

# ASEN 6060

## ADVANCED ASTRODYNAMICS

### Week 7 Discussion, Part 2

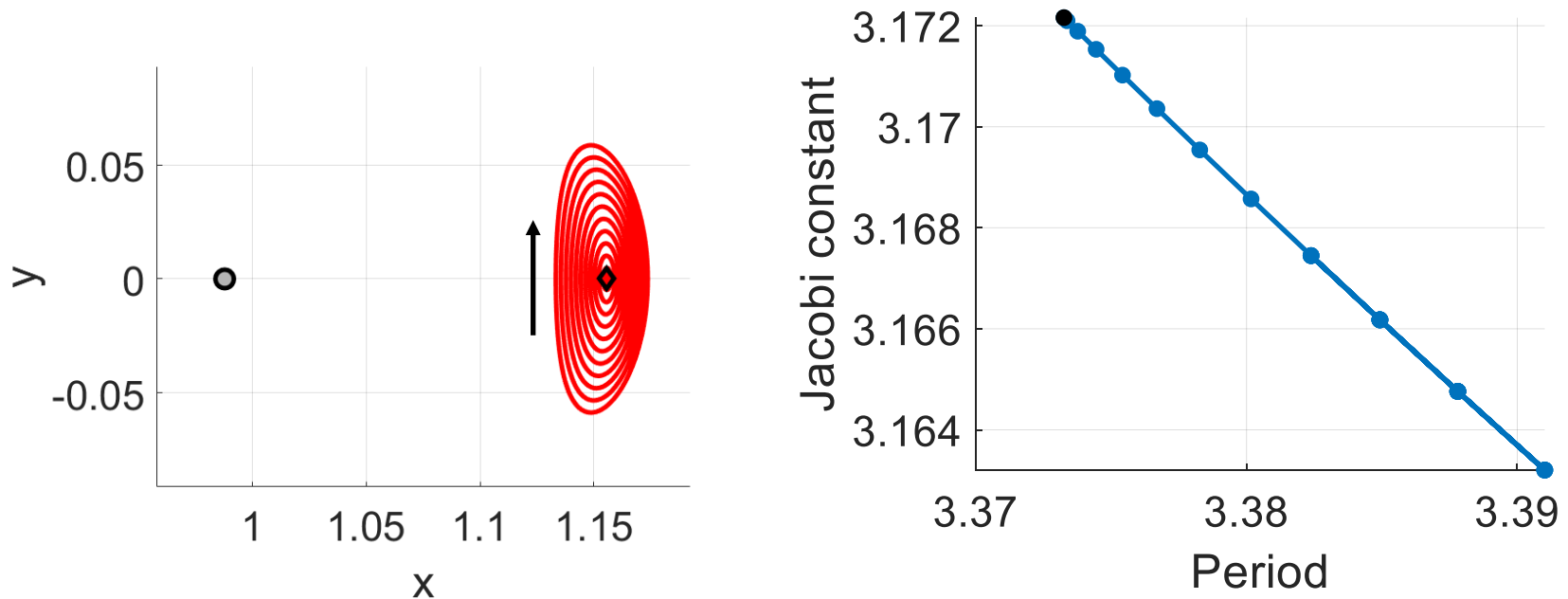
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#### Objectives:

- Gain intuition that will be useful for creating, debugging and assessing implementation of continuation algorithms

# Example 1: Computing $L_2$ Lyapunov Orbits

Your colleague is trying to compute orbits along the  $L_2$  Lyapunov orbit family via pseudo-arclength continuation, but they cannot seem to calculate more members than those shown here.

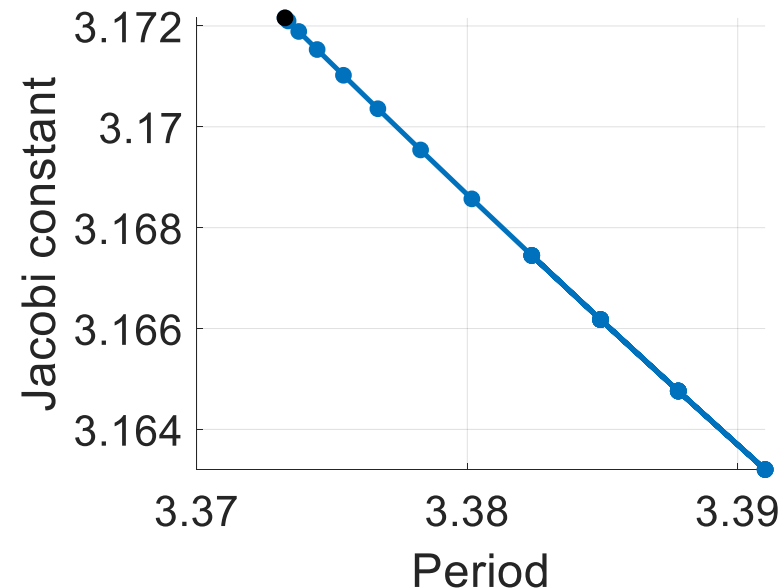
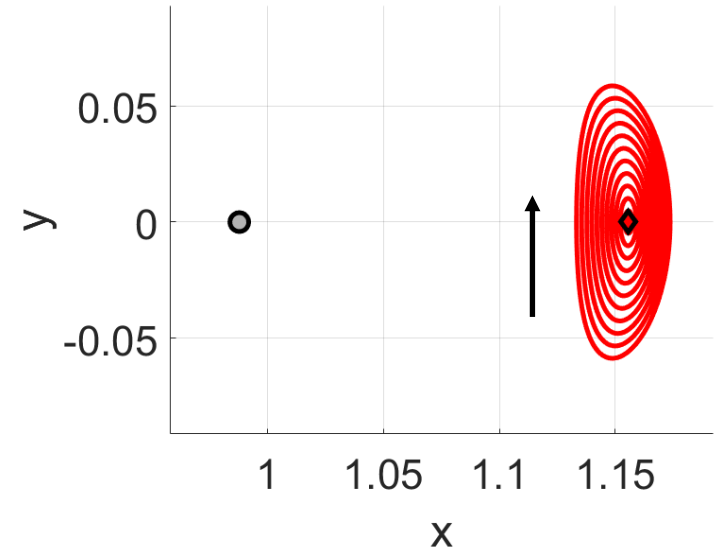


**Question 1:** What information would you ask for to explore the issue in their continuation scheme? Also create a list of ideas for what the problem could be.

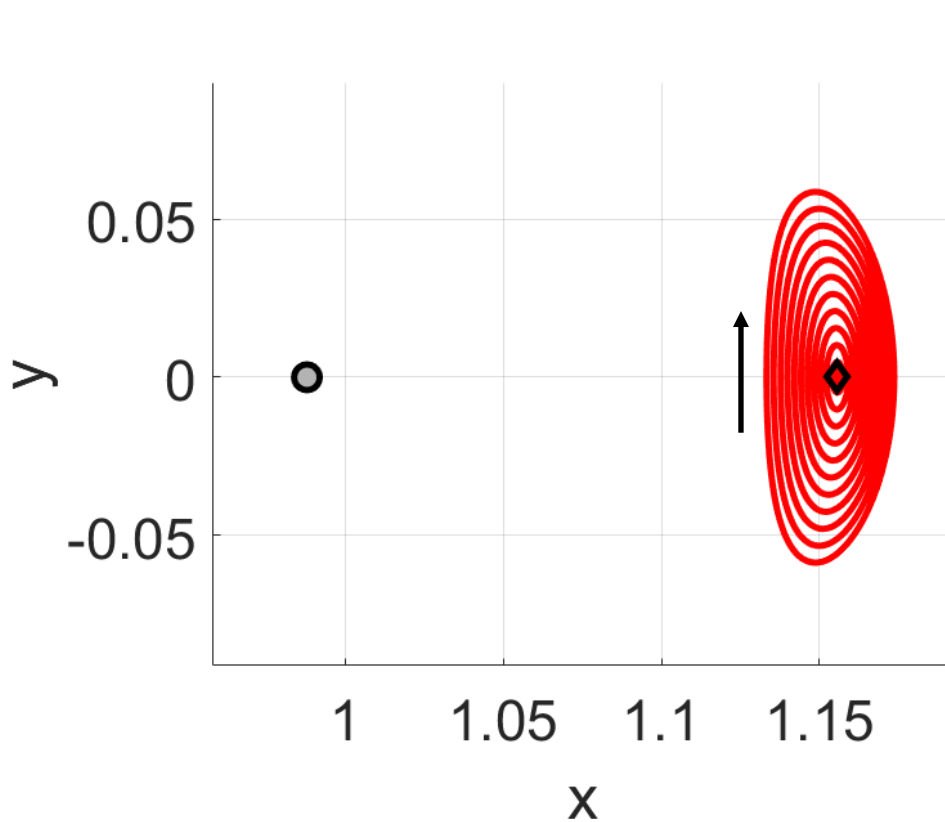
# Question 1

## Group Brainstorming:

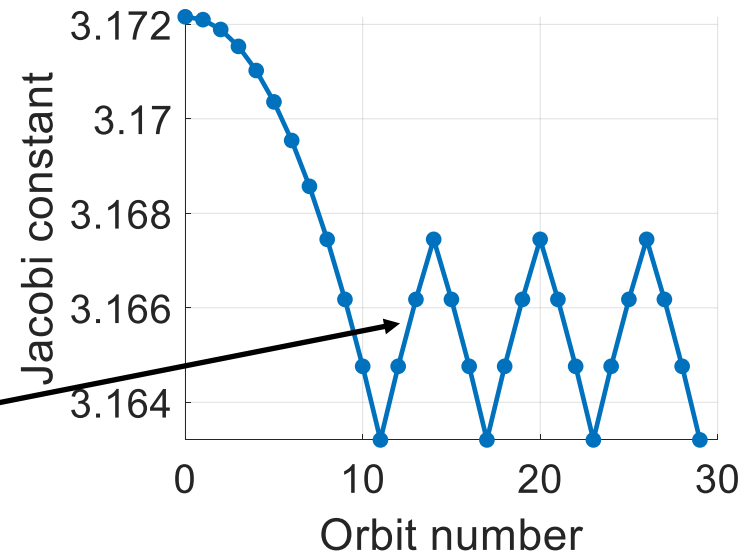
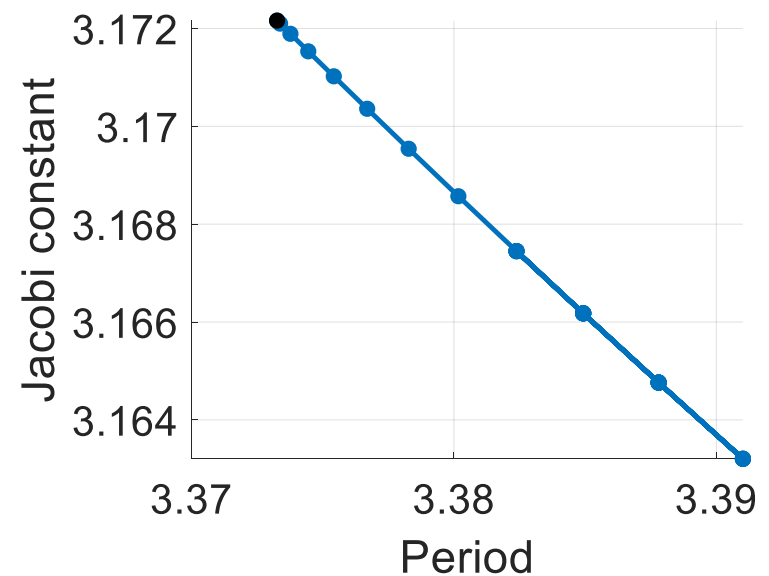
- The step size is too large in this segment of the family? Should we decrease it?
- Is the nullspace vector flipping directions? This could hinder stepping further along the family. Perhaps they are computing the same pair of solutions multiple times?
- What are the termination conditions for both corrections and continuation?
- Reason: nullspace vector flipping direction.



# Example 1: Computing $L_2$ Lyapunov Orbits

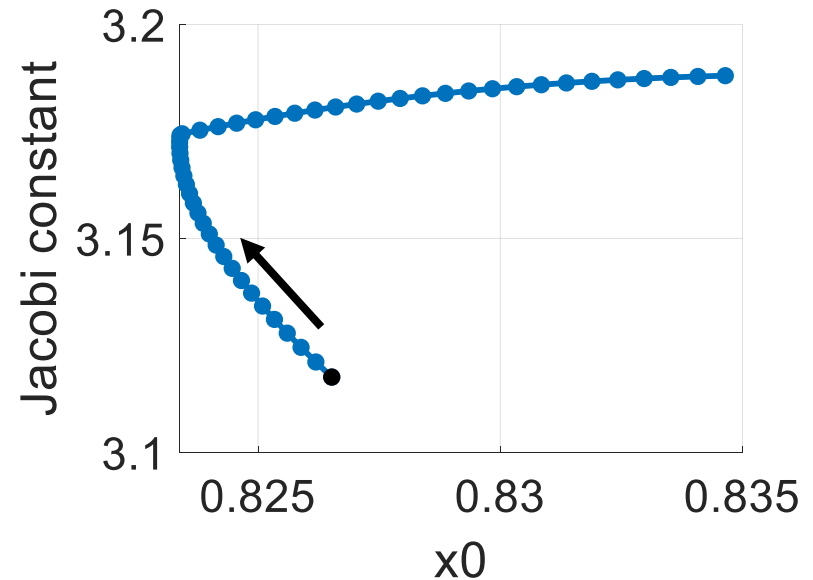
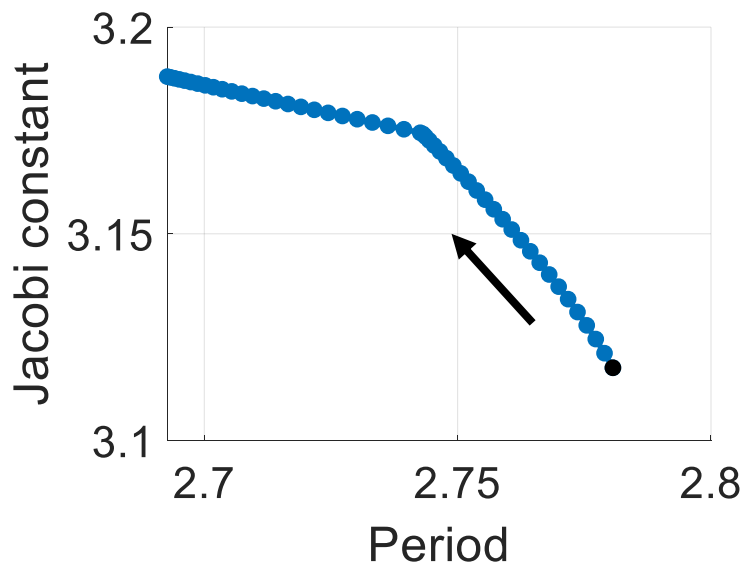


Basis vector for nullspace, as calculated using a Matlab function, is flipping direction



## Example 2: Computing $L_1$ Halo Orbits

Your colleague is trying to compute orbits along the  $L_1$  halo orbit family via pseudo-arclength continuation and they are analyzing the results by studying the characteristics of the computed orbits

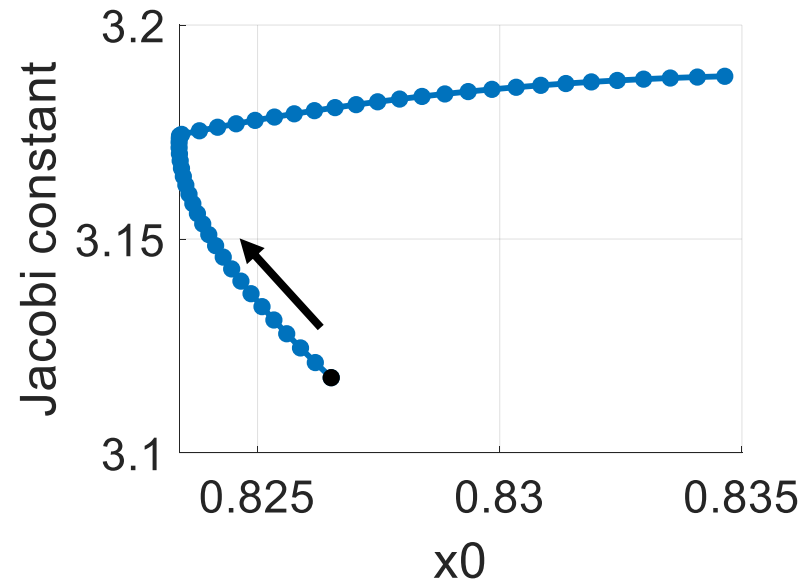
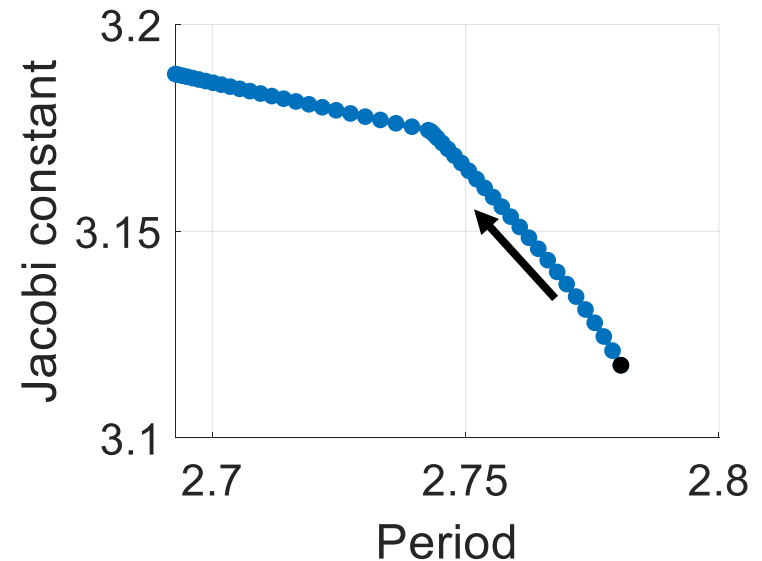


**Question 2:** Do you think their results indicate they have successfully computed the  $L_1$  halo orbit family? Why or why not?

# Question 2

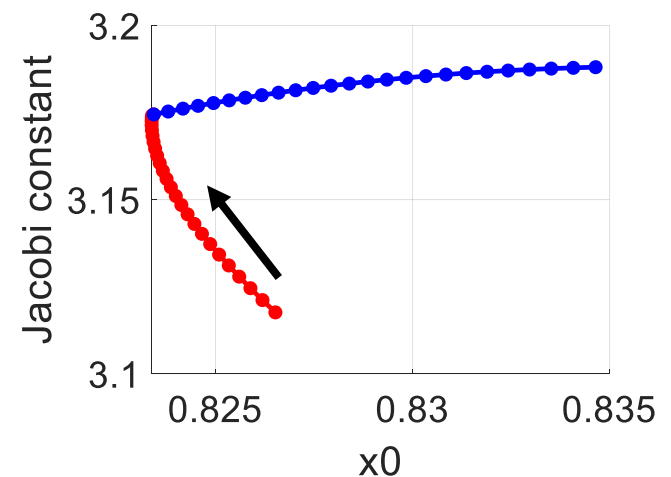
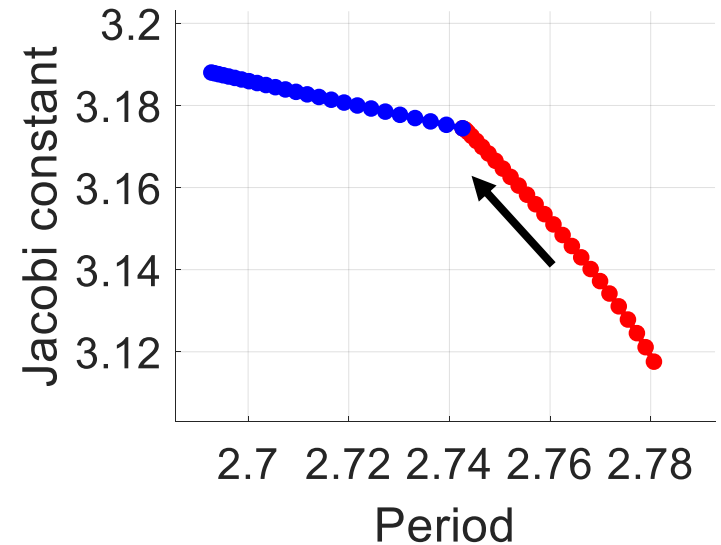
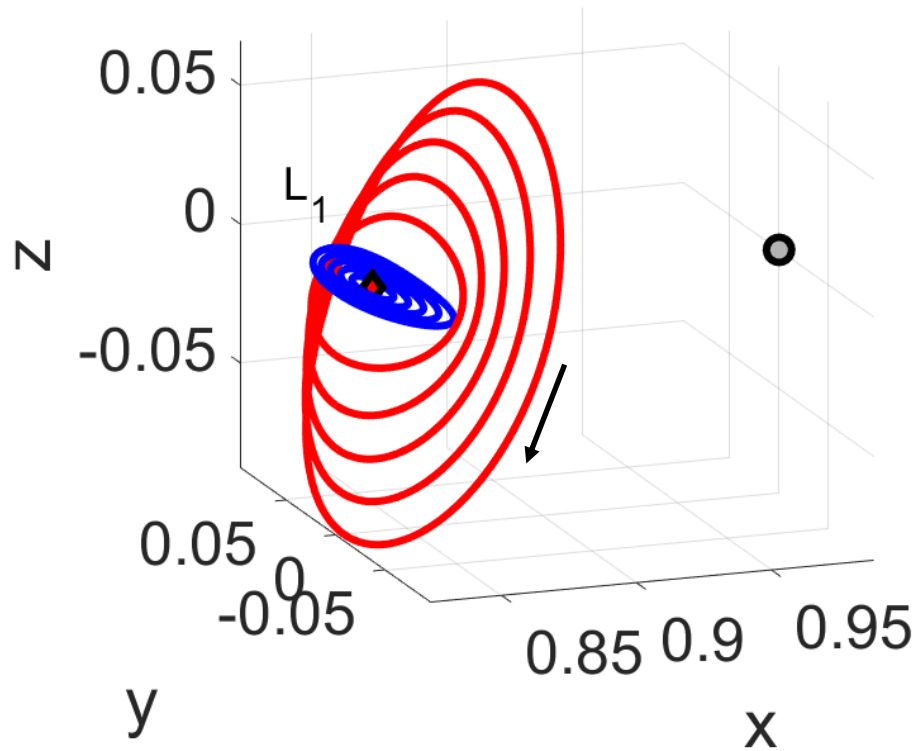
## Group Brainstorming:

- It looks like maybe the Jacobi constant vs Period /  $x_0$  curves might not be smooth – let's ask for a zoomed-in view?
- If it is not smooth, and we expect a set of quantities to vary smoothly, the continuation scheme might be jumping onto a different family.
- If it is a turning point, once we zoom in, it could be smooth
- Why is there a change in density along the curve? Is there an adaptive step size?
- Is there another parameter space that could be helpful?
- Answer: jumped onto a different family during continuation



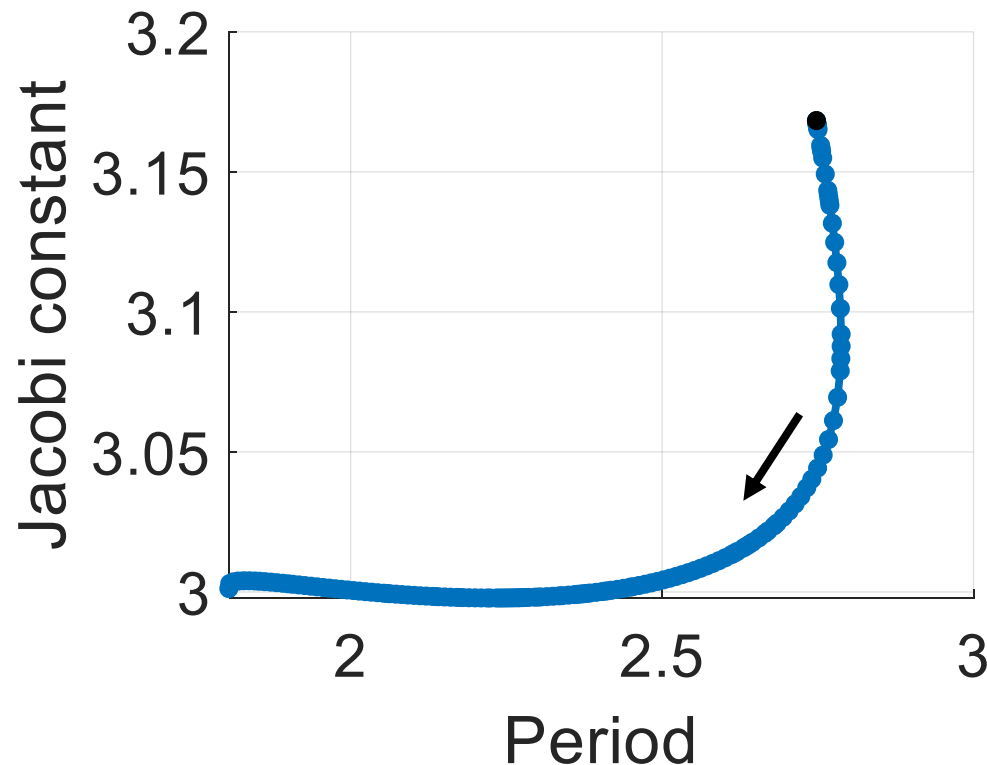
# Example 2: Computing $L_1$ Halo Orbits

Due to a large step size, your colleague is accidentally calculating members along two different families



# *Example 3: Computing $L_1$ Halo Orbits*

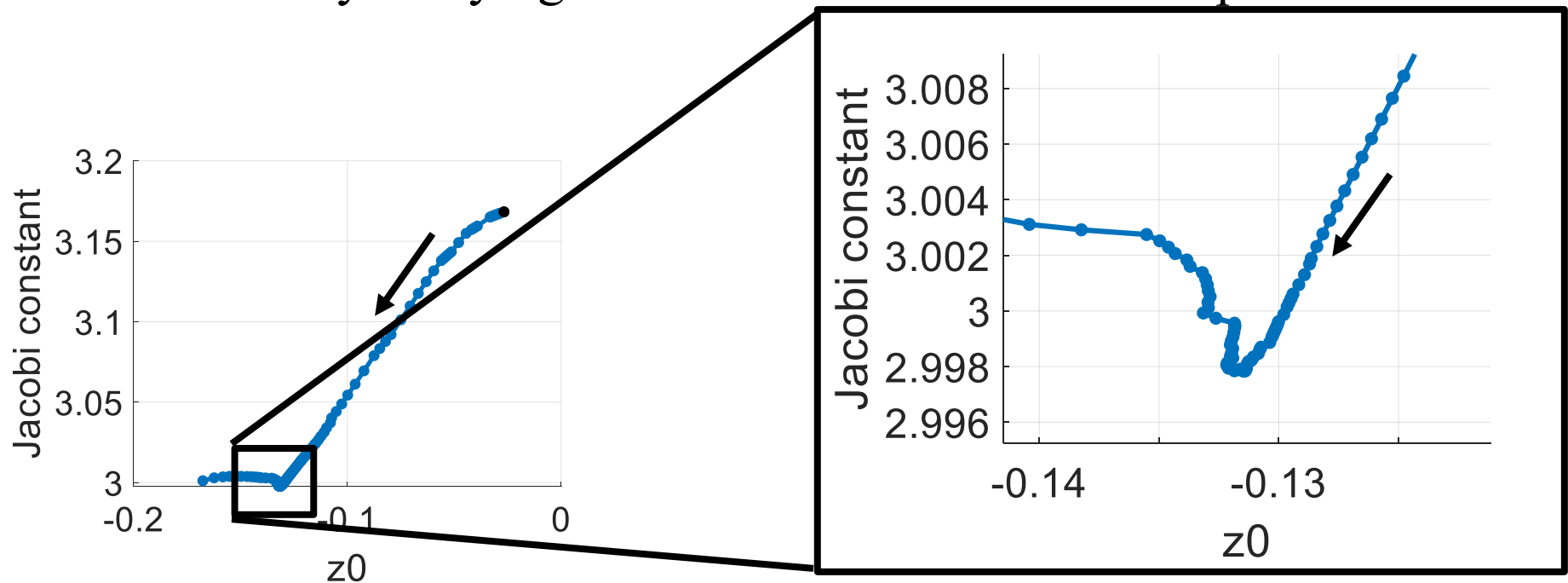
Your colleague is trying to compute orbits along the  $L_1$  halo orbit family via pseudo-arclength continuation and they are analyzing the results by studying the characteristics of the computed orbits





# Example 3: Computing $L_1$ Halo Orbits

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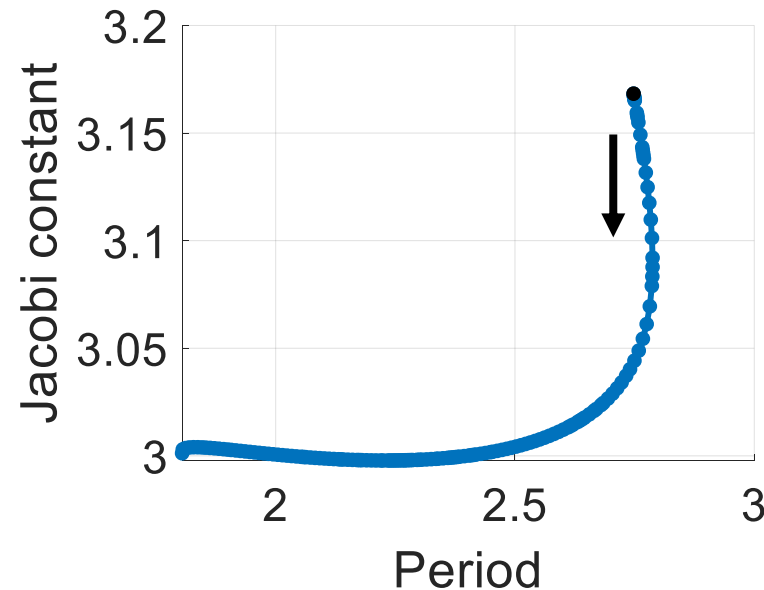
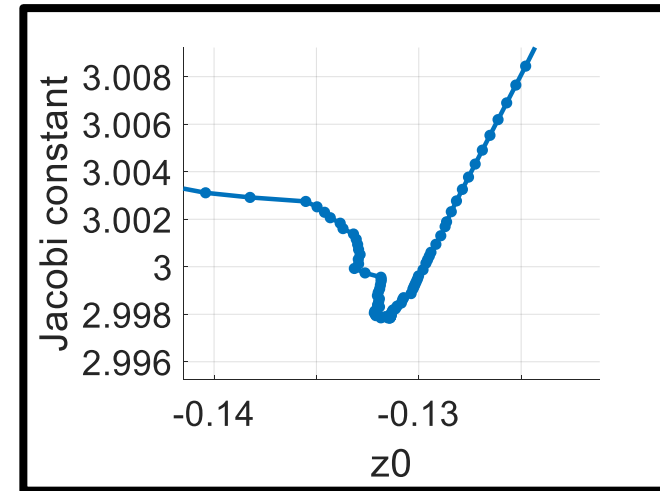
**Question 3:** Create a list of ideas for what could be causing the observed behavior and why.

# Question 3

Group Brainstorming:

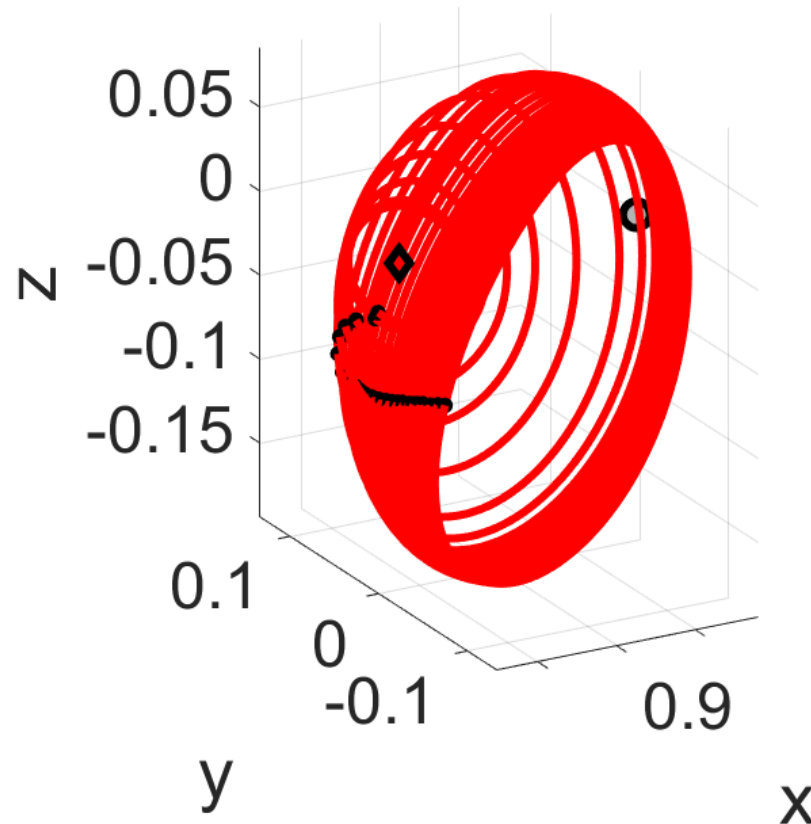
- Are the orbits getting closer to P2 and is there some kind of inaccuracy in computation or are the orbits intersecting P2? Could we check the initial and final states? Can we check  $\text{norm}(F) < \text{tol}$  and  $|x_f - x_0|$  (state vectors)  $< \text{tol}$ ?
- Are we close to an intersection of two or more families and maybe different members are being computed?
- Is the nullspace multi-dimensional?
  - Is there an error?

Answer: the formulation does not constrain the location of the initial condition, so we cannot expect  $z_0$  to vary smoothly. But we see CJ and period vary smoothly, as expected. So they are correctly computing members of the same family.



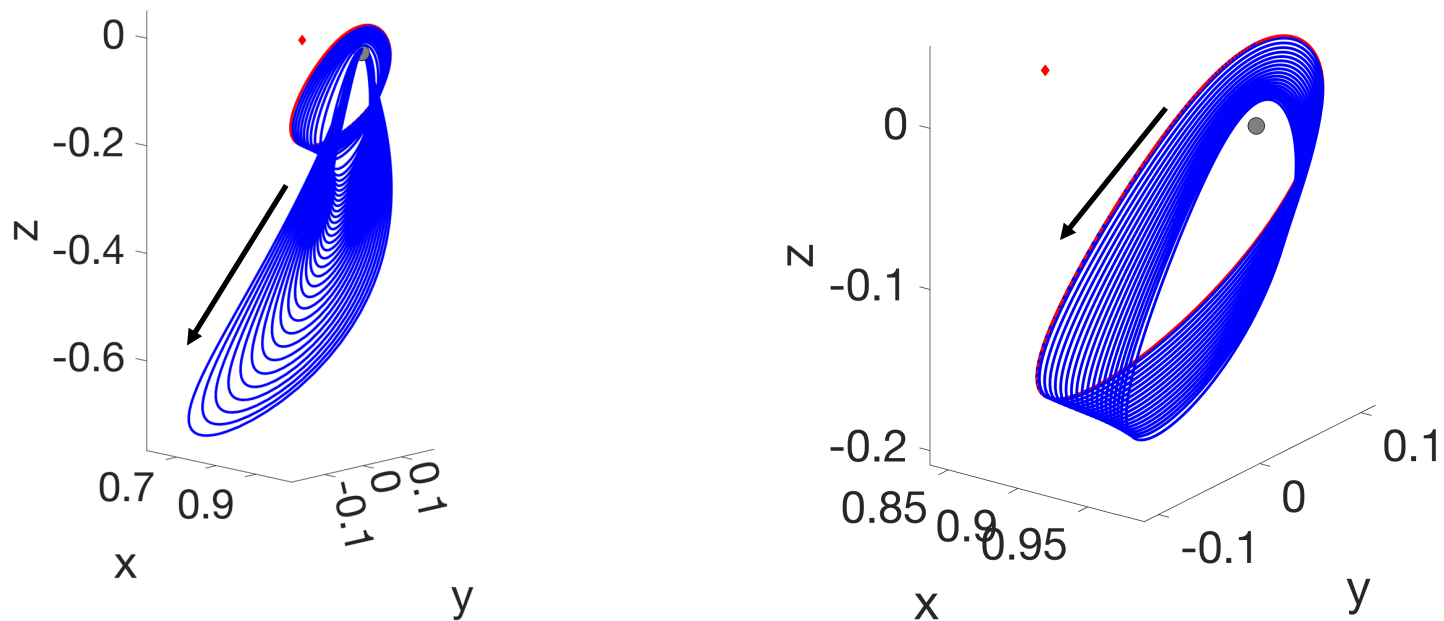
# *Example 3: Computing $L_1$ Halo Orbits*

In some formulations, initial condition may shift along a periodic orbit, so  $z_0$  may not vary smoothly



# Example 4: Computing $L_1$ Halo Orbits

Two colleagues are trying to compute members of the  $L_1$  halo orbit family that lie close to the Moon. They both used information about the same periodic orbit to form an initial guess for the first orbit and then applied pseudo-arclength continuation. But they recovered two distinct results:



**Question 4:** Create a list of ideas for what could be causing the differences and why.

# Question 4

## Group Brainstorming:

- Is the figure on the right using different termination conditions? Max number of iterations of continuation / number of solutions? Minimum distance from Moon constrained to surface?
- Is corrections failing where the problem becomes more numerically sensitive? Differences in integration schemes/tolerances? Step size? Corrections tolerance?
- Is the nullspace vector oscillating in its direction, causing computation of the same segment of the family?
- What formulation of  $F$ ,  $V$  are they using?
- Are they using different initial state locations?
- Answer: The one on the left/top uses an initial state near apolune whereas the one on the right/bottom uses an initial state near perilune

