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$\boxed{HW 1}$

Problem 1 → Given: Lucy orbit information relative to the sun at time t_1

$r = 1.6599 \times 10^8$ km, $a = 2.3132 \times 10^8$ km, $v_r = -11.6485$ km/s

Assumptions: 2-Body problem, $\mu_{sun} = 1.32712428 \times 10^{11}$ km³/s²,

$\mu_{jupiter} = 1.268 \times 10^8$ km³/s², no additional perturbations

a] $\mathcal{E} = -\frac{\mu_{sun}}{2a} = \boxed{-286.8590 \text{ km}^2/\text{s}^2 = \mathcal{E}}$ (∵ Orbit is heliocentric can use $\mathcal{E} = -\frac{\mu}{2a}$)

$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \rightarrow v = \sqrt{\mu_{sun}\left(\frac{2}{r} - \frac{1}{a}\right)} = \boxed{32.0207 \frac{\text{km}}{\text{s}} = v}$

b] $\boxed{\vec{r} = 1.6599 \times 10^8 \hat{r} + 0\hat{\theta} + 0\hat{h} \text{ km}}$

$\vec{v} = -11.6485\hat{r} + x\hat{\theta} \frac{\text{km}}{\text{s}}$, $|\vec{v}| = 32.0207 \frac{\text{km}}{\text{s}}$

∴ $x = \sqrt{v^2 - v_r^2} = 29.8268 \frac{\text{km}}{\text{s}}$ (>0 because v_θ cannot be <0)

$\boxed{\vec{v} = -11.6485\hat{r} + 29.8268\hat{\theta} + 0\hat{h} \text{ km/s}}$

c] $\tan \phi_{fpa} = \frac{v_r}{v_\theta} \rightarrow \phi_{fpa} = -21.3325°$

$h = rv\cos(\phi_{fpa}) = \boxed{4.9510 \times 10^9 \frac{\text{km}^2}{\text{s}} = h}$

$p = h^2/\mu_{sun} = \boxed{1.8470 \times 10^8 \text{ km} = p}$ - (semi-latus rectum)

$p = a(1-e^2) \rightarrow e = \sqrt{1 - p/a} = \boxed{0.4489 = e}$ (>0 because eccentricity >0 always)

$r = \frac{h^2/\mu}{1 + e\cos\theta^*} \rightarrow \theta^* = \pm 75.4550°$ ∵ $v_r < 0$, $\phi_{fpa} < 0$, $\theta^* < 0$ ∴ $\boxed{\theta = -75.4550°}$

$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_{sun}}} = \boxed{6.0680 \times 10^7 \text{ s} = \mathbb{P}}$ - (orbit period)

$\boxed{\because e < 1, \text{ the orbit is an ellipse}}$

at time t_2, in inertial frame with axes $\hat{x}\hat{y}\hat{z}$,

$\vec{R} = 1.0751 \times 10^8 \hat{x} - 1.2647 \times 10^8 \hat{y} + 1.3644 \times 10^5 \hat{z}$ km

$\vec{V} = 1.5180 \times 10\hat{x} + 2.8193 \times 10\hat{y} + 1.0504 \times 10^{-2}\hat{z}$ km/s

d] $\vec{h} = \vec{R} \times \vec{V} = -0.0052 \times 10^9 \hat{x} + 0.0009 \times 10^9 \hat{y} + 4.9508 \times 10^9 \hat{z}$ km²/s

$|\vec{h}| = \boxed{h = 4.9508 \times 10^9 \text{ km}^2/\text{s}}$

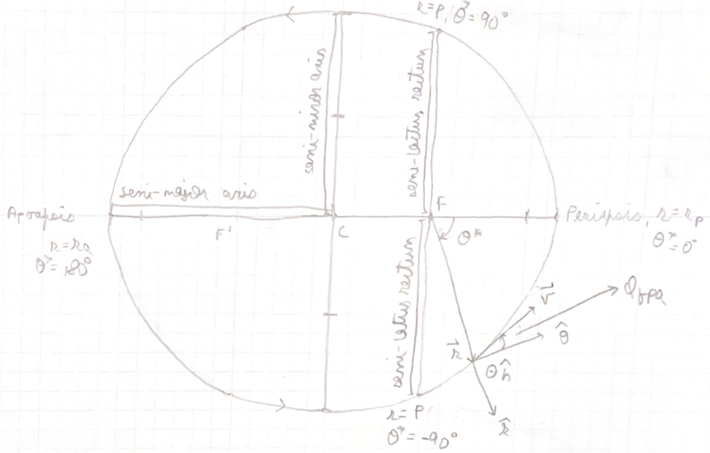
$\vec{e} = \frac{\vec{V} \times \vec{h}}{\mu_{sun}} - \frac{\vec{R}}{|\vec{R}|} = 0.4041\hat{x} + 0.1956\hat{y} + 3.8514 \times 10^{-4}\hat{z}$, $|\vec{e}| = \boxed{e = 0.4489}$

$\mathcal{E} = \frac{1}{2}|\vec{V}|^2 - \frac{\mu_{sun}}{|\vec{R}|} = \boxed{-286.8759 \frac{\text{km}^2}{\text{s}^2} = \mathcal{E}}$

$h, \mathcal{E}, e$ are $\boxed{\text{consistent}}$ with previously calculated values because these are constants.

e

1 tick = 2.0×10^8 km
$b = a\sqrt{1-e^2} = 2.0670 \times 10^8$ km
$r_p = 1.2747 \times 10^8$ km
$a = 2.3132 \times 10^8$ km
$p = 1.847 \times 10^8$



f) $r = \frac{h^2/\mu}{1+e\cos\theta^*}$, $r_{max}(\theta^* = -180^\circ) = 3.3517 \times 10^8$ km, $r_{min}(\theta^* = 0) = 1.2747 \times 10^8$ km

$\therefore r_{Lucy} = 1.6599 \times 10^8$ km ($> r_{min}$ and $< r_{max}$), Lucy is not at periapsis or apoapsis. Also, $r_{Lucy} < p$ and since, $\theta^* > -90^\circ$, Lucy is on the bottom half of the orbit, and past the semi-latus rectum going towards the periapsis.

g) Assumptions for 2 Body Problem:

① $mass_{Lucy} \lll M_{sun}$ ← True

② Inertial frames chosen ← Assuming true from given statements

③ Lucy and sun are spherical bodies with uniform density ← Lucy is probably not spherical, and sun's gravitational field is not uniform.

④ No other forces except gravity ← Solar radiation pressure may be a big contributor to dynamics here. Additionally, Dinkinesh, Selam, Earth may contribute n-body effects to the spacecraft. Other celestial bodies may also contribute to this 2-body system depending on their positions.

Given all of the above, 2BP is a good first step to approximate for the dynamical system. But, more work needs to be done to increase the fidelity of the approximation.

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Problem 2 → Given: Voyager flyby of Jupiter in March 1979, hyperbolic orbit

$v_\infty = 10.7527$ km/s, $\theta^*_\infty = 139.3724°$

Assumptions: 2-Body Problem, $\mu_{jupiter} = 1.268 \times 10^8 \frac{km^3}{s^2}$, no additional perturbations

a] $V_\infty = \sqrt{\frac{\mu}{|a|}} \rightarrow |a| = \frac{\mu}{V_\infty^2} = 1.0967 \times 10^6$ km. By convention $a<0 \rightarrow \boxed{a = -1.0967 \times 10^6 \text{ km}}$

$\theta^*_\infty = \pm \cos^{-1}\left(\frac{-1}{e}\right) \rightarrow -\frac{1}{e} = \cos(\theta^*_\infty) \rightarrow e = \frac{-1}{\cos(\theta^*_\infty)} = \boxed{1.3176 = e}$

$\delta/2 + 90° = \theta^*_\infty \rightarrow \delta = 2(\theta^*_\infty - 90°) = \boxed{98.7448° = \delta}$

b] $r = \frac{|a|(e^2-1)}{1+e\cos\theta^*}$, for r_p, $\theta^* = 0 \rightarrow \therefore r_p = \frac{|a|(e^2-1)}{1+e} = \boxed{3.4831 \times 10^5 \text{ km} = r_p}$

$\mathcal{E} = \frac{v^2}{2} - \frac{\mu_{jupiter}}{r} = \frac{V_\infty^2}{2}$, at $r_p \rightarrow \frac{V_p^2}{2} - \frac{\mu_{jupiter}}{r_p} = \frac{V_\infty^2}{2} \rightarrow V_p = \sqrt{\left(\frac{V_\infty^2}{2} + \frac{\mu_{jupiter}}{r_p}\right)2}$

$\boxed{V_p = 29.0468 \text{ km/s}}$

c] 2BP assumptions:

Assumptions	Does it hold?
(1) $M_{voyager1} \lll M_{jupiter}$	True
$M_{voyager}$ is constant	Probably true during flyby
(2) coordinate system is inertial	True
(3) Treat satellite, Jupiter as point mass	Jupiter is oblate gravitational field is not constant. So, assumption does not hold
(4) Gravity because of Jupiter-Voyager1 is only force	Jupiter has 95 moons these add 3rd Body effects to this system. Asteroids and sun may also contribute to this. So, assumption doesn't hold

As given above, some assumptions don't completely hold true. ∴ The 2BP does not paint the full picture. However, it is a very good first step to approximate this dynamical system.