1

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ASEN 5050
Fall 2024

$\boxed{HW2}$

Problem 1 → Given: $G m_{mars} = 4.305 \times 10^{4} \frac{km^3}{s^2}$, $r_{eq, mars} = 3397.2$ km

On June 1st, 2024, in Mars-centered inertial frame,

$\vec{r}_1 = 3.62067 \times 10^{3} \hat{x} - 3.19925 \times 10^{2} \hat{y} - 4.20645 \times 10^{2} \hat{z}$ km, $r_1 = |\vec{r}_1|$

$\vec{v}_1 = -4.28843 \times 10^{-1} \hat{x} - 3.00176 \times 10^{-2} \hat{y} - 3.39801 \hat{z}$ km/s, $v_1 = |\vec{v}_1|$

Assume: 2 Body problem, $m_{s/c} << M_{mars}$ → ∴ $\mu_{mars} = G(\cancel{m_{s/c}}^{0} + M_{mars}) = G M_{mars}$

a) $\vec{h} = \vec{r}_1 \times \vec{v}_1$

$= 1.0745 \times 10^{3} \hat{x} + 1.2483 \times 10^{4} \hat{y} - 245.8814 \hat{z}$ $\frac{km^2}{s}$

$\vec{n} = \hat{z} \times \vec{h}_1 = -1.2483 \times 10^{4} \hat{x} + 1.0745 \times 10^{3} \hat{y} + 0 \hat{z}$ km²/s

$\hat{n} = \vec{n}/|\vec{n}| = -0.9963 \hat{x} + 0.0858 \hat{y} + 0 \hat{z}$

$\vec{e} = \left[\left(v_1^2 - \mu_{mars}/r_1\right)\vec{r}_1 + (\vec{r}_1 \cdot \vec{v}_1)\vec{v}_1\right]\frac{1}{\mu_{mars}} = -0.0040 \hat{x} + 1.7421 \times 10^{-4} \hat{y} - 0.0086 \hat{z}$

$e = |\vec{e}| = \boxed{0.0095 = e}$

$\mathcal{E} = v_1^2/2 - \mu_{mars}/r = -5.8898$ km²/s²

$a = -\mu_{mars}/2\mathcal{E} = \boxed{3.6485 \times 10^{3} \text{ km} = a}$

$i = \arccos\left(\frac{\vec{h} \cdot \hat{z}}{|\vec{h}|}\right) = \boxed{91.1242^{\circ} = i}$ ← By definition $i \geq 0$

$\Omega = \arccos(\hat{n} \cdot \hat{x}) = \boxed{175.0805^{\circ} = \Omega}$ ← $\Omega > 0$ because $\hat{n} \cdot \hat{y} > 0$

$\omega = \arccos\left(\frac{\vec{n} \cdot \vec{e}}{|\vec{n}||\vec{e}|}\right) = \boxed{-65.1600^{\circ} = \omega}$ ← $\omega < 0$ because $\hat{e} \cdot \hat{z} < 0$

$\theta^* = \arccos\left(\frac{\vec{e} \cdot \vec{r}}{|\vec{e}||\vec{r}|}\right) = \boxed{-108.2374^{\circ} = \theta^*}$ ← $\theta^* < 0$ because $\vec{r} \cdot \vec{v} = -113.7397 < 0$

b) ① $\vec{r}_{1, \hat{r}\hat{\theta}\hat{h}} = [C]^T \vec{r}_1$

$$C = \begin{bmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\Omega C_i C_\theta & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{bmatrix} = \begin{bmatrix} 0.9895 & -0.1162 & 0.0857 \\ -0.0874 & -0.0096 & 0.9961 \\ -0.1150 & -0.9932 & -0.0196 \end{bmatrix}$$

where $S/C_x = \sin(x)/\cos(x)$, $\theta = \theta^* + \omega$

$\boxed{\vec{r}_{1, \hat{r}\hat{\theta}\hat{h}} = 3.6590 \times 10^{3} \hat{r} + 0 \hat{\theta} + 0 \hat{h} \text{ km}}$

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$$\boxed{\vec{V}_{1,\hat{r}\hat{\theta}\hat{h}} = [C]^T \vec{V}_1 = -0.0311\hat{r} + 3.4250\hat{\theta} + 0\hat{h} \ \text{km/s}}$$

b] (2) $\boxed{\vec{r}_{1,\hat{r}\hat{\theta}\hat{h}} = r_1\hat{r} = 3.6590 \times 10^3 \hat{r} + 0\hat{\theta} + 0\hat{h} \ \text{km}}$

$$v_r = \frac{\mu_{mars}}{h} \cdot e\sin(\theta^*) = -0.0311 \ \text{km/s}$$

$$v_\theta = \frac{\mu_{mars}}{h}(1 + e\cos\theta^*) = 3.4250 \ \text{km/s}$$

$$\rightarrow \vec{V}_{1,\hat{r}\hat{\theta}\hat{h}} = v_r\hat{r} + v_\theta\hat{\theta}$$

$$\boxed{\vec{V}_{1,\hat{r}\hat{\theta}\hat{h}} = -0.0311\hat{r} + 3.4250\hat{\theta} + 0\hat{h} \ \text{km/s}}$$

c] At perigee $\rightarrow \theta^* = 0°$. Assume s/c followed orbit as calculated using 2 Body problem equations of motion.

$$r_p = a(1-e) = 3.6137 \times 10^3 \ \text{km}$$

$$\vec{r}_{p,\hat{r}\hat{\theta}\hat{h}} = r_p\hat{r} = 3.6137 \times 10^3 \hat{r} + 0\hat{\theta} + 0\hat{h} \ \text{km}$$

$$v_{r,p} = 0, \quad v_{\theta,p} = \frac{\mu_{mars}}{h}(1 + e\cos(\theta^*)) = 3.4679 \ \text{km/s}$$

$$\vec{V}_{p,\hat{r}\hat{\theta}\hat{h}} = v_r\hat{r} + v_\theta\hat{\theta} + 0\hat{h} = 0\hat{r} + 3.4679\hat{\theta} + 0\hat{h} \ \text{km/s}$$

$$C = \begin{bmatrix} -0.4201 & -0.9034 & 0.0857 \\ 0.0183 & 0.0860 & 0.9961 \\ -0.9073 & 0.4200 & -0.0196 \end{bmatrix}$$

$$\vec{r}_{p,xyz} = [C]\vec{r}_{p,\hat{r}\hat{\theta}\hat{h}} = \boxed{-1.5180 \times 10^3 \hat{x} + 66.0771\hat{y} - 3.2787 \times 10^3 \hat{z} \ \text{km} = \vec{r}_{p,xyz}}$$

$$\vec{V}_{p,xyz} = [C]\vec{V}_{p,\hat{r}\hat{\theta}\hat{h}} = \boxed{-3.1330\hat{x} + 0.2984\hat{y} + 1.4565\hat{z} \ \text{km/s} = \vec{V}_{p,xyz}}$$

Problem 2 $\rightarrow$ Given: $Gm_{moon} = 4.902799 \times 10^3 \ \text{km}^3/\text{s}^2$, $r_{eq,moon} = 1738$ km

$\hat{n} = 0.6428\hat{x} - 0.7660\hat{y} + 0\hat{z}$, $\hat{h} = -0.3237\hat{x} - 0.2717\hat{y} + 0.9063\hat{z}$,

$\vec{e} = 0.0475\hat{x} + 0.3755\hat{y} + 0.1295\hat{z}$

Assume: 2 Body problem, $m_{s/c} \ll m_{moon} \rightarrow \therefore \mu_{moon} = G(m_{s/c} + m_{moon}) \approx Gm_{moon}$

a] $e = |\vec{e}| = \boxed{0.400 = e}$

$i = \text{acos}(\hat{h} \cdot \hat{z}) = \boxed{25.0011° = i}$ $\leftarrow$ By definition $i \geq 0$

$\Omega = \text{acos}(\hat{n} \cdot \hat{x}) = \boxed{-49.9991° = \Omega}$ $\leftarrow \Omega < 0$ because $\hat{n} \cdot \hat{y} < 0$

$\omega = \text{acos}\left(\frac{\hat{n} \cdot \vec{e}}{|\vec{e}|}\right) = \boxed{129.9932° = \omega}$ $\leftarrow \omega > 0$ because $\vec{e} \cdot \hat{z} > 0$

b] s/c is at ascending node, $r = 4070.6$ km

θ^* is angle between perigee and spacecraft.

w is angle between perigee and ascending node.

$\therefore$ when s/c is at ascending node, $\theta^* = -w$.

$\therefore \theta^* = -129.9932^\circ$

$r = \dfrac{h^2/\mu_{moon}}{1 + e\cos\theta^*} \rightarrow h = \sqrt{r(1+e\cos\theta^*)\cdot\mu_{moon}}$

$\therefore h = 3.8505 \times 10^3 \text{ km}^2/\text{s}$ ← h is positive by definition

$a(1-e^2) = h^2/\mu_{moon} \rightarrow a = \dfrac{h^2}{\mu_{moon}(1-e^2)} = \boxed{3.6002 \times 10^3 \text{ km} = a}$

Exp. No.	Experiment/Subject		Date	02
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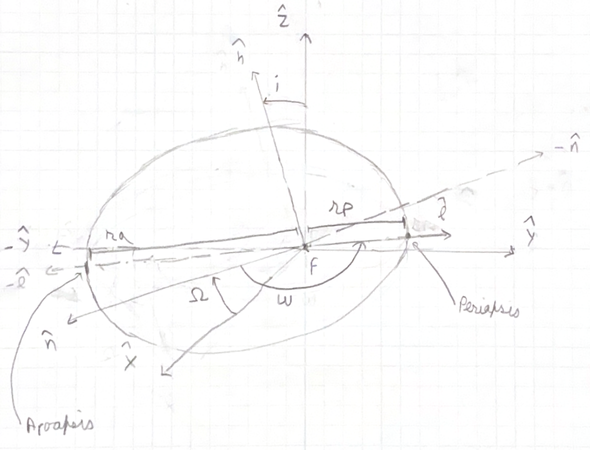

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