

影像資料壓縮

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壹、影像資料壓縮基本原理

●影像壓縮之主要應用方向：

- 影像之儲存
- 影像之傳輸
- 編碼與解碼原理應用於影像特徵抽取

一、基本原理：

1 · 定義：

(1). 資料壓縮(Data Compression)是指在仍足以表達某程度資訊內容條件下，進行減少該資訊的資料量過程。

(2). 相對資料多餘量(Relative Data Redundancy) R_D

$$R_D = 1 - 1/C_R$$

$$C_R = n_1/n_2$$

C_R 稱為壓縮比(Compression Ratio)

2 · 數位影像壓縮之資料多餘量：

- (1). 編碼多餘量(Coding Redundancy)
- (2). 像素間多餘量(Interpixel Redundancy)
- (3). 心理視覺多餘量(Psychovisual Redundancy)

二、編碼多餘量(Coding Redundancy)

由統計特性圖中計算影像中各灰度值之機率為：

$$p_r(r_k) = n_k/n \quad k = 0, 1, 2, \dots, L-1$$

若每一灰度值分別需以 $l(r_k)$ 位元來代表，則影像中每一影像元素平均需要 L_{avg} 表示之：

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p_r(r_k)$$

一張 $M \times N$ 影像之編碼則需要 $M \times N \times L_{avg}$ 位元

1 · 自然編碼(Natural Coding)

2 · 依機率分佈編碼

Table 6.1 Variable-Length Coding Example

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

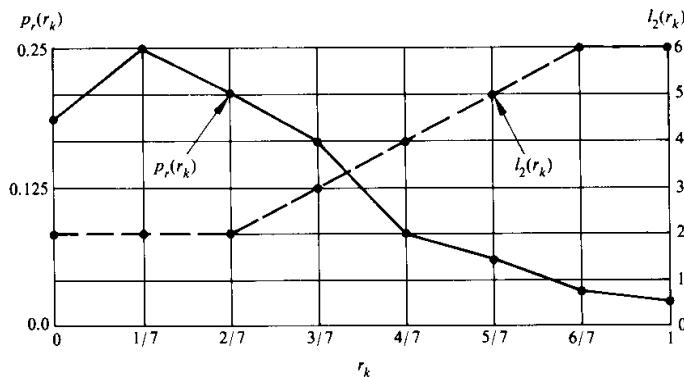


Figure 6.1 Graphic representation of the fundamental basis of data compression through variable-length coding.

$L_{avg} = 2.7$ bits, 而自然編碼為 3.0 bits，故 $C_R = 3.0/2.7$ ，
 $R_D = 1 - 2.7/3.0 = 0.099$

三、像素間多餘量(Interpixel Redundancy)

1 · 自動相關(Autocorrelation)

$$A(\Delta n) = \frac{1}{N - \Delta n} \sum_{y=0}^{N-1-\Delta n} f(x, y)f(x, y + \Delta n)$$

$$\gamma(\Delta n) = A(\Delta n) / A(0)$$

2 · 影像元素多餘量之比較：

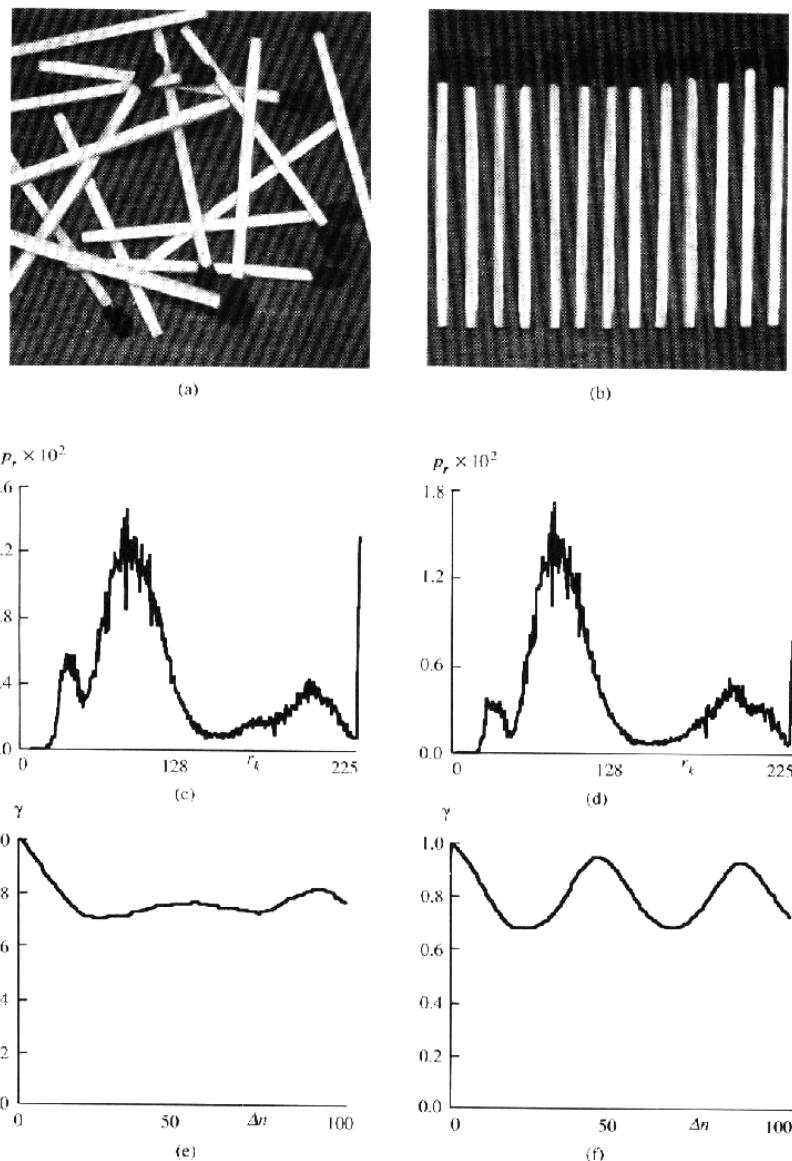


Figure 6.2 Two images and their gray-level histograms and normalized autocorrelation coefficients along one line.

3 · 範例：

映射(Mapping)之應用

$$f(x,0), f(x,1), \dots, f(x,N-1) \rightarrow (g_1, r_1), (g_2, r_2), \dots$$

$$C_R = \frac{(1024)(343)(1)}{(12166)(11)} = 2.63$$

$$R_D = 1 - 1/2.63 = 0.62$$

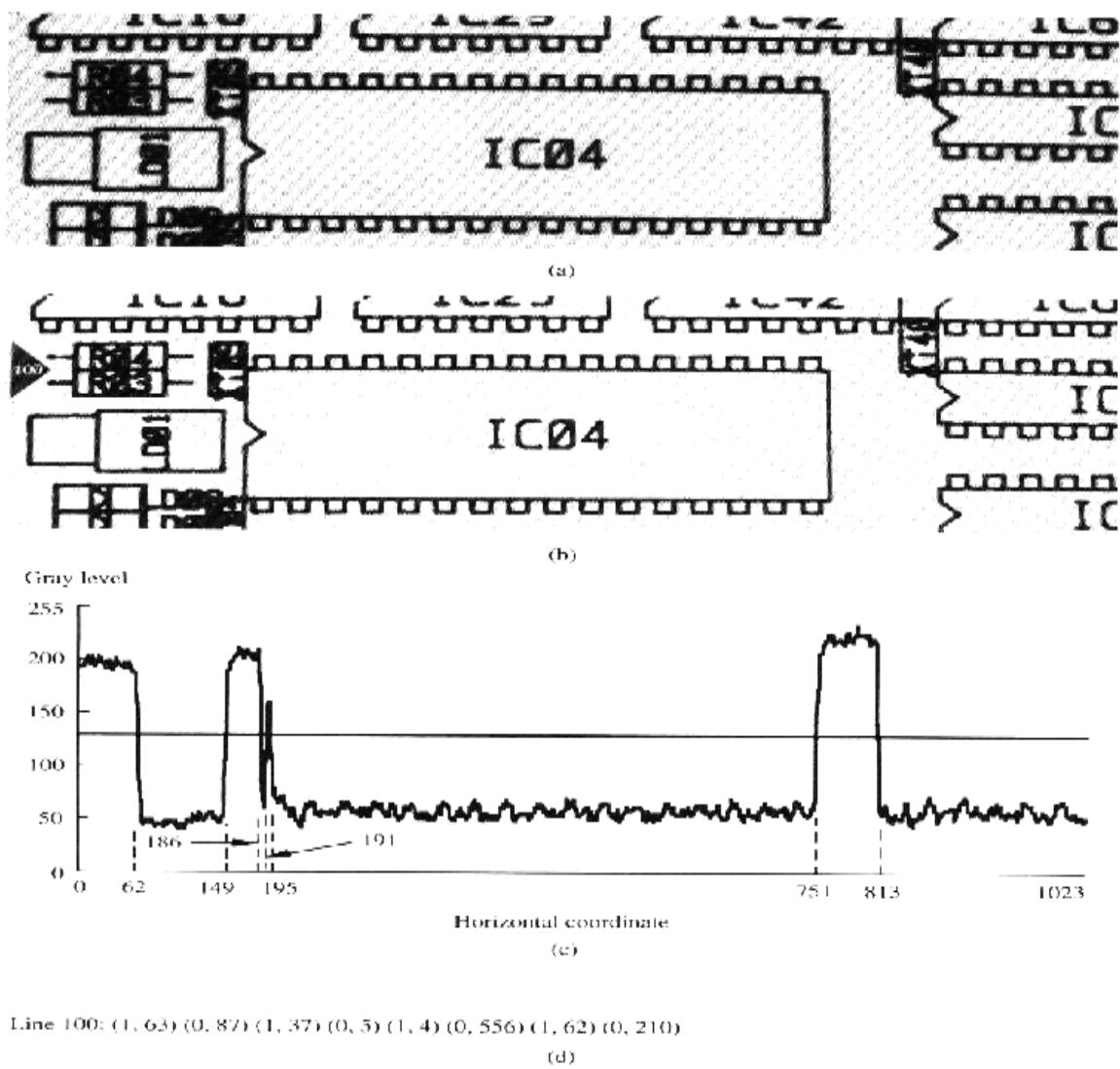


Figure 6.3 Illustration of run-length coding: (a) original image; (b) binary image with line 100 marked; (c) line profile and binarization threshold; (d) run-length code.

四、心理視覺多餘量(Psychovisual Redundancy)

- 1 · 心理視覺多餘量與影像之量化(Quantization)相關，此種多餘量之去除主要是針對不影響視覺效果之影像資訊。
 - 2 · 心理視覺多餘量之去除為不可逆之程序，換言之，此種影像壓縮會造成影像資訊之損失。
 - 3 · 改良灰度值量化法(Improved Gray-scale, IGS)

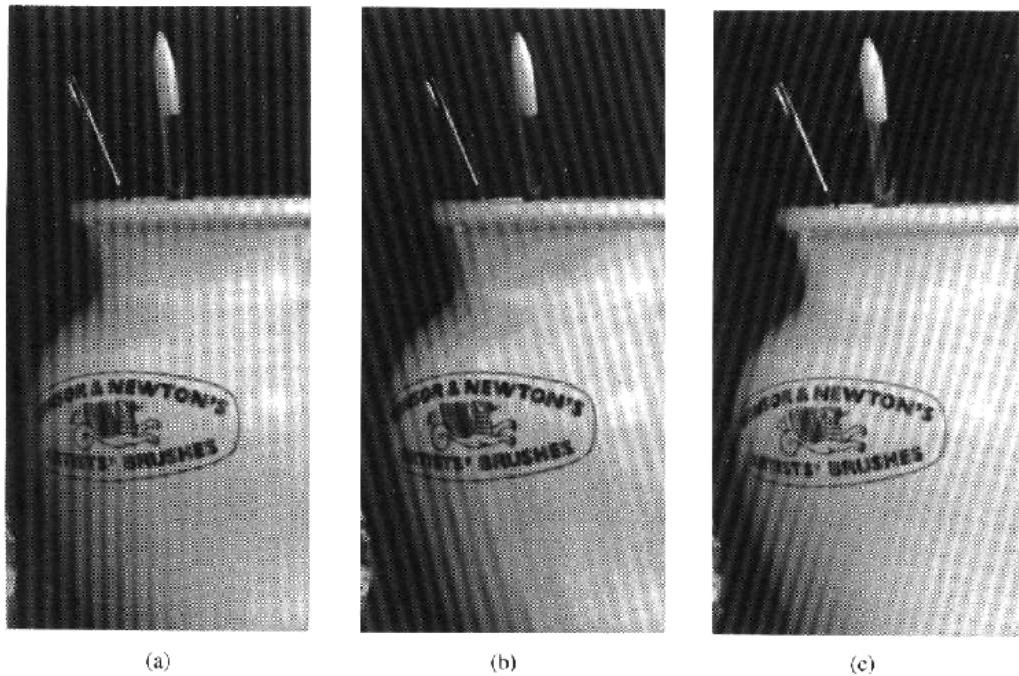


Figure 6.4 (a) Original image; (b) uniform quantization to 16 levels; (c) IGS quantization to 16 levels.

Table 6.2 IGS Quantization Procedure

<i>Pixel</i>	<i>Gray Level</i>	<i>Sum</i>	<i>IGS Code</i>
$i - 1$	N/A	0000 0000	N/A
i	0110 1100	0110 1100	0110
$i + 1$	1000 1011	1001 0111	1001
$i + 2$	1000 0111	1000 1110	1000
$i + 3$	1111 0100	1111 0100	1111

五、存真之標準(Fidelity Criteria)

1 · 客觀性存真標準(Objective Fidelity Criteria)

(1).以平均平方根誤差為判斷標準：

$$e(x,y) = g(x,y) - f(x,y)$$

↑ ↑ ↑
誤差 解碼影像 原始影像

$$e_{rms} = \left\{ \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [g(x,y) - f(x,y)]^2 \right\}^{1/2}$$

(2).以輸出影像對雜訊比之平均平方根為判斷標準：

$$g(x,y) = f(x,y) + e(x,y)$$

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} g(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [g(x,y) - f(x,y)]^2}$$

$$SNR_{rms} = (SNR_{ms})^{1/2}$$

2 · 主觀性存真標準(Subjective Fidelity Criteria)

(1).應用訂定之絕對尺度(Absolute Rating Scale)

(2).應用配對比較之方法(Pair Comparison)

**Table 6.3 Television Allocations Study Organization Rating Scale
(From Frendendall and Behrend [1960])**

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

貳、影像壓縮模式

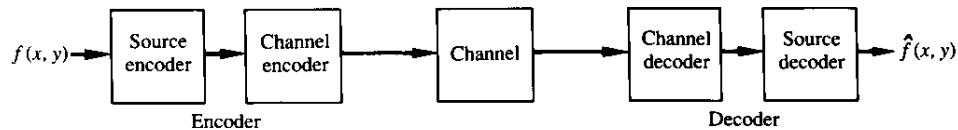


Figure 6.5 A general compression system model.

一、原始資料編碼器與解碼器：

- 1 · 原始資料編碼器之主要功能為減少或去除前述之資料編碼、像素間與心理視覺多餘量。
- 2 · 映射器(Mapper) — 減少像素間多餘量 — 可逆程序
量化器(Quantizer) — 減少心理視覺多餘量 — 不可逆程序
符號編碼器(Symbol Encoder) — 減少編碼多餘量 — 可逆程序

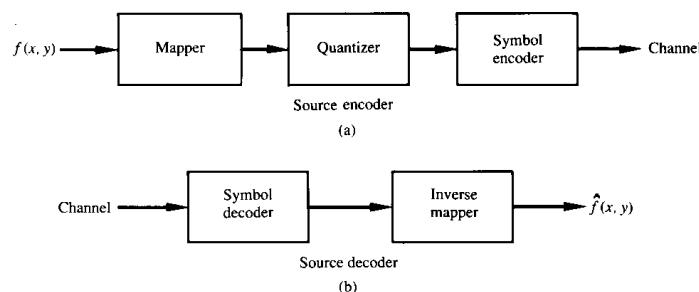


Figure 6.6 (a) Source encoder and (b) source decoder model.

二、管道編碼器與解碼器：

- 1 · 管道編碼器之功能為減少管道雜訊對影像資訊之影響。
- 2 · Hamming 編碼法：

[例] 7-bit Hamming (7,4) code

$$\begin{aligned}
 \text{編碼 : } & h_1 = b_3 \oplus b_2 \oplus b_0, \quad h_3 = b_3 \\
 & h_2 = b_3 \oplus b_1 \oplus b_0, \quad h_5 = b_2 \\
 & h_4 = b_2 \oplus b_1 \oplus b_0, \quad h_6 = b_1 \\
 & \qquad\qquad\qquad h_7 = b_0
 \end{aligned}$$

$$\text{解碼 : } c_1 = h_1 \oplus h_3 \oplus h_5 \oplus h_7$$

$$c_2 = h_2 \oplus h_3 \oplus h_6 \oplus h_7$$

$$c_4 = h_4 \oplus h_5 \oplus h_6 \oplus h_7$$

若 $c_4c_2c_1$ 不為零，則表示有錯誤，即 $h_1\dots h_5h_6h_7$ 之第 $c_4c_2c_1$ 位元(十進位)取補數，再由 $h_3h_5h_6h_7$ 解碼得 $b_3b_2b_1b_0$

參、資訊理論基礎

一、量測資訊量(Measuring Information)

$$I(E) = \log \frac{1}{P(E)} = -\log P(E)$$

$I(E)$ 稱為隨機事件 E 之資訊本體(Self-information), $P(E)$

為隨機事件 E 之發生機率 ($P(E) = 1, I(E) = 0.0$)

對數 \log 之基底決定量測資訊量之單位。

以擲銅板於地之事件為例： $I(E) = -\log_2 1/2 = 1 \text{ bit}$

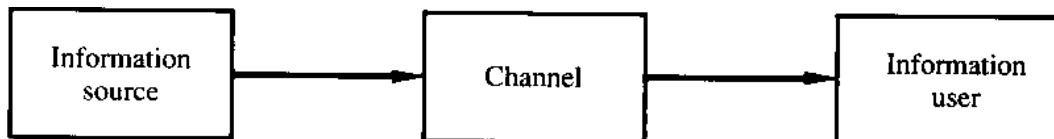
二、資訊管道(Information Channel)

當資訊本體由資訊源傳遞至資訊使用者時，我們稱資訊源與資訊使用者間為資訊管道所連接。

1 · 以資訊源符號集合 A 與機率向量 z 表示資訊源

Source Alphabet $A = \{a_1, a_2, \dots, a_J\}$

$$Z = [P(a_1), P(a_2), \dots, P(a_J)]^T$$



Ensemble (A, z)

$$A = \{a_j\}$$

$$z = [P(a_1), P(a_2), \dots, P(a_J)]^T$$

$$Q = [q_k]$$

Ensemble (B, v)

$$B = \{b_k\}$$

$$v = [P(b_1), P(b_2), \dots, P(b_K)]^T$$

Figure 6.7 A simple information system.

2 · 資訊源之 Entropy (or Uncertainty)

$$H(Z) = -\sum_{j=1}^J P(a_j) \log P(a_j)$$

3 · 資訊源、管道與資訊使用者間之關係：

$$\nu = Q z$$

其中 Q 稱為管道矩陣：

$$Q = \begin{bmatrix} P(b_1 | a_1) & P(b_1 | a_2) & \cdots & P(b_1 | a_J) \\ P(b_2 | a_1) & P(b_2 | a_2) & \cdots & P(b_2 | a_J) \\ \vdots & \vdots & \cdots & \vdots \\ P(b_k | a_1) & P(b_k | a_2) & \cdots & P(b_k | a_J) \end{bmatrix}$$

4 · 相對資訊(Mutual information)：

$$\begin{aligned} I(z, v) &= H(z) - H(z | v) \\ &= \sum_{j=1}^J \sum_{k=1}^K P(a_j, b_k) \log \frac{P(a_j, b_k)}{P(a_j)P(b_k)} \end{aligned}$$

5 · 管道之能力(Capacity)：

$$C = \max[I(z, \nu)]$$

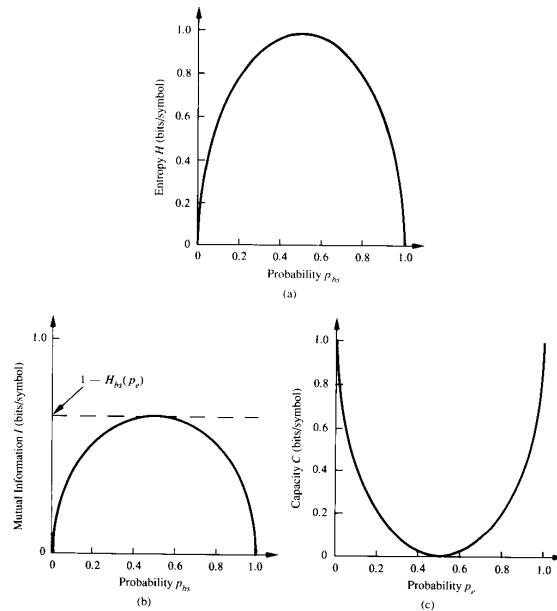


Figure 6.8 Three binary information functions: (a) the binary entropy function; (b) the mutual information of a binary symmetric channel (BSC); (c) the capacity of the BSC.

三、資訊理論之影像壓縮應用

首先考慮以下之 4×8 影像，其灰階為 8-bit (256)

21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243

若考慮此影像為 8-bit 灰階資訊源所產生之影像，而資訊源產生像素資料之過程為隨機，則此例中之資訊源符號 (Source Symbol) 為 256 個不同灰階，若產生各灰階之機率皆相同，則其 Entropy 為 8 bits/pixel，總 Entropy 為 $4 \times 8 \times 8 = 256$ bits。而此影像為資訊源所能產生所有 $2^{8 \times 4 \times 8}$ 個影像中之一個影像。

若考慮影像中像素灰度值出現之機率來模擬產生此影像之資訊源行為：

灰度值	機率	出現頻率
21	12	3/8
95	4	1/8
169	4	1/8
243	12	3/8

則其 Entropy 可計算得 1.81 bits/pixel，此稱為 First-order Estimate。

若將影像中若干相鄰影像元素合組為一單元 (Block)，則可以更逼近計算資訊源本身真正之 Entropy，以上面影像為例，若將兩影像元素合為一組，則有下表之關係

灰度值對	機率	出現頻率
(21,21)	8	1/4
(21,95)	4	1/8
(95,169)	4	1/8
(169,243)	4	1/8
(243,243)	8	1/4
(243,21)	4	1/8

則其 Entropy 可計算得 1.25 bits/pixel ，此稱為 Second-order Estimate。

由 First-order Estimate 與 Second-order Estimate 間之差值 $1.81 - 1.25 = 0.56 \text{ bits/pixel}$ 顯示可另以映射 (Mapping) 方法來壓縮影像，將影像之像素值重新以相鄰像素灰度值差計算，求得 Difference 影像：

21	0	0	74	74	74	0	0
21	0	0	74	74	74	0	0
21	0	0	74	74	74	0	0
21	0	0	74	74	74	0	0

此影像之灰度值統計為：

灰度值差	機率	出現頻率
0	12	$1/2$
21	4	$1/8$
74	12	$3/8$

則其 Entropy 可計算得 1.41 bits/pixel ，此值較 Second-order Estimate 之 1.25 bits 為高，故理論上應可再找出其他更好之映射方法。

肆、全存真影像壓縮(Error-free Compression)

一、不等長編碼(Variable-length Coding)

1. Huffman 編碼法：

- Huffman 編碼法為依資訊源符號出現機率，在對資訊源符號逐一編碼條件下(The symbols be coded one at a time)，最佳之編碼方法
- Huffman 編碼法之解碼過程為即時(Instantaneous)且為唯一(Uniquely Decodable)之解碼。

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

Figure 6.11 Huffman source reductions.

Original source			Source reduction			
Sym.	Prob.	Code	1	2	3	4
a_2	0.4	1	0.4 1	0.4 1	0.4 1	0.6 0
a_6	0.3	00	0.3 00	0.3 00	0.3 00	0.4 1
a_1	0.1	011	0.1 011	0.2 010	0.3 01	
a_4	0.1	0100	0.1 0100	0.1 011		
a_3	0.06	01010	0.1 0101			
a_5	0.04	01011				

Figure 6.12 Huffman code assignment procedure.

2 · 其他不等長編碼法

(1).Huffman 截短編碼法(Truncated Huffman Code) :

僅對出現機率最高之 ψ 個符號進行 Huffman 編碼，其他符號則以前置碼(Prefix)加上等長碼(如自然編碼)代表。

(2).B 編碼(B-code) :

B 編碼法適合於資訊源符號之出現機率符合下列次方法則(Power Law)之編碼作業：

$$P(a_j) = j^{-\beta}$$

- B 編碼不是即時可解碼。
- B 編碼分別由連結位元及資料位元組成。
- 連結位元(Continuation Bit)亦可用以記錄次度值。
- 連結位元改變時亦表示代表新的符號碼(Code Word)之開始。
- 就程式撰寫而言，B 編碼法較 Huffman 編碼簡易。

(3).偏移編碼(Shift Code) :

偏移編碼亦適合於資訊源符號出現機率符合次方法則之編碼作業。

- 首先依符號出現機率由小至大重新排列符號次序。
- 將排序後之符號等分為數個區(Block)。
- 對每一區均以同法編碼(例如以自然編碼法)。
- 對各區編好之碼加上偏移碼(Shift-up, Shift-down)

(4).Huffman 偏移編碼(Huffman Shift Code)

此編碼法與前述之偏移編碼法類同，但其偏移碼與區內碼是依 Huffman 編碼法則決定。

Table 6.5 Variable-Length Codes

<i>Source Symbol</i>	<i>Prob- ability</i>	<i>Binary Code</i>	<i>Huffman</i>	<i>Truncated Huffman</i>	<i>B_z-Code</i>	<i>Binary Shift</i>	<i>Huffman Shift</i>
<i>Block 1</i>							
a_1	0.2	00000	10	11	C00	000	10
a_2	0.1	00001	110	011	C01	001	11
a_3	0.1	00010	111	0000	C10	010	110
a_4	0.06	00011	0101	0101	C11	011	100
a_5	0.05	00100	00000	00010	C00C00	100	101
a_6	0.05	00101	00001	00011	C00C01	101	1110
a_7	0.05	00110	00010	00100	C00C10	110	1111
<i>Block 2</i>							
a_8	0.04	00111	00011	00101	C00C11	111 000	00 10
a_9	0.04	01000	00110	00110	C01C00	111 001	00 11
a_{10}	0.04	01001	00111	00111	C01C01	111 010	00 110
a_{11}	0.04	01010	00100	01000	C01C10	111 011	00 100
a_{12}	0.03	01011	01001	01001	C01C11	111 100	00 101
a_{13}	0.03	01100	01110	10 0000	C10C00	111 101	00 1110
a_{14}	0.03	01101	01111	10 0001	C10C01	111 110	00 1111
<i>Block 3</i>							
a_{15}	0.03	01110	01100	10 0010	C10C10	111 111 000	00 00 10
a_{16}	0.02	01111	010000	10 0011	C10C11	111 111 001	00 00 11
a_{17}	0.02	10000	010001	10 0100	C11C00	111 111 010	00 00 110
a_{18}	0.02	10001	001010	10 0101	C11C01	111 111 011	00 00 100
a_{19}	0.02	10010	001011	10 0110	C11C10	111 111 100	00 00 101
a_{20}	0.02	10011	011010	10 0111	C11C11	111 111 101	00 00 1110
a_{21}	0.01	10100	011011	10 1000	C00C00C00	111 111 110	00 00 1111
<i>Entropy</i> 4.0							
<i>Average Length</i>		5.0	4.05	4.24	4.65	4.59	4.13

3 · 算術編碼法(Arithmetic Coding)

- 算術編碼法與前述編碼法不同處在於其編碼與資訊源符號無一對一之對應，而是對整個序列之符號編碼給予一個介於 0 至 1 間之區間編碼。
- 符號序列愈長，則區間範圍愈小，而代表區間之小數點以下有效位數亦增加。
- 隨著符號序列長度之增加，其算數編碼長度之 entropy 趨近於無雜訊編碼理論之極限(效率愈高)。

Table 6.6 Arithmetic Coding Example

<i>Source Symbol</i>	<i>Probability</i>	<i>Initial Subinterval</i>
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)

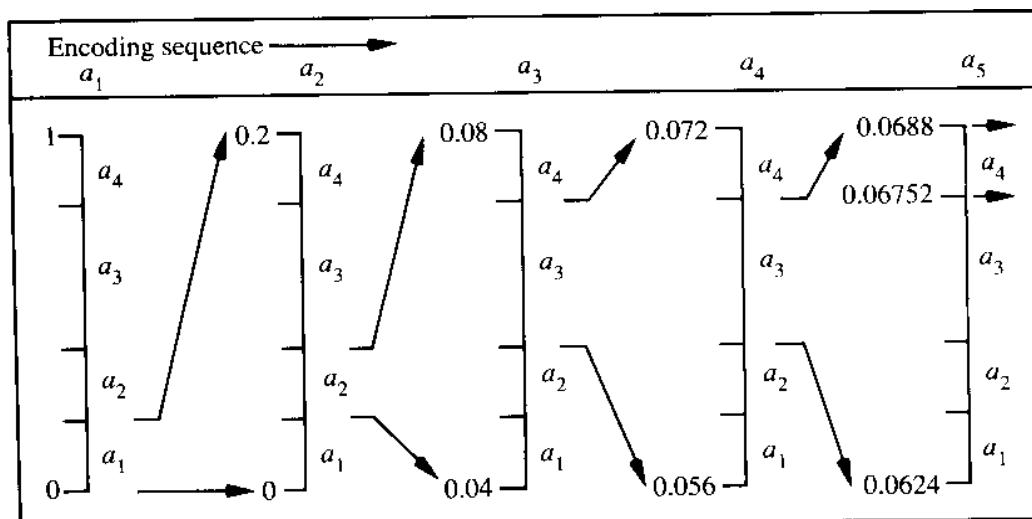


Figure 6.13 Arithmetic coding procedure.

二、位元面編碼(Bit-plane Coding)

1 · 位元面之分解

(1). 依二進位多項式分解

$$a_{m-1}2^{m-1} + a_{m-2}2^{m-2} + \dots + a_12^1 + a_02^0$$

此種分解方法之缺點為微小灰度值變化可能大幅影響及相鄰位元面之變動。

[例] 127 (01111111) , 128 (10000000)

(2). 依灰度碼(Gray Code)分解

$$g_i = a_i \oplus a_{i+1}$$

$$g_{m-1} = a_{m-1}$$

此種分解方法受微小灰度值變化之影響較小。

[例] 127 (01000000) , 128 (11000000)



(a)

This indenture made this first day of our Lord one thousand nine hundred six between John C. of King and State of Tennessee Andrew Jackson of the County of Davidson of the other part and John Donelson for a sum of two thousand and paid the sum of one thousand dollars and by these presents John C. of King and John Jackson his heirs and a certain tract or parcels of land containing thousand acres more or less and has been

(b)

Figure 6.14 A 1024 × 1024 (a) 8-bit monochrome image and (b) binary image.

2 · 固定面積編碼(Constant Area Coding, CAC)

- (1). White Block Skipping (WBS)
- (2). 1xn Block with WBS code sequence
- (3). 連續區塊分割法。

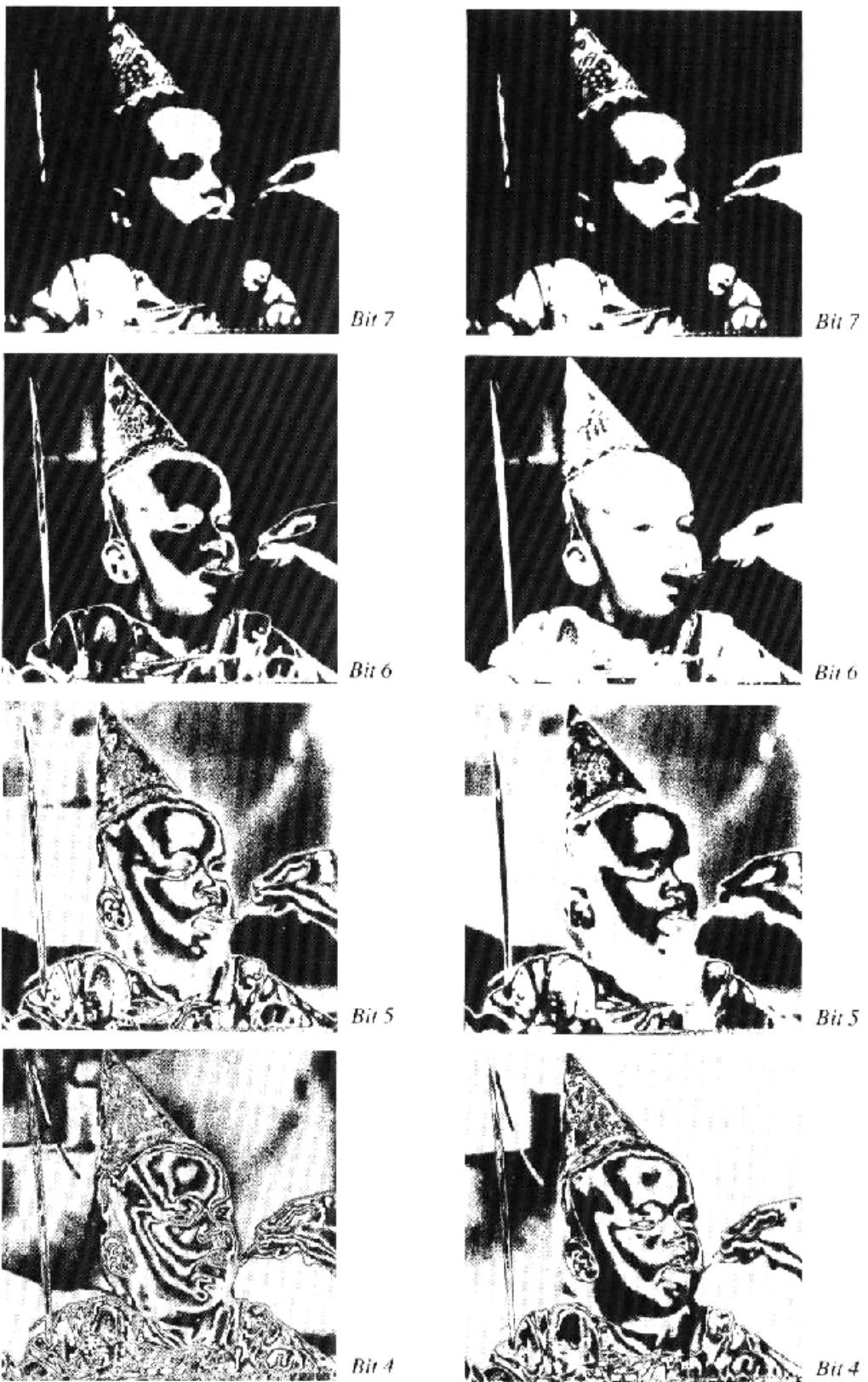


Figure 6.15 The four most significant binary (left column) and Gray (right column) coded bit planes of the image in Fig. 6.14(a).

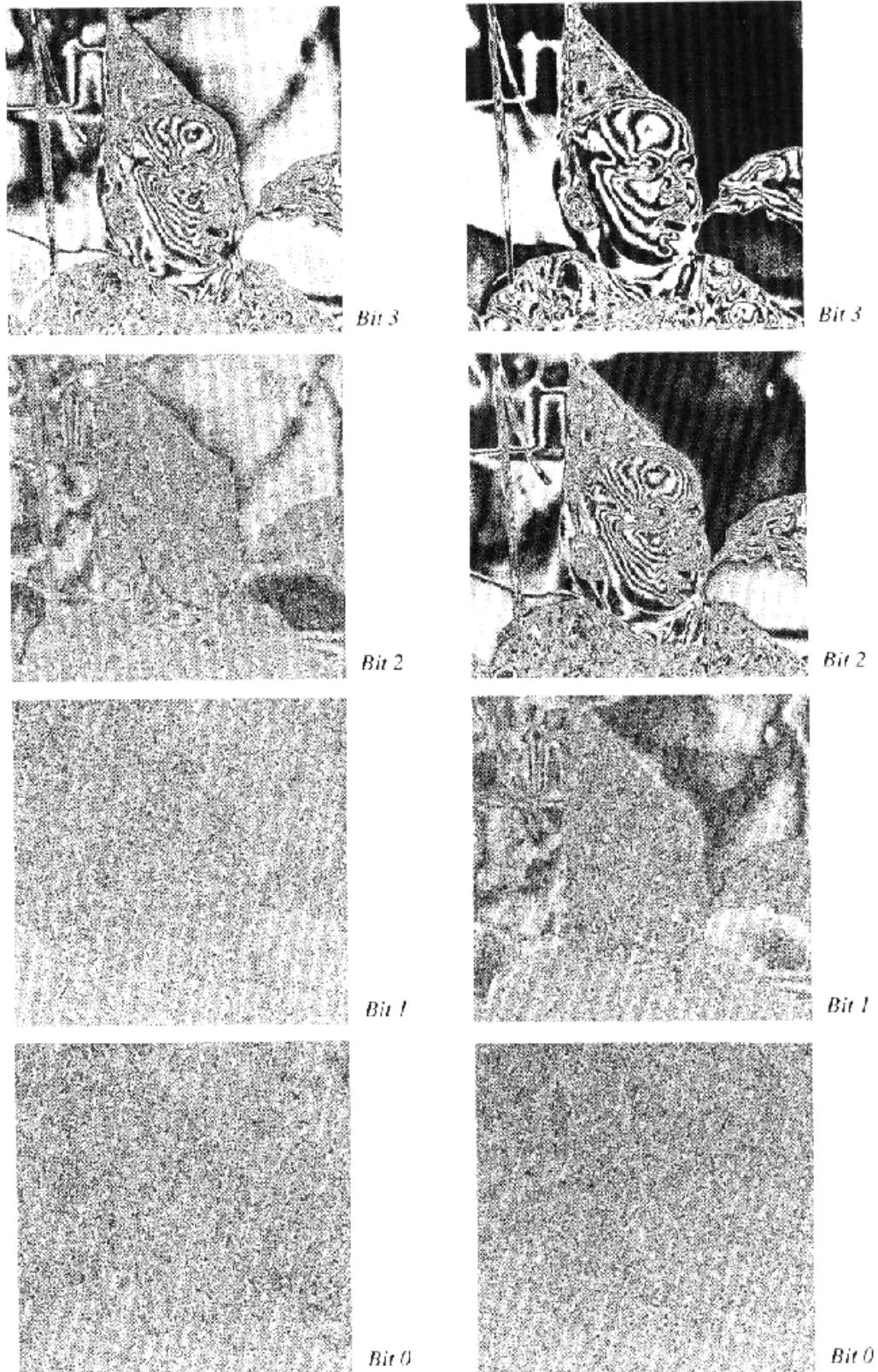


Figure 6.16 The four least significant binary (left column) and Gray (right column) coded bit planes of the image in Fig. 6.14(a).

3 · 一維區段編碼(One-dimensional RLC)

- (1). RLC 為 FAX 標準編碼方法。
- (2). 除以 RLC 法進行有效率之編碼外，通常亦配合不等長編碼法提高其效率。
- (3). Approximate Run-length Entropy

$$H_{RL} = \frac{H_0 + H_1}{L_0 + L_1}$$

4 · 二維區段編碼(Two-dimensional RLC)

- (1). 二維區段編碼法適合於具有少而大面積區塊之影像編碼。
- (2). 相對位址編碼(Relative Address Coding, RAC)

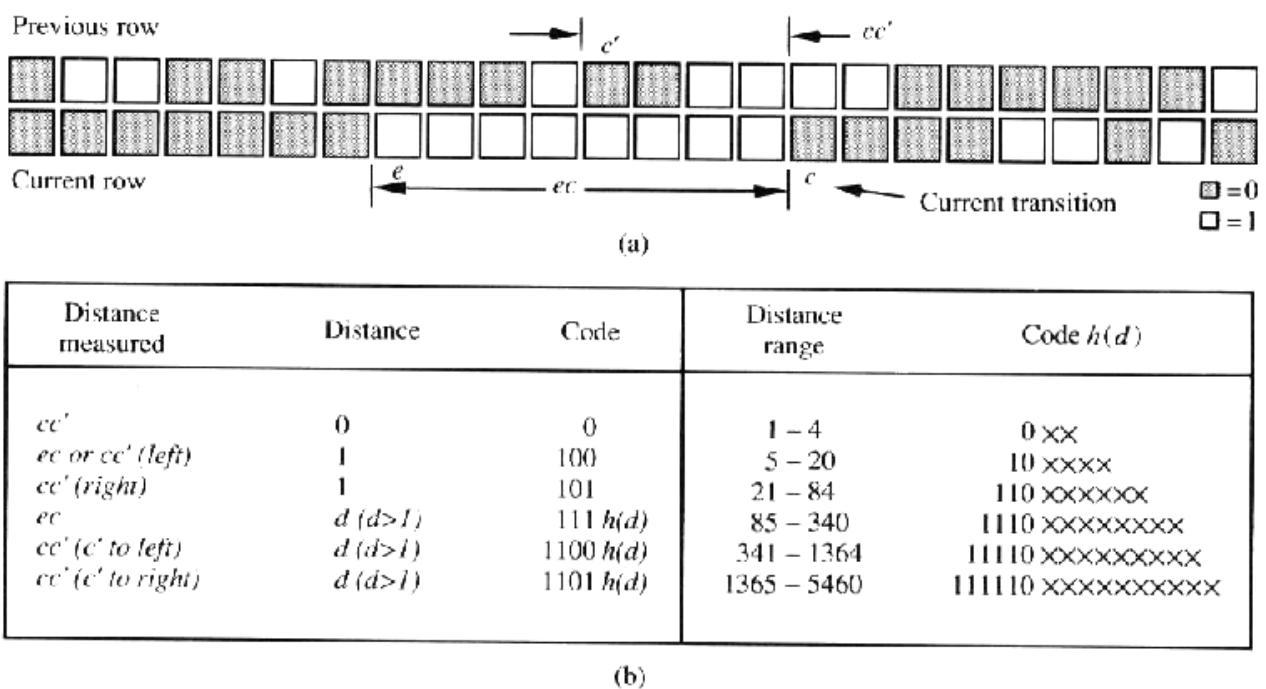
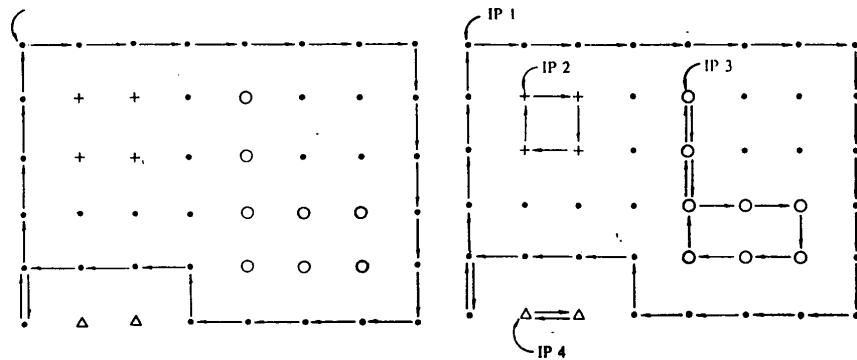


Figure 6.17 A relative address coding (RAC) illustration.

5 · 輪廓追蹤與編碼(Contour Tracing and Coding)

(1). 直接輪廓追蹤編碼法



(2). Predictive Differential Quantizing, PDQ

\triangle' , \triangle'' , Merge, Start

(3). Double Delta Coding, DDC

\triangle' , \triangle'' , Merge, Start

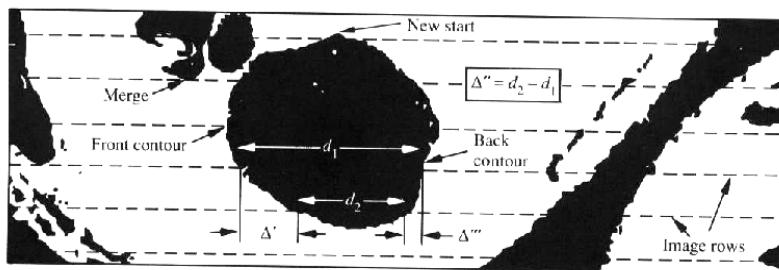


Figure 6.18 Parameters of the PDQ algorithm.

Table 6.7 Error-Free Bit-Plane Coding Results for Fig. 6.14(a): $H \approx 6.82$ Bits/Pixel

Method	Bit-Plane Code Rate (Bits/Pixel)								Code Rate	Compression Ratio
	7	6	5	4	3	2	1	0		
<i>Binary Bit-Plane Coding</i>										
CBC (4×4)	0.14	0.24	0.60	0.79	0.99	—	—	—	5.75	1.4: 1
RLC	0.09	0.19	0.51	0.68	0.87	1.00	1.00	1.00	5.33	1.5: 1
PDQ	0.07	0.18	0.79	—	—	—	—	—	6.04	1.3: 1
DDC	0.07	0.18	0.79	—	—	—	—	—	6.03	1.3: 1
RAC	0.06	0.15	0.62	0.91	—	—	—	—	5.17	1.4: 1
<i>Gray Bit-Plane Coding</i>										
CBC (4×4)	0.14	0.18	0.48	0.40	0.61	0.98	—	—	4.80	1.7: 1
RLC	0.09	0.13	0.40	0.33	0.51	0.85	1.00	1.00	4.29	1.9: 1
PDQ	0.07	0.12	0.61	0.40	0.82	—	—	—	5.02	1.6: 1
DDC	0.07	0.11	0.61	0.40	0.81	—	—	—	5.00	1.6: 1
RAC	0.06	0.10	0.49	0.31	0.62	—	—	—	4.05	1.8: 1

Table 6.8 Error-Free Binary Image Compression Results for Fig. 6.14(b): $H \approx 0.55$ Bits/Pixel

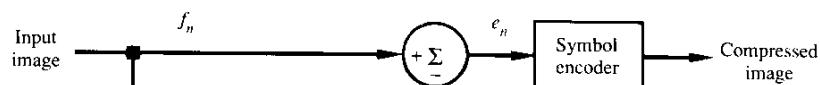
	WBS (1×8)	WBS (4×4)	RLC	PDQ	DDC	RAC
Code Rate (bits/pixel)	0.48	0.39	0.32	0.23	0.22	0.23
Compression Ratio	2.1: 1	2.6: 1	3.1: 1	4.4: 1	4.7: 1	4.4: 1

6 · 全存真預測編碼(Lossless Predictive Coding)

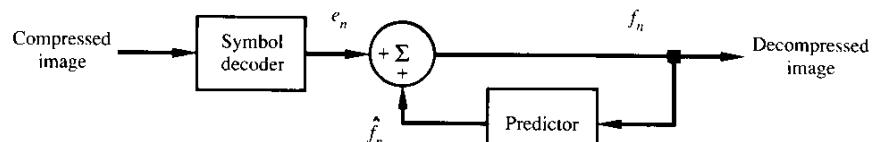
$$e_n = f_n - \hat{f}_n$$

$$\hat{f}_n = \text{round} \left[\sum_{i=1}^m \alpha_i f_{n-i} \right]$$

$$\hat{f}_n(x, y) = \text{round} \left[\sum_{i=1}^m \alpha_i f(x, y - i) \right]$$



(a)



(b)

Figure 6.19 A lossless predictive coding model: (a) encoder; (b) decoder.

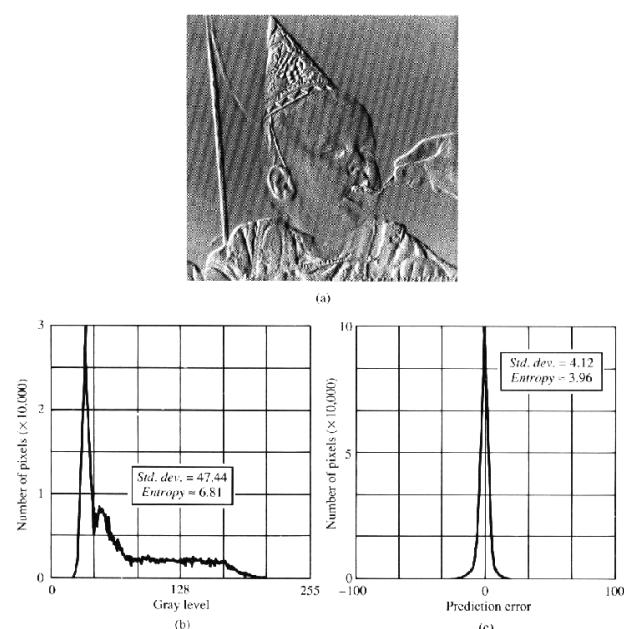


Figure 6.20 (a) The prediction error image resulting from Eq. (6.4-9); (b) the original image's gray-level histogram; (c) the prediction error histogram.

五、部分存真影像壓縮(Lossy Compression)

一、部分存真預測編碼(Lossy Predictive Coding)

$$\dot{\hat{f}}_n = \dot{e}_n + \hat{f}_{n-1}$$

1 · Delta Modulation (DM)

$$\hat{f}_n = \alpha \dot{f}_{n-1}$$

$$\dot{e}_n = \begin{cases} +\zeta & \text{for } e_0 > 0 \\ -\zeta & \text{otherwise} \end{cases}$$

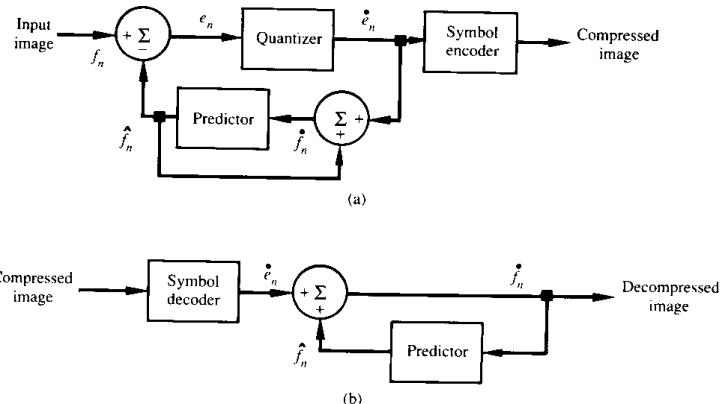
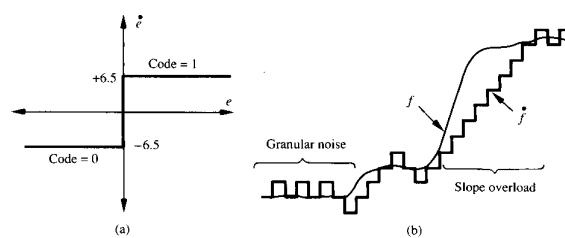


Figure 6.21 A lossy predictive coding model: (a) encoder; (b) decoder.



Input		Encoder				Decoder		Error
n	f	\hat{f}	e	\dot{e}	\hat{f}	\hat{f}	\hat{f}	$ f - \hat{f} $
0	14	—	—	—	14.0	—	14.0	0.0
1	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
2	14	20.5	-6.5	-6.5	14.0	20.5	14.0	0.0
3	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
•	•	•	•	•	•	•	•	•
14	29	20.5	8.5	6.5	27.0	20.5	27.0	2.0
15	37	27.0	10.0	6.5	33.5	27.0	33.5	3.5
16	47	33.5	13.5	6.5	40.0	33.5	40.0	7.0
17	62	40.0	22.0	6.5	46.5	40.0	46.5	15.5
18	75	46.5	28.5	6.5	53.0	46.5	53.0	22.0
19	77	53.0	24.0	6.5	59.6	53.0	59.6	17.5
•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•

Figure 6.22 A delta modulation (DM) example.

2 · 最佳化預測(Optimal Predictors)

● Differential Pulse Code Modulation, DPCM

$$E\{e_n^2\} = E\{\left[f_n - \sum_{i=1}^m \alpha_i f_{n-i}\right]^2\}$$

$$\begin{aligned}\hat{f}(x, y) &= \alpha_1 f(x, y-1) + \alpha_2 f(x-1, y-1) \\ &\quad + \alpha_3 f(x-1, y) + \alpha_4 f(x-1, y+1)\end{aligned}$$

[例] :

$$\hat{f}(x, y) = 0.97 f(x, y-1)$$

$$\hat{f}(x, y) = 0.5 f(x, y-1) + 0.5 f(x-1, y)$$

$$\hat{f}(x, y) = 0.75 f(x, y-1) + 0.75 f(x-1, y) - 0.5 f(x-1, y-1)$$

$$\hat{f}(x, y) = \begin{cases} 0.97 f(x, y-1) & \text{if } \Delta h \leq \Delta v \\ 0.97 f(x-1, y) & \text{otherwise} \end{cases}$$

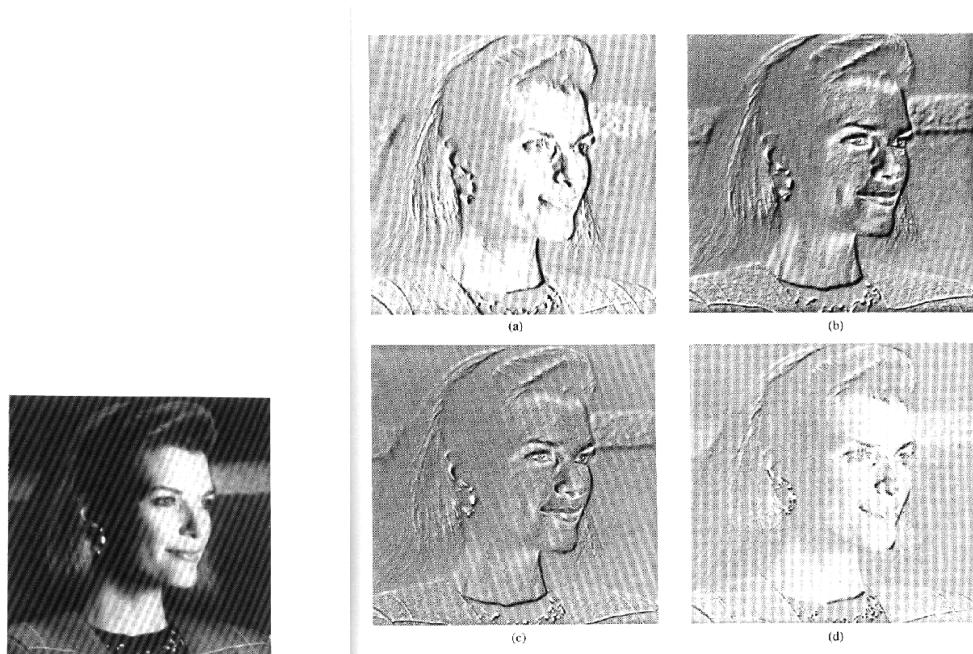


Figure 6.23 A 512 × 512 8-bit monochrome image.

Figure 6.24 A comparison of four linear prediction techniques.

3 · 最佳化量化(Optimal Quantization)

形成 $E\{ (s - t_i)^2 \}$ 最小誤差之條件為：

$$\int_{s_{i-1}}^{s_i} (s - t_i) p(s) ds = 0 \quad i = 1, 2, \dots, L/2$$

$$S_i = \begin{cases} 0 & i = 0 \\ (t_i + t_{i+1}) / 2 & i = 1, 2, \dots, L/2 - 1 \\ \infty & i = L/2 \end{cases}$$

$$S_{-i} = -S_i \quad t_{-i} = -t_i$$

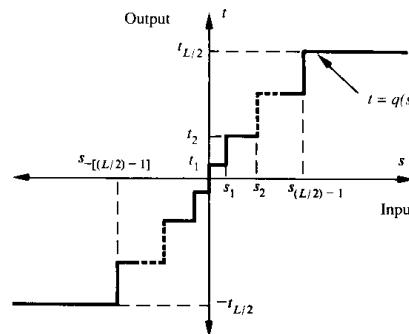


Figure 6.25 A typical quantization function.

Table 6.9 Lloyd-Max Quantizers for a Laplacian Probability Density Function of Unit Variance

Levels <i>i</i>	2		4		8	
	<i>s_i</i>	<i>t_i</i>	<i>s_i</i>	<i>t_i</i>	<i>s_i</i>	<i>t_i</i>
1	∞	0.707	1.102	0.395	0.504	0.222
2			∞	1.810	1.181	0.785
3					2.285	1.576
4					∞	2.994
θ	1.414		1.087		0.731	

Table 6.10 Lossy DPCM RMSE Summary

<i>Predictor</i>	<i>Lloyd-Max Quantizer</i>			<i>Adaptive Quantizer</i>		
	2-level	4-level	8-level	2-level	4-level	8-level
Eq. (6.5-16)	30.88	6.86	4.08	7.49	3.22	1.55
Eq. (6.5-17)	14.59	6.94	4.09	7.53	2.49	1.12
Eq. (6.5-18)	9.90	4.30	2.31	4.61	1.70	0.76
Eq. (6.5-19)	38.18	9.25	3.36	11.46	2.56	1.14
<i>Compression</i>	8.00:1	4.00:1	2.70:1	7.11:1	3.77:1	2.56:1



Figure 6.26 DPCM result images: (a) 1.0; (b) 1.125; (c) 2.0; (d) 2.125; (e) 3.0; (f) 3.125 bits/pixel.

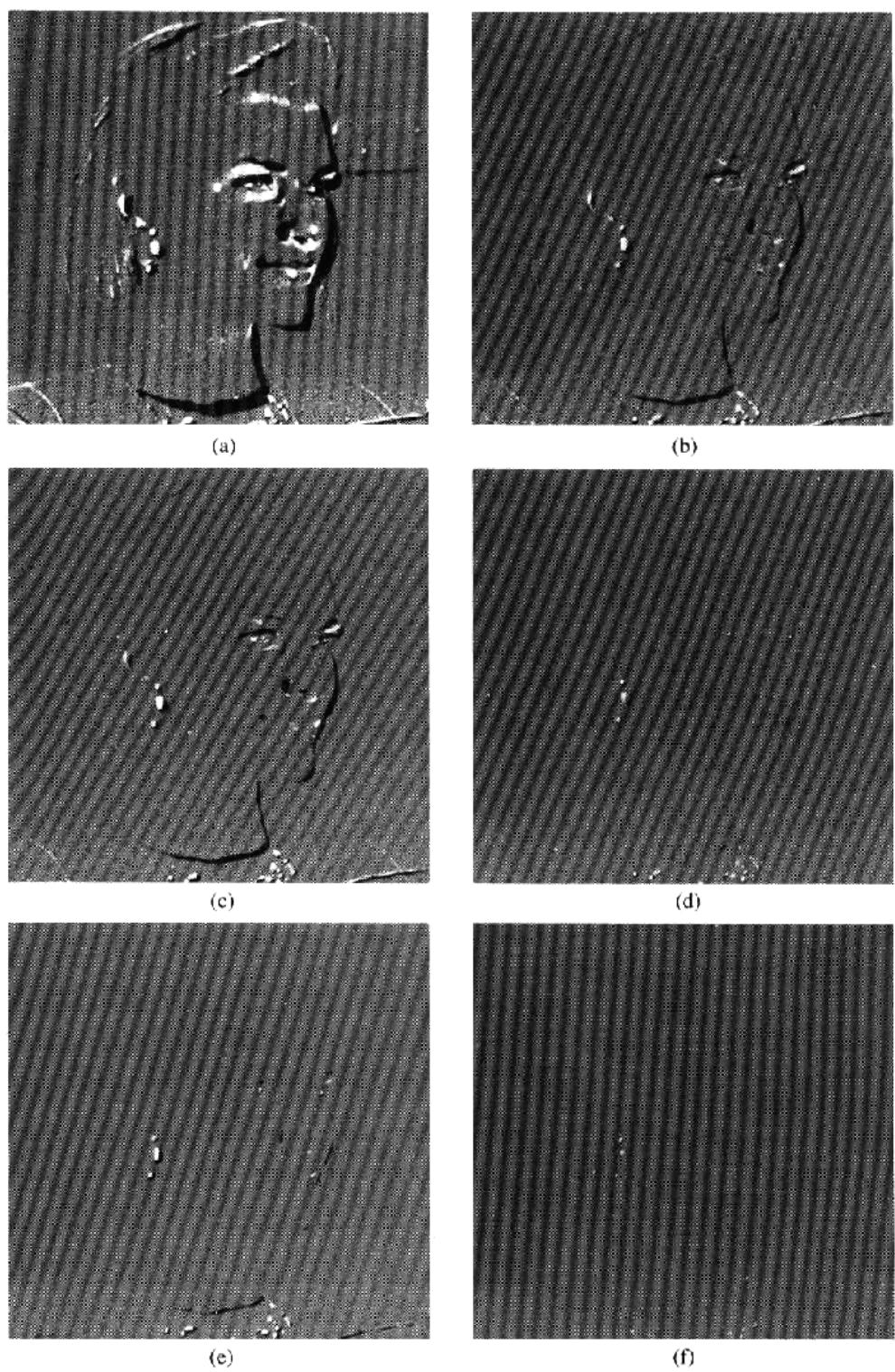


Figure 6.27 The scaled ($\times 8$) DPCM error images that correspond to Figs. 6.26(a)–(f).

二、轉換編碼(Transform Coding)

●轉換編碼過程乃利用線性轉換將影像映射至頻率領域中之一組轉換係數(Transform Coefficient)，再進行量化與編碼。

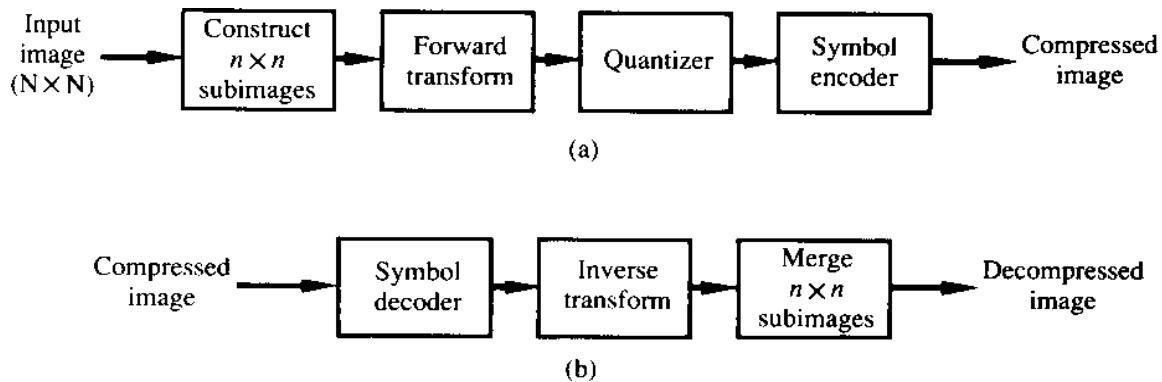


Figure 6.28 A transform coding system: (a) encoder; (b) decoder.

1 · 轉換之選擇(Transform Selection)

- (1).Karhunen-Loeve 轉換 (KLT)
- (2).Discrete Fourier 轉換 (DFT)
- (3).Discrete Cosine 轉換 (DCT)
- (4).Walsh-Hadamard 轉換 (WHT)

$$f(x, y) = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} T(u, v) h(x, y, u, v)$$

$$F = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} T(u, v) H_{uv}$$

$$H_{uv} = \begin{bmatrix} h(0,0,u,v) & h(0,1,u,v) & \cdots & h(0,n-1,u,v) \\ h(1,0,u,v) & h(1,1,u,v) & \cdots & h(1,n-1,u,v) \\ \vdots & \vdots & \ddots & \vdots \\ h(n-1,0,u,v) & h(n-1,1,u,v) & \cdots & h(n-1,n-1,u,v) \end{bmatrix}$$

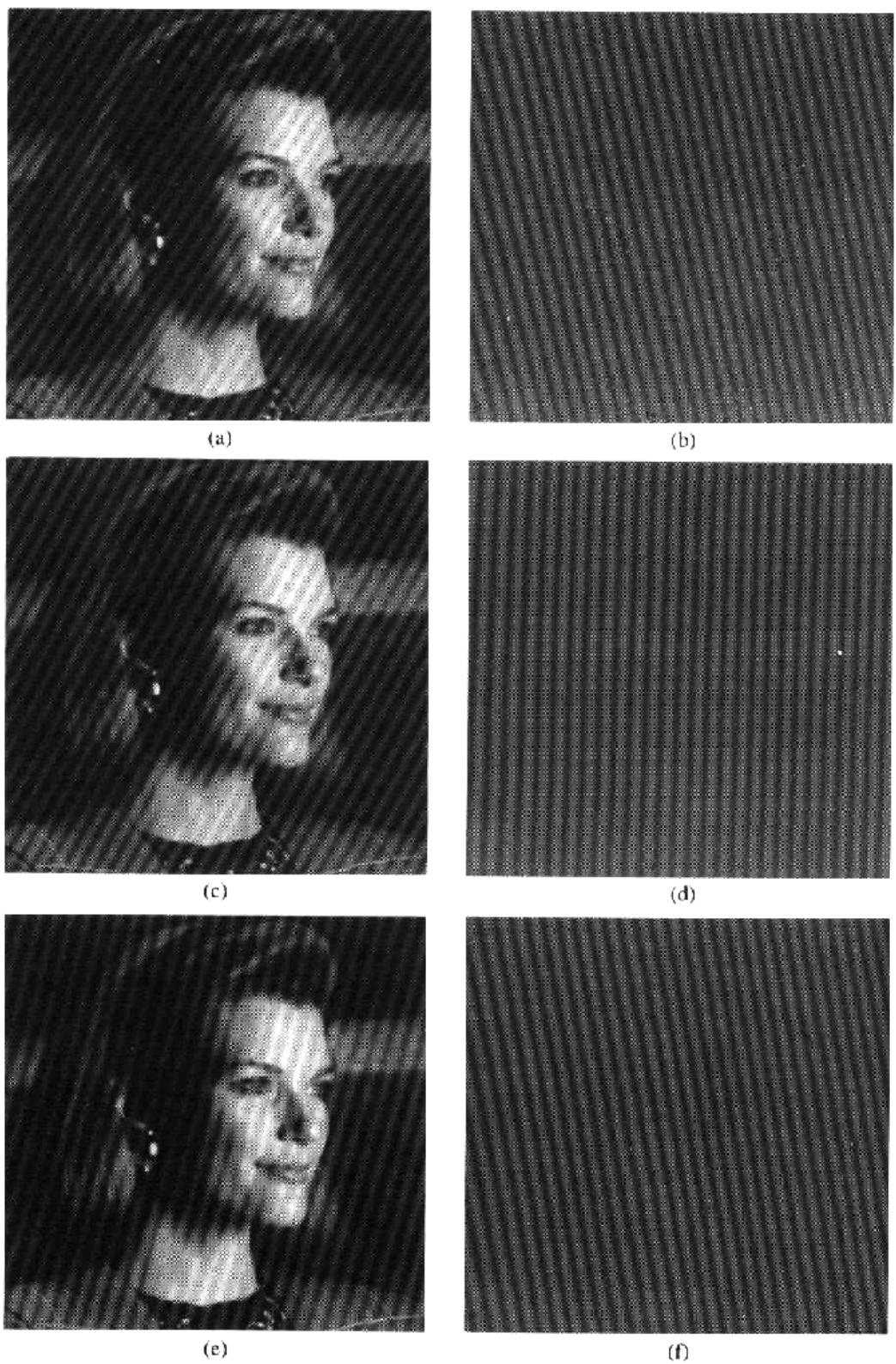


Figure 6.29 Approximations of Fig. 6.23 using the (a) Fourier, (c) Hadamard, and (e) cosine transforms, together with the corresponding scaled error images.

2 · 遮罩函數(Masking Function)

$$m(u, v) = \begin{cases} 0, & \text{若 } T(u, v) \text{ 符合某特定刪除條件} \\ 1, & \text{其他情形} \end{cases}$$

$$\hat{F} = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} T(u, v) m(u, v) H_{uv}$$

3 · 區塊效應(Blocking Artifact)

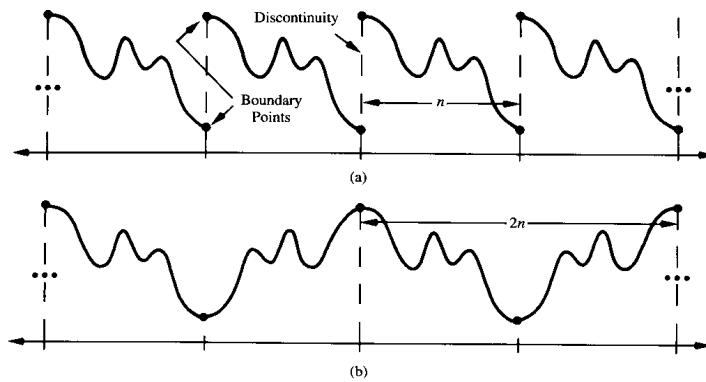


Figure 6.30 The periodicity implicit in the 1-D (a) DFT and (b) DCT.

4 · 次影像(Subimage)大小之選擇：

- 次影像大小增加時通常增加壓縮與計算之複雜性。
- 常用之次影像大小為 8×8 與 16×16 。

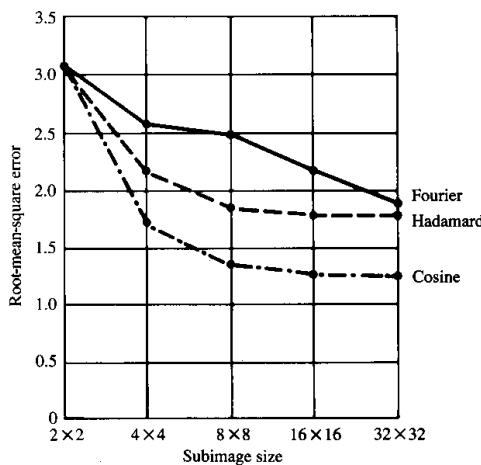


Figure 6.31 Reconstruction error versus subimage size.

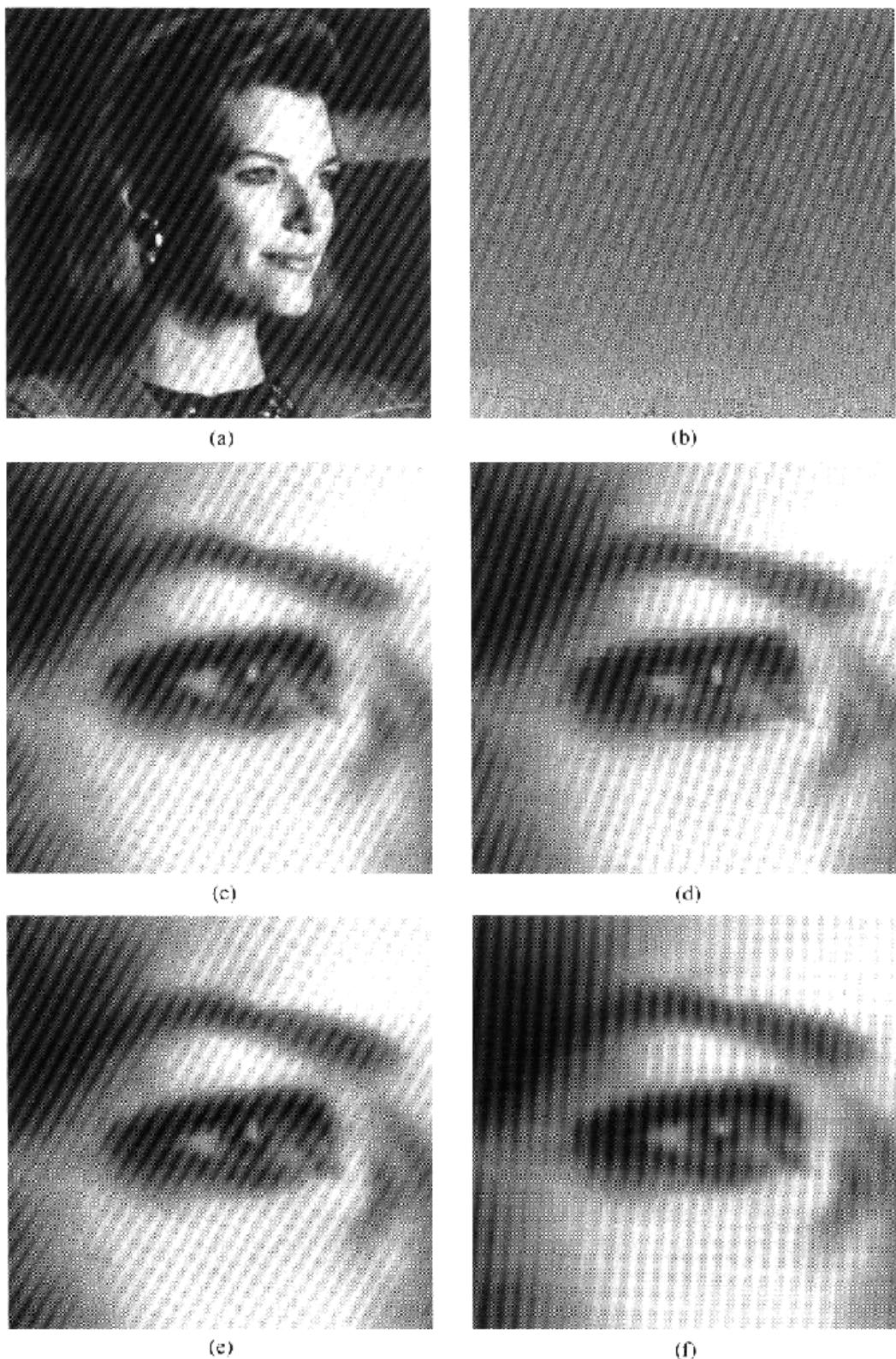
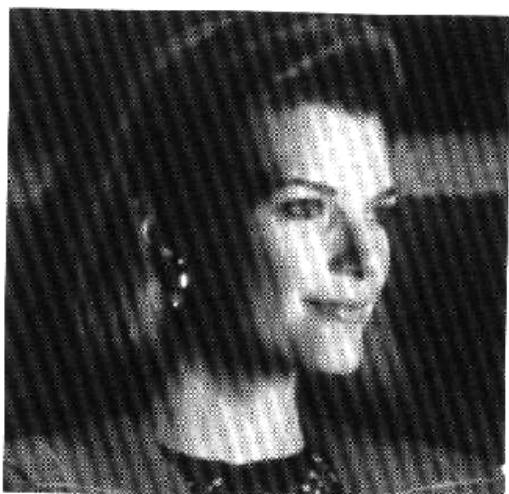


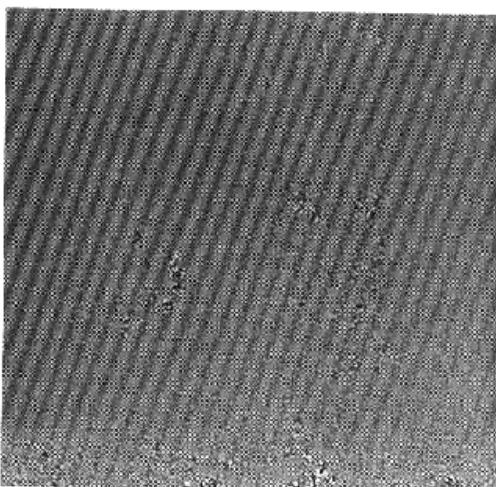
Figure 6.32 Approximations of Fig. 6.23 using 25 percent of the DCT coefficients: (a) and (b) 8×8 subimage results; (c) zoomed original; (d) 2×2 result; (e) 4×4 result; and (f) 8×8 result.



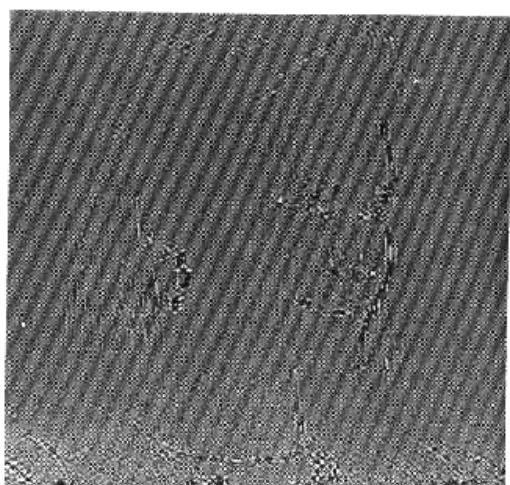
(a)



(b)



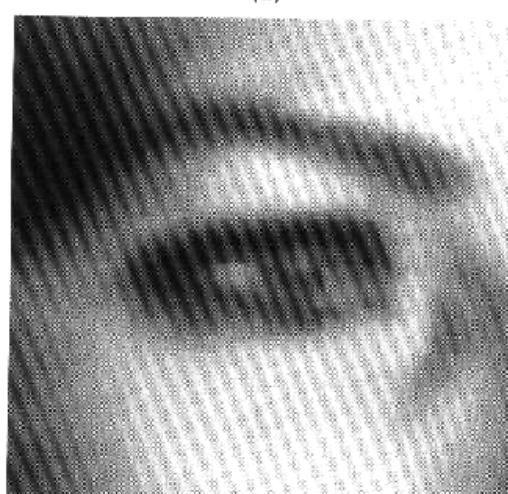
(c)



(d)



(e)



(f)

Figure 6.33 Approximations of Fig. 6.23 using 12.5 percent of the 8×8 DCT coefficients: (a), (c), and (e) threshold coding results; (b), (d), and (f) zonal coding results.

5 · 係數之選擇：

- (1).Bit Allocation - Truncating, Quantizing, Coding。
- (2).Zonal Coding - Maximum Variance。

(a)	(b)
(c)	(d)

Figure 6.34 A typical (a) zonal mask, (b) zonal bit allocation, (c) threshold mask, and (d) thresholded coefficient ordering sequence. Shading highlights the coefficients that are retained.

- (3).Threshold Coding - Maximum Magnitude。

- Single Global Threshold。
- N-largest Coding。
- Normalized Array Coding。

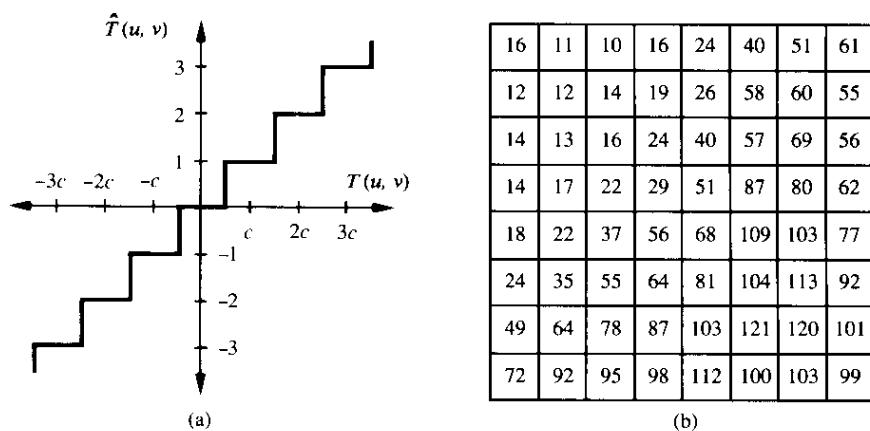


Figure 6.35 (a) A threshold coding quantization curve (see Eq. 6.5-30); (b) a typical normalization matrix.

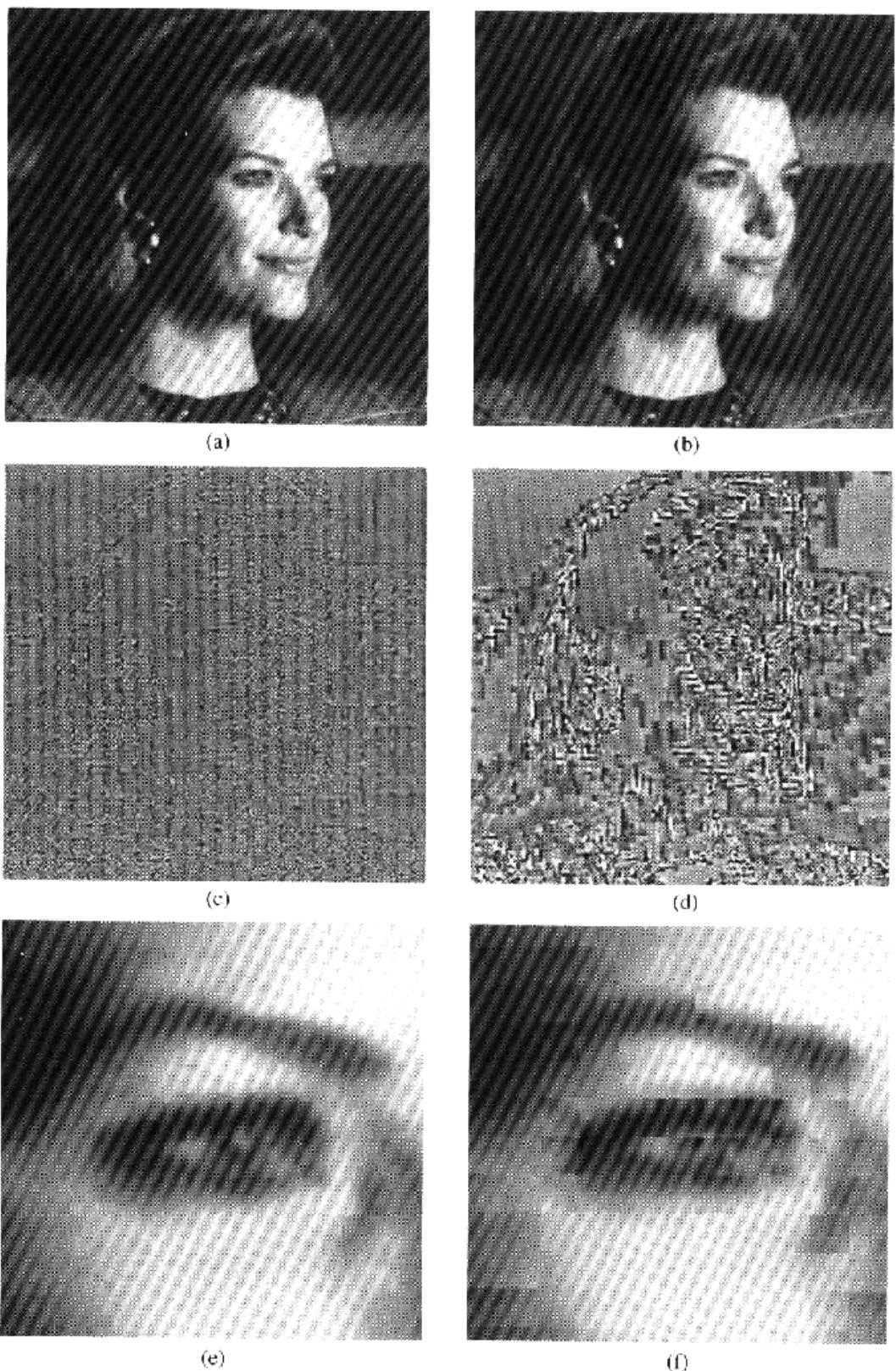


Figure 6.36 (a), (c), and (e) Approximations of Fig. 6.23 using the DCT and normalization array of Fig. 6.35(b); (b), (d), and (f) similar results for $4\mathbf{Z}$.

陸、影像資料壓縮標準

一、二元影像壓縮標準

1 · ISO、CCITT 與 JBIG

2 · 二元影像之一維壓縮標準

(1).Terminating Codes + Makeup Codes。

(2).每橫行以白色影像元素為起始(即使碼長為 0)。

(3).000000000001 表示 EOL 與影像起始。

(4).影像結束以連續六個 EOL 表示。

3 · 二元影像之二維壓縮

(1).參考線(Reference Line)。

(2).影像第一橫行之參考線為全白像素組成。

(3).參考元素(Reference Element), a_0

(4).Pass Mode, Vertical Mode, Horizontal Mode。

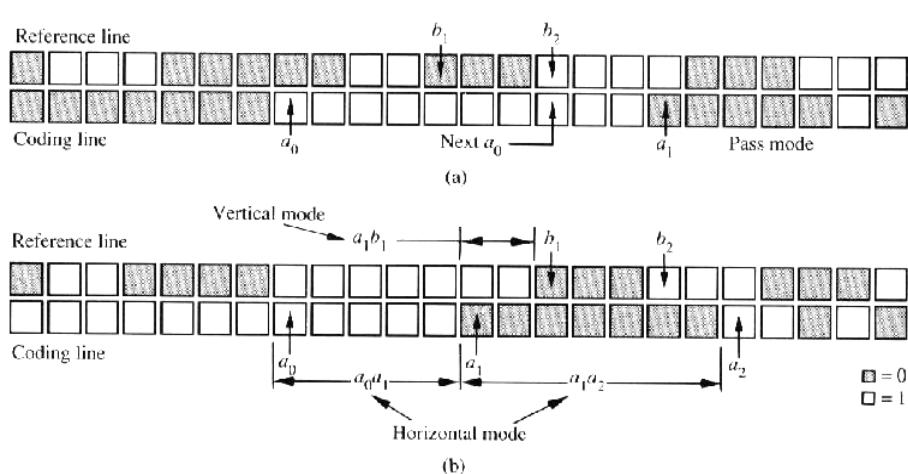


Figure 6.38 CCITT (a) pass mode and (b) horizontal and vertical mode coding parameters.

Table 6.13 CCITT Two-Dimensional Code Table

Mode	Code Word
Pass	0001
Horizontal	$001 + M(a_0a_1) + M(a_1a_2)$
Vertical	
a_i below b_j	1
a_i one to the right of b_j	011
a_i two to the right of b_j	000011
a_i three to the right of b_j	0000011
a_i one to the left of b_j	010
a_i two to the left of b_j	000010
a_i three to the left of b_j	0000010
Extension	0000001xxx

Table 6.11 CCITT Terminating Codes

<i>Run Length</i>	<i>White Code Word</i>	<i>Black Code Word</i>	<i>Run Length</i>	<i>White Code Word</i>	<i>Black Code Word</i>
0	00110101	0000110111	32	00011011	000001101010
1	000111	010	33	00010010	000001101011
2	0111	11	34	00010011	000011010010
3	1000	10	35	00010100	000011010011
4	1011	011	36	00010101	000011010100
5	1100	0011	37	00010110	000011010101
6	1110	0010	38	00010111	000011010110
7	1111	00011	39	00101000	000011010111
8	10011	000101	40	00101001	000001101100
9	10100	000100	41	00101010	000001101101
10	00111	0000100	42	00101011	000011011010
11	01000	0000101	43	00101100	000011011011
12	001000	0000111	44	00101101	000001010100
13	000011	00000100	45	00000100	000001010101
14	110100	00000111	46	00000101	000001010110
15	110101	0000011000	47	00001010	000001010111
16	101010	00000010111	48	00001011	000001100100
17	101011	00000011000	49	01010010	000001100101
18	0100111	00000001000	50	01010011	000001010010
19	0001100	000001100111	51	01010100	000001010011
20	0001000	000001101000	52	01010101	000000100100
21	0010111	000001101100	53	00100100	000000110111
22	0000011	000000110111	54	00100101	0000000111000
23	0000100	000000101000	55	01011000	0000000100111
24	0101000	000000010111	56	01011001	0000000101000
25	0101011	000000011000	57	01011010	0000001011000
26	0010011	0000011001010	58	01011011	0000001011001
27	0100100	0000011001011	59	01001010	0000000101011
28	0011000	0000011001100	60	01001011	0000000101100
29	000000010	0000011001101	61	00110010	0000001011010
30	000000011	0000001101000	62	00110011	0000001100110
31	00011010	0000001101001	63	00110100	0000001100111

Table 6.12 CCITT Makeup Codes

<i>Run Length</i>	<i>White Code Word</i>	<i>Black Code Word</i>	<i>Run Length</i>	<i>White Code Word</i>	<i>Black Code Word</i>
64	11011	0000001111	960	011010100	0000001110011
128	10010	0000011001000	1024	011010101	0000001110100
192	010111	0000011001001	1088	011010110	0000001110101
256	0110111	0000001011011	1152	011010111	00000001110110
320	00110110	0000000110011	1216	011011000	00000001110111
384	00110111	0000000110100	1280	011011001	00000001010010
448	01100100	0000000110101	1344	011011010	00000001010011
512	01100101	00000001101100	1408	011011011	00000001010100
576	01101000	00000001101101	1472	010011000	00000001010101
640	01100111	00000001001010	1536	010011001	00000001011010
704	011001100	00000001001011	1600	010011010	00000001011011
768	011001101	00000001001100	1664	011000	00000001100100
832	011010010	00000001001101	1728	010011011	00000001100101
896	011010011	00000001110010			
		<i>Code Word</i>			<i>Code Word</i>
1792	000000001000		2240	0000000010110	
1856	000000001100		2304	0000000010111	
1920	000000001101		2368	0000000011100	
1984	0000000010010		2432	0000000011101	
2048	0000000010011		2496	0000000011110	
2112	0000000010100		2560	0000000011111	
2176	0000000010101				

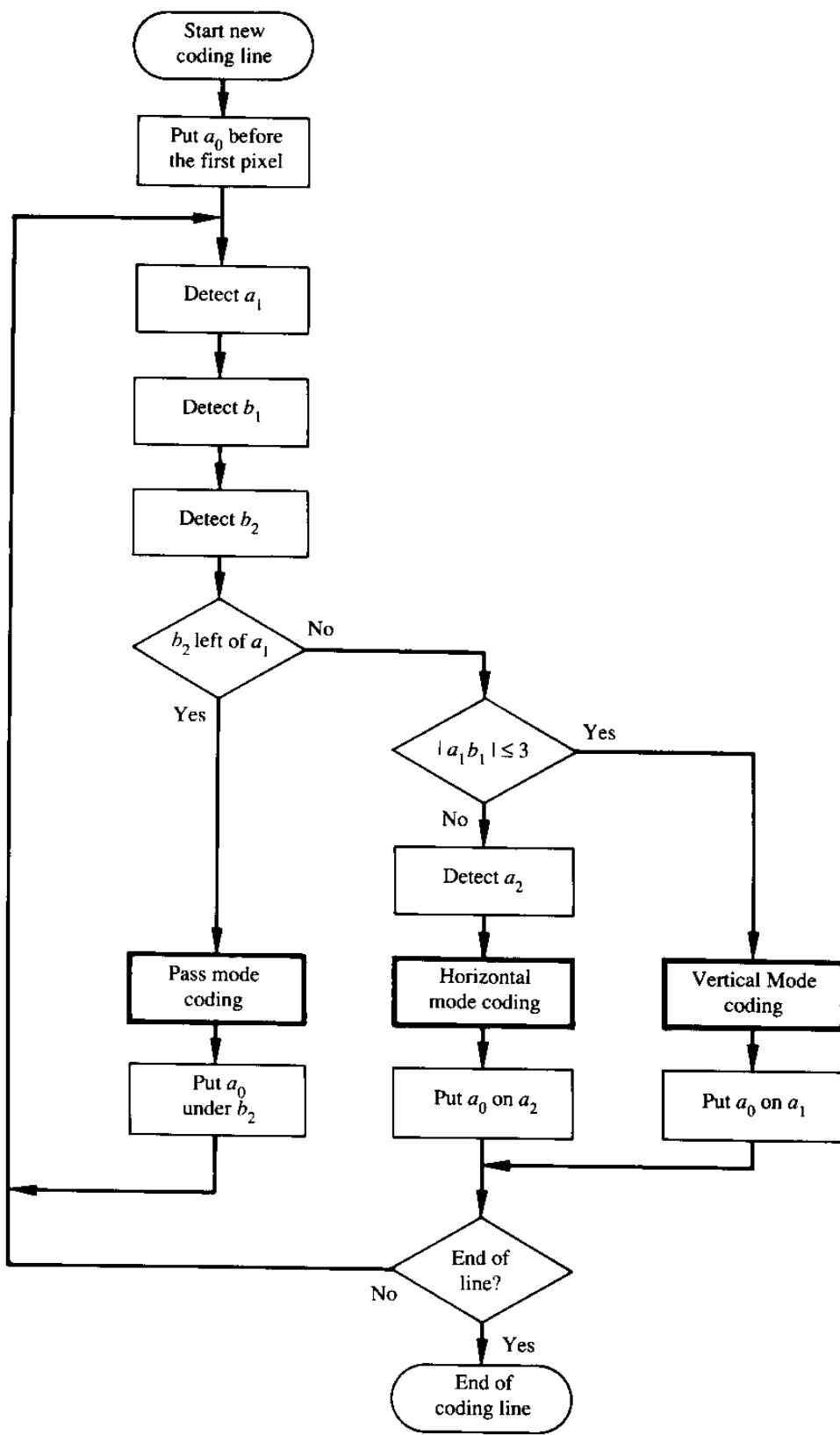


Figure 6.37 CCITT 2-D coding procedure. The notation $|a_i b_i|$ denotes the absolute value of the distance between changing elements a_i and b_i .

二、具次階影像壓縮標準

1 · 單一固定影像(Still-frame)壓縮標準

JPEG (Joint Photographic Experts Group)標準

(1).Lossy baseline coding system。

壓縮過程：

- A. DCT 運算
- B. 量化
- C. 不等長編碼

(2).Extended coding system。

(3).Lossless independent coding system

2 · 連續影像壓縮標準

(1). H.261 標準

(2). MPEG I 標準

(3). MPEG II 標準

Table 6.14 JPEG Coefficient Coding Categories

Range	DC Difference Category	AC Category
0	0	N/A
-1, 1	1	1
-3, -2, 2, 3	2	2
-7, ..., -4, 4, ..., 7	3	3
-15, ..., -8, 8, ..., 15	4	4
-31, ..., -16, 16, ..., 31	5	5
-63, ..., -32, 32, ..., 63	6	6
-127, ..., -64, 64, ..., 127	7	7
-255, ..., -128, 128, ..., 255	8	8
-511, ..., -256, 256, ..., 511	9	9
-1023, ..., -512, 512, ..., 1023	A	A
-2047, ..., -1024, 1024, ..., 2047	B	B
-4095, ..., -2048, 2048, ..., 4095	C	C
-8191, ..., -4096, 4096, ..., 8191	D	D
-16383, ..., -8192, 8192, ..., 16383	E	E
-32767, ..., -16384, 16384, ..., 32767	F	N/A

Table 6.15 JPEG Default DC Code (Luminance)

Category	Base Code	Length	Category	Base Code	Length
0	010	3	6	1110	10
1	011	4	7	11110	12
2	100	5	8	111110	14
3	00	5	9	1111110	16
4	101	7	A	11111110	18
5	110	8	B	111111110	20

Table 6.16 JPEG Default AC Code (Luminance)

<i>Run/ Category</i>	<i>Base Code</i>	<i>Length</i>	<i>Run/ Category</i>	<i>Base Code</i>	<i>Length</i>
0/0	1010 (= EOB)	4			
0/1	00	3	8/1	11111010	9
0/2	01	4	8/2	11111111000000	17
0/3	100	6	8/3	111111110110111	19
0/4	1011	8	8/4	111111110111000	20
0/5	11010	10	8/5	111111110111001	21
0/6	111000	12	8/6	111111110111010	22
0/7	1111000	14	8/7	111111110111011	23
0/8	1111110110	18	8/8	111111110111100	24
0/9	111111110000010	25	8/9	111111110111101	25
0/A	111111110000011	26	8/A	111111110111110	26
1/1	1100	5	9/1	111111000	10
1/2	111001	8	9/2	111111110111111	18
1/3	1111001	10	9/3	1111111111000000	19
1/4	111110110	13	9/4	1111111111000001	20
1/5	1111110110	16	9/5	1111111111000010	21
1/6	111111110000100	22	9/6	1111111111000011	22
1/7	111111110000101	23	9/7	1111111111000100	23
1/8	111111110000110	24	9/8	1111111111000101	24
1/9	111111110000111	25	9/9	1111111111000110	25
1/A	111111110001000	26	9/A	1111111111000111	26
2/1	11011	6	A/1	111111001	10
2/2	1111000	10	A/2	111111111001000	18
2/3	1111110111	13	A/3	1111111111001001	19
2/4	111111110001001	20	A/4	1111111111001010	20
2/5	111111110001010	21	A/5	1111111111001011	21
2/6	111111110001011	22	A/6	1111111111001100	22
2/7	111111110001100	23	A/7	1111111111001101	23
2/8	111111110001101	24	A/8	1111111111001110	24
2/9	111111110001110	25	A/9	1111111111001111	25
2/A	111111110001111	26	A/A	1111111111010000	26
3/1	111010	7	B/1	111111010	10
3/2	111110111	11	B/2	1111111111010001	18
3/3	11111110111	14	B/3	1111111111010010	19
3/4	111111110010000	20	B/4	1111111111010011	20
3/5	111111110010001	21	B/5	1111111111010100	21
3/6	111111110010010	22	B/6	1111111111010101	22
3/7	111111110010011	23	B/7	1111111111010110	23
3/8	111111110010100	24	B/8	1111111111010111	24
3/9	111111110010101	25	B/9	1111111111011000	25
3/A	111111110010110	26	B/A	1111111111011001	26

Table 6.16 Continued

<i>Run/ Category</i>	<i>Base Code</i>	<i>Length</i>	<i>Run/ Category</i>	<i>Base Code</i>	<i>Length</i>
4/1	111011	7	C/1	1111111010	11
4/2	1111111000	12	C/2	1111111111011010	18
4/3	111111110010111	19	C/3	1111111111011011	19
4/4	1111111110011000	20	C/4	11111111111011100	20
4/5	1111111110011001	21	C/5	11111111111011101	21
4/6	1111111110011010	22	C/6	11111111111011110	22
4/7	1111111110011011	23	C/7	11111111111011111	23
4/8	1111111110011100	24	C/8	11111111111100000	24
4/9	1111111110011101	25	C/9	11111111111100001	25
4/A	1111111110011110	26	C/A	11111111111100010	26
5/1	1111010	8	D/1	11111111010	12
5/2	1111111001	12	D/2	111111111100011	18
5/3	111111111001111	19	D/3	1111111111100100	19
5/4	1111111110100000	20	D/4	1111111111100101	20
5/5	1111111110100001	21	D/5	1111111111100110	21
5/6	1111111110100010	22	D/6	1111111111100111	22
5/7	1111111110100011	23	D/7	1111111111101000	23
5/8	1111111110100100	24	D/8	1111111111101001	24
5/9	1111111110100101	25	D/9	1111111111101010	25
5/A	1111111110100110	26	D/A	1111111111101011	26
6/1	1111011	8	E/1	111111110110	13
6/2	11111111000	13	E/2	1111111111101100	18
6/3	1111111110100111	19	E/3	1111111111101101	19
6/4	1111111110101000	20	E/4	1111111111101110	20
6/5	1111111110101001	21	E/5	1111111111101111	21
6/6	1111111110101010	22	E/6	1111111111110000	22
6/7	1111111110101011	23	E/7	1111111111110001	23
6/8	1111111110101100	24	E/8	1111111111110010	24
6/9	1111111110101101	25	E/9	1111111111110011	25
6/A	1111111110101110	26	E/A	1111111111110100	26
7/1	11111001	9	F/0	111111110111	12
7/2	11111111001	13	F/1	1111111111110101	17
7/3	1111111110101111	19	F/2	1111111111110110	18
7/4	1111111110110000	20	F/3	1111111111110111	19
7/5	1111111110110001	21	F/4	1111111111111000	20
7/6	1111111110110010	22	F/5	1111111111111001	21
7/7	1111111110110011	23	F/6	1111111111111010	22
7/8	1111111110110100	24	F/7	1111111111111011	23
7/9	1111111110110101	25	F/8	1111111111111100	24
7/A	1111111110110110	26	F/9	1111111111111101	25
			F/A	1111111111111110	26

[範例]：壓縮過程

原始影像

52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

減 2^7

-76	-73	-67	-62	-58	-67	-64	-55
-65	-69	-62	-38	-19	-43	-59	-56
-66	-69	-60	-15	16	-24	-62	+55
-65	-70	-57	-6	26	-22	-58	-59
-61	-67	-60	-24	-2	-40	-60	-58
-49	-63	-68	-58	-51	-65	-70	+53
-43	-57	-64	-69	-73	-67	-63	-45
-41	-49	-59	-60	-63	-52	-50	-34

DCT轉換

-415	-29	-62	25	55	-20	-1	3
7	-21	-62	9	11	-7	-6	6
-46	8	77	-25	-30	10	7	-5
-50	13	35	-15	-9	6	0	3
11	-8	-13	-2	-1	1	-4	1
-10	1	3	-3	-1	0	2	-1
-4	-1	2	-1	2	-3	1	-2
-1	-1	-1	-2	-1	-1	0	-1

Normalization

-26	-3	-6	2	2	0	0	0
1	-2	-4	0	0	0	0	0
-3	1	5	-1	-1	0	0	0
-4	1	2	-1	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Zigzag排序

[-26 -3 1 -3 -2 -6 2 -4 1 -4 1 1 5 0 2 0
0 -1 2 0 0 0 0 -1 -1 EOBI]

最後編碼

1010110 0100 001 0100 0101 100001 0110 100011 001 100011 001
001 100101 11100110 110110 0110 11110100 000 1010

[範例]：解壓縮過程

解碼

-26	-3	-6	2	2	0	0	0
1	-2	-4	0	0	0	0	0
-3	1	5	-1	-1	0	0	0
-4	1	2	-1	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

乘上 Z

-416	-33	-60	32	48	0	0	0
12	-24	-56	0	0	0	0	0
-42	13	80	-24	-40	0	0	0
-56	17	44	-29	0	0	0	0
18	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

DCT反轉換

-70	-64	-61	-64	-69	-66	-58	-50
-72	-73	-61	-39	-30	-40	-54	-59
-68	-78	-58	-9	13	-12	-48	-64
-59	-77	-57	0	22	-13	-51	-60
-54	-75	-64	-23	-13	-44	-63	-56
-52	-71	-72	-54	-54	-71	-71	-54
-45	-59	-70	-68	-67	-67	-61	-50
-35	-47	-61	-66	-60	-48	-44	-44

加 2^7

58	64	67	64	59	62	70	78
56	55	67	89	98	88	74	69
60	50	70	119	141	116	80	64
69	51	71	128	149	115	77	68
74	53	64	105	115	84	65	72
76	57	56	74	75	57	57	74
83	69	59	60	61	61	67	78
93	81	67	62	69	80	84	84

誤差

-6	-9	-6	2	11	-1	-6	-5
7	4	-1	1	11	-3	-5	3
2	9	-2	-6	-3	-12	-14	9
-6	7	0	-4	-5	-9	-7	1
-7	8	4	-1	11	4	3	-2
3	8	4	-4	2	11	1	1
2	2	5	-1	-6	0	-2	5
-6	-2	2	6	-4	-4	-6	10