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ASEN 5050
Fall 2024
Jash Bhalavat

Midterm 1

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work and I have followed all the exam policies.

J.C.B   Jash Bhalavat   10/6/2024

10/6/2024 – 11:08 AM MST   10/6/2024 – 5:54 PM MST

Problem 1 → Given: $G m_{mars} = 4.305 \times 10^4 \ km^3/s^2$, $r_{eq,mars} = 3397.2 \ km$,
$h = 2.4799 \times 10^4 \ km^2/s$, @ $t_1 \rightarrow r_1 = 15{,}000 \ km$, $v_1 = 1.8111 \ km/s$, $\phi_{fpa} < 0°$,
@ $t_2 \rightarrow r_2 = 19{,}000 \ km$, $v_{r,2} < 0$

Assumptions → $m_{s/c} << M_{mars} \rightarrow \mu \approx G M_{mars}$, 2-Body problem

a) $\mathcal{E} = \frac{v_1^2}{2} - \frac{\mu}{r_1} = -1.2300 \ km^2/s^2$

$a = -\frac{\mu}{2\mathcal{E}} = \boxed{1.7501 \times 10^4 \ km = a}$

b) $h = \sqrt{\mu a (1-e^2)} \rightarrow h^2/(\mu a) = 1 - e^2 \rightarrow e^2 = 1 - \frac{h^2}{\mu a} \rightarrow e = \pm\sqrt{1 - h^2/\mu a}$

$\boxed{e = 0.4286}$ ← +ve because $e \geq 0$ by definition

c) $\because e < 1$, orbit is elliptical $\rightarrow \therefore r_1 = a(1 - e\cos E_1)$

$\frac{r_1}{a} = 1 - e\cos E_1 \rightarrow e\cos E_1 = 1 - \frac{r_1}{a} \rightarrow E_1 = \cos^{-1}\left(\frac{1}{e}\left(1 - \frac{r_1}{a}\right)\right)$

$E_1 = \pm 1.2309^{rad} \rightarrow \because \phi_{fpa}$ @ $t_1 < 0$, s/c is moving towards periapsis and $\therefore E < 0 \rightarrow \boxed{E_1 = -1.2309 \ rad}$

$r_1 = \frac{h^2/\mu}{1 + e\cos\theta_1^*} \rightarrow \theta_1^* = \cos^{-1}\left[\left(\frac{h^2/\mu}{r_1} - 1\right) \cdot \frac{1}{e}\right] = \pm 96.3807°$

$\because \phi_{fpa}$ @ $t_1 < 0$, $\theta_1^* < 0 \rightarrow \boxed{\theta_1^* = -96.3807°}$

d] $E_2 = \cos^{-1}\left(\frac{1}{e}\left(1-\frac{r_2}{a}\right)\right) = \pm 1.7720$ rad

$V_{r_2} < 0 \rightarrow \phi_{fpa_{1,2}} < 0 \rightarrow E_2 < 0 \rightarrow \boxed{\therefore E_2 = -1.7720 \text{ rad}}$

$\theta_2^* = \cos^{-1}\left[\frac{1}{e}\left(\frac{h^2/\mu}{r_2} - 1\right)\right] = \pm 125.3740°$

$\because V_{r_2} < 0 \rightarrow \phi_{fpa_{1,2}} < 0 \rightarrow \theta_2^* < 0 \rightarrow \therefore \boxed{\theta_2^* = -125.3740°}$

e] $P = 2\pi\sqrt{a^3/\mu} = 7.0109 \times 10^4$ s , $n = \frac{2\pi}{P} = 8.9621 \times 10^{-5} \frac{1}{s}$

$t_1 - t_p = \frac{1}{n}(E_1 - e\sin(E_1)) = -9.2259 \times 10^3$ s $\rightarrow < 0$ indicates time until periapsis

$t_2 - t_p = \frac{1}{n}(E_2 - e\sin(E_2)) = -1.5087 \times 10^4$ s $\rightarrow < 0$ indicates time until periapsis

Time past periapsis for $t_2 \rightarrow P - |t_2 - t_p| = 5.5022 \times 10^4$ s

Assuming <1 revolution between t_2 and t_1

$t_2 - t_1 =$ t_1 to periapsis + periapsis to t_2

$= |t_1 - t_p| + P - |t_2 - t_p| = \boxed{6.4248 \times 10^4 \text{ s} = t_2 - t_1}$

f] $r_p = a(1-e) = 9.9995 \times 10^3$ km

$r_a = a(1+e) = 2.5002 \times 10^4$ km

$b = a\sqrt{1-e^2} = 1.5812 \times 10^4$ km

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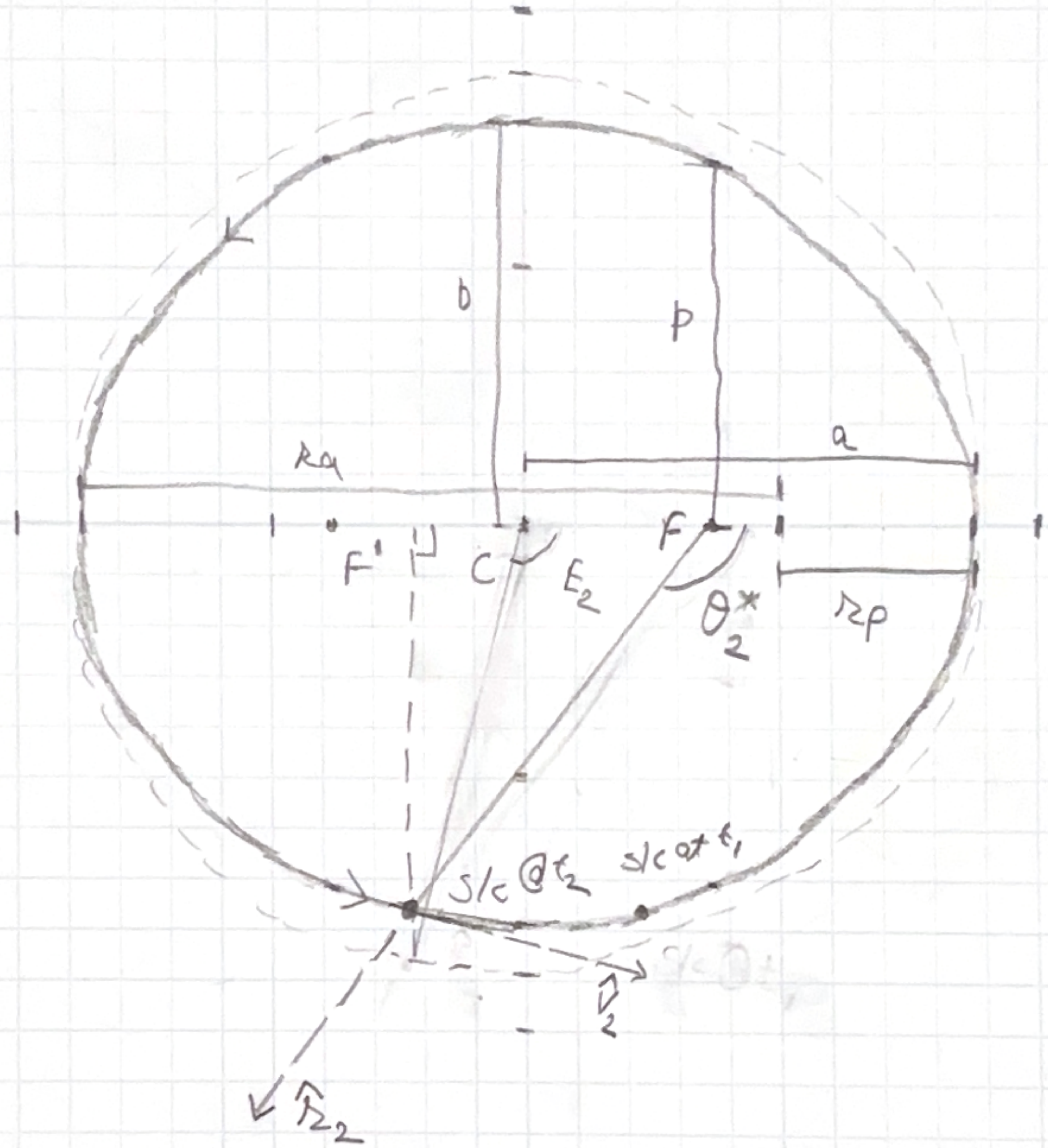
f) 1 tick = 1×10^4 km

dotted circle is auxiliary circle



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Problem 2 → Given: $G M_{mars} = 4.305\times10^{4}\ km^3/s^2$, $r_{eq,mars} = 3397.2\ km$

$\vec{R}_3 = -7.6650\times10^{3}\hat{x} + 6.5468\times10^{3}\hat{y} - 4.5740\times10^{2}\hat{z}\ km$

$\vec{V}_3 = 1.6334\hat{x} + 0.1226\hat{y} - 1.9455\hat{z}\ km/s$

→ Mars-centered inertial frame

Assumptions → 2-Body problem, $m_{s/c} \ll M_{mars}$ → $\mu \approx G M_{mars}$

a) $R_3 = |\vec{R}_3| = 1.0091\times10^{4}\ km$, $V_3 = |\vec{V}_3| = 2.5432\ km/s$

$\vec{h} = \vec{R}_3 \times \vec{V}_3 = -1.2681\times10^{4}\hat{x} - 1.5659\times10^{4}\hat{y} - 1.1633\times10^{4}\hat{z}\ km^2/s$

$|\vec{h}| = h = 2.3267\times10^{4}\ km^2/s$

$\vec{n} = \hat{z}\times\vec{h} = 1.5659\times10^{4}\hat{x} - 1.2681\times10^{4}\hat{y} + 0\hat{z}\ km^2/s$

$\hat{n} = \frac{1}{|\vec{n}|}\cdot\vec{n} = 0.7771\hat{x} - 0.6293\hat{y} + 0\hat{z}$

$\vec{e} = \frac{1}{\mu}\left[\left(V_3^2 - \frac{\mu}{R_3}\right)R^3 - (\vec{R}_3\cdot\vec{V}_3)\vec{V}_3\right] = 0.0188\hat{x} + 0.3657\hat{y} - 0.5127\hat{z}$

$e = |\vec{e}| = \boxed{0.6300 = e}$

$\mathcal{E} = V_3^2/2 - \mu/R^3 = -1.0323\ km^2/s^2$

$a = -\frac{\mu}{2\mathcal{E}} = \boxed{2.0851\times10^{4}\ km = a}$

$i = \pm a\cos[(\vec{h}\cdot\hat{z})/h] = \boxed{119.9995^\circ = i}$ ← $i > 0$ by definition, no quadrant check needed

$\Omega = a\cos(\hat{n}\cdot\hat{x}) = \pm 39.0^\circ \rightarrow \because \hat{n}\cdot\hat{y} = -0.6293 < 0$

$\therefore \Omega < 0 \rightarrow \boxed{\Omega = -39.0^\circ}$

$\omega = a\cos[(\hat{n}\cdot\vec{e})/e] = \pm 110.0018^\circ \rightarrow \because \vec{e}\cdot\hat{z} = -.5127 < 0 \rightarrow \therefore \omega < 0$

$\boxed{\omega = -110.0018^\circ}$

$\theta_3^* = a\cos\left[\frac{\vec{e}\cdot\vec{R}_3}{e\cdot R_3}\right] = \pm 66.9979^\circ \rightarrow \because \vec{R}_3\cdot\vec{V}_3 = -1.0828\times10^{4}\ km^2/s < 0$

$\therefore \theta_3^* < 0 \rightarrow \boxed{\theta_3^* = -66.9979^\circ}$

b) $P = 2\pi\sqrt{\frac{a^3}{\mu}} = 9.1179\times10^{4}\ s$, $n = \frac{2\pi}{P} = 6.8911\times10^{-5}\ \frac{1}{s}$

$E_3 = 2\cdot\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_3^*}{2}\right)\right] = -0.6109\ rad$ ← No quadrant check required because $\tan^{-1}$ range is $[-\pi/2, \pi/2]$ → $2\tan^{-1}$ range is $[-\pi, \pi]$.

$(t_3 - t_p) = \frac{1}{n}(E_3 - e\sin(E_3)) = -3.6210\times10^{3}\ s < 0$ → time until periapsis

$t_4 = t_3 + 2$ hours $\rightarrow$ $t_4 - t_p = t_3 - t_p + 2$ hours $= 3.5790 \times 10^3$ s

Kepler's equation $\rightarrow$ $t_4 - t_p = \frac{1}{n}(E_4 - e\sin E_4)$

Using Newton's method to solve for E_4. Done in Matlab using code written for HW 4.

$E_4 = 0.6049$ rad

$\theta_4^* = 2 \cdot \tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E_4}{2}\right)\right) = 66.4461°$ ← No quadrant check needed for $\tan^{-1}$, but $\because E_4 > 0 \rightarrow \theta_4^* > 0$ so sanity check works out

Using values from part a →

$p = h^2/\mu = 1.2575 \times 10^4$ km

$\Delta\theta^* = \theta_4^* - \theta_3^* = 133.4440°$

$R_3 = 1.0091 \times 10^4$ km, $R_4 = \frac{p}{1+e\cos\theta_4^*} = 1.0046 \times 10^4$ km

$f = 1 - \frac{R_4}{p}(1 - \cos\Delta\theta^*) = -0.3482$

$g = \frac{R_3 R_4}{\sqrt{\mu p}}[\sin\Delta\theta^*] = 3.1632 \times 10^3$ s

$\dot{f} = \sqrt{\frac{\mu}{p}}\tan\left(\frac{\Delta\theta^*}{2}\right)\left(\frac{1-\cos\Delta\theta^*}{p} - \frac{1}{R_4} - \frac{1}{R_3}\right) = -2.7714 \times 10^{-4}\ \frac{1}{s}$

$\dot{g} = 1 - \left(\frac{R_3}{R_4}\right)(1 - \cos\Delta\theta^*) = -0.3542$

$\vec{R}_4 = f\vec{R}_3 + g\vec{V}_3 = \boxed{7.8359 \times 10^3\hat{x} - 1.8919 \times 10^3\hat{y} - 5.9948 \times 10^3\hat{z} \text{ km} = \vec{R}_4}$

$\vec{V}_4 = \dot{f}\vec{R}_3 + \dot{g}\vec{V}_3 = \boxed{1.5456\hat{x} - 1.8578\hat{y} + 0.8159\hat{z} \text{ km/s} = \vec{V}_4}$

c) From part a → $i = 119.9995°$

If a spacecraft is in a polar orbit and the $\hat{x}\hat{y}$ plane is equatorial, the inclination has to be $90°$. $\because i \neq 90°$, the $\hat{x}\hat{y}$ plane is not equatorial.

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Problem 3 → Given: $GM_{mars} = 4.305\times10^4\ km^3/s^2$, $R_{eq,mars} = 3397.2\ km$

$\hat{r}_{descending} = -0.64279\hat{x} - 0.76604\hat{y} + 0\hat{z}$

$\hat{r}_{apoapsis} = -0.0297\hat{x} - 0.97508\hat{y} - 0.21985\hat{z}$

Assumptions → $\mu = G(M_{mars} + M_{s/c}) \rightarrow M_{s/c} \ll M_{mars} \rightarrow \mu \approx GM_{mars}$

2-Body problem

a) $\hat{n}$ is direction of ascending node and exactly 180° apart from descending node in the $\hat{x}\hat{y}$ plane.

$\therefore \hat{n} = -\hat{r}_{descending} = 0.64279\hat{x} + 0.76604\hat{y} + 0\hat{z}$

$\Omega = \arccos(\hat{n}\cdot\hat{x}) = \arccos(0.64279) = \pm 49.9998°$

$\because \hat{n}\cdot\hat{y} = 0.76604 > 0 \rightarrow \Omega > 0 \rightarrow \boxed{\Omega = 49.9998°}$

b) $\hat{e}$ is direction of periapsis and exactly 180° apart from apoapsis in the orbital plane.

$\therefore \hat{e} = -\hat{r}_{apoapsis} = 0.0297\hat{x} + 0.97508\hat{y} + 0.21985\hat{z}$

ω is angle between $\hat{n}$ and $\vec{e}$

$\omega = \arccos(\hat{n}\cdot\hat{e}) = \arccos(0.7660) = \pm 40.0003°$

$\because \hat{e}\cdot\hat{z} = 0.21985 > 0 \rightarrow \omega > 0 \rightarrow \boxed{\omega = 40.0003°}$