①

ASEN 5050
Fall 2024
Jash Bhalavat

Midterm 2

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work and I have followed all the exam policies.

J.C.B. Jash Bhalavat

11/16/2024, 9:45AM MST 11/16/2024, 3:47 PM MST

Problem 1 → Given: s/c orbiting moon, $r_{p,i} = 1850$ km, $r_{a,i} = 5400$ km, $r_i = 3500$ km, apoapsis → periapsis, $r_{p,f} = 3000$ km, $r_{a,f} = 5400$ km, s/c implemented min. magnitude maneuver

Assumptions: 2 Body Problem assumptions, $\mu = G(m_{s/c} + M_{moon}) \rightarrow m_{s/c} << M_{moon}$

$\therefore \mu \approx \mu_{moon} = 4.902799 \times 10^3 \ km^3/s^2$, $\vec{h}$ direction does not change

Orbit i → a) $a_i = \frac{1}{2}(r_{p,i} + r_{a,i}) = 3625$ km

$e_i = (r_{a,i} - r_{p,i})/(r_{a,i} + r_{p,i}) = 0.4897$

$p_i = a_i(1 - e_i^2) = 2.7559 \times 10^3$ km

$h_i = \sqrt{p_i \cdot \mu_{moon}} = 3.6758 \times 10^3 \ km^2/s$

Time immediately before maneuver → t_1 → where $r_1 = 3500$ km

$\theta_1^* = \pm a\cos\left(\frac{1}{e_i}\left(\frac{p_i}{r_1} - 1\right)\right) = \pm 115.7347°$

$\because$ s/c going from apoapsis to periapsis → $\theta^* < 0$ → $\therefore \theta_1^* = -115.7347°$

$v_{r,1} = \frac{\mu_{moon}}{h_i} e_i \sin(\theta_1^*) = -0.5883$ km/s

$v_{\theta,1} = \frac{\mu_{moon}}{h_i}(1 + e_i\cos(\theta_1^*)) = 1.0502$ km/s

$\rightarrow \vec{v}_1 = -0.5883 \frac{km}{s}\hat{r} + 1.0502 \frac{km}{s}\hat{\theta} + 0\hat{h}$

Orbit f → $a_f = \frac{1}{2}(r_{p,f} + r_{a,f}) = 4200$ km

$e_f = (r_{a,f} - r_{p,f})/(r_{a,f} + r_{p,f}) = 0.2857$

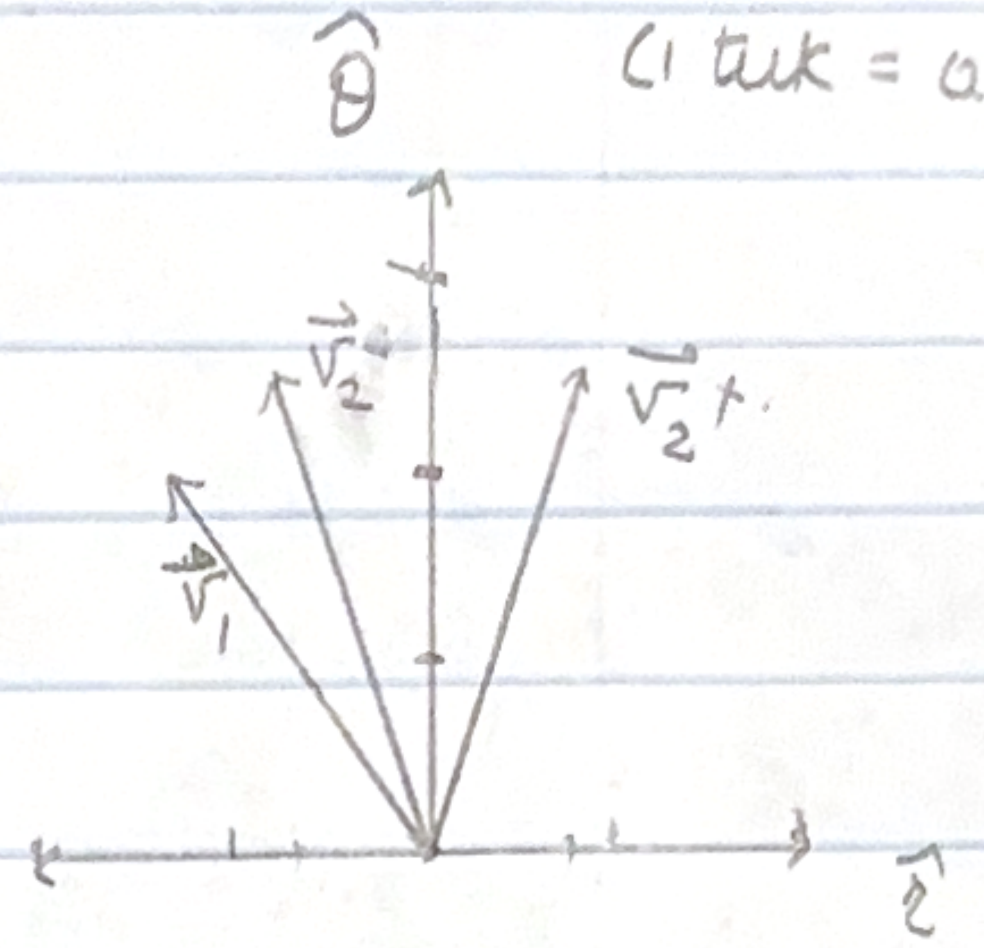
$p_f = a_f(1 - e_f^2) = 3.8571 \times 10^3$ km

$h_f = \sqrt{p_f \cdot \mu_{moon}} = 4.3487 \times 10^3 \ km^2/s$

Time immediately after maneuver t_2 → where $r_2 = r_1 = 3500$ km

$\theta_2^* = \pm a\cos\left(\frac{1}{e_f}\left(\frac{p_f}{r_2} - 1\right)\right) = \pm 69.0752°$

Let's draw velocity diagram to help choose θ_2^*



$\because v_\theta > 0$

$v_{\theta,2} = \frac{\mu_{moon}}{h_f}(1 + e_f \cos\theta_2^*) = 1.2425$ km/s

$v_{r,2} = \pm \frac{\mu_{moon}}{h_f}(e_f \sin\theta_2^*) = \pm 0.3009$ km/s

As seen in the diagram, $\Delta\vec{v} \left(= \vec{v}_2 - \vec{v}_1\right)$ would be minimal if $v_{r,2} < 0$. Since it is given that the s/c chooses the maneuver with the minimum magnitude, $v_{r,2} < 0 \rightarrow \theta_2^* < 0$

$\therefore \theta_2^* = -69.0752°$

$\therefore \vec{v}_2 = -0.3009 \frac{km}{s}\hat{r} + 1.2425 \frac{km}{s}\hat{\theta} + 0\hat{h}$

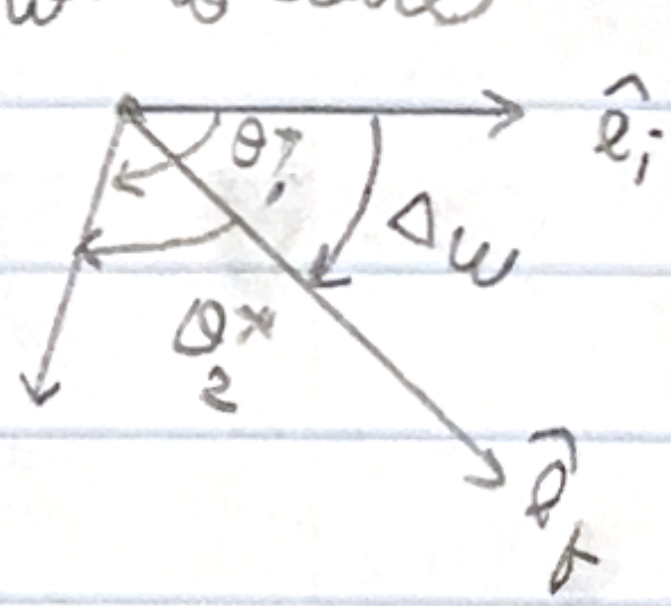
$\therefore \Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = 0.2847 \frac{km}{s}\hat{r} + 0.1922 \frac{km}{s}\hat{\theta} + 0\hat{h}$

$\therefore |\Delta\vec{v}| = \boxed{\Delta v = 0.3458 \text{ km/s}}$

b) Maneuver is coplanar ($\vec{h}$ does not change direction). $\therefore$ change in true anomaly is equal to change in ω. The representative diagram below says that: $\theta_2^* + \Delta\omega = \theta_1^*$

$\therefore \Delta\omega = \theta_1^* - \theta_2^* = \boxed{-46.6596°}$

(not to scale)



where the negative indicates that the periapsis shifted in the clockwise direction (if $\vec{h}$ is out of the page). The magnitude of $\Delta\omega$ is $\boxed{46.6596°}$

c) No, it's not a guarantee that if a $(1.1)\Delta v$ burn is executed, the orbit will have an $r_p \geq 3000$ km because the direction of Δv is also important. If the $1.1 \times \Delta v$ is executed exactly in the opposite direction as $\vec{v}_1$, it will reduce the size of the orbit. That's just one example of how the direction of Δv is also important to increase orbit size.

ASEN 5050
Fall 2024
Jash Bhalavat

midterm 2

Problem 2 →

Given: $m_i = 2500$ km, $m_{prop} = 500$ kg, $I_{sp} = 300$ s, moon X coplanar orbit w/s/c, Prograde orbit, circular orbit, $\mu_X = 4000\ km^3/s^2$

Assumptions: 2 Body problem, $m_{s/c} \ll m_{jupiter}$, $m_{s/c} \ll m_X$

$\therefore$ Jupiter, s/c 2BP → $\mu = \mu_{jupiter} = 1.268\times10^8\ km^3/s^2$

$\therefore$ s/c, X 2BP → $\mu = \mu_X = 4000\ km^3/s^2$

a) $\vec{v}_{in} = 2.2789\ \frac{km}{s}\ \hat{r} + 5.8841\ \frac{km}{s}\ \hat{\theta} + 0\hat{h} \rightarrow |\vec{v}_{in}| = 6.3100$ km/s

$a_{in} = 1.8\times10^6$ km

$\mathcal{E}_{in} = -\mu_{jupiter}/(2a_{in}) = -35.2222\ km^2/s^2$

$\mathcal{E}_{in} = \frac{v_{in}^2}{2} - \frac{\mu_{jupiter}}{r_{in}} \rightarrow r_{in} = (\mu_{jupiter})/(v_{in}^2/2 - \mathcal{E}_{in}) = \boxed{2.3\times10^6\ km = r_{in}} = a_X$

b) For X, Jupiter 2 BP → assume → $m_X \ll m_{jupiter}$ → $\mu = \mu_{jupiter} = 1.268e8\ \frac{km^3}{s^2}$

$\therefore \vec{v}_X = \sqrt{\frac{\mu_{jupiter}}{a_X}}\ \hat{\theta} = 7.4250\ \hat{\theta}$ km/s

$\vec{v}_{\infty,in} = \vec{v}_{in} - \vec{v}_X = \boxed{2.2789\ \hat{r} - 1.5409\ \hat{\theta} + 0\hat{h}\ km/s = \vec{v}_{\infty,in}}$

c) $v_{out} = 7.6759$ km/s, $\phi_{fpa,out} = 20.91°$

$v_{r,out} = v_{out}\sin(\phi_{fpa,out}) = 2.7395$ km/s

$v_{\theta,out} = v_{out}\cos(\phi_{fpa,out}) = 7.1704$ km/s

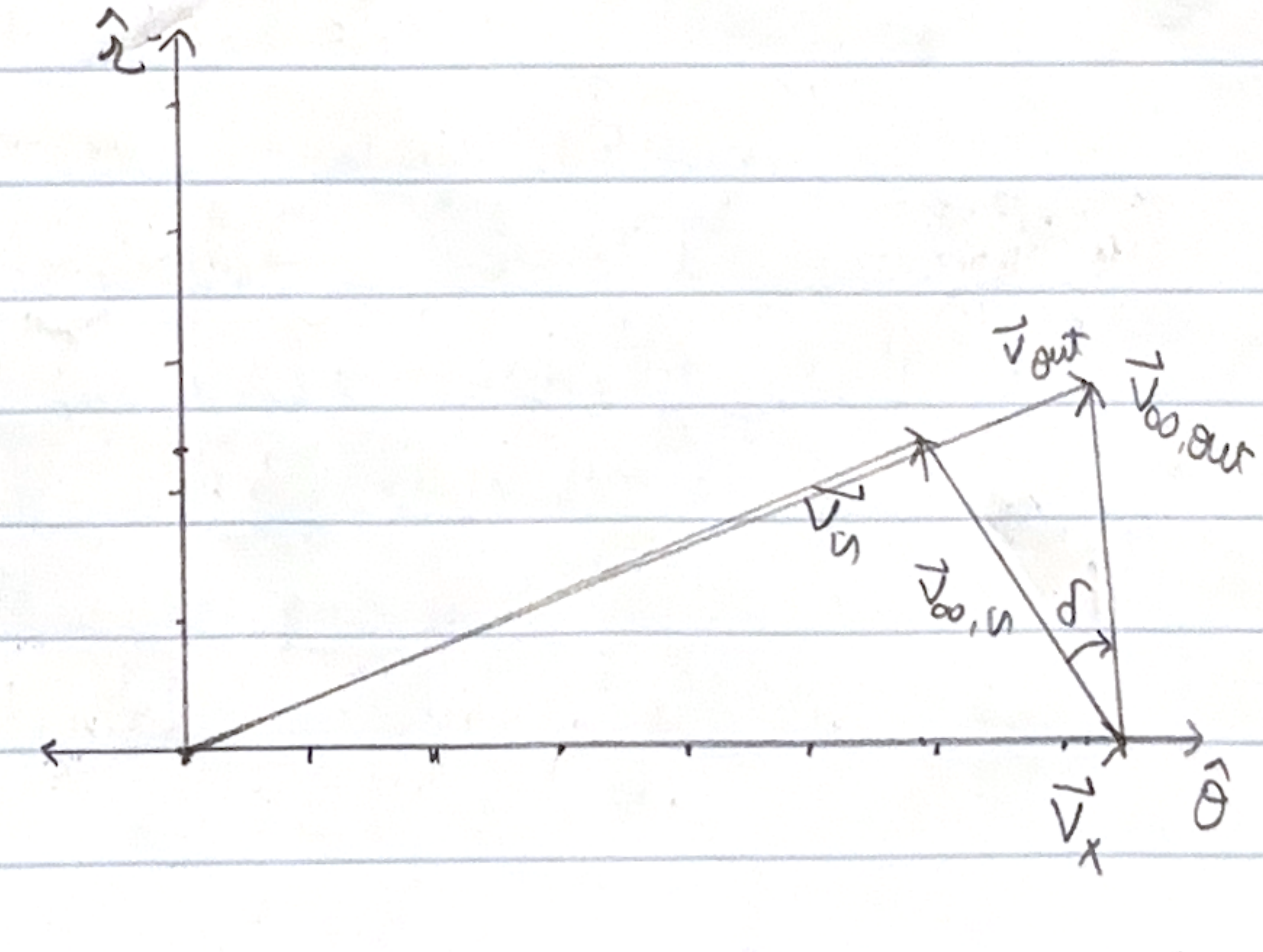
→ $\boxed{\vec{v}_{out} = 2.7395\ \hat{r} + 7.1704\ \hat{\theta} + 0\hat{h}\ \frac{km}{s}}$

d) $v_{\infty,out} = \vec{v}_{out} - \vec{v}_X = \boxed{2.7395\ \hat{r} - 0.2546\ \hat{\theta} + 0\hat{h}\ km/s = \vec{v}_{\infty,out}}$

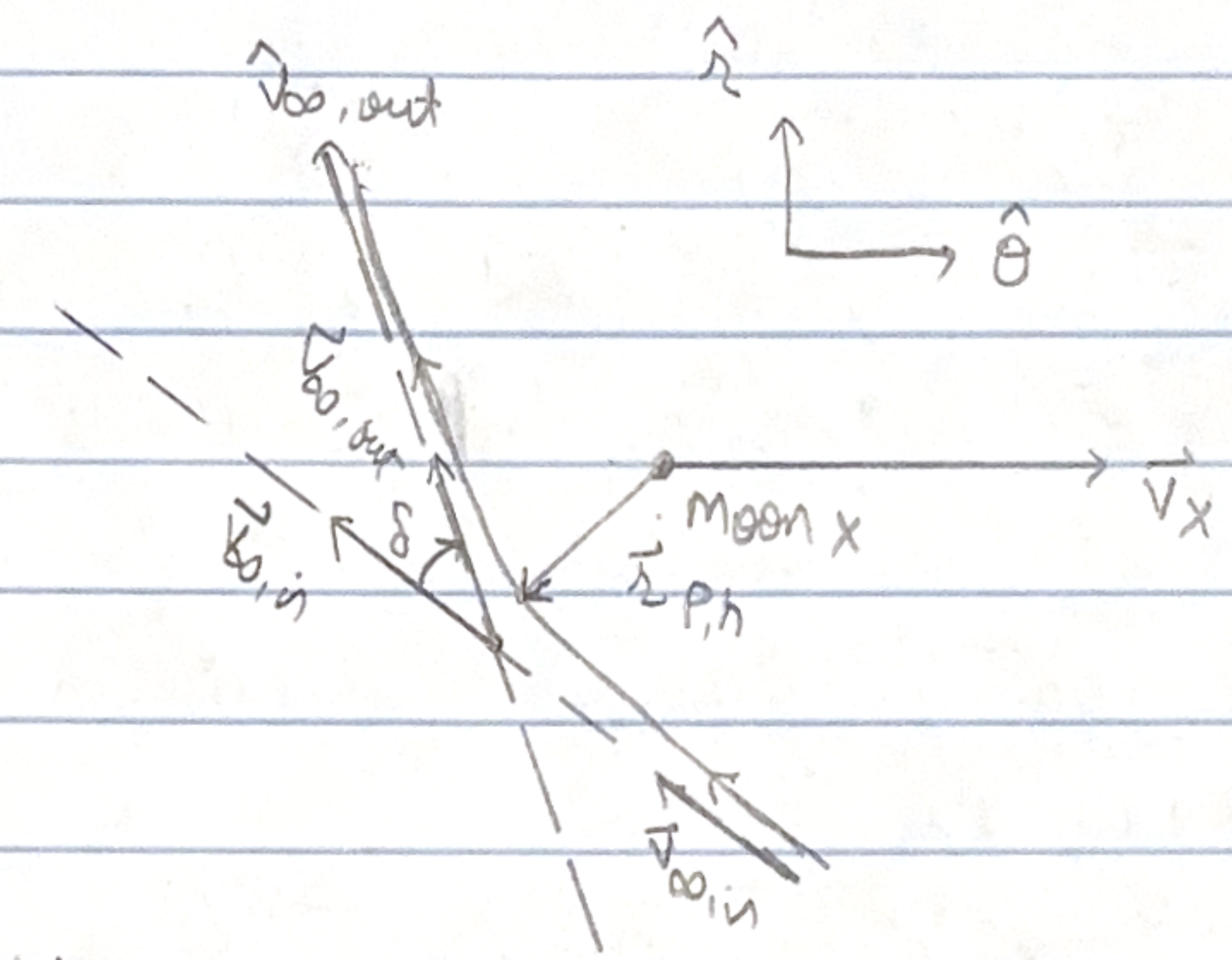
e)

(1 tick = 1 km/s)

① $\vec{v}_{in}$, $\vec{v}_{out}$
② $\vec{v}_{\infty,in}$, $\vec{v}_{\infty,out}$
③ $\vec{v}_X$
④ δ
⑤ $\hat{r}\hat{\theta}$



f]



g] $|\vec{V}_{in}| = 6.31$ km/s $= V_{in}$, $|\vec{V}_{out}| = 7.6759$ km/s $= V_{out}$

$\because V_{out} > V_{in}$, s/c orbit energy has to increase $\rightarrow \therefore$ s/c has to pass behind moon X during the flyby.

h] $V_{\infty,in} = 2.7509$ km/s $\neq V_{\infty,out} = 2.7513$ km/s

(It is given in the problem statement that this is acceptable) For the rest of this problem, using $|V_\infty| = 2.75$ km/s $= V_\infty$

$\mathcal{E}_h = V_\infty^2/2 = 3.7813$ km²/s²

$a_h = -\mu_X/(2\mathcal{E}_h) = -528.9256$ km

$\Delta\vec{V}_{eq} = \vec{V}_{out} - \vec{V}_{in} = 0.4606\,\hat{r} + 1.2863\,\hat{\theta} + 0\,\hat{h}$ km/s $\rightarrow \Delta V_{eq} = 1.3663$ km/s

$\Delta V_{eq} = 2V_\infty \sin(\delta/2) \rightarrow \delta = \sin^{-1}\left(\frac{\Delta V_{eq}}{2V_\infty}\right)\cdot 2 = 28.7673°$

$\delta = 2\sin^{-1}\left(\frac{1}{e_h}\right) \rightarrow e_h = \frac{1}{\sin(\delta/2)} = 4.0255$

$r_{p,h} = a_h(1-e_h) = \boxed{1.6003\times 10^3 \text{ km} = r_{p,h}}$

i] Calculated in part h, $\rightarrow \boxed{\Delta V_{eq} = 1.3663 \text{ km/s}} = \Delta V$ for an impulsive maneuver

j] $m_{prop,needed} = m_i\left(1 - e^{\frac{-\Delta V}{g_0 I_{sp}}}\right)$ ← convert ΔV to m/s $\rightarrow \Delta V = 1000 \cdot \Delta V_{eq}$ $= 1366.3$ m/s

$= 928.4773$ kg $> m_{prop}$ ($= 500$ kg)

$\therefore$ The s/c couldn't have performed this impulsive maneuver

$\therefore$ Theory I is more probable

③

ASEN 5050
Fall 2024
Josh Bhalavat

midterm 2

Problem 3 → Given: Interplanetary S/C, Jupiter flyby, $a_{jupiter} = 5.202603191$ AU,
S/C orbit coplanar with Jupiter orbit, AU = 149597870.7 km
$\vec{V}_{in} = 3.2476\,\hat{r} + 13.1175\,\hat{\theta}$ km/s, $\vec{V}_{out} = 3.3426\,\hat{r} + 13.7356\,\hat{\theta}$ km/s

Assumptions: 2 Body Problem, $e_{jupiter} = 0$

S/C sun 2BP → $m_{S/C} \ll m_{sun} \rightarrow \mu = \mu_{sun} = 1.32712428 \times 10^{11}$ km³/s²

S/C jupiter 2BP → $m_{S/C} \ll m_{jupiter} \rightarrow \mu = \mu_{jupiter} = 1.268 \times 10^{8}$ km³/s²

jupiter sun 2BP → $m_{jupiter} \ll m_{sun} \rightarrow \mu = \mu_{sun} = 1.32712428 \times 10^{11}$ km³/s²

$$\vec{V}_{jupiter} = \sqrt{\frac{\mu_{sun}}{a_{jupiter}}}\,\hat{\theta} = 13.0582\,\hat{\theta} \text{ km/s}$$

$\vec{V}_{\infty,in} = \vec{V}_{in} - \vec{V}_{jupiter} = 3.2476\,\hat{r} + 0.0593\,\hat{\theta} + 0\,\hat{h}$ km/s

$\vec{V}_{\infty,out} = \vec{V}_{out} - \vec{V}_{jupiter} = 3.3426\,\hat{r} + 0.6774\,\hat{\theta} + 0\,\hat{h}$ km/s

$V_{\infty,in} = |\vec{V}_{\infty,in}| = 3.2481$ km/s

$V_{\infty,out} = |\vec{V}_{\infty,out}| = 3.4106$ km/s

For a natural flyby, $V_{\infty,in} = V_{\infty,out} \rightarrow \because V_{\infty,in} \neq V_{\infty,out}$, the given velocity vectors cannot correspond to a S/C before and after a natural flyby of Jupiter.

Furthermore, using the same process as problem 2 to find $r_{p,h}$

$\mathcal{E}_h = V_{\infty,in}^2/2 \rightarrow a_h = -\frac{\mu_{jupiter}}{2\mathcal{E}_h} \rightarrow \Delta\vec{V}_{eq} = \vec{V}_{out} - \vec{V}_{in} \rightarrow \delta = \sin^{-1}\left(\frac{\Delta V_{eq}}{2V_{\infty,in}}\right) \rightarrow e_h = \frac{1}{\sin(\delta/2)}$

$\rightarrow r_{p,h} = a_h(1 - e_h) = 1.1283 \times 10^{8}$ km which is $> R_{SOI,jupiter} = 4.82 \times 10^{7}$ km

this further disproves that the velocity vectors are not due to a natural jupiter flyby because the flyby r_p is outside Jupiter's sphere of influence.