### **Table of Contents**

```
clear; clc; close all;
% ASEN 5010 - HW 3, Problem 4
% Spring 2025
% Jash Bhalavat
mass = 1; % kg
v_{CN_P_0} = [0, 0, 0]'; % m/s
v_CN_N_0 = v_CN_P_0;
omega_PN_P_0 = [0, 0, 0.85]'; % rad/sec
euler_PN_0 = [0, 0, 0]'; % rad
g = 9.81; % km/s2
I_c_P = [32.5, 32.5, 5]'; % kg-m2
r_NozzleC_P = [0, 0, -3]'; % m
r_NoseC_P = [0, 0, 4]'; % m
r_CN_0 = [0, 0, 0]'; % [m]
tspan = [0, 60]; % sec
function thrust P = thrust(time)
  % Thrust in Principal frame
  thruster_misalignment = deg2rad(2.5); % rad
  thruster_mag = 15; % N
  thruster_duration = 10; % sec
  thrust_parallel = 0; % [N] Passes CoM
  thrust_perp = 0; % [N]
  if time <= thruster_duration</pre>
     thrust_parallel = thruster_mag * cos(thruster_misalignment);
     thrust_perp = thruster_mag * sin(thruster_misalignment);
  end
  thrust_P = [0, -thrust_perp, thrust_parallel]'; % [N]
end
function out = L_c_P(time, r)
  % Torque at CoM in P frame
  thrust_at_time_P = thrust(time);
  out = cross(r, thrust_at_time_P); % [Nm]
end
```

## **EOM**

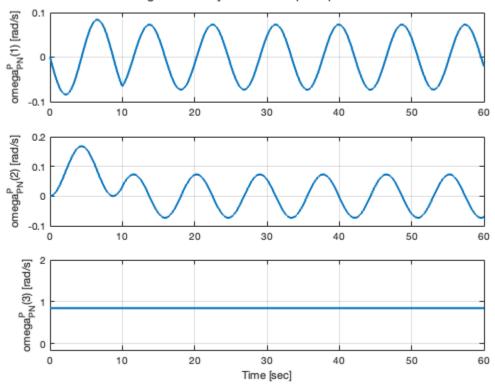
```
state0 = [r_CN_N_0; v_CN_N_0; omega_PN_P_0; euler_PN_0];
[tout_state, state_out] = ode45(@(t, state)state_eom(t, state, mass, g,
I c P, r NozzleC P), [tspan(1), tspan(2)], state0);
function state dot = state eom(time, state, mass, q, I, r NozzleC P)
    % state - [x_N, y_N, z_N, x_N_dot, y_N_dot, z_N_dot, omega_PN_P(1),
omega_PN_P(2),
    % omega_PN_P(3), alpha, beta, gamma]'
    % state_dot - [x_N_dot, y_N_dot, z_N_dot, x_N_dotdot, y_N_dotdot,
z_N_dotdot, omega_PN_P(1), omega_PN_P(2),
    % omega_PN_P(3), alpha, beta, gamma]'
    gravity_N = [0 \ 0 \ -g]';
    r_CN_N = state(1:3);
    v CN N = state(4:6);
    omega_PN_P = state(7:9);
    euler PN = state(10:12);
    r_CN_N_dot = v_CN_N;
    PN = R3(euler_PN(3))*R2(euler_PN(2))*R1(euler_PN(1));
    thrust_at_time_P = thrust(time);
    thrust at time N = PN' * thrust at time P;
    v_CN_N_dot = (thrust_at_time_N + mass*gravity_N)/mass;
    L_c_P_at_time = L_c_P(time, r_NozzleC_P);
    omega_PN_P_dot(1,1) = -1/I(1) * (I(3) - I(2)) *
omega_PN_P(2)*omega_PN_P(3) + L_c_P_at_time(1)/I(1);
    omega_PN_P_dot(2,1) = -1/I(2) * (I(1) - I(3)) *
omega_PN_P(1)*omega_PN_P(3) + L_c_P_at_time(2)/I(2);
    omega_PN_P_dot(3,1) = -1/I(3) * (I(2) - I(1)) *
omega_PN_P(1)*omega_PN_P(2) + L_c_P_at_time(3)/I(3);
    B theta = 1/\cos(\text{euler PN}(2)) .* [\cos(\text{euler PN}(3)), -\sin(\text{euler PN}(3)), 0;
                cos(euler_PN(2))*sin(euler_PN(3)),
cos(euler_PN(2))*cos(euler_PN(3)), 0;
                -sin(euler_PN(2))*cos(euler_PN(3)),
sin(euler_PN(2))*sin(euler_PN(3)), cos(euler_PN(2))];
    euler_PN_dot = B_theta * omega_PN_P;
    state_dot = [r_CN_N_dot; v_CN_N_dot; omega_PN_P_dot; euler_PN_dot];
end
```

### Part a

```
omega_PN_P_out = state_out(:,7:9);
```

```
figure()
subplot(3,1,1)
plot(tout_state, omega_PN_P_out(:,1), 'LineWidth',2)
ylabel("omega_{PN}^P(1) [rad/s]")
grid on
subplot(3,1,2)
plot(tout_state, omega_PN_P_out(:,2), 'LineWidth',2)
ylabel("omega_{PN}^P(2) [rad/s]")
grid on
subplot(3,1,3)
plot(tout_state, omega_PN_P_out(:,3), 'LineWidth',2)
ylabel("omega_{PN}^P(3) [rad/s]")
xlabel("Time [sec]")
grid on
sgtitle("Angular velocity of rocket in principal frame")
```

#### Angular velocity of rocket in principal frame



## Part b

```
v_CN_N_out = state_out(:, 4:6);
for i = 1:length(tout_state)
    euler_PN = state_out(i, 10:12);
    PN = R3(euler_PN(3))*R2(euler_PN(2))*R1(euler_PN(1));
    v_CN_P_out(i,:) = (PN * v_CN_N_out(i,:)')';
```

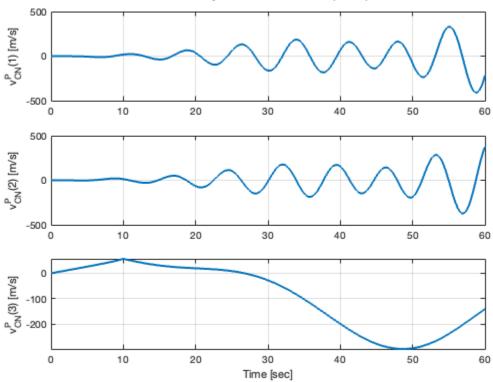
#### end

```
figure()
subplot(3,1,1)
plot(tout_state, v_CN_P_out(:,1), 'LineWidth',2)
ylabel("v_{CN}^P(1) [m/s]")
grid on

subplot(3,1,2)
plot(tout_state, v_CN_P_out(:,2), 'LineWidth',2)
ylabel("v_{CN}^P(2) [m/s]")
grid on

subplot(3,1,3)
plot(tout_state, v_CN_P_out(:,3), 'LineWidth',2)
ylabel("v_{CN}^P(3) [m/s]")
xlabel("Time [sec]")
grid on
sgtitle("Translational velocity of rocket's CoM in principal frame")
```



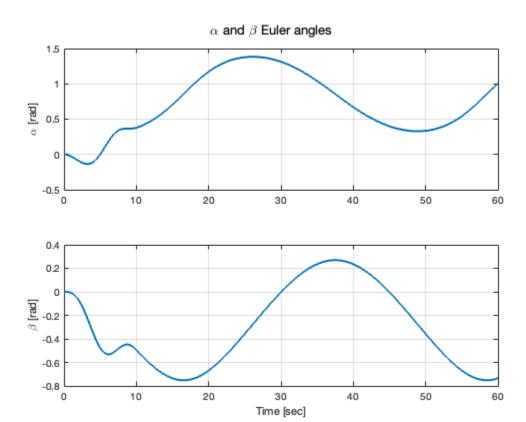


# Part c

```
euler_PN_out = state_out(:, 10:12);
figure()
subplot(2,1,1)
```

```
plot(tout_state, euler_PN_out(:,1), 'LineWidth',2)
ylabel("\alpha [rad]")
grid on

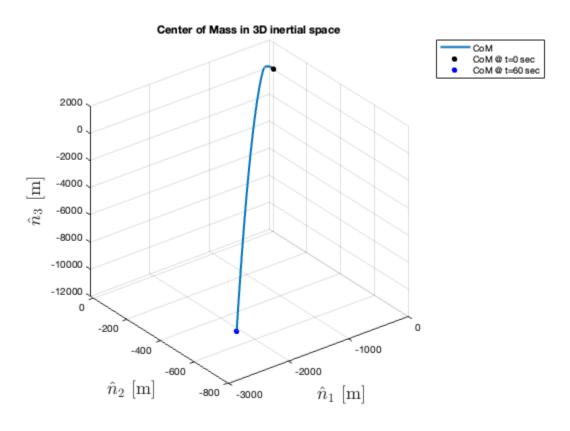
subplot(2,1,2)
plot(tout_state, euler_PN_out(:,2), 'LineWidth',2)
xlabel("Time [sec]")
ylabel("\beta [rad]")
grid on
sgtitle("\alpha and \beta Euler angles")
```

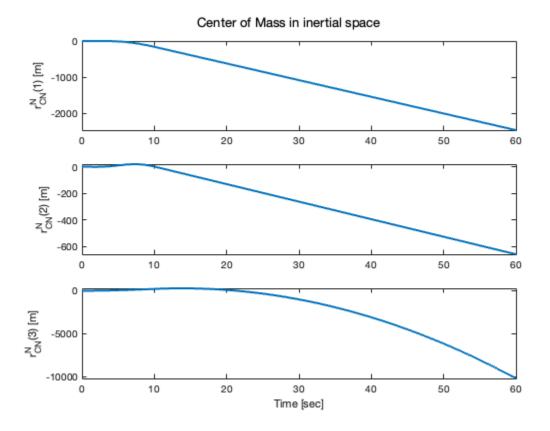


## Part d

```
figure()
plot3(state_out(:,1), state_out(:,2), state_out(:,3), 'LineWidth',2)
xlabel('$$\hat{n}_{1}$$ [m]','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{n}_{2}$$ [m]','Interpreter','Latex', 'FontSize',18)
zlabel('$$\hat{n}_{3}$$ [m]','Interpreter','Latex', 'FontSize',18)
hold on
scatter3(state_out(1,1), state_out(1,2), state_out(1,3), 'filled', 'black')
scatter3(state_out(end,1), state_out(end,2), state_out(end,3), 'filled',
'blue')
legend("CoM", "CoM @ t=0 sec", "CoM @ t=60 sec")
grid on
title("Center of Mass in 3D inertial space")
```

```
figure()
subplot(3,1,1)
plot(tout_state, state_out(:,1), 'LineWidth',2)
ylabel("r_{CN}^N(1) [m]")
subplot(3,1,2)
plot(tout_state, state_out(:,2), 'LineWidth',2)
ylabel("r_{CN}^N(2) [m]")
subplot(3,1,3)
plot(tout_state, state_out(:,3), 'LineWidth',2)
ylabel("r_{CN}^N(3) [m]")
xlabel("Time [sec]")
sgtitle("Center of Mass in inertial space")
```



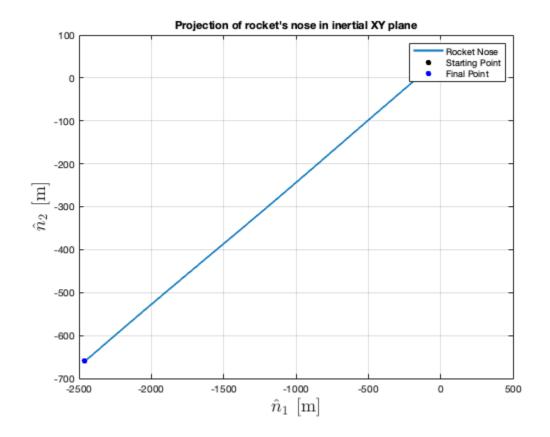


## Part e

```
euler_PN = state_out(:,10:12);
r_CN_N = state_out(:,1:3);

for i = 1:length(tout_state)
    PN = R3(euler_PN(i, 3))*R2(euler_PN(i, 2))*R1(euler_PN(i, 1));
    r_NoseN_N(i,:) = (r_CN_N(i,:)' + PN' * r_NoseC_P)';
end

figure()
plot(r_NoseN_N(:,1), r_NoseN_N(:,2), 'LineWidth',2)
hold on
scatter(r_NoseN_N(1,1), r_NoseN_N(1,2), 'filled', 'black')
scatter(r_NoseN_N(end,1), r_NoseN_N(end,2), 'filled', 'blue')
legend("Rocket Nose", "Starting Point", "Final Point")
xlabel('$$\hat{n}_{1}$$ [m]','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{n}_{2}$$ [m]','Interpreter','Latex', 'FontSize',18)
title("Projection of rocket's nose in inertial XY plane")
grid on
```



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