ASEN 6060 Spring 2025: Homework 4

Due: 3/18/2025 at 9:00pm MT

Notes:

- Use the mass ratio calculated in HW 1 for the Earth-Moon system (if there were errors in your calculations, correct them first)
- In your writeup, show all your working and include a discussion (see the syllabus for more information about expected components of a homework submission)
- For all problems, copy the text of the scripts you create and/or use to the end of your submission for each problem or subproblem as appropriate.

Problem 1:

In this problem, we will assess the stability of orbits along families of periodic orbits. This may also be a good opportunity to correct any errors or issues in your scripts from HW 3.

a) Use your corrections and pseudo-arclength continuation scheme to calculate members of the following three families in the Earth-Moon CR3BP (calculate as large a region of the family as you can):
   i. L_2 Lyapunov orbit family: use the linearization scheme you studied in HW 2 to construct your own initial guess
   ii. L_2 halo orbit family: use the following orbit period and state vector $\bar{x}_0 = [1.180462, 0, -0.0209998, 0, -0.158363, 0]^T, T = 3.411921$
   iii. L_2 axial orbit family: use the following orbit period and state vector $\bar{x}_0 = [1.0301513, 0, 0, 0, 0.7030025, 0.1552945]^T, T = 4.312367$

   Plot the computed members in the configuration space with each family in a new figure. Add any additional annotations that may be useful.

b) For the first L_2 halo orbit that you calculate from the provided initial guess, compute and report the monodromy matrix, the eigenvalues, and associated eigenvectors. Qualitatively describe, in your own words, the stability of this periodic orbit and discuss the implications for the structure of the solution space in the local neighborhood of this periodic orbit.

c) Repeat this process for each of the computed periodic orbits along the three families by first calculating the eigenvalues of the monodromy matrix. Using this information, devise and display figures that summarize the stability of members along each family of periodic orbits as a function of the orbit period. (Please be sure to create separate figures for each family and ensure that someone else can clearly read any annotations). Using these figures, qualitatively describe, in your own words, the stability of the computed members of these families and discuss the implications for the structure of the solution space in the local neighborhood of the periodic orbits. Identify and discuss the significant bifurcations that occur along these families.

Problem 2:

a) Create a script to compute segments of the global stable and unstable manifolds associated with a general three-dimensional periodic orbit. Attach this script to this problem. Describe the mathematical and theoretical components of your manifold computation procedure, using equations where appropriate. Also report and justify the selected values for all necessary parameters.
b) Compute and plot the stable and unstable manifolds associated with an L_1 Lyapunov orbit in the Earth-Moon CR3BP with an orbit period of approximately 2.72 nondimensional time units. For the stable/unstable half-manifolds directed towards the Moon, integrate only until the first event where $x = 1 - \mu$. For the stable/unstable half-manifolds directed towards the Earth, integrate only until the first event where $x = -\mu$. Plot states along the stable manifold in blue and states along the unstable manifold in red. Describe the characteristics of the computed manifolds in your own words.
c) Repeat part b) for the stable and unstable half-manifolds directed towards the Moon only; however, this time generate each half-manifold for an integration time of 6 nondimensional time units. Describe the computed manifolds in your own words and discuss the complexity of visualization/analysis for this more complex set of trajectories.