ASEN 6060
ADVANCED ASTRODYNAMICS
Computing Periodic Orbit Families, part 2

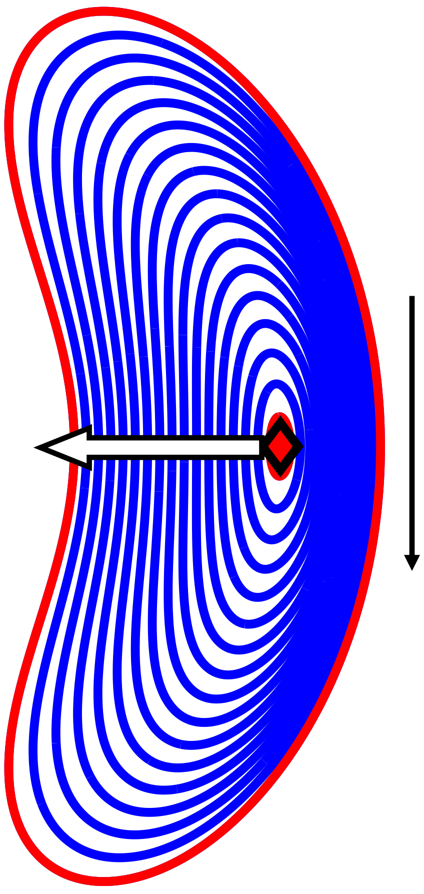
Objectives:

- Define and formulate continuation schemes for effectively computing families of periodic orbits

Credit: these notes were developed in collaboration with Dr. Ian Elliott in 2020, and have since been updated

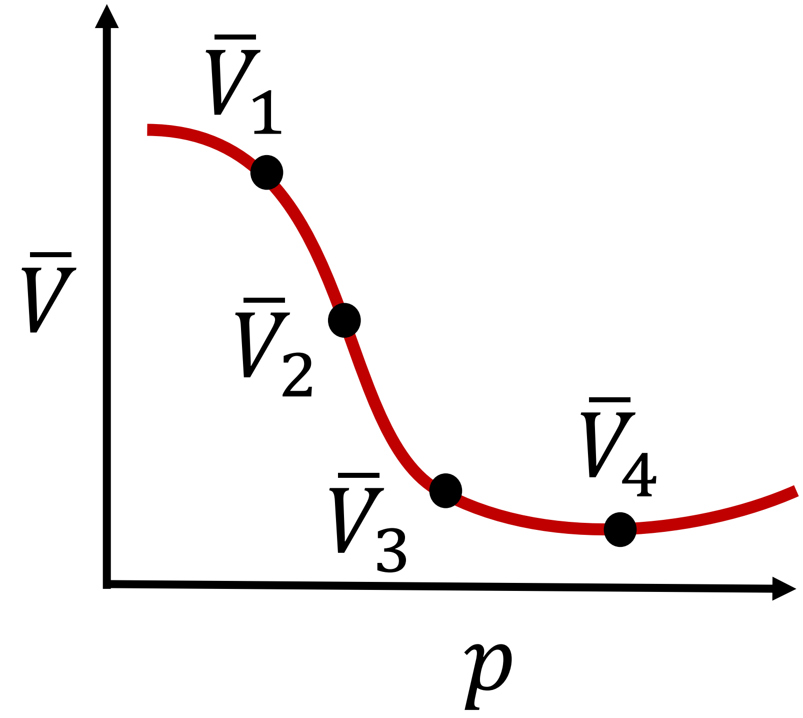
Continuation Methods

- Periodic orbits exist in continuous one-parameter families of orbits in the CR3BP
- Computing orbit families is useful for exploring solution space
- Continuation methods use a periodic orbit to calculate a nearby member, repeating the process along a family of orbits
- More efficient than generating distinct new initial guesses for multiple members along a family



Continuing Along Periodic Orbit Family

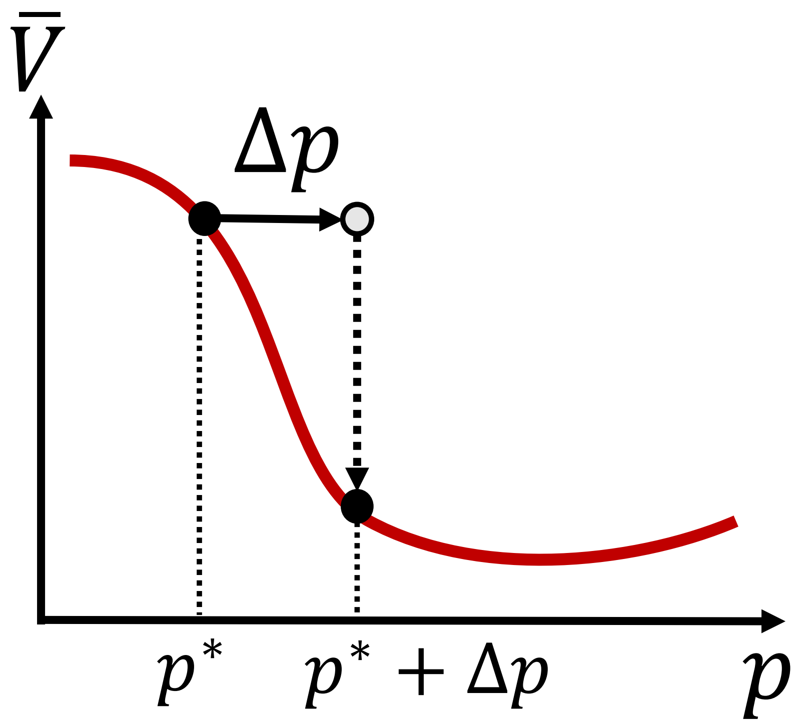
Fundamental steps of continuation:



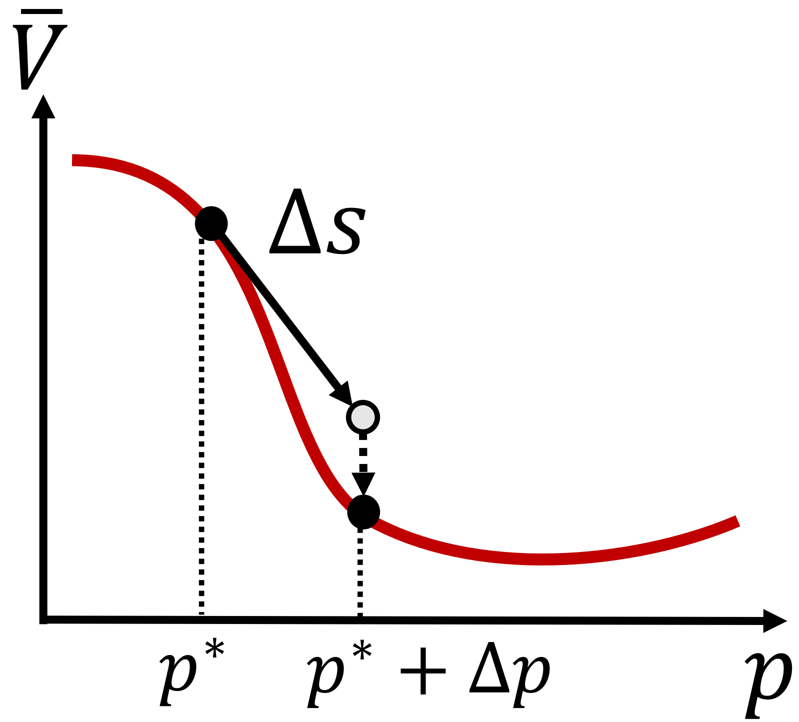
Continuation Methods

Multiple approaches for implementing continuation, including:

Natural Parameter Continuation



Pseudo-Arclength Continuation



Continuation Methods

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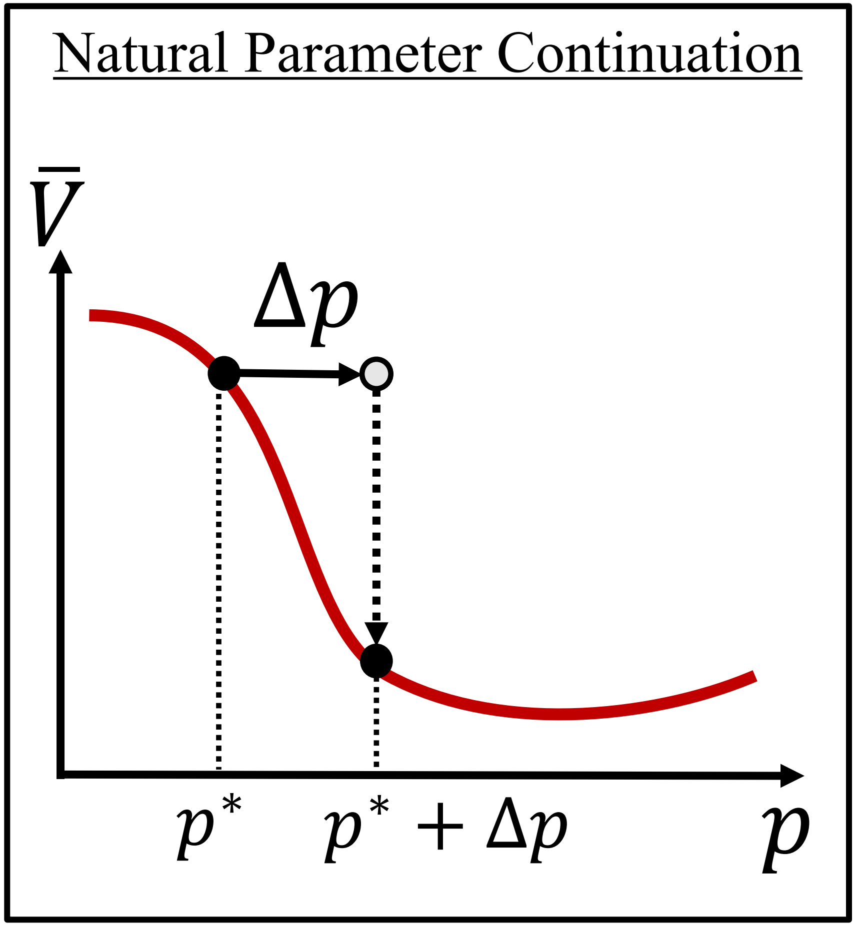


Natural Parameter Continuation

Step along the family by

Apply a corrections scheme, augmented with an additional constraint

Example parameters:

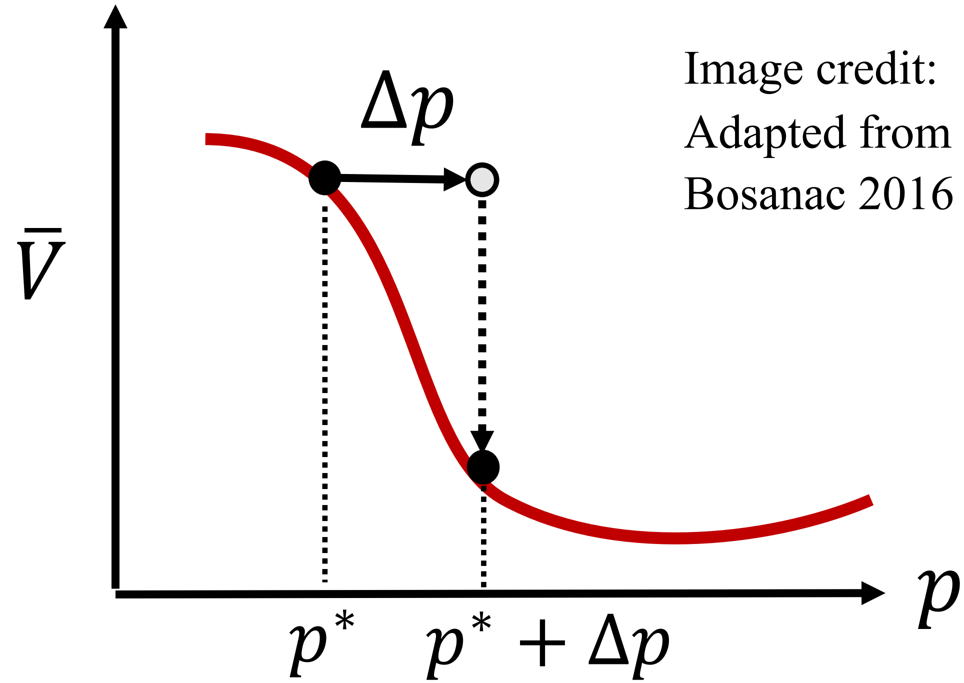


Image credit: Adapted from Bosanac 2016

Natural Parameter Continuation

Constrain difference between current value of parameter, p, and its value along the previous solution, p^*, with specified step Δp:

$$\bar{H}(\bar{V}) = \begin{bmatrix} F_1 \\ \vdots \\ F_m \\ \\ \end{bmatrix}$$

Example: Natural Parameter Cont. in T

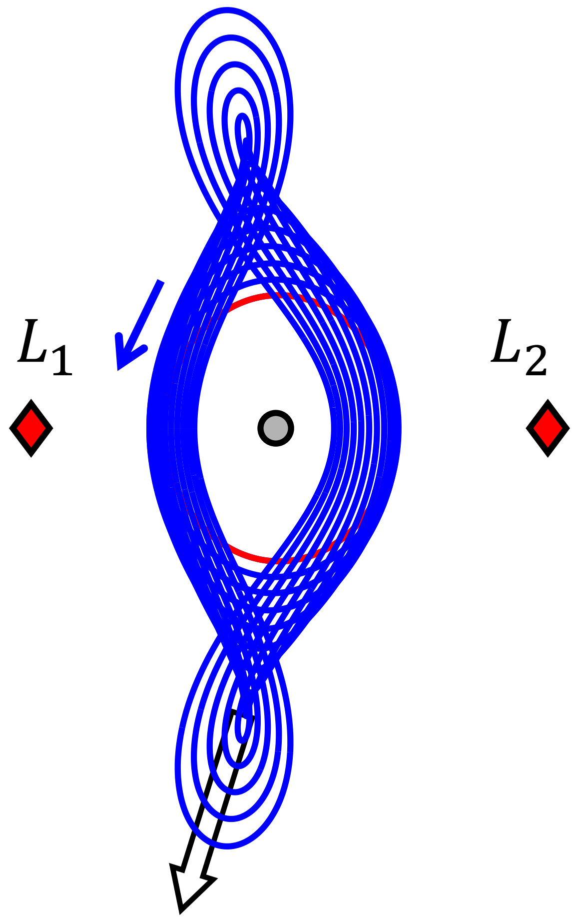
$$\bar{V} = \begin{bmatrix} \bar{x}_0 \\ T \end{bmatrix} \qquad \bar{F}(\bar{V}) = \begin{bmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \\ \dot{x}_f - \dot{x}_0 \\ \dot{z}_f - \dot{z}_0 \\ y_0 \end{bmatrix} \longrightarrow \bar{H}(\bar{V}) = \begin{bmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \\ \dot{x}_f - \dot{x}_0 \\ \dot{z}_f - \dot{z}_0 \\ y_0 \\ (\qquad\qquad) \end{bmatrix}$$

$$D\bar{F}(\bar{V}) \longrightarrow D\bar{H}(\bar{V})$$

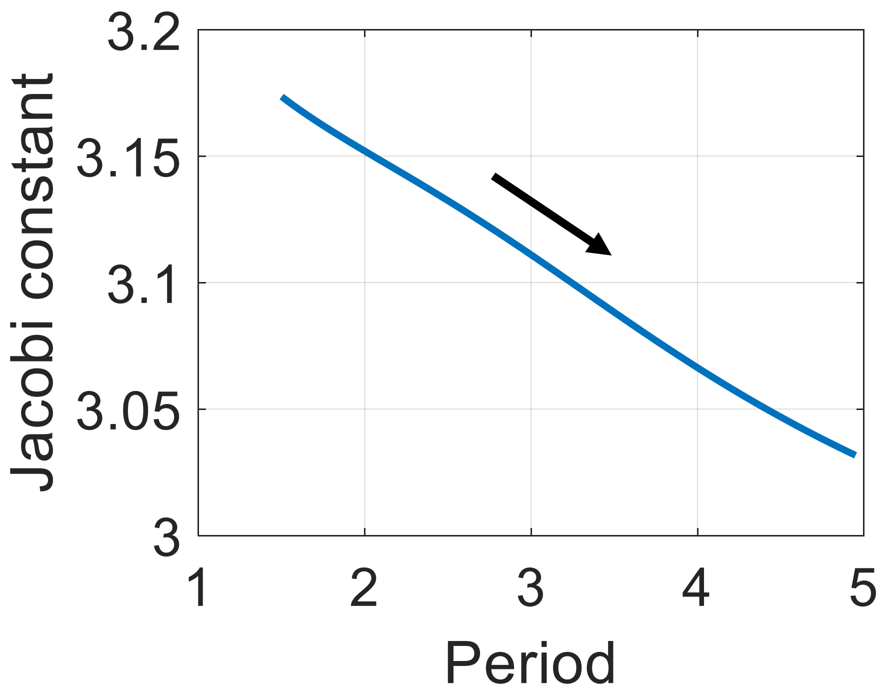
$$\bar{V}_{i+1} = \bar{V}_i - D\bar{F}(\bar{V}_i)^T \left[D\bar{F}(\bar{V}_i) D\bar{F}(\bar{V}_i)^T \right]^{-1} \bar{F}(\bar{V}_i) \longrightarrow \bar{V}_{i+1} = \bar{V}_i - D\bar{H}(\bar{V})^{-1} \bar{H}(\bar{V}_i)$$

E.g.: Distant Prograde Orbit (DPO) Family

Solutions along DPO orbit family found via natural parameter continuation using the orbit period

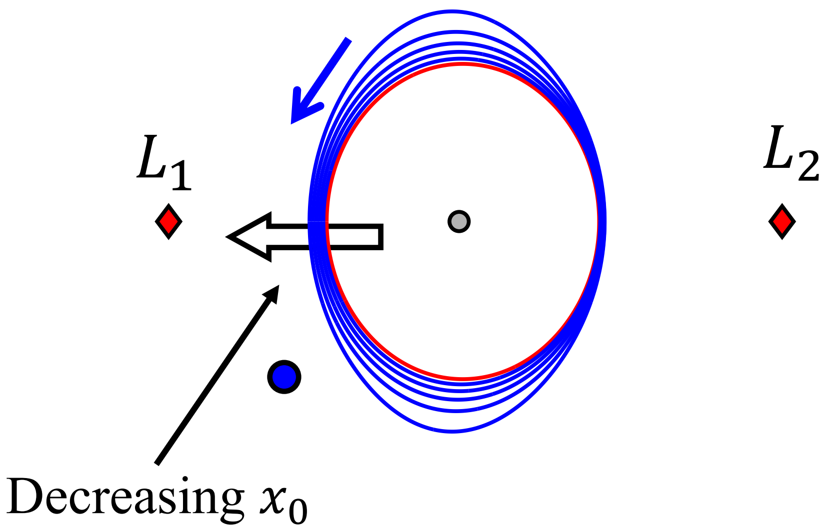


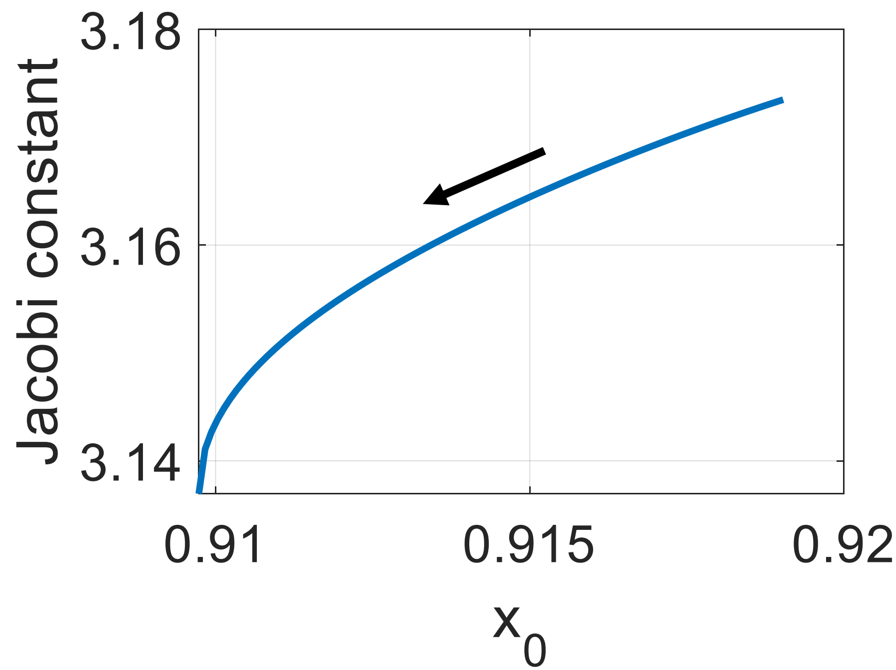
Increasing orbit period



E.g.: Distant Prograde Orbit (DPO) Family

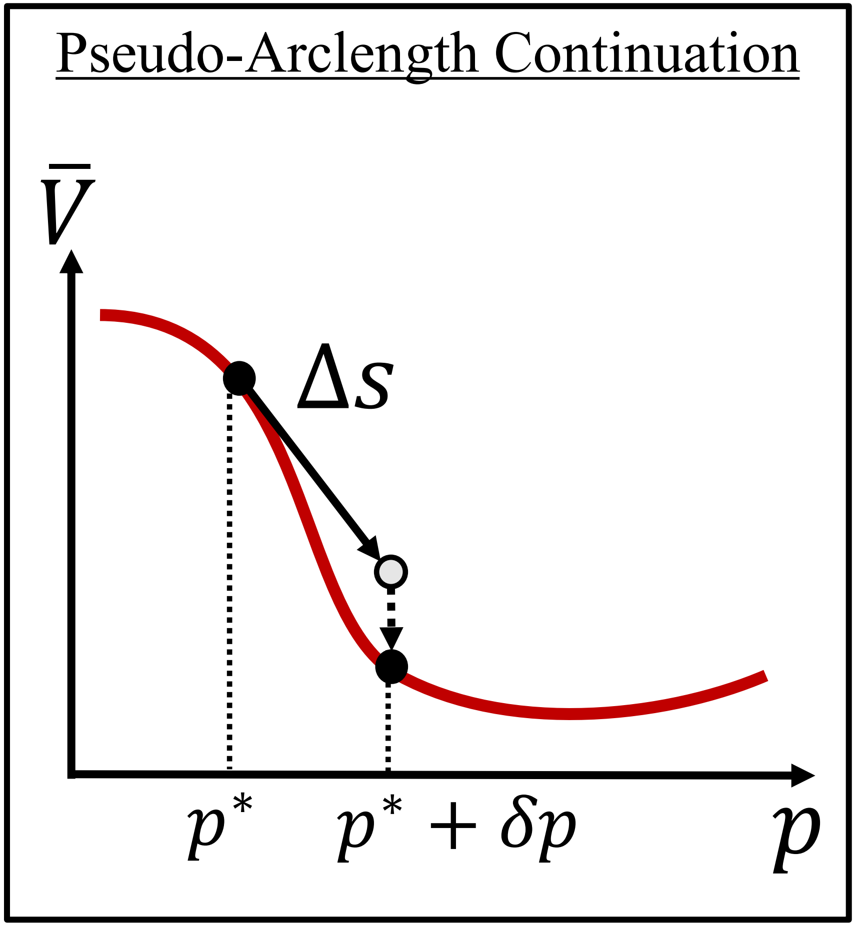
Same initial orbit, continuing along decreasing x_0





Continuation Methods

Multiple approaches for implementing continuation, including:



Pseudo-Arclength Continuation

Predict nearby member of the family via gradient of solution curve

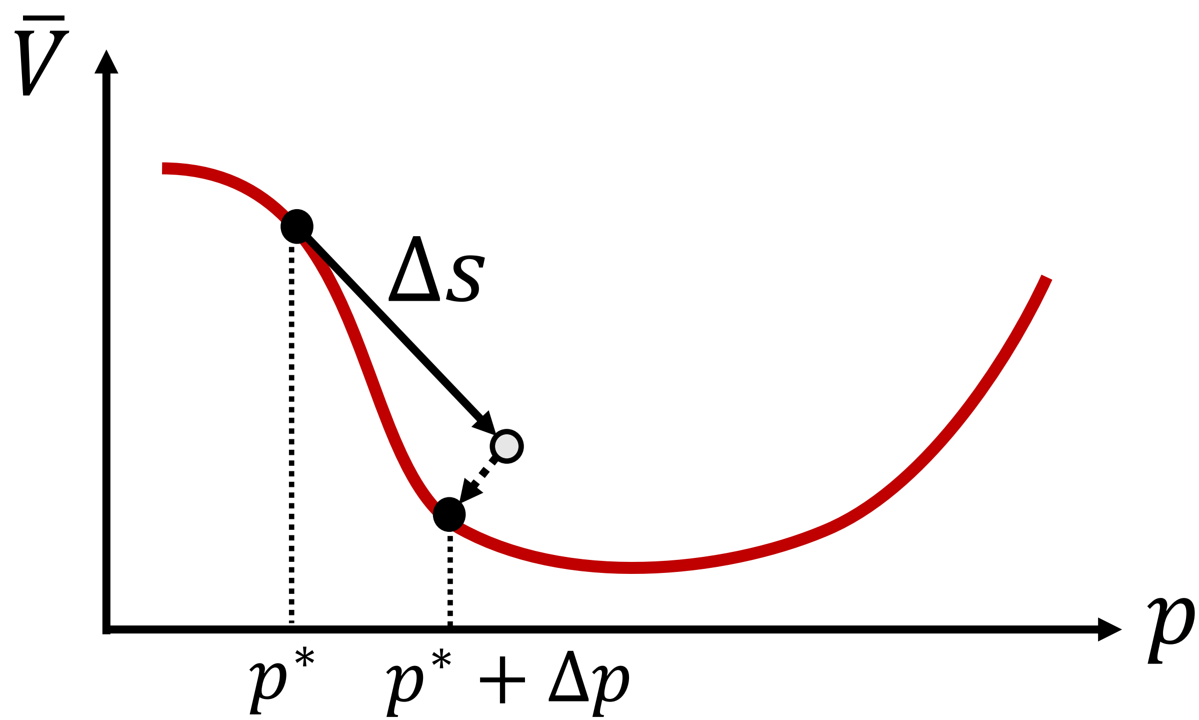


Image credit:
Adapted from
Bosanac 2016

Pseudo-Arclength Continuation

Formulate single shooting algorithm to produce 1-parameter family $(n - m = 1)$. Consider one formulation:

$$\bar{V} = \begin{bmatrix} \bar{x}_0 \\ T \end{bmatrix} \qquad \bar{F}(\bar{V}) = \begin{bmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \\ \dot{x}_f - \dot{x}_0 \\ \dot{z}_f - \dot{z}_0 \\ y_0 \end{bmatrix}$$

Apply update equation to initial guess until first solution computed:

$$\bar{V}_{i+1} = \bar{V}_i - D\bar{F}(\bar{V}_i)^T \left[D\bar{F}(\bar{V}_i) D\bar{F}(\bar{V}_i)^T \right]^{-1} \bar{F}(\bar{V}_i)$$

Pseudo-Arclength Continuation

Select an appropriate step size Δs. The initial guess of next orbit is calculated as:

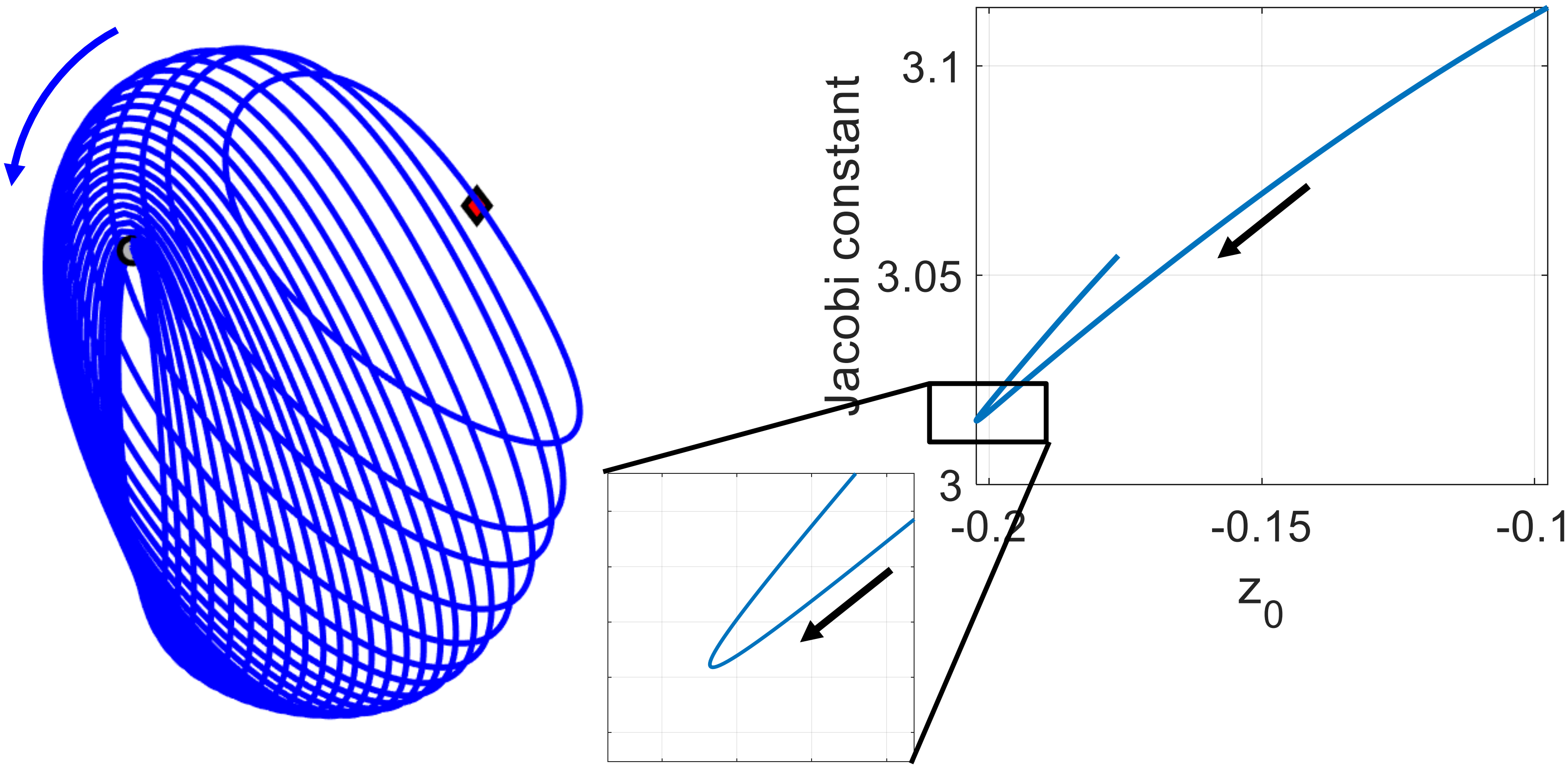
Construct new constraint vector to limit distance between subsequent solutions via projection onto tangent space:

$$\bar{H}(\bar{V}) = \begin{bmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \\ \dot{x}_f - \dot{x}_0 \\ \dot{z}_f - \dot{z}_0 \\ y_0 \\ \\ \end{bmatrix}$$

Iterate the following update equation until next solution is computed:

E.g.: Southern L_2 Halo Orbit Family

Pseudo-arclength continuation performs well despite turning points in natural parameters along the family as orbits approach the Moon.



Pseudo-Arclength Implementation Notes