ASEN 6060
ADVANCED ASTRODYNAMICS
Additional Dynamical Models, Pt 2

Objectives:

- Briefly summarize some significant components of ephemeris models

Ephemeris Models

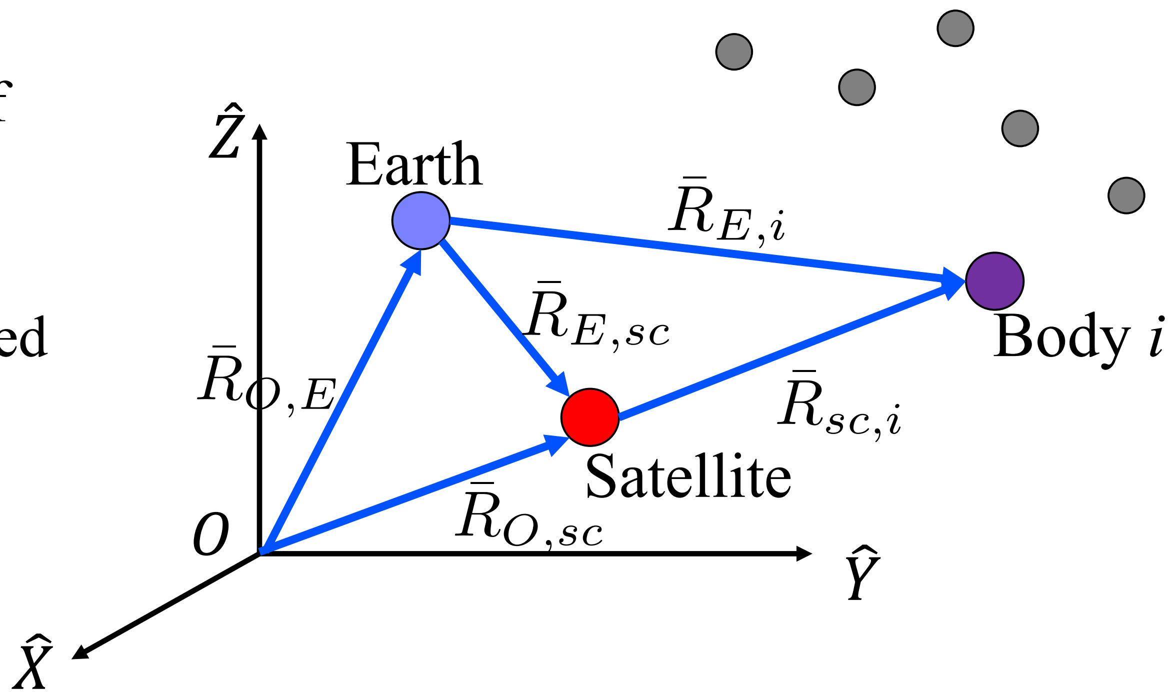
- Offer higher-fidelity representation of dynamical environment
  - Designer must specify necessary accuracy based on design/analysis needs and region of solar system
  - Multiple formulations may be possible, must always specify components and parameter sources
- Modeling each contribution may be difficult based on known information at specific stage of design/analysis
- Model fidelity also influences computational complexity

Third Body Effects

Consider a satellite located relative to the Earth, influenced by the point mass gravity of *n* additional bodies (not inc. Earth)

$\hat{X}\hat{Y}\hat{Z}$ Axes of inertial frame

O Inertially-fixed origin



Third Body Effects

Write the equations of motion for the satellite and the Earth:

$$\ddot{\bar{R}}_{O,sc} = -\frac{Gm_E}{R_{E,sc}^3}\bar{R}_{E,sc} - \sum_{i=1}^{n}\left(\frac{Gm_i}{R_{i,sc}^3}\bar{R}_{i,sc}\right)$$

$$\ddot{\bar{R}}_{O,E} = -\frac{Gm_{sc}}{R_{sc,E}^3}\bar{R}_{sc,E} - \sum_{i=1}^{n}\left(\frac{Gm_i}{R_{i,E}^3}\bar{R}_{i,E}\right)$$

Subtracting the two: $\ddot{\bar{R}}_{E,sc} = \ddot{\bar{R}}_{O,sc} - \ddot{\bar{R}}_{O,E}$

Rearranging:

$$\ddot{\bar{R}}_{E,sc} = \underbrace{-\frac{G(m_E + m_{sc})}{R_{E,sc}^3}\bar{R}_{E,sc}}_{\text{2BP}} \overbrace{- \sum_{i=1}^{n} Gm_i \left(\frac{\bar{R}_{sc,i}}{R_{sc,i}^3} - \frac{\bar{R}_{E,i}}{R_{E,i}^3}\right)}^{\text{Perturbations due to “third body”}}$$

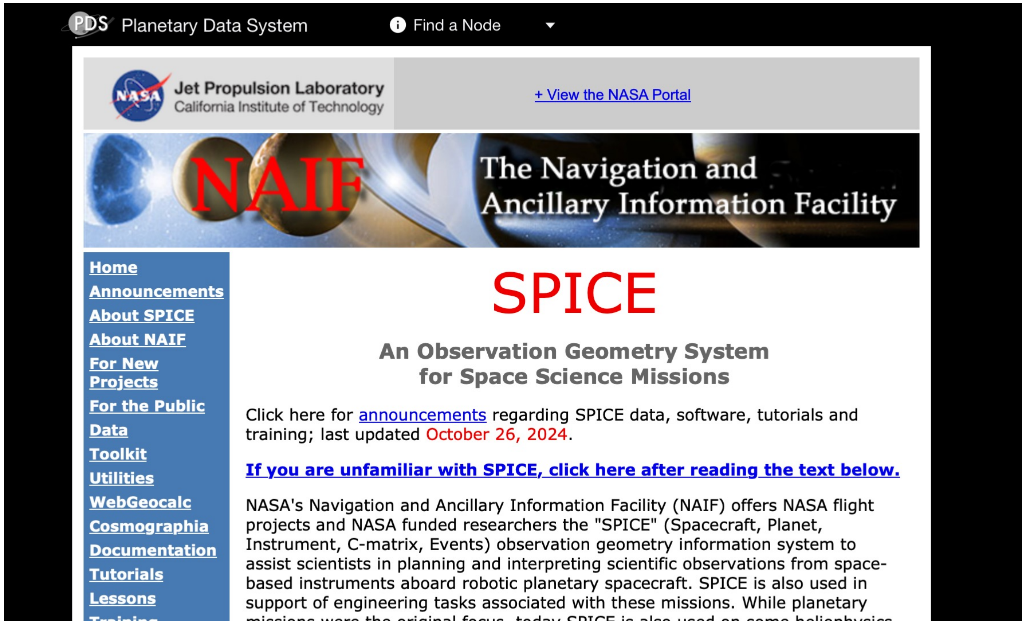
Direct effect of body i on s/c

Indirect effect of body i on Earth

Celestial Body Ephemerides

NASA's Navigation and Ancilliary Information Facility (NAIF) publishes:

- Planetary and lunar ephemeris data in Development Ephemeris (DE) files
- SPICE (Spacecraft, Planet, Instrument, C-matrix, Events) Toolkit with functions to interpret data & compute information



https://naif.jpl.nasa.gov/naif/

Celestial Body Ephemerides

Planetary and lunar ephemeris data:

- Contains information to extract states of celestial bodies at desired epochs via SPICE files, body parameters
- DE files are updated over time, given new labels
- Most recent files are DE440 and DE441 (might see GMAT/STK use DE405 or DE421 depending on settings)

Different DE files may produce small differences in ephemerides:

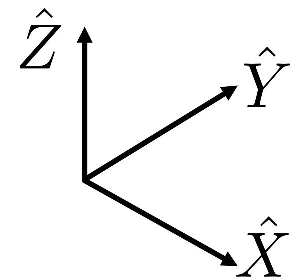
- DE440 uses the solar system barycenter (SSB) calculated from "the Sun, barycenters of eight planetary systems, the Pluto system barycenter, 343 asteroids, 30 KBOs, and a KBO ring representing the main Kuiper belt." (Park et al., 2021, pg. 2)
- SSB shifts by ~100 km in DE440 compared with DE430 (Park et al., 2021, pg. 2)

Coordinate Systems

Reference system: “constants, conventions, models, and parameters, which serve as the necessary basis for the mathematical representation of geometric and physical quantities” (Drewes, 2009)

When defining a reference system, specify:

- Origin
- Axes (e.g., right-handed, orthogonal triad)
- Scale (magnitude of vectors defining axes)
- Time evolution of these quantities



Common types of coordinate systems, used to define position of object within a reference system:

- Cartesian: three components are projection of vector onto axes
- Spherical: two angles and distance
- Elliptic: two angles and height above reference ellipsoid

Ref: Drewes, H., 2009, “Reference Systems, Reference Frames, and the Geodetic Datum," In: Sideris M.G. (eds) Observing our Changing Earth International Association of Geodesy Symposia, Vol 133. Springer, Berlin, Heidelberg.

Coordinate Frames

- A ‘frame’ often denotes an implementation of a ‘system’ (Drewes, 2009)

- Inertial frame:
  - Fixed axes in space and a non-accelerating origin
  - Two standards for specifying ‘inertial’ frames for astrodynamics

- Body-fixed frame:
  - Origin is center of body or on surface
  - Axes rotate with a celestial body
  - Relies on parameters describing the body’s orientation in 3D space

Ref: Drewes, H., 2009, “Reference Systems, Reference Frames, and the Geodetic Datum,” In: Sideris M.G. (eds) Observing our Changing Earth International Association of Geodesy Symposia, Vol 133. Springer, Berlin, Heidelberg.

Modern Inertial Frames

ICRF: International Celestial Reference Frame

- Close representation of an inertial frame
- Calculated by IAU
- Nonrotating with respect to extragalactic radio sources (quasars)
- Updated every several years since inception, with better observational data
- ICRF2 in 2009, ICRF3 in 2019
- Origin at barycenter of solar system.

Geocentric Celestial Reference Frame (GCRF): uses the axes of ICRF and origin at center of Earth

ICRF/GCRF are modern standards for inertial frames
Also use ICRF axes in defining Moon-centered inertial frame

Useful Terminology

Ecliptic: plane containing Earth's mean orbit relative to the Sun
Inclined relative to Earth's mean equatorial plane by angle $\epsilon \approx 23.5^\circ$

Vernal Equinox: direction to ascending node of ecliptic plane relative to Earth's mean equatorial plane

Equatorial plane: plane normal to Earth's spin axis

Local meridian: half a great circle

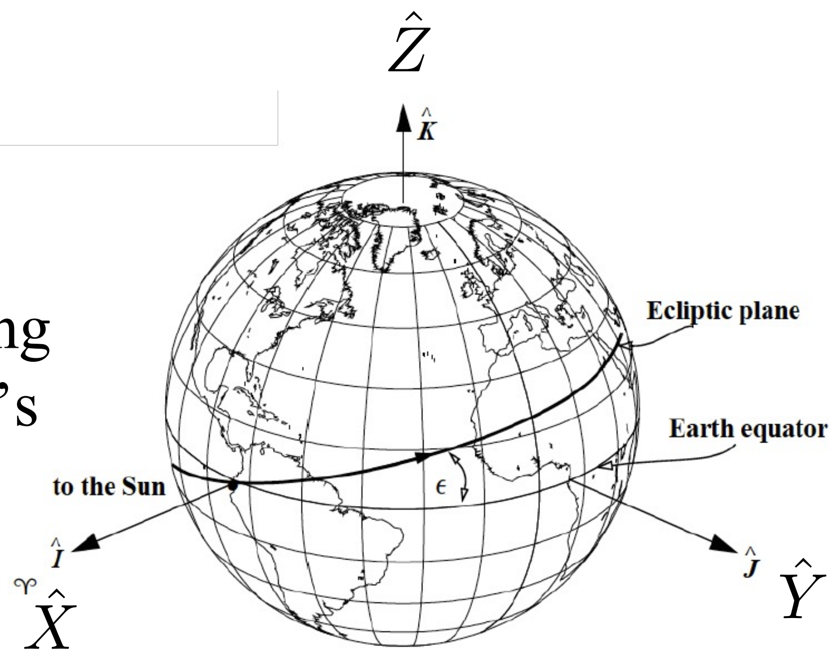
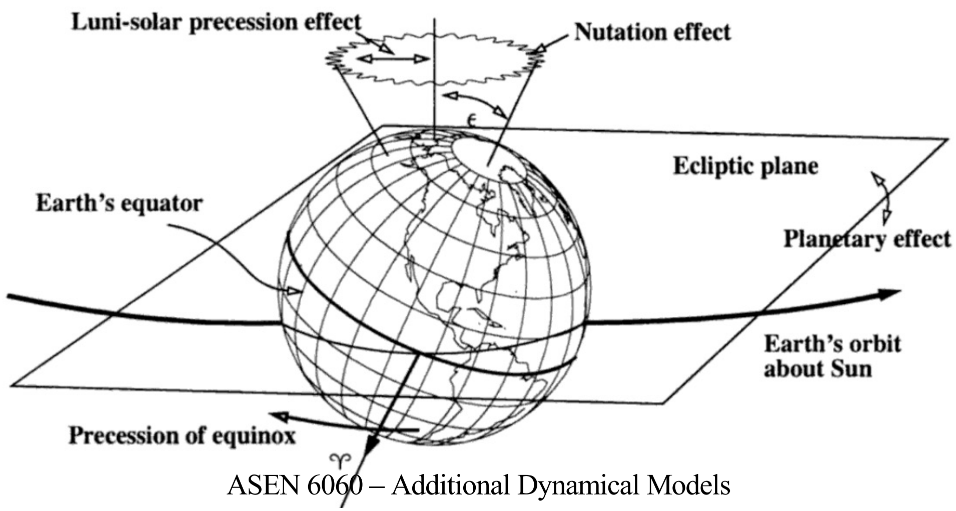


Image credit:
Vallado, 2013

Evolution of Earth Orientation

- Celestial Intermediate Pole (CIP): "axis of Earth rotation…. normal to the true equator" (Vallado, 2022)
- Earth orientation evolves due to the following components:
  – Precession-nutation of CIP
    - Periodic effects with a period >2 days and secular effects
    - Due to gravity of Sun, Moon and planets, the irregular shape and gravity field of Earth, evolution of Moon's orbit
  – Sidereal motion: Rotation of Earth about CIP
  – Polar motion: Shift of the CIP relative to Earth's crust / surface



Additional Definitions

- "True" vs "Mean"
  - Used to describe how reference plane is calculated
  - Mean: includes model of precession
  - True: includes models of precession and nutation
- "… of date" vs "… of epoch"
  - "… of date": uses current epoch at each instant in simulation
  - "… of epoch": at a specified, fixed epoch, e.g., J2000 = Jan 1, 2000 12:00.000 TDB
- Examples:
  - Mean equator of date: orientation of equatorial plane modeled using only precession
  - True equator of date: includes nutation due to lunar and solar perturbations
  - Mean equinox of date: intersection of ecliptic of date and mean equator of date
- Earlier standards for inertial frames used these definitions

Earth-Centered Inertial Frames

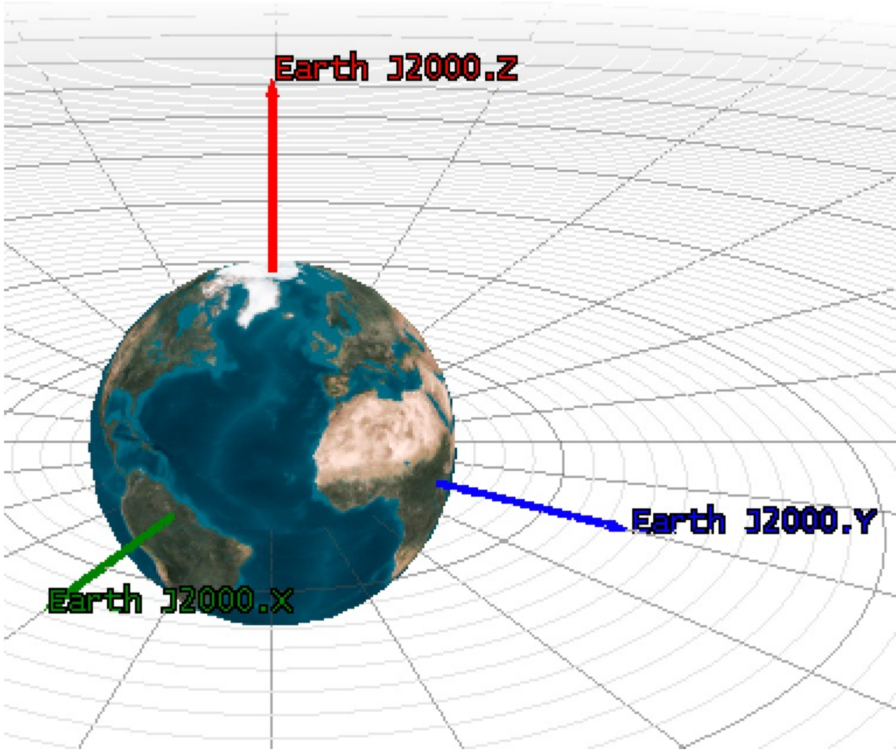
Earth-Centered Mean Equatorial (EME) J2000 system

Origin = Earth center

$\hat{X}$: Aligned with vernal equinox on J2000 epoch

$\hat{Z}$: Aligned with normal to the Earth's mean equatorial plane at the J2000 epoch

$\hat{Y}$: Completes the right-hand triad, lies in equatorial plane

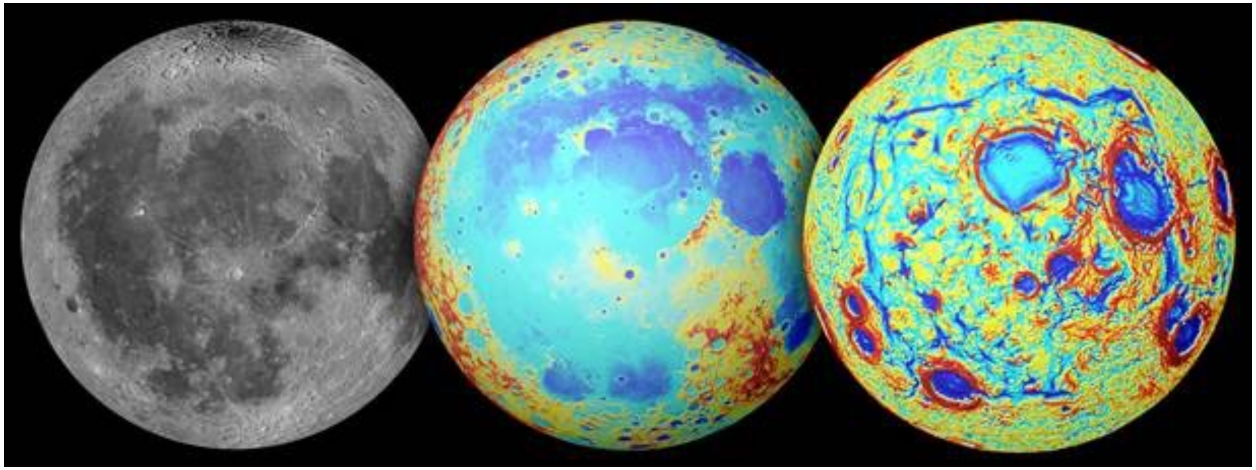


True of Date (TOD) Equatorial Earth-Centered System: uses true equator and true vernal equinox at each specific epoch

Note: GCRF and EMEJ2000 frames differ on order of *mas* → often used interchangeably in trajectory design

Higher-Order Gravitational Contributions

Credit: NASA



Credit: NASA/GSFC/Colorado School of Mines/MIT

Typically represent gravitational field using a potential function which can be constructed using a variety of methods:

- Spherical Harmonics
- Mass point concentrations
- Polyhedral model

Spherical Harmonics Gravity Model

The potential field for an irregular body is described in body-fixed frame using spherical harmonics as:

Radius of central body

Latitude

Longitude

$$U = \frac{\mu}{r}\left[1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l} P_{l,m}[sin(\phi)]\left(C_{l,m}cos(m\lambda) + S_{l,m}sin(m\lambda)\right)\right]$$

Point mass contribution

Distance of s/c from central body

Associated Legendre polynomial

Coefficients for expansion

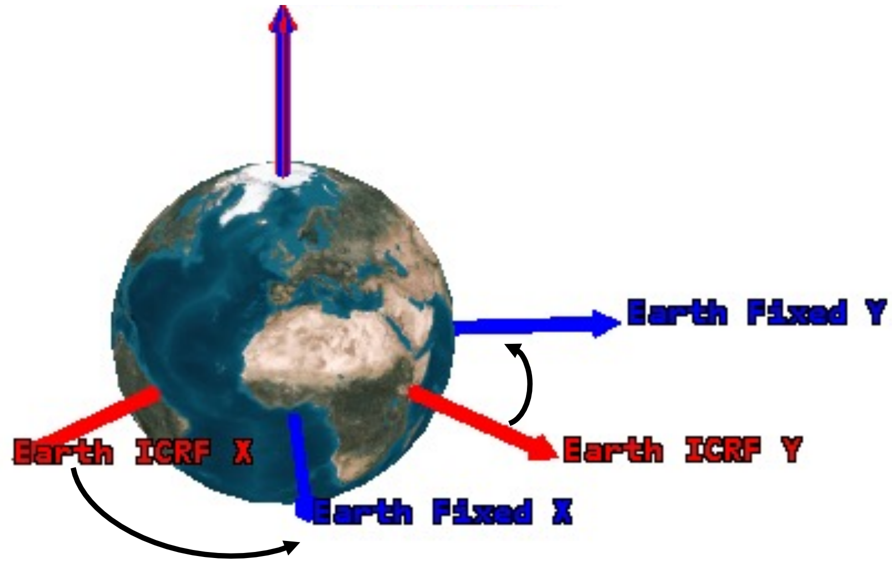
Where:

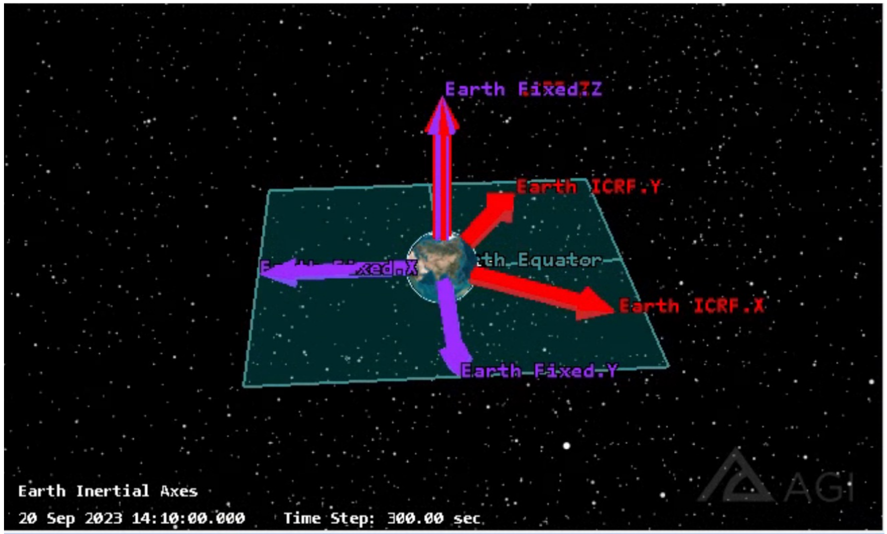
$$P_l[\gamma] = \frac{1}{2^l}\sum_{j=0}^{l}\frac{(-1)^j(2l-2j)!}{j!(l-j)!(l-2j)!}\gamma^{l-2j} \qquad P_{l,m}[\gamma] = (1-\gamma^2)^{m/2}\frac{d^m}{d\gamma^m}P_l\,[\gamma]$$

International Terrestrial Reference Frame (ITRF)

Origin = Earth center

Axes:
- Rotate with the Earth
- Selected to ensure no net rotation relative to horizontal plate motion on the Earth
- Regularly updated with improved modeling of ground stations due to plate tectonics





ITRF is modern standard for an Earth-fixed frame

Transformations Between Frames

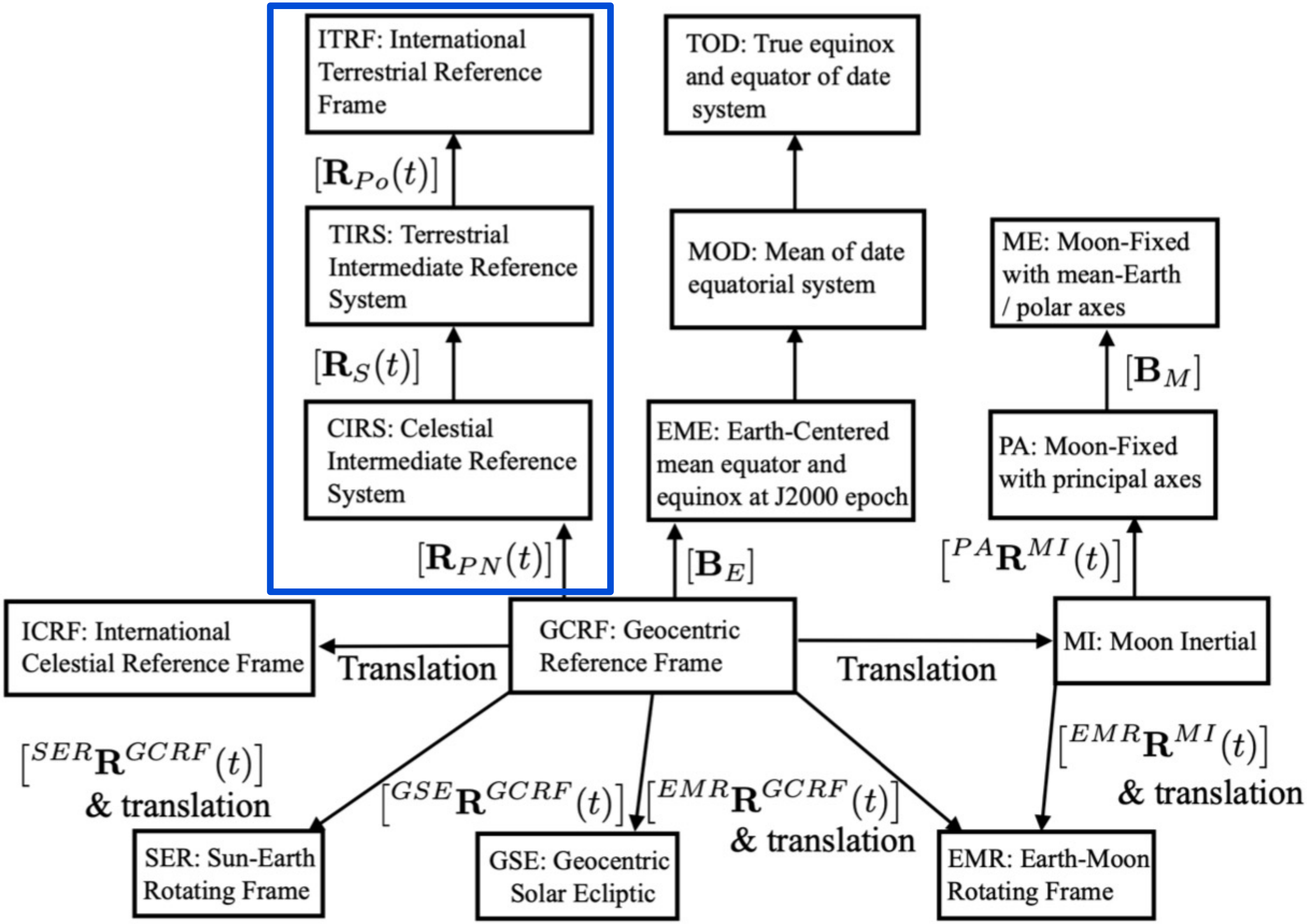


Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

Transformations Between GCRF And ITRF

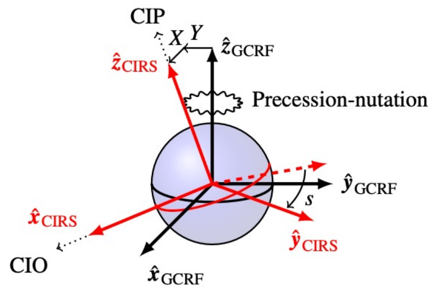
$$\boldsymbol{r}_{E,p}^{ITRF} = \left[{}^{ITRF}\boldsymbol{R}^{GCRF}(t)\right]\boldsymbol{r}_{E,p}^{GCRF} = [\boldsymbol{R}_{Po}(t)]\,[\boldsymbol{R}_{S}(t)]\,[\boldsymbol{R}_{PN}(t)]\,\boldsymbol{r}_{E,p}^{GCRF}$$

1. Precession-nutation:

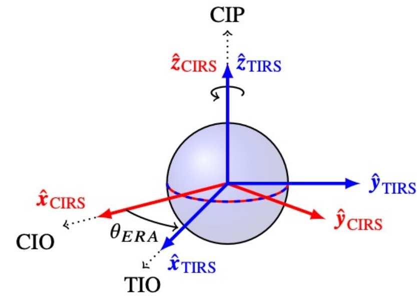
$$[\boldsymbol{R}_{PN}(t)] = [\boldsymbol{R}_3(-s)]\begin{bmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{bmatrix}$$

$$a = \frac{1}{1+\sqrt{1-X^2-Y^2}}$$

X,Y: first 2 coordinates of CIP unit vector in GCRF
s(t): angular difference between first axes of GCRF and intermediate set



(a) Transformation from GCRF (black) to CIRS (red), governed by $[\boldsymbol{R}_{PN}(t)]$ to reflect precession-nutation component



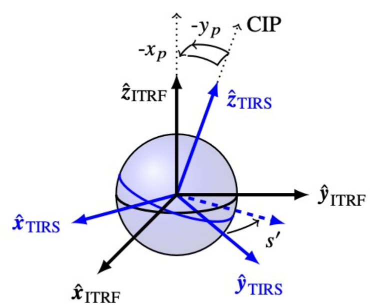
(b) Transformation from CIRS (red) to TIRS (blue), governed by $[\boldsymbol{R}_{S}(t)]$ to reflect the sidereal motion component

2. Sidereal motion:

$$[\boldsymbol{R}_S(t)] = [\boldsymbol{R}_3(\theta_{ERA})]$$

Earth rotation angle:

$$\theta_{ERA} = 2\pi\,(0.7790572732640 + 1.00273781191135448\,(JD_{UT1} - 2451545.0))$$



(c) Transformation from TIRS (blue) to ITRF (black), governed by $[\boldsymbol{R}_{Po}(t)]$ to reflect the polar motion component

3. Polar motion:

$$[\boldsymbol{R}_{Po}(t)] = \left[\boldsymbol{R}_1(-y_p)\right]\left[\boldsymbol{R}_2(-x_p)\right]\left[\boldsymbol{R}_3(s')\right]$$

x_p,y_p: location of CIP via Earth orientation parameters
s'(t): angular diff. between first axis of ITRF and intersection of ITRF/TIRS XY planes

Image credit: Folta, D.; Bosanac, N.; Elliott, I.L.; Mann, L.; Mesarch, R.; Rosales, J., 2022, "Astrodynamics Convention and Modeling Reference for Lunar, Cislunar, and Libration Point Orbits (Version 1.1)", NASA/TP–20220014814

Spherical Harmonics Gravity Model

- To use higher-order gravity field, need expansion coefficients:
  - Access using data generated from missions
  - Updated as improved data is gathered, may influence trajectory generation if using different data files
- For the Moon: Gravity Recovery and Interior Laboratory (GRAIL) mission has produced data with up to degree and order 1200
- Need to specify degree and order of gravity field
  - Tradeoff fidelity of model and computational speed

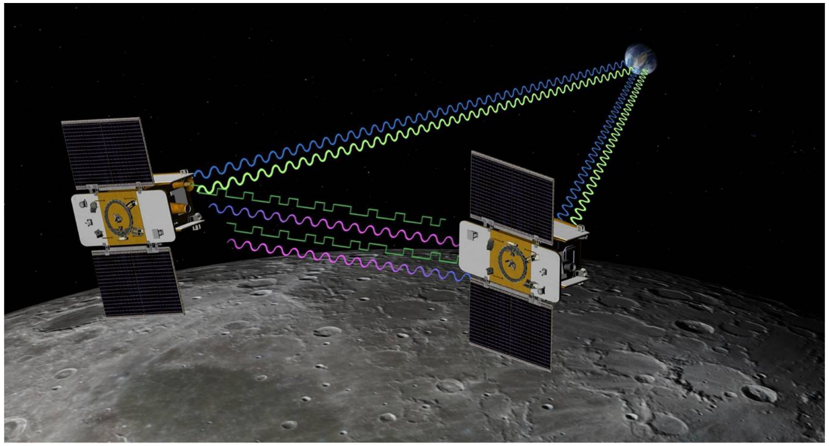


Image credit: NASA

Moon-Centered Frames

Moon Inertial

- Origin at center of Moon
- Use axes of ICRF (other definitions often used too!)

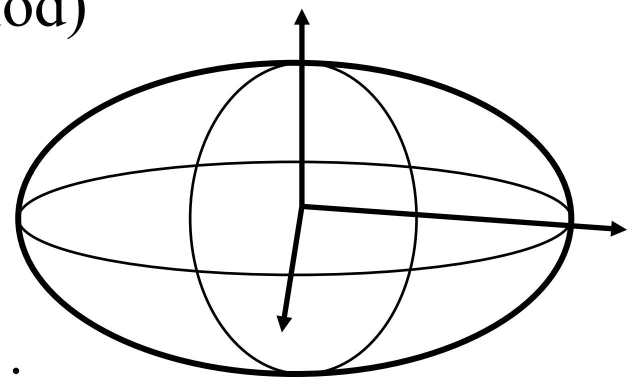
Moon-Fixed

- Origin at center of Moon
- Mean-Earth / Polar (ME) axes:

  $\hat{X}$: Directed from lunar center of mass to mean Earth direction (lunar rotational period matches orbital period)

  $\hat{Z}$: Aligned with mean lunar rotation axis

  $\hat{Y}$: Completes right-handed triad



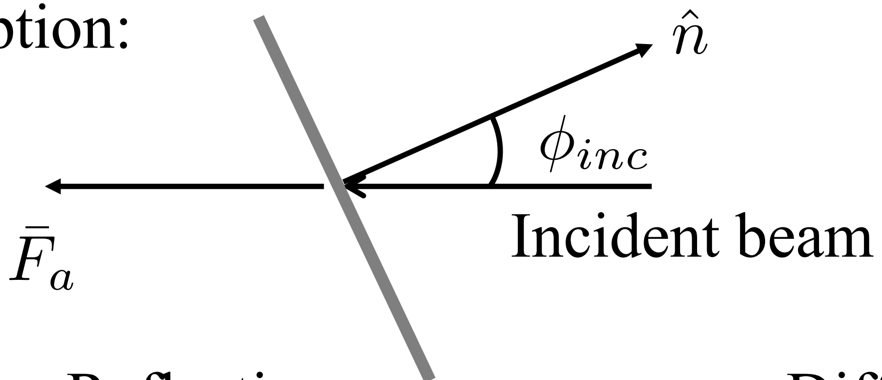
- Lunar principal axes (PA):
  - Principal axes of the Moon, assuming no mantle distortion due to tides and rotation
- Updated over time

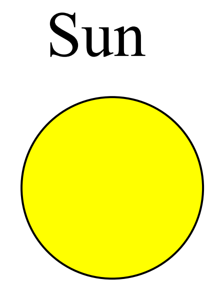
Ref: Lunar Reconnaissance Orbiter Project and Lunar Geodesy and Cartography Working Group, 2008, "A Standardized Lunar Coordinate System for the Lunar Reconnaissance Orbiter and Lunar Datasets," White paper, Version 5.

Solar Radiation Pressure

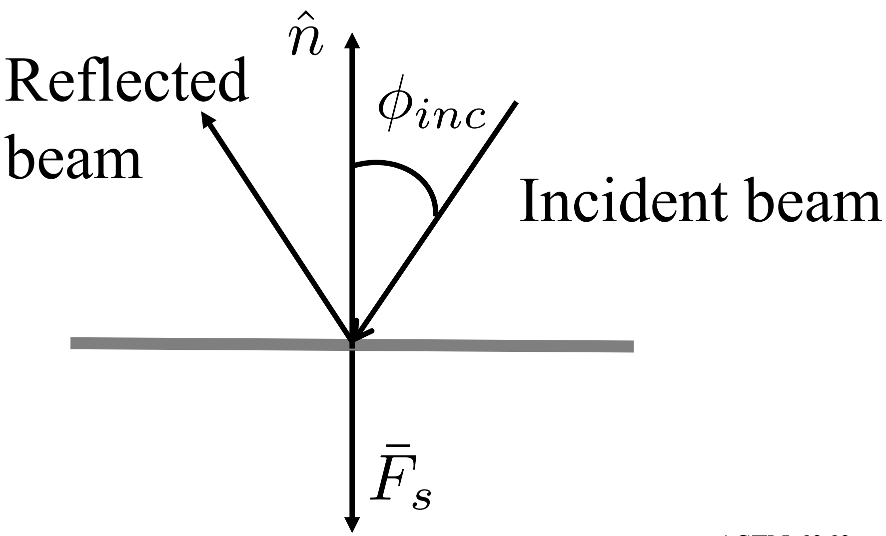
Nonconservative perturbation due to radiation of the Sun imparting force on the satellite via photons.

Absorption:

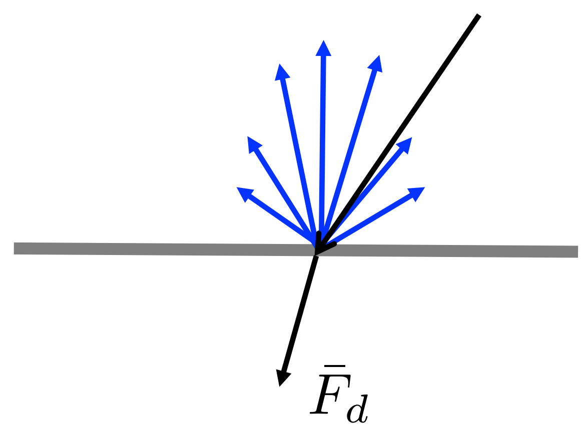




Specular Reflection:

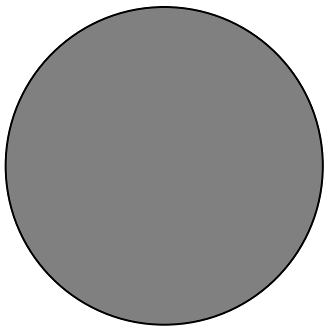


Diffuse Reflection:

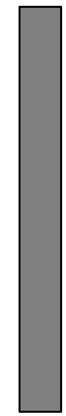


Common SRP Models

Cannonball



Flat Plate



Actual spacecraft

Credit: nasa.gov

Shadow Regions

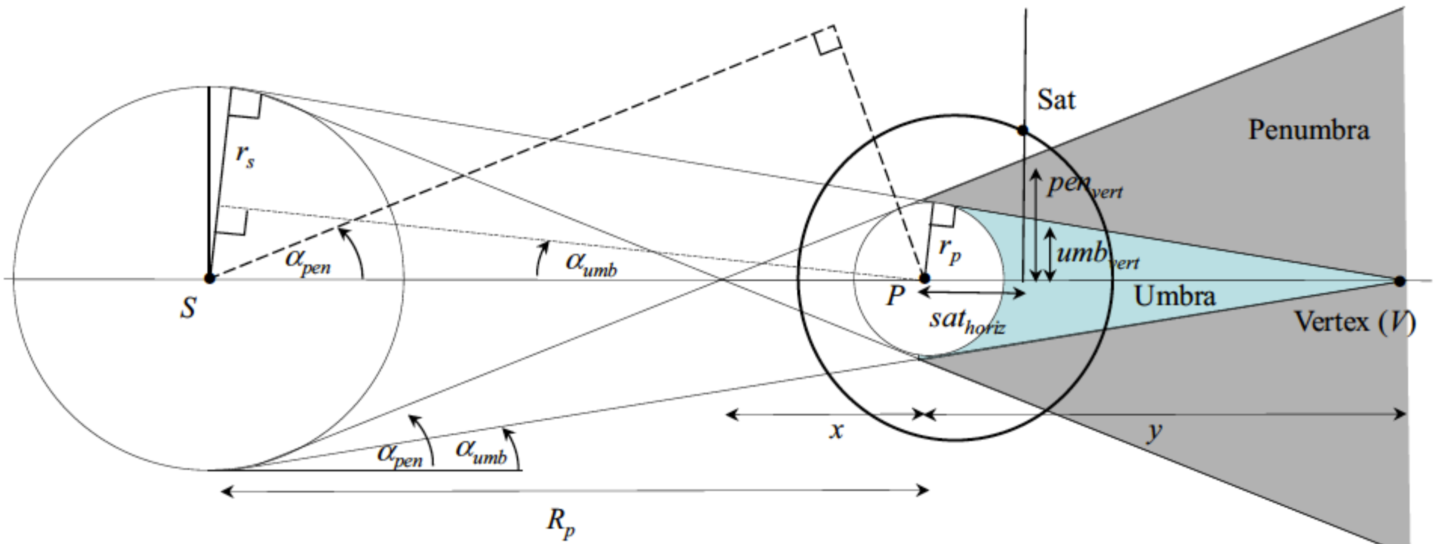


Figure 5-6. **Eclipse Geometry.** The general geometry for eclipses involves a primary body (Earth) and a secondary body (Sun). Eclipses occur within the umbra and the penumbra regions. Notice the scales are greatly exaggerated; the actual angular departure of the penumbra from the umbra region is actually rather small (See also Fig. 5-2). For the Sun-Earth system, $PV \approx 1.384 \times 10^6$ km, or about four times the distance to the Moon. The angles are useful for quick calculations. The dashed line represents an arbitrary orbit. The vertical and horizontal components are used in determining instantaneous shadow positions. The dashed lines and x and y parameters are useful for developing the equations.

Credit: Vallado

Solar Radiation Pressure

Spherical model of a spacecraft

$$\bar{a}_{SRP,c} = \nu P_{SRP} \frac{C_R A \bar{R}_{S,sc}}{M_{sc} R_{S,sc}}$$

ν = eclipse factor, 1 = direct sunlight, 0 = shadow, [1,0] in penumbra

P_{SRP} = Force per unit area due to SRP at 1 AU

M_{sc} = Mass of spacecraft

C_R = Coefficient of reflectivity, C_R = [0, 2] depends on surface material

A = Area facing the Sun

$\bar{R}_{S,sc}$ = Position vector from spacecraft to Sun

Solar Radiation Pressure

Higher-fidelity model with *p* flat plates:

$$\bar{a}_{SRP,fp} = \frac{P_{SRP}}{M_{sc}} \sum_{i=1}^{p} A_i \cos(\theta_i) \left[\left(1 - \rho_s^i\right) \frac{\bar{R}_{S,sc}}{R_{S,sc}} + 2 \left(\rho_s^i \cos(\theta_i) + \frac{\rho_d^i}{3} \right) \hat{\bar{n}}_i \right] H(\theta_i)$$

A_i = surface area of i-th flat plate

$\hat{\bar{n}}_i$ = unit vector perpendicular to the outward-facing surface or reflective side of flat plate

θ_i = angle between $\hat{\bar{n}}_i$ and $\bar{R}_{S,sc}$

$H(\theta_i)$ = binary function to note whether reflective side of the flat plate faces Sun

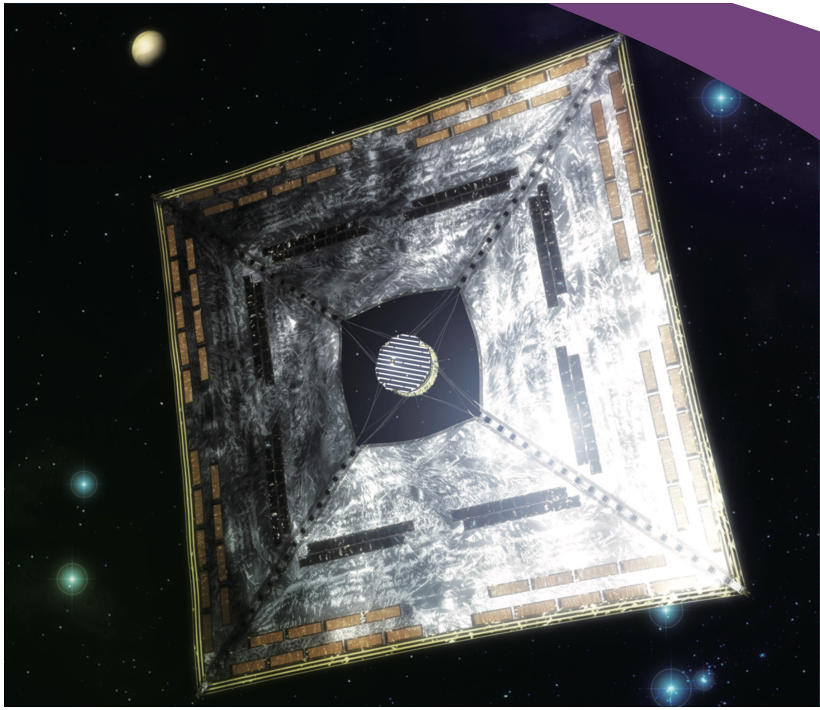
$\rho_a^i, \rho_s^i, \rho_d^i$ = relative contributions for absorption, specular reflection and diffuse reflection of i-th flat plate, each sum to 1

Solar Sails

JAXA – IKAROS

- (Interplanetary Kite-craft Accelerated by Radiation Of the Sun)
- Launched May 21, 2010
- Demonstrated solar sail technology in interplanetary space
- Solar sail ~ 20m diagonal
- JAXA estimates the 196 m^2 sail provides about 1.12mN of thrust

→ estimated via OD



Source: JAXA

Atmospheric Drag

Nonconservative perturbation due to atmospheric particles (energy changes due to friction), depends on satellite velocity.

$$\bar{a}_{drag} = -\frac{1}{2}\frac{C_D A}{m_{sc}}\rho v_{rel}^2 \hat{v}_{rel}$$

c_D = coefficient of drag [~2.2 for s/c]

A = cross-sectional area of satellite, normal to velocity vector

ρ = atmospheric density (time-varying) at s/c altitude

$\bar{v}_{rel}$ = velocity vector relative to rotating atmosphere

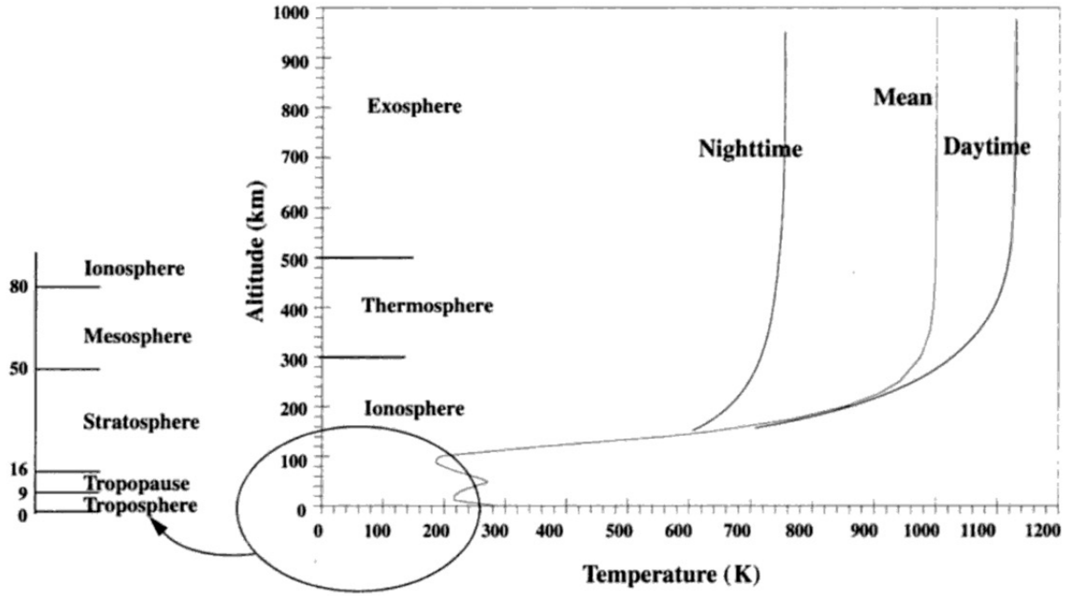
Ballistic coefficient, captures effect of drag on satellite: $m_{sc}/(Ac_D)$

Most significant influence on *a, e*

Atmospheric Drag

How to model density? Depends on solar flux, magnetic field, etc

Atmosphere is complex - various models have been developed.



Use simplified , static exponential model for preliminary analysis:

$$\rho = \rho_0 e^{-\frac{(h_{ellip} - h_0)}{H}}$$

Credit: Vallado

Continuous Thrust

Additional acceleration due to continuous thrust engine:

$$\bar{a}_{thrust} = \frac{f_s f_d F_T}{M_{sc}} \hat{\bar{u}}$$

F_T = Thrust magnitude

M_{sc} = Spacecraft mass

f_s = Thrust scale factor

f_d = Duty cycle factor (fraction of maneuver time that thrusters are activated)

$\hat{\bar{u}}$ = Thrust direction in specified axes

Spacecraft mass decrements: $\dot{m} = f_d \frac{F_T}{I_{sp} g}$

I_{sp} = Specific impulse

g = gravitational acceleration on surface of Earth