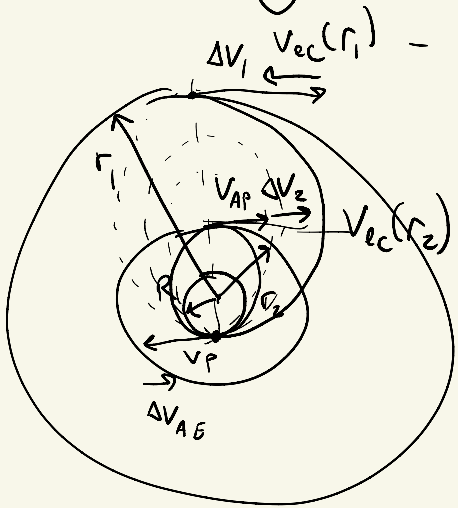
AeroAssist Transfers

Use of drag forces to effect orbit transfers

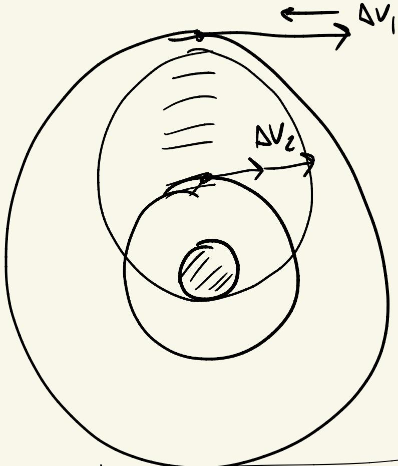


- Only apply for $r_1 > r_2$

- Assume a planetary radius, R, defines the top of the atmosphere

- Periapsis passage can shed kinetic energy from drag & pull down the apoapsis radius.

- When apoapsis $= r_2$, raise periapsis to a circular orbit.

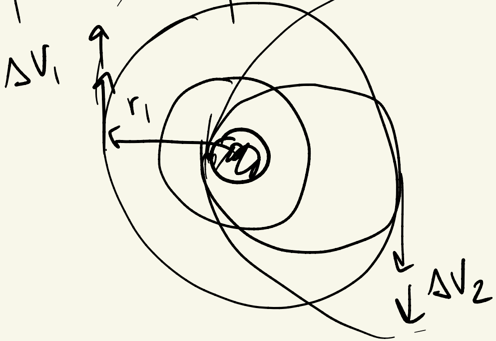


$$\Delta V_1 = \sqrt{\frac{\mu}{r_1}} - \sqrt{\frac{2\mu}{(r_1+R)}\frac{R}{r_1}}$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{(r_2+R)}\frac{R}{r_2}}$$

$$\Delta V_{AE} = \sqrt{\frac{\mu}{r_1}} - \sqrt{\frac{2\mu}{(r_1+R)}\frac{R}{r_1}} + \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{(r_2+R)}\frac{R}{r_2}}$$

A parabolic optim exists....



$$\Delta V_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{2}-1\right)$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{(r_2+R)}\frac{R}{r_2}}$$

$$\Delta V_{AEp} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{2}-1\right) + \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{(r_2+R)}\frac{R}{r_2}}$$

$\Delta V_{AE}(r_1, r_2, R, \mu) \Rightarrow$ Radius "R" puts a scale into the problem ...

Define $\alpha_1 = \frac{r_1}{R}$; $\alpha_2 = \frac{r_2}{R}$ $\qquad \frac{\alpha_1}{\alpha_2} = \mathcal{R} \geq 1 \qquad \alpha_1 \geq \alpha_2$

Normalize $\sqrt{\frac{\mu}{R}}$

$$\bar{J}_{AE}(\alpha_1, \alpha_2) = \frac{1}{\sqrt{\alpha_1}} - \sqrt{\frac{2}{\alpha_1(1+\alpha_1)}} + \frac{1}{\sqrt{\alpha_2}} - \sqrt{\frac{2}{\alpha_2(1+\alpha_2)}}$$

$$\bar{J}_{AEP}(\alpha_1, \alpha_2) = \frac{1}{\sqrt{\alpha_1}}(\sqrt{2}-1) + \frac{1}{\sqrt{\alpha_2}} - \sqrt{\frac{2}{\alpha_2(1+\alpha_2)}}$$

$$\bar{J}_{H}(\alpha_1, \alpha_2) = \frac{1}{\sqrt{\alpha_1}} - \sqrt{\frac{2\alpha_2}{(\alpha_1+\alpha_2)\alpha_1}} + \sqrt{\frac{2\alpha_1}{(\alpha_1+\alpha_2)\alpha_2}} - \frac{1}{\sqrt{\alpha_2}}$$

$$g_{I}(\alpha_1, \alpha_2) = J_H(\alpha_1, \alpha_2) - J_{AE}(\alpha_1, \alpha_2) \geq 0$$

$$g_{II}(\alpha_1, \alpha_2) = J_H(\alpha_1, \alpha_2) - J_{AEp}(\alpha_1, \alpha_2) \geq 0$$

$$g_{III}(\alpha_1, \alpha_2) = J_{AE}(\alpha_1, \alpha_2) - J_{AEp}(\alpha_1, \alpha_2)$$

Easiest ... is $g_{III}(\alpha_1, \alpha_2) = 0$

$$\boxed{g_{III} = \sqrt{\frac{2}{\alpha_1}}\left(\sqrt{2} - 1 - \frac{1}{\sqrt{1+\alpha_1}}\right)} = 0$$

Indep. of α_2

$$\sqrt{1+\alpha_1} = \frac{1}{(\sqrt{2}-1)}\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \sqrt{2}+1 \Rightarrow \alpha_1 + 1 = (\sqrt{2}+1)^2 = 2 + 2\sqrt{2} + 1 = 3 + 2\sqrt{2}$$

$$\boxed{\alpha_1 = 2 + 2\sqrt{2} = 2(\sqrt{2}+1) = 4.83 \ldots}$$

$\Rightarrow$ Whenever $\alpha_1 \geq 2(\sqrt{2}+1)$ $J_{AE} \geq J_{AEp}$

$$g_{II} = J_H - J_{AEp}$$

$$g_{II} = \sqrt{\frac{2}{\alpha_1}}\left(\sqrt{2}-1\right) - \sqrt{\frac{2\alpha_2}{\alpha_1(\alpha_1+\alpha_2)}} + \sqrt{\frac{2\alpha_1}{\alpha_2(\alpha_1+\alpha_2)}} - \frac{2}{\sqrt{\alpha_2}} + \sqrt{\frac{2}{\alpha_2(1+\alpha_2)}}$$

<u>Limits</u> Let $\alpha_1 \to \infty$

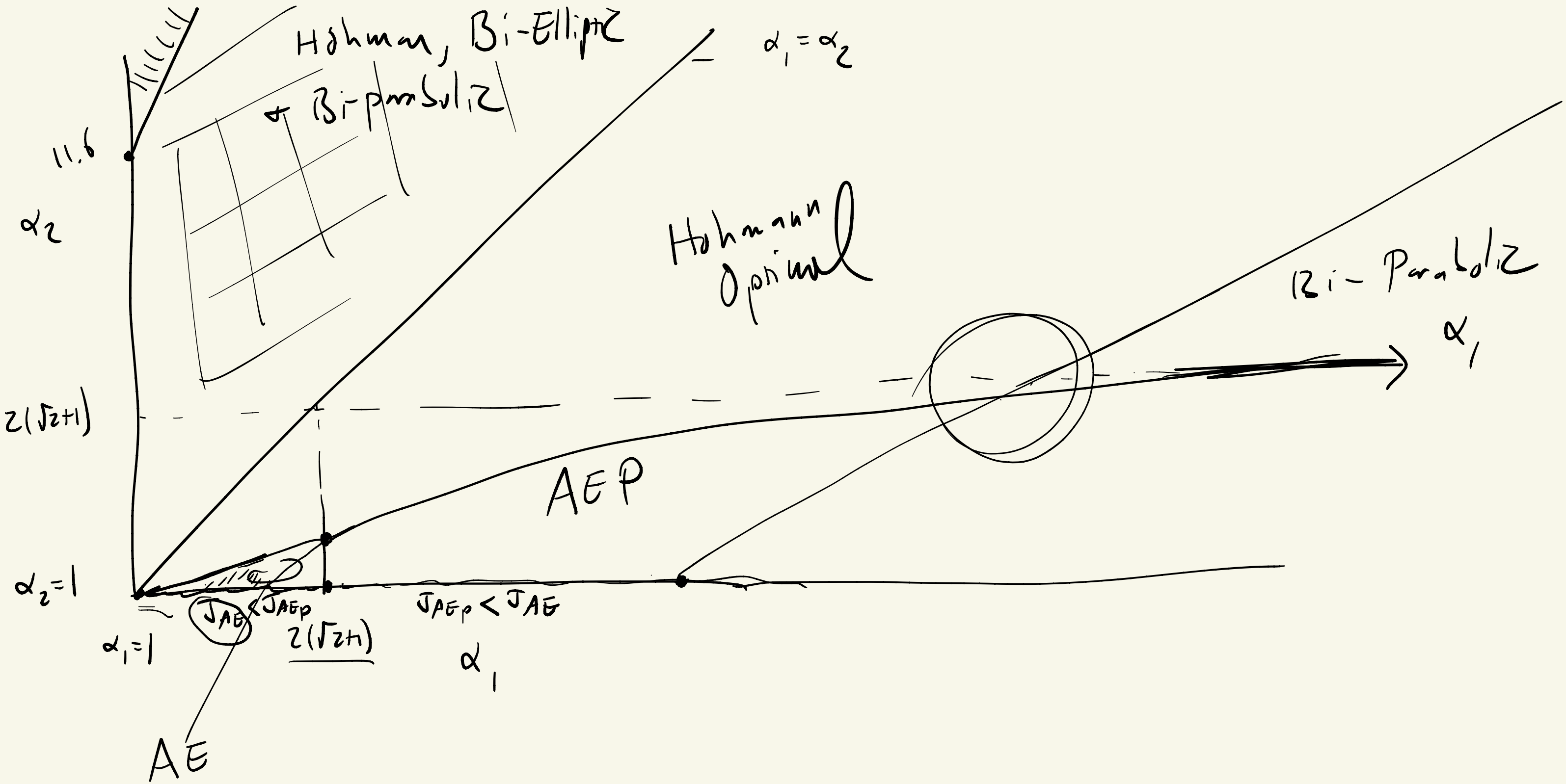
$$\lim_{\alpha_1\to\infty} g_{II}(\alpha_1,\alpha_2) = \sqrt{\frac{2}{\alpha_2}} - \frac{2}{\sqrt{\alpha_2}} + \sqrt{\frac{2}{\alpha_2(1+\alpha_2)}} = 0$$

solution $\alpha_2 = 2(\sqrt{2}+1) \simeq 4.83\ldots$

As $\alpha_1 \gg 1$; if $\alpha_2 \leq 2(\sqrt{2}+1)$ $\quad J_{AEp} \leq J_H$

" $\alpha_2 > 2(\sqrt{2}+1)$ $\quad J_H < J_{AEp}$

IF $\alpha_1 = \alpha_2 = \alpha$ $\quad g_{II}(\alpha,\alpha) = \sqrt{\frac{2}{\alpha}}\left[\frac{1}{\sqrt{1+\alpha}} - 1\right] < 0$



Hohman, Bi-Elliptic
+ Bi-parabolic
$\alpha_1 = \alpha_2$
11.6
α_2
Hohmann
Optimal
Bi-Parabolic
α_1
$2(\sqrt{2}+1)$
AEP
$\alpha_2 = 1$
$J_{AE} < J_{AEp}$
$J_{AEp} < J_{AE}$
$\alpha_1 = 1$
$2(\sqrt{2}+1)$
α_1
AE

Let $\alpha_2 = 1$ + find when $g_{II}(\alpha_1, 1) = 0$

$$g_{II}(\alpha_1, 1) = \sqrt{\frac{2}{\alpha_1}}\left[\sqrt{2} - 1 + \frac{\alpha_1 - 1}{\sqrt{1+\alpha_1}}\right] - 1 = 0$$

$$\left.\begin{array}{l} \alpha_1 = 2.0365\ldots \\ \alpha_2 = 1 \end{array}\right\} \quad J_{AEp} < J_H \text{ for } \begin{array}{l} \alpha_2 = 1 \\ \alpha_1 > 2.0365 \end{array}$$

Finally, consider $g_I = J_H - J_{AE}$

$$g_I = \sqrt{\frac{2}{\alpha_1(1+\alpha_1)}} - \sqrt{\frac{2\alpha_2}{\alpha_1(\alpha_1+\alpha_2)}} + \sqrt{\frac{2\alpha_1}{\alpha_2(\alpha_1+\alpha_2)}} + \sqrt{\frac{2}{\alpha_2(1+\alpha_2)}} - \frac{2}{\sqrt{\alpha_2}}$$

Look at $g_I(\alpha_1, \alpha_2 = 1) = \sqrt{\frac{2\alpha_1}{1+\alpha_1}} - 1 \geq 0 \Rightarrow \alpha_1 \geq 1$

Note .... $g_I(1,1) = 0$, expand analytically in the neighborhood of

$\alpha_1 = \alpha_2 = 1$ ....

$$(1+\epsilon)^{\pm n} = 1 \pm n\epsilon + \cdots$$

$$\alpha_1 = 1 + a\epsilon + \mathcal{O}(\epsilon^2)$$

$$\alpha_2 = 1 + \epsilon + \mathcal{O}(\epsilon^2)$$

$g_I(1+a\epsilon, 1+\epsilon) = 0 \quad \epsilon \ll 1$, $\Rightarrow$ solve for "a"

$a = 3 \quad g_I(1+3\epsilon, 1+\epsilon) = 0 \quad \epsilon \ll 1$

Finally, compare AEP w/ Bi-Parabolic

J_{AEP} ..., $J_{BP} = \dfrac{(\sqrt{2}-1)}{\sqrt{\alpha_1}} + \dfrac{(\sqrt{2}-1)}{\sqrt{\alpha_2}}$

$$\boxed{g_{IV} = J_{AEP} - J_{BP}}$$

$$\boxed{g_{IV}(\alpha_1, \alpha_2) = \sqrt{\frac{2}{\alpha_2}}\left[\sqrt{2} - 1 - \frac{1}{\sqrt{1+\alpha_2}}\right]}$$

Discuss that $J_{BP} \geq J_{AEP}$ for all α_1, if $\alpha_2 \leq 2(\sqrt{2}+1) \sim 4.8$

Practicality (Earth)

$R \sim 6400$ km

LEO $r_1 \sim 7E3$ km

GEO $r_2 \sim 42E3$ km

GPS $r_{GPS} \sim 26E3$ km

Moon $r_3 \sim 384E3$ km

$\frac{r_2}{r_1} \sim 6 \Rightarrow$ Bi-Parabolic doesn't work

$\frac{r_3}{r_2} \sim 9$

$\frac{r_3}{r_1} \sim 55$

CR3BP