

# ASEN 6060

## ADVANCED ASTRODYNAMICS

### Week 12 Discussion

---

#### Objectives:

- Present selected examples of how to visually use Poincaré maps for trajectory analysis and design problems
- Strengthen understanding of visual representation of Poincaré maps

# Example 1: Identify Overlapping Manifolds

Identify states that lie within two hyperbolic invariant manifolds

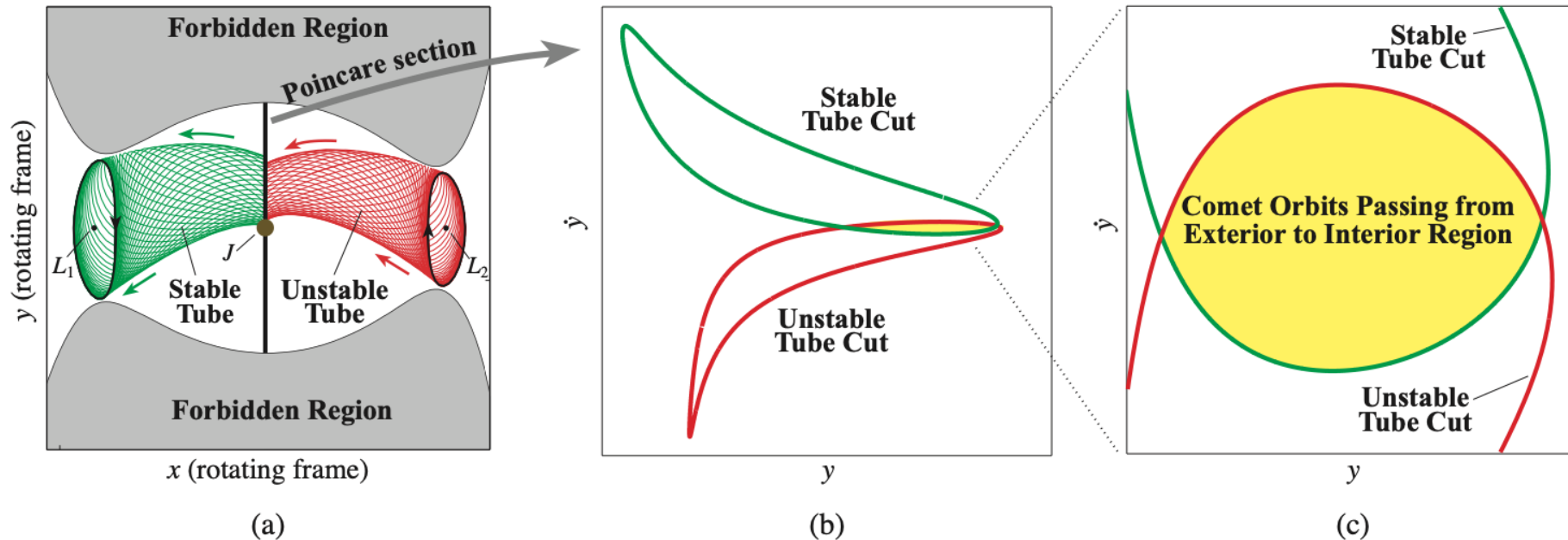


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D. (2001). Resonance and Capture of Jupiter Comets. In: Dynamics of Natural and Artificial Celestial Bodies. Springer, Dordrecht. [https://doi.org/10.1007/978-94-017-1327-6\\_3](https://doi.org/10.1007/978-94-017-1327-6_3)

# Example 2: Periapse Maps

Use to connect characteristics of solution space to known fundamental solutions, e.g., in the Sun-Saturn CR3BP

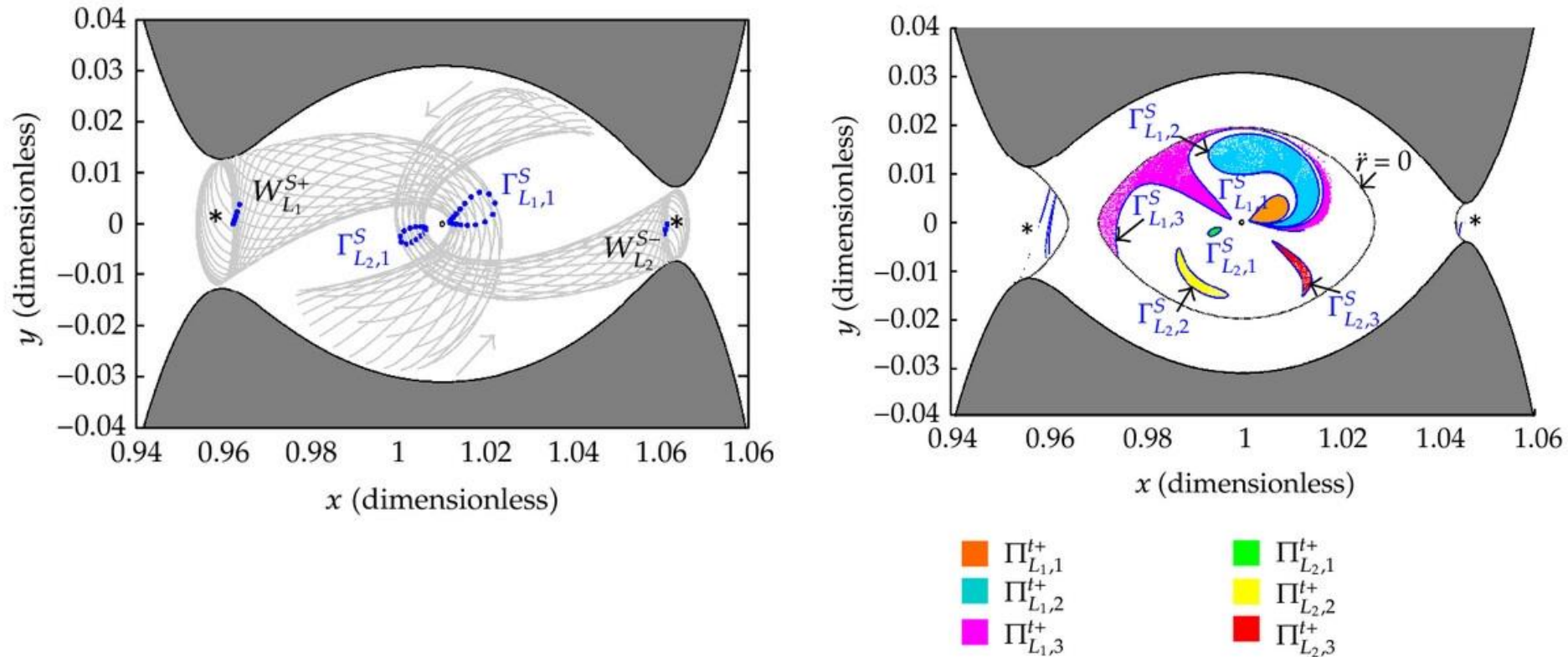


Image credit: Howell, K.C., Davis, D.C., Haapala, A.F., “Application of Periapse Maps for the Design of Trajectories Near the Smaller Primary in Multi-Body Regimes”  
Mathematical Methods Applied to the Celestial Mechanics of Artificial Satellites, 2011

# Example 3: Collinear Eq. Pt. Neighborhood

Poincaré map  
representation of  
intersections of bounded  
motion near  $L_1$  with a  
surface of section defined  
as  $\Sigma : z = 0$

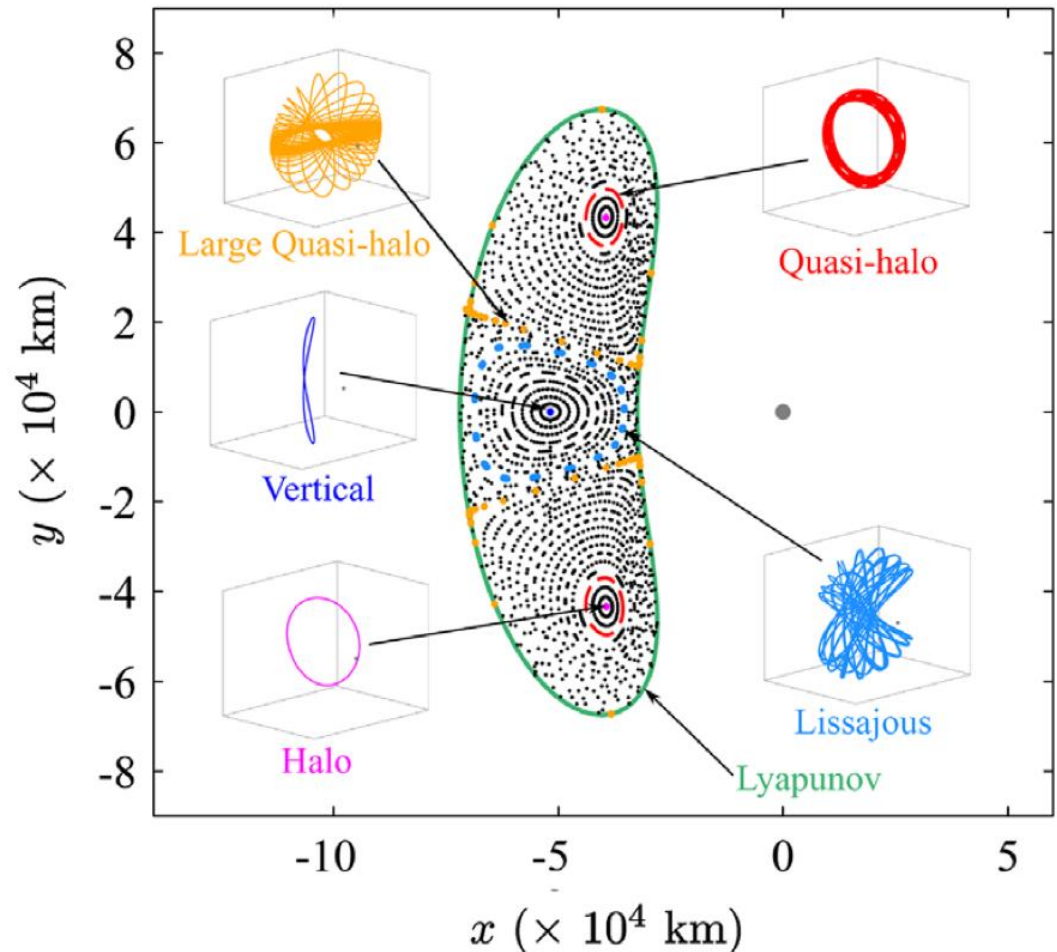


Image credit: Foltá, D.C., et al. “Earth–Moon Libration Point Orbit Stationkeeping: Theory, Modeling, And Operations,” *Acta Astronautica*, Vol. 94, No. 1, 2014, pp. 421-433

# Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., “patched CR3BP”

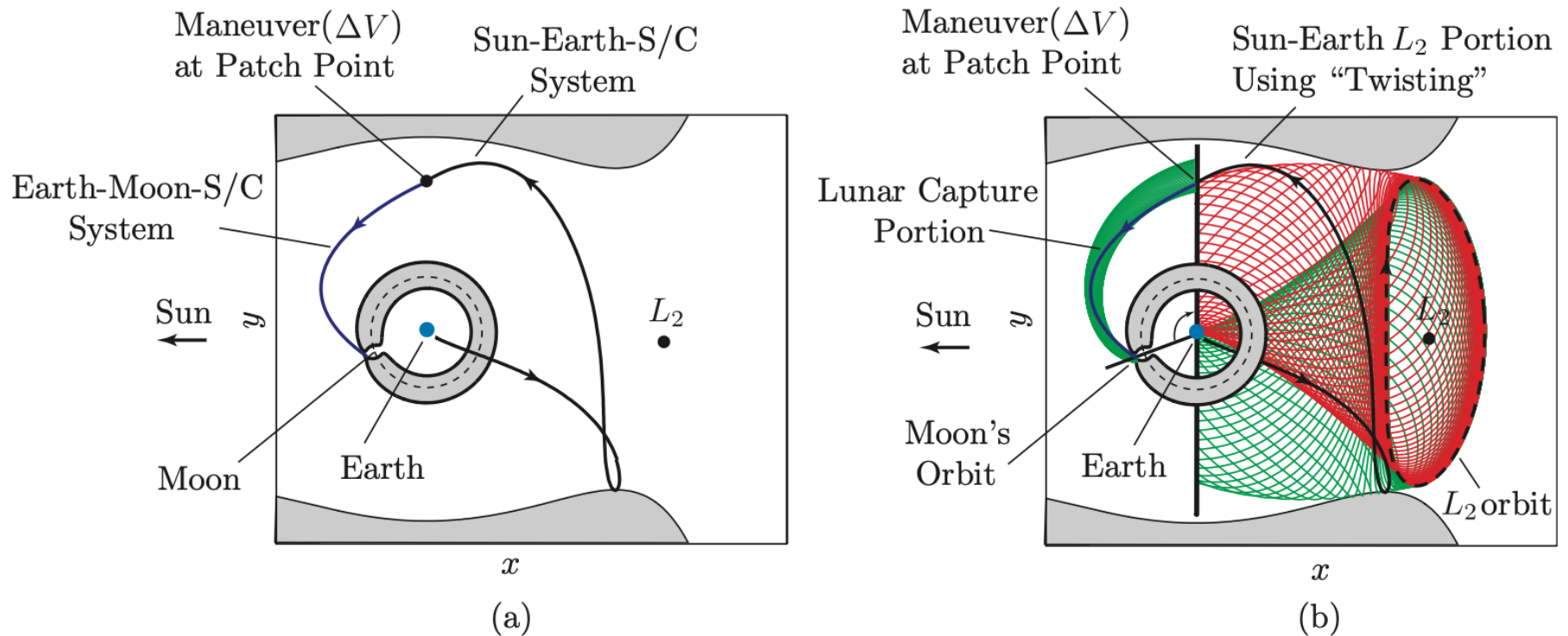
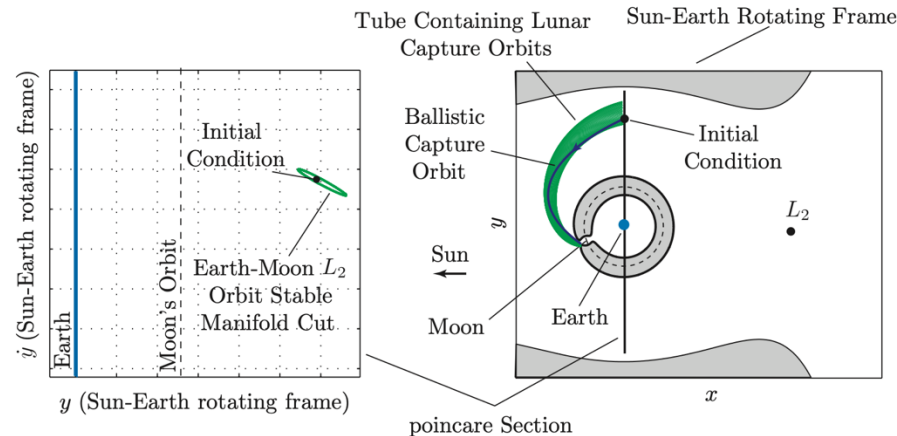


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, “Dynamical Systems, the Three-Body Problem and Space Mission Design”

# Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., “patched CR3BP”

Earth-Moon CR3BP:



Sun-Earth CR3BP:

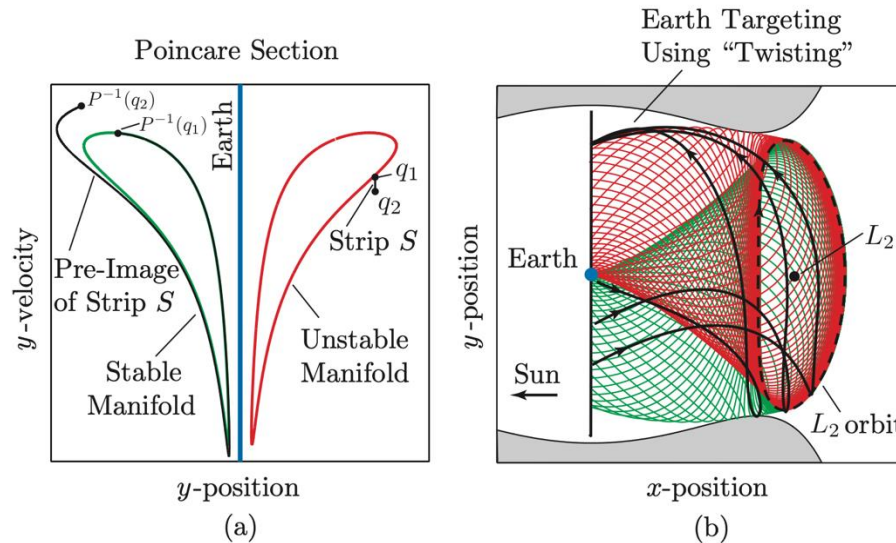
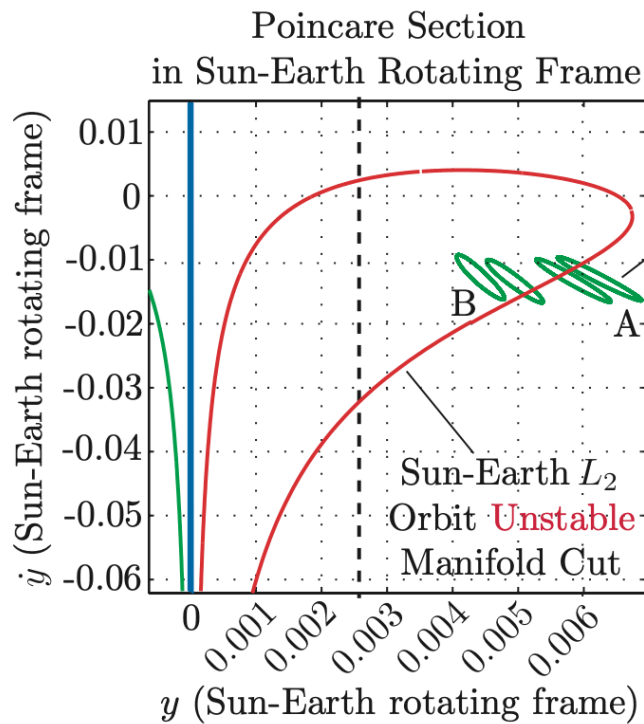


Image credits: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, “Dynamical Systems, the Three-Body Problem and Space Mission Design”

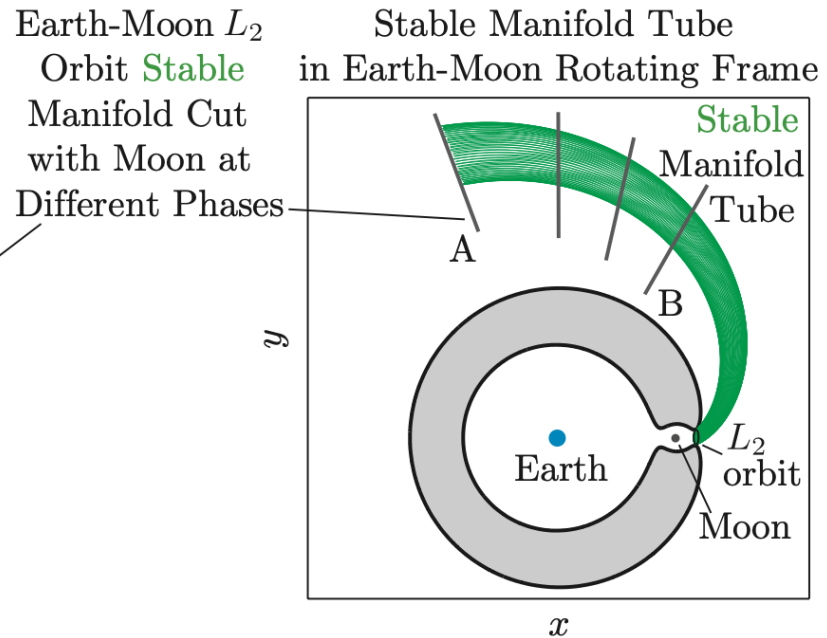


# Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., “patched CR3BP”



(a)



(b)

Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, “Dynamical Systems, the Three-Body Problem and Space Mission Design”

# Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., “patched CR3BP”

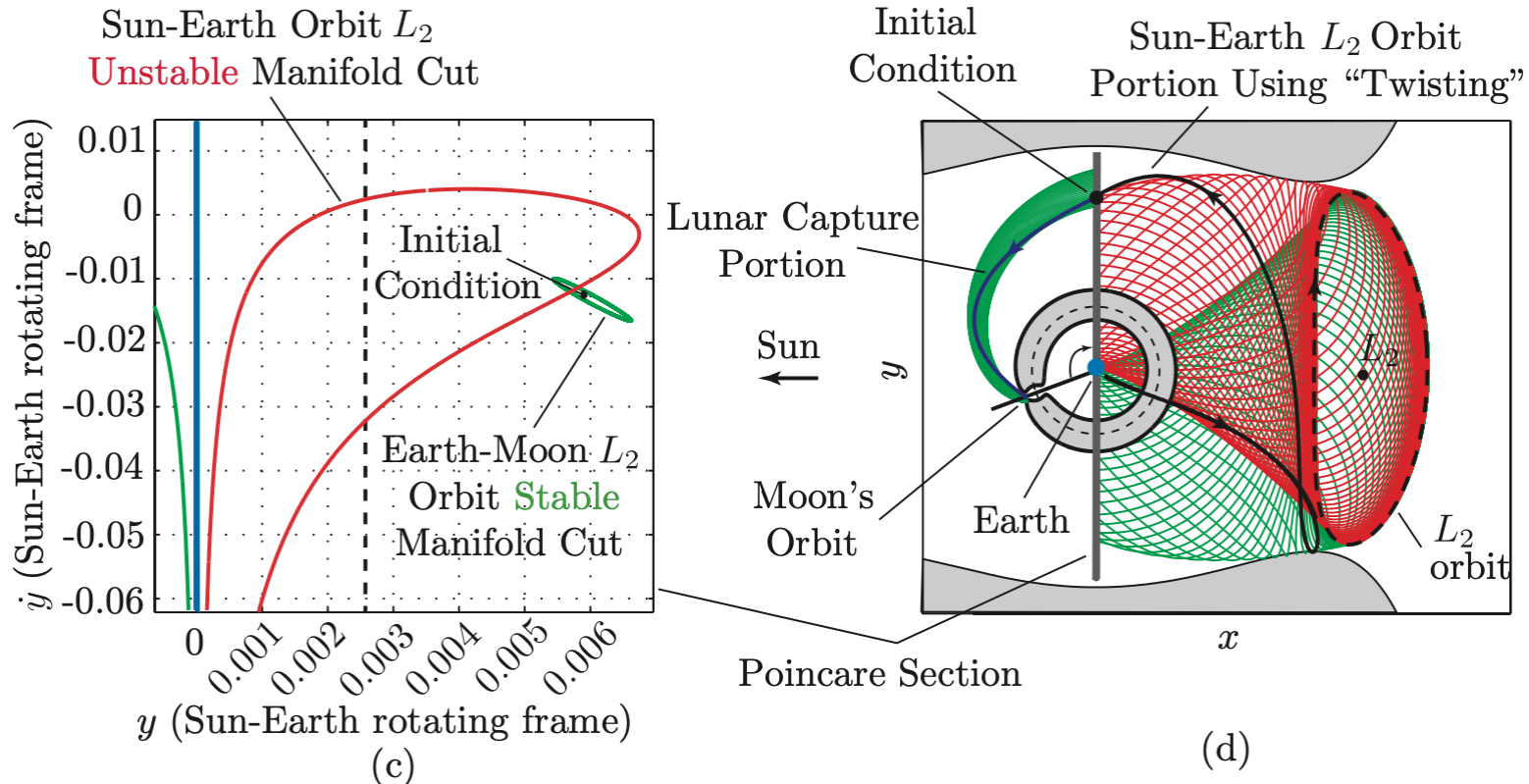
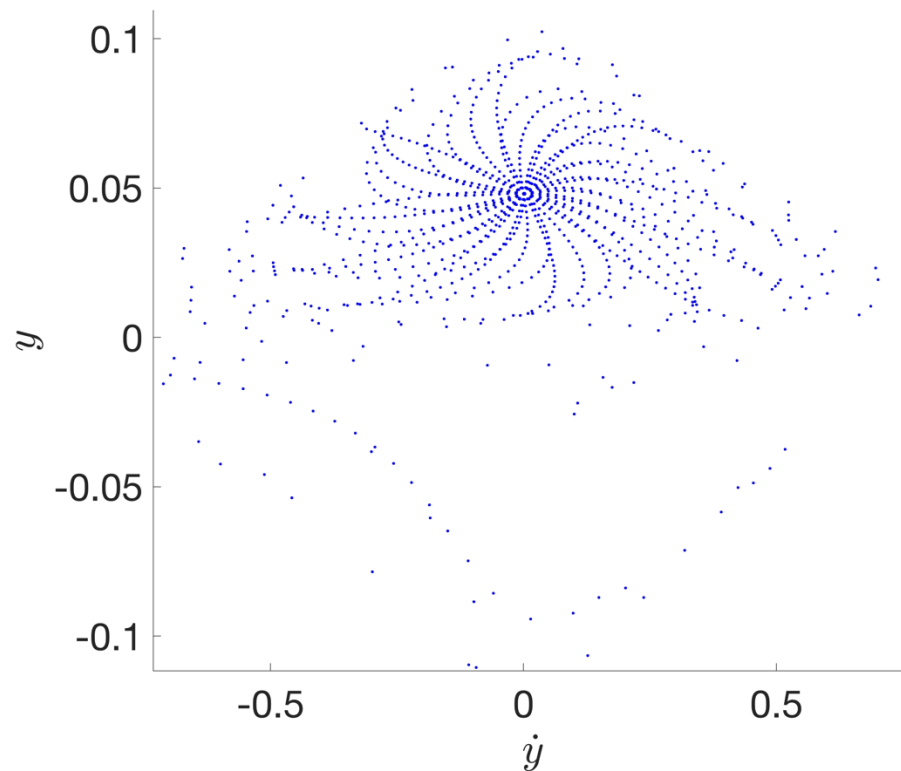


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, “Dynamical Systems, the Three-Body Problem and Space Mission Design”



# Question 1

**Question 1:** Your colleague has created the following Poincaré map and has asked you if it makes sense. What questions would you ask your colleague to help you respond? What are your initial thoughts in response to their question?



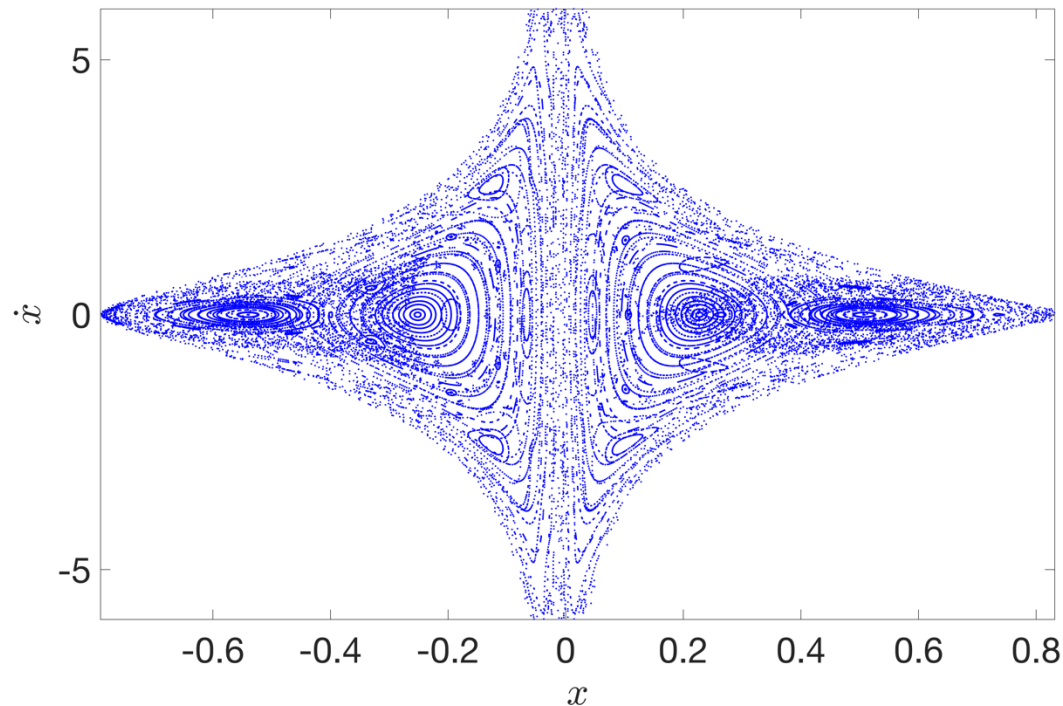
# *Question 1*

---

## **Question 1: Group Brainstorming:**

## Question 2

**Question 2:** Your colleague has created the following Poincaré map and has asked you if it makes sense. What questions would you ask your colleague to help you respond? What are your initial thoughts in response to their question?



# *Question 2*

---

**Question 2:** Group Brainstorming:

# *Question 3*

---

**Question 3:** When generating a Poincaré map representation of trajectories, what can you do to check if the results are accurate?

# *Question 3*

---

## **Question 3: Group Brainstorming:**



# Question 4

---

**Question 4:** Consider the following three nearby initial conditions:

$$\bar{x}_{0,1} = [0.3693, 0, 0, 0, 1.4772, 0]$$

$$\bar{x}_{0,2} = [0.3670, 0, 0, 0, 1.4865, 0]$$

$$\bar{x}_{0,3} = [0.3640, 0, 0, 0, 1.4994, 0]$$

Our goal today is to describe the type of trajectories associated with these initial conditions in as much detail as possible.

- a) Devise a plan for how you could use the tools and concepts we have covered in class to perform this assessment.
- b) Implement your plan and describe the types of these 3 trajectories

# *Question 4*

---

**Question 4a):** Devise a plan for how you could use the tools and concepts we have covered in class to perform this assessment.

Group Brainstorming: