影像處理基本原理

壹、視覺構成

貳、影像模型

參、取樣與量化

肆、影像元素間之基本關係

伍、影像幾何

陸、照相軟片性質

壹、視覺構成

一、人類眼睛的構造：

１．外覆薄膜：

(1).眼角膜與鞏膜外覆(Cornea and Sclera outer cover)

(2).脈絡膜(Choroid)

(3).網膜(Retina)

２．瞳孔(Pupil) - 虹彩膜(Iris Diaphragm)

３．水晶體(Lens) - 60-70%水,約 6%脂質,高蛋白

４．光接收體(Light receptors)

(1).Cones - 明亮視覺 (6-7 millions)

Photopic or Bright-light Vision

(2).Rods - 陰暗視覺 (75-150 millions)

Scotopic or Dim-light Vision

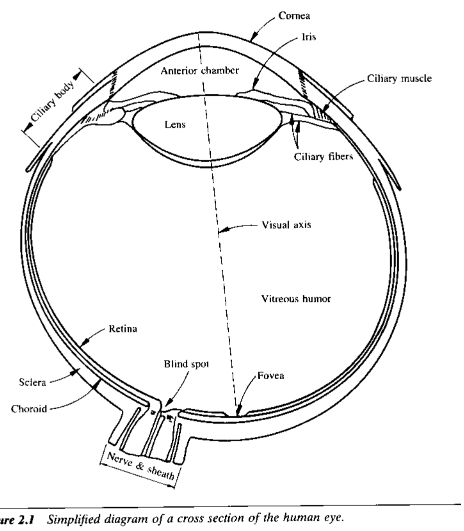


Figure 2.1 *Simplified diagram of a cross section of the human eye.*

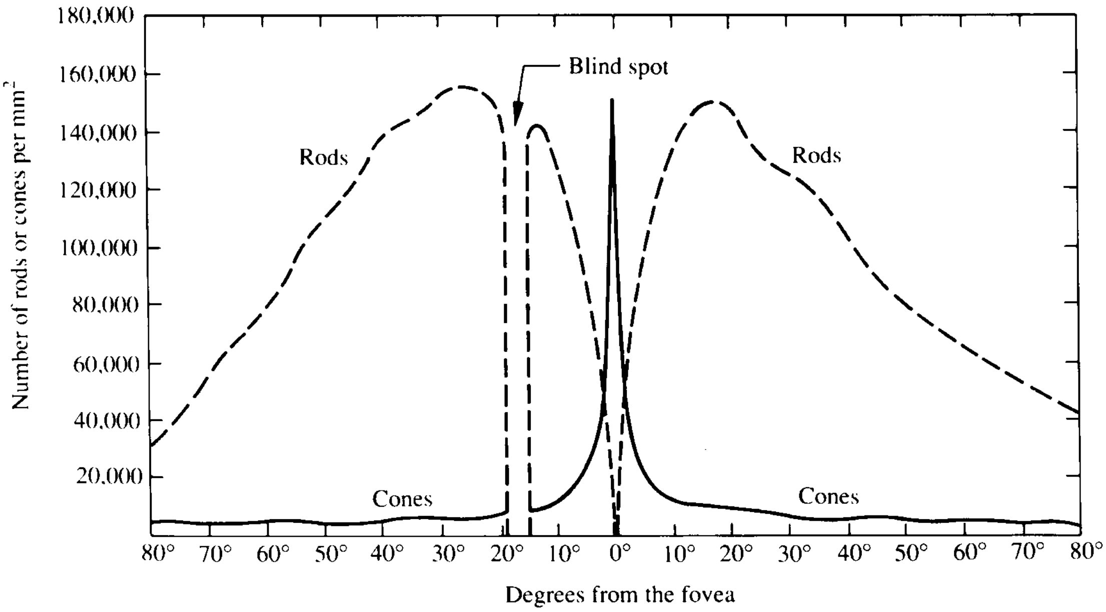


Figure 2.2 *Distribution of rods and cones in the retina. (Adapted from Graham [1965].)*

二、眼球內影像之構成：

●視網膜上的影像主要顯現在 Fovea 區域，由相對位置的光接收體(Light receptor)的激發而能辨知，此辨知過程是將輻射能轉換爲電流脈衝，再由人腦解譯出來。

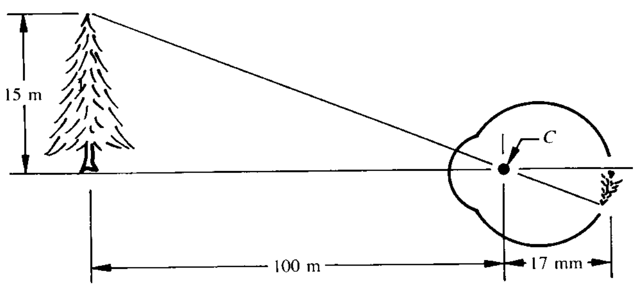


Figure 2.3 *Optical representation of the eye looking at a tree. Point C is the optical center of the lens.*

三、亮度之適應與辨別：

１．主觀明亮度與實際明亮度。

２．明亮度適應(Brightness Adaptation)。

３．韋伯比(Weber Ratio)。

４．同時對比(Simultaneous Contrast)。

５．馬赫帶(Mach Band)效應。

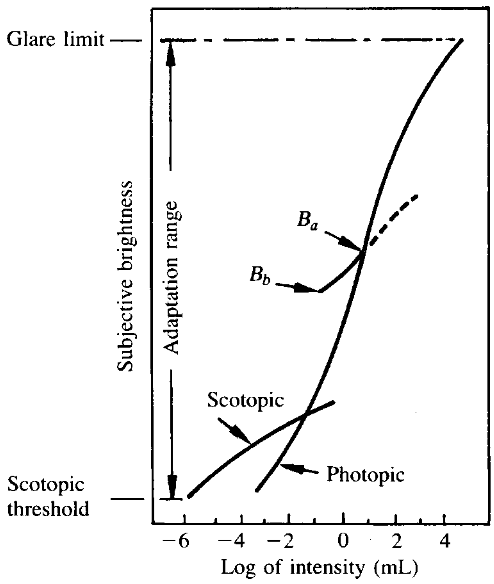


Figure 2.4 *Range of subjective brightness sensations showing a particular adaptation level.*

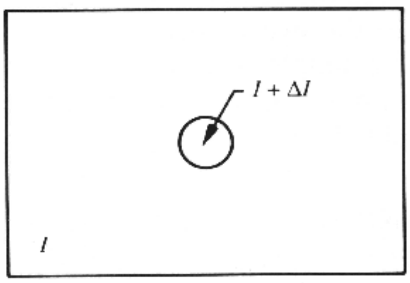


Figure 2.5 *Basic experimental setup used to characterize brightness discrimination.*

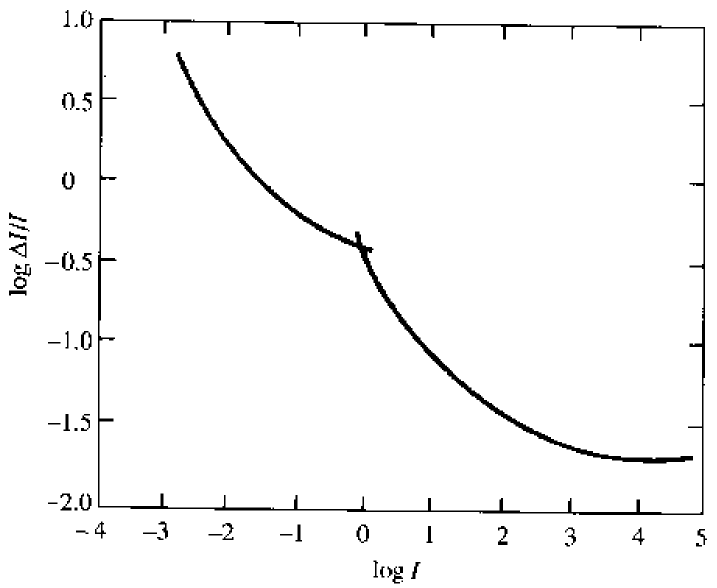
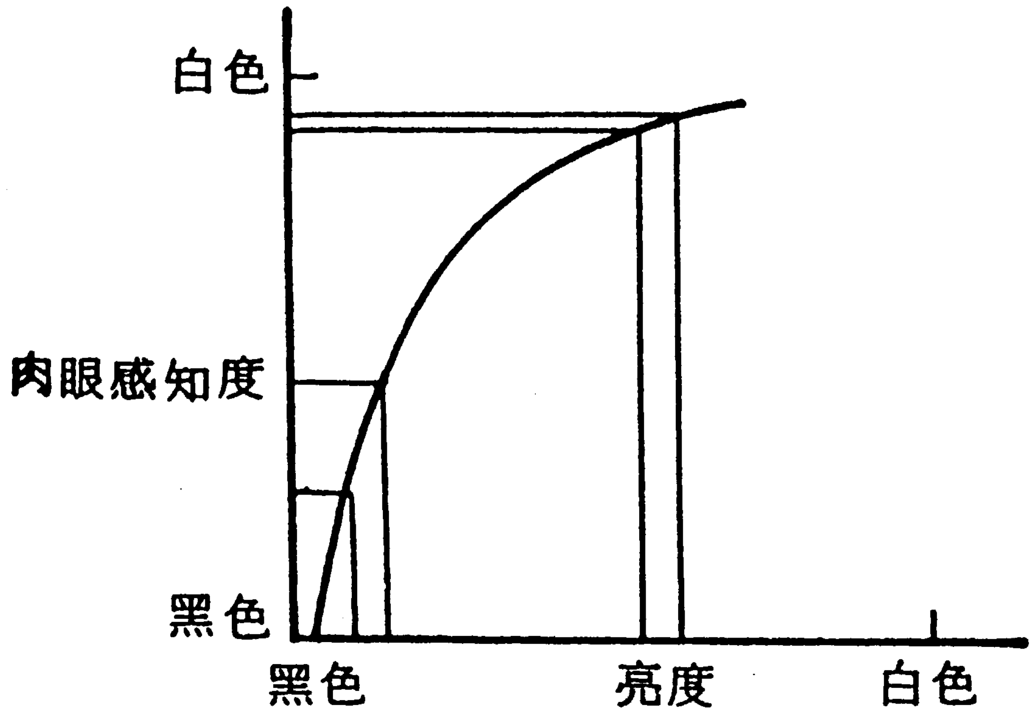


Figure 2.6 *Typical Weber ratio as a function of intensity. (Adapted from Graham [1965].)*



眼睛對於光線亮度的反應

同時對比

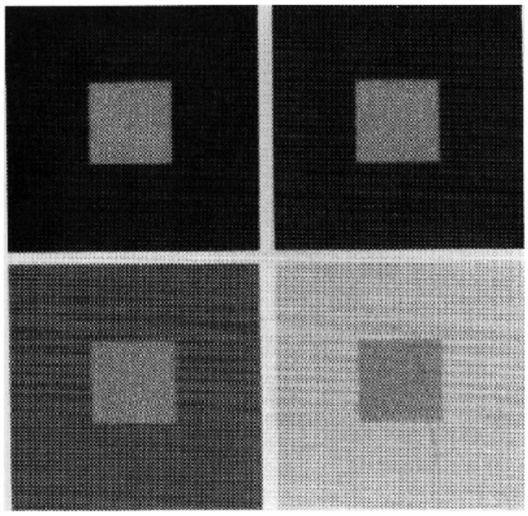


Figure 2.8 Example of simultaneous contrast. All the small squares have exactly the same intensity, but they appear progressively darker as the background becomes lighter.

馬赫帶效應

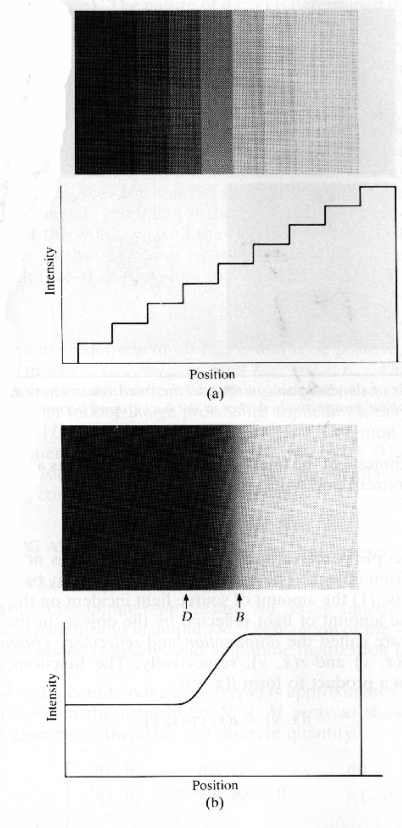
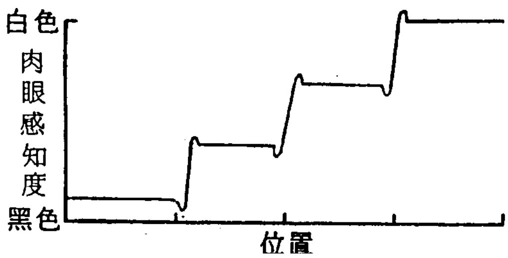


Figure 2.7 Two examples showing that perceived brightness is not a simple function of intensity. (Adapted from Cornsweet [1970].)



四、光之測量與計量單位：

１．Luminous Flux, Luminus Intensity

２．Candela

３．Lux

４．Luminance

５．Lambert (L): 1L = 1/π candle per ㎡

Luminous Flux, Lumen, Candela, Lux

We conclude this introductory section with a brief account of some of the main points in *Photometry*, the science of light measurement. Here we are concerned with the *luminous energy* emitted by a source of light, which stimulates the sensation of vision; and not with any other radiations it may emit, such as infra-red rays, for example, which are invisible.

A source of light such as a lamp emits a continuous stream of luminous energy. We give the name *luminous flux*, symbol Φ, to the 'luminous energy emitted per second'. The unit of luminous flux is the *lumen*, symbol lm. A lumen is a unit of energy per second or power, so it must be related to the watt. Experiment shows that about 621 lumens of green light of wavelength $5{\cdot}540 \times 10^{-10}$ m is equal to 1 watt.

A light source such as a lamp radiates luminous flux in all directions round it. If we consider a small lamp S and a particular direction SA, an amount of luminous flux Φ is radiated in a small cone of 'solid angle' ω drawn round SA with S at the apex. Fig. 17.3 (i). The *luminous intensity*, *I*, of the lamp is defined as the ratio Φ/ω or the 'luminous flux per unit solid angle'. Since a solid angle is measured in steradian, sr, the unit of I is 'lm sr^{-1}'.

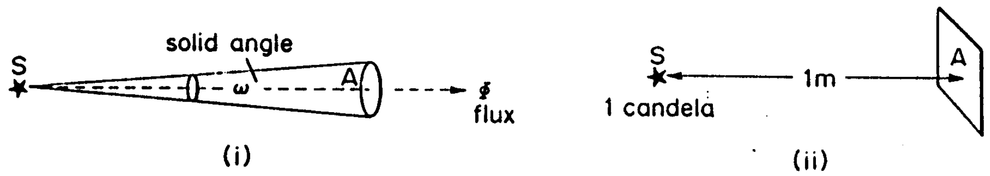


Fig. 17.3 (i) Luminous intensity, (ii) Lux.

A practical unit of luminous intensity is the *candela*, symbol cd. It is defined as the luminous intensity of 1/600 000 metre^2 (1/60 cm^2) of the surface of a black body at the temperature of freezing platinum under 101 325 newton per metre^2 pressure. A standard is maintained at the National Physical Laboratory and here the luminous intensity of manufacturers' lamps are measured in terms of the standard. From above, $I = \Phi/\omega$, so $\Phi = I\omega$. Thus 1 lm = 1 cd sr.

We now consider the surface on which the luminous flux falls. The *illuminance* (or *illumination*), *E*, of a surface is defined as the 'luminous flux per unit area'. If we imagine concentric spheres of different radii r drawn round a small lamp S as centre, the total flux from S will fall on areas equal to $4\pi r^2$. So we can see that the illuminance varies *inversely as the square* of the distance from S. The unit of illuminance is the *lux*, lx. This is the illuminance of a surface A 1 m away from a lamp S of 1 cd when the light falls normally on A. Fig. 17.3 (ii).

The *luminance*, *L*, of a surface is the 'luminous flux per unit area' coming from that surface. The illuminance of white chalk on a blackboard is the same as the surrounding surface. The luminance of the chalk, however, is very much higher than that of the board since the reflection factor of the chalk is much greater than that of the board.

The following table summarises some of the units discussed.

	Symbol	Unit
Luminous flux	Φ	lumen (lm)
Luminous intensity	I	candela (cd)
Illuminance	E	lux (lx)
Luminance	L	cd m^{-2}

貳、影像模型(Image Model)

影像(Image) - 指二維的光強度函數 f(x,y)

$$0 < f(x,y) < \infty \qquad (2.2\text{-}1)$$

f(x,y)可視為由兩個部份組成

$$f(x,y) = i(x,y) \cdot r(x,y) \qquad (2.2\text{-}2)$$

其中 $0 < i(x,y) < \infty$ 為照明(Illumination)

$0 < r(x,y) < 1$ 為反射(Reflectance)

典型的 r(x,y)與 i(x,y)值	
●晴天時太陽之照明	i ≒ 9,000 ft-candle
●陰天時太陽之照明	i ≒ 1,000 ft-candle
●一般辦公室室內之照明	i ≒ 100 ft-candle
●黑色天鵝絨之反射	r ≒ 0.01
●不銹鋼之反射	r ≒ 0.65
●雪之反射	r ≒ 0.93

黑白影像 f 在(x,y)之強度常以灰度(Gray Level, l)稱之：

$$L_{min} < l < L_{max} \qquad (2.2\text{-}5)$$

$L_{min} = i_{min} \cdot r_{min}$ ，

$L_{max} = i_{max} \cdot r_{max}$ ，

$[L_{min}, L_{max}]$稱為灰度區間或明亮度區間(Gray Scale)，

常用之灰度區間為[0,L]，0 為黑色，L 為白色。

參、取樣與量化

一、均勻取樣與量化：

１．影像取樣(Image sampling) - 空間座標之數位化

２．灰度量化(Gray-level Quantization) - 強度或明亮度之數位化

二、數位影像元素與解析度：

１．數位影像(Digital Image)

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & . & . & . & f(0,M-1) \\ f(1,0) & f(1,1) & . & . & . & f(1,M-1) \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ f(N-1,0) & f(N-1,1) & . & . & . & f(N-1,M-1) \end{bmatrix} \qquad (2.3\text{-}1)$$

$$N = 2^n, \qquad M = 2^k \qquad (2.3\text{-}2)$$

$$G = 2^m \qquad (2.3\text{-}2)$$

Table 2.1 Number of Storage Bits for Various Values of *N* and *m*

N \ *m*	1	2	3	4	5	6	7	8
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1,024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608

Table 2.2 Number of 8-bit Bytes of Storage for Various Values of *N* and *m*

N \ *m*	1	2	3	4	5	6	7	8
32	128	256	512	512	1,024	1,024	1,024	1,024
64	512	1,024	2,048	2,048	4,096	4,096	4,096	4,096
128	2,048	4,096	8,192	8,192	16,384	16,384	16,384	16,384
256	8,192	16,384	32,768	32,768	65,536	65,536	65,536	65,536
512	32,768	65,536	131,072	131,072	262,144	262,144	262,144	262,144
1,024	131,072	262,144	393,216	524,288	655,360	786,432	917,504	1,048,576

２．灰度層次與影像元素數量對解析度之影響

影像元素數量之影響

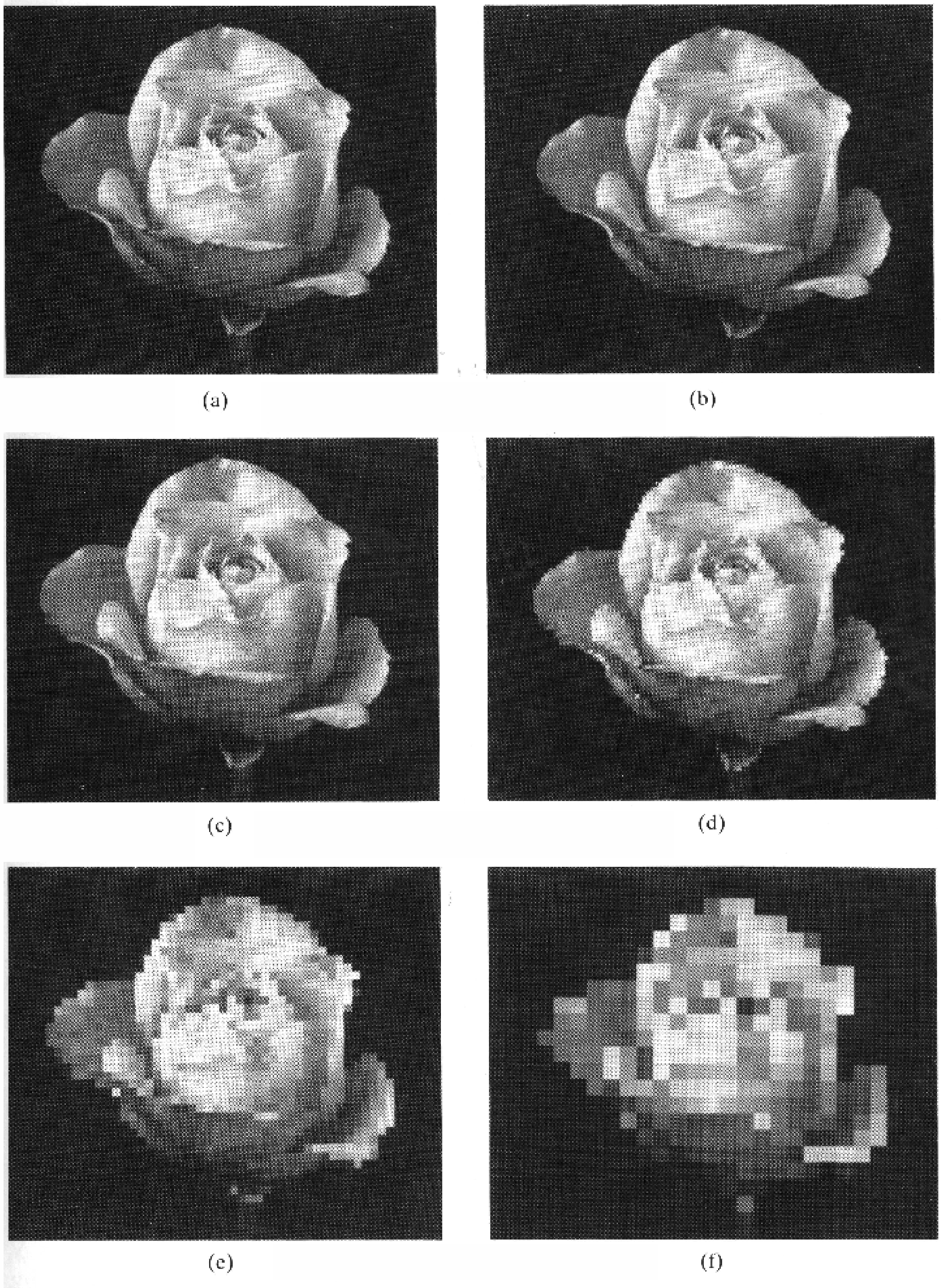


Figure 2.9 Effects of reducing spatial resolution.

灰度層次對解析度之影響

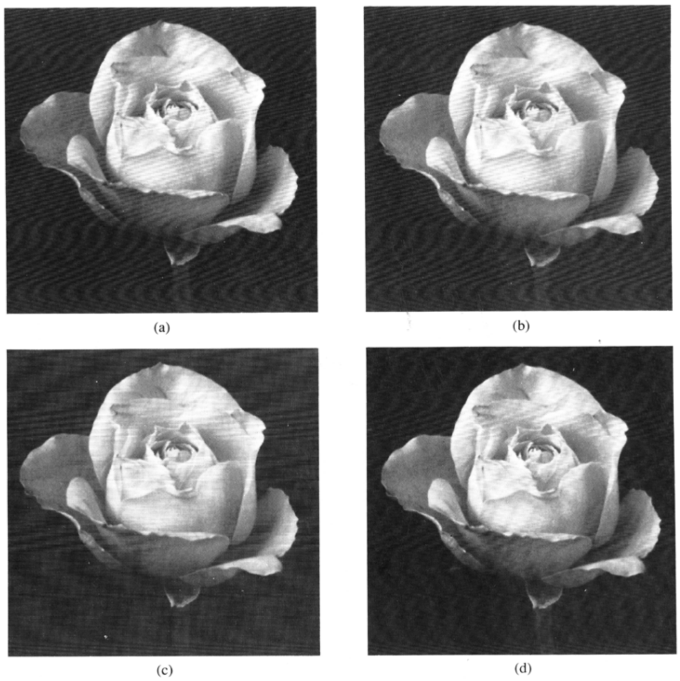


Figure 2.10 *A 1024 × 1024 image displayed in 256, 128, 64, 32, 16, 8, 4, and 2 levels, respectively.*

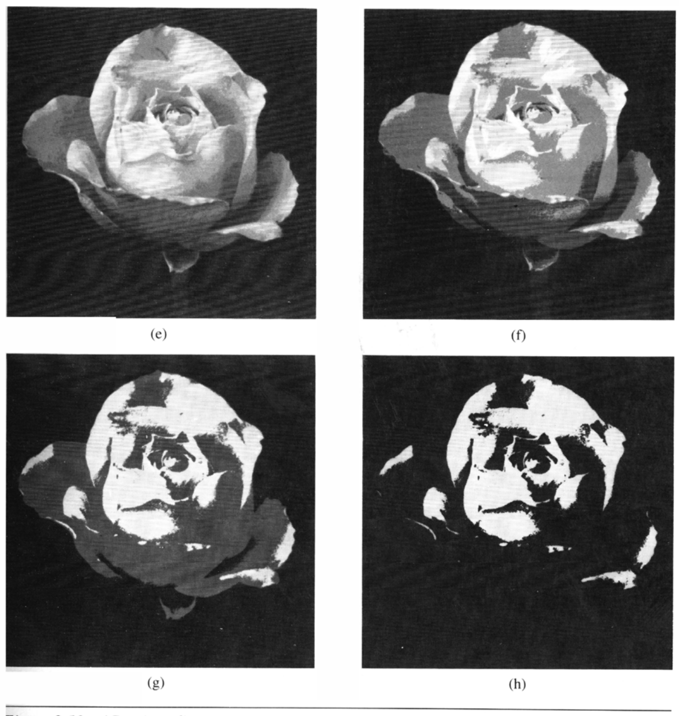
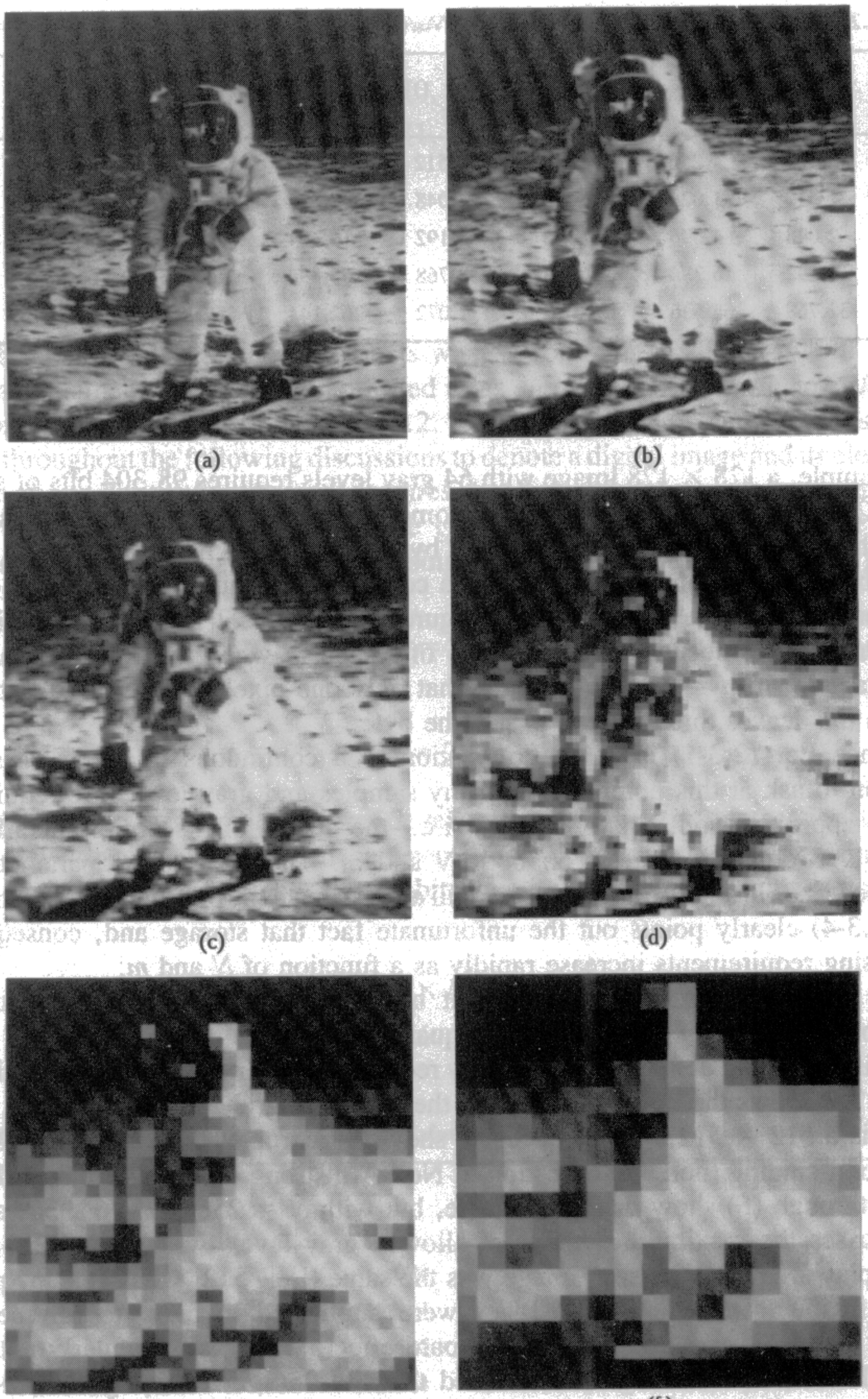


Figure 2.10 *(Continued)*

影像元素數量之影響



(a) (b) (c) (d) (e) (f)

Figure 2.7 Effects of reducing sampling-grid size.

灰度層次對解析度之影響

Figure 2.8 A 512 × 512 image displayed in 256, 128, 64, 32, 16, 8, 4, and 2 levels, respectively.

Figure 2.8 (Continued.)

Isopreference Curve 實驗

Figure 2.11 *Test images used in evaluating subjective image quality. (From Huang [1965].)*

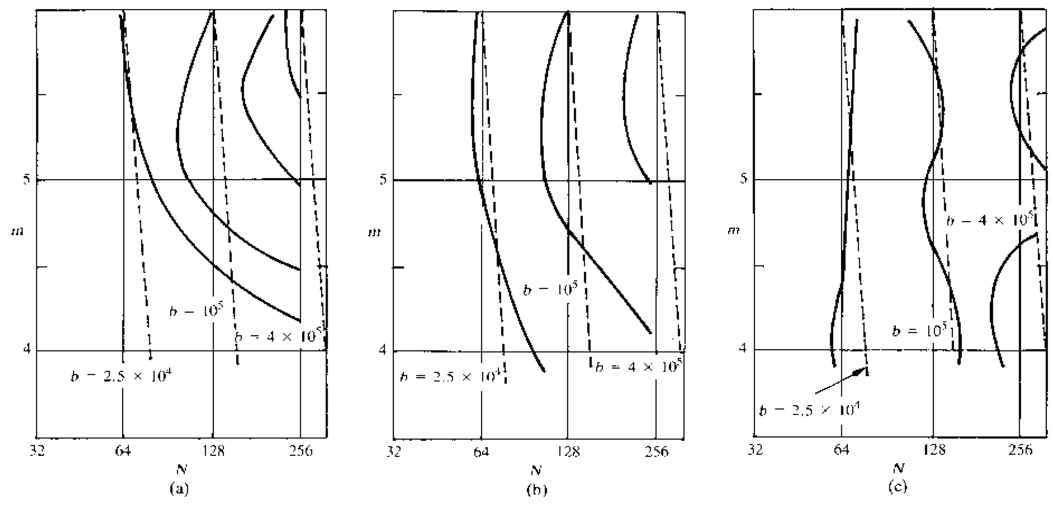


Figure 2.12 *Isopreference curves for (a) face, (b) cameraman, and (c) crowd. (From Huang [1965].)*

三、不均勻取樣與量化：

１．取樣之頻率。

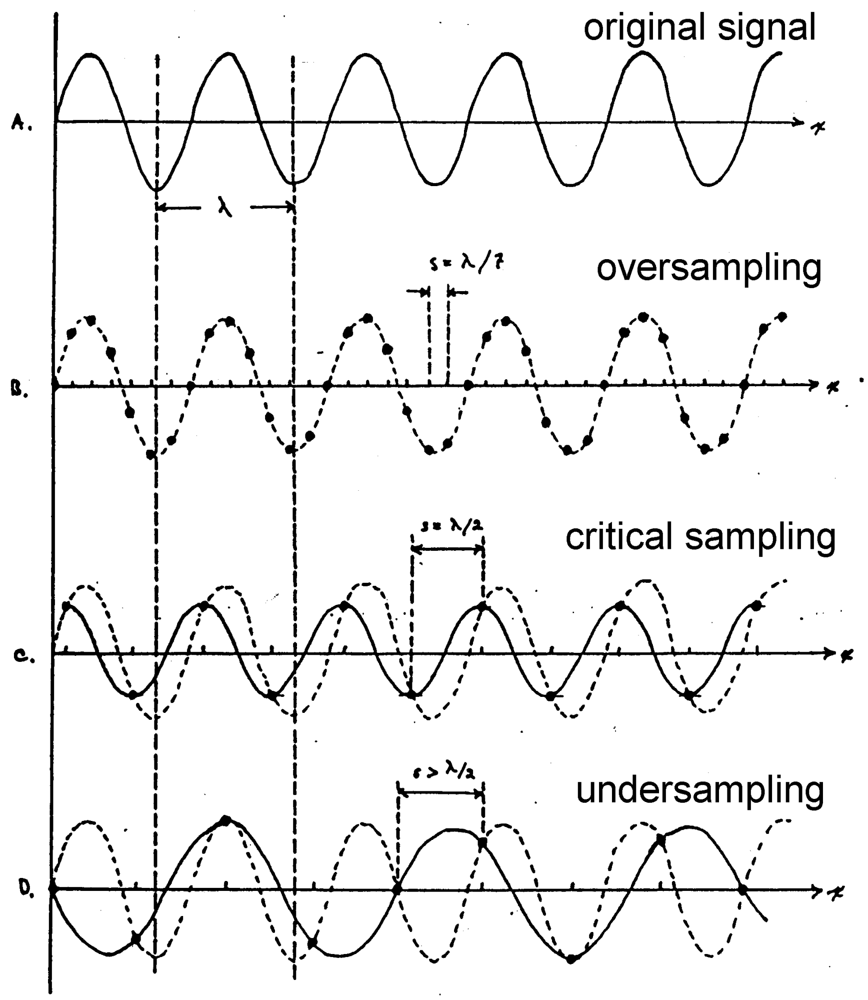
２．量化之考慮：

(1).考慮影像內明暗度變化大之邊界。

(2).考慮影像內灰度出現之頻率(考慮 Histogram)。

四、奈奎斯特原則(Nyquist Criterion)：

●取樣之頻率必須比待測訊號的頻率至少高兩倍以上



五、二維影像之取樣：

１．空間頻率(Spatial Frequency)

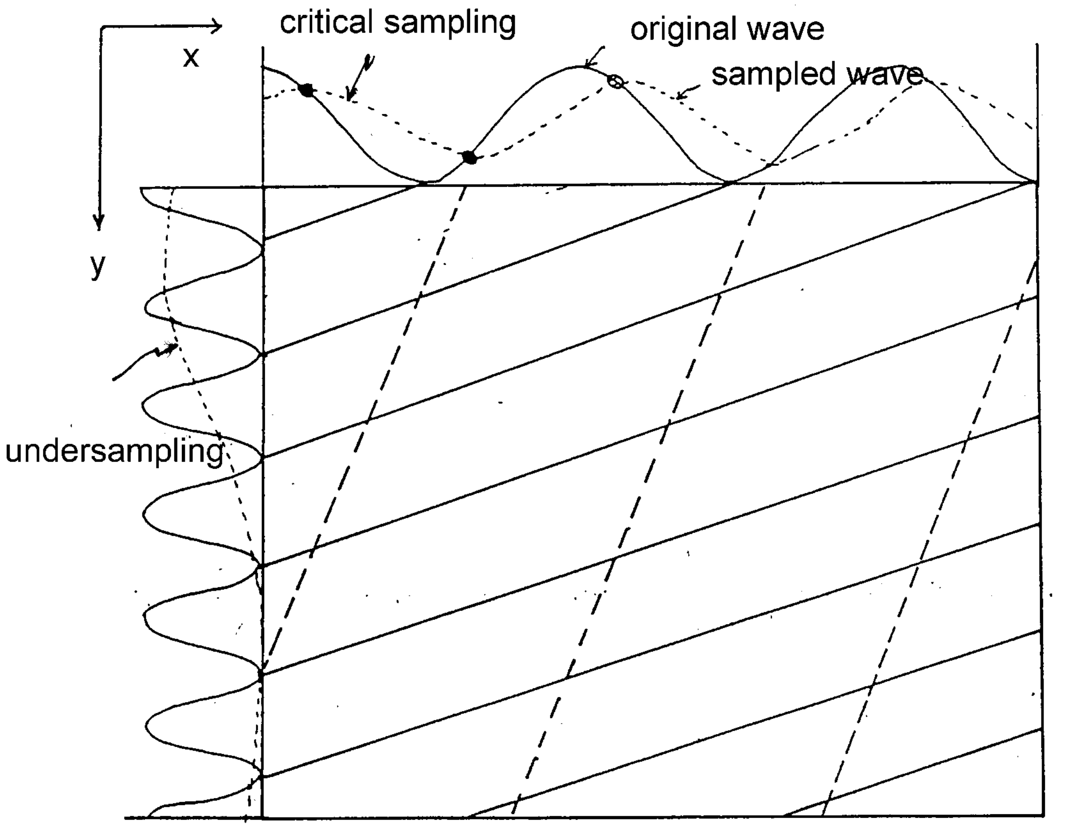
$K_x, K_y \qquad K = (K_x^2 + K_y^2)^{1/2}$

２．取樣區間(Sampling Interval)

$S_x, S_y \qquad S = (S_x^2 + S_y^2)^{1/2}$

３．假合(Aliasing)或虛假現象

４．假合常造成影像之模糊或增加影像之雜訊，要減少假合（通常很難完全避免）則必須同時考慮實景之空間頻率以及影像之取樣空間（解析度）。



original image = f(x,y) = A [1 + cos(2*pi* (kx (x - x0) + ky (y - y0)))]

肆、影像元素間之基本關係

一、影像元素之鄰近點(Neighbors)

１．基本符號定義

$f(x,y)$， p， q， S

２．四相鄰 $N_4(p)$：(4-neighbors of p)

$(x+1, y)$， $(x-1, y)$， $(x, y+1)$， $(x, y-1)$

３．四對角相鄰 $N_D(p)$：(4 diagonal neighbors of p)

$(x+1, y+1)$， $(x-1, y-1)$， $(x-1, y+1)$， $(x+1, y-1)$

４．八相鄰 $N_8(p)$：(8-neighbors of p)

$$N_8(p) = N_4(p) \cup N_D(p)$$

				s
			q	
		p	r	

二、相連性質(Connectivity)：

● 影像元素間之相連性質爲建立影像內各物體間邊界之重要觀念。爲決定兩個影像元素是否相連，則可設定特定之條件，考慮元素間是否“相近”，或灰度值間可否有符合近似之條件，常用的幾種相連性定義如下所述：

首先定義 V 集合：令 V 表灰度值之集合，用以定義相連性。

１．四相連(4-connectivity)：

若 q 在 $N_4(p)$ 中且 p 和 q 的值在 V 中，則 p 和 q 稱爲四相連。

２．八相連(8-connectivity)：

若 q 在 $N_8(p)$ 中且 p 和 q 的值在 V 中，則 p 和 q 稱爲八相連。

３．m 相連(m-connectivity)：

若 p 和 q 的值在 V 中，且滿足以下條件：

(1).q 在 $N_4(p)$ 中，或

(2).q 在 $N_D(p)$ 中且 $N_4(p) \cap N_4(q)$ 爲空集合。

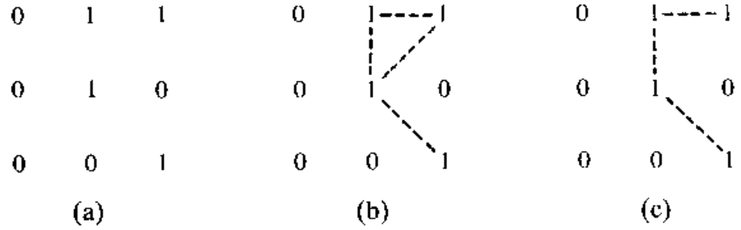


Figure 2.13 (a) Arrangement of pixels; (b) 8-neighbors of the center pixel; (c) m-neighbors of the same pixel. The dashed lines are paths between that pixel and its neighbors.

三、有關相連性質的一些定義：

１．p 和 q 若是相連的話，則稱它們爲相鄰(Adjacent)。

２．兩個影像子集合 S_1 ,S_2 中若有部分影像元素相鄰的話，則稱 S_1 和 S_2 相鄰。

３．由 p 至 q 之路徑(Path)是一序列的相鄰影像元素。

$$(x_0, y_0),\quad (x_1, y_1), \ldots , (x_n, y_n),$$

$$p \quad \rightarrow \quad q$$

n 爲路徑的長度(Length)

四、相連影像元素之標記(Labeling)

１．掃描 → 檢查灰度值 → 標記 → 排序 → 尋找相等之標記 → 二次掃描 → 再標記

２．二元影像中個體之計數應用。

五、關係，同等與轉移範圍

Relation，Equivalence，and Transitive Closure

１．二元關係(Binary relation)

點集合 A = $\{p_1, p_2, p_3, p_4\}$

$$\begin{array}{ll} p_1 & p_2 \\ p_3 & \\ & p_4 \end{array}$$

對點集合 A 之四相連二元關係 R 爲

$$R = \{ (p_1, p_2), (p_2, p_1), (p_1, p_3), (p_3, p_1) \}$$

２．同等關係(Equivalence relation)

符合以下三個性質之關係稱爲同等關係

(1).反身性(Reflexive)：　aRa；

(2).對稱性(Symmetric)：aRb → bRa；

(3).轉移性(Transitive)：　aRb 且 bRc → aRc。

３．矩陣表示式與轉移範圍

$$
C = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{d} & \mathrm{e} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}
\qquad
C^{+} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{d} & \mathrm{e} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}
$$

由矩陣Ｂ求取$Ｂ^{+}$之方法

(1).矩陣序列：

$$Ｂ^{+} = Ｂ + ＢＢ + ＢＢＢ + \ldots + (Ｂ)^{n}$$

乘法以 AND 替代，加法以 OR 替代。

(2).Warshall 演算法：

Step 1: Set j = 1.

Step 2: For i = 1,2,...,n, if b(i,j) = 1, then, for k = 1,2,...,n, set b(i,k) = b(i,k) + b(j,k).

Step 3: Increment j by 1.

Step 4: If j ≦ n, go to step 2; otherwise go to step 5.

Step 5: Stop. The result is $Ｂ^{+}$ in place of Ｂ.

(3).影像處理之意義

六、影像中之距離(Distance measures)

１．距離函數(Distance function)之定義（基本條件）:

設 p, q, z 之座標為(x,y), (s,t), (u,v)

(1). $D(p,q) \geqq 0$ ($D(p,q) = 0$ iff $p = q$)

(2). $D(p,q) = D(q,p)$, and

(3). $D(p,z) \leqq D(p,q) + D(q,z)$

２．歐氏距離(Euclidean distance)

$$D_e(p,q) = [\,(x - s)^2 + (y - t)^2\,]^{1/2}$$

３．D_4 距離(D_4 distance)

$$D_4 = |x - s| + |y - t|$$

４．D_8 距離(D_8 distance)

$$D_8 = \max(|x - s|, |y - t|)$$

５．D_4 和 D_8 距離與相連性無關，亦即不受影像元素之灰度值影響，但考慮到 m-相連之路徑長時則與灰度值息息相關，例如：

	p_3	P_4
p_1	p_2	
p		

	0	1
0	1	
1		

if $V=\{1\}$
m-distance = 2

	0	1
1	1	
1		

if $V=\{1\}$
m-distance = 3

七、數學與邏輯運算(Arithmetic/Logic Operation)

１．數學運算：

加法： p + q , 減法： p - q ,
乘法： p * q , 除法： p ÷ q

２．邏輯運算：（應用於二元影像）

AND:	p AND q
OR:	p OR q
COMPLEMENT:	NOT q
EXCLUSIVE OR:	XOR q

３．運算方式：

(1).個別影像元素處理(Pixel-by-pixel operation)
(2).鄰域處理(Neighborhood oriented operation)

４．遮罩運算(Mask operation)：

遮罩運算之目的是讓一個影像元素的值成爲它及相鄰區域內影像元素的函數。

$$z = \frac{1}{9}(z_1 + z_2 + \ldots + z_9) = \frac{1}{9}\sum_{i=1}^{9} z_i$$

$$z = (w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9) = \sum_{i=1}^{9} w_i z_i$$

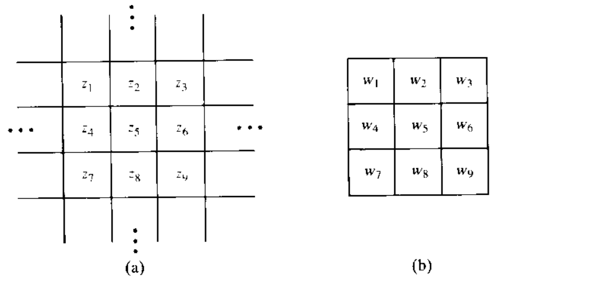


Figure 2.15 *(a) Subarea of an image showing pixel values; (b) a 3 × 3 mask with general coefficients.*

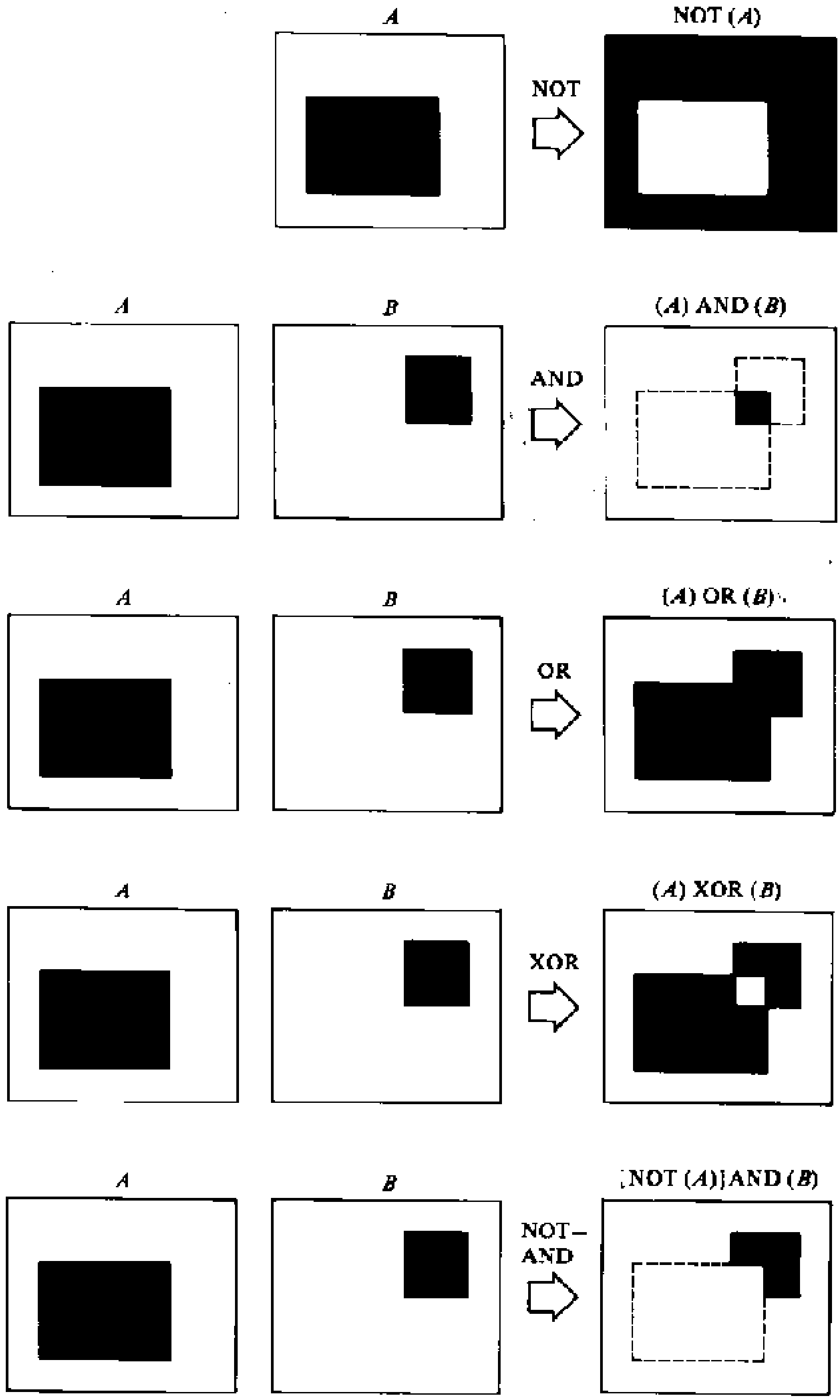


Figure 2.14 *Some examples of logic operations on binary images.*

伍、影像幾合(Image Geometry)

一、基本影像轉換

１．平移(Translation)：

將一點由座標(X,Y,Z)以位移向量(X0, Y0, Z0)平移至新座標(X*, Y*, Z*)，其關係式為：

$$X^* = X + X_0$$
$$Y^* = Y + Y_0$$
$$Z^* = Z + Z_0$$

以矩陣表示之形式為：

$$\begin{bmatrix} X^* \\ Y^* \\ Z^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_0 \\ 0 & 1 & 0 & Y_0 \\ 0 & 0 & 1 & Z_0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

或為：

$$\begin{bmatrix} X^* \\ Y^* \\ Z^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_0 \\ 0 & 1 & 0 & Y_0 \\ 0 & 0 & 1 & Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

或為：ｖ*＝ Ａｖ

２．改變比例(Scaling)：

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

S_x, S_y, S_z 分別為 X, Y, Z 軸向之比例係數(Scaling factor)

３．旋轉(Rotation)：

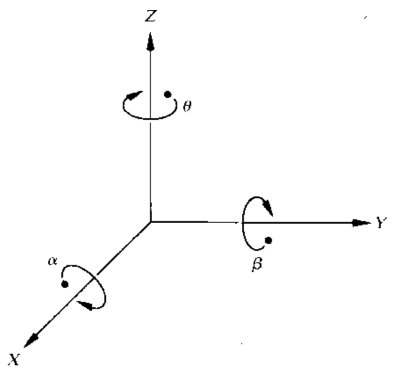


Figure 2.16 *Rotation of a point about each of the coordinate axes. Angles are measured clockwise when looking along the rotation axis toward the origin.*

* 由軸面向原點依順時針方向旋轉之角度為正角度。

對 Z 軸旋轉

$$S_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

對 X 軸旋轉

$$S_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

對 Y 軸旋轉

$$S_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

４．結合與反轉換(Concatenation and Inverse Transform)

(1).幾個轉換可以結合而以一個轉換矩陣來表示，例如ｖ點經過平移、改變比例和對 Z 軸旋轉，可以下式表示：

$$v^* = R_\theta(S(Tv)) = Av$$

而Ａ即是一個 4 x 4 矩陣 $A = R_\theta ST$

請注意$A = R_\theta ST$之次序是不可以隨意互換

多點之轉換也可以應用相同原理，以下式表之：

$$V^* = AV$$

而Ｖ為一個 4 x m 矩陣代表 m 個點

(2).平移之反轉換

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3).旋轉之反轉換：

$$S_\theta^{\cdot 2} = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

二、透視轉換(Perspective Transform)

●透視轉換是將三維空間的點投影到平面上，基本上透視轉換和前述之基本轉換是不同的，因透視轉換爲非線性轉換

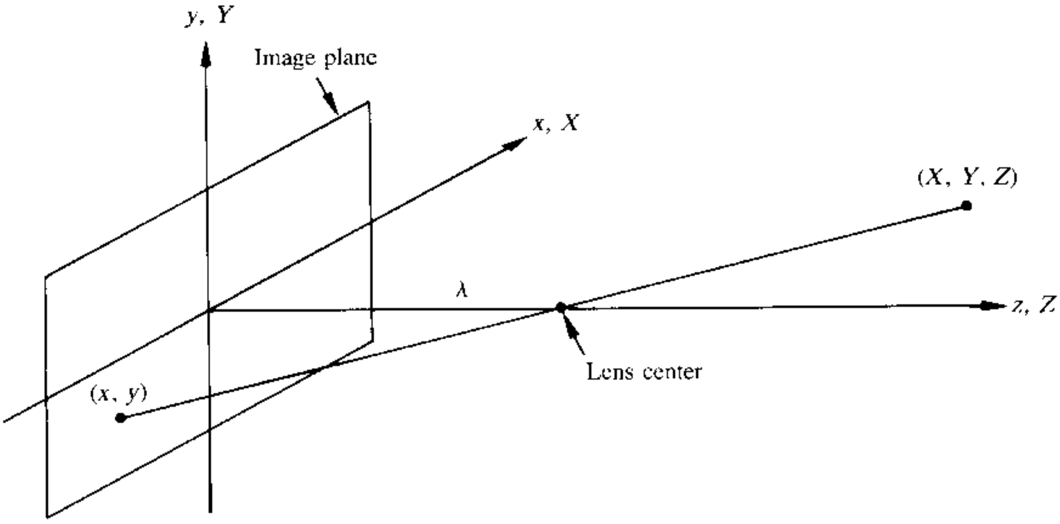


Figure 2.17 Basic model of the imaging process. The camera coordinate system (x, y, z) is aligned with the world coordinate system (X, Y, Z).

1．基本關係式：

$$\frac{x}{\lambda} = -\frac{X}{Z-\lambda} = \frac{X}{\lambda - Z} \qquad \frac{y}{\lambda} = -\frac{Y}{Z-\lambda} = \frac{Y}{\lambda - Z}$$

由以上二式可求得點之投影座標

$$x = -\frac{\lambda X}{\lambda - Z} \qquad \mathrm{y} = -\frac{\lambda Y}{\lambda - Z}$$

２．透視轉換之矩陣表示法：

(1).利用齊次座標(Homogeneous coordinates)轉換與表示：

$$w=\begin{bmatrix}X\\Y\\Z\end{bmatrix}\qquad \mathrm{w_h}=\begin{bmatrix}kX\\kY\\kZ\\k\end{bmatrix}$$

(2).定義透視轉換矩陣：

$$P=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-1/\lambda&1\end{bmatrix}$$

(3).進行轉換：

$$c_h=Pw_h=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\frac{-1}{\lambda}&1\end{bmatrix}\begin{bmatrix}kX\\kY\\kZ\\k\end{bmatrix}=\begin{bmatrix}kX\\kY\\kZ\\\frac{-kZ}{\lambda}+k\end{bmatrix}$$

(4).由齊次座標轉換回原座標系統：

$$c=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}\frac{\lambda X}{\lambda-Z}\\\frac{\lambda Y}{\lambda-Z}\\\frac{\lambda Z}{\lambda-Z}\end{bmatrix}$$

* z is a free variable for inverse perspective transform

３．透視轉換之反轉換：

$$w_h = P^{-1} c_h$$

逆矩陣P^{-1}可以由上式求出，如下式：

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\lambda} & 1 \end{bmatrix}$$

但是影像平面上之一點(x_0, y_0)或$(x_0, y_0, 0)$經由上面式子轉換之結果將無法得到三維空間之投射$(z=0)$，如下所示：

$$c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ 0 \\ 1 \end{bmatrix} \qquad \mathrm{w_h} = \begin{bmatrix} kx_0 \\ ky_0 \\ 0 \\ k \end{bmatrix} \qquad \mathrm{w} = \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ 0 \end{bmatrix}$$

故必須假藉一自由變數 z 以求得三維之反轉換

$$c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ kz \\ k \end{bmatrix} \qquad \mathrm{w_h} = \begin{bmatrix} kx_0 \\ ky_0 \\ kz \\ \frac{kz}{\lambda} + k \end{bmatrix} \qquad \mathrm{w} = \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix} = \begin{bmatrix} \frac{\lambda x_0}{\lambda + z} \\ \frac{\lambda y_0}{\lambda + z} \\ \frac{\lambda z}{\lambda + z} \end{bmatrix}$$

以 Z 表示 z 則可得

$$X = \frac{x_0}{\lambda}(\lambda - \mathrm{Z}) \qquad \mathrm{Y} = \frac{y_0}{\lambda}(\lambda - \mathrm{Z})$$

三、相機模型(Camera Model)

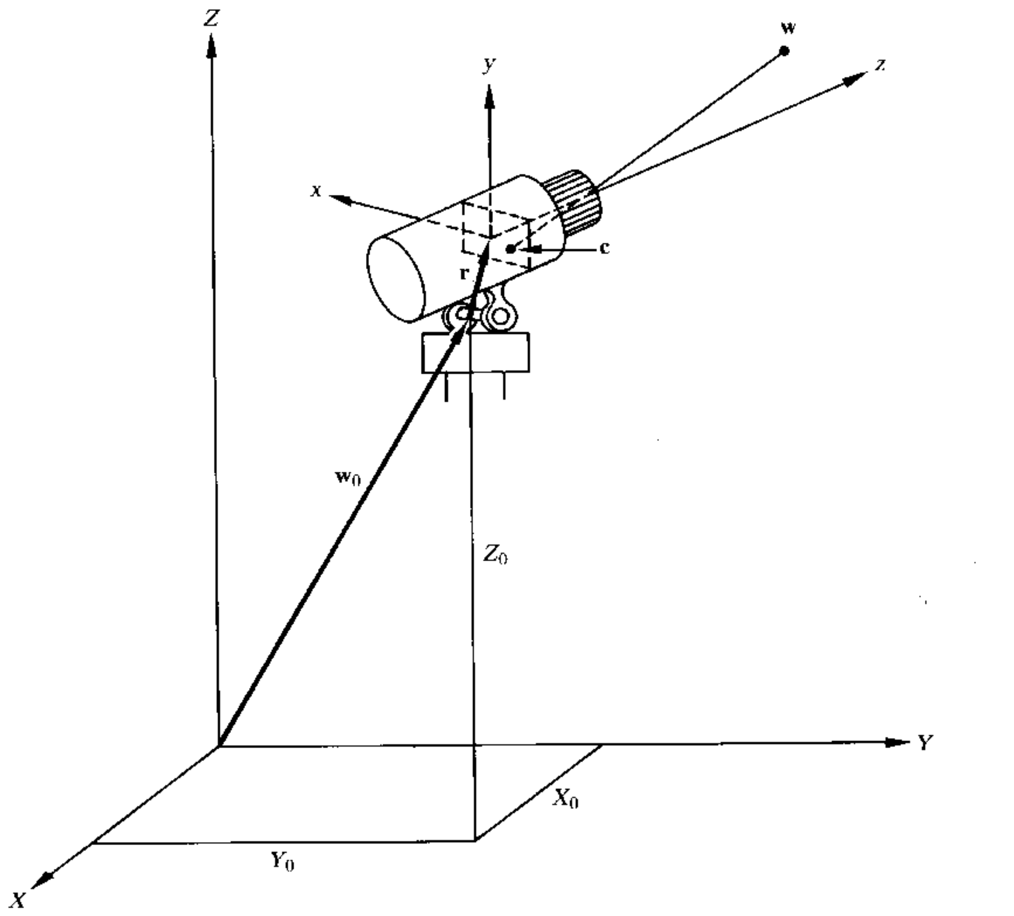


Figure 2.18 Imaging geometry with two coordinate systems. (From Fu, Gonzalez, and Lee [1987].)

前述之轉換有一基本假設，即是影像座標與體座標是一致的(原點相同且沒有旋轉)。上圖之相機模型則加入了較複雜之座標系統間的關係，雖然兩個座標系統不一致，但是前述之原理仍可以應用於這個相機模型中，只要經過適當的體座標系統轉換，令兩座標系統之原點和座標軸一致即可，以下爲使兩個座標系統一致的步驟：

１．首先將體座標系統之原點移至環架(Gimbal)中心：

$$G=\begin{bmatrix}1&0&0&-X_0\\0&1&0&-Y_0\\0&0&1&-Z_0\\0&0&0&1\end{bmatrix}$$

２．分別對 X 軸及 Z 軸進行旋轉：

（請注意逆時針方向角度為正值）

$$S=\begin{bmatrix}\cos\theta&\sin\theta&0&0\\-\sin\theta\cos\alpha&\cos\theta\cos\alpha&\sin\alpha&0\\\sin\theta\sin\alpha&-\cos\theta\sin\alpha&\cos\alpha&0\\0&0&0&1\end{bmatrix}$$

$R = R_\alpha R_\theta$

３．再將座標系統之原點移至影像平面中心：

$$C=\begin{bmatrix}1&0&0&-r_1\\0&1&0&-r_2\\0&0&1&-r_3\\0&0&0&1\end{bmatrix}$$

４．進行透視轉換：

$c_h = P\,C\,R\,G\,w_h$

$$x=\lambda\frac{(X-X_0)\cos\theta+(Y-Y_0)\sin\theta-r_1}{-(X-X_0)\sin\theta\sin\alpha+(Y-Y_0)\cos\theta\sin\alpha-(Z-Z_0)\cos\alpha+r_3+\lambda}\quad(2.5\text{-}42)$$

$$y=\lambda\frac{-(X-X_0)\sin\theta\cos\alpha+(Y-Y_0)\cos\theta\cos\alpha+(Z-Z_0)\sin\alpha-r_2}{-(X-X_0)\sin\theta\sin\alpha+(Y-Y_0)\cos\theta\sin\alpha-(Z-Z_0)\cos\alpha+r_3+\lambda}\quad(2.5\text{-}43)$$

５．一個實例：

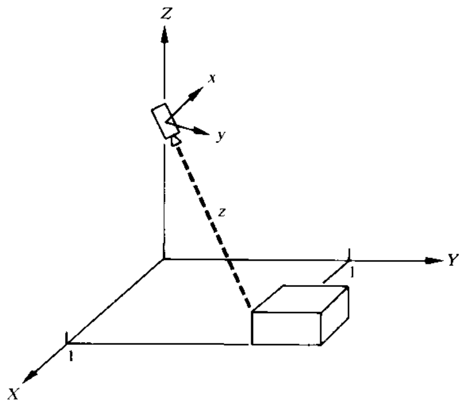


Figure 2.19 Camera viewing a 3-D scene. (From Fu, Gonzalez, and Lee [1987].)

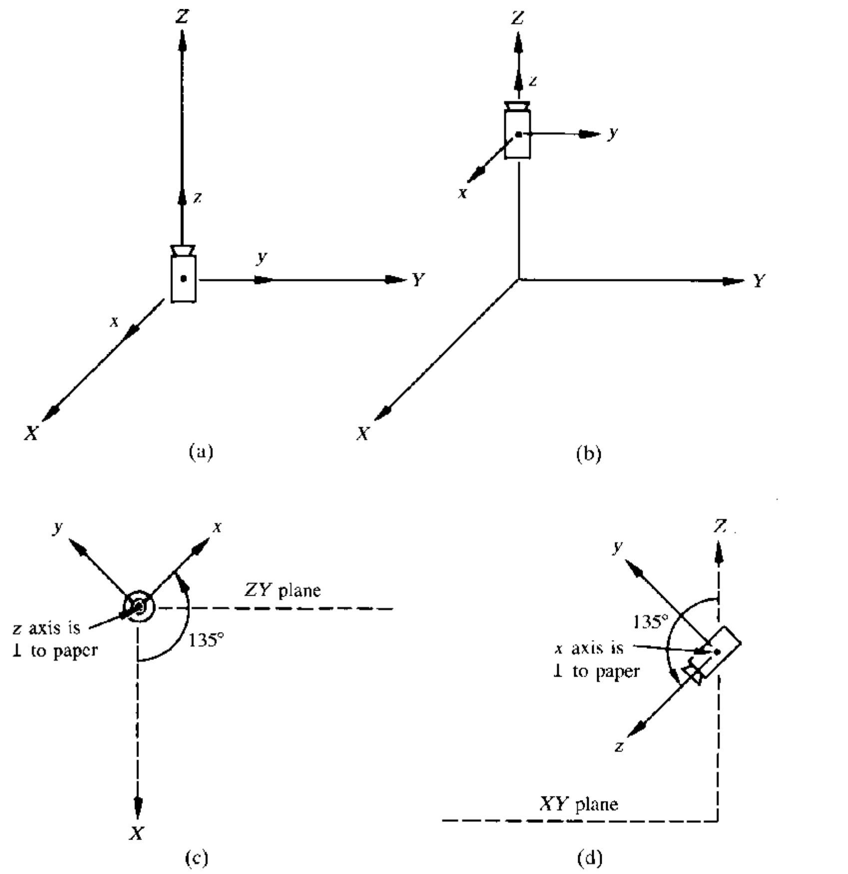


Figure 2.20 (a) Camera in normal position; (b) gimbal center displaced from origin; (c) observer view of rotation about z axis to determine pan angle; (d) observer view of rotation about x axis for tilt. (From Fu, Gonzalez, and Lee [1987].)

$X_0 = 0\text{ m},\ Y_0 = 0\text{ m},\ Z_0 = 1\text{ m},\ \alpha = 135°,\ \theta = 135°,\ \mathrm{r}_1 = 0.03\text{ m},\ \mathrm{r}_2 = \mathrm{r}_3 = 0.02\text{ m},\ \lambda = 35\text{mm}$

$$x = \lambda \frac{-0.03}{-153 + \lambda} \quad \mathrm{y} = \lambda \frac{-0.42}{-153 + \lambda}$$

四、相機校準方法(Camera Calibration)

應用已知點座標來計算照相機參數(Offset, tilt, pan...)的計算程序一般稱爲相機校準。

令　Ａ = ＰＣＲＧ,　c_h = Ａw_h,　k = 1, 則

$$\begin{bmatrix} c_{h1} \\ c_{h2} \\ c_{h3} \\ c_{h4} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

展開得

$$xc_{h4} = a_{11}X + a_{12}Y + a_{13}Z + a_{14}$$
$$yc_{h4} = a_{21}X + a_{22}Y + a_{23}Z + a_{24}$$
$$c_{h4} = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

將 c_{h4} 代入可得

$$a_{11}X + a_{12}Y + a_{13}Z - a_{41}xX - a_{42}xY - a_{43}xZ - a_{44}x + a_{14} = 0$$
$$a_{21}X + a_{22}Y + a_{23}Z - a_{41}yX - a_{42}yY - a_{43}yZ - a_{44}y + a_{24} = 0$$

因此由上面二式可知有 12 個係數待求得，故校準之目的即是求得這些係數，求取係數之步驟如下：

１．取得六個或六個以上(m≧6)之點的已知座標 (X_i, Y_i, Z_i),　i = 1,2,...,m。

２．以照相機取得已知點之影像座標(x_i, y_i), i = 1,2,...,m。

３．代入上面之聯立式中求解係數。

五、立體取像(Stereo Imaging)

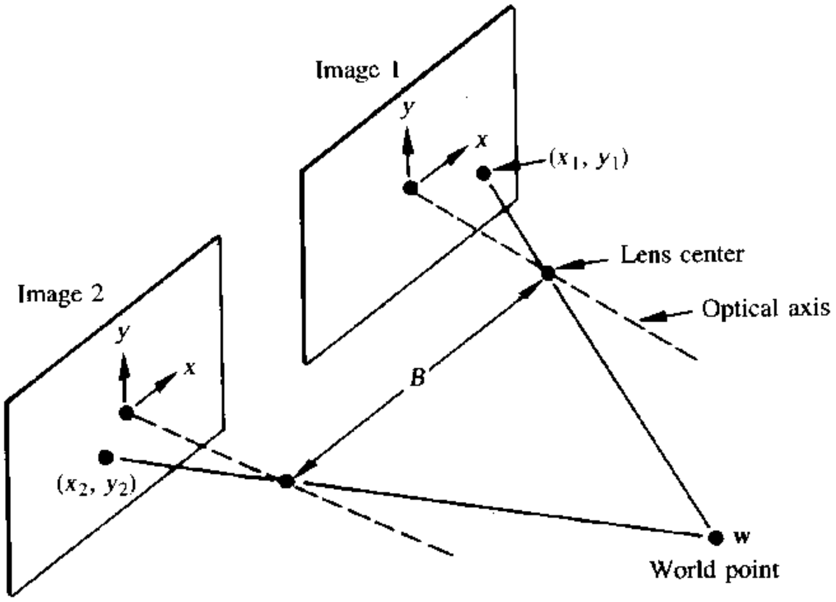


Figure 2.21 *Model of the stereo imaging process. (From Fu, Gonzalez, and Lee [1987].)*

$$X_1 = \frac{x}{\lambda}(\lambda - Z_1) \qquad (2.5\text{-}50)$$

$$X_2 = \frac{x_2}{\lambda}(\lambda - Z_2) \qquad (2.5\text{-}51)$$

$$X_2 = X_1 + B \qquad (2.5\text{-}52)$$

$$Z_2 = Z_1 = Z \qquad (2.5\text{-}53)$$

$$X_1 = \frac{x}{\lambda}(\lambda - Z) \qquad (2.5\text{-}54)$$

$$X_1 + B = \frac{x_2}{\lambda}(\lambda - Z) \qquad (2.5\text{-}55)$$

$$Z = \lambda - \frac{\lambda B}{x_2 - x_1} \qquad (2.5\text{-}56)$$

陸、照相軟片(Photographic Film)

一、軟片結構及曝光(Film Structure and Exposure)：

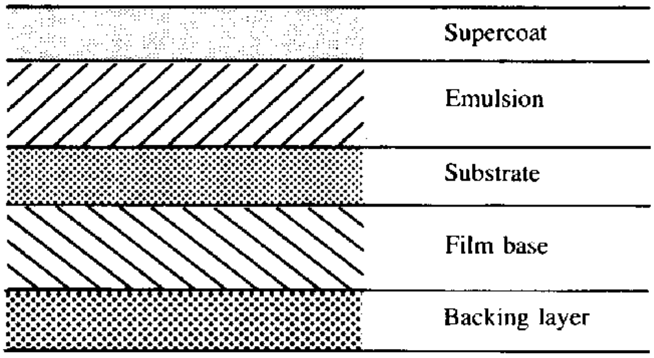


Figure 2.23 *Structure of modern black-and-white film.*

二、軟片特性：

1 ．對比(Contrast)

(1).曝光(Exposure)： E = IT

(2).Hurter and Driffield 曲線 (H&D Curve)

(3).Film Gamma (γ)： 0.7 ～ 1.0 一般用途

1.5 ～ 10.0 高反差或對比

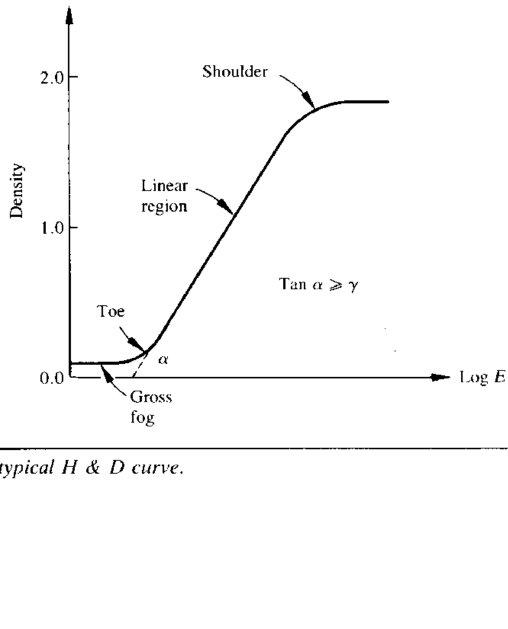


Figure 2.24 *A typical H & D curve.*

２．速度(Speed)

(1). ASA scale： 50 100 200 (Linear)

↕ ↕ ↕

(2). DIN scale： 18 21 24 (Logarithmic)

３．粒子粗細(Graininess)

４．解析能力(Resolving Power)

三、光圈與快門設定(Diaphragm and Shutter Settings)

１．f-number： 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22

(stop number) f-number 與光進入之量成反比

２．快門速度： 1, 1/2, 1/4, 1/8, 1/15, 1/30

1/60, 1/125, 1/250, 1/500

３．景深： "Pin-hole" characteristic