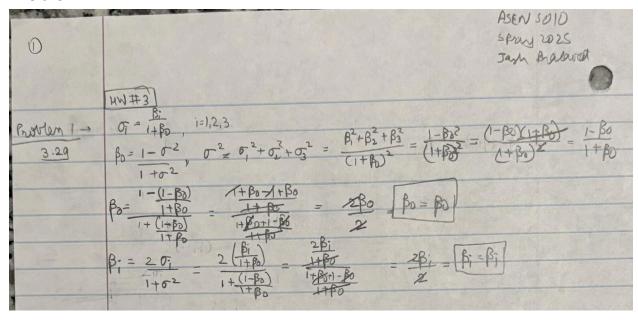
# ASEN 5010 - HW 3 Spring 2025 Jash Bhalavat

#### **Problem 1**



# Problem 2

Paldona	Egn 316 - 5= - 02 + 2 (1+02) 00 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
3.34	Egn. 3.160 > 0 = 4 (1-03) I3x3 + 2 (0) + 200 ) 4
3-147 n , vector form	$ \frac{F_{0} \times 3 \times 1}{\sigma} \Rightarrow \sigma_{1}^{2} = \frac{1}{\sigma^{2}}, i=1,2,3 $ $ \frac{\sigma}{\sigma} = \frac{\sigma}{\sigma^{2}}, \sigma^{2} = \sigma^{2} = \sigma^{2} $ $ \frac{\sigma}{\sigma} = \frac{\sigma}{\sigma^{2}}, \sigma^{2} = \sigma^{2} = \sigma^{2} $ $ \frac{\sigma}{\sigma^{2}} = \frac{\sigma^{2}}{\sigma^{2}}, \sigma^{2} = \sigma^{2} = \sigma^{2} $ $ \frac{\sigma}{\sigma^{2}} = \frac{\sigma^{2}}{\sigma^{2}}, \sigma^{2} = \sigma^{2} $ $ \frac{\sigma}{\sigma^{2}} = \frac{\sigma^{2}}{\sigma^{2}}, \sigma^{2} = \sigma^{2} $ $ \frac{\sigma^{2}}{\sigma^{2}} = \frac{\sigma^{2}}{\sigma^{2}}, $
	$ \frac{1}{\sigma^{4/2}} \qquad \frac{1}{\sigma^{4/2$
	Sul 5 - 5 = 52 +
(20'[x]) is	0 veranse = - 0 10 1 [07. I3x3 - 20 [0] + 20 00 ] 5 W
I is crossed with	verence = - \( \omega \) \( \frac{1}{2} \left[ \sigma^T \] \( \frac{1} \left[ \sigma^T \] \( \frac{1}{2} \le
lota, b, c be eli	
	i.e. 5 to [I3x3] = 5 6 5 o T. Use this is (1)
	$\frac{\dot{\sigma}^{5} = -\dot{\sigma}}{\sigma^{4}} + (\sigma)(\frac{1}{2}[\sigma^{T}]_{3\times3} + \sigma^{T}\sigma^{T}\sigma]^{\beta}\omega = -\dot{\sigma} + \frac{1}{2}(1 + \sigma^{T}\sigma)\sigma\sigma^{T}\omega$
2	$ \frac{\dot{\sigma}^{5}}{\sigma^{4}} + \frac{1}{2} \frac{(1+\sigma^{2})}{\sigma^{4}} \underline{\sigma} \underline{\sigma}^{T} \underline{\beta} \underline{w} $
¥	

# Problem 3

FIODI <del>C</del> III 3	
	Jam Braland
	AS EN SO D
•	SPLLY 2025
•	Hw#3
Proven 3 -1	C-center of mass, P-Point where rod towns tre
4.7	surface with it slips L 18/7
	Ic= 12 m2, J= 12 m2+ m2 (Paralle gris theorem) It I LA
	= 1 M2
the inter	0= = 100, 0= - = 0 sho = = = 0 sho = = = 0 sho
	x= 2500, x= 2000, x= 2000 / 7
	n= coo 2, -sin 0 20
sum of forces	$m\ddot{s} \hat{n}_{2} = (N - mg) \hat{n}_{2} \rightarrow N = mg - \frac{mL\theta^{2}}{2} \Leftrightarrow \theta = \frac{mL\theta}{2} \Leftrightarrow \theta = \frac{mL\theta}{2} \Leftrightarrow \theta$
	@ Point of suppage -> Sum of porces in x -> mx n, = fg = MNn,
	M= mx 20 [= (- 0 sin 0 + 0 con 0)]
	N M[9- \frac{1}{2}(\text{\tiny{\text{\ti}}}\text{\tin}\text{\tert{\text{\tetx{\text{\texi}\text{\texi}\text{\text{\tetx{\texi}\text{\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\text{\texi{\text{\text{\texi}
Energy function	V(8) = mg/s = mg/2 4000
•	T(0,0) = 252 + 2×2 + 1/2 02 = 2 [ 2 502 0 0 2 + 1/2 0 20 0 0 + 24 m2 0 2
	$E(t_0) = V_0 + T_0 = \frac{MSL}{2}$
	$E(t_0) = V(0) + 7(0,0) = \frac{mox}{2} = \frac{mox}{2} + \frac{m(t_0)^2}{8} + \frac{m(t_0)^2}{24} \rightarrow \frac{3}{2} = \frac{9}{2} + \frac{100}{5} + \frac{100}{5}$
	$\frac{1}{8} = \frac{30}{L}(1-400) - 0$
Torque & -	Lp= Ip = = = = = = = = = = = = = = = = =
10-00	= = = mgsin 0 ez
	$\theta = \frac{1}{2} \text{ masin } \theta - \frac{3}{100} = \frac{39}{21} \sin \theta - \beta$
Plus (2), (2) into	$\theta = \frac{5}{2} \text{ mosin } \theta - \frac{3}{30} = \frac{39}{21} \sin \theta - 8$ $\mu = \frac{1}{2} \left[ -\frac{39}{2} (1 - (100) \sin \theta + \frac{39}{21} \sin \theta \cos \theta) \right] - \frac{38}{2} (\sin \theta - \sin \theta \cos \theta) + \frac{38}{21} \sin \theta \cos \theta$
	5- \(\frac{1}{2}\left(\frac{1}{2}\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot
	$\mu = -\frac{3}{2}\sin\theta + 3\sin\theta\cos\theta$
	$1 - \frac{3}{2} \omega_0 + \frac{3}{2} \omega_0 - \frac{3}{2} \omega_0^2 \Theta$
•	

#### **Problem 4**

This problem can be divided into 3 different equations of motions: translational, rotational, and Euler angles. All these equations of motion are differential equations (either 1st order or 2nd order) and can be numerically integrated using Matlab's ODE45 solver. At the least, the solver requires an initial condition and a function that implements the equation of motion. All three sets of equations can be augmented into one vector to compactly script in Matlab. The augmented state vector is as follows:

$$state = \left[ x_{CN}^{N}, y_{CN}^{N}, z_{CN}^{N}, x_{CN}^{N}, x_{CN}^{N}, y_{CN}^{N}, z_{CN}^{N}, \omega_{PN,1}^{N}, \omega_{PN,2}^{N}, \omega_{PN,3}^{N}, \alpha, \beta, \gamma \right]^{T}$$

The equations of motion for each of the 3 sets are as follows:

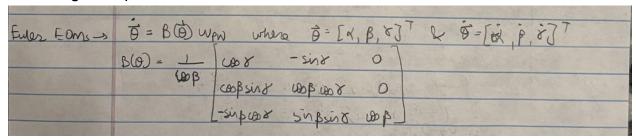
Translational Equations of Motion:

frollen 4 ->	Translational EOMo:
	NX=[x, y,z, x, y, z] →NX=[x, y, z, x, y, z] where all these elements
	represent com from inertial origin in inertial frame
	Voirs owner- partial trepren:
	mx or = Tx Ty Tz are components of theust is irestead frome
	m 500 = 15 m= 1kg g= 9.81 ==
	m'z = NTz-mg
	Thrust is principal frame = [0, -T(D) 500, T(D) 500 T(D)
	T(t)=15N (qt510) &ON (4t>10A), 0=25°
	NT can be found - T = [NP] T = [PN] T
	where PN(t) = R3(8(t)) R(B(t)) R(A(t))

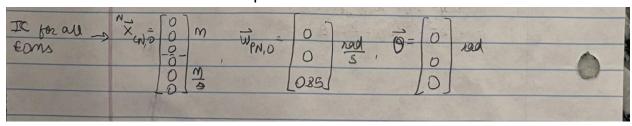
#### **Rotational Equations of Motion:**

Rotational,	PLC=PRNoysle X Tit) where rnoysle = [0 0,-3] m
£0mb	I, wp. () = - (I33 - I22) WP. (1) WP. (1) + L. (1) 7 where I = 325 5 m2
4 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	In WANGED = - (In-Iss) WANGO WANGO + PUCO > IN-38.5 kg m?
	Is was = - (I2-I1) Was (1) Was (2) + L( (3) ] Is = 5 kg m2

Euler angles Equations of Motion:

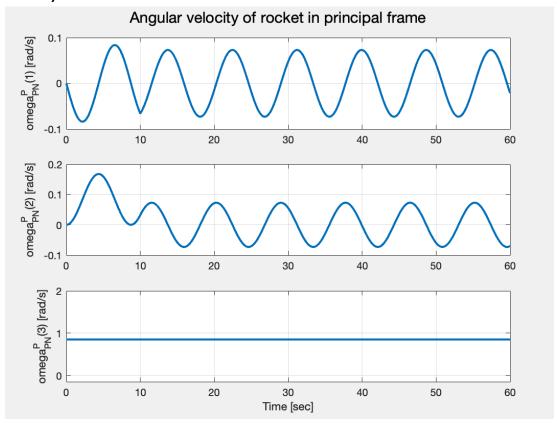


All the initial conditions for the equations of motion are as follows:

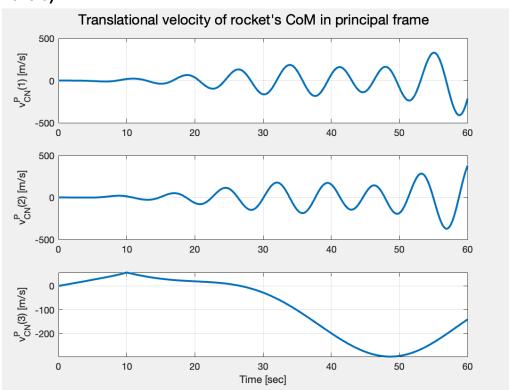


The plots are as follows:

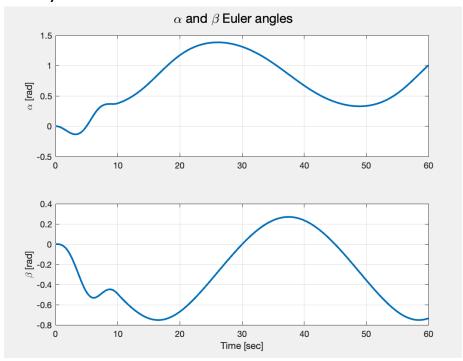
# Part a)



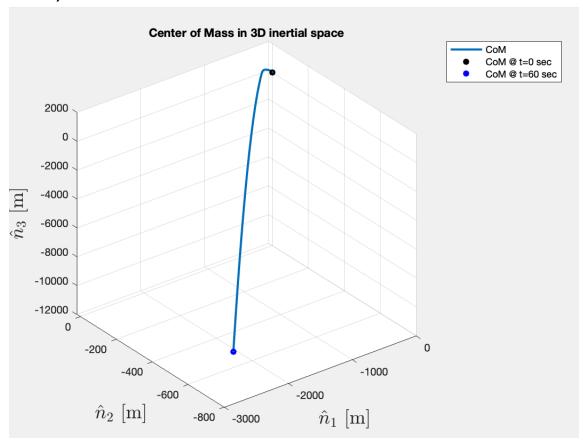
### Part b)



# Part c)



# Part d)

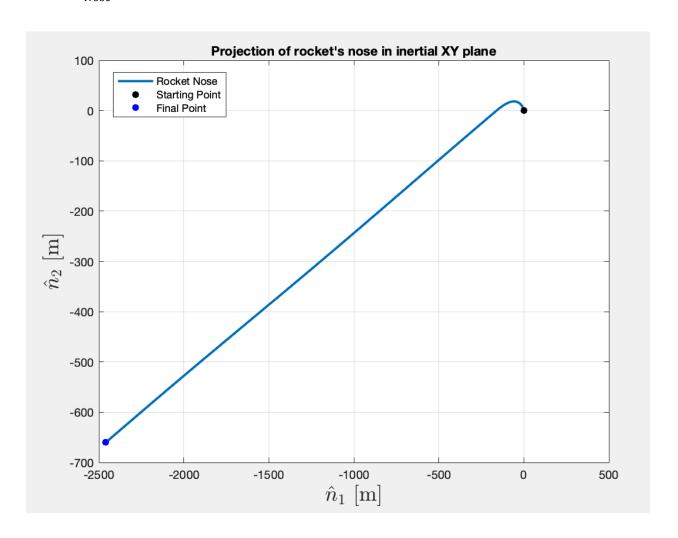


# Part e)

Rocket's nose in inertial XY plane is calculated by this equation:

$$r_{Nose}^{N} = r_{CN}^{N} + \left[PN\right]^{T} * r_{Nose}^{P}$$

where 
$$r_{Nose}^P = [0, 0, 4] meters$$



#### **Table of Contents**

```
clear; clc; close all;
% ASEN 5010 - HW 3, Problem 4
% Spring 2025
% Jash Bhalavat
mass = 1; % kg
v_{N_p} = [0, 0, 0]'; % m/s
v_CN_N_0 = v_CN_P_0;
omega_PN_P_0 = [0, 0, 0.85]'; % rad/sec
euler_PN_0 = [0, 0, 0]'; % rad
g = 9.81; % km/s2
I_c_P = [32.5, 32.5, 5]'; % kg-m2
r_NozzleC_P = [0, 0, -3]'; % m
r NoseC P = [0, 0, 4]'; % m
r_{N_0} = [0, 0, 0]'; % [m]
tspan = [0, 60]; % sec
function thrust P = thrust(time)
  % Thrust in Principal frame
  thruster_misalignment = deg2rad(2.5); % rad
  thruster_mag = 15; % N
  thruster duration = 10; % sec
  thrust_parallel = 0; % [N] Passes CoM
  thrust_perp = 0; % [N]
  if time <= thruster_duration</pre>
     thrust_parallel = thruster_mag * cos(thruster_misalignment);
     thrust_perp = thruster_mag * sin(thruster_misalignment);
  end
  thrust_P = [0, -thrust_perp, thrust_parallel]'; % [N]
end
function out = L_c_P(time, r)
  % Torque at CoM in P frame
  thrust_at_time_P = thrust(time);
  out = cross(r, thrust_at_time_P); % [Nm]
end
```

### **EOM**

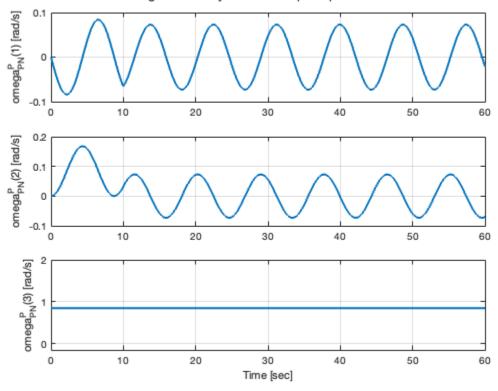
```
state0 = [r_CN_N_0; v_CN_N_0; omega_PN_P_0; euler_PN_0];
[tout state, state out] = ode45(@(t, state)state eom(t, state, mass, g,
I c P, r NozzleC P), [tspan(1), tspan(2)], state0);
function state dot = state eom(time, state, mass, q, I, r NozzleC P)
    % state - [x_N, y_N, z_N, x_N_dot, y_N_dot, z_N_dot, omega_PN_P(1),
omega PN P(2),
    % omega PN P(3), alpha, beta, gamma]'
    % state dot - [x N dot, y N dot, z N dot, x N dotdot, y N dotdot,
z N dotdot, omega PN P(1), omega PN P(2),
    % omega PN P(3), alpha, beta, gamma]'
    gravity N = [0 \ 0 \ -g]';
    r CN N = state(1:3);
    v \in N = state(4:6);
    omega PN P = state(7:9);
    euler PN = state(10:12);
    r CN N dot = v CN N;
    PN = R3(euler PN(3))*R2(euler PN(2))*R1(euler PN(1));
    thrust at time P = thrust(time);
    thrust at time N = PN' * thrust at time P;
    v CN N dot = (thrust at time N + mass*gravity N)/mass;
    L c P at time = L c P(time, r NozzleC P);
    omega PN P dot(1,1) = -1/I(1) * (I(3) - I(2)) *
omega PN P(2)*omega PN P(3) + L c P at time(1)/I(1);
    omega_PN_P_dot(2,1) = -1/I(2) * (I(1) - I(3)) *
omega PN P(1)*omega PN P(3) + L c P at time(2)/I(2);
    omega PN P dot(3,1) = -1/I(3) * (I(2) - I(1)) *
omega PN P(1)*omega PN P(2) + L c P at time(3)/I(3);
    B theta = 1/\cos(\text{euler PN}(2)) .* [\cos(\text{euler PN}(3)), -\sin(\text{euler PN}(3)), 0];
                cos(euler PN(2))*sin(euler PN(3)),
cos(euler PN(2))*cos(euler PN(3)), 0;
                -sin(euler PN(2))*cos(euler PN(3)),
sin(euler PN(2))*sin(euler PN(3)), cos(euler PN(2))];
    euler PN dot = B theta * omega PN P;
    state dot = [r CN N dot; v CN N dot; omega PN P dot; euler PN dot];
end
```

### Part a

```
omega_PN_P_out = state_out(:,7:9);
```

```
figure()
subplot(3,1,1)
plot(tout_state, omega_PN_P_out(:,1), 'LineWidth',2)
ylabel("omega_{PN}^P(1) [rad/s]")
grid on
subplot(3,1,2)
plot(tout_state, omega_PN_P_out(:,2), 'LineWidth',2)
ylabel("omega_{PN}^P(2) [rad/s]")
grid on
subplot(3,1,3)
plot(tout_state, omega_PN_P_out(:,3), 'LineWidth',2)
ylabel("omega_{PN}^P(3) [rad/s]")
xlabel("Time [sec]")
grid on
sgtitle("Angular velocity of rocket in principal frame")
```

#### Angular velocity of rocket in principal frame



### Part b

```
v_CN_N_out = state_out(:, 4:6);
for i = 1:length(tout_state)
    euler_PN = state_out(i, 10:12);
    PN = R3(euler_PN(3))*R2(euler_PN(2))*R1(euler_PN(1));
    v_CN_P_out(i,:) = (PN * v_CN_N_out(i,:)')';
```

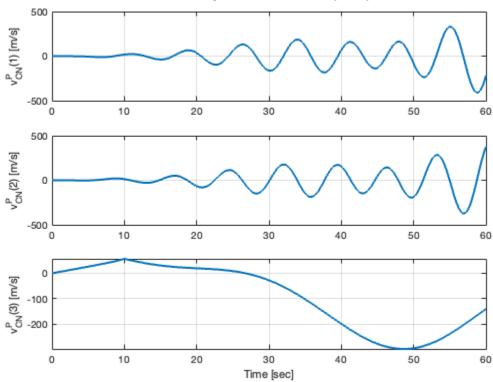
#### end

```
figure()
subplot(3,1,1)
plot(tout_state, v_CN_P_out(:,1), 'LineWidth',2)
ylabel("v_{CN}^P(1) [m/s]")
grid on

subplot(3,1,2)
plot(tout_state, v_CN_P_out(:,2), 'LineWidth',2)
ylabel("v_{CN}^P(2) [m/s]")
grid on

subplot(3,1,3)
plot(tout_state, v_CN_P_out(:,3), 'LineWidth',2)
ylabel("v_{CN}^P(3) [m/s]")
xlabel("v_{CN}^P(3) [m/s]")
xlabel("Time [sec]")
grid on
sgtitle("Translational velocity of rocket's COM in principal frame")
```



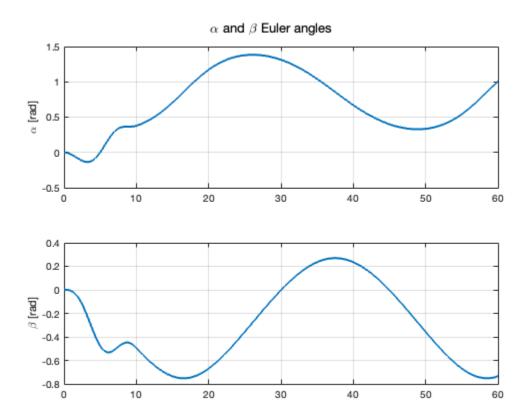


# Part c

```
euler_PN_out = state_out(:, 10:12);
figure()
subplot(2,1,1)
```

```
plot(tout_state, euler_PN_out(:,1), 'LineWidth',2)
ylabel("\alpha [rad]")
grid on

subplot(2,1,2)
plot(tout_state, euler_PN_out(:,2), 'LineWidth',2)
xlabel("Time [sec]")
ylabel("\beta [rad]")
grid on
sgtitle("\alpha and \beta Euler angles")
```

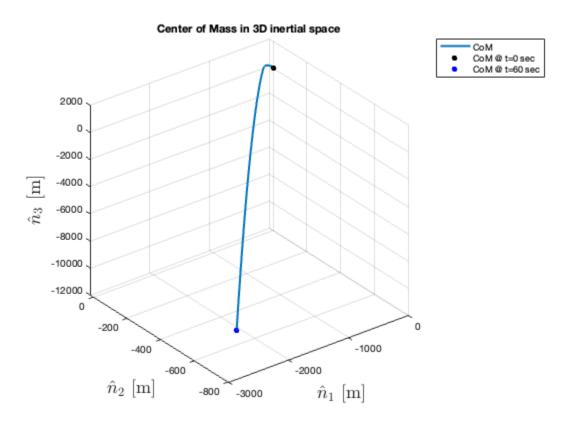


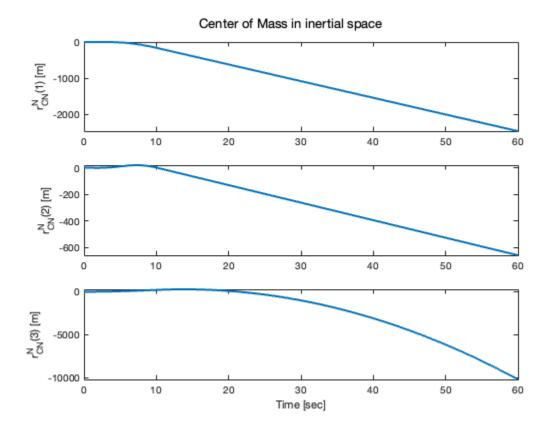
#### Part d

```
figure()
plot3(state_out(:,1), state_out(:,2), state_out(:,3), 'LineWidth',2)
xlabel('$$\hat{n}_{1}$$ [m]','Interpreter','Latex', 'FontSize',18)
ylabel('$$\hat{n}_{2}$$ [m]','Interpreter','Latex', 'FontSize',18)
zlabel('$$\hat{n}_{3}$$ [m]','Interpreter','Latex', 'FontSize',18)
hold on
scatter3(state_out(1,1), state_out(1,2), state_out(1,3), 'filled', 'black')
scatter3(state_out(end,1), state_out(end,2), state_out(end,3), 'filled',
'blue')
legend("CoM", "CoM @ t=0 sec", "CoM @ t=60 sec")
grid on
title("Center of Mass in 3D inertial space")
```

Time [sec]

```
figure()
subplot(3,1,1)
plot(tout_state, state_out(:,1), 'LineWidth',2)
ylabel("r_{CN}^N(1) [m]")
subplot(3,1,2)
plot(tout_state, state_out(:,2), 'LineWidth',2)
ylabel("r_{CN}^N(2) [m]")
subplot(3,1,3)
plot(tout_state, state_out(:,3), 'LineWidth',2)
ylabel("r_{CN}^N(3) [m]")
xlabel("Time [sec]")
sgtitle("Center of Mass in inertial space")
```



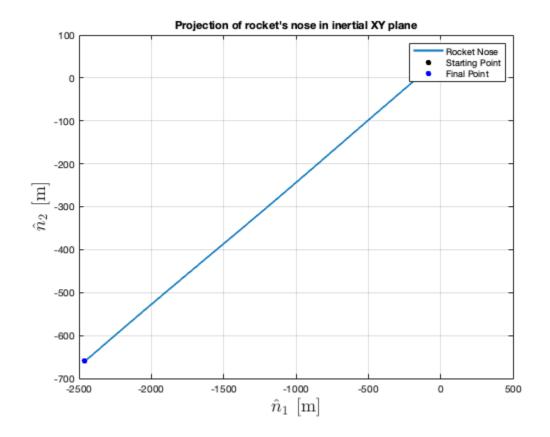


### Part e

```
euler_PN = state_out(:,10:12);
r_CN_N = state_out(:,1:3);

for i = 1:length(tout_state)
    PN = R3(euler_PN(i, 3))*R2(euler_PN(i, 2))*R1(euler_PN(i, 1));
    r_NoseN_N(i,:) = (r_CN_N(i,:)' + PN' * r_NoseC_P)';
end

figure()
plot(r_NoseN_N(:,1), r_NoseN_N(:,2), 'LineWidth',2)
hold on
scatter(r_NoseN_N(1,1), r_NoseN_N(1,2), 'filled', 'black')
scatter(r_NoseN_N(end,1), r_NoseN_N(end,2), 'filled', 'blue')
legend("Rocket Nose", "Starting Point", "Final Point")
xlabel('$$\hat{n}_{1}$$ [m]', 'Interpreter', 'Latex', 'FontSize',18)
ylabel('$$\hat{n}_{2}$$ [m]', 'Interpreter', 'Latex', 'FontSize',18)
title("Projection of rocket's nose in inertial XY plane")
grid on
```



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