ASEN 5050, Fall 2024, Jash Bhalavat

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ASEN 5050
Fall 2024
Jash Bhalavat

$\boxed{HW3}$

Problem 1 → Given: $GM_{saturn} = 3.794 \times 10^7 \ km^3/s^2$, $R_{saturn,eq} = 60,268$ km

Assumptions: $M_{s/c} \ll M_{saturn} \rightarrow \therefore GM_{saturn} = \mu_{saturn}$, s/c does not break up in atmosphere. Saturn is perfect sphere with radius = $R_{saturn,eq}$. 2 Body Problem

Given: saturn-centered inertial frame at t_1

$\vec{r}_1 = -720{,}000\hat{x} + 670{,}000\hat{y} + 310{,}000\hat{z}$ km

$\vec{v}_1 = 2.160\hat{x} - 3.360\hat{y} + 0.620\hat{z}$ km/s

a] $|\vec{r}_1| = 1.0312 \times 10^6$ km , $|\vec{v}_1| = 4.0422$ km/s

$|\vec{h}| = h = |\vec{r}_1 \times \vec{v}_1| = |1.4570 \times 10^6\hat{x} + 1.1160 \times 10^6\hat{y} + 0.9720 \times 10^6\hat{z}| = 2.0768 \times 10^6 \ \frac{km^2}{s}$

$\hat{n} = \hat{z} \times \vec{h}/h = -0.6081\hat{x} + 0.7939\hat{y} + 0\hat{z}$

$\vec{e} = \frac{1}{\mu_{saturn}}\left[\left(v_1^2 - \frac{\mu}{r_1}\right)\vec{r}_1 - (\vec{r}_1 \cdot \vec{v}_1)\vec{v}_1\right] = 0.5939\hat{x} - 0.6812\hat{y} - 0.1080\hat{z}$

$e = |\vec{e}| = \boxed{0.9102 = e}$

$\mathcal{E} = \frac{v_1^2}{2} - \frac{\mu_{saturn}}{r_1} = -28.6218 \ \frac{km^2}{s^2}$

$a = -\frac{\mu_{saturn}}{2\mathcal{E}} = \boxed{6.6287 \times 10^5 \ km = a}$

$i = \cos^{-1}\left(\frac{\vec{h} \cdot \hat{z}}{h}\right) = \boxed{62.0936° = i}$ ← always +ve by definition

$\Omega = \cos^{-1}(\hat{n} \cdot \hat{x}) = \boxed{127.4506° = \Omega}$ ← +ve because $\hat{n} \cdot \hat{y} > 0$

$\omega = \cos^{-1}\left(\frac{\hat{n} \cdot \vec{e}}{e}\right) = \boxed{-172.2803° = \omega}$ ← −ve because $\vec{e} \cdot \hat{z} < 0$

$\theta^* = \cos^{-1}\left(\frac{\vec{e} \cdot \vec{r}_1}{e \cdot r}\right) = \boxed{-167.8322° = \theta^*}$ ← −ve because $\vec{r}_1 \cdot \vec{v}_1 < 0$

Let t_2 be time at impact on surface

$r_2 = \frac{h^2/\mu_{saturn}}{1 + e\cos\theta_2^*} = R_{saturn,eq} \rightarrow 1 + e\cos\theta_2^* = \frac{h^2/\mu_{saturn}}{R_{saturn,eq}} \rightarrow \cos(\theta_2^*) = \frac{\left(\frac{h^2/\mu_{sat}}{R_{sat,eq}} - 1\right)}{e}$

$\therefore \theta_2^* = \pm 13.1687°$

θ_1^* is −ve. That means s/c has crossed apoapsis and approaching periapsis. Periapsis is closest point of orbit. If θ_2^* is +ve, it would mean that s/c impacts saturn after crossing the closest point (periapsis). That is not possible. $\therefore$ θ_2^* has to be negative. → $\theta_2^* = -13.1687°$

$h, a, i, e, \Omega, \omega$ remain constant. $\therefore \vec{r}_2, \vec{v}_2$ can be calculated.

$$v_{2r} = \frac{\mu}{h} e \sin\theta_2^* = -3.7882 \text{ km/s}$$
$$v_{2\theta} = \frac{\mu}{h}(1+e\cos\theta_2^*) = 34.4594 \text{ km/s}$$
$$\rightarrow \vec{v}_{2,\hat{r}\hat{\theta}\hat{h}} = -3.7882\hat{r} + 34.4594\hat{\theta} + 0\hat{h} \text{ km/s}$$

$$\vec{r}_{2,\hat{r}\hat{\theta}\hat{h}} = r_{sat,eq}\hat{r} + 0\hat{\theta} + 0\hat{h} = 60{,}268\hat{r} + 0\hat{\theta} + 0\hat{h} \text{ km}$$

$$\theta = \theta_2^* \mp \omega = -185.4490°$$

$$\vec{r}_{2,xyz} = R_3(-\Omega)R_1(-i)R_3(-\theta)\vec{r}_{2,\hat{r}\hat{\theta}\hat{h}} = \boxed{3.4356\times10^4\hat{x} - 4.9258\times10^4\hat{y} + 5.0676\times10^3\hat{z} \text{ km} = \vec{r}_{2,xyz}}$$

$$\vec{v}_{2,xyz} = R_3(-\Omega)R_1(-i)R_3(-\theta)\vec{v}_{2,\hat{r}\hat{\theta}\hat{h}}$$
$$\boxed{\vec{v}_{2,xyz} = 12.5761\hat{x} + 10.2611\hat{y} - 30.6325\hat{z} \text{ km/s}}$$

6] 2-Body problem assumptions:

① $m_{s/c} \ll m_{saturn}$ – Holds

$m_{s/c}$ is constant – NO, as the s/c hits the atmosphere, it will break up

② coordinate system is inertial – Holds

③ Saturn is point mass – Saturn is very oblate and since orbit is not equatorial ($i \neq 0$), it will perturb.

④ Gravity because of saturn–s/c is only force – NO saturn has 146 moons. They will affect s/c orbit. As s/c passes through atmosphere, drag will also affect orbit. Saturn's rings will also affect orbit

Since, this is an impact orbit, 2BP is not the best approximation for this system. This is a great start, but needs a lot more work to approximate this dynamical environment.

Problem 2

- Part b, c

(2)

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2BP

Problem 2 → Assume: $GM_{mars} = 42828.3143\ km^3/s^2 \leftarrow m_{S/C} \ll m_{mars}$, $\mu = GM_{mars}$

Given: $a = 6463.8$ km, $e = 0.45454$, $i = 74.924°$, $\Omega = 1.2410°$, $\omega = 353.31°$, $\theta^* = 199.38°$, $(\omega = -6.69°)$ [Because our range is [-180°, 180°]]

b) $P = 2\pi\sqrt{a^3/\mu_{mars}} = \boxed{1.5778 \times 10^4\ sec = P}$

$r_p = a(1-e) = \boxed{3.5257 \times 10^3\ km = r_p}$

c) $a(1-e^2) = h^2/\mu_{mars} \rightarrow h = \sqrt{a\mu_{mars}(1-e^2)} = 1.4820 \times 10^4\ km^2/s$

$v_r = \frac{\mu_{mars}}{h} e\sin(\theta^*) = -0.4359$ km/s

$v_\theta = \frac{\mu_{mars}}{h}(1+e\cos(\theta^*)) = 1.6507$ km/s

$\rightarrow \vec{v}_{\hat{r}\hat{\theta}\hat{h}} = -0.4359\,\hat{r} + 1.6507\,\hat{\theta} + 0\,\hat{h}$ km/s

$r = \frac{h^2/\mu_{mars}}{1+e\cos\theta^*} = 8.9779 \times 10^3\ km \rightarrow \vec{r}_{\hat{r}\hat{\theta}\hat{h}} = 8.9779 \times 10^3\,\hat{r} + 0\,\hat{\theta} + 0\,\hat{h}$ km

(our range is [-180°, 180°]) $\theta^* = 199.38° \rightarrow = -160.62°$, $\omega = 353.31° = -6.69°$

$\theta = \theta^* + \omega = -167.31°$

$\vec{r}_{xyz} = R_3(-\Omega)R_1(-i)R_3(-\theta)\cdot\vec{r}_{\hat{r}\hat{\theta}\hat{h}} = \boxed{-8.7455 \times 10^3\,\hat{x} - 0.702 \times 10^3\,\hat{y} - 1.9044 \times 10^3\,\hat{z}\ km = \vec{r}_{xyz}}$

$\vec{v}_{xyz} = R_3(-\Omega)R_1(-i)R_3(-\theta)\cdot\vec{v}_{\hat{r}\hat{\theta}\hat{h}} = \boxed{0.7962\,\hat{x} - 0.3768\,\hat{y} - 1.4625\,\hat{z} = \vec{v}_{xyz}}$

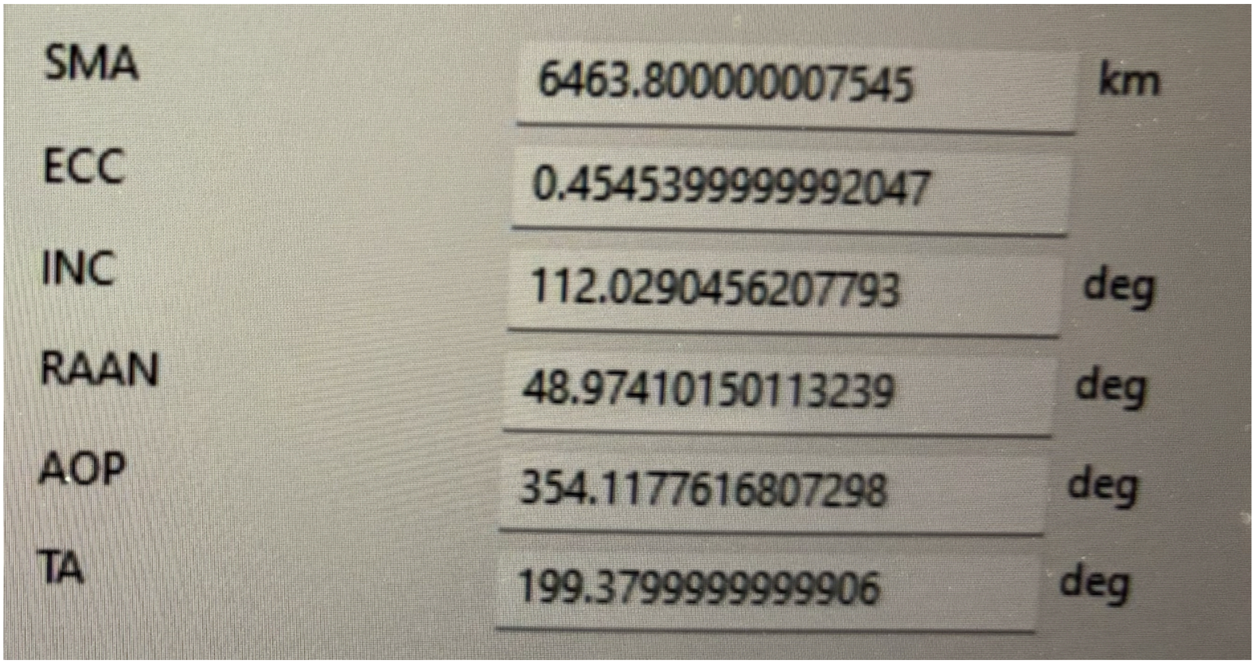
GMAT marsICRF Initial condition:

$\vec{r}_{xyz} = -8.7455 \times 10^3\,\hat{x} - 0.7025 \times 10^3\,\hat{y} - 1.9044 \times 10^3\,\hat{z}$ km

$\vec{v}_{xyz} = 0.7962\,\hat{x} - 0.3768\,\hat{y} - 1.4625\,\hat{z}$ km/s

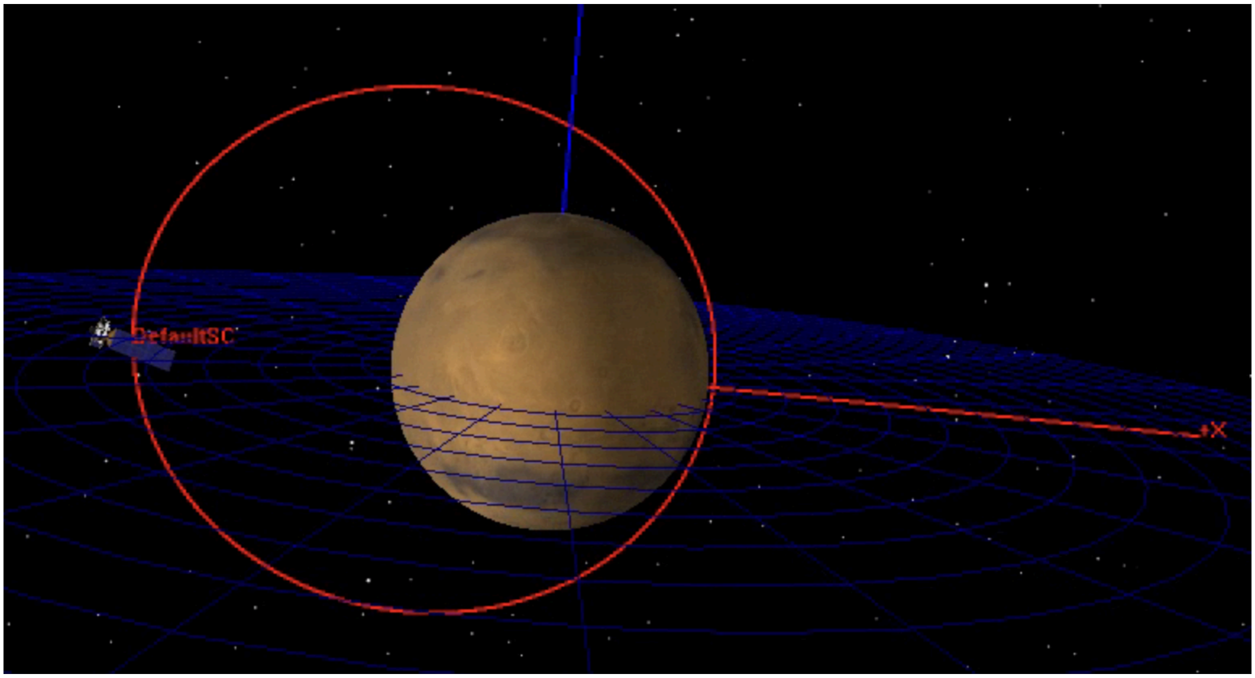
These are very consistent with the calculated $\vec{r}, \vec{v}$ in inertial frame. 1)

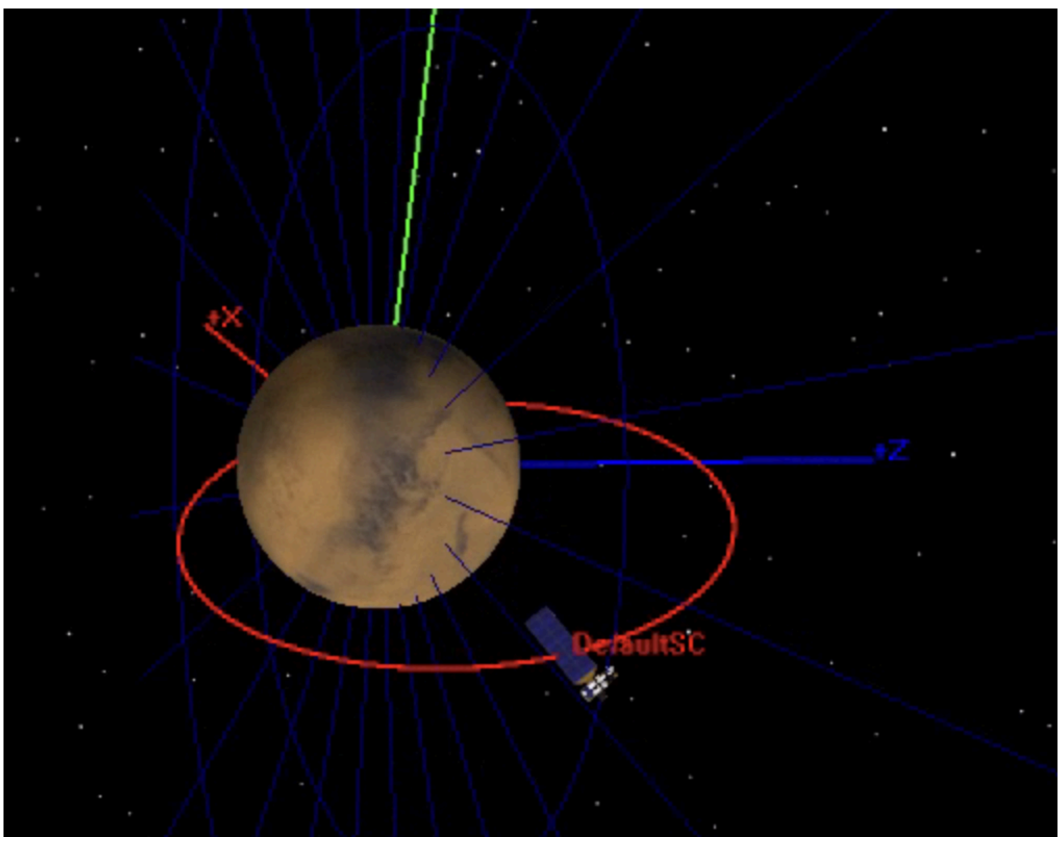
- Part d



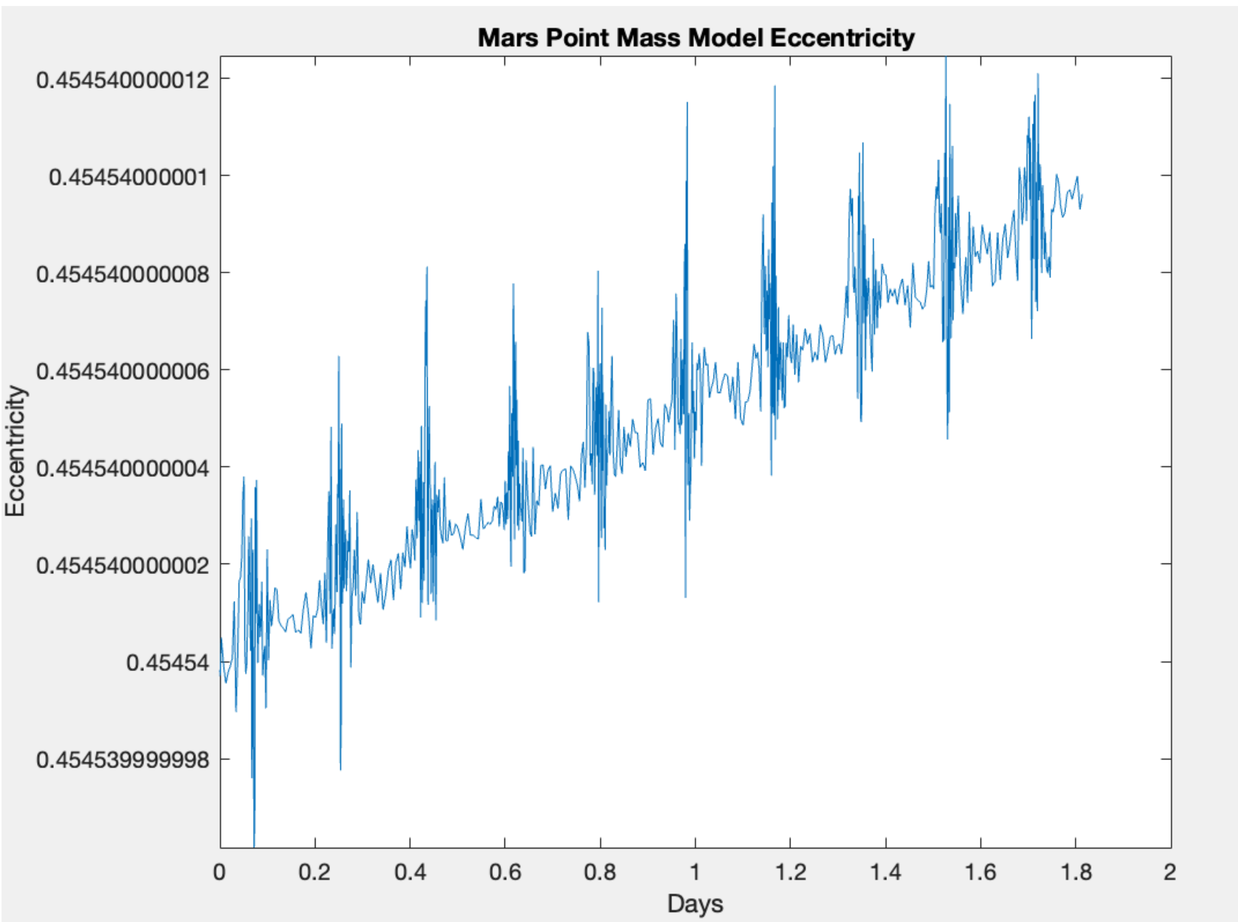
RAAN has increased to 354.11° which indicates that the first axis hasn't changed much, because the RAAN went from 1.24° to -5.89°. That's a negligible difference. But, the inclination has increased to 112.03° which indicates that that orbit is not retrograde and the XY plane has moved closer to the negative angular momentum vector.

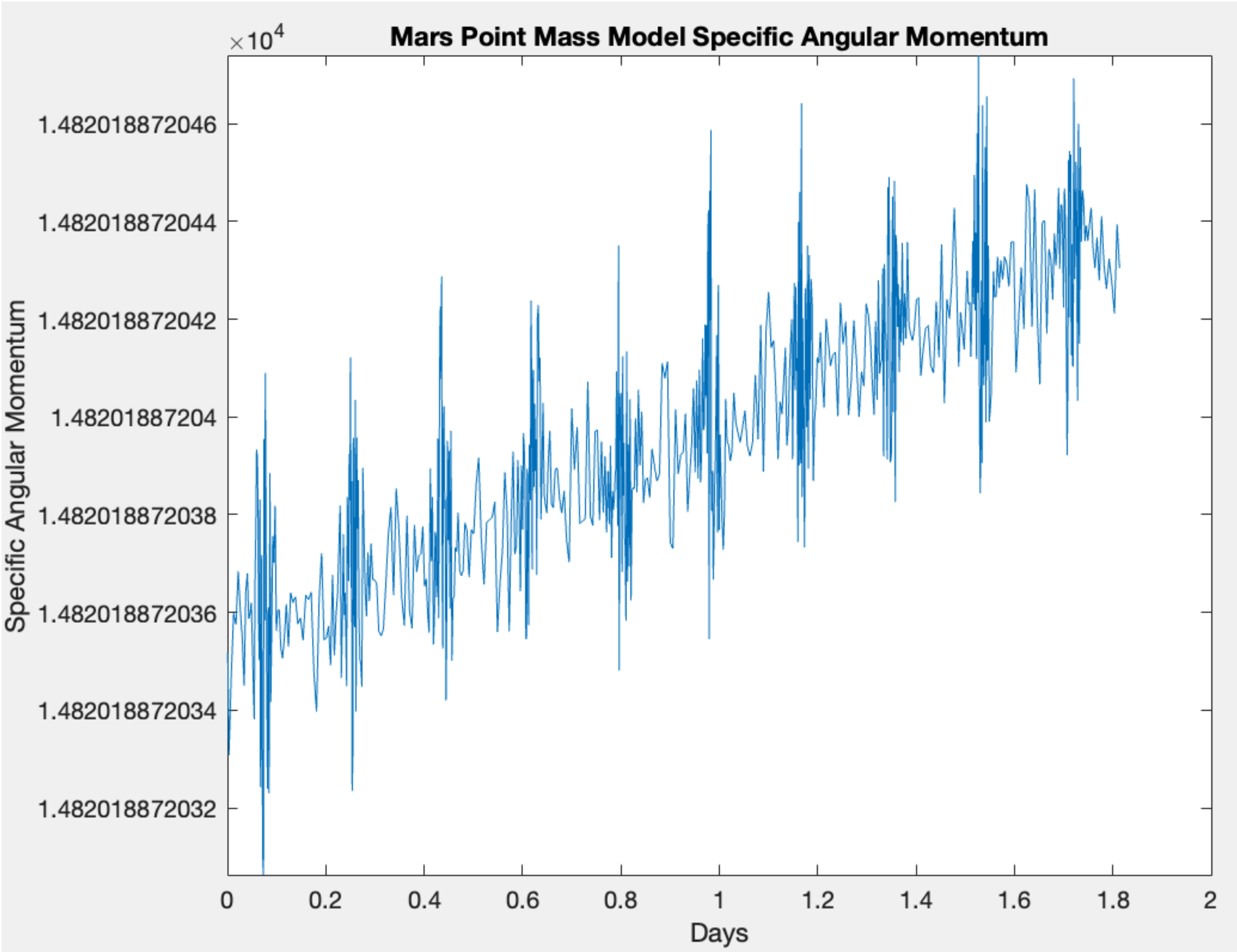
- Part e



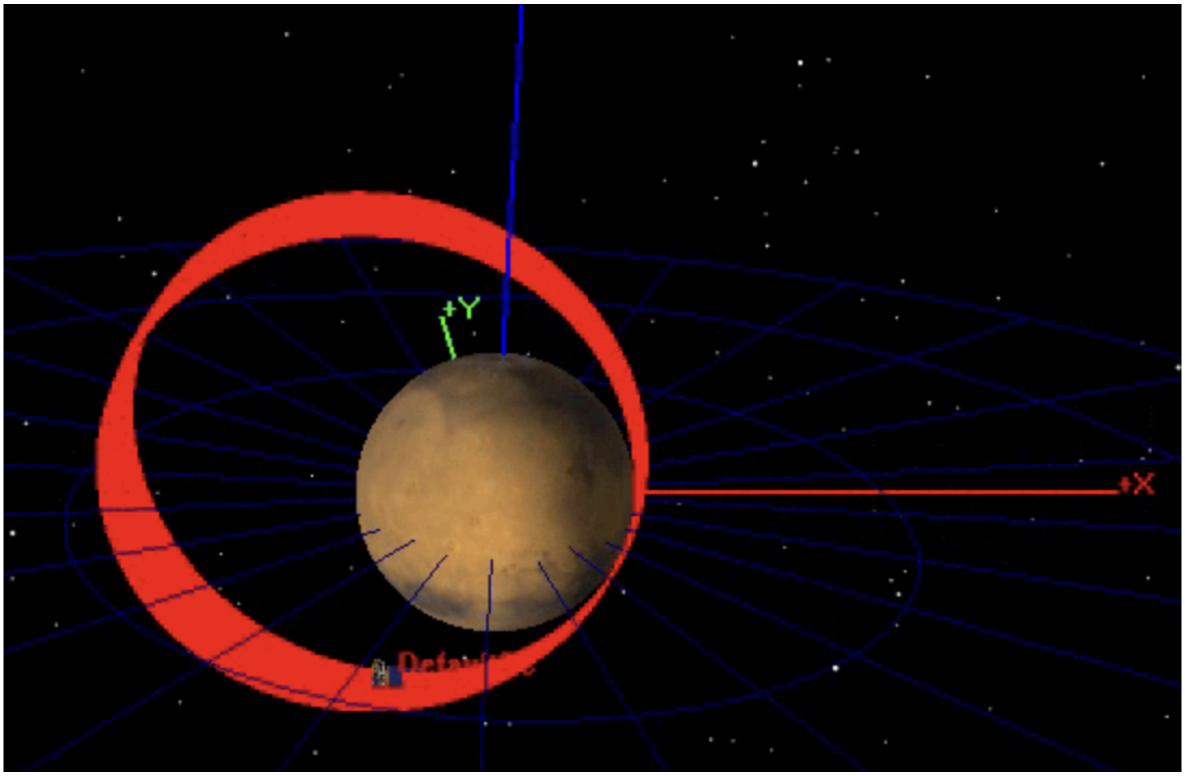


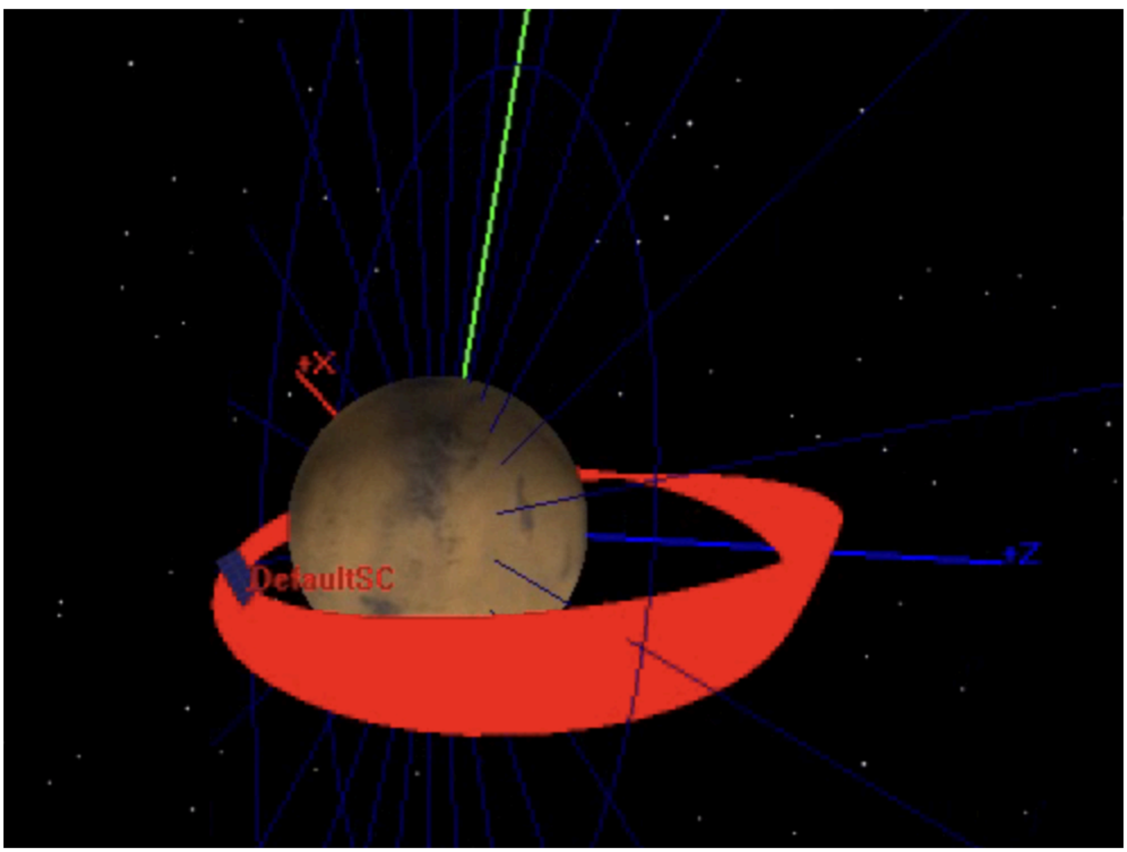
- Part f
  - The orbit is elliptical. It is prograde with a periapsis near the +X axis. It is not equatorial, but closer to a polar orbit. Its periapsis (3.5257e3 km) is very close the surface of Mars (3.3972e3 km) - about 128.5 km. Given the true anomaly, the spacecraft starts close to apoapsis and is moving towards periapsis. The orbit meets its periapsis closer to the equatorial plane of Mars.
- Part g
  - Eccentricity and Specific Angular Momentum are very consistent with slight increase in magnitude (plots below). Given that a point mass model is used, it makes sense that orbital elements that are supposed to be constant over an orbit in a 2-body problem are consistent. The slight increase can be attributed to 3rd body effects of other celestial bodies.




Mars Point Mass Model Specific Angular Momentum
×10^4
1.482018872046
1.482018872044
1.482018872042
1.48201887204
1.482018872038
1.482018872036
1.482018872034
1.482018872032
Specific Angular Momentum
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Days

- Part h





- Part i
  - Multiple orbital elements seem to change over time
    - A - the orbit’s apoapsis and periapsis change
    - I - the orbit’s inclination changes as time goes by
  - The orbital plane shifts over time
    - As seen in the pictures below, the apoapsis changes a lot more than periapsis indicating that the orbital plane shifts around an axis closer to the focus of the orbit (near Mars)

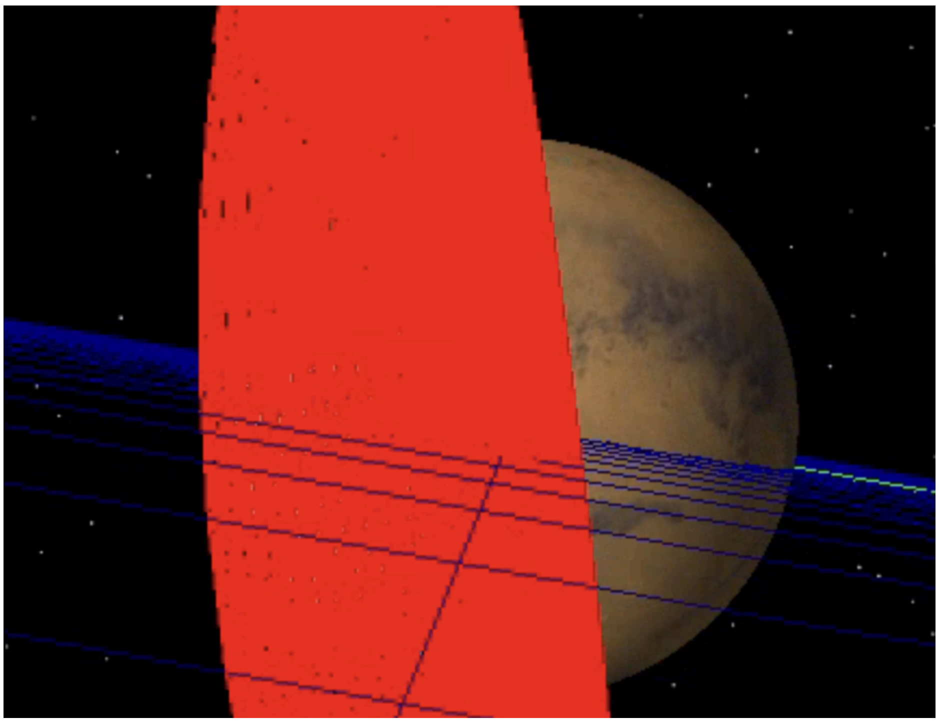


Figure showing change in apoapsis over time

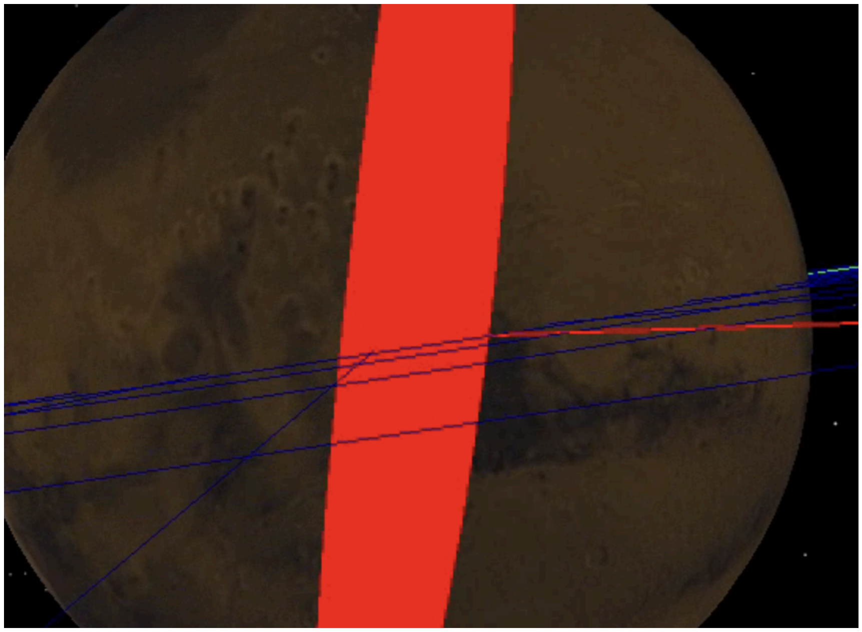
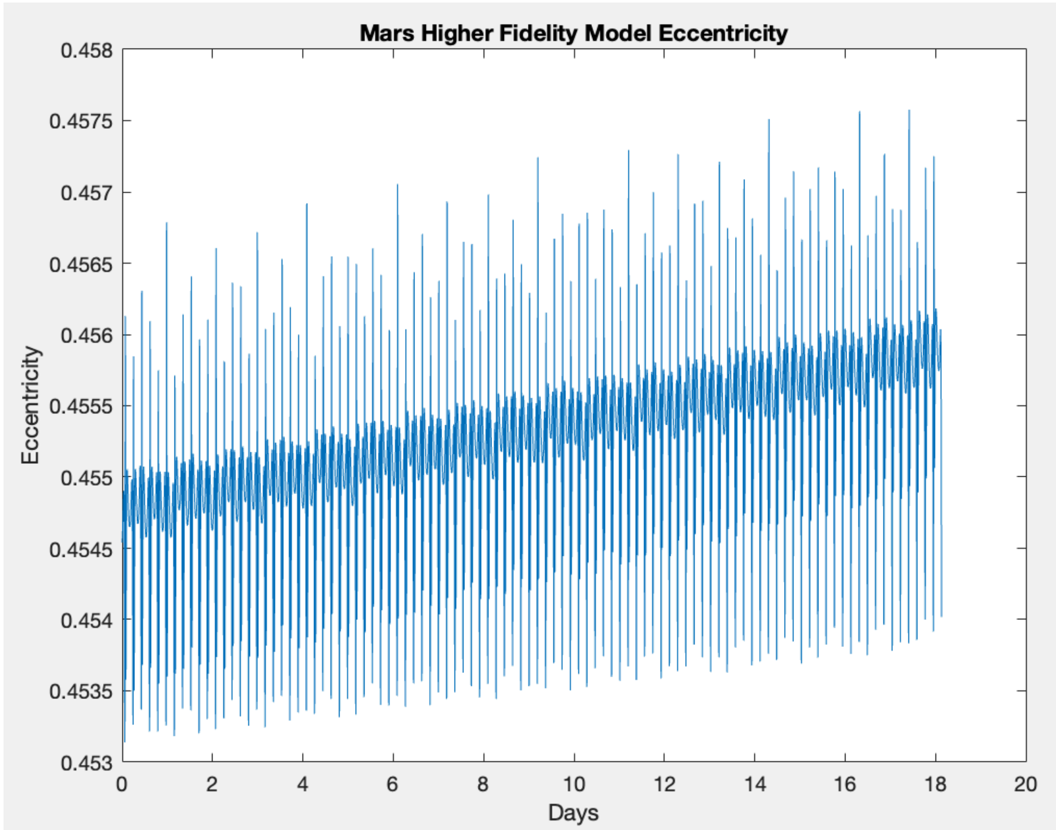
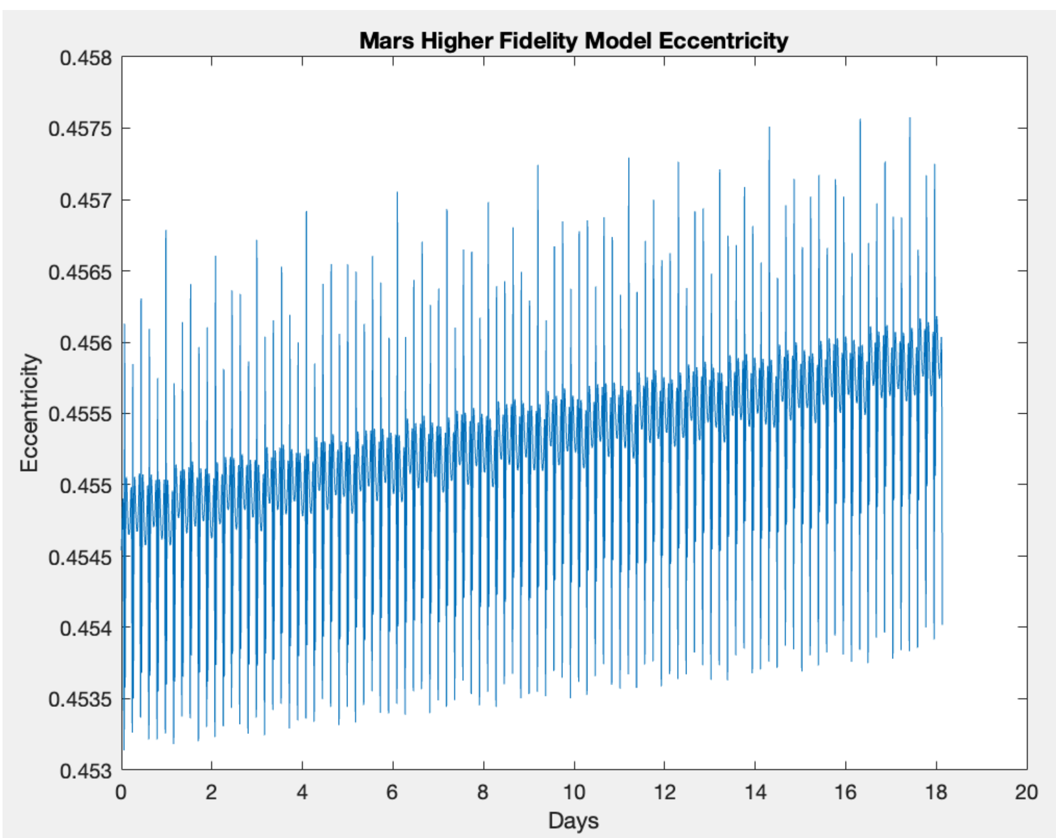


Figure showing change in periapsis over time

- Part j
  - It is expected that the eccentric and specific angular momentum (plots below) change more when a higher fidelity gravitational model is used instead of the point mass model and that's what's observed. A higher fidelity gravitational model ensures that the orbit is no longer following a 2-body problem and that introduces perturbations to the orbit which lead to non-consistent orbital elements. Hence, elements that were previously assumed to be consistent over an orbital period (such as energy, eccentricity, angular momentum, etc.) are now changing relatively drastically.


Mars Higher Fidelity Model Eccentricity
Eccentricity
Days
0.458
0.4575
0.457
0.4565
0.456
0.4555
0.455
0.4545
0.454
0.4535
0.453
0
2
4
6
8
10
12
14
16
18
20


Mars Higher Fidelity Model Eccentricity
Eccentricity
Days
0.458
0.4575
0.457
0.4565
0.456
0.4555
0.455
0.4545
0.454
0.4535
0.453
0
2
4
6
8
10
12
14
16
18
20

- Part k
    - The epoch is 13 Feb 2019 00:00:00.000. As seen on MAVEN's website, it looks like Maven uses "Deep Dips" to lower its apoapsis to prepare for the Mars 2020 rover missions and provide relay capabilities. This means that gravity is not the only force during this period (which violates one of the assumptions made for a 2-body problem). That's why the 2-body problem is not a sufficiently good approximation of the dynamical environment to reliably design the trajectory of the MAVEN spacecraft.