CS146 - Assignment LBA

March 22, 2020

0.1 Questions to answer

What is the basic average price for each product? You need to think carefully about how to anchor the basic price for each product since this will depend on the currency used as well as the distribution of prices.

How much does each of the following factors modify the basic price of the product (up or down)?

```
Brand of the grocery store.
The geographical location of the grocery store.
```

Does price variation by geographical location correlate with variation in rental prices in Buenos Aires, or not?

You can find a 2016 map of average rental prices in Buenos Aires, organized by Metro station, here.

[418]:
```
#import libraries
import pystan
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as sts

sns.set()
pd.set_option('display.max_columns', 50)
```

1 Data Processing

[611]:
```
#read data
data = pd.read_csv('lba_data.csv',encoding='"UTF-8"')
```

[612]:
```
#drop name and address cols
data = data.drop(['Your name', 'Grocery store street address'], axis=1)

#normalising data according to weight/count/volume
for i in range(3,60,2):
```

```
    data[data.columns[i]] =  data[data.columns[i]] / data[data.columns[i-1]]

#dropping quantity lists.
cols = list(range(2,61,2))
data = data.drop(data.columns[cols],axis=1)

#creating column names list
col_names = ['Apples', 'Bananas','Tomatoes','Potatoes','Flour', 'Rice', 'Milk',␣
↪'Butter', 'Eggs', 'Chicken']
#col_names = [col_names] * 3
col_names = [val for val in col_names for _ in range(3)]

#adding price and quantity to col names
for i in range(0,30):
    col_names[i] = col_names[i] + ' Price'

#adding each product indexing
for j in range(0,3):
    for i in range(j,30,3):
        col_names[i] = col_names[i] +  ' ' + str(range(1,4)[int(j)])

#rename cols in original data set
column_indices = list(range(2,32))
old_names = data.columns[column_indices]
data.rename(columns=dict(zip(old_names, col_names)), inplace=True)
```

[613]:
```
#numerical code for product types and location/brand multipliers
store_di = { 1: 'Supermercados Día', 2: 'Jumbo', 3: 'Carrefour', 4: 'Safeway',␣
↪5: 'Wellcome'}

neigh_di = { 1:"Almagro",2:"Balvanera",3:"Caballito",4:"Colegiales",5:
↪"Monserrat",
             6:"Palermo",7:"Recoleta",8:"Retiro",9:"San Francisco", 10:"Taipei"}

product_di = {1:"Apples",2:"Bananas",3:"Tomatoes",4:"Potatoes",5:"Flour",
             6:"Rice",7:"Milk",8:"Butter",9:"Eggs",10:"Chicken"}

#inverse mappings for column coding store/neighboorhood
inv_store_di = {v: k for k, v in store_di.items()}
inv_neigh_di = {v: k for k, v in neigh_di.items()}
data['Grocery store'] = data['Grocery store'].map(inv_store_di)
data['Neighboorhood'] = data['Neighboorhood'].map(inv_neigh_di)

#cleaning error
data.at[29,'Grocery store']= 4

data.tail()
```

[613]:

	Grocery store	Neighboorhood	Apples Price 1	Apples Price 2
35	3.0	10	306.666667	493.333333
36	5.0	10	230.000000	260.000000
37	5.0	10	460.000000	593.333333
38	3.0	10	384.615385	226.562500
39	5.0	10	1380.000000	350.000000

	Apples Price 3	Bananas Price 1	Bananas Price 2	Bananas Price 3
35	520.000000	100.000000	118.000000	NaN
36	350.000000	140.000000	NaN	NaN
37	115.000000	140.000000	196.666667	NaN
38	325.000000	99.099099	118.000000	NaN
39	211.111111	98.000000	193.333333	140.0

	Tomatoes Price 1	Tomatoes Price 2	Tomatoes Price 3	Potatoes Price 1
35	196.666667	130.000000	326.666667	590.000000
36	220.000000	NaN	NaN	196.666667
37	300.000000	280.000000	220.000000	196.666667
38	236.000000	326.666667	222.222222	110.000000
39	173.333333	440.000000	306.666667	184.615385

	Potatoes Price 2	Potatoes Price 3	Flour Price 1	Flour Price 2
35	130.000000	NaN	78.000000	NaN
36	NaN	NaN	120.000000	110.0
37	163.333333	138.000000	162.352941	140.0
38	118.000000	130.000000	78.000000	130.0
39	147.500000	196.666667	118.000000	162.4

	Flour Price 3	Rice Price 1	Rice Price 2	Rice Price 3	Milk Price 1
35	NaN	NaN	NaN	NaN	181.043663
36	NaN	NaN	NaN	NaN	182.800000
37	118.0	191.2	139.0	238.666667	241.379310
38	190.0	247.5	264.0	153.333333	174.474960
39	NaN	NaN	NaN	NaN	160.215054

	Milk Price 2	Milk Price 3	Butter Price 1	Butter Price 2
35	170.000000	174.474960	1550.0	814.977974
36	174.400000	182.800000	1400.0	1700.000000
37	187.103594	189.217759	1600.0	1080.000000
38	107.784431	168.000000	1100.0	1550.000000
39	166.000000	0.174475	1600.0	1080.000000

	Butter Price 3	Eggs Price 1	Eggs Price 2	Eggs Price 3	Chicken Price 1
35	NaN	27.000000	23.8	17.000000	433.333333
36	814.977974	15.800000	15.8	17.800000	712.000000
37	1150.000000	295.454545	230.0	263.333333	498.402556
38	814.977974	490.909091	450.0	675.000000	433.333333

```
39      900.000000     13.800000           9.6      10.600000       632.000000

    Chicken Price 2  Chicken Price 3
35       460.000000            138.0
36      1187.500000            278.0
37       926.666667            190.0
38       460.000000            176.0
39       560.000000              NaN
```

[635]:

```
#preprocess data into lists of length: number of observed prices collected
prices = []
stores = []
products = []
neighs = []

for i in range(1,11):
    #iterate through each product
    product = product_di[i]

    #compile dataframe of each product's price and multipliers dropping Nans
    price1 = data[[product+' Price 1','Grocery store','Neighboorhood']].dropna()
    price2 = data[[product+' Price 2','Grocery store','Neighboorhood']].dropna()
    price3 = data[[product+' Price 3','Grocery store','Neighboorhood']].dropna()

    #for each price of each product, compile into a list
    price = price1[product+' Price 1'].values.tolist() +  price2[product+'␣
 ↪Price 2'].values.tolist() + price3[product+' Price 3'].values.tolist()

    #corresponding list of product key
    product = len(price)*[i]

    #for each neighboorhood/store of each product, compile into a list
    neigh = price1['Neighboorhood'].values.tolist() + price2['Neighboorhood'].
 ↪values.tolist() + price3['Neighboorhood'].values.tolist()
    store = price1['Grocery store'].values.tolist() + price2['Grocery store'].
 ↪values.tolist() + price3['Grocery store'].values.tolist()

    #add lists into master list of processed data
    prices += price
    stores += store
    products += product
    neighs += neigh

#making sure stores list was int not float.
stores = list(map(int, stores))
```

2 Inference With Stan

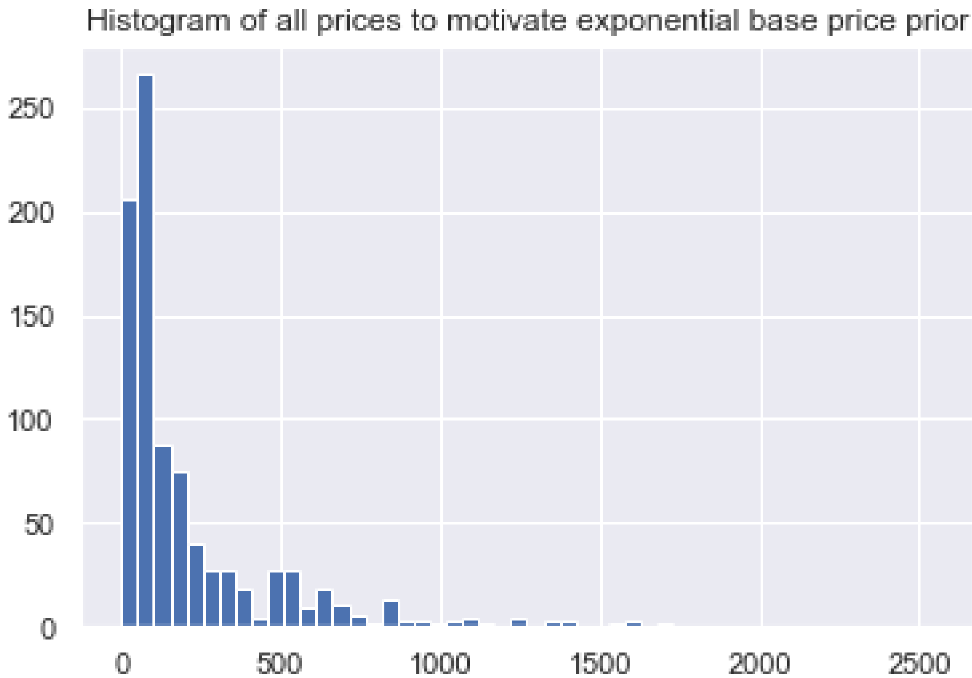
2.0.1 Model Choice

- My model uses a partially hierarchical structure, comprising of a exponential prior draw from a gamma hyperprior in order to capture noise around pricing of different products and two cauchy priors with fixed parameters. The likelihood function is represented by a normal distribution where the mean is represented by base price * store multiplier * neighbourhood multiplier and sigma that is derived from a gamma hyperprior to capture the different variations of uncertainty for each product pricing.

2.0.2 Model Assumptions

- The cauchy distribution was chosen as thh prior for the store and neighbourhood multipliers. The choice of the cauchy distribution was due to the cauchy's fat tails, which could represent a broad prior capturing a wider range of store and neighbourhood multiplier values. Given the diversity of neighbourhoods and brands we are attempting to model, a cauchy is an ideal candidate for a generative prior. The multipliers are centered around 1 to represent an average multiplier effect of 1.
- The exponential distribution was used to as the prior for the base prices. The choice of the exponential distribution was motivated by the exponential shape of the distribution of all prices observed at cursory glance (as shown by the histogram below). The lambda parameter that was used to generate the priors was devided from a gamma hyperprior with parameters (1,200), chosen in part to scale the prices to the Argentinian peso.
- The likelihood function is normal because of the assumption that prices of goods should be normally distributed if we observe enough samples. The sigma is derived from a gamma hyperprior with parameters (1,2) to scale the uncertainty to the Argentinian peso and to allow a broad range of prices.

[675]:
```
plt.hist(prices, bins = 50)
plt.title("Histogram of all prices to motivate exponential base price prior")
plt.show()
```



[685]:
```
#Build the stan model
stan_code = """

// The data block contains all known quantities - typically the observed
// data and any constant hyperparameters.
data {

    int<lower=1> n;
    real<lower=0> Prices[n];                        // number of observed prices␣
 ↪collected
    int<lower=1, upper=10> Product[n];              // number of products
    int<lower=1, upper=5> Store[n];              // number of stores
    int<lower=1, upper=10> Neigh[n];              // number of  neighboorhood

}

// The parameters block contains all unknown quantities - typically the
// parameters of the model. Stan will generate samples from the posterior
// distributions over all parameters.

parameters {

    real<lower=0> base_price[10];       //vector containing base prices of each␣
 ↪product
```

```
    real<lower=0> mul_store[5];        //vector containing store multipliers
    real<lower=0> mul_neigh[10];        //vector containing neighboorhood␣
↪multipliers
    real<lower=0> sigma;
    real<lower=0> lambda;

}

// The model block contains all probability distributions in the model.
// This of this as specifying the generative model for the scenario.
model {

    sigma ~ gamma(1, 2);      //generate random noise for sigma parameter of␣
↪normal distribution
    lambda ~ gamma(1, 200);      //generate noise around lambda parameter of␣
↪base price exponential


    for (i in 1:10) { //base price prior
        base_price[Product[i]] ~ exponential(lambda);   //base price
    };

    for (i in 1:5) { //store prior
        mul_store[Store[i]] ~ cauchy(1, 0.5);  //store multiplier
    };

    for (i in 1:10) { //neighboorhood prior
        mul_neigh[Neigh[i]] ~ cauchy(1, 0.5);    //neighbourhood multiplier
    }

    for(i in 1:n) { //likelihood function
        Prices[i] ~ normal(base_price[Product[i]] * mul_store[Store[i]] *␣
↪mul_neigh[Neigh[i]], sigma);
  }

}

"""

#compile the model
stan_model = pystan.StanModel(model_code=stan_code)
```

```
INFO:pystan:COMPILING THE C++ CODE FOR MODEL
anon_model_b411f0838ff5e1a42abf40c5d96cd1c8 NOW.
```

[477]:
```
#stan input data
stan_data = {
```

```
                'n': len(prices),
                'Prices': prices,
                'Product': products,
                'Store': stores,
                'Neigh': neighs,
            }
```

[686]:
```
# Fitting stan model to the data. This will generate samples from the posterior␣
 ↪over all parameters of the model.
stan_results = stan_model.sampling(data=stan_data)
posterior = stan_results.extract()
```

```
WARNING:pystan:n_eff / iter below 0.001 indicates that the effective sample size
has likely been overestimated
WARNING:pystan:Rhat above 1.1 or below 0.9 indicates that the chains very likely
have not mixed
WARNING:pystan:76 of 4000 iterations ended with a divergence (1.9 %).
WARNING:pystan:Try running with adapt_delta larger than 0.8 to remove the
divergences.
WARNING:pystan:3924 of 4000 iterations saturated the maximum tree depth of 10
(98.1 %)
WARNING:pystan:Run again with max_treedepth larger than 10 to avoid saturation
```

3 Questions to Answer

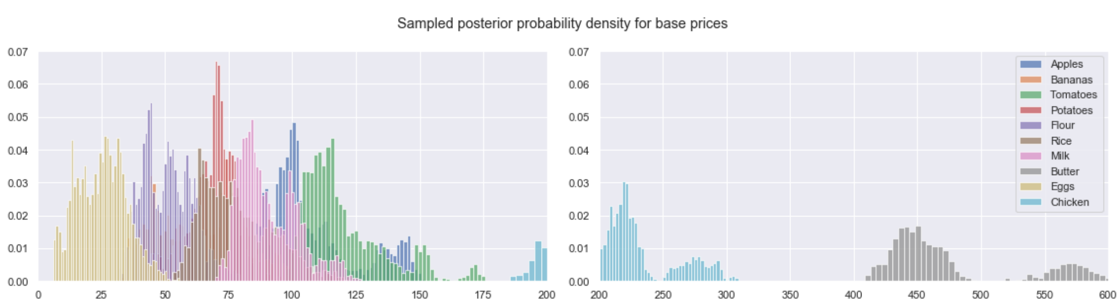
3.1 What is the basic average price for each product?

An individual egg is the cheapest good at an average base price of 25 pesos, while the most expensive good is a kilogram of butter. Basic average price for each product can be observed in the table below.

[694]:
```
print(stan_results.stansummary(pars=['base_price'], probs=[0.025, 0.5, 0.975]))

plt.figure(figsize = (16,8))
plt.suptitle('Sampled posterior probability density for base prices')

plt.subplot(2, 2, 1)
for i in range(10):
    plt.hist(posterior['base_price'][:,i],bins=50, density = True, alpha = 0.7,␣
 ↪label = product_di[i+1])
plt.xlim((0,200))## What is the basic average price for each product?

plt.subplot(2, 2, 2)
for i in range(0,10):
    plt.hist(posterior['base_price'][:,i],bins=50, density = True, alpha = 0.7,␣
 ↪label = product_di[i+1])
```

```
plt.xlim((200,600))## What is the basic average price for each product?
plt.legend()

plt.tight_layout(rect=[0, 0.03, 1, 0.95])

plt.show()
```

```
Inference for Stan model: anon_model_b411f0838ff5e1a42abf40c5d96cd1c8.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

                 mean se_mean     sd   2.5%    50%  97.5%  n_eff   Rhat
base_price[1]  107.92   10.98  17.41  86.05 101.75 145.84      3   2.71
base_price[2]   63.51   10.34  16.64   36.1  65.18  94.38      3    2.3
base_price[3]  120.23    9.19  16.23 100.93 115.19 167.84      3   2.37
base_price[4]    68.7    2.64   9.28  47.01  69.88  86.93     12   1.72
base_price[5]   51.01    4.69   9.66  35.33  50.77  70.87      4   1.64
base_price[6]   78.26     8.5  13.92  57.84  75.74 109.22      3   2.18
base_price[7]   90.59     6.2  11.57  74.93   87.5  118.0      3   1.93
base_price[8]   479.0   38.46  56.36  416.9 455.92  598.2      2   4.52
base_price[9]   25.09    5.35   9.64   7.63  25.51  44.91      3   1.86
base_price[10] 230.99   19.47  28.95 194.43 221.18 293.68      2   3.54

Samples were drawn using NUTS at Sun Mar 22 18:23:35 2020.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

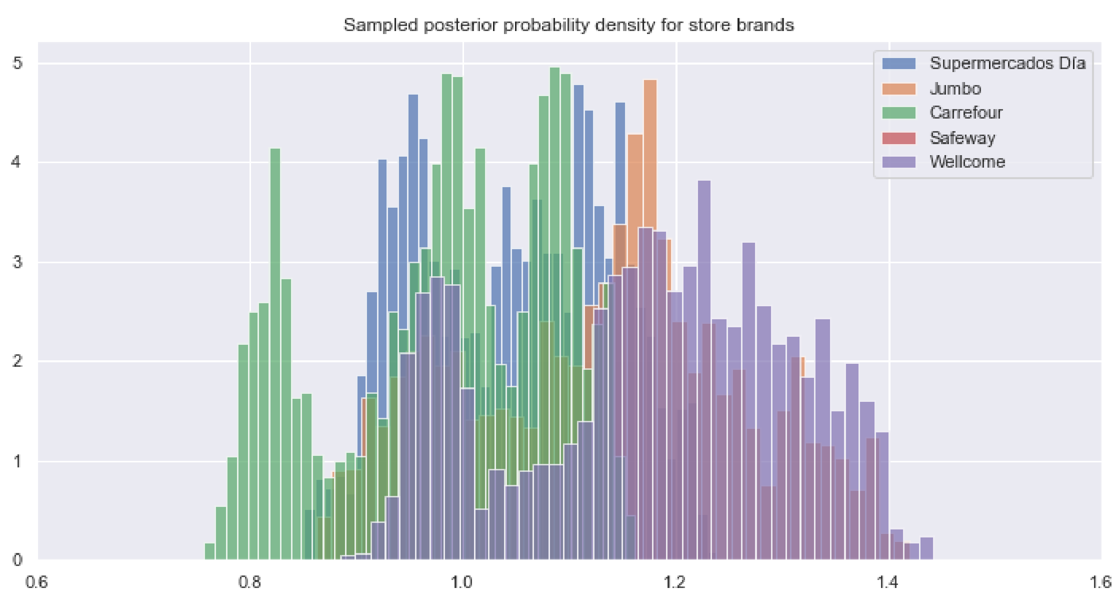


3.2 How much does each brand of the grocery store modify the basic price of the product (up or down)?

Carrefour is the cheapest brand modifier, while Safeway is the most expensive. I neglected to graph Safeway on the plot below and the multiplier mean for Safeway suggest that I failed to accurately model Safeway's multiplier. The unreasonably large Safeway multipler shows how prices in San Francisco are much more expensive than Buenoes Aires or Taipei.Wellcome is the next most expensive modifer, and shares much overlap with the other three Argentinian based brands.

[693]:
```
print(stan_results.stansummary(pars=['mul_store'], probs=[0.025, 0.5, 0.975]))


plt.figure(figsize = (12,6))
plt.title('Sampled posterior probability density for store brands')
for i in range(5):
    plt.hist(posterior['mul_store'][:,i],bins=40, density=True, alpha = 0.7,␣
 ↪label = store_di[i+1])
plt.xlim((0.6,1.6))
plt.legend()

plt.show()
```

```
Inference for Stan model: anon_model_b411f0838ff5e1a42abf40c5d96cd1c8.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

                 mean se_mean     sd     2.5%      50%    97.5%  n_eff   Rhat
mul_store[1]     1.05    0.06   0.09     0.89     1.05     1.21      3   2.56
mul_store[2]     1.13    0.08   0.13      0.9     1.15     1.38      3   2.69
mul_store[3]     0.98    0.07    0.1     0.79     0.99     1.14      2   3.07
mul_store[4] 1.7e301     nan    inf 2.5e-301 3.7e-154  8.0e222    nan    nan
mul_store[5]     1.18    0.08   0.13     0.95     1.19     1.39      2   2.79

Samples were drawn using NUTS at Sun Mar 22 18:23:35 2020.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```



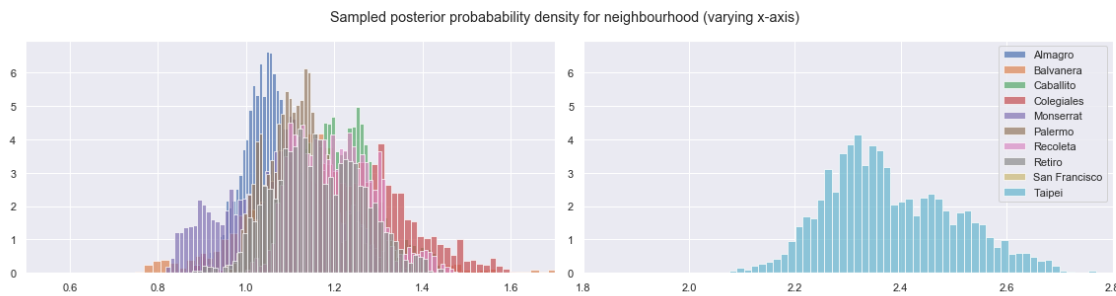
3.3 How much does the geographical location of the grocery store modify the basic price of the product (up or down)?

Most of the Argentinian neighbourhoods share similar multipliers, with Montserrat being the cheapest and Colegiales being the most expensive within Buenos Aires. Taipei is approximately twice as expensive as Buenoes Aires, and San Francisco I failed to model accurately again. The unreasonably large San Francisco multipler shows how prices in San Francisco are much more expensive than Buenoes Aires or Taipei.

[698]:
```
print(stan_results.stansummary(pars=['mul_neigh'], probs=[0.025, 0.5, 0.975]))
Carrefour is the cheapest brand modifier, while Safeway is the most expensive.␣
↪I neglected to graph Safeway on the plot below and the multiplier mean for␣
↪Safeway suggest that I failed to accurately model Safeway's multiplier.␣
↪Wellcome is the next most expensive modifer, and shares much overlap with␣
↪the other three Argentinian based brands. figsize = (16,8))
plt.suptitle('Sampled posterior probabability density for neighbourhood␣
↪(varying x-axis)')

plt.subplot(2, 2, 1)
for i in range(10):
    plt.hist(posterior['mul_neigh'][:,i], bins=50, density=True, alpha = 0.7,␣
↪label = neigh_di[i+1])
plt.xlim((0.5,1.7))

plt.subplot(2, 2, 2)
for i in range(10):
    plt.hist(posterior['mul_neigh'][:,i], bins=50, density=True, alpha = 0.7,␣
↪label = neigh_di[i+1])
plt.xlim((1.8,2.8))
plt.legend()

plt.tight_layout(rect=[0, 0.03, 1, 0.95])

plt.show()
```

```
Inference for Stan model: anon_model_b411f0838ff5e1a42abf40c5d96cd1c8.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

                mean se_mean     sd    2.5%     50%   97.5%  n_eff   Rhat
mul_neigh[1]    1.07    0.04   0.08    0.94    1.06    1.24      4   1.58
mul_neigh[2]    1.15    0.03   0.14    0.84    1.15    1.45     20   1.17
mul_neigh[3]     1.2    0.02   0.09    1.02     1.2    1.39     19   1.28
mul_neigh[4]    1.25    0.04   0.13    0.99    1.25    1.51      9   1.46
```

```
mul_neigh[5]      1.07    0.06   0.12     0.85    1.08    1.28      3   1.95
mul_neigh[6]      1.13    0.03   0.08      1.0    1.12    1.32      6    1.7
mul_neigh[7]       1.2    0.04   0.09     1.06    1.19     1.4      5   1.44
mul_neigh[8]      1.16    0.02    0.1     0.99    1.16    1.36     19   1.29
mul_neigh[9]   5.0e303     nan    inf 4.0e-223 6.3e153 1.0e301    nan    nan
mul_neigh[10]     2.38    0.03   0.12     2.18    2.36    2.64     23   1.16

Samples were drawn using NUTS at Sun Mar 22 18:23:35 2020.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```



3.4 Does price variation by geographical location correlate with variation in rental prices in Buenos Aires, or not?

- Price variation of products by geographical location is moderately correlated with variation in rental prices given a Pearson correlation coefficient (R-Square) of 0.44.

[784]:
```python
#rent prices of each neighbourhood using metro station data
Almagro = [4167,4400,4080,4260,4827]
Balvanera = [3500,3850,3580,3750,3750,3880,4480,4600,4250]
Caballito = [4300,4600,3800,4030]
Colegiales = [5600,5121]
Monserrat = [4060,4500,3860,3620,3850]
Palermo = [6050,5100,5633,5720,5114,5055]
Recoleta = [5675,4475,4031]
Retiro = [4600,4700]

neighs = [Almagro,Balvanera,Caballito,Colegiales,Monserrat,Palermo,Recoleta,Retiro]

#calculate means of each neighbourhood
rent_means = []
for i in neighs:
        rent_means.append(np.mean(i))
```

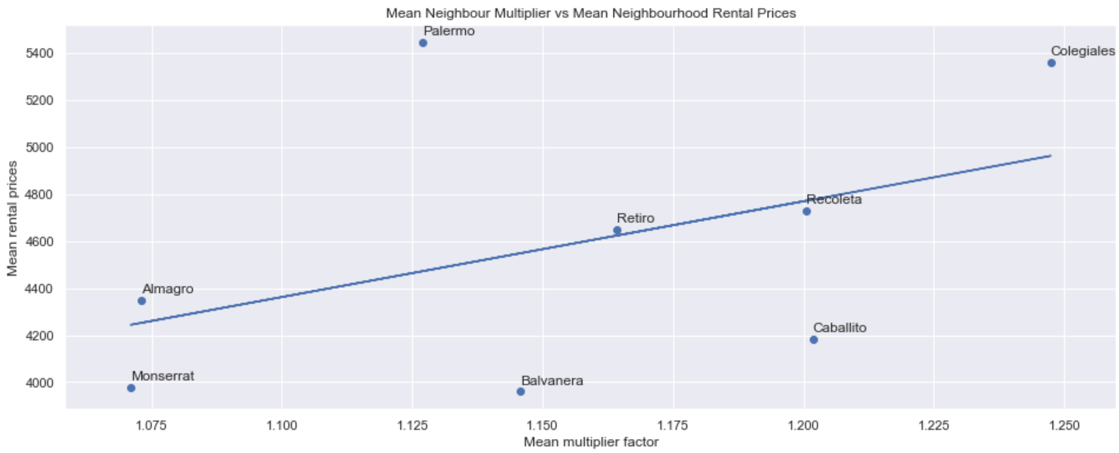
```
neighs_labels = ['Almagro','Balvanera','Caballito' ,'Colegiales',
                 'Monserrat','Palermo','Recoleta','Retiro']

rent = pd.DataFrame((np.mean(posterior['mul_neigh'], axis=0)[:8]).
 ↪tolist(),columns =['multiplier_means'])
rent['rents_means'] = rent_means
rent['neighbourhoods'] = neighs_labels
rent
```

[784]:

	multiplier_means	rents_means	neighbourhoods
0	1.072989	4346.800000	Almagro
1	1.145719	3960.000000	Balvanera
2	1.201835	4182.500000	Caballito
3	1.247411	5360.500000	Colegiales
4	1.071019	3978.000000	Monserrat
5	1.127070	5445.333333	Palermo
6	1.200492	4727.000000	Recoleta
7	1.164271	4650.000000	Retiro

[797]:
```
print('R-Square Correlation Coefficient:',rent['multiplier_means'].
 ↪corr(rent['rents_means']))
x = rent['multiplier_means']
y = rent['rents_means']
b, m = np.polynomial.polynomial.polyfit(x, y, 1)

plt.figure(figsize = (16,6))
plt.scatter(x,y)
plt.plot(x, b + m * x)
plt.xlabel("Mean multiplier factor")
plt.ylabel("Mean rental prices")
plt.title("Mean Neighbour Multiplier vs Mean Neighbourhood Rental Prices")
for i, lab in enumerate(neighs_labels):
    plt.annotate(lab, (x[i], y[i]+30))
```

```
R-Square Correlation Coefficient: 0.4414725532478215
```


Mean Neighbour Multiplier vs Mean Neighbourhood Rental Prices
Palermo
Colegiales
Recoleta
Retiro
Almagro
Caballito
Monserrat
Balvanera
Mean rental prices
Mean multiplier factor
5400
5200
5000
4800
4600
4400
4200
4000
1.075
1.100
1.125
1.150
1.175
1.200
1.225
1.250