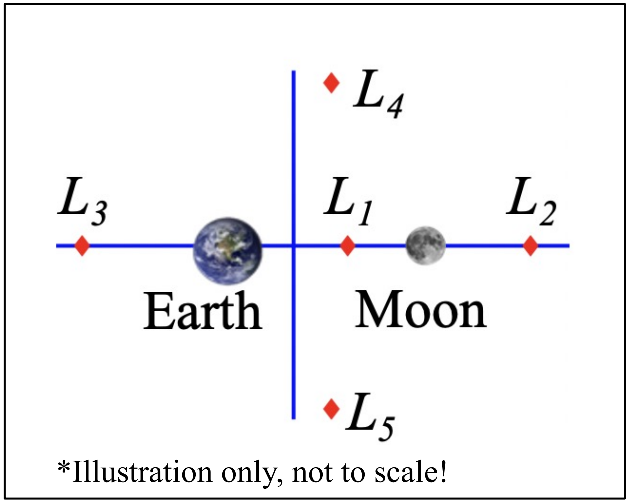
ASEN 5050
ADVANCED ASTRODYNAMICS
Multi-Body Dynamics

Objectives:

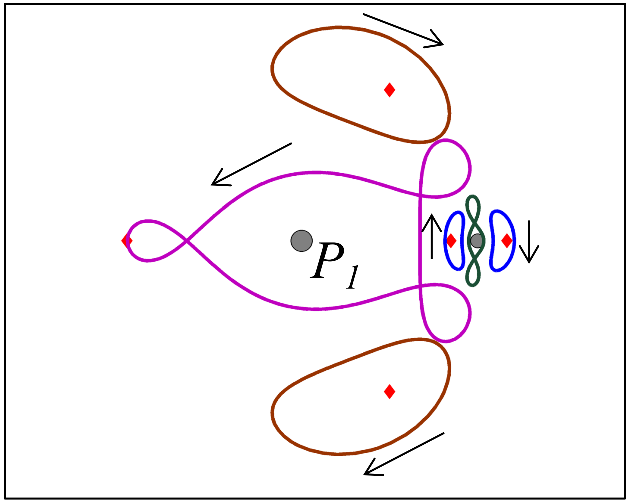
- Overview different types of fundamental solutions
- Overview the trajectory design process
- Present interesting trajectory design examples

Fundamental Solutions

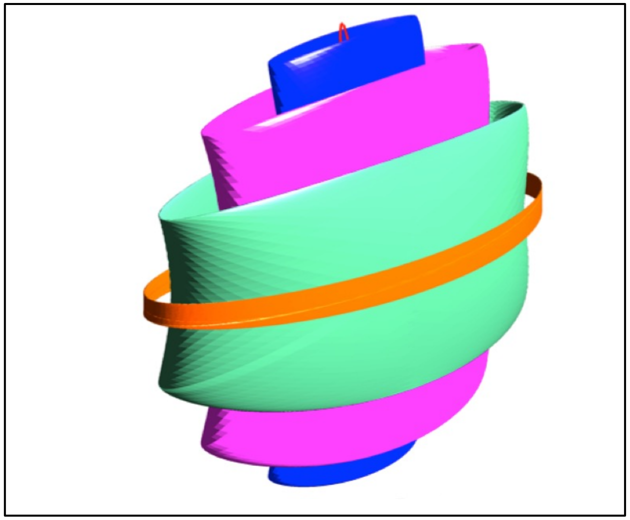
Equilibrium Points



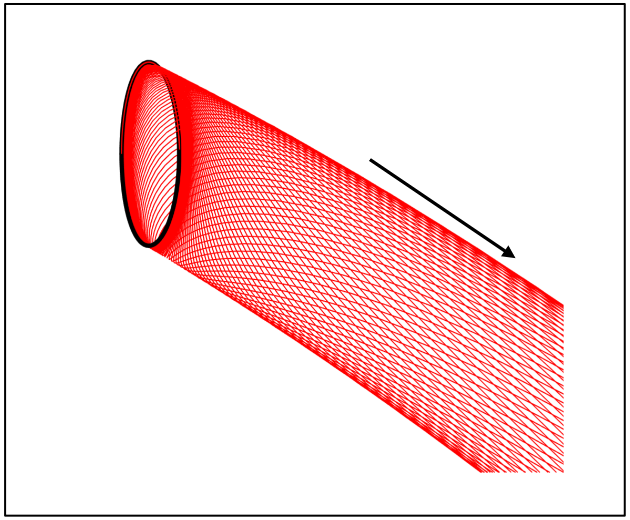
Periodic Orbit



Quasi-Periodic Trajectory tracing out the surface of a torus



Stable/ Unstable Manifolds



Locating Equilibrium Points

Recall the equations of motion:

$$\ddot{x} - 2\dot{y} = \frac{\partial U^*}{\partial x} \qquad \ddot{y} + 2\dot{x} = \frac{\partial U^*}{\partial y} \qquad \ddot{z} = \frac{\partial U^*}{\partial z}$$

Where $U^* = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$

$$r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2} \qquad r_2 = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$$

Equilibrium points are stationary solutions in the rotating frame, i.e.,

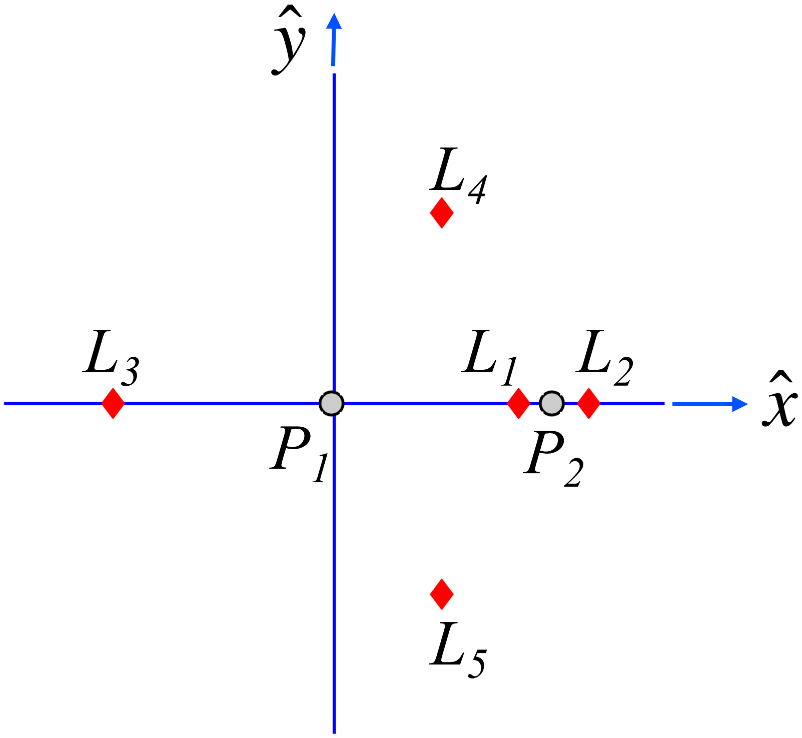
$$\dot{x} = \dot{y} = \dot{z} = 0 \qquad \ddot{x} = \ddot{y} = \ddot{z} = 0$$

Equilibrium Points in the CR3BP

There are five equilibrium points in the CR3BP:

- L_1, L_2, L_3 are collinear equilibrium points
- L_4, L_5 are triangular or equilateral equilibrium points

Note: also called Lagrange points, libration points



Periodic Orbits

A periodic orbit is a nonconstant trajectory that repeats in the rotating frame after a minimal period *T*: $\bar{x}_{PO}(t) = \bar{x}_{PO}(t+T)$

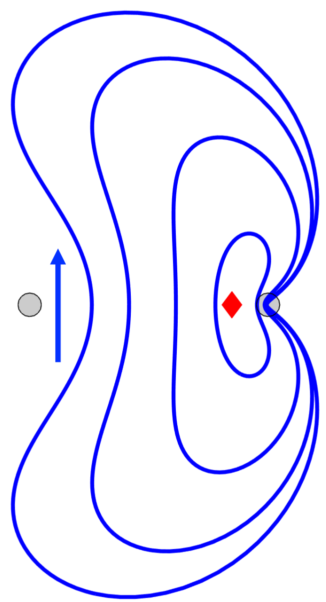
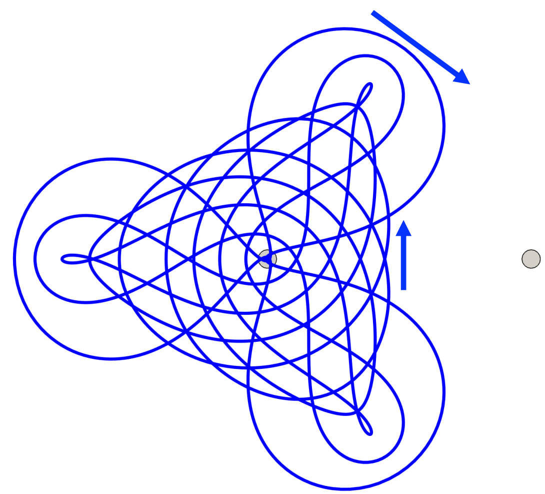
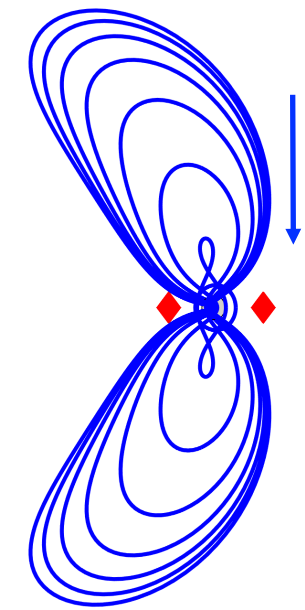
Periodic orbits can exist in dynamical systems that are autonomous (e.g., CR3BP) or periodic (e.g., ER3BP, orbit period constrained)

In the autonomous CR3BP:

- a periodic orbit is completely specified by the period *T* and a (nonunique) state, $\bar{x}_{PO}$
- Periodicity is defined in the rotating frame
- Calculate numerically using corrections algorithm
- Periodic orbits exist in continuous families (sets of solutions with a continuously varying set of initial conditions, orbit period, and other parameters)

Periodic Orbit Families in the CR3BP

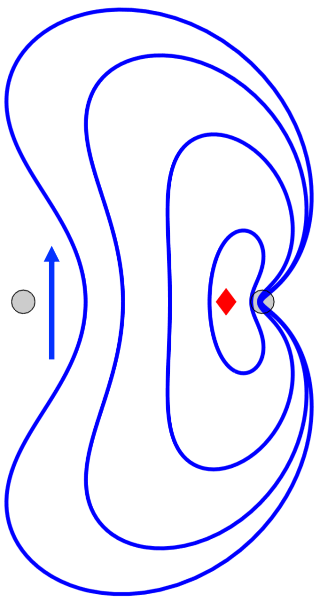
Some prominent periodic orbit families include:

Libration point orbits	Resonant orbits	Primary-centered orbits
		

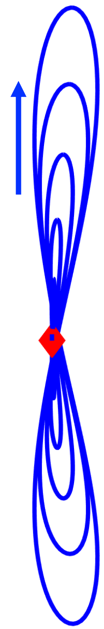
Selected Libration Point Orbits

- Lyapunov and vertical orbits emanate from L_1, L_2, L_3
- Halo and axial orbits emanate from bifurcations along Lyapunov and/or vertical orbit families (i.e., they intersect them)

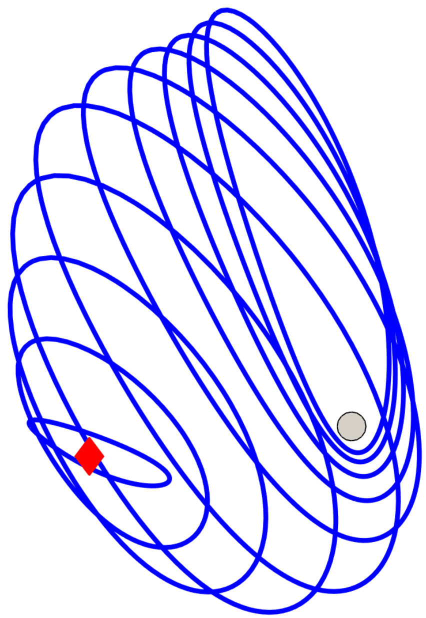
L_1 Lyapunov orbits



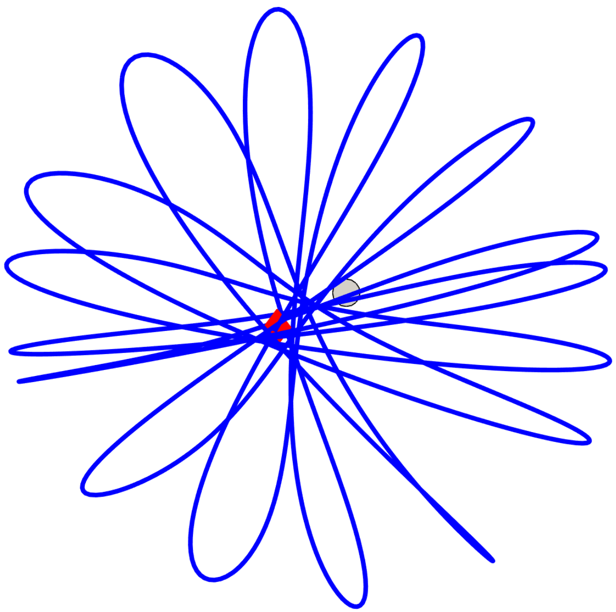
L_1 vertical orbits



L_1 halo orbits



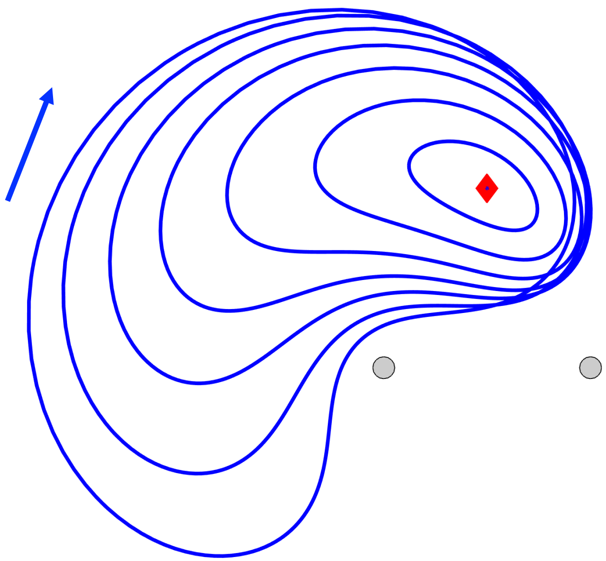
L_1 axial orbits



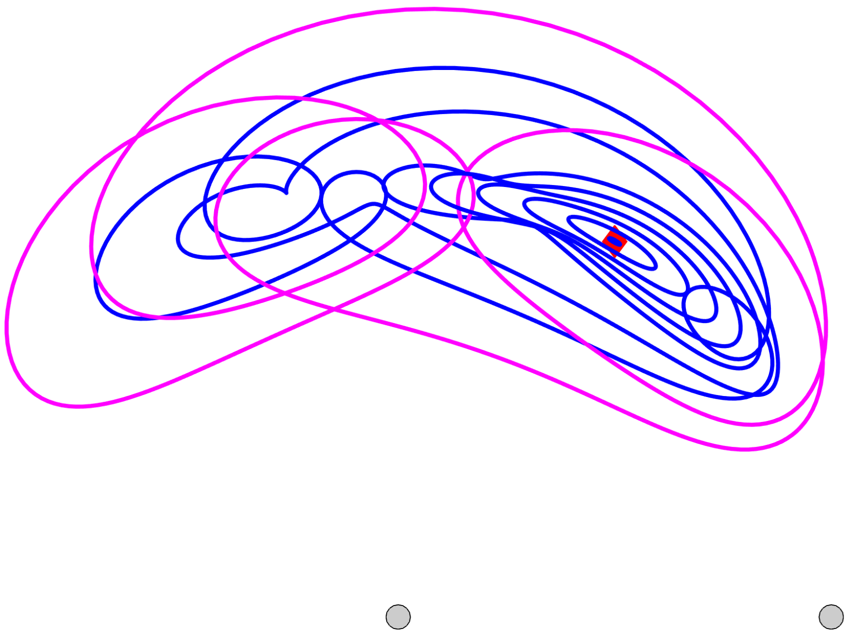
Selected Libration Point Orbits

- Similar families about L_5 exist and are symmetric about the *x*-axis

L_4 short period orbits



L_4 long period orbits



Selected Resonant Orbits

$p{:}q$ resonance: P_3 completes p revolutions in the same time interval that P_2 completes q revolutions in its orbit relative to P_1

→Not all orbits in the family possess exact resonant period

Interior resonance: $p > q$, orbit primarily about P_1

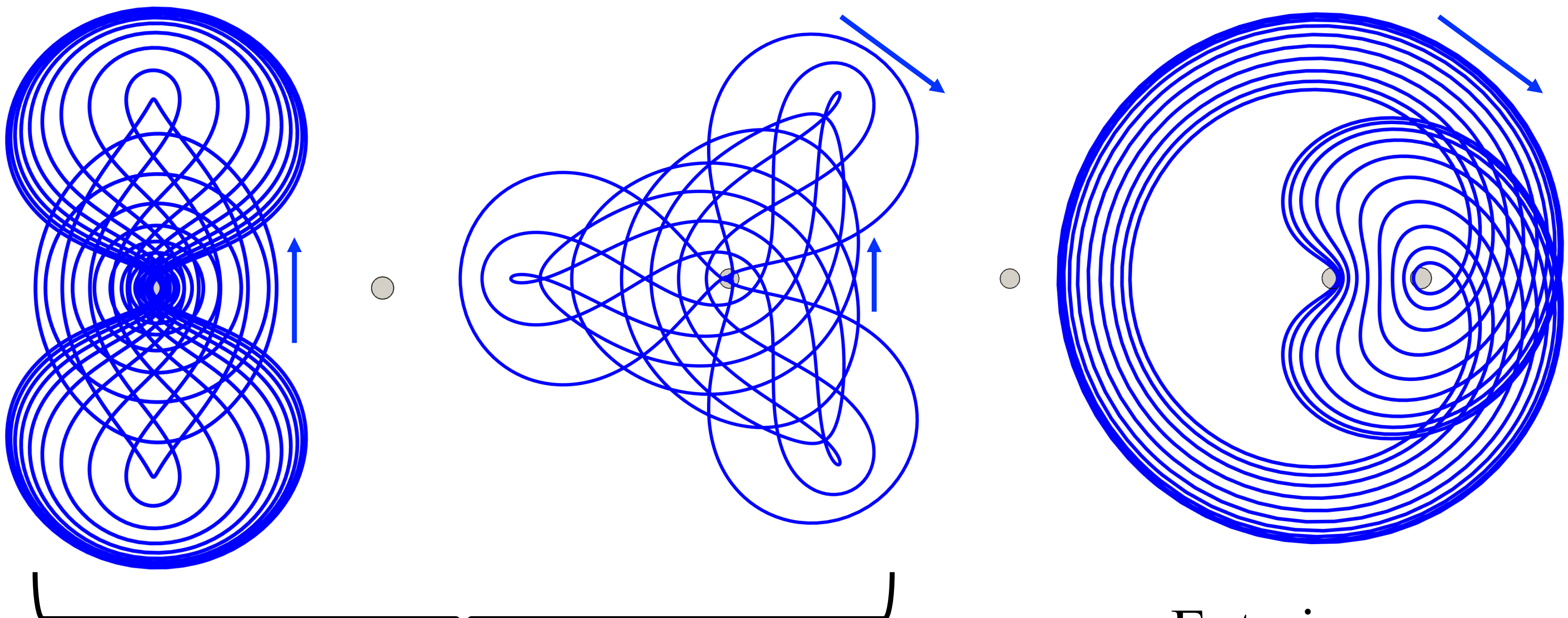
Exterior resonance $p < q$, orbit performs revolution/s in the exterior region with 1 or more close passes to the primaries or equilibrium points possible

Selected Resonant Orbits

2:1 resonant orbit family

3:1 resonant orbit family

1:2 resonant orbit family

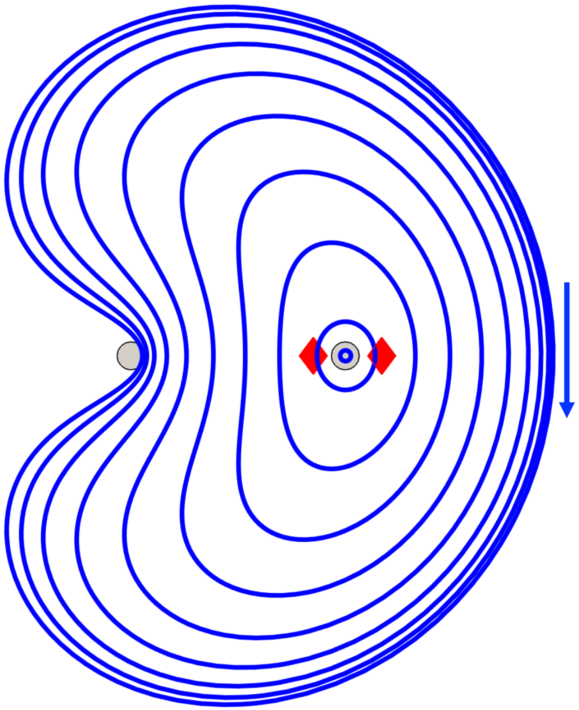


Interior resonant orbit families

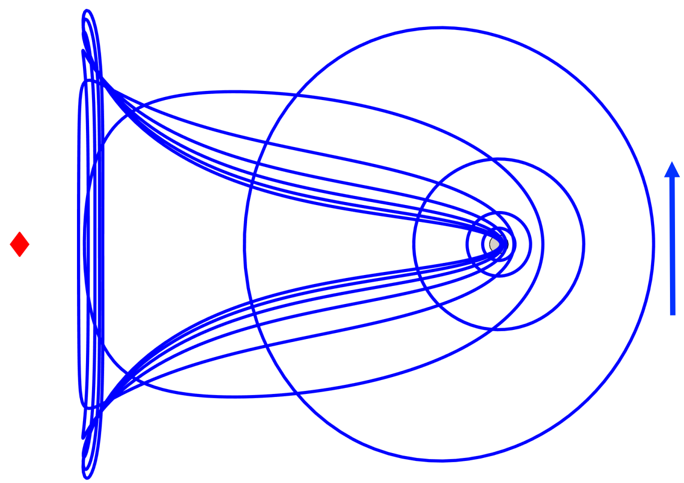
Exterior resonant orbit family

Selected Primary-Centered Orbits

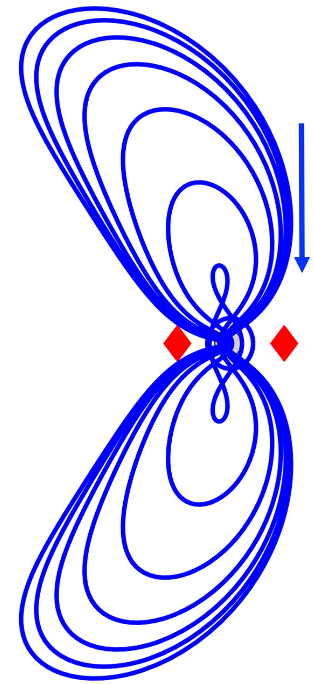
Distant retrograde orbits



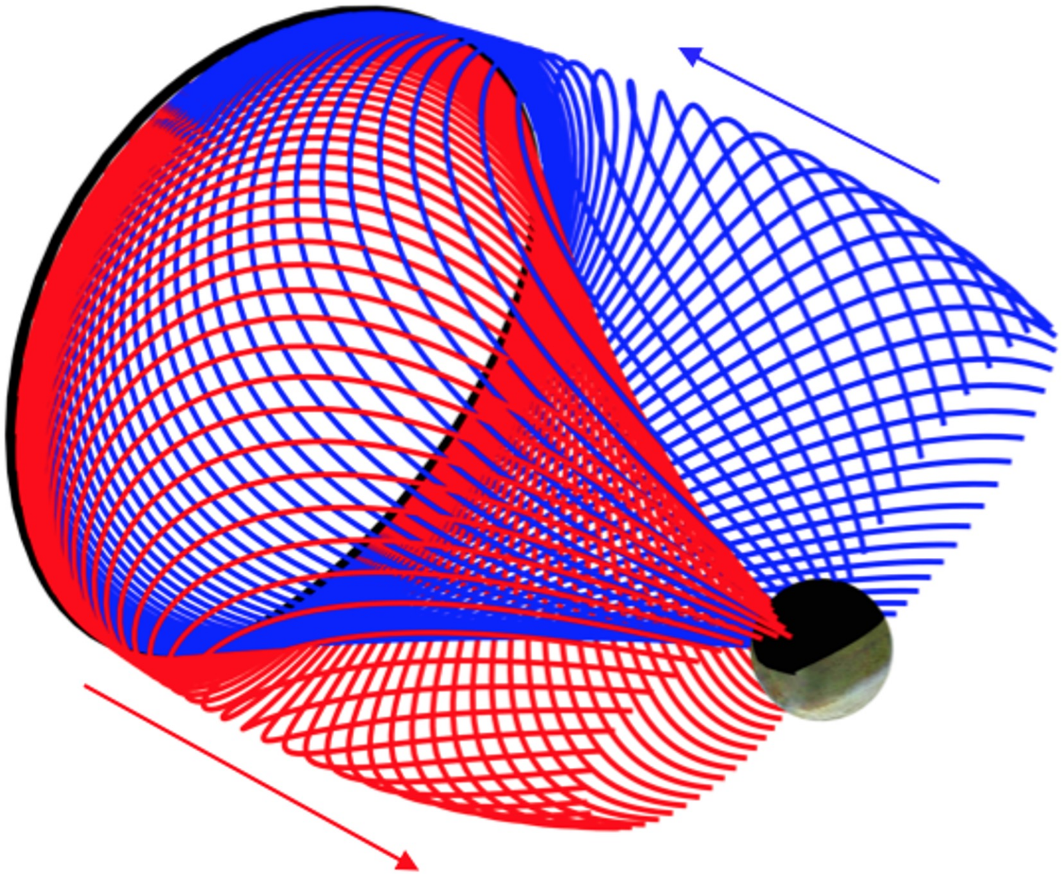
Low prograde orbits



Distant prograde orbits

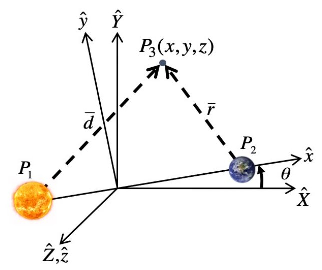


Stable/Unstable Manifolds of Periodic Orbits



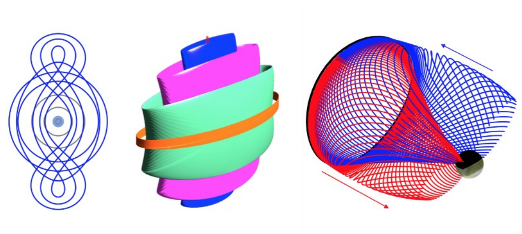
Generating Trajectories in Earth-Moon System

Define low-fidelity model

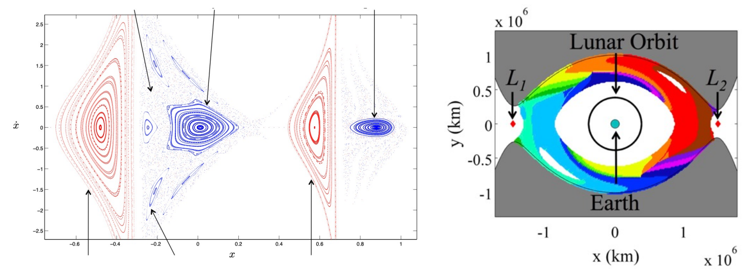


Analyze underlying solution space

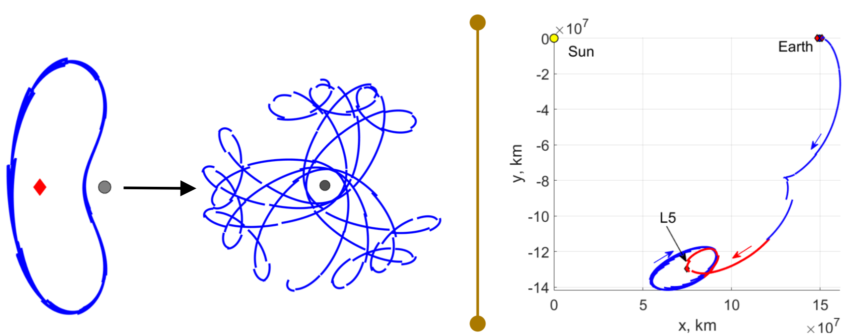
Fundamental solutions



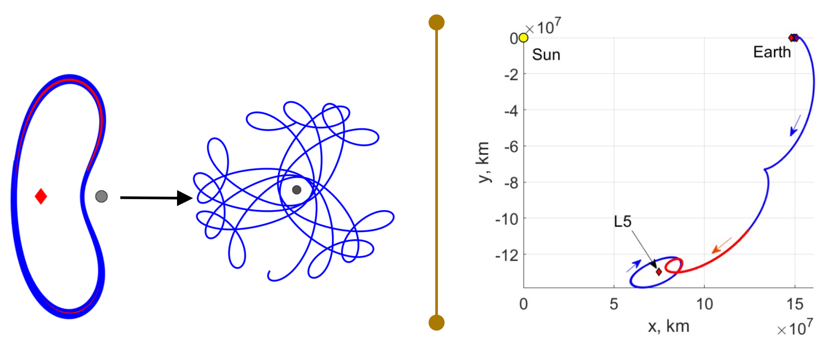
Poincaré map



Form initial guess

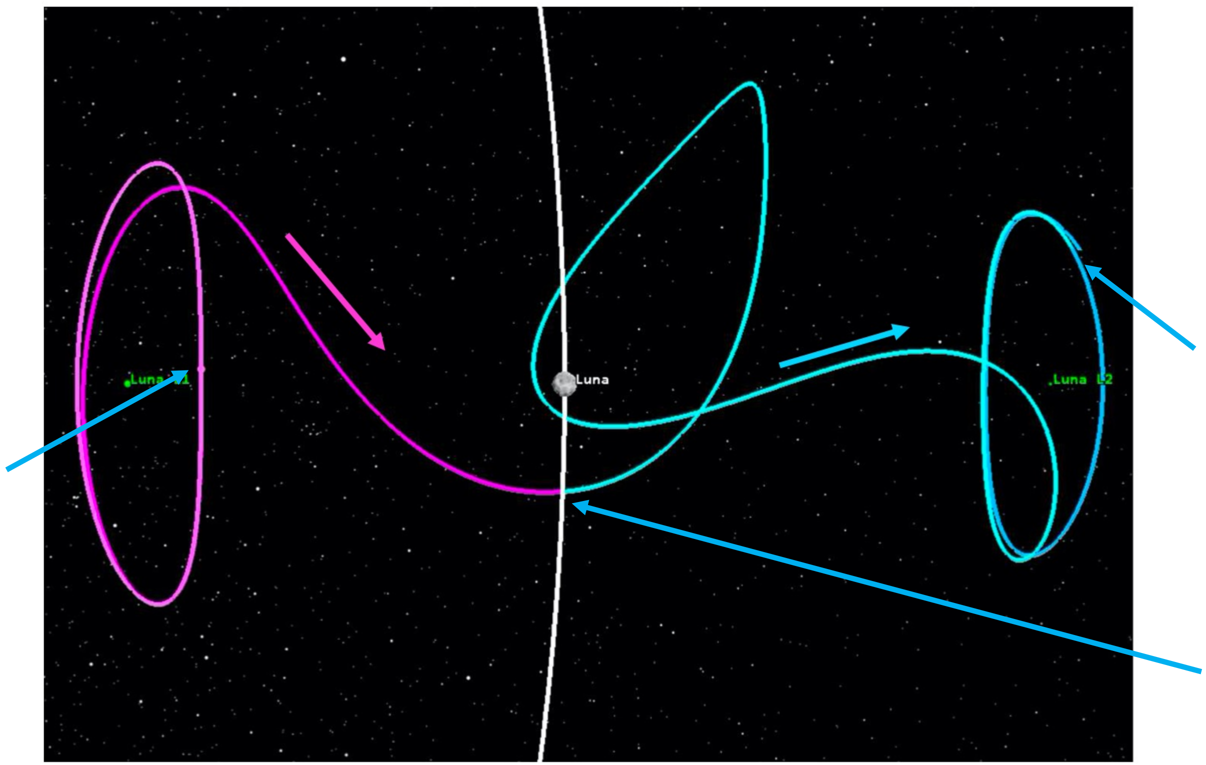


Corrections to recover solution



Transfer Design via Stable/Unstable Manifolds

Can use the hyperbolic invariant manifolds to design initial guess for transfers between periodic orbits with maneuvers



(a) Transfer from L_1 Lyapunov to L_2 Lyapunov orbit

Image credit: Short, Haapala, Bosanac, 2020, “Technical Implementation of the Circular Restricted Three-Body Model in STK Astrogator”, AAS/AIAA Astrodynamics Specialist Conference.

Transfer Design

Selected planar transfers between L_1 and L_2 Lyapunov orbits at one energy level:

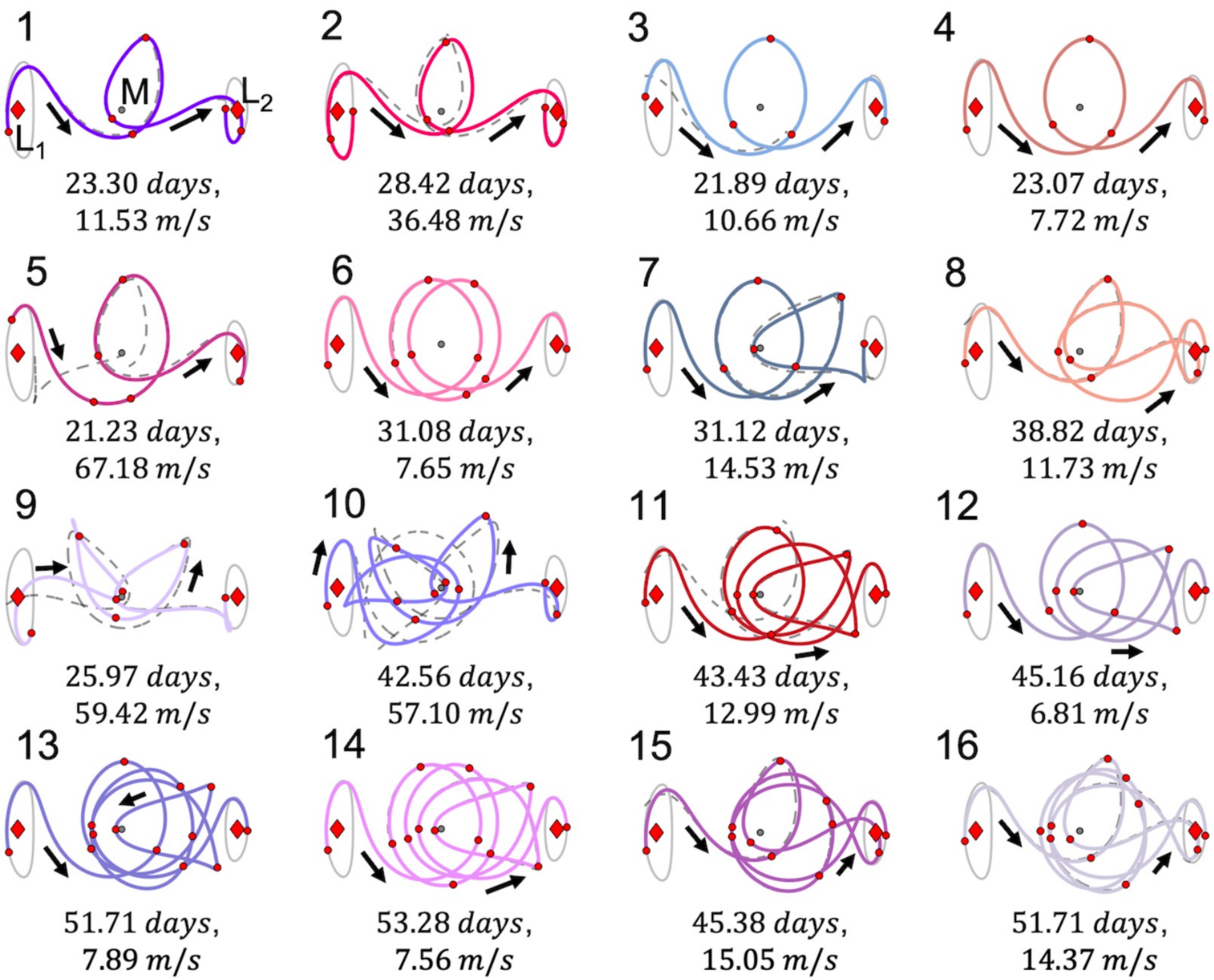
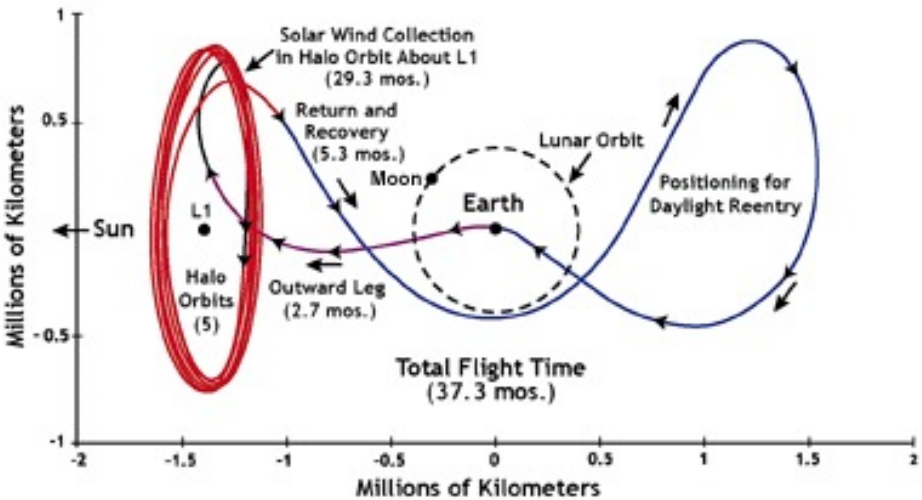


Image credits: Smith & Bosanac, 2023, JAS

Flying the CR3BP

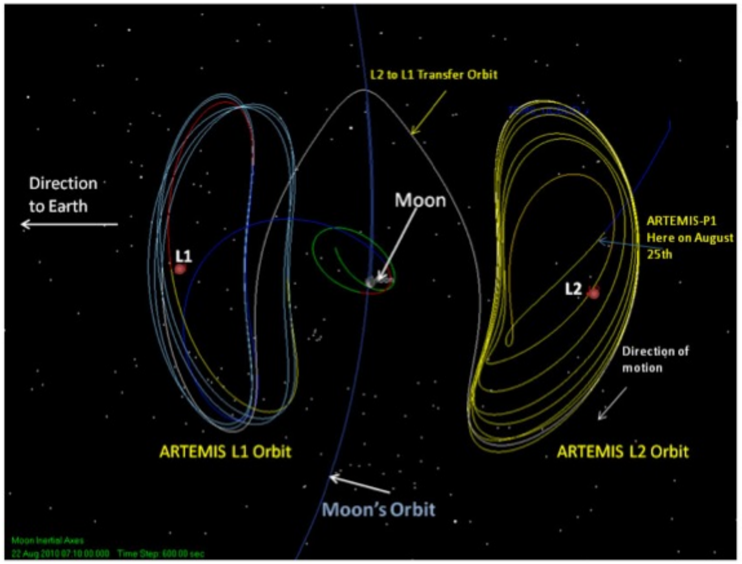
Examples of flown missions:

GENESIS (2001)



Credit: NASA/JPL-Caltech
Purdue University

ARTEMIS (2009, Ext. of THEMIS)



Credit: NASA/GSFC
Purdue University

Flying the CR3BP

TESS: Transiting Exoplanet Survey Satellite

Credit: Image credit: Dichmann, Parker, Williams, Mendelsohn, 2014, "Trajectory Design for the Transiting Exoplanet Survey Satellite"

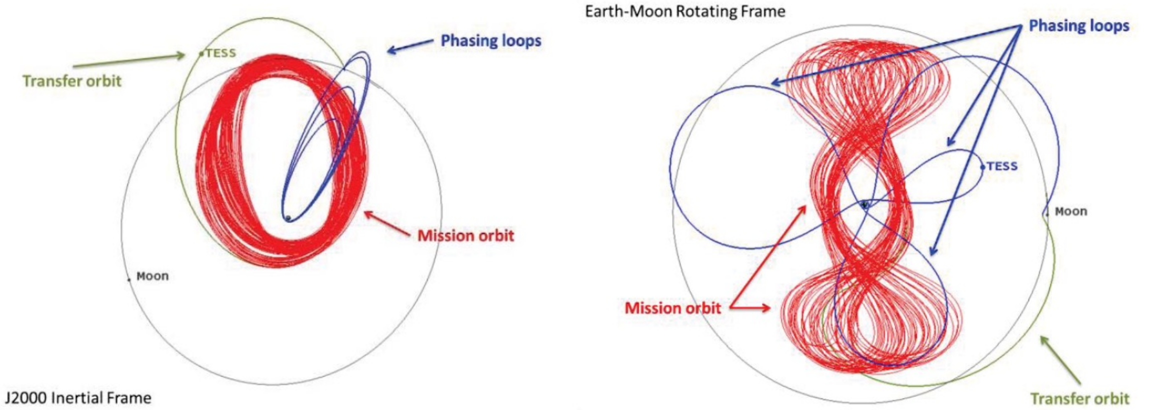
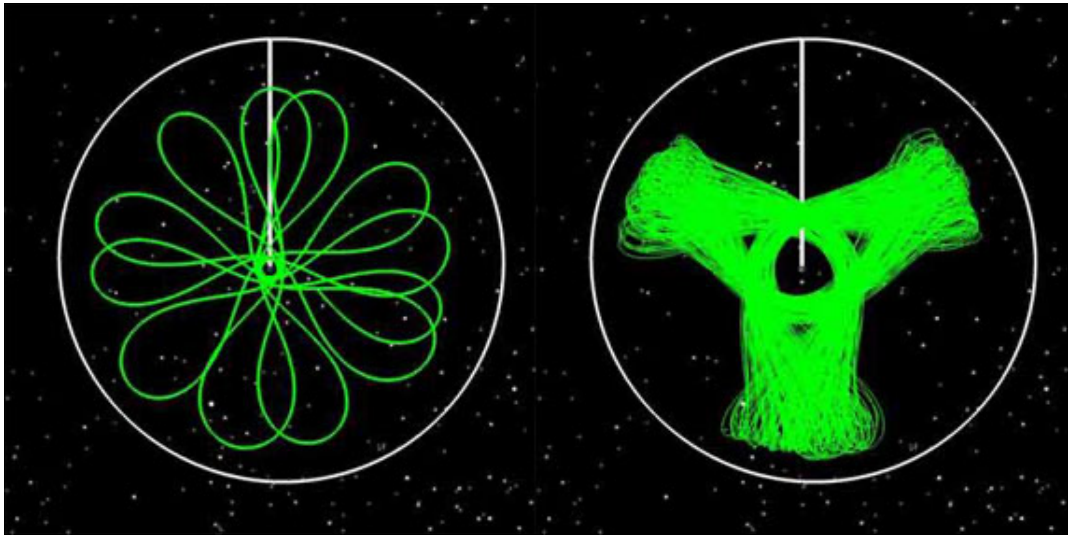


Figure 9: (a) Propagation in J2000 inertial frame; (b) Propagation in Earth-Moon rotating frame

IBEX: Interstellar Boundary Explorer

Image Credits: Applied Defense Solution

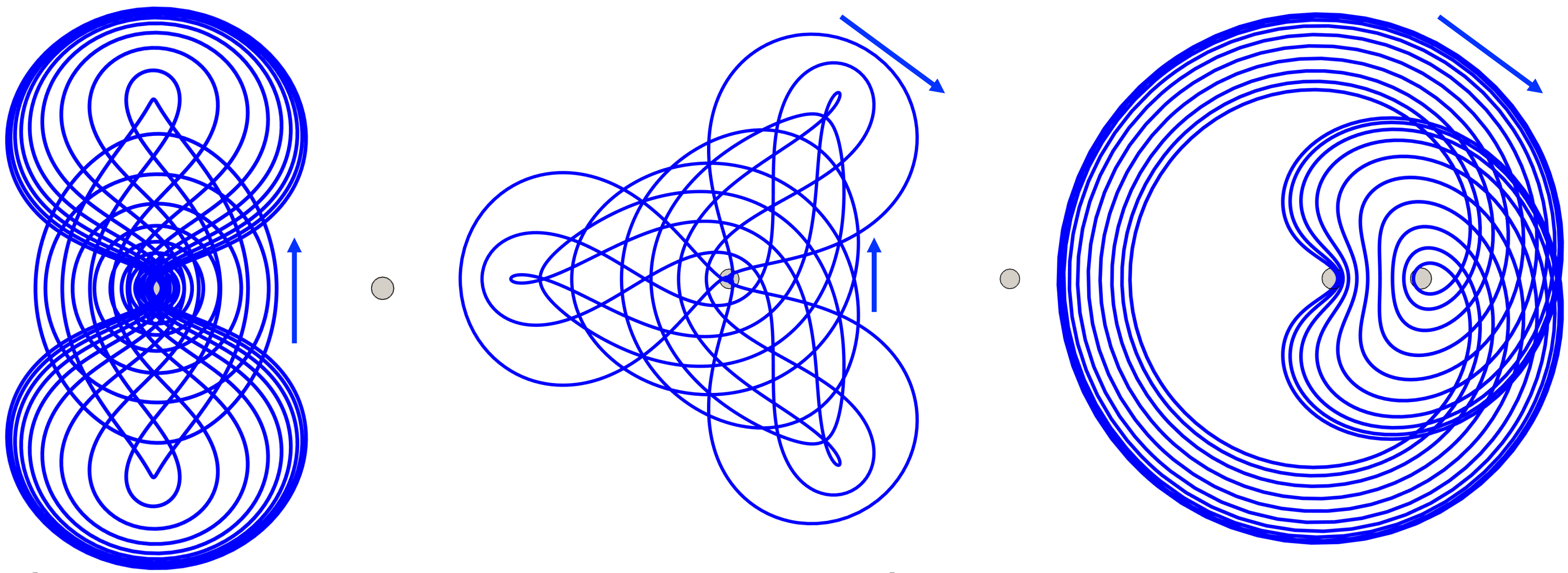


Selected Resonant Orbits

2:1 resonant orbit family

3:1 resonant orbit family

1:2 resonant orbit family



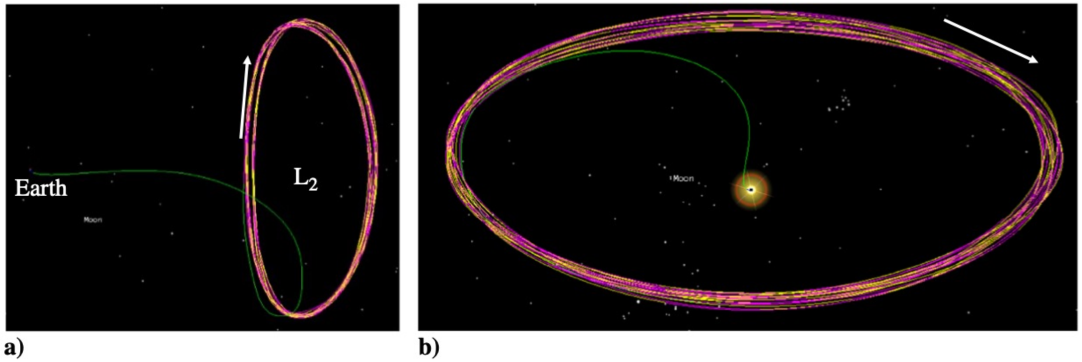
Interior resonant orbit families

Exterior resonant orbit family

Mission Applications

Nancy Grace Roman Space Telescope

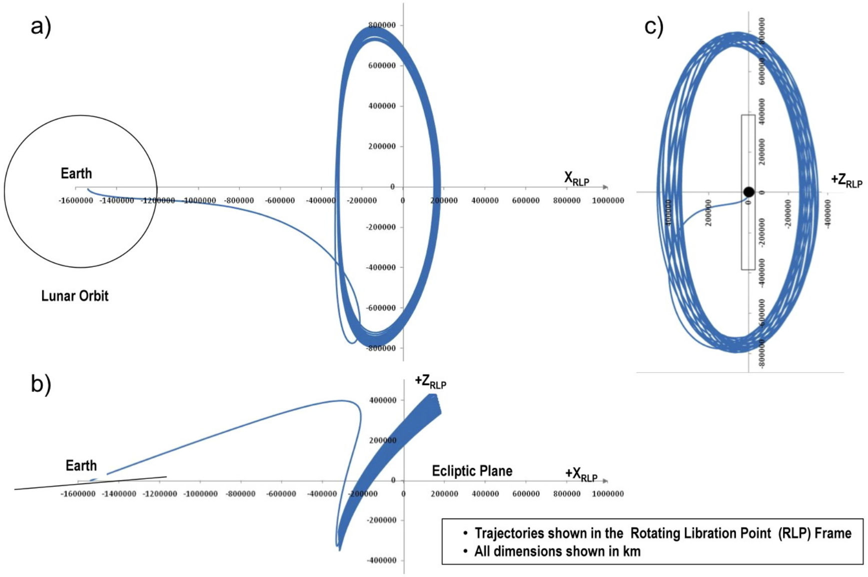
Image credit: Bosanac, Webster, Howell, Folta, 2019, “Trajectory Design for the Wide Field Infrared Survey Telescope” Journal of Guidance, Control and Dynamics



James Webb Space Telescope

Image credit: James Webb Space Telescope User Documentation

Nice quote from this website:
“It is incorrect to say that JWST "will be at L2." Rather, *JWST will orbit around L2*.”



Artemis 1

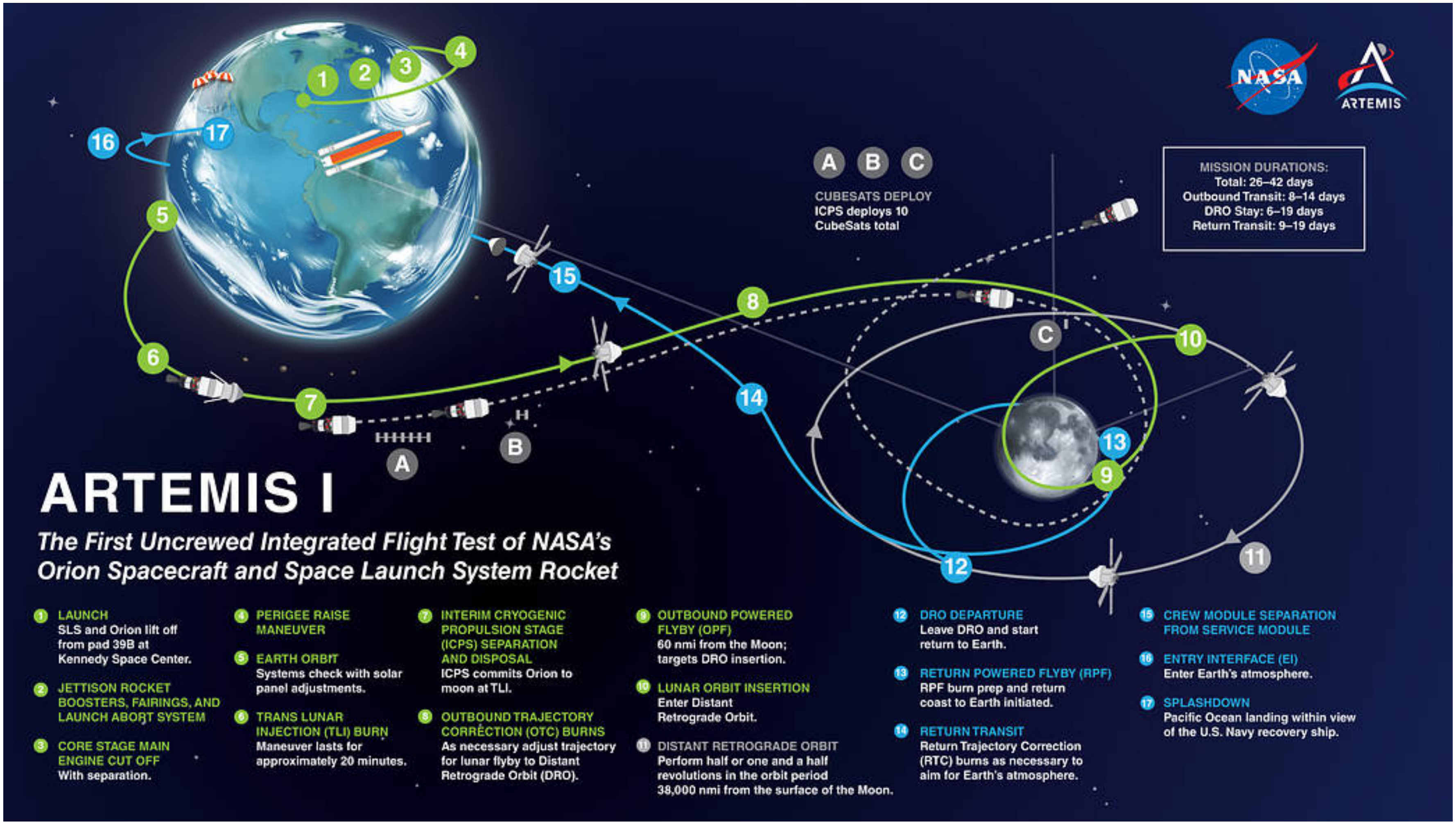
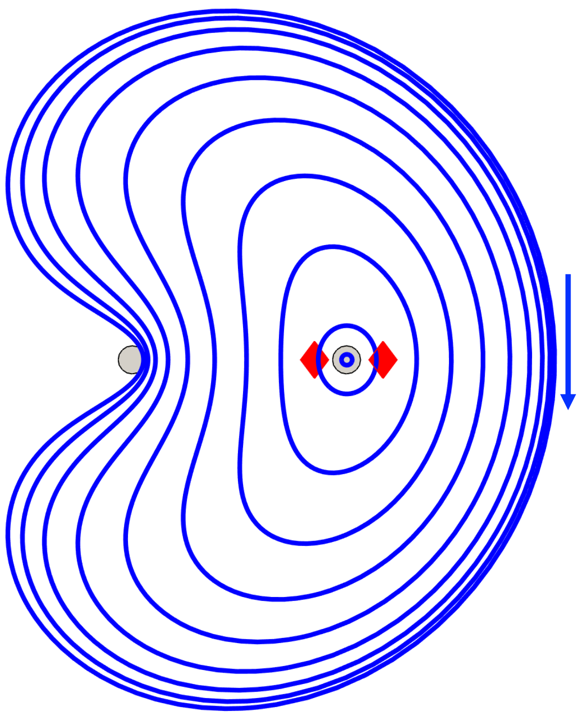


Image credit: nasa.gov

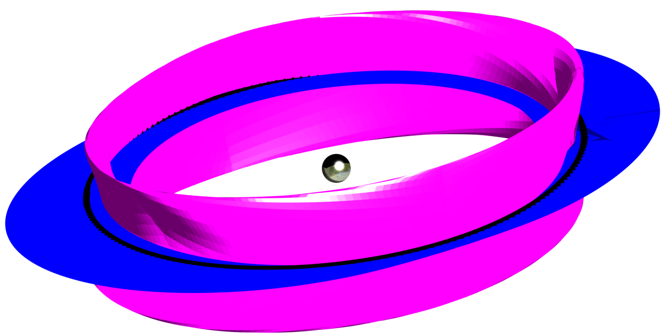
Distant Retrograde Orbits

Motion is clockwise in rotating frame

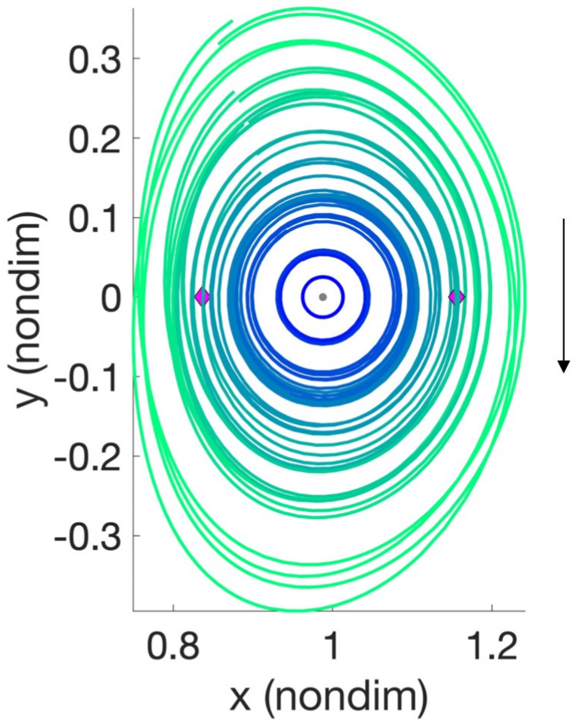
Periodic Orbits in CR3BP



Torus in CR3BP

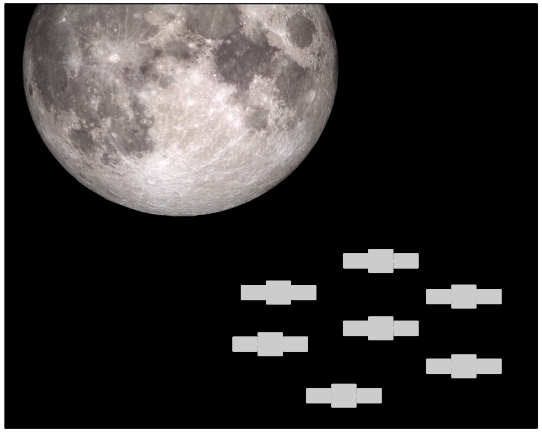


Bounded Motion in Ephemeris Model



Looking Forward

- Expanding exploration, infrastructure, technology demonstrations, and science in multi-body environments throughout our solar system
- Increasing presence in cislunar space
- Small satellite exploration
- Further understanding of motion of celestial bodies



Moon image credit: NASA/GSFC/ASU

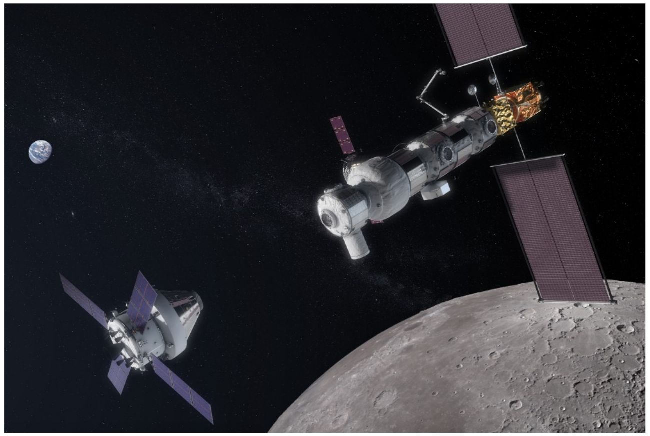
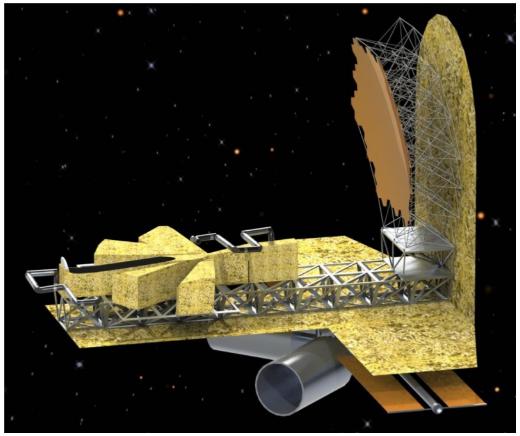


Image credit: NASA



Credit: iSAT ConOps Graphical Storyboard, 2019, Mukherjee, Mick, Naasz, et. al.