ASEN 6060 Spring 2025: Homework 2
Due: 02/11/2025 at 9pm MT

Notes:

- Use the value of μ calculated in HW 1 for the Earth-Moon system (if there were errors in your calculations, correct them first)
- In your writeup, show all your working and include a discussion (see the syllabus for more information about expected components of a homework submission)
- Submit any scripts used for this homework in Gradescope along with your writeup

Problem 1:

a) Write a script to compute the location of each equilibrium point in the CR3BP. In your writeup, discuss the configuration of your script, and copy the text of the script to the end of your submission for this subproblem.
b) Create a table listing the locations and associated Jacobi constants for each equilibrium point in the Earth-Moon CR3BP; report these values using 15 digits. (Optional addition: consider comparing the locations you calculate in this problem to your predictions from HW 1!)
c) Discuss the effect of the mass ratio on the locations of each equilibrium point. Generate and use any combination of data, figures, calculations and/or theory to support your argument.

Problem 2:

Use the characteristic equation of the variational equations to calculate the in-plane and out-of-plane eigenvalues of each of the five equilibrium points for the Earth-Moon and Sun-Earth systems (use the parameter file from HW 1 to calculate the mass ratio for the Sun-Earth system). Report this information in a table (reporting 15 digits for each number) and identify the eigenvalues associated with the in-plane and out-of-plane modes.

Problem 3:

Calculate the critical value of the mass ratio that corresponds to a change in the stability of L_4 and L_5 in the CR3BP. Generate and use relevant data, analytical expressions, and/or figures to thoroughly justify your response.

Problem 4:

Following the general approach covered in the course lectures, let's derive the analytical expressions for the initial variations relative to a collinear equilibrium point that supply a useful initial guess for a Lyapunov orbit. Use the in-plane variational equations, formulated relative to a collinear equilibrium point, and assume that in-plane variations possess the form:
$\xi(t) = \sum_{j=1}^{4} A_j e^{\lambda_j t}$ $\qquad$ $\eta(t) = \sum_{j=1}^{4} B_j e^{\lambda_j t}$
Assume that (λ_1, λ_2) are the real modes and (λ_3, λ_4) are the oscillatory modes for in-plane motion.

(a) Derive expressions for the A_j coefficients in terms of the initial conditions $\xi_0, \eta_0, \dot{\xi}_0, \dot{\eta}_0$, the two modes (λ_1, λ_3), and the second derivatives of the pseudo-potential function. It helps to follow the process we summarized in class. You may use a symbolic toolbox in software such as Matlab to assist you with the single step of inverting a matrix of variables. For any other steps, show your working by hand.

(b) Derive analytical expressions for the initial values for $(\dot{\xi}_0, \dot{\eta}_0)$ such that only the oscillatory modes are excited. You should recover the following result:

$$\dot{\eta}_0 = \alpha_3 \lambda_3 \xi_0 \qquad\qquad \dot{\xi}_0 = \frac{\lambda_3 \eta_0}{\alpha_3}$$

(c) In the Earth-Moon system and relative to the L_1 equilibrium point, set $\xi_0 = -0.001$ and $\eta_0 = 0$. Then, calculate $(\dot{\xi}_0, \dot{\eta}_0)$ when only the planar oscillatory modes are activated. In the linearized system, propagate this initial condition for one revolution around the equilibrium point. (Hint: Recall that for an oscillatory mode with eigenvalue $\lambda = i\omega$, the period of oscillation is $P = 2\pi/\omega$.) Then, calculate the associated initial state vector in the rotating frame and relative to the barycenter as $\bar{x}_0 = [x_e + \xi_0, y_e + \eta_0, z_e, \dot{x}_e + \dot{\xi}_0, \dot{y}_e + \dot{\eta}_0, \dot{z}_e]$. Propagate this initial condition for the same time interval but in the nonlinear CR3BP. Plot the two trajectories on the same figure and discuss the difference between them.

(d) Do you think the trajectory generated in Problem 4c) in the nonlinear CR3BP might supply a good initial guess for a planar periodic orbit around L_1 (i.e., an L_1 Lyapunov orbit)? Justify your response. If you think they might supply a poor initial guess, also discuss how you could generate trajectories from this linearization-based analysis that might supply a better initial guess.

(e) Let's try generating an initial guess for an L_1 Lyapunov orbit using an alternate approach: using the eigenvectors associated with the oscillatory modes in the linear system. Calculate and report the complex eigenvectors associated with (λ_3, λ_4); the two eigenvectors should be a complex conjugate pair. Calculate and report the real and imaginary components of these complex eigenvectors; these components supply us with a set of real basis vectors that describe the associated eigenspace. Discuss the connection between the real and/or imaginary components of these eigenvectors to the process of selecting various initial variations to generate a periodic path relative to L_1 in the linearized system. Also compare the real and/or imaginary components of these eigenvectors to the initial variation you calculated in part c).

Problem 5:

(Note: I recommend completing this problem only after completing Problem 4)

a) Calculate the eigenvalues and eigenvectors of the A_{2D} matrix (defined in class) associated with the in-plane variational equations, formulated relative to L_4, in the Earth-Moon system.

b) Use these eigenvalues and eigenvectors to calculate an initial variation that will supply an initial guess for each of: 1) a short period orbit near L_4 and 2) a long period orbit near L_4. (Hint: an initial guess for a short period orbit is generated by exciting the in-plane oscillatory mode with the shortest period or, equivalently, the highest frequency). Select this initial variation such that the vector of the position components, $[\xi_0, \eta_0]$, possesses a magnitude of 0.02 nondimensional units.

c) In the linearized system, propagate these initial variations for one revolution around the equilibrium point. (Hint: Recall that for an oscillatory mode with eigenvalue $\lambda = i\omega$, the period of oscillation is $P = 2\pi/\omega$.) Then, calculate the associated initial state vectors in the rotating frame and relative to the barycenter as $\bar{x}_0 = [x_e + \xi_0, y_e + \eta_0, z_e, \dot{x}_e + \dot{\xi}_0, \dot{y}_e + \dot{\eta}_0, \dot{z}_e]$. Propagate these initial conditions for the same time interval but in the nonlinear CR3BP. For each type of orbit, plot the two trajectories associated with the same initial variation on the same figure and discuss the difference between them; plot the trajectories associated with short period and long period motion on separate figures.

d) Do you think the trajectories generated in Problem 5f) in the nonlinear CR3BP might supply good initial guesses for planar periodic orbits around L_4? Justify your response. If you think they might supply poor initial guesses, also discuss how you could generate trajectories from this linearization-based analysis that might supply better initial guesses.