3, 4, 10\}$, $A \cap C = \{4\}$.
$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$.

36. (a) $A \cap (A \cup B) = A$, $[\because A \subseteq B \cup A]$.

37. (b) It is obvious.

38. (a) $B \cup C = \{a, b, c, d, e\}$
$A \cap (B \cup C) = \{a, b, c\} \cap \{a, b, c, d, e\} = \{a, b, c\}$.

39. (a) $A \cap (B - A) = \phi$, $[\because x \in B - A \Rightarrow x \notin A]$.

40. (c) $A \cap (A \cup B)' = A \cap (A' \cap B')$, $\quad (\because (A \cup B)' = A' \cap B')$
$= (A \cap A') \cap B'$, (by associative law)
$= \phi \cap B'$, $\quad (\because A \cap A' = \phi)$
$= \phi$.

41. (b) $B' = \{1, 2, 3, 4, 5, 8, 9, 10\}$

$A \cap B' = \{1, 2, 5\} \cap \{1, 2, 3, 4, 5, 8, 9, 10\} = \{1, 2, 5\} = A$

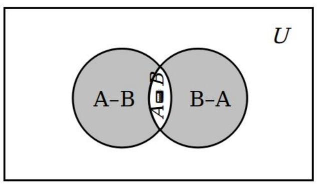
42. (b) It is obvious.

43. (c) $N_5 \cap N_7 = N_{35}$,
[$\because$ 5 and 7 are relatively prime numbers].

44. (a) $3N = \{x \in N : x$ is a multiple of 3}
$7N = \{x \in N : x$ is a multiple of 7}
$3N \cap 7N = \{x \in$ is a multiple of 3 and 7}
$= \{x \in N : x$ is a multiple of 3 and 7}
$= \{x \in N : x$ is a multiple of 21} = 21N.

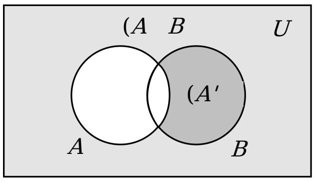
45. (d) It is obvious.

46. (a) From *Venn-Euler's* diagram,



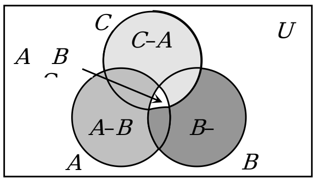
$\therefore (A - B) \cup (B - A) \cup (A \cap B) = A \cup B$.

47. (a) From *Venn-Euler's* Diagram,



$\therefore\ (A \cup B)' \cup (A' \cap B) = A'$.

48. (c) From *Venn-Euler's* Diagram,



Clearly, $\{(A-B)\cup(B-C)\cup(C-A)\}' = A \cap B \cap C$.

49. (c) Since $A \subseteq B$, $\therefore A \cup B = B$.

So, $n(A \cup B) = n(B) = 6$.

50. (c) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$0.25 = 0.16 + 0.14 - n(A \cap B)$

$n(A \cap B) = 0.30 - 0.25 = 0.05$.

51. (c) Since A and B are disjoint, $A \cap B = \phi$

$n(A \cap B) = 0$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A) + n(B) - 0 = n(A) + n(B)$.

52. (b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

53. (a) Minimum value of $n = 100 - (30 + 20 + 25 + 15))$

$= 100 - 90 = 10$.

54. (d) $n(C) = 224, n(H) = 240, n(B) = 336$

$n(H \cap B) = 64$, $n(B \cap C) = 80$

$n(H \cap C) = 40$, $n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^C) = n[(C \cup H \cup B)^c]$

$= n(\cup) - n(C \cup H \cup B)$

$= 800 - [n(C) + n(H) + n(B) - n(H \cap C)$

$-n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800 - [224 + 240 + 336 - 64 - 80 - 40 + 24]$

$= 800 - 640 = 160$.

55. (c) Let A denote the set of Americans who like cheese and let B denote the set of Americans who like apples.

Let Population of American be 100.

Then $n(A) = 63, n(B) = 76$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 63 + 76 - n(A \cap B)$

$n(A \cup B) + n(A \cap B) = 139$

$n(A \cap B) = 139 - n(A \cup B)$

But $n(A \cup B) \le 100$

$-n(A \cup B) \ge -100$

$139 - n(A \cup B) \ge 139 - 100 = 39$

$n(A \cap B) \ge 39$ *i.e.*, $39 \le n(A \cap B)$ .....(i)

Again, $A \cap B \subseteq A, A \cap B \subseteq B$

$n(A \cap B) \le n(A) = 63$ and $n(A \cap B) \le n(B) = 76$

$n(A \cap B) \le 63$

.....(ii)

Then, $39 \le n(A \cap B) \le 63$ $\quad 39 \le x \le 63$.

56. (a) Let $n(P) =$ Number of teachers in Physics.

$n(M) =$ Number of teachers in Maths

$n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$20 = n(P) + 12 - 4$ $\quad n(P) = 12$.

57. (a) Let B, H, F denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then we are given $n(B) = 21, n(H) = 26, n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$.

We have to find $n(B \cup H \cup F)$.

To find this, we use the formula

$n(B \cup H \cup F) = n(B) + n(H) + n(F)$

$-n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

Hence,

$n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

Thus these are 43 members in all.

58. (d) $n(M) = 55, n(P) = 67, n(M \cup P) = 100$

Now, $n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$100 = 55 + 67 - n(M \cap P)$

$n(M \cap P) = 122 - 100 = 22$

Now n (P only) $= n(P) - n(M \cap P) = 67 - 22 = 45$.

59. (c) In general, $A \times B \ne B \times A$

$A \times B = B \times A$ is true, if $A = B$.

60. (b) From De' morgan's law, $(A \cap B)' = A' \cup B'$.

61. (d) $A - B = \{x : x \in A \text{ and } x \notin B\}$

$= \{x : x \in A \text{ and } x \in B^c\} = A \cap B^c$.

62. (a) It is obvious.

63. (a) From De' morgan's law, $A - (B \cap C) = (A - B) \cup (A - C)$.

64. (b) From Distributive law, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

65. (b) $A - B = \{1\}$ and $B - C = \{4\}$

$(A - B) \times (B - C) = \{(1, 4)\}$.

66. (a) It is obvious.

67. (b) $A \cup B = \{1, 2, 3, 8\}$; $A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{(1, 3), (2, 3), (3, 3), (8, 3)\}$.

68. (c) $A - B = \{3\}, A \cap B = \{2, 5\}$

$(A - B) \times (A \cap B) = \{(3, 2); (3, 5)\}$.

69. (a) Given $n(N) = 12$, $n(P) = 16$, $n(H) = 18$, $n(N \cup P \cup H) = 30$

From, $n(N \cup P \cup H) = n(N) + n(P) + n(H) - n(N \cap P)$

$-n(P \cap H) - n(N \cap H) + n(N \cap P \cap H)$

$n(N \cap P) + n(P \cap H) + n(N \cap H) = 16$

Now, number of pupils taking two subjects

$= n(N \cap P) + n(P \cap H) + n(N \cap H) - 3n(N \cap P \cap H)$

$= 16 - 0 = 16$.

70. (e) $n(A) = 4$, $n(B) = 3$

$n(A) \times n(B) \times n(C) = n(A \times B \times C)$

$4 \times 3 \times n(C) = 24 \qquad n(C) = \frac{24}{12} = 2$.

71. (c) Given set is $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$

We can see that, $2(\pm 2)^2 + 3(\pm 3)^2 = 35$

and $2(\pm 4)^2 + 3(\pm 1)^2 = 35$

(2, 3), (2, –3), (–2, –3), (–2, 3), (4, 1), (4, –1), (–4, –1), (–4, 1) are 8 elements of the set. $n = 8$.

72. (c) It is obvious.

73. (a) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cap C = \{3, 4, 6\}$.

74. (d) It is obvious.

75. (d) Let the original set contains $(2n+1)$ elements, then subsets of this set containing more than n elements, *i.e.*, subsets containing $(n+1)$ elements, $(n+2)$ elements, ... $(2n+1)$ elements.

Required number of subsets

$= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$

$= {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + ... + {}^{2n+1}C_1 + {}^{2n+1}C_0$

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_{n-1} + {}^{2n+1}C_n$

$= \frac{1}{2}\left[(1+1)^{2n+1}\right] = \frac{1}{2}[2^{2n+1}] = 2^{2n}$.

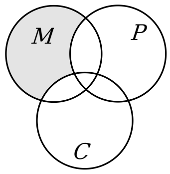
76. (a) It is obvious.

77. (b) $A = \{4, 8, 12, 16, 20, 24, ...\}$

$B = \{6, 12, 18, 24, 30, ...\}$

$A \subset B = \{12, 24, ...\}$ = {x : x is a multiple of 12}.

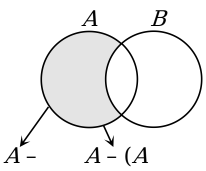
78. (c) $n(M$ alone$) = n(M) - n(M \cap C) - n(M \cap P) + n(M \cap P \cap C)$



$= 100 - 28 - 30 + 18 = 60$.

79. (d) $A - B = A - (A \cap B)$ is correct.

$A = (A \cap B) \cup (A - B)$ is correct.



(3) is false.

(1) and (2) are true.

80. (b) $n((A \times B) \cap (B \times A))$

$= n((A \cap B) \times (B \cap A)) = n(A \cap B).n(B \cap A)$

$= n(A \cap B).n(A \cap B) = (99)(99) = 99^2$.

81. (d) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 12 + 9 - 4 = 17$

Now, $n((A \cup B)^C) = n(U) - n(A \cup B) = 20 - 17 = 3$.

Relations

1. (a) $n(A \times A) = n(A).n(A) = 3^2 = 9$

So, the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times A$ is a relation over the set A.

2. (a,b,c) R_4 is not a relation from X to Y, because (7, 9) $\in R_4$ but (7, 9) $\notin X \times Y$.

3. (c) Here $n(A \times B)$ = 2 × 3 = 6

Since every subset of $A \times B$ defines a relation from A to B, number of relation from A to B is equal to number of subsets of $A \times B = 2^6 = 64$.

4. (b) $R = \{(a, b) : a, b \in N, a - b = 3\} = \{((n+3), n) : n \in N\}$

$= \{(4, 1), (5, 2), (6, 3), ...\}$.

5. (b) R = {(2, 1), (4, 2), (6, 3),......}.

So, R^{-1} = {(1, 2), (2, 4), (3, 6),.....}.

6. (a) Since (1, 1); (2, 2); (3, 3) R therefore R is reflexive. (1, 2) R but (2, 1) R, therefore R is not symmetric. It can be easily seen that R is transitive.

7. (b) Since $x < y, y < z \Rightarrow x < z \forall x, y, z \in N$

$\therefore$ $xRy, yRz \Rightarrow xRz$, $\therefore$ Relation is transitive,

$\therefore$ $x < y$ does not give $y < x$,

$\therefore$ Relation is not symmetric.

Since $x < x$ does not hold, hence relation is not reflexive.

8. (b) Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

9. (b) Since R is an equivalence relation on set A, therefore (a, a) R for all $a \in A$. Hence, R has at least n ordered pairs.

10. (a) For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

xRx for all x. So, R is reflexive.

R is not symmetric, because $\sqrt{2}R1$ but $1R\sqrt{2}$,

R is not transitive also because $\sqrt{2}$ $R1$ and $1R2\sqrt{2}$ but $\sqrt{2}$ $R2\sqrt{2}$.

11. (b) Clearly, the relation is symmetric but it is neither reflexive nor transitive.

12. (a) It is obvious.

13. (b) It is obvious.

14. (c) We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)}

R^{-1} = {(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)}

Hence RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}.

15. (d) A relation from P to Q is a subset of $P \times Q$.

16. (c) $R = A \times B$.

17. (c) Number of relations on the set A = Number of subsets of $A\times A=2^{n^2}$, $[\because n(A\times A)=n^2]$.

18. (a) It is obvious.

19. (a) Since R is reflexive relation on A, therefore $(a,a)\in R$ for all $a\in A$.

The minimum number of ordered pairs in R is n.

Hence, $m\geq n$.

20. (d) Here $R=\{(x,y):|x^2-y^2|<16\}$

and given $A=\{1,2,3,4,5\}$

$R=\{(1,2)(1,3)(1,4);(2,1)(2,2)(2,3)(2,4);(3,1)(3,2)$
$(3,3)(3,4);(4,1)(4,2)(4,3);(4,4)(4,5),(5,4)(5,5)\}$.

21. (d) Given, $xRy\Rightarrow x$ is relatively prime to y.

Domain of $R=\{2,3,4,5\}$.

22. (c) R be a relation on N defined by $x+2y=8$.

$R\{(2,3);(4,2);(6,1)\}$

Hence, Domain of $R=\{2,4,6\}$.

23. (c) $\because R=\{(x,y)|\ x,y\in Z, x^2+y^2\leq 4\}$

$\therefore\ R=\{(-2,0),(-1,0),(-1,1),(0,-1)(0,1),(0,2),(0,-2)$
$(1,0),(1,1),(2,0)\}$

Hence, Domain of $R=\{-2,-1,0,1,2\}$.

24. (a) R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y=x-3\Rightarrow x-y=3$

R = {11, 8},{13, 10}.

Hence, $R^{-1}=\{8,11\};\{10,13\}$.

25. (a) It is obvious.

26. (b) It is obvious.

27. (a,b) (1, 1)(2, 2)(3, 3)(4, 4) $\in R$; R is reflexive.

$\because$ (1, 2) (3, 1) $\in R$ and also (2, 1)(1, 3) $\in R$.

Hence, R is symmetric. But clearly R is not transitive.

28. (b) For any integer n, we have $n|\ n\Rightarrow nRn$

So, nRn for all $n\in Z\Rightarrow R$ is reflexive

Now 2|6 but 6+2, (2,6) $\in R$ but (6, 2) $\notin R$

So, R is not symmetric.

Let $(m,n)\in R$ and $(n,p)\in R$.

Then $\left.\begin{array}{l}(m,n)\in R\Rightarrow m|\ n\\ (n,p)\in R\Rightarrow n|\ p\end{array}\right]\Rightarrow m|\ p\Rightarrow(m,p)\in R$

So, R is transitive.

Hence, R is reflexive and transitive but it is not symmetric.

29. (a) For any $a\in N$, we find that $a|a$, therefore R is reflexive but R is not transitive, because aRb does not imply that bRa.

30. (b) Let $(a,b)\in R$

Then, $(a,b)\in R\Rightarrow(b,a)\in R^{-1}$, [By def. of R^{-1}]

$(b,a)\in R$, $[\because R=R^{-1}]$

So, R is symmetric.

31. (c) It is obvious.

32. (b) The relation is not symmetric, because $A\subset B$ does not imply that $B\subset A$. But it is anti-symmetric because $A\subset B$ and $B\subset A$ $A=B$.

33. (c) It is obvious.

34. (c) Since $x\not< x$, therefore R is not reflexive. Also $x<y$ does not imply that $y<x$, So R is not symmetric. Let xRy and yRz. Then, $x<y$ and $y<z$ $x<z$ *i.e.,* xRz. Hence R is transitive.

35. (b,c) x is a brother of y, y is also brother of x. So, it is symmetric. Clearly it is transitive.

36. (c) Since (1, 1) $\notin R$ so, is not reflexive.

Now (1, 2) $\in R$ but (2,1) $\notin R$, therefore R is not symmetric Clearly R is transitive.

37. (b) The void relation R on A is not reflexive as (a, a) $\notin R$ for any $a\in A$. The void relation is symmetric and transitive.

38. (b) For any $a\in R$, we have $a\geq a$, Therefore the relation R is reflexive but it is not symmetric as (2, 1) $\in R$ but (1, 2) $\notin R$. The relation R is transitive also, because $(a,b)\in R,(b,c)\in R$ imply that $a\geq b$ and $b\geq c$ which is turn imply that $a\geq c$.

39. (a) It is obvious.

40. (c) It is obvious.

41. (d) It is obvious.

42. (d) It is obvious.

43. (d) It is obvious.

44. (c) $x\equiv 3(\text{mod}7)$ $x-3=7p,(p\in z)$

$\Rightarrow$ $x=7p+3, p\in z$ *i.e.,* $\{7p+3: p\in z\}$.

45. (b) Given, R and S are relations on set A.

$R\subseteq A\times A$ and $S\subseteq A\times A$ $R\cap C\subseteq A\times A$

$R\cap S$ is also a relation on A.

Reflexivity : Let a be an arbitrary element of A.

Then, $a\in A$ $(a,a)\in R$ and $(a,a)\in S$,

[$\because R$ and S are reflexive]

$(a,a)\in R\cap S$

Thus, $(a,a)\in R\cap S$ for all $a\in A$.

So, $R\cap S$ is a reflexive relation on A.

Symmetry : Let $a,b\in A$ such that $(a,b)\in R\cap S$.

Then, $(a,b)\in R\cap S$ $(a,b)\in R$ and $(a,b)\in S$

$(b,a)\in R$ and $(b,a)\in S$,

[$\because R$ and S are symmetric]

$(b,a)\in R\cap S$

Thus, $(a,b)\in R\cap S$

$(b,a)\in R\cap S$ for all $(a,b)\in R\cap S$.

So, $R\cap S$ is symmetric on A.

Transitivity : Let $a,b,c\in A$ such that

$(a,b)\in R\cap S$ and $(b,c)\in R\cap S$. Then,

$(a,b)\in R\cap S$ and $(b,c)\in R\cap S$

$\{((a,b)\in R \text{ and} (a,b)\in S)\}$

and $\{((b,c)\in R \text{and} (b,c)\in S\}$

$\{(a,b)\in R,(b,c)\in R\}$ and $\{(a,b)\in S,(b,c)\in S\}$

$(a,c)\in R$ and $(a,c)\in S$

$\left[\begin{array}{l}\because R \text{ and } S \text{ are transitive So} \\ (a,b)\in R \text{ and } (b,c)\in R \Rightarrow (a,c)\in R \\ (a,b)\in S \text{ and } (b,c)\in S \Rightarrow (a,c)\in S\end{array}\right.$

$(a,c)\in R\cap S$

Thus, $(a,b)\in R\cap S$

and $(b,c)\in R\cap S \Rightarrow (a,c)\in R\cap S$.

So, $R\cap S$ is transitive on *A.*

Hence, *R* is an equivalence relation on *A*.

46. (b, c, d) These are fundamental concepts.

47. (c) Here $R=\{(1,3),(2,2);(3,2)\}$, $S=\{(2,1);(3,2);(2,3)\}$

Then $RoS=\{(2,3);(3,2);(2,2)\}$.

48. (b) Here $\alpha R\beta \Leftrightarrow \alpha\perp\beta$ $\quad \alpha\perp\beta \Leftrightarrow \beta\perp\alpha$

Hence, *R* is symmetric.

49. (d) We have $(a,b)R(a,b)$ for all $(a,b)\in N\times N$

Since $a+b=b+a$. Hence, *R* is reflexive.

R is symmetric for we have $(a,b)R(c,d)$

$a+d=b+c$

$d+a=c+b$ $\quad c+b=d+a \Rightarrow (c,d)R(e,f)$.

Then by definition of *R*, we have

$a+d=b+c$ and $c+f=d+e$,

whence by addition, we get

$a+d+c+f=b+c+d+e$ or $a+f=b+e$

Hence, $(a,b)\ R(e,f)$

Thus, $(a,b)\ R(c,d)$ and $(c,d)R(e,f) \Rightarrow (a,b)R(e,f)$.

50. (a,b,c,d) It is obvious.

51. (c) Here (3, 3), (6, 6), (9, 9), (12, 12), [Reflexive]; (3, 6), (6, 12), (3, 12), [Transitive].

Hence, reflexive and transitive only.

52. (b) It is obvious.

53. (c) Given *A* = {1, 2, 3, 4}

R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}

(2, 3) *R* but (3, 2) *R*. Hence *R* is not symmetric.

R is not reflexive as (1, 1) *R*.

R is not a function as (2, 4) *R* and (2, 3) *R*.

R is not transitive as (1, 3) *R* and (3, 1) *R* but (1, 1) *R*.

54. (d) Total number of reflexive relations in a set with *n* elements $=2^n$.

Therefore, total number of reflexive relation set with 4 elements $=2^4$.

55. (a) Since $1+a.a=1+a^2>0, \forall a\in S$, $\quad (a,a)\in R$

R is reflexive.

Also $(a,b)\in R$ $\quad 1+ab>0$ $\quad 1+ba>0$ $\quad (b,a)\in R$,

R is symmetric.

$\because (a,b)\in R$ and $(b,c)\in R$ need not imply $(a,c)\in R$. Hence, *R* is not transitive.

56. (a) $A=\{2,4,6\}$; $B=\{2,3,5\}$

A *B* contains $3\times 3=9$ elements.

Hence, number of relations from *A* to *B* $=2^9$.

Critical Thinking Questions

1. (a) Since $8^n-7n-1=(7+1)^n-7n-1$

$=7^n+{}^nC_1 7^{n-1}+{}^nC_2 7^{n-2}+.....+{}^nC_{n-1}7+{}^nC_n-7n-1$

$={}^nC_2 7^2+{}^nC_3 7^3+..+{}^nC_n 7^n$, $({}^nC_0={}^nC_n, {}^nC_1={}^nC_{n-1}$ etc.)

$=49[{}^nC_2+{}^nC_3(7)+......+{}^nC_n 7^{n-2}]$

$\therefore$ 8^n-7n-1 is a multiple of 49 for $n\geq 2$

For $n=1$, $8^n-7n-1=8-7-1=0$;

For $n=2$, $8^n-7n-1=64-14-1=49$

$\therefore$ 8^n-7n-1 is a multiple of 49 for all $n\in N$.

$\therefore$ *X* contains elements which are multiples of 49 and clearly *Y* contains all multiplies of 49.

$\therefore$ $X\subseteq Y$.

2. (b) $N_3\cap N_4=\{3,6,9,12,15......\}\cap\{4,8,12,16,20,.....\}$

= {12, 24, 36......} = N_{12}.

Trick : $N_3\cap N_4=N_{12}$

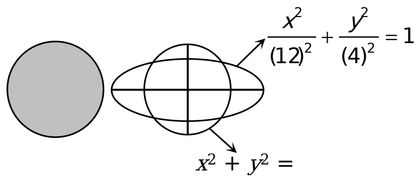
[$\because$ 3, 4 are relatively prime numbers].

3. (b) *n*(*A* *B*) = *n*(*A*) + *n*(*B*) – *n*(*A* *B*) = $3+6-n(A\cap B)$

Since, maximum number of elements in $A\cap B=3$

$\therefore$ Minimum number of elements in $A\cup B=9-3=6$.

4. (d) *A* = Set of all values (*x, y*) : $x^2+y^2=25=5^2$



$B=\frac{x^2}{144}+\frac{y^2}{16}=1$ *i.e.*, $\frac{x^2}{(12)^2}+\frac{y^2}{(4)^2}=1$.

Clearly, *A* *B* consists of four points.

5. (b) $A-B=A$ $\quad B^c=A$ $\quad \overline{B}$.

6. (a) Clearly, *A* = {2, 3}, *B* = {2, 4}, *C* = {4, 5}

B *C* = {4}

$\therefore$ *A* × (*B* *C*) = {(2, 4); (3, 4)}.

7. (c) Let number of newspapers be *x*. If every students reads one newspaper, the number of students would be $x(60)=60x$

Since, every students reads 5 newspapers

Numbers of students $=\frac{x\times 60}{5}=300$, $x=25$.

8. (c) Here *A* and *B* sets having 2 elements in common, so $A\times B$ and $B\times A$ have 2^2 *i.e.*, 4 elements in common.

Hence, $n[(A\times B)\cap(B\times A)]=4$.

9. (c) $(x,y)\in R \Leftrightarrow (y,x)\in R^{-1}$, $\therefore$ $R^{-1}=\{(3,1),(5,1),(1,2)\}$.

10. (a) $|a-a|=0<1$ $\therefore a\,R\,a\ \forall a\in R$

$\therefore$ *R* is reflexive.

Again *a R b* $\quad |a-b|\leq 1 \Rightarrow |b-a|\leq 1 \Rightarrow bRa$

$\therefore$ *R* is symmetric, Again $1R\frac{1}{2}$ and $\frac{1}{2}R1$ but $\frac{1}{2}\neq 1$

$\therefore$ R is not anti-symmetric.

Further, 1 R 2 and 2 R 3 but $1 \not R 3$, $[\because |1-3| = 2 > 1]$

$\therefore$ R is not transitive.

11. (d) Since $A \subseteq A$. $\therefore$ Relation '$\subseteq$' is reflexive.

Since $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$

$\therefore$ Relation '$\subseteq$' is transitive.

But $A \subseteq B$, $B \subseteq A$, $\therefore$ Relation is not symmetric.

12. (d) Since n | n for all $n \in N$, therefore R is reflexive. Since 2 | 6 but $6 \nmid 2$, therefore R is not symmetric.

Let n R m and m R p $n|m$ and $m|p$ $n|p$ nRp. So, R is transitive.

13. (a) Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2)\}$, $S = \{(2, 2)\ (2, 3)\}$ be transitive relations on A.

Then R $S = \{(1, 1); (1, 2); (2, 2); (2, 3)\}$

Obviously, R S is not transitive. Since $(1, 2) \in R$ S and $(2, 3) \in R \cup S$ but $(1, 3) \notin R \cup S$.

14. (a) We first find R^{-1}, we have

$R^{-1} = \{(5, 4); (4, 1); (6, 4); (6, 7); (7, 3)\}$. We now obtain the elements of $R^{-1}oR$ we first pick the element of R and then of R^{-1}. Since $(4, 5) \in R$ and $(5, 4) \in R^{-1}$, we have $(4, 4) \in R^{-1}oR$

Similarly, $(1, 4) \in R, (4, 1) \in R^{-1} \Rightarrow (1, 1) \in R^{-1}oR$

$(4, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (4, 4) \in R^{-1}oR,$

$(4, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (4, 7) \in R^{-1}oR$

$(7, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (7, 4) \in R^{-1}oR,$

$(7, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (7, 7) \in R^{-1}oR$

$(3, 7) \in R, (7, 3) \in R^{-1} \Rightarrow (3, 3) \in R^{-1}oR,$

Hence, $R^{-1}oR = \{(1, 1); (4, 4); (4, 7); (7, 4), (7, 7); (3, 3)\}$.

15. (d) On the set N of natural numbers,

$R = \{(x, y) : x, y \in N, 2x + y = 41\}$.

Since $(1, 1) \notin R$ as $2.1 + 1 = 3 \neq 41$. So, R is not reflexive.

$(1, 39) \in R$ but $(39, 1) \notin R$. So R is not symmetric

(20, 1)

(1, 39 $\in R$. But $(20, 39) \notin R$, So R is not transitive.