(1)

ASEN 5050
Fall 2024
Jash Bhalavat

HW 5

Problem 1 → Given: Lunar orbit, $a_1 = 7045$ km, $e_1 = 0.23$, $\theta_1^* = -142^\circ$, Maneuver is coplanar

Assumptions → 2-Body problem, impulsive maneuver.

$\mu = G(m_{moon} + \cancel{m_{s/c}}) \approx G m_{moon} = 4902.799 \ km^3/s^2$

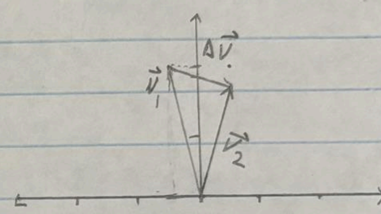
a) $p = a_1(1-e_1^2) = h_1^2/\mu \rightarrow h_1 = \sqrt{\mu(a_1)(1-e_1^2)} = 5.7195 \times 10^3 \ km^2/s$

$v_{r,1} = \frac{\mu}{h_1} e_1 \sin(\theta_1^*) = -0.1214 \ km/s$

$v_{\theta_1} = \frac{\mu}{h_1}(1 + e_1 \cos\theta_1^*) = 0.7018 \ km/s$

$\rightarrow \vec{v} = [v_{r_1}, v_{\theta_1}, 0]$, $\boxed{\vec{v}_1 = -0.1214\hat{r} + 0.7018\hat{\theta} + 0\hat{h} \ km/s}$

b) $\Delta\vec{v} = 0.3\hat{r} - 0.1\hat{\theta} \ km/s$



c) $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \rightarrow \vec{v}_2 = \vec{v}_1 + \Delta\vec{v} = -0.1214\hat{r} + 0.7018\hat{\theta} + 0.3\hat{r} - 0.1\hat{\theta}$ km/s

$\boxed{\vec{v}_2 = 0.1786\hat{r} + 0.6018\hat{\theta} + 0\hat{h} \ km/s}$, $v_2 = |\vec{v}_2| = 0.6278 \ km/s$

d) $r_1 = \frac{h_1^2/\mu}{1 + e_1\cos\theta_1^*} = 8.1493 \times 10^3$ km → Assuming impulsive burn → $r_1 = r_2$

$\mathcal{E}_2 = v_2^2/2 - \mu/r_2 = -0.4046 \ km^2/s^2$, $a_2 = -\frac{\mu}{2\mathcal{E}_2} = \boxed{6.0494 \times 10^3 \ km = a_2}$

$r = (h^2/\mu)/(1 + e\cos\theta^*)$, $v_\theta = \left(\frac{\mu}{h}\right)(1 + e\cos\theta^*) \rightarrow 1 + e\cos\theta^* = \frac{v_\theta h}{\mu} \rightarrow r = \frac{h^2/\mu}{(v_\theta h)/\mu}$

$r = \frac{h^2}{\mu} \cdot \frac{\mu}{v_\theta h} = h/v_\theta \rightarrow h = r v_\theta \rightarrow h_2 = r_2 v_{\theta_2} = 4.9046 \times 10^3 \ km^2/s$

$\mathcal{E} = \frac{\mu^2(e^2-1)}{2h^2} \rightarrow e^2 - 1 = \frac{2\mathcal{E}h^2}{\mu^2} \rightarrow e_2 = \pm\sqrt{1 + \frac{2\mathcal{E}h_2^2}{\mu^2}}$ ← $e \geq 0$ by definition

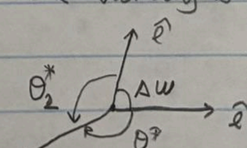
$\boxed{e_2 = 0.4362}$

$\cos\theta_2^* = \left(\frac{h_2^2/\mu}{r_2} - 1\right)/e_2 \rightarrow \theta_2^* = \pm 155.8187^\circ$ ($\because v_{r_2} > 0 \rightarrow \phi_{fpa} > 0 \rightarrow \theta_2^* > 0$)

$\therefore \boxed{\theta_2^* = 155.8187^\circ}$

e) Maneuver is coplanar (given) → ∴ change in true anomaly is equal to the change in periapsis. The representative diagram on the right says → $|\theta_1^*| + |\theta_2^*| + \Delta\omega = 360^\circ$

$\therefore \Delta\omega = 360^\circ - |\theta_1^*| - |\theta_2^*| = \boxed{62.1813^\circ = \Delta\omega}$



f] $m_{tot} = 1224$ kg, $m_{prop} = 156$ kg, $I_{sp} = 212$ s, $g_0 = 9.81$ m/s^2

$m_{prop,needed} = m_{tot}\left(1 - e^{\frac{-\Delta V}{I_{sp} g_0}}\right) = \boxed{172.6541 \text{ kg} = m_{prop,needed}}$

$\because m_{prop\,needed} > m_{prop} \rightarrow$ The LRO $\boxed{\text{could not implement}}$ the maneuver specified in part b.

Problem 2 → Given: Mars orbit, $t_1 \rightarrow r_1 = 6500$ km, $E_1 = \frac{\pi}{2}$ rad, $r_{p_1} = 5915$ km,

$t_2 \rightarrow r_{p,2} = 5712$ km, $r_{a,2} = 7888$ km periapsis → apoapsis, after maneuver

Assumptions: 2-Body Problem, Impulsive maneuver,

$\mu = G(M_{mars} + \cancel{m_{s/c}}) \approx GM_{mars}$ ($\because m_{s/c} \ll M_{mars}$), $\mu = 4.305 \times 10^4$ km^3/s^2

a] at $t_1 \rightarrow E = \frac{\pi}{2}$ rad $\rightarrow$ s/c is at b $\rightarrow r_b = a$

$\therefore r_1 = a_1 = 6500$ km, $r_{p_1} = a(1-e_1) \rightarrow e_1 = 1 - \frac{r_{p,1}}{a_1} = 0.09$

$p_1 = a_1(1-e_1^2) = h_1^2/\mu \rightarrow h_1 = \sqrt{\mu a_1(1-e_1^2)} = 1.6660 \times 10^4$ km^2/s

$\cos\theta_1^* = \left(\frac{h_1^2/\mu}{r_1} - 1\right)/e_1 \rightarrow \theta_1^* = \pm 95.1636°$ ($\because E > 0 \rightarrow \theta^* > 0$)

$\therefore \theta_1^* = 95.1636°$

$v_{r,1} = \frac{\mu}{h_1} e_1 \sin(\theta_1^*) = 0.2316$ km/s

$v_{\theta,1} = \frac{\mu}{h_1}(1 + e_1\cos(\theta_1^*)) = 2.5631$ km/s

$\rightarrow \boxed{\vec{v}_1 = 0.2316\,\hat{r} + 2.5613\,\hat{\theta} + 0\hat{h} \text{ km/s}}$

b] $r_2 = r_1 = 6500$ km (Impulsive maneuver)

$e_2 = \frac{r_{a,2} - r_{p,2}}{r_{a_2} + r_{p,2}} = 0.16$, $r_{a,2} = a_2(1+e_2) \rightarrow a_2 = \frac{r_{a2}}{1+e_2} = 6.8 \times 10^3$ km

$p_2 = a_2(1-e_2^2) = h_2^2/\mu \rightarrow h_2 = 1.6889 \times 10^4$ km^2/s

$\cos(\theta_2^*) = \left(\frac{h_2^2/\mu}{r_2} - 1\right)\frac{1}{e_2} \rightarrow \theta_2^* = \pm 83.0457°$ ($\because$ s/c moving from periapsis to apoapsis $\theta^* > 0$)

$\therefore \theta_2^* = 83.0457°$

$v_{r,2} = \frac{\mu}{h_2} e_2 \sin(\theta_2^*) = 0.4048$ km/s

$v_{\theta,2} = \frac{\mu}{h_2}(1 + e_2\cos(\theta_2^*)) = 2.5983$ km/s

$\rightarrow \vec{v}_2 = 0.4048\,\hat{r} + 2.5983\,\hat{\theta} + 0\hat{h}$ km/s

$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1 = 0.1734\,\hat{r} + 0.0353\,\hat{\theta} + 0\hat{h}$ km/s

$\boxed{\Delta V = |\Delta\vec{V}| = 0.1768 \text{ km/s}}$

(2)

ASEN 5050
Fall 2024
Jash Bharawat

HW 5

$AU = 149,597,870.7\ km$

Problem 3 → Given: $r_{initial} = a_{earth} = 1.0000010178\ AU$, $r_{final} = a_{saturn} = 9.554909595\ AU$

Assumptions: 2-Body problem, $\mu = G(M_{sun} + M_{s/c}) \approx G M_{sun} = 1.3271 2428\ \frac{km^3}{s^2}$ ($M_{s/c} << M_{sun}$)

a] $a_{trans} = \frac{r_i + r_f}{2} = 7.8950 \times 10^8\ km$

Algorithm 36 is Vallado →

$V_i = \sqrt{\frac{\mu}{r_i}} = 29.7847\ km/s$, $V_f = \sqrt{\frac{\mu}{r_f}} = 9.6356\ \frac{km}{s}$,

$V_{trans,a} = \sqrt{\frac{2\mu}{r_i} - \frac{\mu}{a_{trans}}} = 40.0769\ km/s$, $V_{trans,b} = \sqrt{\frac{2\mu}{r_f} - \frac{\mu}{a_{trans}}} = 4.1944\ km/s$

$\Delta V_a = V_{trans,a} - V_i = 10.2922\ km/s$, $\Delta V_b = V_f - V_{trans,b} = 5.4412\ km/s$

$\Delta V_{tot} = |\Delta V_a| + |\Delta V_b| = \boxed{15.73345\ km/s = \Delta V_{tot}}$

$TOF = \pi \sqrt{\frac{a_{trans}^3}{\mu}} = \boxed{1.9130 \times 10^5\ s = TOF}$

b] $n_f = \sqrt{\mu / a_{trans}^3} = 6.7411 \times 10^{-9}\ rad/s \rightarrow \alpha = n_f \cdot TOF = 1.2896\ rad$

Phasing angle $= \phi = \pi - \alpha = 1.8520\ rad = \boxed{106.1128^\circ = \phi}$

c] $r_b = 11\ AU \rightarrow a_{t,1} = \frac{r_i + r_b}{2} = 8.9759 \times 10^8\ km$, $a_{t,2} = \frac{r_b + r_f}{2} = 1.5375 \times 10^9\ km$

Algorithm 37 in Vallado →

$V_{t,1,a} = \sqrt{\frac{2\mu}{r_i} - \frac{\mu}{a_{t,1}}} = 40.3286\ \frac{km}{s}$, $V_{t,1,b} = \sqrt{\frac{2\mu}{r_b} - \frac{\mu}{a_{t,1}}} = 3.6662\ km/s$

$V_{t,2,b} = \sqrt{\frac{2\mu}{r_b} - \frac{\mu}{a_{t,2}}} = 8.6590\ \frac{km}{s}$, $V_{t,2,c} = \sqrt{\frac{2\mu}{r_f} - \frac{\mu}{a_{t,2}}} = 9.9686\ km/s$

From a] → $V_i = 29.7847\ km/s$, $V_f = 9.6356\ km/s$

$\Delta V_a = V_{t,1,a} - V_i = 10.5440\ km/s$

$\Delta V_b = V_{t,2,b} - V_{t,1,b} = 4.9927\ km/s$

$\Delta V_c = V_f - V_{t,2,c} = -0.3330\ km/s$

→ $\Delta V_{tot} = |\Delta V_a| + |\Delta V_b| + |\Delta V_c|$

$\boxed{\Delta V_{tot} = 15.86967\ km/s}$

$TOF = \pi \sqrt{a_{t1}^3 / \mu} + \pi \sqrt{a_{t2}^3 / \mu} = \boxed{7.5197 \times 10^8\ s = TOF}$

d] $\Delta V_{hohmann} < \Delta V_{bi-elliptical}$

$TOF_{hohmann} < TOF_{bi-elliptical}$

TOF for a bi-elliptic transfer is bound to be greater because the s/c travels through 2 arcs usually larger than the final orbit

$\because R = \frac{r_{final}}{r_{initial}} = 9.5549 < 11.94 \rightarrow$ Hohmann transfer is cheaper

$\therefore$ It makes sense that $\Delta V_{hohmann} < \Delta V_{bi-elliptical}$

- Problem 4
  - Part a
    - Specific Angular Momentum - 14928.3142 km/s
    - Specific Energy - -3.2446 km^2/s^2
    - Eccentric Anomaly - 180.0°
    - UTC Modified Date - 27113.5 Days
  - Part b
    - True Anomaly - 317.1620°
    - Eccentric Anomaly - 333.1636°
    - Altitude - 499.9999 km
    - Time of Flight along the propagated trajectory segment - 27113.5863 - 27113.5 days = 7464.0515 seconds
  - Part c
    - Spacecraft below 500 km altitude - 27113.6019 - 27113.5864 days = 1343.3657 seconds

(3)

ASEN 5050
Fall 2024
Jash Bhalavat

$\boxed{\text{HW 5}}$

Problem 4 → Given: $a = 6600$ km, $e = 0.46$, $E_1 = 333.1636°$, $E_2 = 26.55°$

Assumptions: 2-Body problem, $\mu = G(M_{s/c} + M_{mars})$ $(\because M_{s/c} << M_{mars}) \rightarrow \mu = GM_{mars}$

$\mu = 42828.3143 \ km^3/s^2$ (From GMAT)

Part C → $E_1 = 333.1636° - 360° \longrightarrow$ Convert to rad $\rightarrow -0.4684$ rad

$E_2 = 0.4634$ rad

$P = 2\pi\sqrt{a^3/\mu} = 1.6279 \times 10^4 s \longrightarrow n = \frac{2\pi}{P} = 3.8597 \times 10^{-4} \frac{rad}{s}$

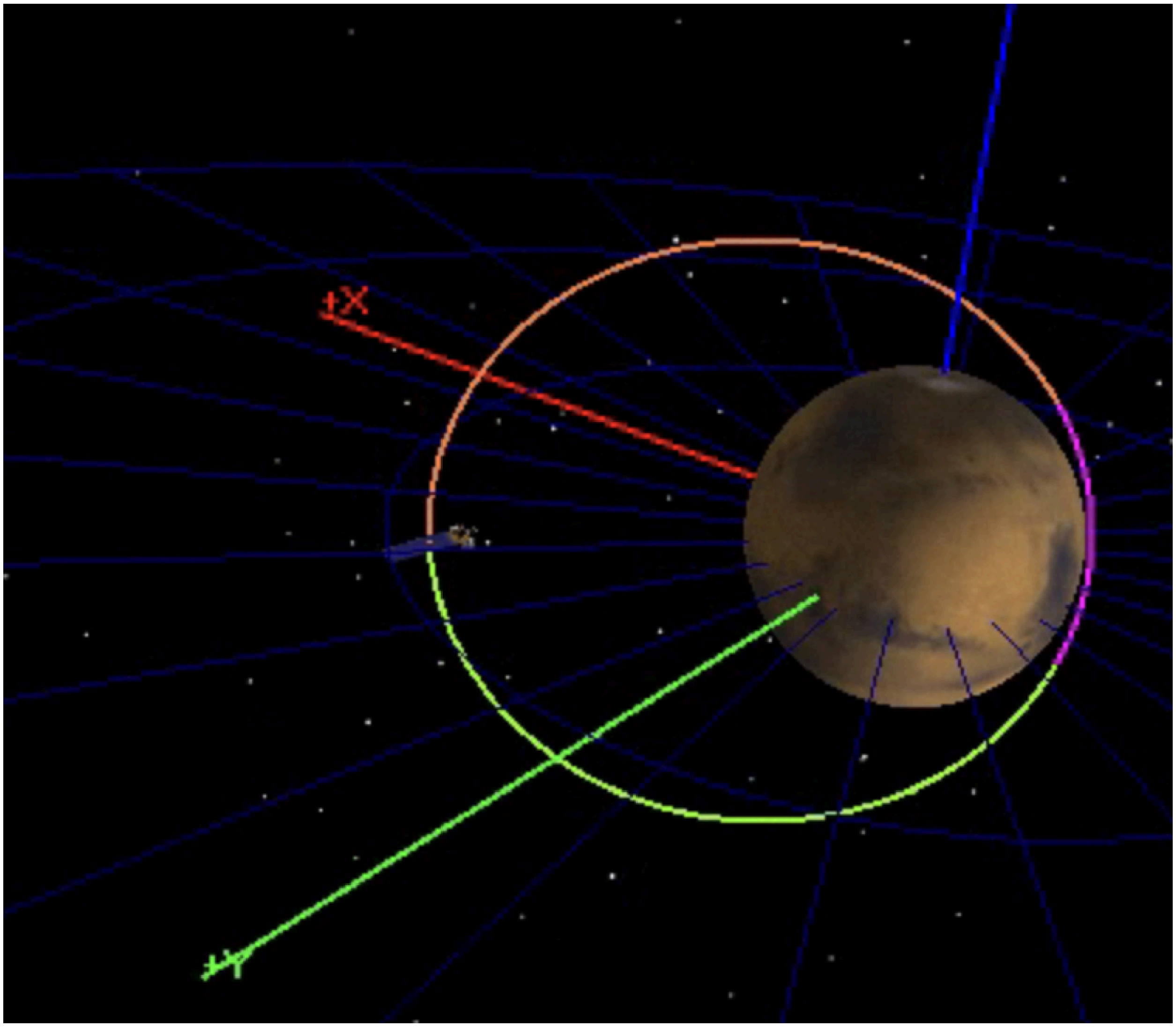
Kepler's eqn → $n(t - t_p) = E - e\sin(E)$

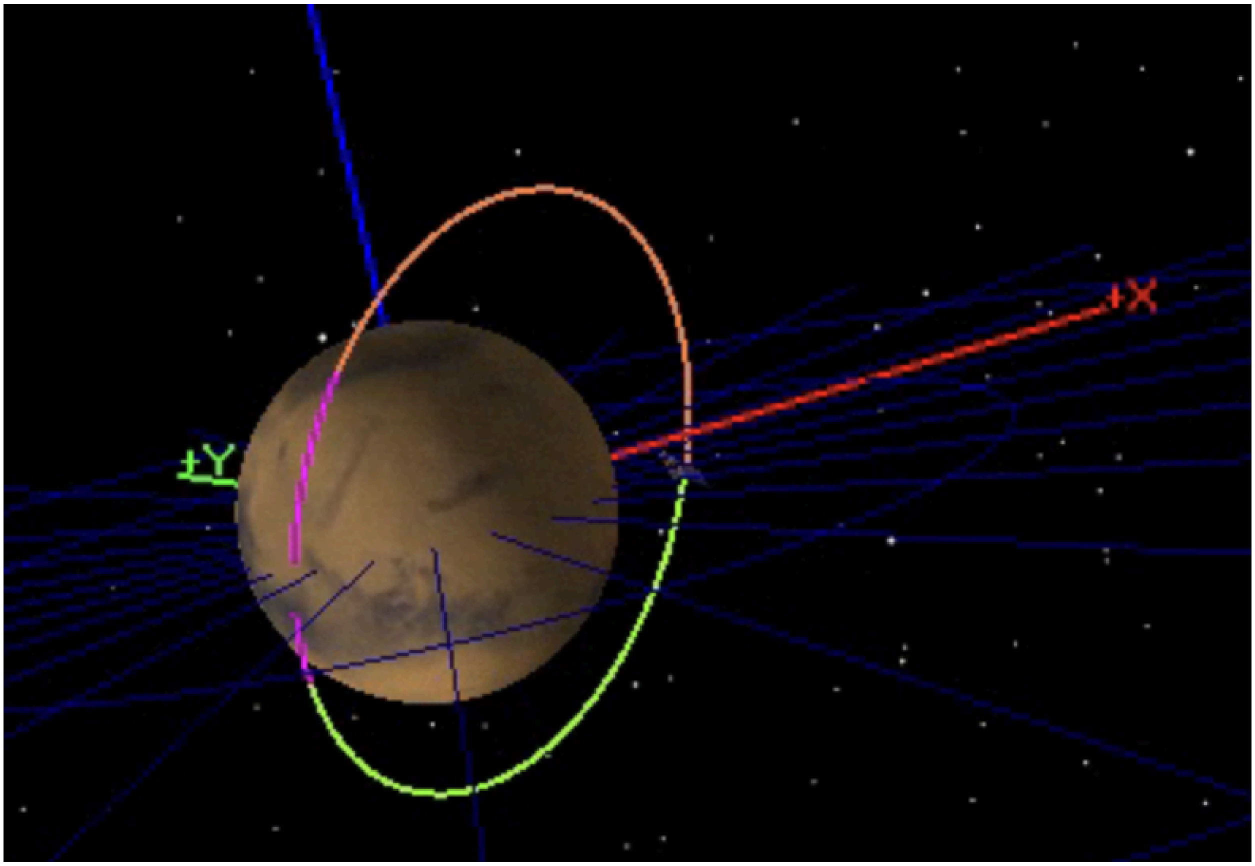
$t_{21} = \frac{T}{2\pi}[(E_2 - e\sin E_2) - (E_1 - e\sin E_1)]$ (since < 1 revolution)

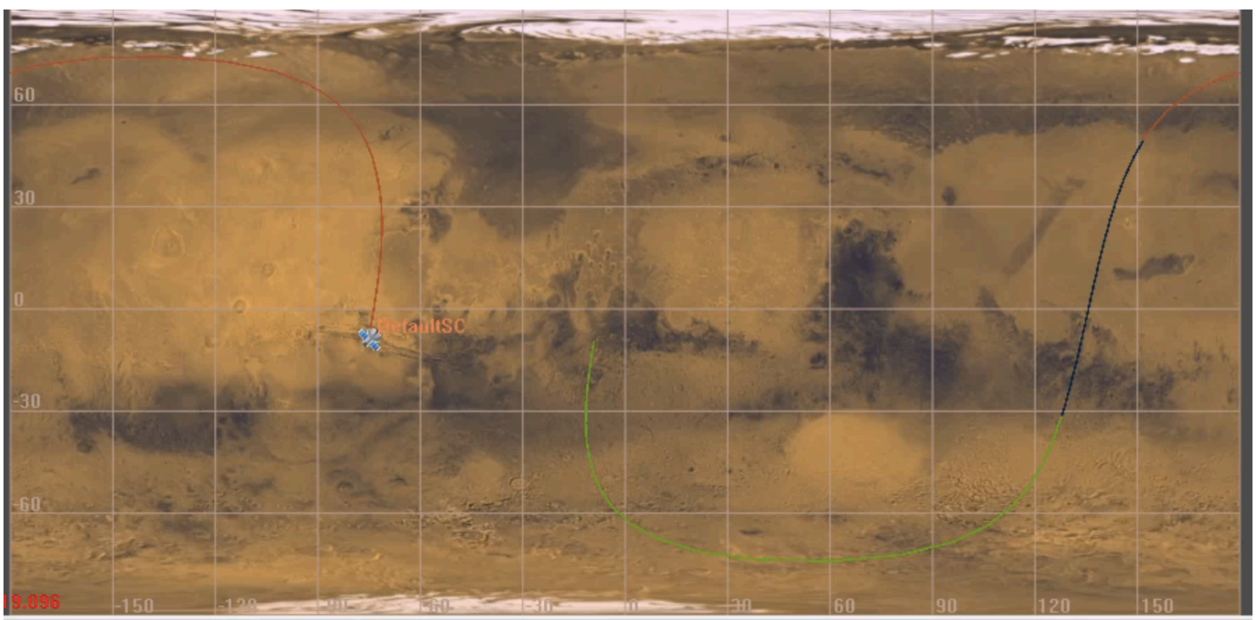
$= 1.3434 \times 10^4$ seconds

    - Using Kepler's equation, the time spent under 500 km altitude is 1.3434e4 seconds which is very close to the GMAT output suggesting that Kepler's equation is a good approximation which indicates that the 2-body approximation holds for this short propagation of the orbit.

- Part d







- The above picture shows the ground track of the spacecraft and the black track is the spacecraft below 500 km. As seen in the picture below, Elysium Volcanic Region falls under that ground track. Hence, the spacecraft may be able to observe that during its orbit.