ASEN 6060 Spring 2025: Homework 1
Due: 01/28/2025 at 9pm MT

Notes:

- Use the physical parameters for celestial bodies available in the "Homework" module on the Canvas page.
- In your writeup, show all your working and/or discussion (see the syllabus for more information about expected components of a homework submission)
- The scripts and computations you develop in this homework will serve as a foundation for your work in this entire course.

Problem 1:

a) Calculate the mass ratio and characteristic quantities for each of the Earth-Moon and Sun-Earth systems.
b) Discuss and justify whether you expect the CR3BP to be a reasonable approximation of the dynamical environment governing the motion of a spacecraft in each of the Earth-Moon and Sun-Earth systems. Hint: consider the assumptions used when formulating the CR3BP as well as the properties of the actual systems.

Problem 2:

a) Write a script to numerically integrate the nondimensional equations of motion for the CR3BP, formulated in the rotating frame, from a specified initial state for a selected integration time. In your writeup, discuss and justify your specific choice of integration scheme, the definition of any tolerances used during numerical integration, and the value you selected for these tolerances. Copy the text of your script to the end of this subproblem.
b) Numerically integrate the following initial states forward in time for the stated integration times in the Earth-Moon CR3BP:

   i. Initial State #1: $\bar{x} = [0.98, 0, 0, 0, 1.2, 0]$ nondimensional units, Integration Time: 2 nondimensional units
   ii. Initial State #2: $\bar{x} = [0.98, 0, 0, 0, 1.7, 0]$ nondimensional units, Integration Time: 8 nondimensional units
   iii. Initial State #3: $\bar{x} = [0.12, 0, 0, 0, 3.45, 0]$ nondimensional units, Integration Time: 25 nondimensional units
   iv. Initial State #4: $\bar{x} = [0.12, 0, 0, 0, 3.48, 0]$ nondimensional units, Integration Time: 25 nondimensional units

   Plot the associated nondimensional trajectories in the configuration space. Add any annotations that you think would be useful to the plot of each trajectory. Discuss the characteristics of each trajectory in the configuration space.
c) Use relevant additional information, quantities, or data to assess whether each of the four numerically-generated trajectories from part b) is likely to closely approximate a true

solution to the equations of motion in the CR3BP. (Hint: consider the concepts we have covered in class that may help you perform this assessment)

d) Convert initial state #3 and the associated integration time into dimensional quantities. In addition, how far is the spacecraft located from the Earth at the initial time? Also express the integration time in terms of the number of orbital periods of the primary system.

Problem 3:

a) Write a second script to numerically integrate the equations of motion for the CR3BP until a stopping condition is satisfied (another way to state this is "until a specific event occurs"). In this problem, let's define the stopping condition as the first occurrence of y=0 along the trajectory, with $\dot{y} > 0$. Submit only the text of your script for this subproblem submission; no writeup is necessary here.

b) Numerically integrate initial state #3 listed in Problem 2b) until the stopping condition defined in Problem 3a). List the state vector at the stopping condition and the associated integration time.

Problem 4:

a) Write a script to compute the zero velocity curves in the xy-plane of the rotating frame at a single value of the Jacobi constant. In your writeup, discuss the procedure you used to calculate this information and justify the selection of any parameters or checks that govern this procedure, where relevant. Copy the text of your script to your submission for this subproblem.

b) Plot, in the rotating frame, the zero velocity curves in the xy-plane in the Earth-Moon CR3BP at the following Jacobi constants: 1) 3.189, 2) 3.173, 3) 3.013, and 4) 2.995. Add any additional annotations that you think might be useful to these plots.

c) For each plot of the zero velocity curves, discuss in your own words the implications of these regions on the possible itineraries of natural trajectories at each energy level within the Earth-Moon CR3BP.

d) Use your zero velocity curve script to estimate the location and Jacobi constant of each of the five equilibrium points to 2 decimal points. Also be sure to describe in 1-2 sentences the approach you use to leverage this script to estimate these values. Note: in the next homework, you will accurately calculate and report this information to ~15 significant figures; however, in this homework problem, you are simply estimating the position vectors and Jacobi constants in this problem using the zero velocity curves.