ASEN 5044
Fall 2024
Jash Bhalavat

AQ3 $\rightarrow$ $\mathbb{C} = [G, FG, F^2G, \dots, F^{n-1}G]$

a] $F = \begin{bmatrix} 0.9975 & 0.05 & 0.0012 & 2\times10^{-5} \\ -0.099 & 0.9975 & 0.0497 & 0.0012 \\ 0.0012 & 2\times10^{-5} & 0.9975 & 0.05 \\ 0.0499 & 0.0012 & -0.0999 & 0.9975 \end{bmatrix}$ $\quad G = \begin{bmatrix} -0.0012 & 2\times10^{-7} \\ -0.0499 & 2\times10^{-5} \\ 0.0012 & 0.0012 \\ 0.0499 & 0.05 \end{bmatrix}$

$$\mathbb{C} = \begin{bmatrix} -0.0012 & 2\times10^{-7} & -0.0037 & 3\times10^{-6} & -0.0062 & 1.68\times10^{-5} & -0.0086 & 4\times10^{-5} \\ -0.0499 & 2\times10^{-5} & -0.0496 & 1\times10^{-4} & -0.0488 & 3\times10^{-4} & -0.0477 & 7\times10^{-4} \\ 0.0012 & 0.0012 & 0.0037 & 0.0037 & 0.0062 & 0.0062 & 0.0086 & 0.0087 \\ 0.0499 & 0.05 & 0.0496 & 0.0497 & 0.048 & 0.0492 & 0.0477 & 0.0485 \end{bmatrix}$$

$\text{rank}(\mathbb{C}) = 4 \longrightarrow = n$ $\therefore$ the system is reachable

b] $G_{u_1} = [-0.0012, -0.0499, 0.0012, 0.0049]^T$

$\mathbb{C} = \begin{bmatrix} -0.0012 & -0.0037 & -0.0062 & -0.0086 \\ -0.0499 & -0.0496 & -0.0488 & -0.0477 \\ 0.0012 & 0.037 & 0.0062 & 0.0086 \\ 0.0499 & 0.0496 & 0.0488 & 0.0477 \end{bmatrix} \rightarrow \text{Rank}(\mathbb{C}) = 2 \leftarrow$ Not fully reachable

$G_{u_2} = [2.6\times10^{-7}, 2.0\times10^{-5}, 0.0012, 0.05]^T$

$\mathbb{C} = \begin{bmatrix} 2.6\times10^{-7} & 3.9\times10^{-6} & 1\times10^{-5} & 4.5\times10^{-5} \\ 2.0\times10^{-5} & 1.4\times10^{-4} & 3\times10^{-4} & 7\times10^{-4} \\ 0.0012 & 0.0037 & 0.0062 & 0.0087 \\ 0.0500 & 0.0497 & 0.0492 & 0.0485 \end{bmatrix} \rightarrow \text{Rank}(\mathbb{C}) = 4 \leftarrow$ Fully reachable

c] $x_1 = Fx_0 + Gu_0 \rightarrow Gu_0 = x_1 - Fx_0$

$x_2 = Fx_1 + Gu_1 = F^2x_0 + FGu_0 + Gu_1 \rightarrow FGu_0 + Gu_1 = x_2 - F^2x_0$

$x_3 = Fx_2 + Gu_2 = F^3x_0 + F^2Gu_0 + FGu_1 + Gu_2 \rightarrow F^2Gu_0 + FGu_1 + Gu_2 = x_3 - F^3x_0$

$$\begin{bmatrix} G & 0 & & 0 \\ FG & G & & 0 \\ F^2G & FG & \ddots & \\ \vdots & \vdots & & \\ F^{(n-1)}G & F^{(n-2)}G & \dots & G \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix} - \begin{bmatrix} F \\ F^2 \\ F^3 \\ \vdots \\ F^N \end{bmatrix} x_s \rightarrow \underline{\overline{FG}}\, u = X - \overline{F} x_s$$

Overdetermined system

$$\boxed{u = \left[(\overline{FG})^T(\overline{FG})\right]^{-1}(\overline{FG})^T(X - \overline{F}x_s)}$$

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AQ3 → d] Let's take only u_0 for example

$\underline{FG} = G_2 \rightarrow G\,u_0 = x_1 - Fx_s$

Then, there can be an infinite inputs to take the system from x_s to x_1. Hence, this is also true for a finite number of input vectors such as in part C.