ASEN 6060
ADVANCED ASTRODYNAMICS
Designing and Correcting Transfers

Objectives:

- Briefly introduce using hyperbolic invariant manifolds in transfer design
- Overview of multiple shooting approach to numerically correcting transfers in CR3BP

Useful reading: Koon, Lo, Marsden, Ross, 2006, “Dynamical Systems, the Three-Body Problem and Space Mission Design”

Transfer Design

- Hyperbolic invariant manifolds are valuable in studying transport within multi-body systems
  - Govern natural motions within the system
  - Can be useful for constructing initial guesses for controlled trajectories (in some scenarios, the corrected trajectories that exist near the initial guess may require low propulsive requirements)

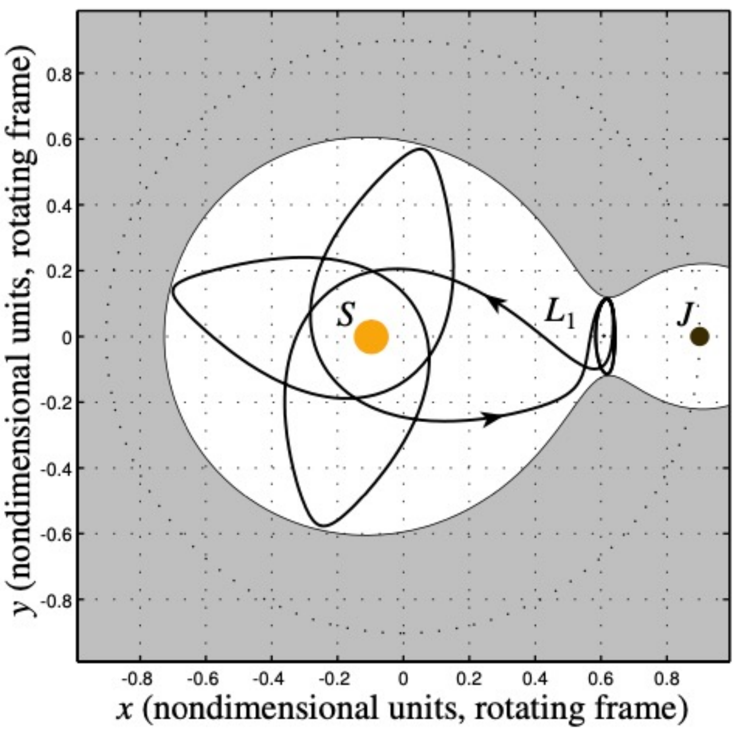
- Note: Using hyperbolic invariant manifolds for initial guess construction is not the only approach. For various propulsion systems and itineraries, other strategies may be more useful; this is an area of extensive existing and ongoing advancement.

Natural Transfers

Trajectories that lie along stable and unstable manifolds associated with either the same or different periodic orbits may intersect

Natural Transfers

Homoclinic connection



Heteroclinic connection

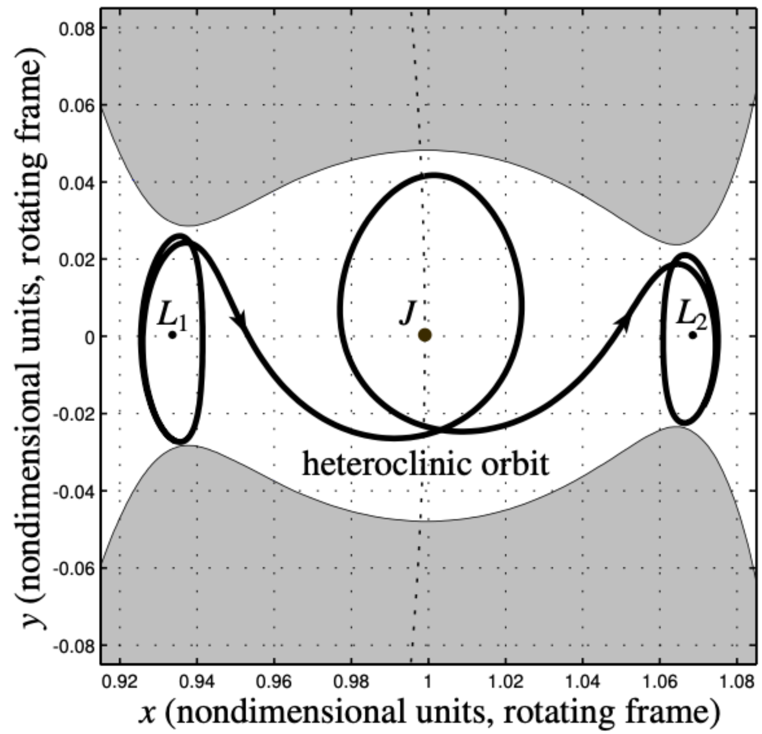
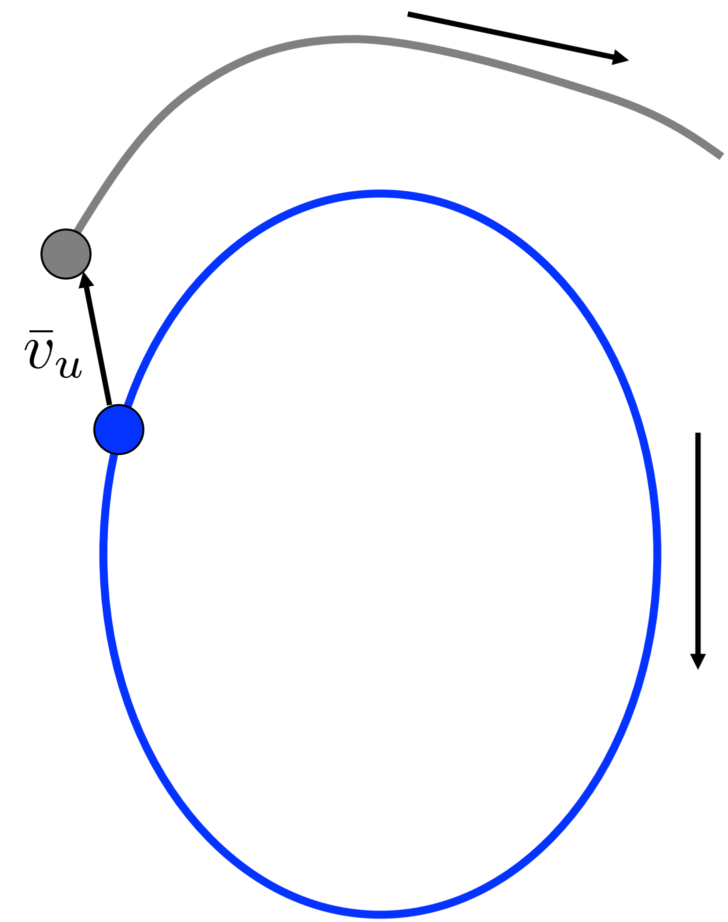


Image credit: Koon, Lo, Marsden, Ross, 2006, "Dynamical Systems, the Three-Body Problem and Space Mission Design"

Transfer Design via Stable/Unstable Manifolds

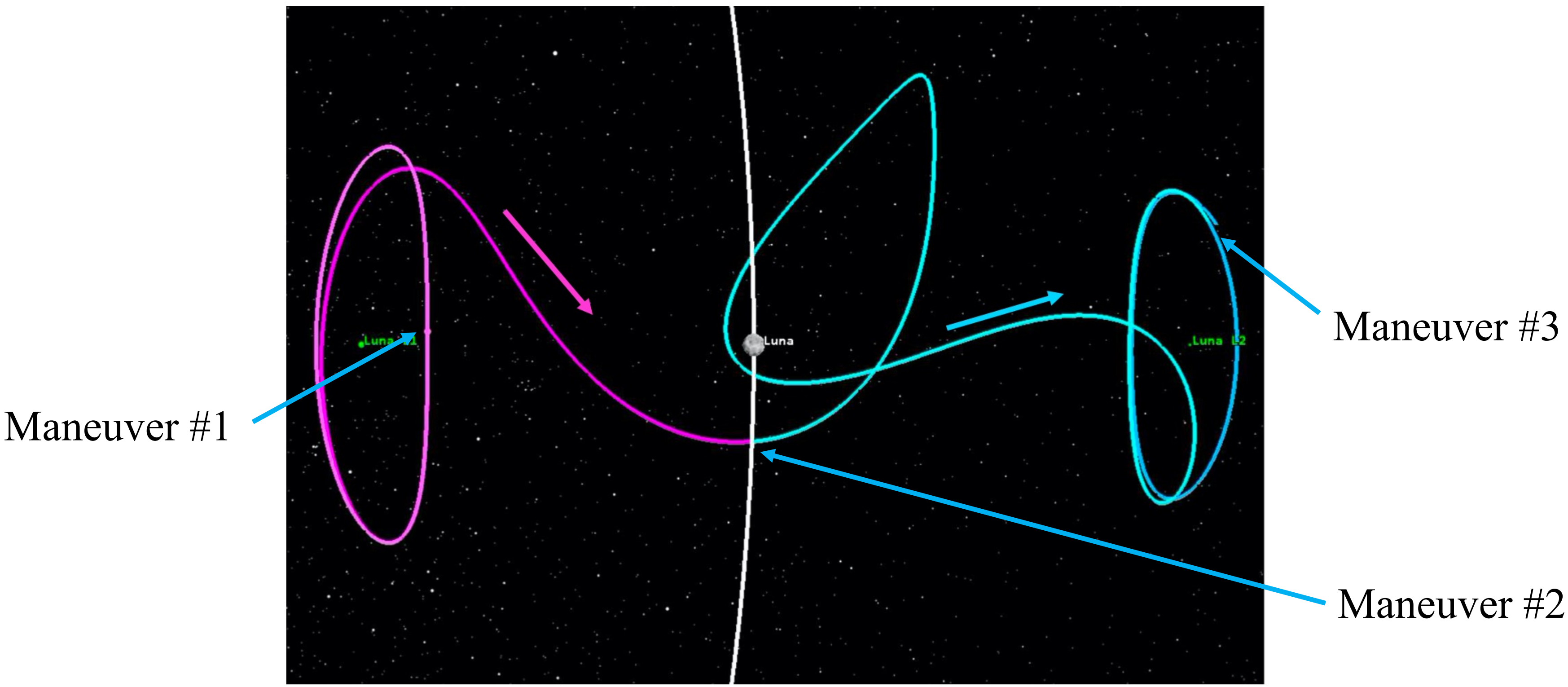
Recall that the hyperbolic invariant manifolds approach/depart the periodic orbit as $t \to \pm\infty$



Note: Conceptual diagram! Your step along the associated eigenvector should not be this big!

Transfer Design via Stable/Unstable Manifolds

Can use the hyperbolic invariant manifolds to design initial guess for transfers between periodic orbits with maneuvers



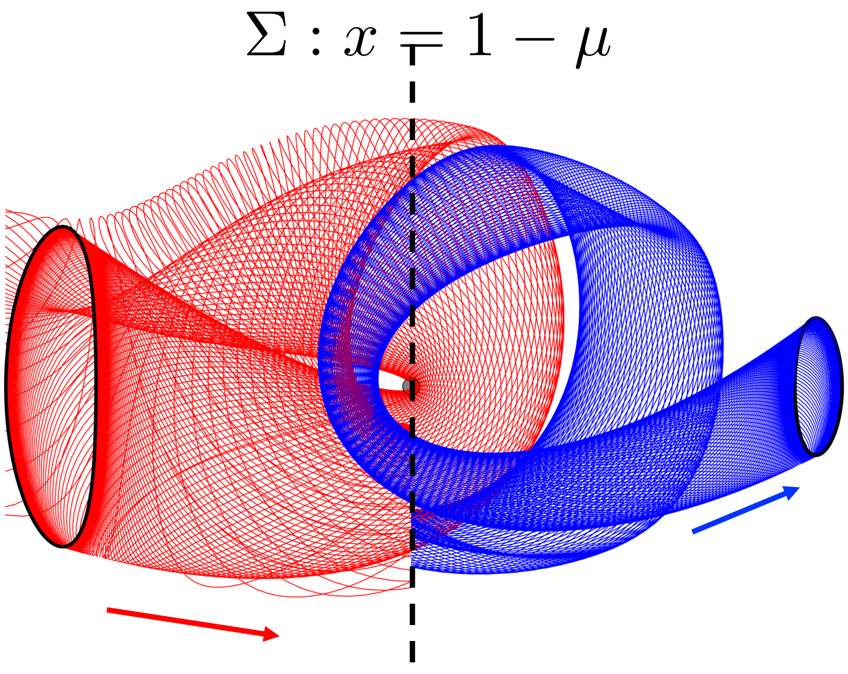
(a) Transfer from L_1 Lyapunov to L_2 Lyapunov orbit

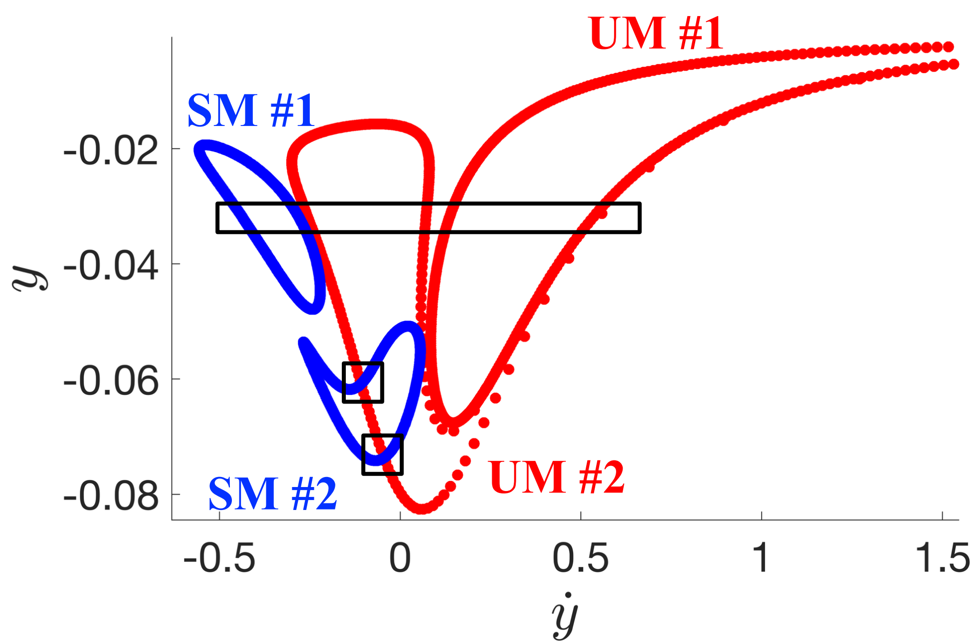
Image credit: Short, Haapala, Bosanac, 2020, “Technical Implementation of the Circular Restricted Three-Body Model in STK Astrogator”, AAS/AIAA Astrodynamics Specialist Conference.

Transfer Design via Stable/Unstable Manifolds

Use a Poincaré map representation to analyze the hyperbolic invariant manifolds to construct an initial guess

E.g.: unstable manifold of L_1 Lyapunov orbit (red), stable manifold of L_2 Lyapunov orbit (blue) at same Jacobi constant

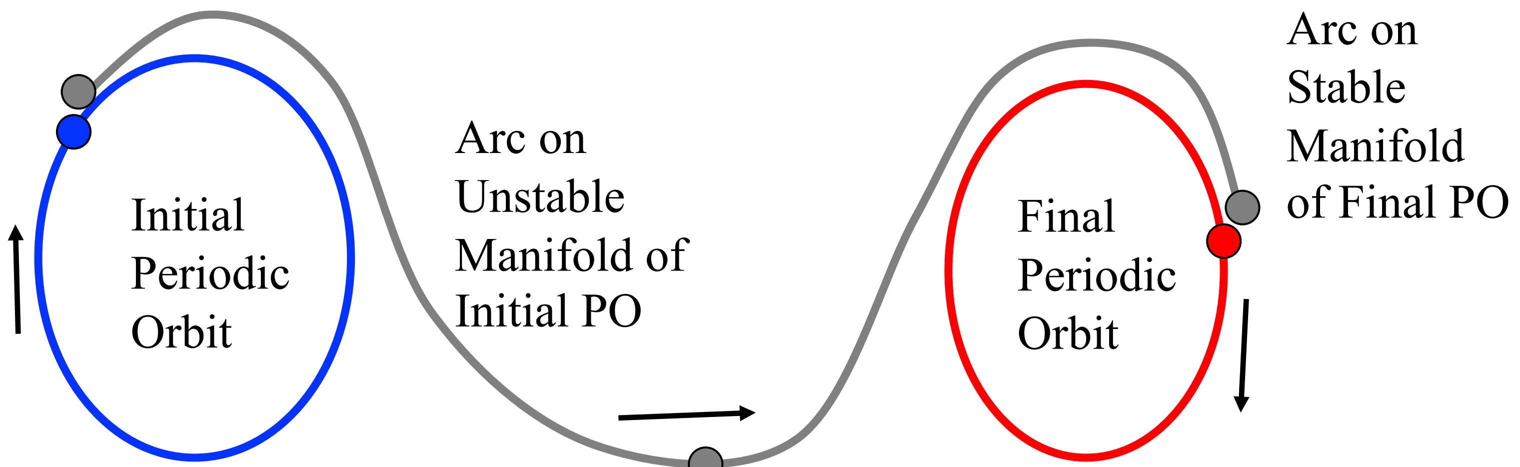




Use initial guess in corrections scheme to recover continuous transfer

Transfer Design Considerations

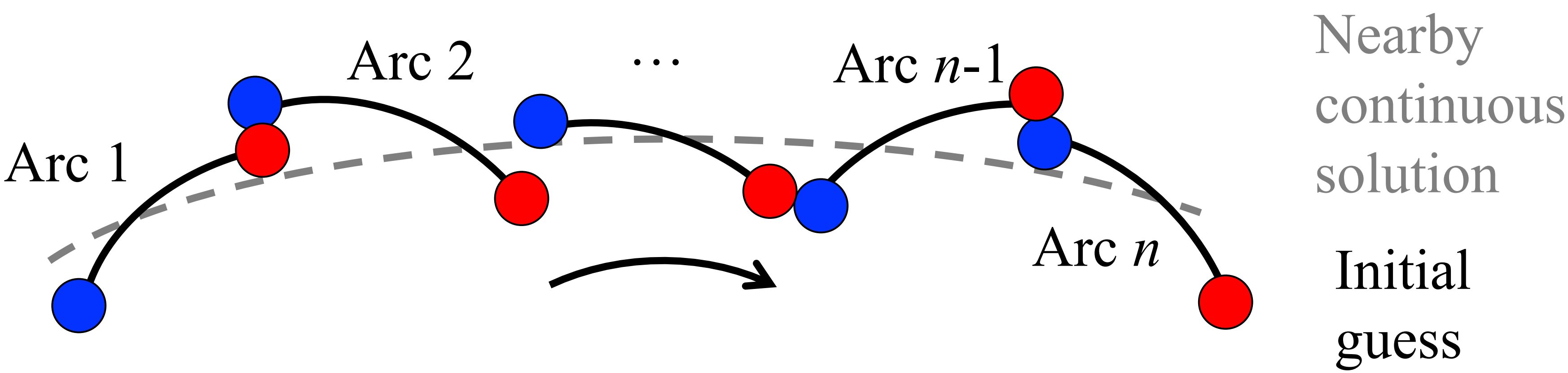
Consider transfer between two periodic orbits



Note: Conceptual diagram!

Corrections via Multiple-Shooting

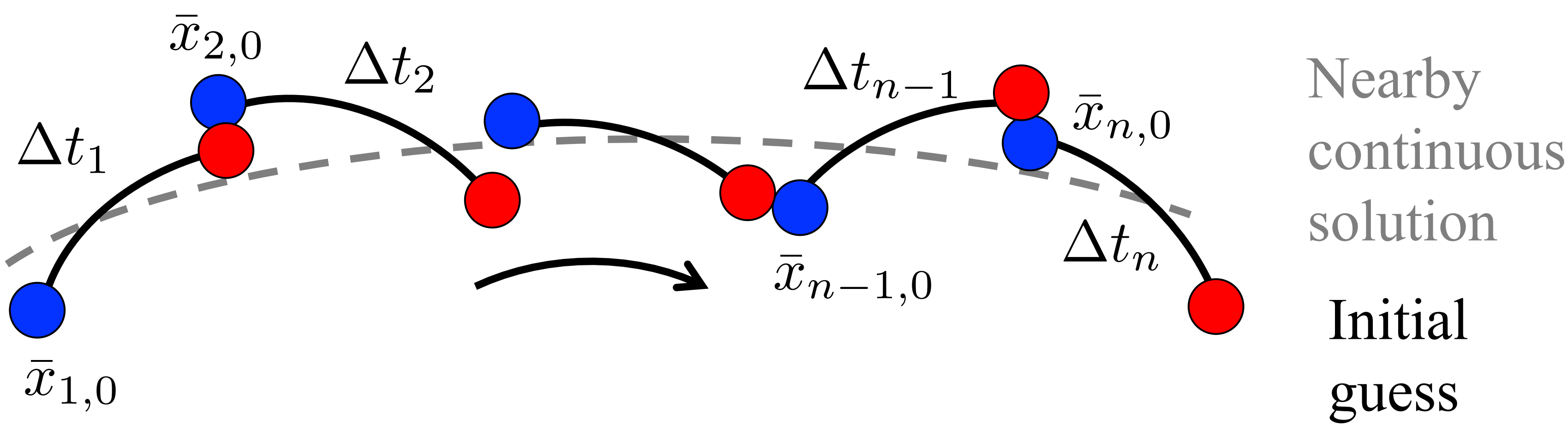
Discretize an arc into multiple segments and simultaneously adjust description of all segments to recover a continuous trajectory (that may also satisfy additional constraints). Distributing the corrections problem across a trajectory is more robust than single-shooting



Note: many formulations based on applications of interest. In the next few slides, focus on recovering a natural trajectory

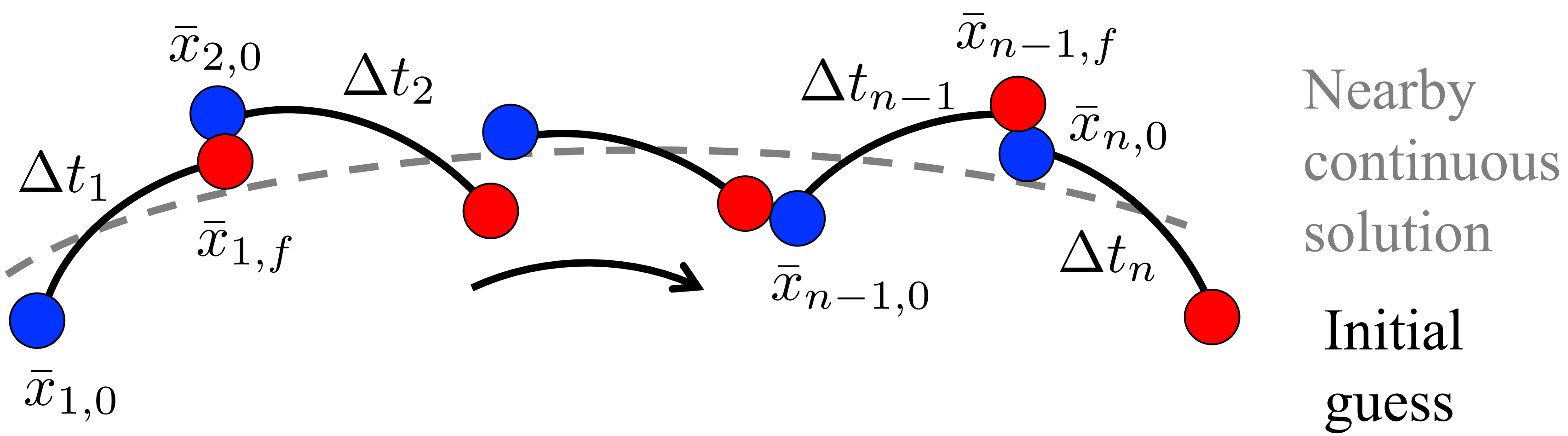
Corrections via Multiple-Shooting

Describe initial guess via state at nodes at beginning of each segment and integration time



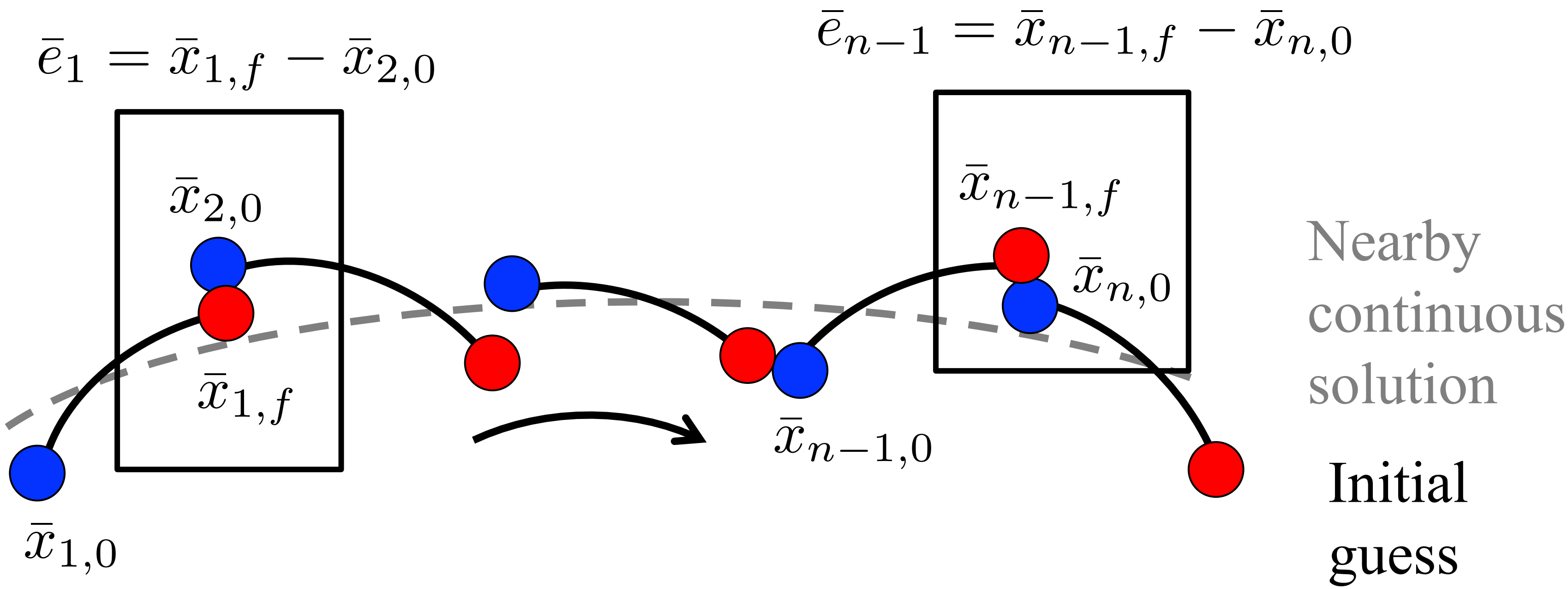
Corrections via Multiple-Shooting

Integrate along each arc from initial state for integration time



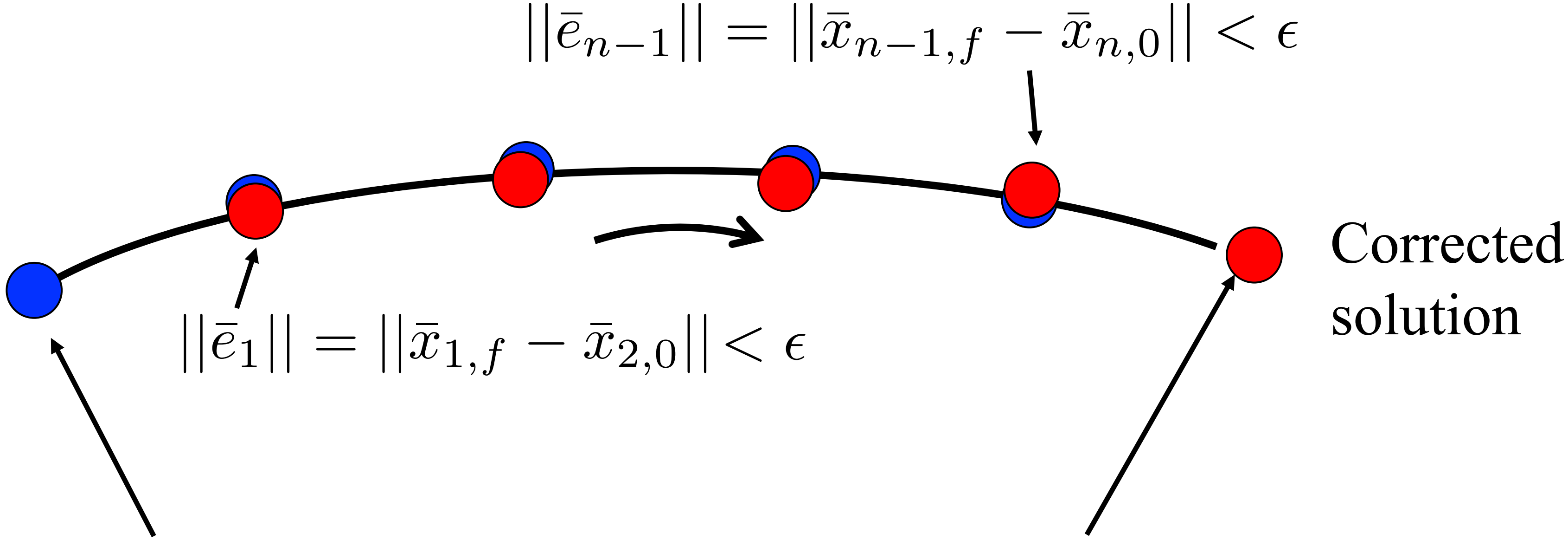
Corrections via Multiple-Shooting

Assess error between state at end of each arc and state at beginning of next arc



Corrections via Multiple-Shooting

Use a free variable and constraint vector formulation to simultaneously adjust nodes and integration times for all arcs to drive errors to zero (within suitable tolerance)



Could also add constraint for initial and final nodes to equal desired states!

Free Variable Vector

Free variable vector describes trajectory, discretized into n segments, by encoding state at each of the n nodes and integration times

(Remember: there are multiple formulations of useful free variable and constraint vectors that depend on problem of interest!)

Constraint Vector

In this example, constraint variable vector encodes state continuity and 1 additional constraint on state at end of trajectory

(Remember: there are multiple formulations of useful free variable and constraint vectors that depend on problem of interest!)

Iterative Updates via Newton's Method

Define an initial guess for $\bar{V}$ (Not always a straightforward step!)

Adjust free variable vector $\bar{V}$ using update equation derived via Newton's method. Recall that when there are more free variables than constraints:

$$\bar{V}_{i+1} = \bar{V}_i - D\bar{F}(\bar{V}_i)^{\mathrm{T}}[D\bar{F}(\bar{V}_i) \cdot D\bar{F}(\bar{V}_i)^T]^{-1}\bar{F}(\bar{V}_i)$$

Continue to update $\bar{V}$ until $||\bar{F}(\bar{V})|| < tol$

DF Matrix

Form $D\bar{F}(\bar{V})$ matrix using derivative of these constraints with respect to free variables

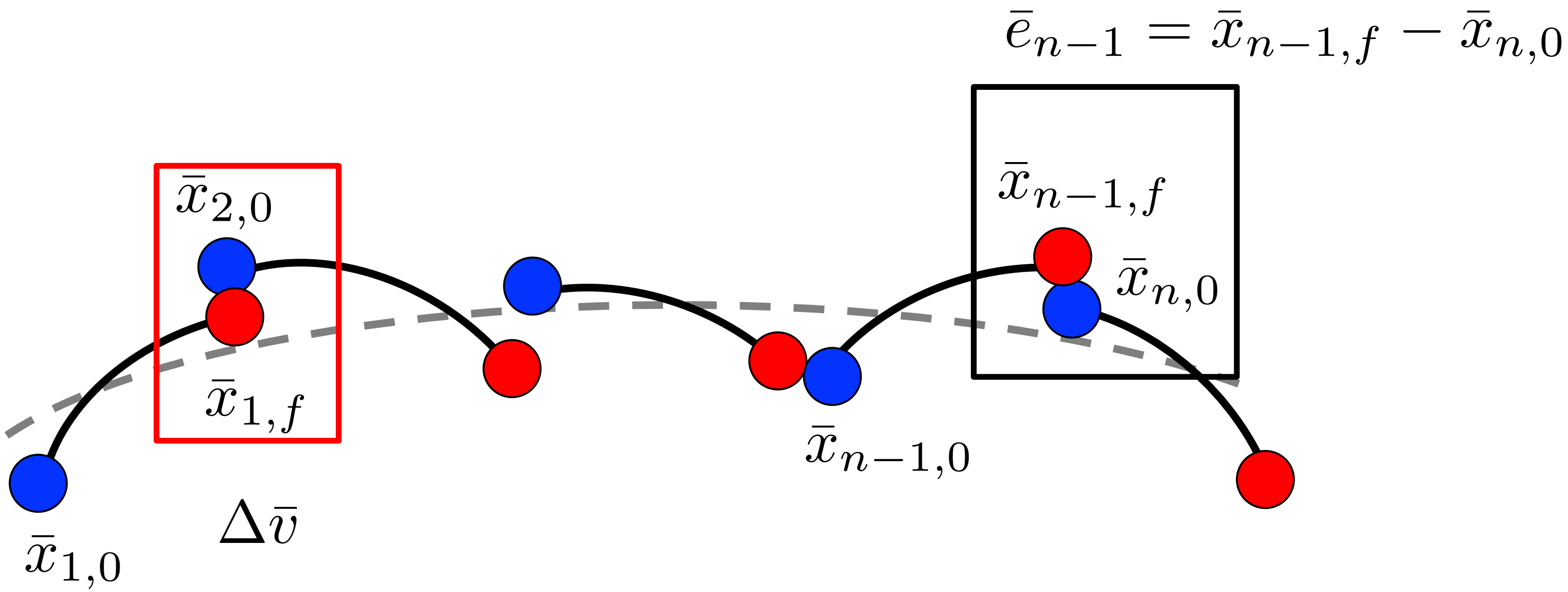
$$D\bar{F}(\bar{V}) = \frac{\partial \bar{F}}{\partial \bar{V}}$$

DF Matrix

$$D\bar{F}(\bar{V}) = \frac{\partial \bar{F}}{\partial \bar{V}}$$

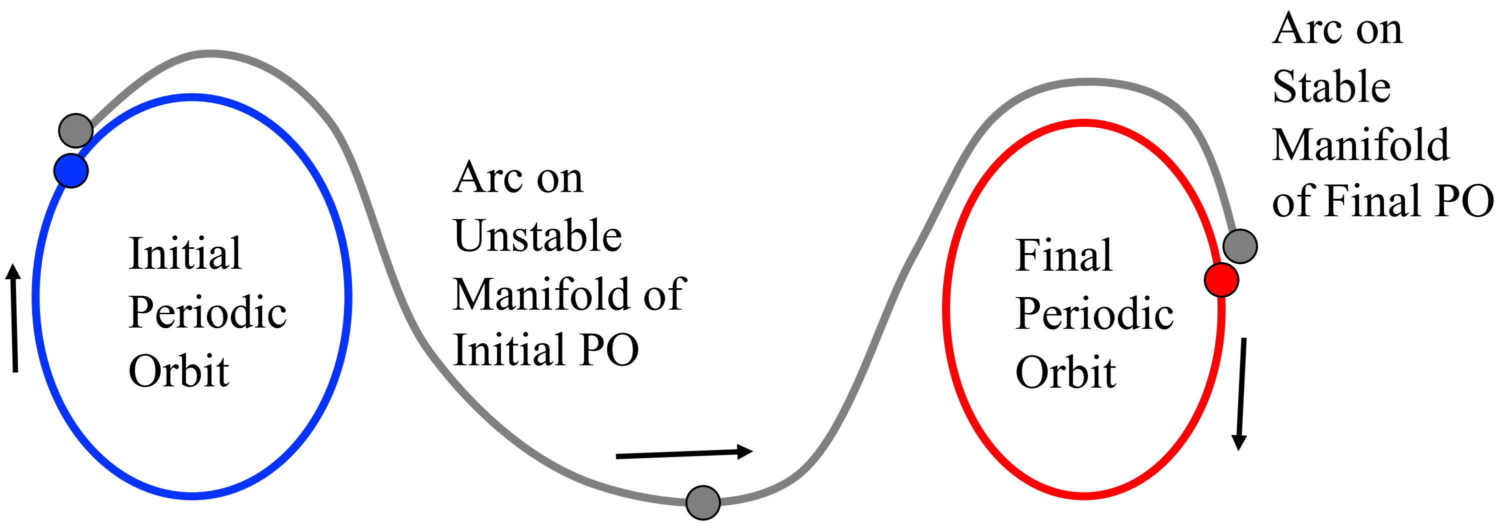
Incorporating Maneuvers

Recall: maneuvers correspond to an instantaneous change in velocity, but not position.



Transfer Design Considerations

- How to formulate corrections problem via multiple-shooting?



Note: Conceptual diagram, not to scale!