ASEN 6060 Spring 2025: Homework 5

Due: 4/8/2025 at 9:00pm MT

Notes:

- Use the mass ratio calculated in HW 1 for the Earth-Moon system (if there were errors in your calculations, correct them first)
- In your writeup, show all your working and include a discussion (see the syllabus for more information about expected components of a homework submission)
- For all problems, copy the text of the scripts you create and/or use to the end of your submission for each subproblem as appropriate.
- Note: although this homework deadline is scheduled after Spring Break, you are not expected or required to work on it during Spring Break.

Problem 1:

In this problem, you will construct a Poincaré map to capture planar trajectories in the Moon vicinity in the Earth-Moon CR3BP. Construct your map with the following configuration:

- One-sided map with an $x = 1 - \mu$ surface of section, capturing only positive crossings with $\dot{x} > 0$.
- Seed your initial conditions directly from the surface of section.
- The Jacobi constant of the initial conditions will equal 3.175, the $\dot{x}$ component of velocity is positive, and the initial conditions lie within the zero velocity curves.
- Select a number of initial conditions that is greater than or equal to 100 and offers enough resolution to understand the structure of the solution space when analyzing the map.
- Propagate the trajectories associated with each initial condition in the Earth-Moon CR3BP and record 300 crossings of the surface of section (or more, if you think it is needed to sufficiently reveal the structure of the solution space). Warning: it may take a little while to generate these trajectories, so start early!

a) Describe, using equations/mathematical notation (where applicable) and in your own words: 1) the approach you used to calculate each initial condition, 2) how you selected the number as well as minimum and maximum position coordinates of the initial conditions, and 3) how you determined the number of crossings to record. Attach your script to this subproblem.

b) Using this configuration and instructions provided at the beginning of this problem, plot the associated Poincaré map to display the $(y, \dot{y})$ components of the state at each crossing.

c) Discuss the patterns you observe in each region of the map and the types of trajectories that produce these patterns. Use this information to describe in detail the characteristics of the solution space for planar trajectories at this Jacobi constant.

Problem 2:

a) Calculate an L_1 Lyapunov orbit and an L_2 Lyapunov orbit that each possess a Jacobi constant equal to 3.17, to within 5 decimal places. If needed, you may use the following initial guesses to start the corrections and/or continuation process:

$$\bar{x}_{0,1} = [0.8213849,0,0,0,0.1475143,0]^T, T_1 = 2.763299$$
$$\bar{x}_{0,2} = [1.164855,0,0,0,-0.0516671,0]^T, T_2 = 3.377214$$

For each orbit, report the state vector at one location along with the orbit period after corrections.

b) Generate the unstable half-manifold directed towards the Moon for the L_1 Lyapunov orbit you calculated in part a) and record up to <u>two positive</u> crossings of the $x = 1 - \mu$ surface of section, with $\dot{x} > 0$. Display these crossings on a Poincaré map that displays the $(y, \dot{y})$ components of the state at each crossing using <u>red</u> markers.

c) Generate the stable half-manifold directed towards the Moon for the L_2 Lyapunov orbit you calculated in part a) and record up to <u>two positive</u> crossings of the $x = 1 - \mu$ surface of section, with $\dot{x} > 0$. Display these crossings on the same Poincaré map as part b) that displays the $(y, \dot{y})$ components of the state at each crossing; for this set of map crossings, use <u>blue</u> markers.

d) Analyze the information contained on the Poincaré map you have constructed and discuss how you could potentially use this information to design initial guesses for transfers from an L_1 Lyapunov orbit to an L_2 Lyapunov orbit at a Jacobi constant of 3.17. Which map crossings could you use to construct these initial guesses? How could you generate initial guesses for transfers with distinct geometries? Consider both natural and maneuver-enabled transfers.

Note: Problem 2 will be a great preparation for the upcoming final project!