# ASEN 5050
# SPACEFLIGHT DYNAMICS
# Coordinate Systems

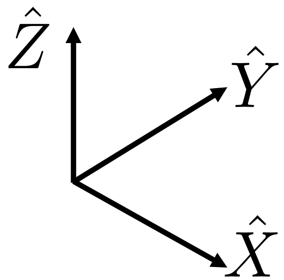
Objectives:

- Define common coordinate systems for specifying spacecraft state

# Coordinate Systems

Reference system: “constants, conventions, models, and parameters, which serve as the necessary basis for the mathematical representation of geometric and physical quantities” (Drewes, 2009)

When defining a reference system, specify:




- Origin
- Axes (e.g., right-handed, orthogonal triad)
- Scale (magnitude of vectors defining axes)
- Time evolution of these quantities

Common types of coordinate systems, used to define position of object within a reference system:

- Cartesian: three components are projection of vector onto axes
- Spherical: two angles and distance
- Elliptic: two angles and height above reference ellipsoid

Ref: Drewes, H., 2009, “Reference Systems, Reference Frames, and the Geodetic Datum," In: Sideris M.G. (eds) Observing our Changing Earth International Association of Geodesy Symposia, Vol 133. Springer, Berlin, Heidelberg.

# *Coordinate Frames*

- A ‘frame’ often denotes an implementation of a ‘system’ (Drewes, 2009)

- Inertial frame:
  - Fixed axes in space and a non-accelerating origin
  - Two standards for specifying ‘inertial’ frames for astrodynamics

- Body-fixed frame:
  - Origin is center of body or on surface
  - Axes rotate with a celestial body
  - Relies on parameters describing the body’s orientation in 3D space

Ref: Drewes, H., 2009, “Reference Systems, Reference Frames, and the Geodetic Datum," In: Sideris M.G. (eds) Observing our Changing Earth International Association of Geodesy Symposia, Vol 133. Springer, Berlin, Heidelberg.

# Useful Terminology

**Ecliptic:** plane containing Earth's mean orbit relative to the Sun
Inclined relative to Earth's mean equatorial plane by angle $\epsilon \approx 23.5^\circ$

**Vernal Equinox:** direction to ascending node of ecliptic plane relative to Earth's mean equatorial plane

**Equatorial plane:** plane normal to Earth's spin axis

**Local meridian:** half a great circle

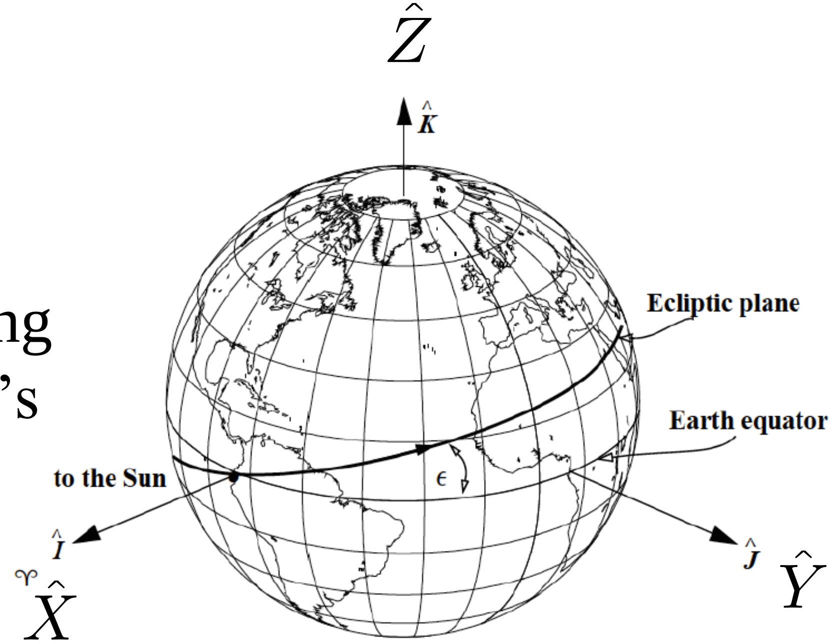
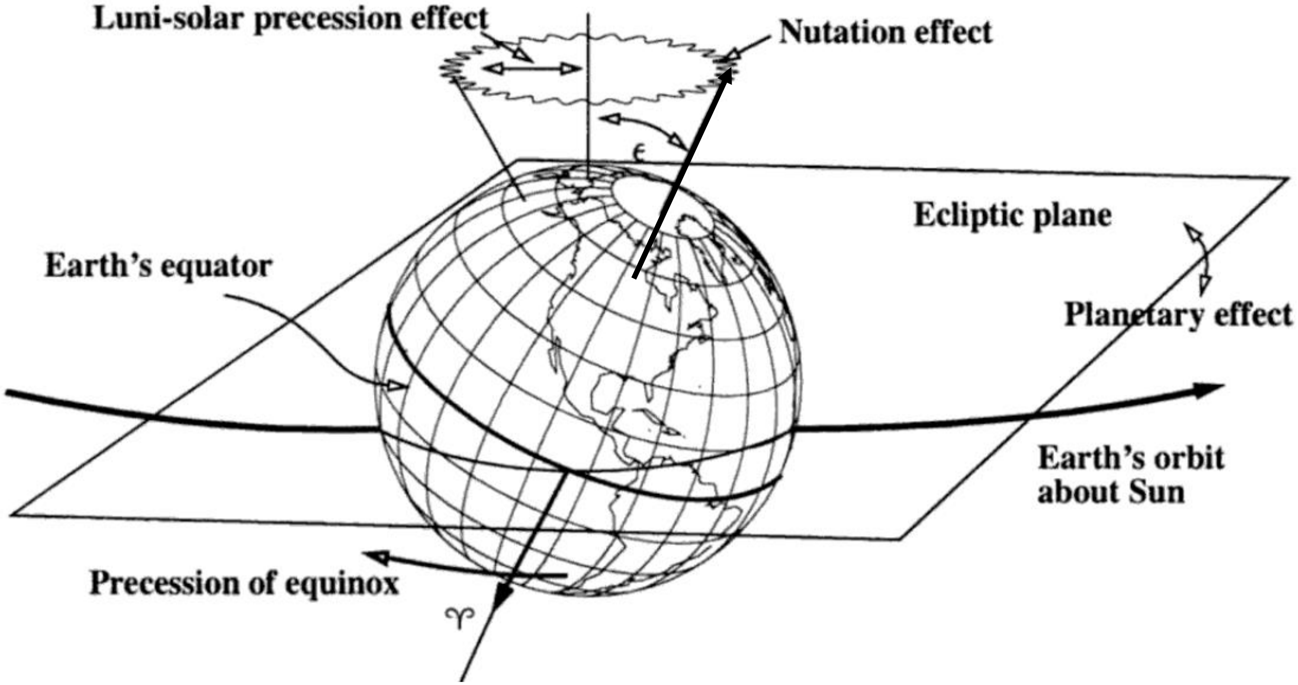



Image credit: Vallado, 2013

# *Useful Terminology*

Over time, the vernal equinox precesses westward around the normal to the ecliptic plane

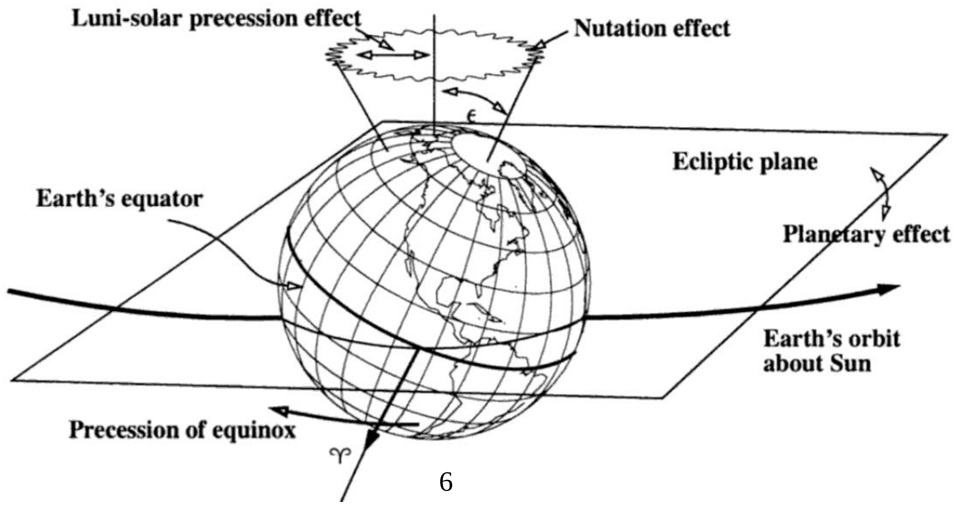



Shifts by approx. 1.4 deg/century

Image credit:
Vallado, 2013

# Evolution of Earth Orientation

- Celestial Intermediate Pole (CIP): “axis of Earth rotation…. normal to the true equator” (Vallado, 2022)
- Earth orientation evolves due to the following components:
  – Precession-nutation of CIP
    - Periodic effects with a period >2 days and secular effects
    - Due to gravity of Sun, Moon and planets, the irregular shape and gravity field of Earth, evolution of Moon’s orbit
  – Sidereal motion: Rotation of Earth about CIP
  – Polar motion: Shift of the CIP relative to Earth’s crust / surface



# *Useful Terminology*

- “True” vs “Mean”
  - Used to describe how reference plane is calculated
  - Mean: includes model of precession
  - True: includes models of precession and nutation

- “… of date” vs “… of epoch”
  - “… of date”: uses current epoch at each instant in simulation
  - “… of epoch”: at a specified, fixed epoch, e.g., J2000 = Jan 1, 2000 12:00.000 TDB

- Examples:
  - Mean equator of date: orientation of equatorial plane modeled using only precession
  - True equator of date: includes nutation due to lunar and solar perturbations

# *Earth-Centered Inertial Frames*

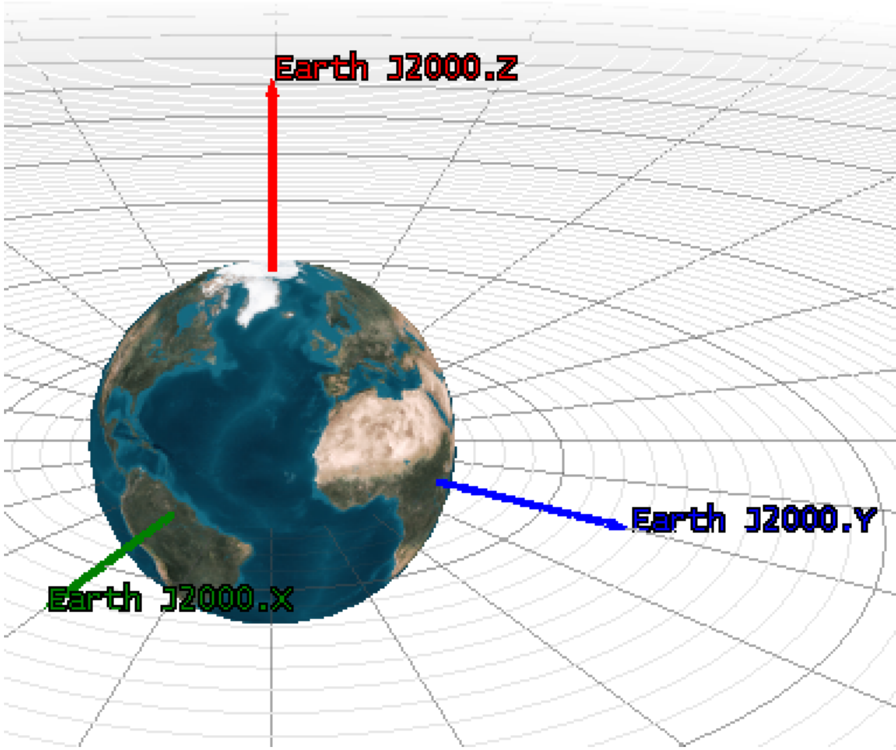
**Earth-Centered Mean Equatorial (EME) J2000 system**

Origin = Earth center

$\hat{X}$ : Aligned with vernal equinox on J2000 epoch

$\hat{Z}$ : Aligned with normal to the Earth's mean equatorial plane at the J2000 epoch

$\hat{Y}$ : Completes the right-hand triad, lies in equatorial plane




**True of Date (TOD) Equatorial Earth-Centered System:** uses true equator and true vernal equinox at each specific epoch

**Mean of Date (MOD) Equatorial Earth-Centered System:** uses mean equator and mean equinox at each specific epoch

# *Inertial Frames*

**ICRF: International Celestial Reference Frame**

- Close representation of an inertial frame
- Origin at solar system barycenter
- Frame managed by International Earth Rotation Service (IERS)
- Nonrotating with respect to extragalactic radio sources (quasars)
- Updated every several years since inception, with better observational data
- First two axes defined to lie in plane close to mean Earth equatorial plane on J2000 epoch, first axis close to vernal equinox
- ICRF2 in 2009, ICRF3 in 2019

Define similar frame, **Geocentric Celestial Reference System (GCRF)** that uses ICRF axes and origin at center of Earth

Both are modern standards for inertial frames

# *Transformations Between Frames*

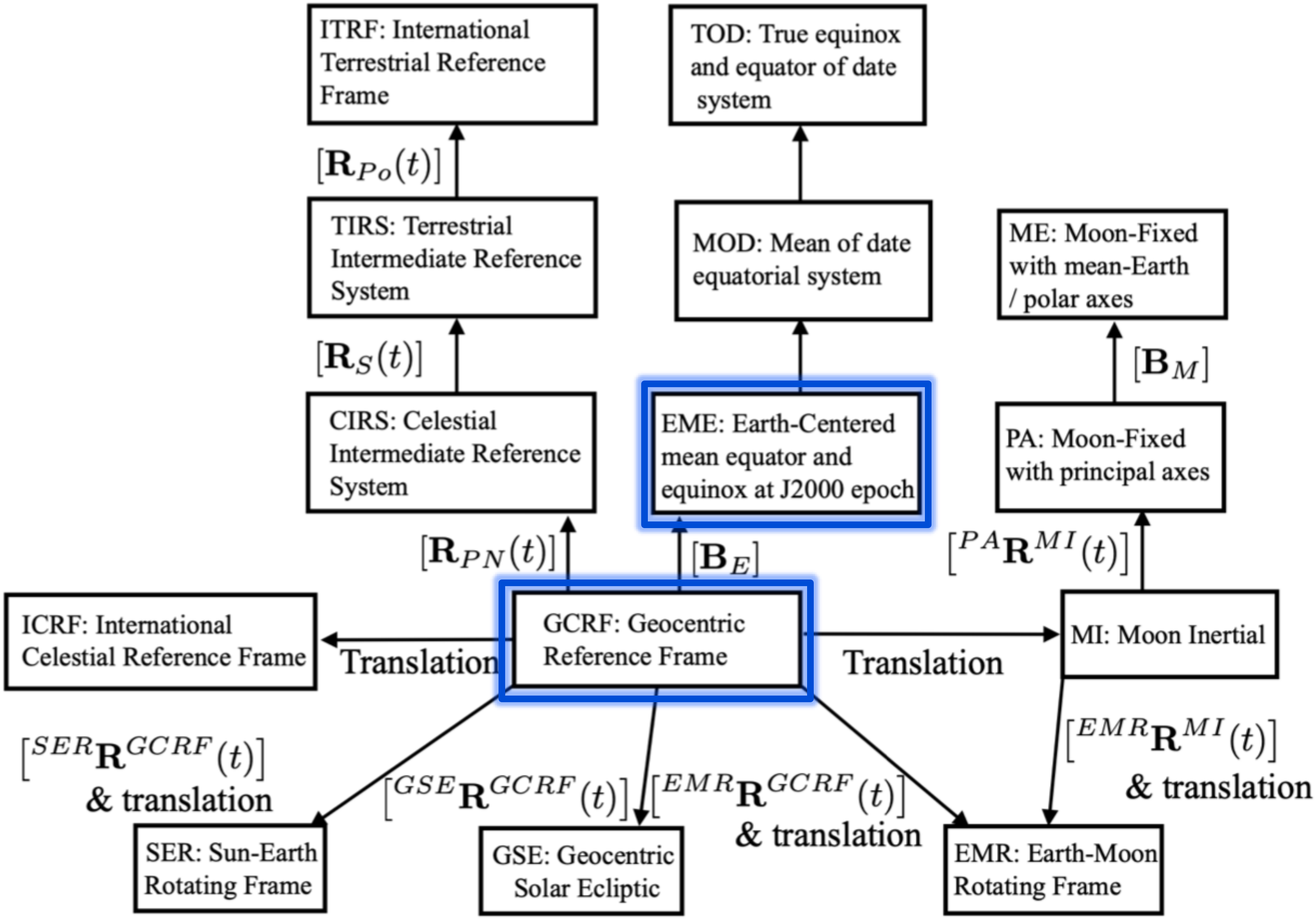



Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

# *Transformations Between GCRF & EMEJ2000*

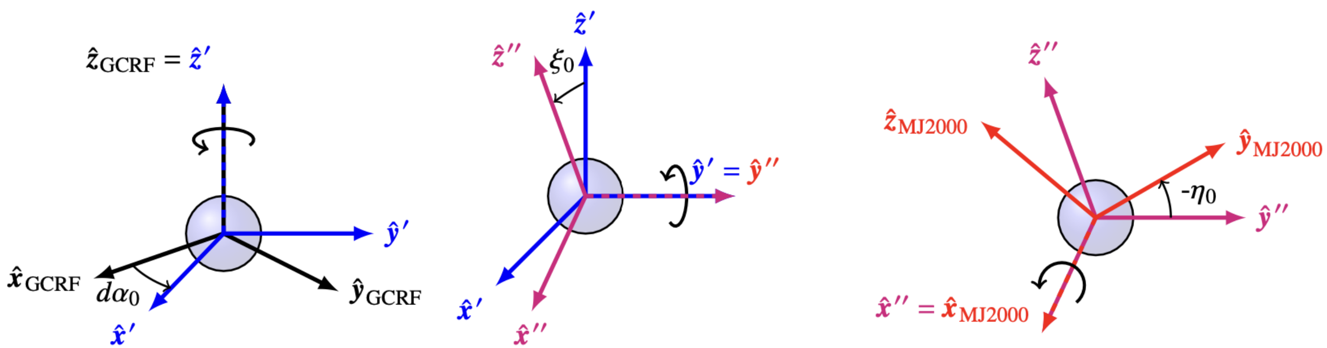
$$\boldsymbol{r}_{E,p}^{MJ2000} = \left[{}^{MJ2000}\boldsymbol{R}^{GCRF}\right]\boldsymbol{r}_{E,p}^{GCRF} = [\boldsymbol{B}_E]\,\boldsymbol{r}_{E,p}^{GCRF}$$

$$[\boldsymbol{B}_E] = [\boldsymbol{R}_1(-\eta_0)]\,[\boldsymbol{R}_2(\xi_0)]\,[\boldsymbol{R}_3(d\alpha_0)]$$

$$[\boldsymbol{B}_E] \approx \begin{bmatrix} 1-0.5\left(d\alpha_0^2+\xi_0^2\right) & d\alpha_0 & -\xi_0 \\ -d\alpha_0-\eta_0\xi_0 & 1-0.5\left(d\alpha_0^2+\eta_0^2\right) & -\eta_0 \\ \xi_0-\eta_0 d\alpha_0 & \eta_0+\xi_0 d\alpha_0 & 1-0.5\left(\eta_0^2+\xi_0^2\right) \end{bmatrix} \quad (36)$$

Frames differ on order of *mas* → often used interchangeably in trajectory design

$d\alpha_0 = -14.6\text{mas} = 7.0783 \times 10^{-8}$ rad $\quad \xi_0 = -16.6170$ mas $= -8.0561 \times 10^{-8}$ rad $\quad \eta_0 = -6.8192$ mas




(a) $[\boldsymbol{R}_3(d\alpha_0)]$ (b) $[\boldsymbol{R}_2(\xi_0)]$ (c) $[\boldsymbol{R}_1(-\eta_0)]$

**Fig. 6 Conceptual representation of the simple rotations captured within the frame bias matrix to transform from the GCRF to the Earth-Centered Mean Equator and Equinox at J2000 inertial system.**

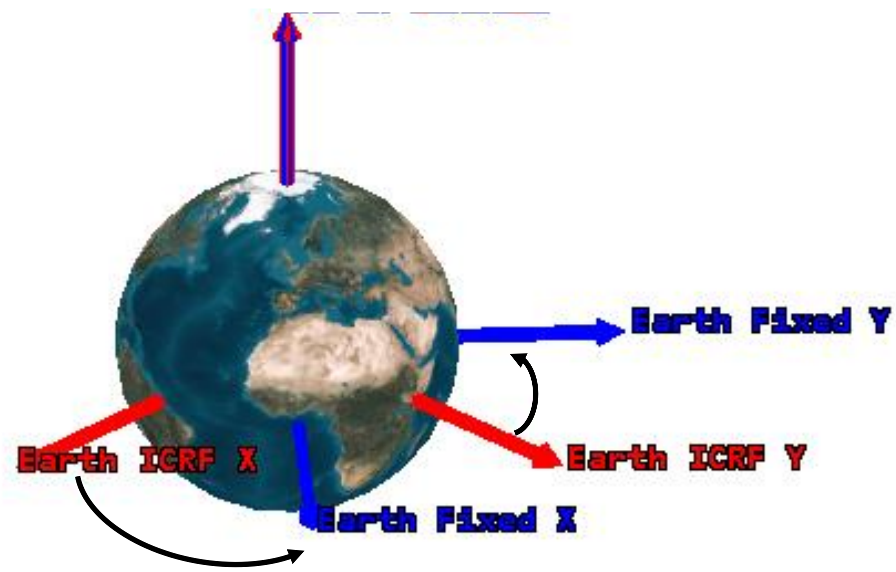
Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

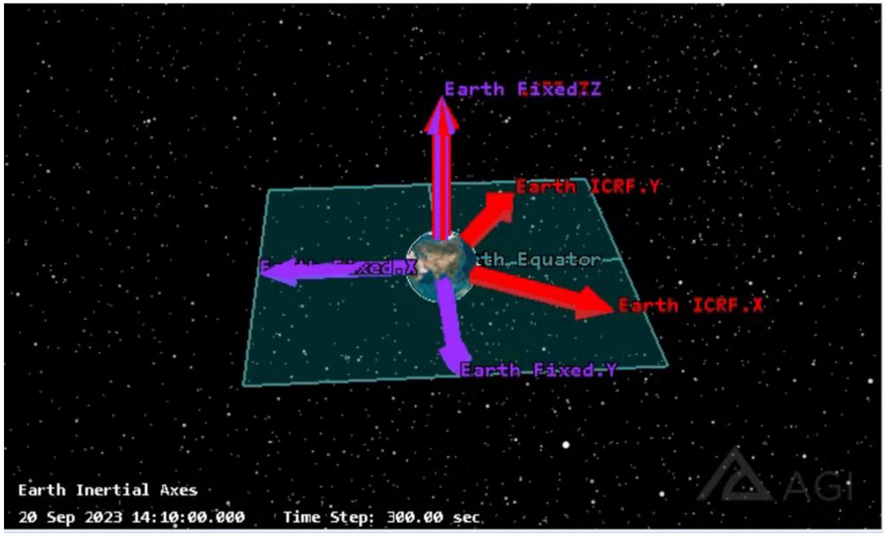
# *International Terrestrial Reference Frame (ITRF)*

Origin = Earth center

Axes:
- Rotate with the Earth
- Selected to ensure no net rotation relative to horizontal plate motion on the Earth
- Regularly updated with improved modeling of ground stations due to plate tectonics

ITRF is modern standard for an Earth-fixed frame



# Transformations Between Frames

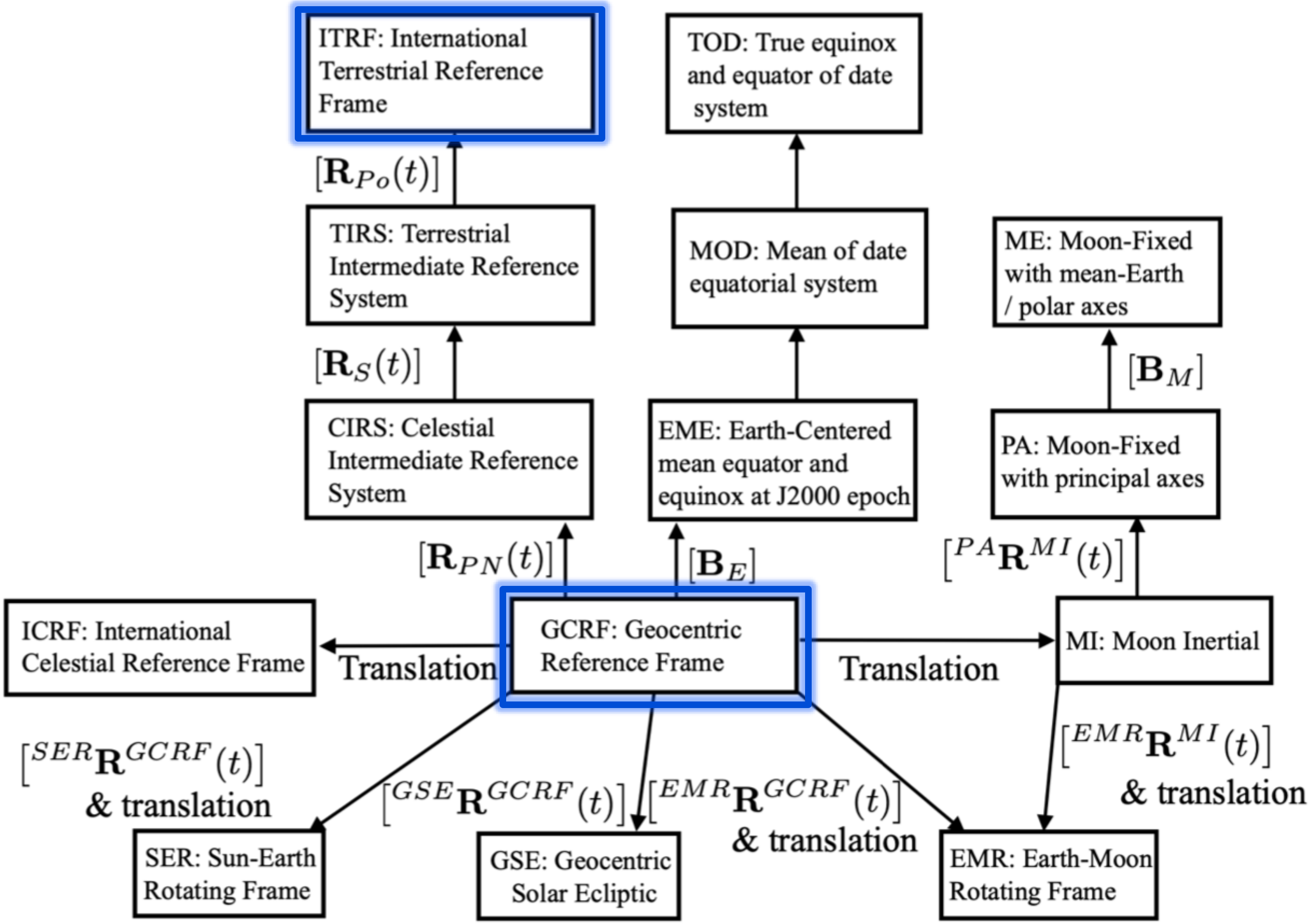



Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

# Transformations Between GCRF And ITRF

$$\boldsymbol{r}_{E,p}^{ITRF} = \left[{}^{ITRF}\boldsymbol{R}^{GCRF}(t)\right]\boldsymbol{r}_{E,p}^{GCRF} = [\boldsymbol{R}_{Po}(t)]\,[\boldsymbol{R}_S(t)]\,[\boldsymbol{R}_{PN}(t)]\,\boldsymbol{r}_{E,p}^{GCRF}$$

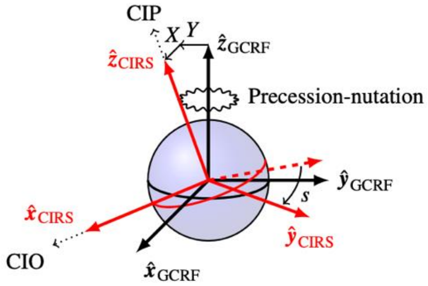
1. Precession-nutation:

$$[\boldsymbol{R}_{PN}(t)] = [\boldsymbol{R}_3(-s)]\begin{bmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{bmatrix}$$

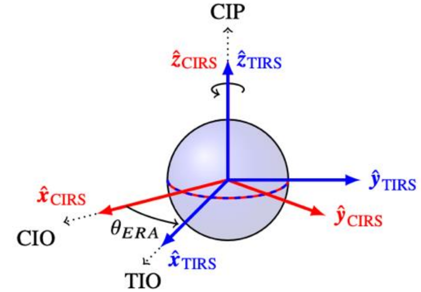
$$a = \frac{1}{1+\sqrt{1-X^2-Y^2}}$$

X,Y: first 2 coordinates of CIP unit vector in GCRF

s(t): angular difference between first axes of GCRF and intermediate set




(a) Transformation from GCRF (black) to CIRS (red), governed by $[\boldsymbol{R}_{PN}(t)]$ to reflect precession-nutation component



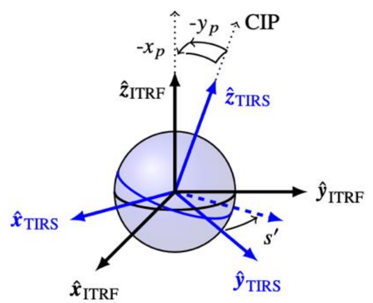

(b) Transformation from CIRS (red) to TIRS (blue), governed by $[\boldsymbol{R}_S(t)]$ to reflect the sidereal motion component

2. Sidereal motion:

$$[\boldsymbol{R}_S(t)] = [\boldsymbol{R}_3(\theta_{ERA})]$$

Earth rotation angle:

$$\theta_{ERA} = 2\pi\,(0.7790572732640 + 1.00273781191135448\,(JD_{UT1} - 2451545.0))$$




(c) Transformation from TIRS (blue) to ITRF (black), governed by $[\boldsymbol{R}_{Po}(t)]$ to reflect the polar motion component

3. Polar motion:

$$[\boldsymbol{R}_{Po}(t)] = \left[\boldsymbol{R}_1(-y_p)\right]\left[\boldsymbol{R}_2(-x_p)\right][\boldsymbol{R}_3(s')]$$

$x_p, y_p$: location of CIP via Earth orientation parameters

s'(t): angular diff. between first axis of ITRF and intersection of ITRF/TIRS XY planes

Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

# Moon-Centered Frames

**Moon Inertial**

- Origin at center of Moon
- Use axes of ICRF (other definitions often used too!)

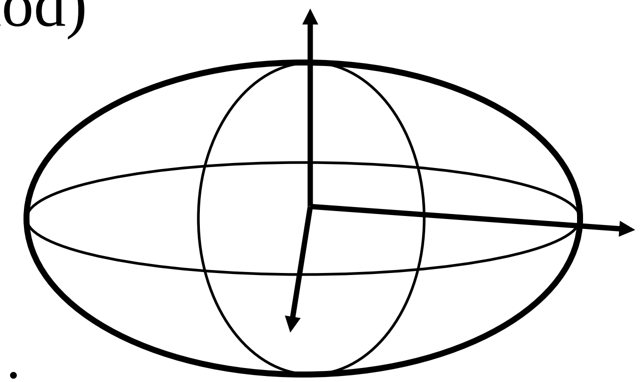
**Moon-Fixed**

- Origin at center of Moon
- Mean-Earth / Polar (ME) axes:

  $\hat{X}$ : Directed from lunar center of mass to mean Earth direction (lunar rotational period matches orbital period)

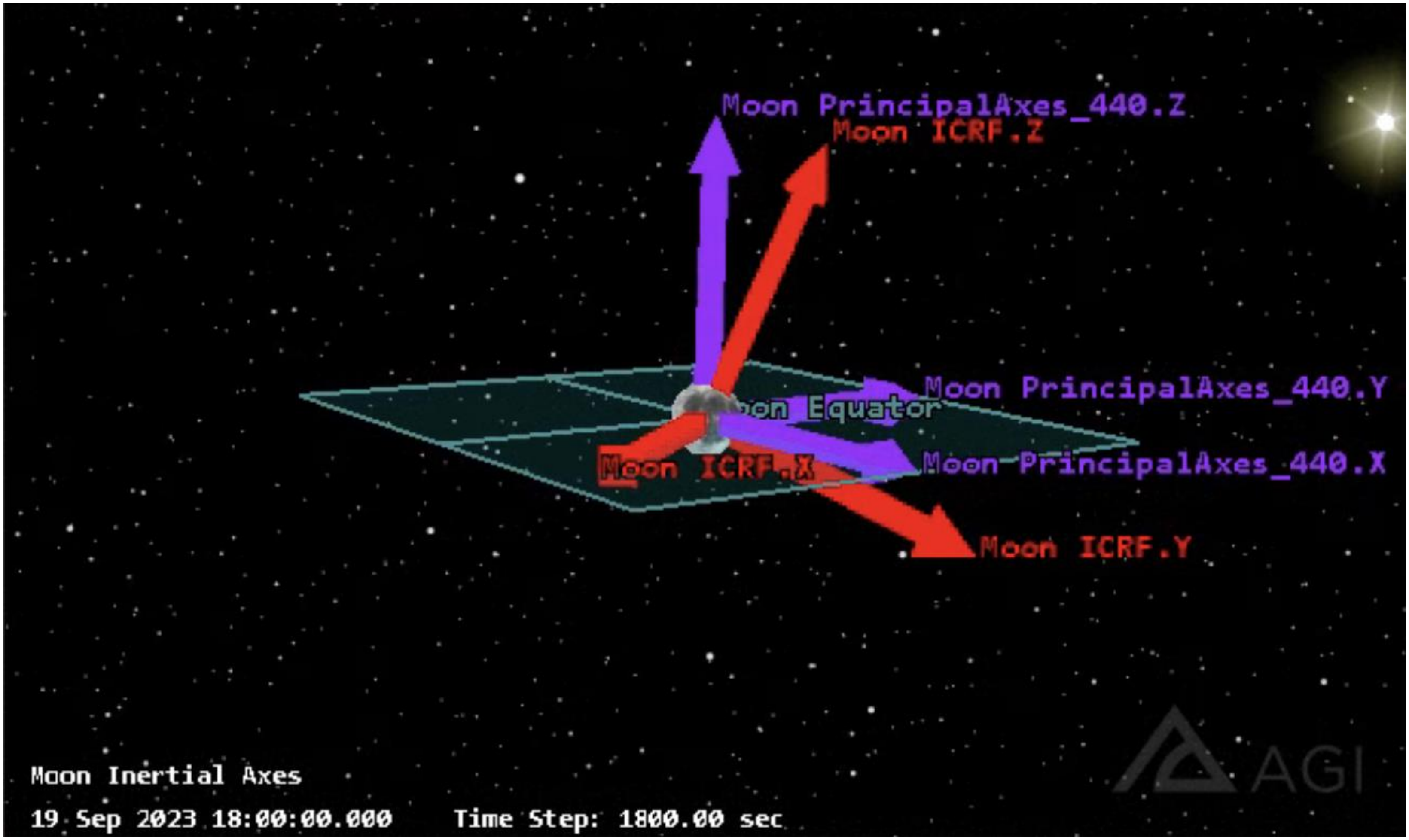
  $\hat{Z}$ : Aligned with mean lunar rotation axis

  $\hat{Y}$ : Completes right-handed triad
- Lunar principal axes (PA):
  - Principal axes of the Moon, assuming no mantle distortion due to tides and rotation
- Updated over time



Ref: Lunar Reconnaissance Orbiter Project and Lunar Geodesy and Cartography Working Group, 2008, "A Standardized Lunar Coordinate System for the Lunar Reconnaissance Orbiter and Lunar Datasets," White paper, Version 5.

# Transformations Between Moon-Defined Axes



# *Transformations Between Frames*

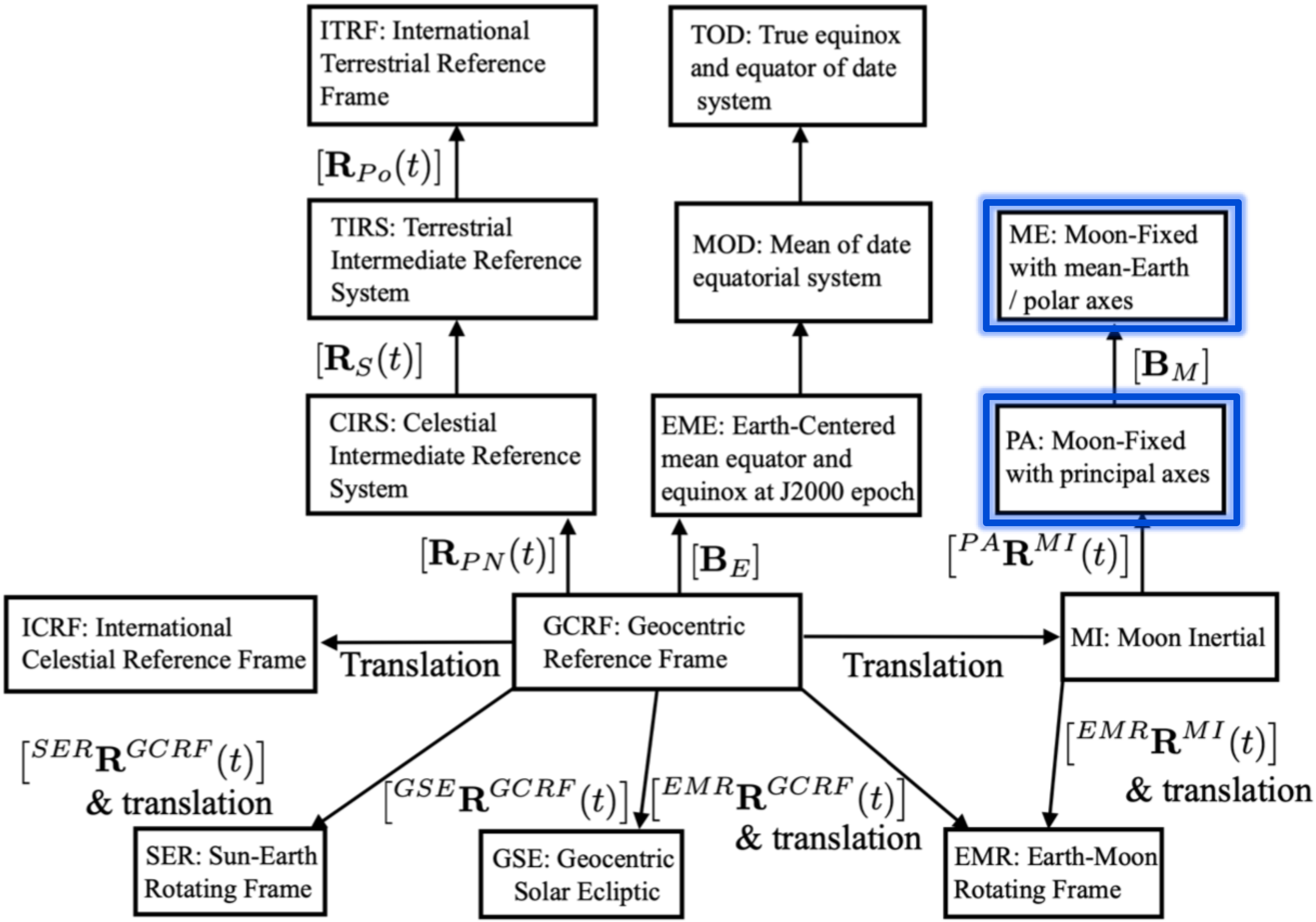
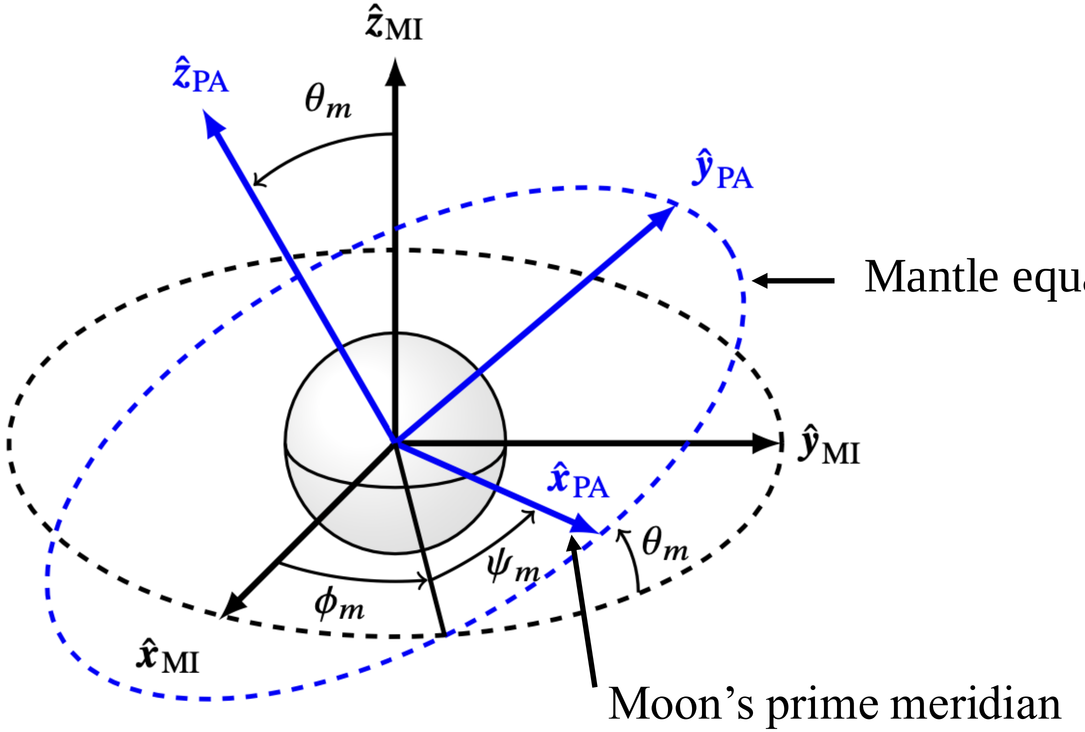



Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

# *Transformations Between Moon-Defined Axes*

$$\boldsymbol{r}^{PA}_{M,p} = \left[{}^{PA}\boldsymbol{R}^{MI}(t)\right] \boldsymbol{r}^{MI}_{M,p} \qquad \left[{}^{PA}\boldsymbol{R}^{MI}(t)\right] = [\boldsymbol{R}_3(\psi_m)]\,[\boldsymbol{R}_1(\theta_m)]\,[\boldsymbol{R}_3(\phi_m)]$$




$$\boldsymbol{r}^{ME}_{M,p} = \left[{}^{ME}\boldsymbol{R}^{PA}\right] \boldsymbol{r}^{PA}_{M,p} = [\boldsymbol{B}_M]\, \boldsymbol{r}^{PA}_{M,p}$$

$$[\boldsymbol{B}_M] = [\boldsymbol{R}_1(-0.2785 \text{ arcsec})]\,[\boldsymbol{R}_2(-78.6944 \text{ arcsec})]\,[\boldsymbol{R}_3(-67.8526 \text{ arcsec})]$$

Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

# *Lat, Long, RA, Dec*

Latitude, Longitude: $\phi, \lambda$
- Earth equatorial plane as reference
- Longitude positive east from Greenwich meridian

Ecliptic Latitude, Longitude: $\phi_{ec}, \lambda_{ec}$
- Ecliptic as reference plane
- Ecliptic longitude positive east from vernal equinox at a given epoch

Right Ascension, Declination: $\alpha, \delta$
- Earth equatorial plane as reference
- Right ascension positive east from vernal equinox at a given epoch

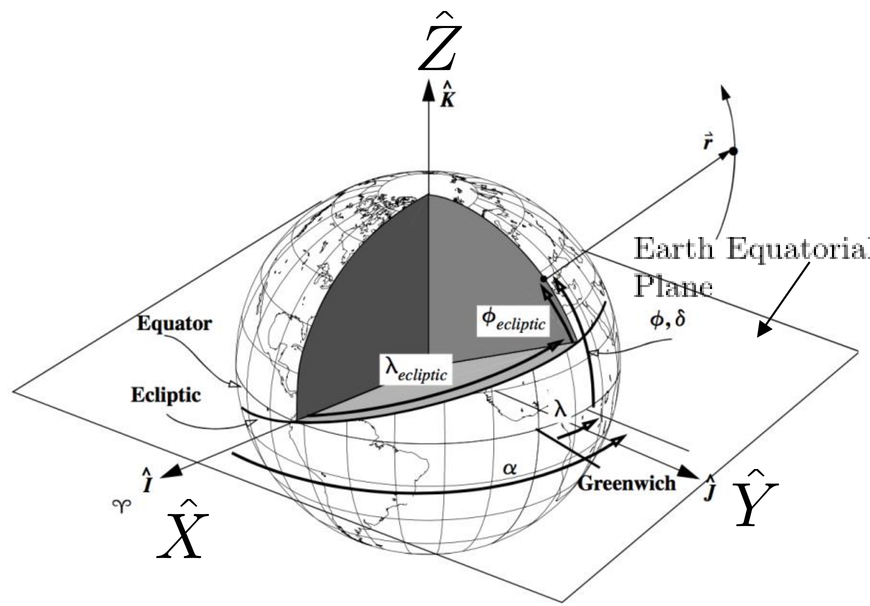



**Figure 3-8. Using Right Ascension and Declination or Longitude and Latitude to Measure Location.** Notice that both terrestrial latitude and longitude, $\phi$ and $\lambda$, and right ascension and declination, $\alpha$ and $\delta$, reference the Earth's equator, whereas ecliptic latitude and longitude are celestial coordinates that reference the ecliptic. Declination and terrestrial latitude are the same, but right ascension and terrestrial longitude reference different starting locations.

Image credit: Vallado, 2013

# *Latitude Definitions*

Geocentric latitude $\phi_{gc}$: uses Earth center as reference point

$$\phi_{gc} = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right) \qquad \lambda = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\bar{r} = r\cos(\phi_{gc})\cos(\lambda)\hat{X} + r\cos(\phi_{gc})\sin(\lambda)\hat{Y} + r\sin(\phi_{gc})\hat{Z}$$

Geodetic latitude $\phi_{gd}$: uses vector that is normal to surface of ellipsoid model of Earth

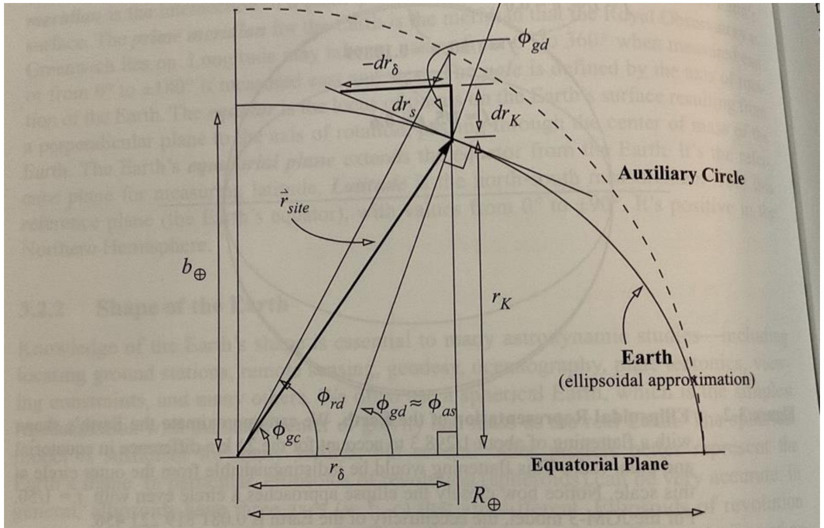



**Figure 3-3. Latitude Geometry (exaggerated scale).** The geodetic latitude, $\phi_{gd}$, makes an angle perpendicular to the surface and the equatorial plane, whereas the geocentric latitude, $\phi_{gc}$, is referenced to the center of the Earth. Astronomic latitude, $\phi_{as}$, is very close to $\phi_{gd}$. The center of the ellipsoid differs slightly for $\phi_{gd}$ and $\phi_{as}$. Reduced latitude, $\phi_{rd}$, is used only to derive relations.

Image credit: Vallado, 2022

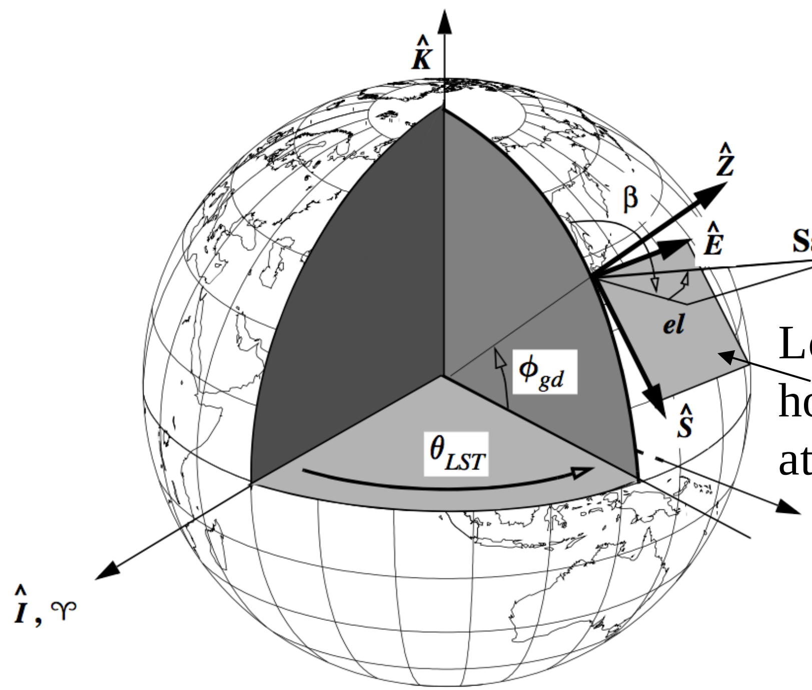
# Topocentric Horizon System

**Topocentric Horizon System, SEZ**

Origin = site on Earth surface
$\hat{S}$ : Points due South
$\hat{E}$ : Points due East
$\hat{Z}$ : Points radially outwards
SEZ frame rotates with site

Locate the site on surface via latitude ($\phi$) and longitude ($\theta$)

Locate spacecraft via azimuth ($\beta$), elevation (*el*) and range.



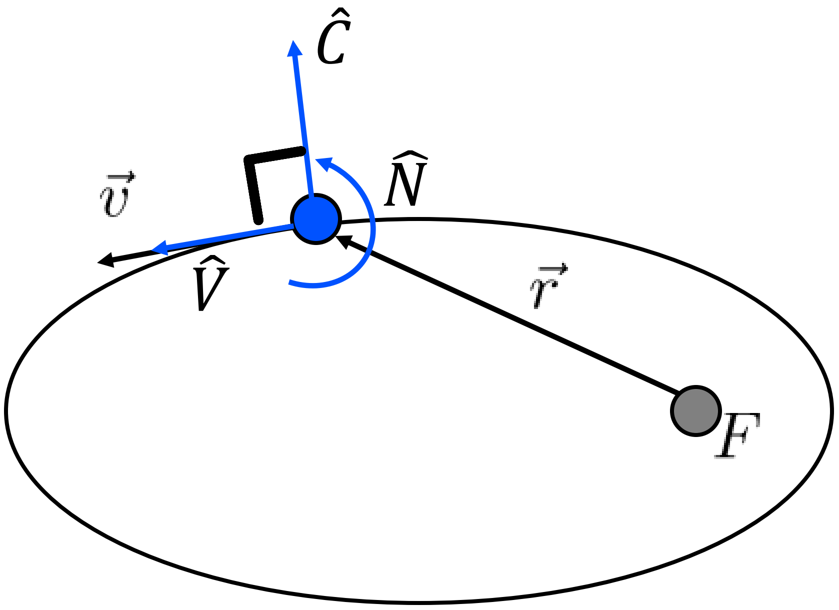

Local horizon at site

Image credit: Vallado, 2013

# *Satellite-Specific Axes*

VNC:
Velocity/Normal/Conormal
Also called VNB
$\hat{V}$ : Along velocity vector
$\hat{N}$ : Along orbit normal
$\hat{C}$ : Completes right-handed triad

Useful for maneuver design or low-thrust trajectories



# Coordinate Systems

Summary table in Vallado, Table 3-1 (pg 159, 4$^{th}$ ed.; pg. 161, 5$^{th}$ ed.):

| System | Symbol | Origin | Fundamental Plane | Principal Direction | Example Use |
|---|---|---|---|---|---|
| **Interplanetary systems** | | | | | |
| Heliocentric | XYZ | Sun | Ecliptic | Vernal equinox | Patched conic |
| Solar system* | $(XYZ)_{ICRF}$ | Barycenter | varies | varies | Planetary motion |
| **Earth-based systems** | | | | | |
| Geocentric† | IJK | Earth | Earth equator | Vernal equinox | General |
| Earth | $(IJK)_{GCRF}$ | Earth | varies | varies | Perturbations |
| Body-fixed | $(IJK)_{ITRF\text{-}\#\#}$ | Earth | Earth equator | Greenwich meridian | Observations |
| Earth-Moon (synodic) | $(IJK)_S$ | Barycenter | Invariable plane | Earth | Restricted three-body |
| Topocentric horizon | SEZ | Site | Local horizon | South | Radar observations |
| Topocentric equatorial | $(IJK)_t$ | Site | Parallel to Earth equator | Vernal equinox | Optical observations |
| **Satellite-based systems** | | | | | |
| Perifocal** | PQW | Earth | Satellite orbit | Periapsis | Processing |
| Satellite radial | RSW†† | Satellite | Satellite orbit | Radial vector | Relative motion, Perturbations |
| Satellite normal | NTW | Satellite | Satellite orbit | Normal to velocity vector | Perturbations |
| Equinoctial | EQW | Satellite | Satellite orbit | Calculated vector | Perturbations |

Final takeaway: never assume definition of a frame/axes by its name in software; always use documentation/clarification!

# Useful references


- Optional textbook: Vallado, D, "Fundamentals of Astrodynamics and Applications" 4th or 5th edition, Microcosm Press
- Seidelmann, P.K., 1992, *Explanatory Supplement to the Astronomical Almanac*, University Science Books, Mill Valley, CA.
- Petit, G.; Luzum, B. (eds), 2010, "IERS Conventions (2010)", International Earth Rotation and Reference Systems Service, Technical Note 36.
- Lunar Reconnaissance Orbiter Project and Lunar Geodesy and Cartography Working Group, 2008, "A Standardized Lunar Coordinate System for the Lunar Reconnaissance Orbiter and Lunar Datasets," White paper, Version 5.
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