<u>ASEN 6060 Spring 2025: Homework 3</u>

Due: 3/4/2024 at 9pm MT

Notes:

- Use the mass ratio calculated in HW 1 for the Earth-Moon system (if there were errors in your calculations, correct them first)
- In your writeup, show all your working and include a discussion (see the syllabus for more information about expected components of a homework submission)
- Submit all scripts used for this homework in Gradescope and associate the requested scripts to each applicable subproblem.

Problem 1:

Write a script to simultaneously numerically integrate the state vector and the state transition matrix along a trajectory in the CR3BP. Include the text of this script created in this subproblem in your submission. In your writeup, discuss the setup of your script and list (in mathematical notation) the first-order differential equations that you are integrating. If you have made any significant changes to the numerical integration scheme and/or tolerances used in HW 1 as you work through this problem, please update us on those changes in your discussion so that we can give you any additional feedback.

Problem 2:

a) Write a script to compute a general three-dimensional periodic orbit via single shooting, taking as an input an initial guess for a free variable vector and the system parameters, and outputting the free variable vector corresponding to a solution. For this script, use the approach labeled "General Variable-Time Formulation" in the lecture notes so that you can compute any general periodic orbit (use the modified constraint formulation). Mathematically define, discuss, and justify the free variable and constraint vectors you have formulated and derive the *DF* matrix in your writeup. Also define and discuss any additional details related to your numerical corrections procedure, implementation, and any governing parameters. Include the text of the corrections scripts created in this subproblem in your submission.
b) Using the approach from HW 2, use linearization around L_1 in the Earth-Moon system to create an initial guess for an L_1 Lyapunov orbit. Report the initial guess, discuss why you selected it and comment on whether you think it is a reasonable initial guess. Note: please try to balance selecting this initial guess to be small enough to produce a decent initial guess, but large enough to have some initial discontinuity so that you can see the corrector work over multiple iterations.
c) Correct the initial guess and graphically display the norm of the constraint vector at each iteration (I recommend displaying this quantity on a log10 scale for visual clarity). Use this plot to discuss in detail whether you think you implemented your corrections algorithm correctly and whether it was successful.
d) Plot the corrected solution and the initial guess in the *xy*-plane of the rotating frame in the same figure; add to this plot any additional annotations that may be useful. Report the

initial state vector and orbit period associated with the final solution that you compute via your corrections algorithm.

Problem 3:

a) Implement natural parameter continuation to compute additional periodic orbits along the Earth-Moon L_1 Lyapunov family using the *x*-coordinate of one of the crossings of the *x*-axis as the natural parameter; continue to generate periodic orbits along this family as far as you can. Discuss your numerical implementation of this continuation method and the selection of any governing parameters. Include the text of the continuation scripts created in this subproblem in your submission.

b) Plot a sample of the orbits you computed along this family (i.e., rather than plot every single orbit that is computed, it often helps to plot every *n*th orbit for clearer visualization). In a separate figure, represent the family in a two-parameter space with the orbital period (in days) on the horizontal axis and the Jacobi constant on the vertical axis. Discuss how far along this family you have computed orbits with this continuation method and the step size you selected.

Problem 4:

Use the corrections algorithm you developed in Problem 2 to compute an Earth-Moon L_1 halo orbit using the following initial guess for a nondimensional state along the orbit and the orbit period:

$\bar{x}_0 = [0.82340, 0, -0.026755, 0, 0.13742, 0]^T$ and $T = 2.7477$

Note that I have provided a truncated state vector in this homework, but it is typical to report more digits (e.g., at least 12) when listing the state along a corrected periodic orbit.

Correct the initial guess and graphically display the norm of the constraint vector at each iteration. Report the initial state vector and orbit period associated with the final solution that you compute via your corrections algorithm. Assess in detail the performance of your algorithm in successfully recovering an L_1 halo orbit. Plot the corrected solution and the initial guess in the rotating frame in the same figure using different colors; add to this plot any additional annotations that may be useful.

Problem 5:

a) Implement pseudo-arclength continuation to compute additional periodic orbits along the Earth-Moon L_1 halo family, starting from the initial periodic orbit calculated in Problem 4; continue to generate periodic orbits as far as you can along this family. Discuss your numerical implementation of this continuation method and the selected values of any governing parameters. Include the text of the continuation scripts created in this subproblem in your submission.

b) Plot a sample of the orbits you computed along this family (i.e., rather than plot every single orbit that is computed, it often helps to plot every *n*th orbit for clearer visualization). In a separate figure, represent the family in a two-parameter space with the orbital period (in days) on the horizontal axis and the Jacobi constant on the vertical axis. Discuss how far along the Earth-Moon L_1 halo family you have computed orbits with this continuation method and the step size you selected in each direction along the family.