Hausaufgaben für P1a

Online unter `http://github.com/jaseg/Hausaufgaben`

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Aufgabe 1

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\begin{align}
\dot{v} &= -m\gamma v + \frac{g_x}{m}\,; g_x = g\sin(x) \tag{1}\\
\text{Ansatz aus Wikipedia (en)} \Rightarrow A(t) &= -m\gamma \tag{2}\\
b(t) &= \frac{g_x}{m} \tag{3}\\
F(t) &= \int_{t_0}^{t} -m\gamma \mathrm{d}t\,; t_0 = 0 \tag{4}\\
&= -m\gamma t \tag{5}\\
v(t) &= C(t)e^{-m\gamma t} \tag{6}\\
\dot{v} &= \dot{C}e^{-m\gamma t} + \left(-m\gamma C e^{-m\gamma t}\right) \tag{7}\\
&= -m\gamma v + \frac{g_x}{m} \tag{8}\\
\Rightarrow \frac{g_x}{m} = \dot{C}e^{-m\gamma t} &\Leftrightarrow \frac{g_x}{m}e^{m\gamma t} = \dot{C} \tag{9}\\
C &= \int_{t_0}^{t} e^{m\gamma t}\mathrm{d}t \frac{g_x}{m} \tag{10}\\
\Rightarrow C &= \frac{g_x}{m} \cdot \frac{1}{m\gamma}e^{m\gamma t} + c \tag{11}\\
&= \frac{g_x}{\gamma m^2}e^{m\gamma t} + c \tag{12}\\
\Rightarrow v(t) &= \frac{g_x}{\gamma m^2}e^{0} + ce^{-m\gamma t} \tag{13}\\
\dot{v} &= -m\gamma c e^{-m\gamma t} \tag{14}\\
-m\gamma c e^{-m\gamma t} &= \frac{g_x}{m} - m\gamma c e^{-m\gamma t} + \frac{g_x}{m} \tag{15}\\
v(0) = 0 &= \frac{g_x}{\gamma m^2}e^{0} + ce^{-m\gamma t} \tag{16}\\
\Rightarrow c = -\frac{g_x}{\gamma m^2} & \tag{17}\\
x(t) &= \int_{t_0}^{t} v(t)\mathrm{d}t \tag{18}\\
&= \int \frac{g_x}{\gamma m^2} + \frac{g_x}{\gamma m^2}e^{-m\gamma t}\mathrm{d}t \tag{19}\\
&= t\frac{g_x}{\gamma m^2} - \frac{g_x}{m}e^{-m\gamma t} + c_2 \tag{20}\\
x(0) = 0 &= -\frac{g_x}{m} + c_2 \tag{21}\\
\Rightarrow c_2 &= \frac{g_x}{m} \tag{22}\\
x(t) &= t\frac{g_x}{\gamma m^2} - \frac{g_x}{m}e^{-m\gamma t} + \frac{g_x}{m} \tag{23}
\end{align}
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Aufgabe 2

Aufgabe 3

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\begin{aligned}
\operatorname{grad}\left(\frac{a}{r}\right) &= -\frac{a}{r^2} && (24)\\
\operatorname{rot}(\vec{\omega}\times\vec{r}) &= \nabla\times(\vec{\omega}\times\vec{r}) && (25)\\
&= \nabla\times\begin{pmatrix}\omega_2 r_3-\omega_3 r_2\\ \omega_3 r_1-\omega_1 r_3\\ \omega_1 r_2-\omega_2 r_1\end{pmatrix} && (26)\\
&= \begin{pmatrix}-\omega_1-\omega_1\\ -\omega_2-\omega_2\\ -\omega_3-\omega_3\end{pmatrix} && (27)\\
&= -2\vec{\omega} && (28)\\
\operatorname{rot}\begin{pmatrix}e^{-x^2-y^2}\\ e^{-x^2-y^2}\\ z\end{pmatrix} &= \begin{pmatrix}0\\ 0\\ -2ye^{-x^2-y^2}+2xe^{-x^2-y^2}\end{pmatrix} && (29)\\
&= \begin{pmatrix}0\\ 0\\ (2x-2y)e^{-x^2-y^2}\end{pmatrix} && (30)\\
\operatorname{rot}\begin{pmatrix}xe^{\sin(r)}\\ ye^{\sin(r)}\\ ze^{\sin(r)}\end{pmatrix}&;\ r=\sqrt{x^2+y^2+z^2} && (31)\\
&= \vec{0}\text{ , da gilt:} && (32)\\
\left(ze^{\sin(r)}\right)^{(y)} &= yz\frac{e^{\sin(r)}\cos(r)}{r} && (33)
\end{aligned}
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