ASEN 6060
ADVANCED ASTRODYNAMICS
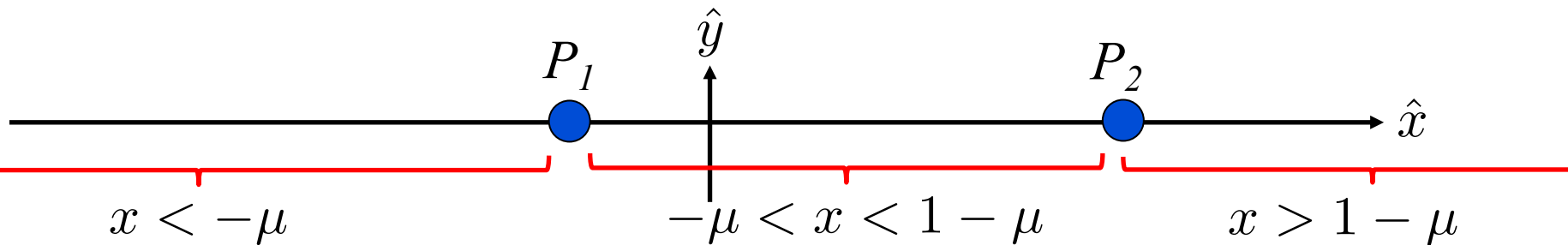## Week 3 Discussion

Objectives:

- Understand important considerations in numerically calculating equilibrium point locations

- Connect the evolution of ZVCs as a function of Jacobi constant to the itinerary and maneuvers performed along a trajectory

Numerically calculating the *x*-coordinates of the collinear equilibrium points

$$x - \frac{(1-\mu)(x+\mu)}{(|x+\mu|)^3} - \frac{\mu(x-1+\mu)}{(|x-1+\mu|)^3} = 0$$

$$x < -\mu \qquad -\mu < x < 1-\mu \qquad x > 1-\mu$$

Question 1: How do you select a suitable initial guess along each of the three intervals?

Question 2: How do you verify if your solution corresponds to an equilibrium point?

Questions 1 and 2

Question 1: How do you select a suitable initial guess along each of the three intervals?

Group Brainstorming:

- Visual analysis of the ZVCs and estimates / plot and analyze U*
- Analytical approximations for some mass ratios
- Add a delta x from P1 and P2, calculate convergence interval?
- Continuation in mu: use Li from nearby mu value as initial guess
- Try other root-finding methods? E.g., bisection

Question 2: How do you verify if your solution corresponds to an equilibrium point?

Group Brainstorming:

- Plug into original function and confirm that it equals zero to within a desired tolerance (what is a good tolerance?)
- Significant digits? 10 or 12 for all

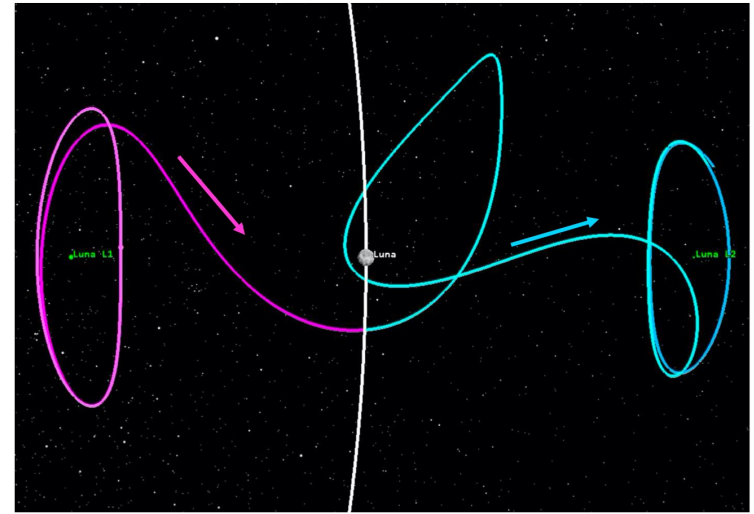# *Transfer Example in Earth-Moon CR3BP*

Maneuver #1

Maneuver #3

Maneuver #2

(a) Transfer from L_1 Lyapunov to L_2 Lyapunov orbit
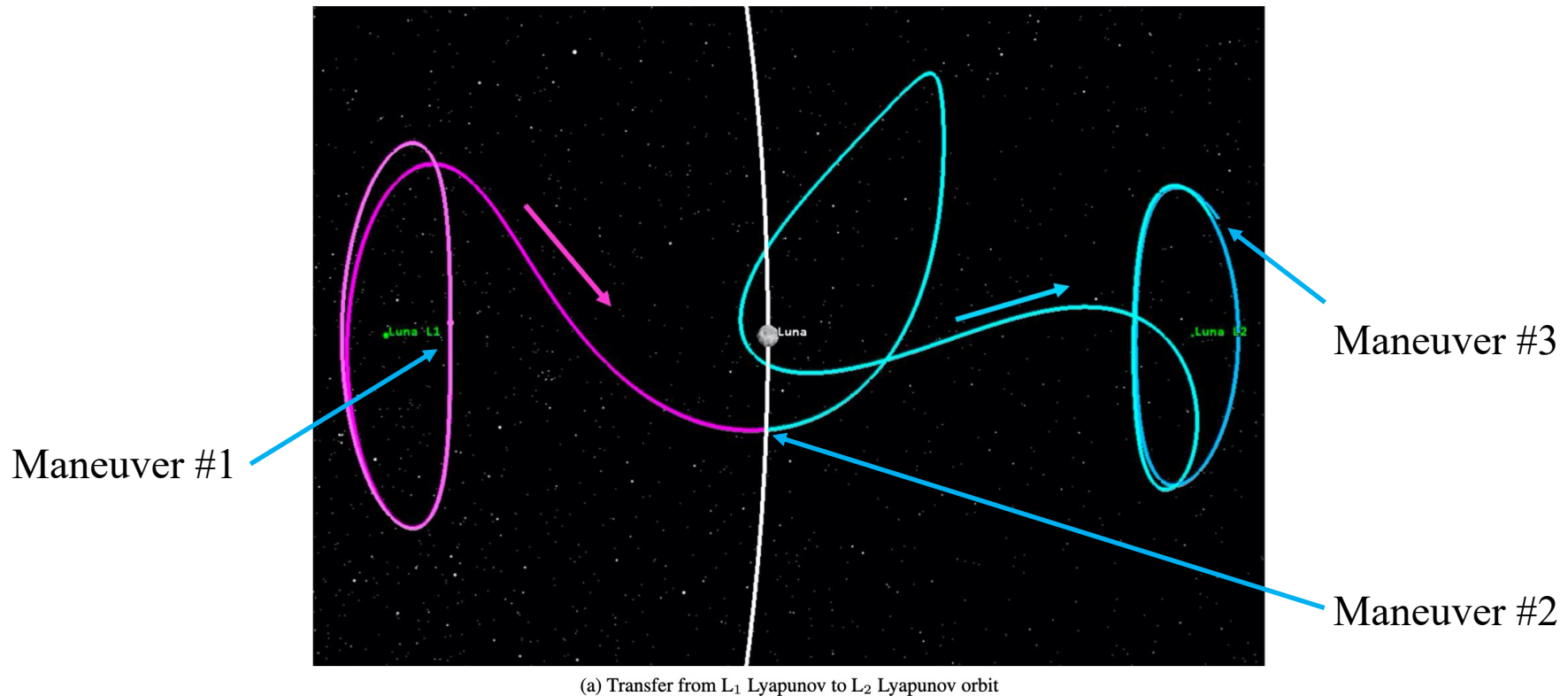
Question 3: Based on your knowledge of the CR3BP, what are the constraints on the value of C along the arcs comprising this transfer?

Image credit: Short, Haapala, Bosanac, 2020, "Technical Implementation of the Circular Restricted Three-Body Model in STK Astrogator", AAS/AIAA Astrodynamics Specialist Conference.
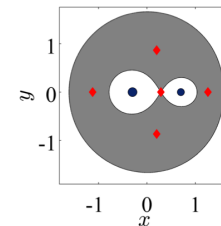
Question 3: Group Brainstorming:
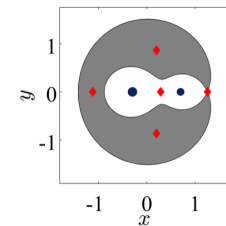
- Arc 1: C < C(L1)

- Arc 2: C < C(L1).

- Arc 3: C < C(L2)

- Arc 4: C < C(L2)

- Determine these by using the constraints from the ZVCs. Cannot visually identify minimum value

- Between each arc, we cannot tell if the Jacobi constant is changing and, if so, how.
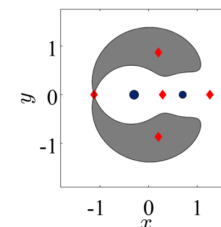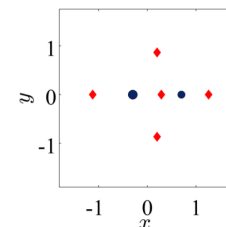

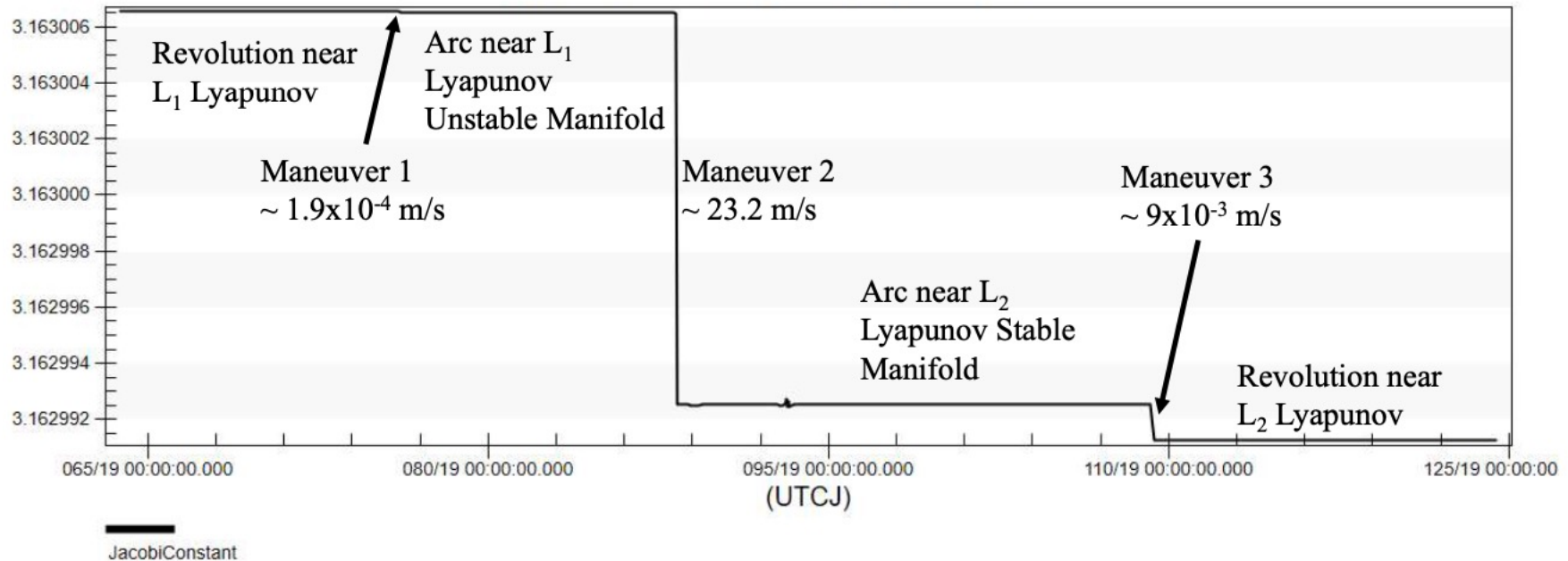(a) Transfer from L_1 Lyapunov to L_2 Lyapunov orbit

(a) $C(L_1)$

(b) $C(L_2)$

(c) $C(L_3)$

(d) $C(L_4) = C(L_5)$

Transfer Example in Earth-Moon CR3BP

Time evolution of C along transfer, calculated in STK

Revolution near L_1 Lyapunov

Arc near L_1 Lyapunov Unstable Manifold

Maneuver 1 $\sim 1.9 \times 10^{-4}$ m/s

Maneuver 2 ~ 23.2 m/s

Maneuver 3 $\sim 9 \times 10^{-3}$ m/s

Arc near L_2 Lyapunov Stable Manifold

Revolution near L_2 Lyapunov

JacobiConstant

(b) Jacobi constant history for transfer

For this example: $C < C(L2)$ throughout

Image credit: Short, Haapala, Bosanac, 2020, "Technical Implementation of the Circular Restricted Three-Body Model in STK Astrogator", AAS/AIAA Astrodynamics Specialist Conference.