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```
clear; clc; close all;

% ASEN 6060 - HW 4, Problem 1
% Spring 2025
% Jash Bhalavat
```

Constants

```
G = 6.67408 * 10^-11; % m3/(kgs2)
G = G / (10^9); % km3/(kgs2)

% Earth
mu_earth = 398600.435507; % km3/s2
a_earth = 149598023; % km
e_earth = 0.016708617;
mass_earth = mu_earth / G; % kg

% Moon
mu_moon = 4902.800118; % km3/s2
a_moon = 384400; % km
e_moon = 0.05490;
mass_moon = mu_moon / G; % kg

% Earth-Moon system
mass_ratio_em = mass_moon / (mass_earth + mass_moon);
m_star_em = mass_earth + mass_moon;
l_star_em = a_moon;
t_star_em = sqrt(l_star_em^3/(G * m_star_em));
mu = mass_ratio_em;
```

Part a - i)

```
% Get L2 Point
% Earth Moon system equilibrium points
```

```
[em_eq_pts, em_eq_validity] = all_eq_points(mu);

% Only looking at L2 eq point planar oscillatory modes
l2_pos = [em_eq_pts(2,:), 0];

l2_in_plane_modes = in_plane_modes(mu, l2_pos);

oscillatory_eval = l2_in_plane_modes(3);
uxx_l2 = u_xx(mu, l2_pos);
uxy_l2 = u_xy(mu, l2_pos);
uyy_l2 = u_yy(mu, l2_pos);
U_star_XX = [uxx_l2, uxy_l2; uxy_l2, uyy_l2];
Omega = [0 2; -2 0];

A2D = [zeros(2), eye(2); U_star_XX, Omega];

[V, D] = eig(A2D);

oscillatory_evec = real(V(:,3));

oscillatory_pos_mag = norm([oscillatory_evec(1), oscillatory_evec(2)]);

pos_mag_req = 0.0001;

oscillatory_mag_factor = pos_mag_req / oscillatory_pos_mag;

oscillatory_ic = oscillatory_evec .* oscillatory_mag_factor;

% Time is one period
t = linspace(0, 2*pi/imag(oscillatory_eval), 1000);

xi_0 = oscillatory_ic(1);
xi_dot_0 = oscillatory_ic(3);
eta_0 = oscillatory_ic(2);
eta_dot_0 = oscillatory_ic(4);
x0 = [l2_pos(1) + xi_0; l2_pos(2) + eta_0; 0; xi_dot_0; eta_dot_0; 0];

TOL = 5e-14;

% Set options for ode113
options = odeset('RelTol', TOL, 'AbsTol', TOL);
[tout, xout] = ode113(@(t, state)CR3BP(state, mu), [0 t(end)], x0, options);

V0 = [x0; t(end)];

V_soln = gen_3d_periodic_orbit_single_shooting(V0, mu, false);

[tout_corrected, xout_corrected] = ode113(@(t, state)CR3BP(state, mu), [0,
V_soln(end)], V_soln(1:6), options);

% V_family_a1 = test_psal_cont(V_soln, mu);
% save("V_family_a1.mat", "V_family_a1")
load("V_family_a1.mat")
```

```
p1_pos = [-mu, 0, 0];
p2_pos = [1-mu, 0, 0];

figure()
scatter(l2_pos(1), l2_pos(2), 'filled', 'red')
hold on
scatter(p1_pos(1), p1_pos(2), 'filled', 'blue')
scatter(p2_pos(1), p2_pos(2), 'filled', 'black')

for i = 1:50:size(V_family_a1, 2)
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0, V_family_a1(7,i)],
V_family_a1(1:6,i), options);
    plot(xout(:,1), xout(:,2), 'LineWidth',2)
end
hold off
legend("L2", "Earth", "Moon")
xlabel('$$\hat{x}$$','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{y}$$','Interpreter','Latex', 'FontSize',18)
grid on
axis equal
title("L2 Lyapunov Orbit Family")
```

Part a - ii)

```
x0 = [1.180462, 0, -0.0209998, 0, -0.158363, 0]';
T = 3.411921;
V0 = [x0; T];

V_soln = gen_3d_periodic_orbit_single_shooting(V0, mu, true);

[tout_corrected, xout_corrected] = ode113(@(t, state)CR3BP(state, mu), [0,
V_soln(end)], V_soln(1:6), options);

% V_family_a2 = test_psal_cont(V_soln, mu);
% save("V_family_a2.mat", "V_family_a2")
load("V_family_a2.mat")
V_family_a2_south = V_family_a2;
load("V_family_a2_pos_delta_s.mat")
V_family_a2_north = V_family_a2;

figure()
p1 = scatter3(l2_pos(1), l2_pos(2), l2_pos(3), 'filled', 'red');
hold on
p2 = scatter3(p2_pos(1), p2_pos(2), p2_pos(3), 'filled', 'black');

for i = 1:250:size(V_family_a2_south, 2)
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0,
V_family_a2_south(7,i)], V_family_a2_south(1:6,i), options);
    p3 = plot3(xout(:,1), xout(:,2), xout(:,3), 'LineWidth',2, 'Color',
'red');
end

for i = 1:250:size(V_family_a2_north, 2)
```

```
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0,
V_family_a2_north(7,i)], V_family_a2_north(1:6,i), options);
    p4 = plot3(xout(:,1), xout(:,2), xout(:,3), 'LineWidth',2, 'Color',
'blue');
end
hold off
p = [p1, p2, p3, p4];
legend(p, "L2", "Moon", "Southern Halo", "Northern Halo")
xlabel('$$\hat{x}$$','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{y}$$','Interpreter','Latex', 'FontSize',18)
zlabel('$$\hat{z}$$','Interpreter','Latex', 'FontSize',18)
grid on
axis equal
title("L2 Halo Orbit Family")
```

Part a - iii)

```
x0 = [1.0301513, 0,0, 0, 0.7030025,0.1552945]';
T = 4.312367;
V0 = [x0; T];

V_soln = gen_3d_periodic_orbit_single_shooting(V0, mu, false);

[tout_corrected, xout_corrected] = ode113(@(t, state)CR3BP(state, mu), [0,
V_soln(end)], V_soln(1:6), options);

V_soln = [xout_corrected(284,:)'; V_soln(end)];

delta_s_neg = -1e-3;
delta_s_pos = 1e-3;
% V_family_a3_neg_ds = pseudo_arc_length_continuation(V_soln, mu,
delta_s_neg);
% V_family_a3_pos_ds = pseudo_arc_length_continuation(V_soln, mu,
delta_s_pos);
load("V_family_a3_neg_ds.mat")
load("V_family_a3_pos_ds.mat")

figure()
scatter3(l2_pos(1), l2_pos(2), l2_pos(3), 'filled', 'red')
hold on
scatter3(p2_pos(1), p2_pos(2), p2_pos(3), 'filled', 'black')

for i = 1:50:size(V_family_a3_neg_ds, 2)
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0,
V_family_a3_neg_ds(7,i)], V_family_a3_neg_ds(1:6,i), options);
    plot3(xout(:,1), xout(:,2), xout(:,3), 'LineWidth',2)
end

for i = 1:50:size(V_family_a3_pos_ds, 2)
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0,
V_family_a3_pos_ds(7,i)], V_family_a3_pos_ds(1:6,i), options);
    plot3(xout(:,1), xout(:,2), xout(:,3), 'LineWidth',2)
end
```

```
hold off
legend("L2", "Moon")
xlabel('$$\hat{x}$$','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{y}$$','Interpreter','Latex', 'FontSize',18)
zlabel('$$\hat{z}$$','Interpreter','Latex', 'FontSize',18)
grid on
axis equal
title("L2 Axial Orbit Family")
% save("V_family_a3.mat", "V_family_a3")
```

Part b - L2 Halo first orbit

```
identity_row = reshape(eye(6), [36,1]);
[tout, xout] = ode113(@(t,state)CR3BP_full(state, mu), [0, V_family_a2(7,1)],
[V_family_a2(1:6,1); identity_row], options);

halo_monodromy = reshape(xout(end,7:42), [6,6])';
[V_halo, D_halo] = eig(halo_monodromy);

[s1, s2, period] = stability_index(V_family_a2_north, mu, identity_row,
options);
figure()
subplot(2,1,1)
plot(period, s1, 'LineWidth', 2)
hold on
scatter(period(1), s1(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S1")
legend("S1", "Starting Point", "Critical Values")
hold off
subplot(2,1,2)
plot(period, s2, 'LineWidth', 2)
hold on
scatter(period(1), s2(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S2")
legend("S2", "Starting Point", "Critical Values")
hold off
sgtitle("S1 and S2 as a function of Period - Halo Orbit (Northern)")
```

Part c - Halo Orbit Family

```
[s1, s2, period] = stability_index(V_family_a2_south, mu, identity_row,
options);
figure()
subplot(2,1,1)
plot(period, s1, 'LineWidth', 2)
hold on
scatter(period(1), s1(1), 'filled', 'red')
```

```
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S1")
legend("S1", "Starting Point", "Critical Values")
hold off
subplot(2,1,2)
plot(period, s2, 'LineWidth', 2)
hold on
scatter(period(1), s2(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S2")
legend("S2", "Starting Point", "Critical Values")
hold off
sgtitle("S1 and S2 as a function of Period - Halo Orbit (Southern)")
```

Part c - L2 Lyapunov orbit family

```
[tout, xout] = ode113(@(t,state)CR3BP_full(state, mu), [0, V_family_a1(7,1)],
[V_family_a1(1:6,1); identity_row], options);

lyapunov_monodromy = reshape(xout(end,7:42), [6,6])';
[V_lyapunov, D_lyapunov] = eig(lyapunov_monodromy);

% [s1, s2, period] = stability_index(V_family_a1, mu, identity_row, options);
figure()
subplot(2,1,1)
plot(period, s1, 'LineWidth', 2)
hold on
scatter(period(1), s1(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S1")
legend("S1", "Starting Point", "Critical Values")
hold off
subplot(2,1,2)
plot(period, s2, 'LineWidth', 2)
hold on
scatter(period(1), s2(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S2")
legend("S2", "Starting Point", "Critical Values")
hold off
sgtitle("S1 and S2 as a function of Period - Lyapunov Orbit Family")
```

Part c - L2 Axial orbit family

```
V_family_a3 = [V_family_a3_neg_ds, flip(V_family_a3_pos_ds)];
% V_family_a3 = V_family_a3_neg_ds;

[tout, xout] = ode113(@(t,state)CR3BP_full(state, mu), [0, V_family_a3(7,1)],
[V_family_a3(1:6,1); identity_row], options);

[s1, s2, period] = stability_index(V_family_a3, mu, identity_row, options);
figure()
subplot(2,1,1)
plot(period, s1, 'LineWidth', 2)
hold on
scatter(period(1), s1(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S1")
legend("S1", "Starting Point", "Critical Values")
hold off
subplot(2,1,2)
plot(period, s2, 'LineWidth', 2)
hold on
scatter(period(1), s2(1), 'filled', 'red')
yline(2, '--r')
yline(-2, '--r')
xlabel("Period [-]")
ylabel("S2")
legend("S2", "Starting Point", "Critical Values")
hold off
sgtitle("S1 and S2 as a function of Period - Axial Orbit Family")
```

Problem 2

```
% ASEN 6060 - HW 4, Problem 2
% Spring 2025
% Jash Bhalavat
```

Part b

```
mu = mass_ratio_em;

% Earth Moon system equilibrium points
[em_eq_pts, em_eq_validity] = all_eq_points(mu);

% Only looking at L1 eq point planar oscillatory modes
l1_pos = [em_eq_pts(1,:), 0];

l1_in_plane_modes = in_plane_modes(mu, l1_pos);

oscillatory_eval = l1_in_plane_modes(3);
uxx_l1 = u_xx(mu, l1_pos);
```

```
uxy_l1 = u_xy(mu, l1_pos);
uyy_l1 = u_yy(mu, l1_pos);
U_star_XX = [uxx_l1, uxy_l1; uxy_l1, uyy_l1];

Omega = [0 2; -2 0];

A2D = [zeros(2), eye(2); U_star_XX, Omega];

[V, D] = eig(A2D);

oscillatory_evec = real(V(:,3));

oscillatory_pos_mag = norm([oscillatory_evec(1), oscillatory_evec(2)]);

pos_mag_req = 0.0001;

oscillatory_mag_factor = pos_mag_req / oscillatory_pos_mag;

oscillatory_ic = oscillatory_evec .* oscillatory_mag_factor;

% Time is one period
t = linspace(0, 2*pi/imag(oscillatory_eval), 1000);

xi_0 = oscillatory_ic(1);
xi_dot_0 = oscillatory_ic(3);
eta_0 = oscillatory_ic(2);
eta_dot_0 = oscillatory_ic(4);
x0 = [l1_pos(1) + xi_0; l1_pos(2) + eta_0; 0; xi_dot_0; eta_dot_0; 0];

% Set options for ode113
options = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
[tout, xout] = ode113(@(t, state)CR3BP(state, mu), [0 t(end)], x0, options);

V0 = [x0; t(end)];

V_soln = gen_3d_periodic_orbit_single_shooting(V0, mu, false);

[tout_corrected, xout_corrected] = ode113(@(t, state)CR3BP_full(state, mu),
[0, V_soln(end)], [V_soln(1:6); reshape(eye(6), [36,1])], options);

% V_family = pseudo_arc_length_continuation(V_soln, mu);
load("V_family_L1_Lyapunov.mat");
% save("V_family_L1_Lyapunov.mat", V_family);

p1_pos = [-mu, 0, 0];
p2_pos = [1-mu, 0, 0];

figure()
scatter(l1_pos(1), l1_pos(2), 'filled', 'red')
hold on
scatter(p1_pos(1), p1_pos(2), 'filled', 'blue')
scatter(p2_pos(1), p2_pos(2), 'filled', 'black')

for i = 1:50:size(V_family, 2)
```

```
    [tout, xout] = ode113(@(t,state)CR3BP(state, mu), [0, V_family(7,i)],
V_family(1:6,i), options);
    plot3(xout(:,1), xout(:,2), xout(:,3), 'LineWidth',2)
end
hold off
legend("L2", "Earth", "Moon")
xlabel('$$\hat{x}$$','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{y}$$','Interpreter','Latex', 'FontSize',18)
grid on
axis equal
title("L1 Lyapunov Orbit Family")


% 2.72 nondim time is 102 index
[tout, xout] = ode113(@(t,state)CR3BP_full(state, mu), [0, V_family(7,102)],
[V_family(1:6,102); reshape(eye(6), [36,1])], options);

manifolds(tout, xout, mu, l1_pos, 10);
title("Stable/Unstable Manifolds associated with L1 Lyapunov Orbit (~2.72 non-
dim period)")
```

Part c

```
manifolds(tout, xout, mu, l1_pos, 6);
title("Stable/Unstable Manifolds (Moon-Bound) associated with L1 Lyapunov
Orbit (6 non-dim period)")
```

Functions

```
function [s1, s2, period] = stability_index(V_family, mu, identity_row,
options)
    counter = 1;

    for i = 1:size(V_family, 2)
        [tout, xout] = ode113(@(t,state)CR3BP_full(state, mu), [0,
V_family(7,i)], [V_family(1:6,i); identity_row], options);
        monodromy_i = reshape(xout(end,7:42), [6,6])';
        [evecs, evals] = eig(monodromy_i);

        % Pass to function
        [s1_i, s2_i] = arrange_evals(evals, evecs);

        if (counter==1)
            s1(counter) = s1_i;
            s2(counter) = s2_i;
        else
            diff1 = abs(s1_i - s1(counter-1));
            diff2 = abs(s1_i - s2(counter-1));
            if diff1 < diff2
                s1(counter) = s1_i;
                s2(counter) = s2_i;
            else
                s1(counter) = s2_i;
```

```
                s2(counter) = s1_i;
            end
        end

        period(counter) = V_family(7,i);
        counter = counter + 1;
    end
end

function manifolds(tout, xout, mu, l1_pos, manifold_time)
    % Script to compute stable/unstable manifolds for a periodic orbit
    % Inputs:
    % tout - discrete time steps
    % xout - 42x1 discrete state vectors
    % mu - system mass ratio
    % l1_pos - equilibrium point position
    % manifold_time - time to propagate manifold forward/backward
    %
    % Outputs:
    % Graph with stable/unstable manifolds

    % Set options for ode113()
    % Part b
    % options = odeset('RelTol', 1e-12, 'AbsTol', 1e-12, 'Events', @(t,y)
eventFn(t, y, mu));

    % Part c - ignore event function
    options = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);

    a = 384400; % [kg] EM average SMA
    d = 50 / a; % [-] Unitless, normalized by a

    period = tout(end);

    p1_pos = [-mu, 0, 0];
    p2_pos = [1-mu, 0, 0];

    figure()
    plot(xout(:,1), xout(:,2), 'black', 'LineWidth', 3)
    hold on
    scatter(l1_pos(1), l1_pos(2), 'filled', 'red')
    scatter(p1_pos(1), p1_pos(2), 'filled', 'blue')
    scatter(p2_pos(1), p2_pos(2), 'filled', ' black')

    % Compute STM - phi(t1+T, t1)
    phi_t1T_t1 = reshape(xout(end,7:42), [6,6])';

    % Begin for loop
    for i = 1:10:length(tout)

        % Compute STM - phi(tj+T, tj)
        phi_tj_t1 = reshape(xout(i, 7:42), [6,6])';
        phi_tjT_tj = phi_tj_t1 * phi_t1T_t1 * inv(phi_tj_t1);
```

```
% Get evals, evecs
[V, D] = eig(phi_tjT_tj);

% Get evals as an array
for j = 1:6
    evals(j) = D(j,j);
end

% Subtract evals by 1 and get 2 minimum indices. These are trivial
% indices
evals_minus_1 = evals - 1;
[min_evals, trivial_index] = mink(evals_minus_1, 2);

% If eval is real and not trivial, assign stable and unstable
% indices
for j = 1:6
    if (isreal(evals(j)) && isnotin(trivial_index, j))
        if evals(j) < 1
            stable_index = j;
        elseif evals(j) > 1
            unstable_index = j;
        end
    end
end

% Get stable/unstable evec and normalize eigenvector by 1st 3 terms
stable_eval = D(stable_index, stable_index);
stable_evec = V(:, stable_index);
stable_pos_norm = norm(stable_evec(1:3));
stable_evec = stable_evec/stable_pos_norm;
% stable_evec(4:6) = -stable_evec(4:6);
unstable_eval = D(unstable_index, unstable_index);
unstable_evec = V(:, unstable_index);
unstable_pos_norm = norm(unstable_evec(1:3));
unstable_evec = unstable_evec/unstable_pos_norm;

% Step into manifold
x_manifold_s_p = xout(i,1:6)' + d * stable_evec;
x_manifold_s_n = xout(i,1:6)' - d * stable_evec;

% ONLY FOR L1
% If x-velocity is positive, moon-bound
% If x-velocity if negative, earth-bound
if (x_manifold_s_p(4) > 0)
    moon_stable = x_manifold_s_p;
    earth_stable = x_manifold_s_n;
else
    moon_stable = x_manifold_s_n;
    earth_stable = x_manifold_s_p;
end
% Repeat for unstable manifolds
x_manifold_u_p = xout(i,1:6)' + d * unstable_evec;
x_manifold_u_n = xout(i,1:6)' - d * unstable_evec;
if (x_manifold_u_p(4) > 0)
```

```
            moon_unstable = x_manifold_u_p;
            earth_unstable = x_manifold_u_n;
        else
            moon_unstable = x_manifold_u_n;
            earth_unstable = x_manifold_u_p;
        end

        % Propagate using the event functions
        [moon_stable_t, moon_stable_x] = ode113(@(t, state)CR3BP(state, mu),
[0, -manifold_time], moon_stable, options);
        [moon_unstable_t, moon_unstable_x] = ode113(@(t, state)CR3BP(state,
mu), [0, manifold_time], moon_unstable, options);
        [earth_stable_t, earth_stable_x] = ode113(@(t, state)CR3BP(state,
mu), [0, -manifold_time], earth_stable, options);
        [earth_unstable_t, earth_unstable_x] = ode113(@(t, state)CR3BP(state,
mu), [0, manifold_time], earth_unstable, options);

        plot(moon_stable_x(:,1), moon_stable_x(:,2), 'blue')
        plot(earth_stable_x(:,1), earth_stable_x(:,2), 'blue')
        plot(moon_unstable_x(:,1), moon_unstable_x(:,2), 'red')
        plot(earth_unstable_x(:,1), earth_unstable_x(:,2), 'red')

    end
    hold off
    legend("Lyapunov Orbit", "L1", "Earth", "Moon")
    grid on
    axis equal
    xlabel('$$\hat{x}$$','Interpreter','Latex', 'FontSize',18)
    ylabel('$$\hat{y}$$','Interpreter','Latex', 'FontSize',18)
end

function [value,isterminal,direction] = eventFn(t,y, mu)
    value = [1-mu-y(1), y(1)-(-mu)];
    isterminal = [1, 1]; % Halt integration when value is 0
    direction = [0, 0]; % When zero is approached from either side
end


function out = isnotin(array, val)
    out = true;
    for el = 1:length(array)
        if val == array(el)
            out = false;
        end
    end
end

function [s1, s2] = arrange_evals(evals_mat, evecs)
    for j = 1:6
        evals(j) = evals_mat(j,j);
    end
    % Get first element and its reciprocal and remove from array
    eval1 = evals(1);
    over_eval1 = 1/eval1;
```

```
    evals(1) = [];

    diff = evals - over_eval1;
    [min_diff, pair1_index] = min(abs(diff));
    pair1 = eval1 + evals(pair1_index);

    evals(pair1_index) = [];

    eval2 = evals(1);
    over_eval2 = 1/eval2;
    evals(1) = [];

    diff = evals - over_eval2;
    [min_diff, pair2_index] = min(abs(diff));
    pair2 = eval2 + evals(pair2_index);

    evals(pair2_index) = [];

    pair3 = evals(1) + evals(2);

    pairs = [pair1, pair2, pair3];
    pairs_minus_2 = pairs - 2;
    [min_pair, min_pair_ind] = min(abs(pairs_minus_2));

    pairs(min_pair_ind) = [];
    s1 = pairs(1);
    s2 = pairs(2);

end
```

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