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ASEN 5050
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Fall 2024

$\boxed{\text{HW 9}}$

Problem 1 →

Given: $e_{s/c} = 0$, $a_{s/c} = 10{,}000$ km, Earth orbiting, Cubesat-1 (CS1) → $x_0 = 5$ m, $\dot{x}_0 = 0 \frac{m}{s}$

CS1, S/C same orbit plane → $z_0 = \dot{z}_0 = 0$ throughout, bounded osc. motion,

$y_{max} = 15$ m

Assumptions: 2BP, $r_{s/c} \gg r_{rel}$ where r_{rel} is relative distance between CS1 & S/C,

$\mu = G(M_{earth} + M_{s/c} + M_{CS1}) \rightarrow M_{CS1} \ll M_{s/c} \ll M_{earth} \rightarrow \mu = G M_{earth} = 3.986004415$ km^3/s^2

a) ∵ CS1 & S/C are in same orbit plane → $z(t) = \dot{z}(t) = 0$

∵ Bounded, oscillatory motion → suppress drift terms from CW eqn's

$x(t) = 4x_0 + \frac{2}{n}\dot{y}_0 + \cancel{\frac{\dot{x}_0}{n}\sin(nt)}^{0} - (3x_0 + \frac{2}{n}\dot{y}_0)\cos(nt)$

$= 4(5) + \frac{2}{n}\dot{y}_0 - (3(5) + \frac{2}{n}\dot{y}_0)\cos(nt)$ — (1)

$y(t) = y_0 - \cancel{\frac{2}{n}\dot{x}_0}^{0} - 3(2nx_0 + \dot{y}_0)t + 2(3x_0 + \frac{2}{n}\dot{y}_0)\sin(nt) + \cancel{\frac{2}{n}\dot{x}_0\cos(nt)}^{0}$

$= y_0 - 3(2n(5) + \dot{y}_0)t + 2(3(5) + \frac{2}{n}\dot{y}_0)\sin(nt)$ — (2)

Drift term → $2n(5) + \dot{y}_0 = 0 \rightarrow \dot{y}_0 = -10n$

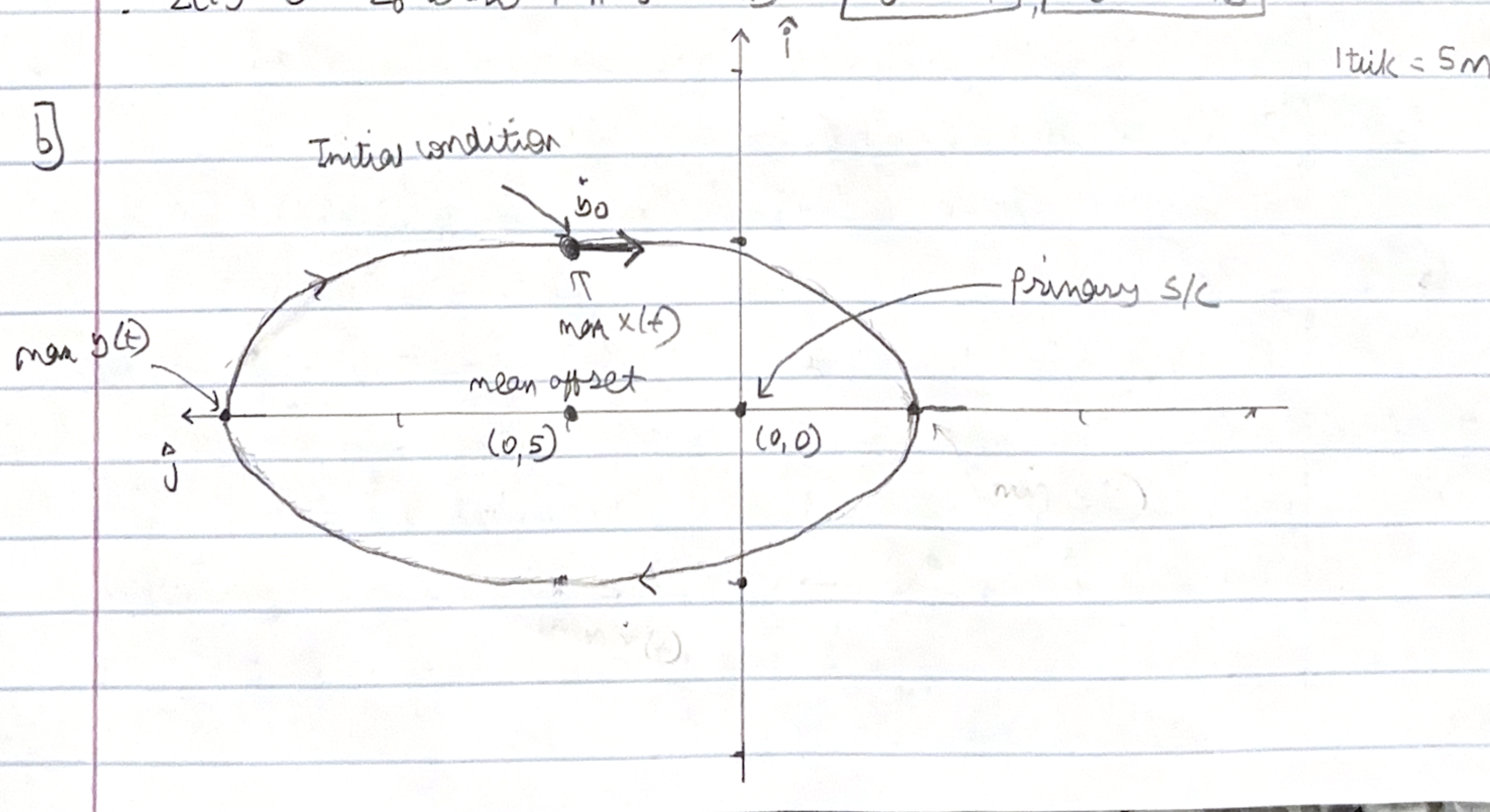
$n = \sqrt{\frac{\mu}{a^3}} = 6.3135 \times 10^{-4} \frac{rad}{s}$ → $\boxed{\dot{y}_0 = -6.3135 \times 10^{-3} \text{ m/s}}$ → Sub. into (2)

$y(t) = y_0 - 10\sin(nt)$ → negative term has to be min. for $y(t)$ to be max.

$y_{max} = 15$ m → $y_0 - \min(10\sin(nt)) = 15 \rightarrow \min(\sin(nt)) = -1 \rightarrow y_0 + 10 = 15 \rightarrow \boxed{y_0 = 5 \text{ m}}$

∵ $z(t) = 0 = z_0\cos(nt) + \frac{1}{n}\dot{z}_0\sin(nt) \rightarrow \boxed{z_0 = 0 \text{ m}}$, $\boxed{\dot{z}_0 = 0 \text{ m/s}}$

b)



$@ t_0 = 0$

Problem 2 → Given: $e_{s/c} = 0$, $a_{s/c} = 10{,}000$ km, CS2 → $x_0 = 2$ m, $y_0 = 2$ m, $z_0 = 0$ m, $\dot{x}_0 = -0.03$ m/s,

$\dot{y}_0 = +0.01$ m/s, $\dot{z}_0 = 0.05$ m/s, $@ t_1 = P/2 \rightarrow x_1 = -2$ m, $y_1 = 2$ m, $z_1 = 0$ m

Assumptions: 2BP, Target vehicle is circular (so CW applies), $R_{s/c} \gg r_{rel}$

$\mu = G(M_E + M_{s/c} + M_{CS2}) \rightarrow M_{CS2} \ll M_{s/c} \ll M_E \rightarrow \mu = 3.986004415 \times 10^5$ km^3/s^2

Final rel. vel. is 0 → $\dot{x}_1 = \dot{y}_1 = \dot{z}_1 = 0$ m/s

a) From Problem 1 → $n = 6.3135 \times 10^{-4}$ rad/s → $P = \frac{2\pi}{n} = 9.9520 \times 10^3$ s → $t_1 = P/2 = 4.976 \times 10^3$ s

$z_1 = 0\text{ m} = \frac{1}{n}\dot{z}_0^+ \sin(n \cdot \frac{P}{2}) + \cancel{z_0\cos(nt)}^{0} = \frac{1}{n}\dot{z}_0^+ \sin(\pi) \rightarrow \dot{z}_0^+ = 0$ m/s

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 4-3\cos(nt) & 0 \\ 6(\sin(nt)-nt) & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \frac{1}{n}\sin(nt) & \frac{2}{n}(1-\cos(nt)) \\ \frac{2}{n}(\cos(nt)-1) & \frac{4}{n}\sin(nt) - 3t \end{bmatrix} \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix}$$

$\vec{r}_t = \phi_{rr}\vec{r}_0 + \phi_{rv}\vec{v}_0 \rightarrow \vec{r}_1 = \phi_{rr}(t_1)\vec{r}_0 + \phi_{rv}(t_1)\vec{v}_0^+$

$\vec{v}_0^+ = \phi_{rv}^{-1}(t_1)\left[\vec{r}_1 - \phi_{rr}(t_1)\vec{r}_0\right]$

where $\vec{v}_0^+ = [\dot{x}_0^+, \dot{y}_0^+]^T$, $\vec{r}_1 = [x_1, y_1]^T$, $\vec{r}_0 = [x_0, y_0]^T$, using matlab

$\vec{v}_0^+ = \begin{bmatrix} \dot{x}_0^+ \\ \dot{y}_0^+ \end{bmatrix} = \begin{bmatrix} 0 \\ -0.0025 \end{bmatrix}$ m/s → $\boxed{\text{Rel. vel. } @ t_0 = \begin{bmatrix} \dot{x}_0^+ \\ \dot{y}_0^+ \\ \dot{z}_0^+ \end{bmatrix} = \begin{bmatrix} 0 \\ -0.0025 \\ 0 \end{bmatrix} \text{ m/s}}$

$\Delta\vec{v}_1 = [\dot{x}_0^+ - \dot{x}_0,\ \dot{y}_0^+ - \dot{y}_0,\ \dot{z}_0^+ - \dot{z}_0]$ m/s $= [0.03, 0.0125, -0.05]$ m/s

$\boxed{|\Delta\vec{v}_1| = 0.0596 \text{ m/s}}$

b) $x(t) = 4x_0 + \frac{2}{n}\dot{y}_0 + \frac{\dot{x}_0}{n}\sin(nt) - 3x_0\cos(nt) - \frac{2}{n}\dot{y}_0\cos(nt)$

$\dot{x}(t) = \dot{x}_0\cos(nt) + 3x_0 n\sin(nt) + 2\dot{y}_0\sin(nt)$

$y(t) = y_0 - \frac{2}{n}\dot{x}_0 - 6nx_0 t - 3\dot{y}_0 t + 6x_0\sin(nt) + \frac{4}{n}\dot{y}_0\sin(nt) + \frac{2}{n}\dot{x}_0\cos(nt)$

$\dot{y}(t) = -6nx_0 - 3\dot{y}_0 + 6x_0 n\cos(nt) + 4\dot{y}_0\cos(nt) - 2\dot{x}_0\sin(nt)$

$z(t) = \frac{1}{n}\dot{z}_0\sin(nt) + z_0\cos(nt) \rightarrow \dot{z}(t) = \dot{z}_0\cos(nt) - z_0 n\sin(nt)$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 3n\sin(nt) & 0 \\ 6n(\cos(nt)-1) & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \cos(nt) & 2\sin(nt) \\ -2\sin(nt) & 4\cos(nt)-3 \end{bmatrix} \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix}$$

$\vec{v}_t = \phi_{vr}\vec{r}_0 + \phi_{vv}\vec{v}_0 \rightarrow \vec{v}_1^- = \phi_{vr}(t_1)\vec{r}_0 + \phi_{vv}(t_1)\vec{v}_0^+$ ← from part a

$\boxed{\vec{v}_1^- = \begin{bmatrix} \dot{x}_1^- \\ \dot{y}_1^- \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0025 \end{bmatrix} \text{ m/s}}$, $\dot{z}_1^- = \cancel{\dot{z}_0^+\cos(nt)}^{0} - \cancel{z_0 n\sin(nt)}^{0} = \boxed{0 \text{ m/s} = \dot{z}_1^-}$

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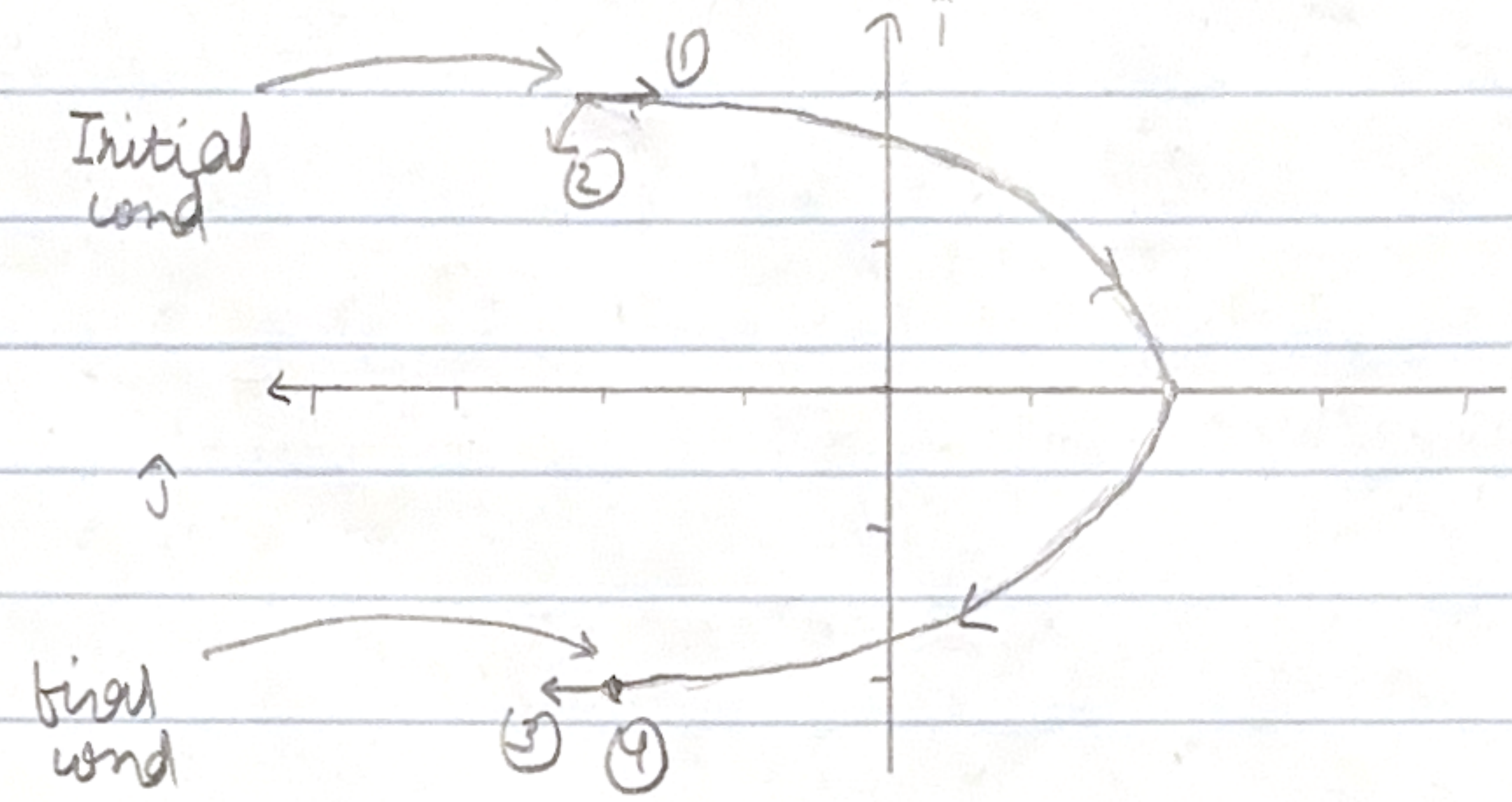
$\Delta \vec{V}_2 = [\dot{x}_1 - \dot{x}_1^-, \dot{y}_1 - \dot{y}_1^-, \dot{z}_1 - \dot{z}_1^-]$ m/s $= [0, 0.0025, 0]$ m/s $\rightarrow |\Delta \vec{V}_2| = 0.0025 \frac{m}{s}$

c)

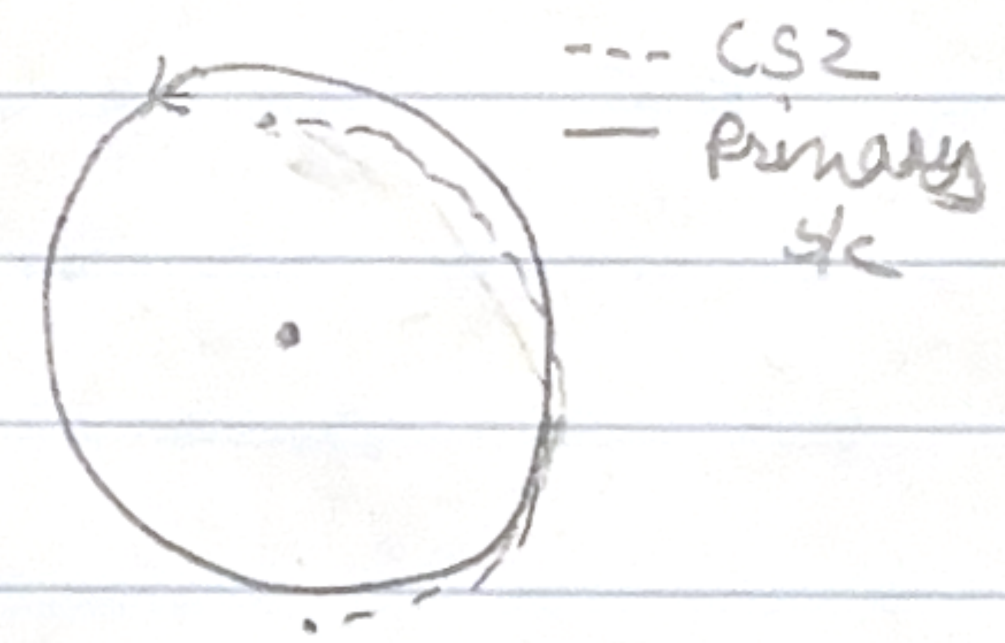


① Velocity post $\Delta \vec{V}_1$
② Velocity pre $\Delta \vec{V}_1$
③ Velocity pre $\Delta \vec{V}_2$
④ Velocity post $\Delta \vec{V}_2$ (it's a dot because vector is $[0,0]^T$)

Earth centered → same plane as primary s/c. Very small e orbit with perigee 2 m closer to earth than primary s/c and apogee 2 m further away like this →

It never meets the primary s/c, and the closest it gets is 2 m.

It performs ~~half~~ of a bounded oscillatory motion during the transfer.



Problem 3 → Given: Orbit around mars

Assumptions: 2BP, $\mu = G(m_{s/c} + M_{mars}) \rightarrow m_{s/c} \ll M_{mars} \rightarrow \mu = GM_{mars} = 4.305 \times 10^4 \frac{km^3}{s^2}$

$P_{rot,mars} = 1.02595675 \text{ days} \left(\frac{86400 \text{ sec}}{1 \text{ day}}\right) = 8.8643 \times 10^4 s$

$W_m = \frac{2\pi \text{ rad}}{8.8643 \times 10^4 s} = 7.0882 \times 10^{-5} \frac{rad}{s}$

$\lambda_\Omega = -W_m P_{s/c}$, visually $\lambda_\Omega = -150° \rightarrow P_{s/c} = 3.6934 \times 10^4 s = 2\pi \sqrt{\frac{a^3}{\mu}}$

$\boxed{a = 1.1415 \times 10^4 \text{ km}}$ – exact value

e – $\boxed{\text{likely between 0 and 1}}$ because uneven distribution on top and bottom of equator. Also, $e \neq 1$ because s/c returns to an ascending node

likely $\boxed{\text{high}}$ because s/c goes through the top (periapsis) very quickly and very slowly moves through apoapsis

i – $\boxed{30^\circ}$ – max. latitude

Ω – $\boxed{\text{cannot be deduced}}$ because $\hat{x}$ axis is unknown

ω – $\boxed{[0^\circ, 180^\circ] - \sim 90^\circ}$ because s/c moves quickly over the top half, so periapsis is $\boxed{\text{in first quadrant}}$

$\nu_{\Omega,0}$ – $\boxed{-60^\circ}$ – initial position, can see visually