# ASEN 6060
# ADVANCED ASTRODYNAMICS
# Week 9 Discussion

Objectives:

- Further explore hyperbolic invariant manifolds of equilibrium points and periodic orbits
- Understand connection between theory and computational results

# *Question 1*

**Question 1:** Assuming a correct implementation, how can you assess the accuracy of your numerical computations for:

1) The monodromy matrix
2) The eigenvalues of the monodromy matrix?

And when are the results 'accurate enough'?

Group Brainstorming:

- Monodromy matrix:
  - Could introduce a variation in the initial state and multiply by M to produce a variation after one period – compare to nonlinear system?
  - Det(M) = 1 (nice scalar check)
  - Finite differencing to approximate
- Should recover two trivial eigenvalues: how close are these to one?
  - How close? 1e-4? Smaller or bigger?
  - How to get them closer to 1? Tighten tolerance on ODE solver / numerical integration. But there are challenges here.
- Check if reciprocal pairs of eigenvalues exist to within some tolerance: check lambda * 1/lambda = 1 to within some tolerance

# *Question 2*

**Question 2:** Consider a monodromy matrix generated along a periodic orbit with large $T$ and close passes of the primaries. How could you potentially increase the accuracy of computing $\mathbf{M}$?

Group Brainstorming:

- Tighten tolerance on ODE solver / numerical integration, improve accuracy of numerical integration scheme. But there are challenges here.

- Break the trajectory down into pieces (mirror theorem or multiple-shooting) and then use our identity that one STM can be expressed as the product of the smaller STMs calculated along successive segments of the trajectory

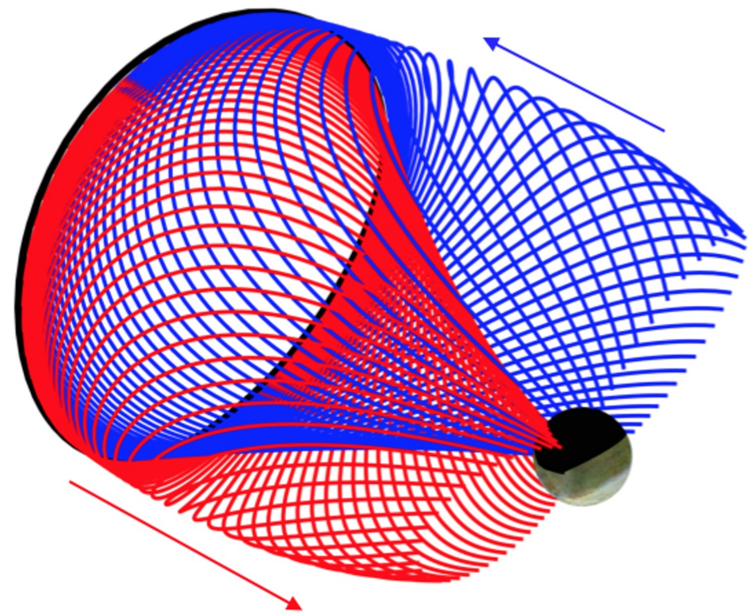
# Question 3

In Homework 4, you are generating segments along the stable and unstable manifolds associated with a periodic orbit in the Earth-Moon CR3BP.

$$\bar{x}^S = \bar{x}_{PO} \pm d\bar{v}^S(\bar{x}_{PO})$$

$$\bar{x}^U = \bar{x}_{PO} \pm d\bar{v}^U(\bar{x}_{PO})$$

→ Requires selecting a value of *d*



**Question 3:** How can you justify that you are, indeed, generating a close approximation of the stable and unstable manifolds associated with a periodic orbit?

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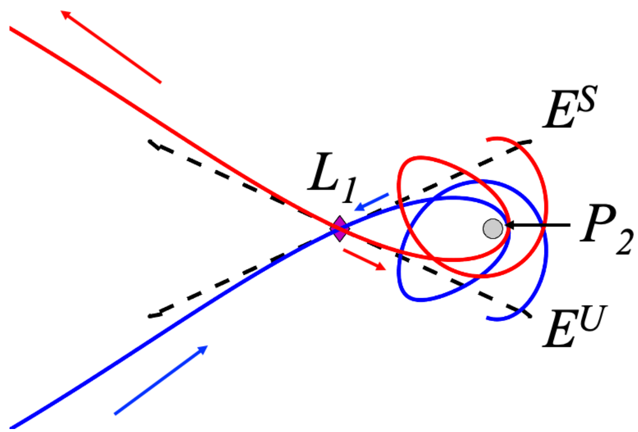
Group Brainstorming:

- How to check accuracy of unstable manifold? If you propagate a state backwards in time, does it approach the reference orbit? Some challenges: how long to integrate for and how to balance error accumulation during numerical integration? This might vary based on stability of the orbit.
- How to check accuracy of stable manifold? Do this as well but propagate forward in time.

# Question 4

**Question 4:** Why are the stable and unstable manifolds of EM $L_1$ symmetric about the x-axis?




Eigenvalues: +/-2.9321, +/- 2.3344i, **+/- 2.2688i**

Eigenvectors for in-plane stable and unstable modes (trunc.):

+2.9321: [0.29325, -0.13493, 0, 0.859815, -0.39562, 0]

-2.9321 : [0.29325, 0.13493, 0, -0.859815, -0.39562, 0]

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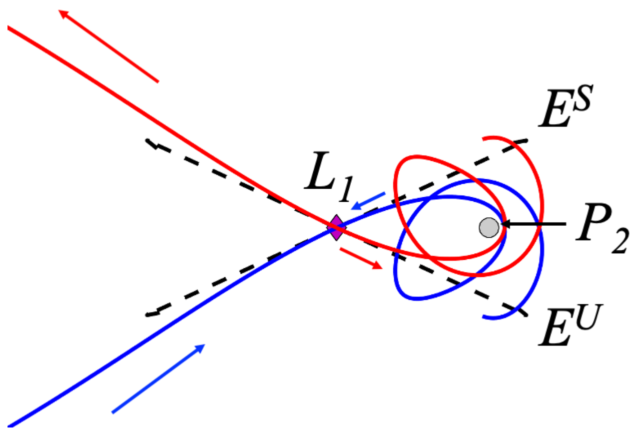



Group Brainstorming:

- Symmetry about the x-axis in the rotating frame (see next slide) and symmetric perturbations from the equilibrium point result in symmetric trajectories (must flip the direction of time)

# Symmetries

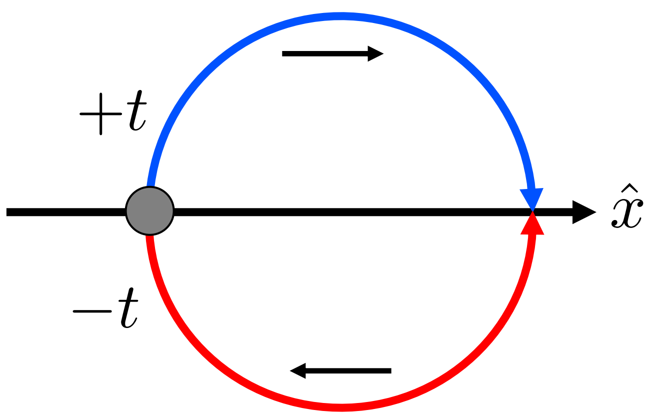
Example 1: $(x, y, z, t) \rightarrow (x, -y, z, -t)$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) = 2\frac{dy}{dt} + x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-1+\mu)}{r_2^3}$$

$$\frac{d}{d(-t)}\left(\frac{dx}{d(-t)}\right) = 2\frac{d(-y)}{d(-t)} + x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-1+\mu)}{r_2^3}$$

$$\frac{d}{d(-t)}\left(\frac{d(-y)}{d(-t)}\right) = -2(-\dot{x}) + (-y) - \frac{(1-\mu)(-y)}{r_1^3} - \frac{\mu(-y)}{r_2^3}$$

$$\frac{d}{d(-t)}\left(\frac{d(z)}{d(-t)}\right) = -\frac{(1-\mu)z}{r_1^3} - \frac{\mu z}{r_2^3}$$



# *Question 5*

Stable/unstable manifolds of periodic orbits approach them asymptotically.

**Question 5:** How can stable/unstable manifolds of two periodic orbits be useful in constructing finite time transfers between those two periodic orbits?