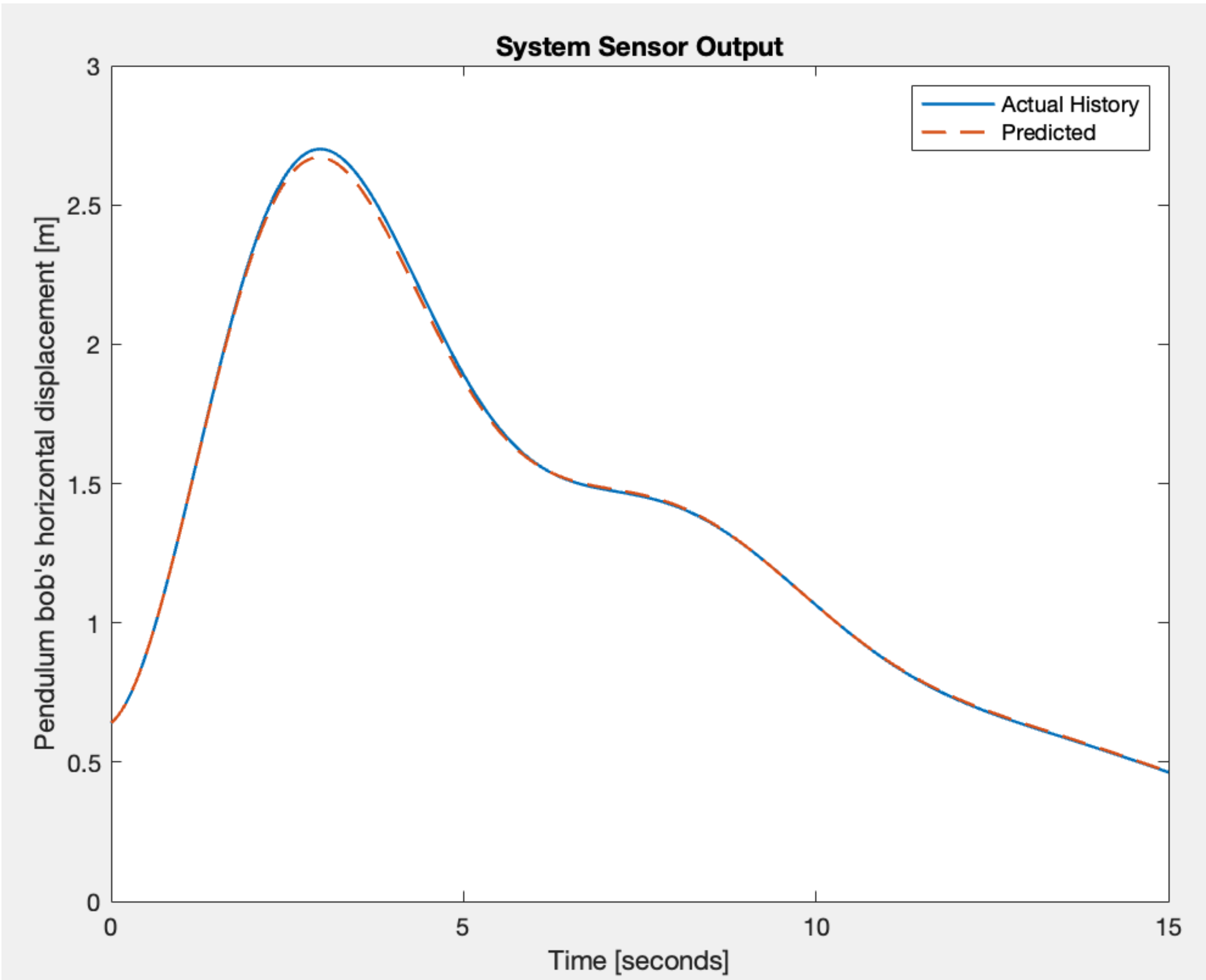
**ASEN 5044, Midterm 1**
**Fall 2024**
**Bhalavat Jash**

- Problem 1
  - Part e




      - The output predicted by the linearized DT closed-loop model is very close to the actual measurements in ‘yNLhist’. The reason that it does not exactly match the actual measurements is because the non-linear model is very accurate around the nominal state and its error increases the farther the system goes away from the nominal state. Additionally, sensor measurements are also not perfect. So, it may also lead to that small deviation from the predicted model.

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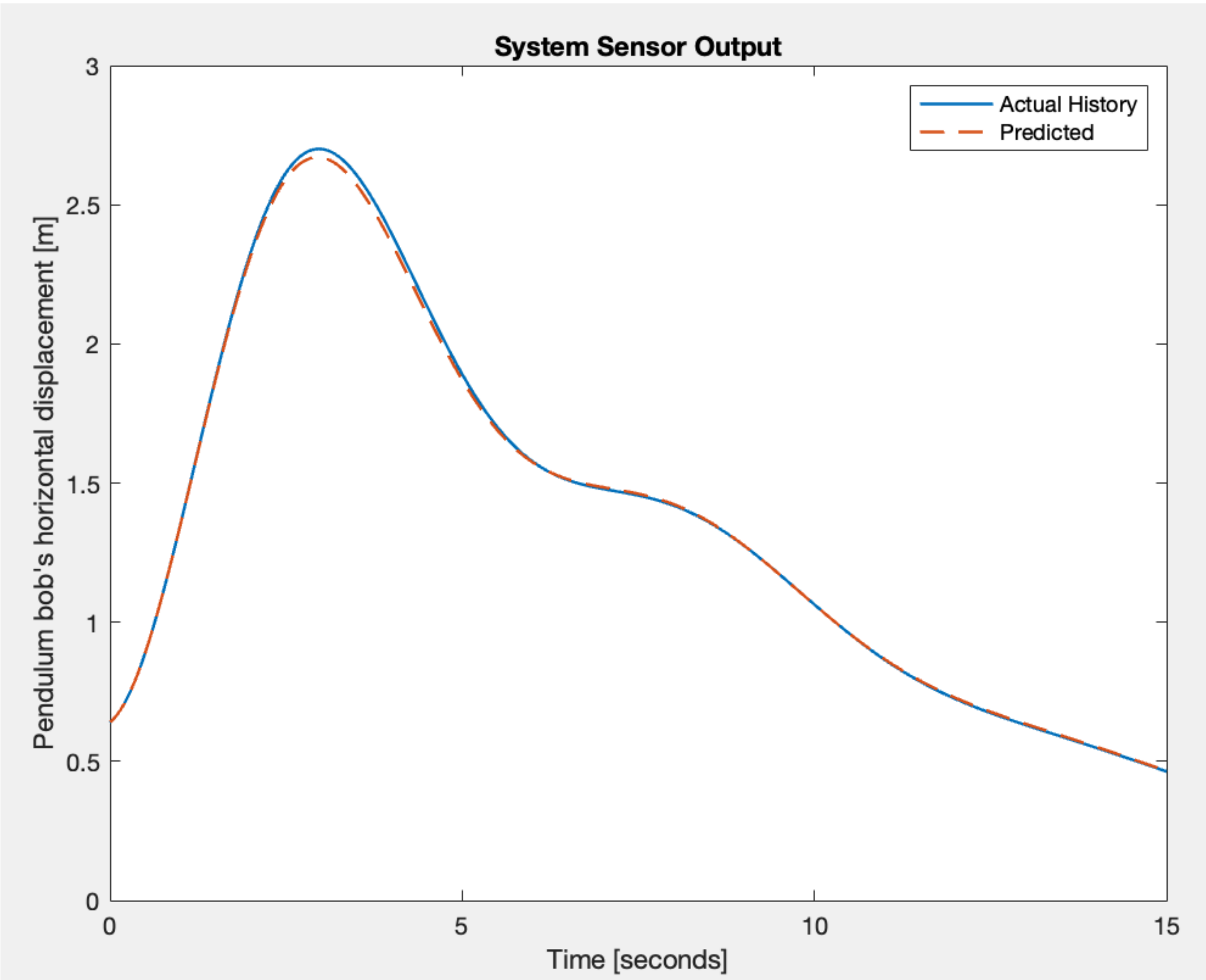


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- Problem 2

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Problem 2 Outcomes of $R_1 \rightarrow 1,2,3,4,5,6$ | Outcomes of $R_2 \rightarrow 1,2,3,4,5,6$

a) For any $R_1$ and $R_2 \rightarrow P(R_1) = P(R_2) = \frac{1}{6}$ because the dice are independent and all outcomes are equally probable

$\therefore P(R_1, R_2) = P(R_1) \cdot P(R_2) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

  - Part a
    - The joint probability distribution table would look like this:

| P(R1 ∧ R2) | R1 = 1 | R1 = 2 | R1 = 3 | R1 = 4 | R1 = 5 | R1 = 6 |
|---|---|---|---|---|---|---|
| R2 = 1 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| R2 = 2 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| R2 = 3 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| R2 = 4 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| R2 = 5 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| R2 = 6 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |

  - Part b

| Joint Probability Distribution Table for X and Y | | | | | | |
|---|---|---|---|---|---|---|
| P(X ∧ Y) | X = 0 | X = 1 | X = 2 | X = 3 | X = 4 | X = 5 |
| Y = 1 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |

| Joint Probability Distribution Table for X and Y | | | | | | |
|---|---|---|---|---|---|---|
| P(X ∧ Y) | X = 0 | X = 1 | X = 2 | X = 3 | X = 4 | X = 5 |
| Y = 2 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 3 | 0 | 0 | 1/36 | 0 | 0 | 0 |
| Y = 4 | 1/36 | 0 | 0 | 1/36 | 0 | 0 |
| Y = 5 | 0 | 0 | 0 | 0 | 1/36 | 0 |
| Y = 6 | 0 | 0 | 0 | 0 | 0 | 1/36 |
| Y = 8 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 9 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 16 | 0 | 0 | 2/36 | 0 | 0 | 0 |
| Y = 25 | 0 | 0 | 0 | 1/36 | 0 | 0 |
| Y = 27 | 1/36 | 0 | 0 | 0 | 0 | 0 |
| Y = 32 | 0 | 0 | 0 | 1/36 | 0 | 0 |
| Y = 36 | 0 | 0 | 0 | 0 | 1/36 | 0 |
| Y = 64 | 0 | 1/36 | 0 | 0 | 1/36 | 0 |
| Y = 81 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 125 | 0 | 0 | 1/36 | 0 | 0 | 0 |
| Y = 216 | 0 | 0 | 0 | 1/36 | 0 | 0 |
| Y = 243 | 0 | 0 | 1/36 | 0 | 0 | 0 |
| Y = 256 | 1/36 | 0 | 0 | 0 | 0 | 0 |
| Y = 625 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 729 | 0 | 0 | 0 | 1/36 | 0 | 0 |
| Y = 1024 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 1296 | 0 | 0 | 1/36 | 0 | 0 | 0 |

| Joint Probability Distribution Table for X and Y | | | | | | |
|---|---|---|---|---|---|---|
| P(X ∧ Y) | X = 0 | X = 1 | X = 2 | X = 3 | X = 4 | X = 5 |
| Y = 3125 | 1/36 | 0 | 0 | 0 | 0 | 0 |
| Y = 4096 | 0 | 0 | 1/36 | 0 | 0 | 0 |
| Y = 7776 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 15625 | 0 | 1/36 | 0 | 0 | 0 | 0 |
| Y = 46656 | 1/36 | 0 | 0 | 0 | 0 | 0 |

- Part c

| Marginal Probabilities Table | | |
|---|---|---|
| Marginal Probability | Description | Value |
| X = 0 | Sum of X = 0 column in part b | 1/6 |
| X = 1 | Sum of X = 1 column in part b | 10/36 |
| X = 2 | Sum of X = 2 column in part b | 8/36 |
| X = 3 | Sum of X = 3 column in part b | 1/6 |
| X = 4 | Sum of X = 4 column in part b | 4/36 |
| X = 5 | Sum of X = 5 column in part b | 2/36 |
| Y = 1 | Sum of Y = 1 row in part b | 1/6 |
| Y = 2 | Sum of Y = 2 row in part b | 1/36 |
| Y = 3 | Sum of Y = 3 row in part b | 1/36 |
| Y = 4 | Sum of Y = 4 row in part b | 2/36 |
| Y = 5 | Sum of Y = 5 row in part b | 1/36 |

| | | |
|---|---|---|
| Y = 6 | Sum of Y = 6 row in part b | 1/36 |
| Y = 8 | Sum of Y = 8 row in part b | 1/36 |
| Y = 9 | Sum of Y = 9 row in part b | 1/36 |
| Y = 16 | Sum of Y = 16 row in part b | 2/36 |
| Y = 25 | Sum of Y = 25 row in part b | 1/36 |
| Y = 27 | Sum of Y = 27 row in part b | 1/36 |
| Y = 32 | Sum of Y = 32 row in part b | 1/36 |
| Y = 36 | Sum of Y = 36 row in part b | 1/36 |
| Y = 64 | Sum of Y = 64 row in part b | 2/36 |
| Y = 81 | Sum of Y = 81 row in part b | 1/36 |
| Y = 125 | Sum of Y = 125 row in part b | 1/36 |
| Y = 216 | Sum of Y = 216 row in part b | 1/36 |
| Y = 243 | Sum of Y = 243 row in part b | 1/36 |
| Y = 256 | Sum of Y = 256 row in part b | 1/36 |
| Y = 625 | Sum of Y = 625 row in part b | 1/36 |
| Y = 729 | Sum of Y = 729 row in part b | 1/36 |
| Y = 1024 | Sum of Y = 1024 row in part b | 1/36 |
| Y = 1296 | Sum of Y = 1296 row in part b | 1/36 |
| Y = 3125 | Sum of Y = 3125 row in part b | 1/36 |
| Y = 4096 | Sum of Y = 4096 row in part b | 1/36 |
| Y = 7776 | Sum of Y = 7776 row in part b | 1/36 |
| Y = 15625 | Sum of Y = 15625 row in part b | 1/36 |
| Y = 46656 | Sum of Y = 46656 row in part b | 1/36 |

- Part d
    - X and Y would be independent if P(X = x & Y = y) = P(X=x) * P(Y=y)
        - This is easy to refute
            - P(X = 1 & Y = 1) from table in part b = 1/36 = 0.028
            - P(X = 1) from table in part c = 10/36
            - P(Y = 1) from table in part c = ⅙
            - P(X = 1) * P(Y = 1) = 10/36 * 1/36 = 0.008
            - Since, 0.028 is not equal to 0.008, X and Y are not independent.
    - Using another way to determine independence:
        - If X and Y were independent, P(Y = y | X = x) = P(Y = y)
        - P(Y = 9 | X = 1) = P (X = 1 & Y = 9) / P(X = 1) = 1/36 / 10/36 = 1/10
        - P(Y = 9) = 1/36
        - Since 1/10 is not equal to 1/36, P(Y = 9 | X = 1) is not equal to P(Y = 9) and X and Y are not independent.