MEM380: Mobile Robots 1

Wk 1 Assignment

Due: Monday, Jan. 10, 11:59pm EST

Objective: The purpose of this assignment is to review some basic linear algebra concepts and to familiarize you with MATLAB .

Interactive MATLAB Tutorial

Go to `http://www.mathworks.com/academia/student_center/tutorials/register.html` and register for a MathWorks account so you can access the MATLAB tutorials. Complete the two tutorials titled: "MATLAB On-Ramp (approximately 30 minutes)" and "MATLAB for Problem Solving (approximately 60 minutes)".

Linear Algebra Review w/ MATLAB

1. (10 pts) Show that the following is true:

(a) Using only variables, *i.e.* no numbers, that if A is an $n \times n$ symmetric matrix, then

$$x^T Ay = y^T Ax$$

where $x, y \in \mathbb{R}^n$.

(b) In MATLAB , let $n = 3$, with x, y, and A given by

$$x = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad y = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \quad A = \begin{pmatrix} -5 & 3 & 0 \\ 3 & -1 & -4 \\ 0 & -4 & 2 \end{pmatrix}.$$

verify that $x^T Ay = y^T Ax$ for the given values of x, y, and A. Include your commands as well as the answers given to/by MATLAB .

2. (20 pts) Show, using only variables, that the following vectors are either linearly independent or linearly dependent if u, v, and w are linearly independent in vector space $\mathbf{V}$.

(a) u, $u + v$, and $u + v + w$.

(b) $u + 2v - w$, $u - 2v - w$, and $4v$.

(c) $u - v$, $v - w$, and $w - u$.

(d) $-u + v + w$, $u - v + w$, and $-u + v - w$.

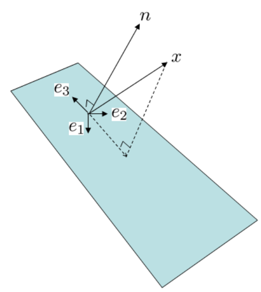

Figure 1: Figure for Problem 3.

3. (20 pts) Suppose two square $n \times n$ matrices A and $\hat{A}$ are similar to one another, *i.e.* $\hat{A} = M^{-1}AM$ for some orthonormal matrix M.

(a) Show, using only variables, that the eigenvalues of $\hat{A}$ are the same as those of A. Do the two matrices have the same eigenvectors?

(b) In MATLAB , let $n = 2$, with A, $\hat{A}$, and M given by

$$A = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}, \quad \hat{A} = \begin{pmatrix} 1 & .5 \\ 0 & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

(c) Compute the eigenvalues and eigenvectors of A by hand. Show your work.

(d) In MATLAB , use `eig(·)` to verify your results. Include your commands as well as the answers given to/by MATLAB . NOTE: If you do not remember how to use the `eig(·)` function, type `help eig` on the MATLAB prompt.

4. (20 pts) Given any matrix A with the singular decomposition $U\Sigma V^T$:

(a) (5 pts) Show, using only variables, that the columns of U are the eigenvectors of AA^T.

(b) (10 pts) In MATLAB , given

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix},$$

compute A^TA and its eigenvalues and unit eigenvectors, compute AA^T and its eigenvalues and unit eigenvectors, and find all entries in the SVD $A = U\Sigma V^T$ using `svd(·)`.

(c) (5 pts) What did you discover about the eigenvalues and unit eigenvectors of A^TA, AA^T, and U, Σ, and V?

(d) **Extra Credit: (+5 pts)** Prove your claim in (c) using only variables.

5. (5 pts) Derive the formula for the orthogonal projection of a vector x onto the plane that is orthogonal to a given vector n, as shown in Figure 1.