ASEN 6060
ADVANCED ASTRODYNAMICS
Week 12 Discussion

Objectives:

- Present selected examples of how to visually use Poincaré maps for trajectory analysis and design problems
- Strengthen understanding of visual representation of Poincaré maps

Example 1: Identify Overlapping Manifolds

Identify states that lie within two hyperbolic invariant manifolds

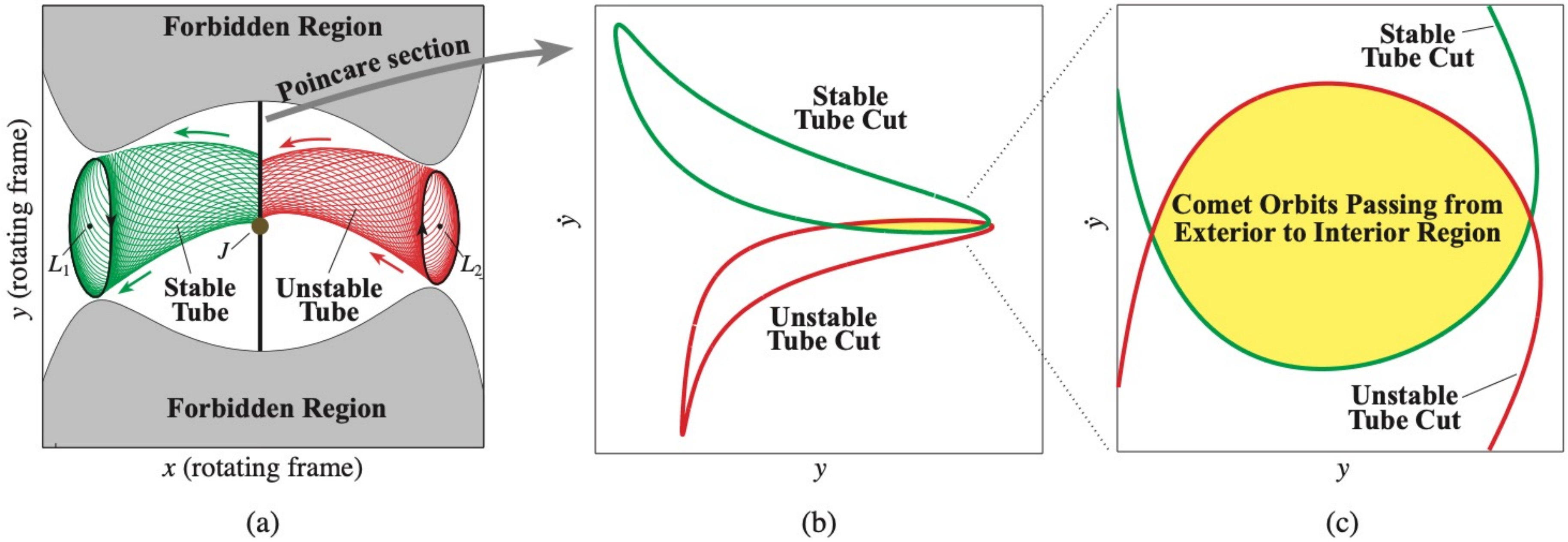


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D. (2001). Resonance and Capture of Jupiter Comets. In: Dynamics of Natural and Artificial Celestial Bodies. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-1327-6_3

Example 2: Periapse Maps

Use to connect characteristics of solution space to known fundamental solutions, e.g., in the Sun-Saturn CR3BP

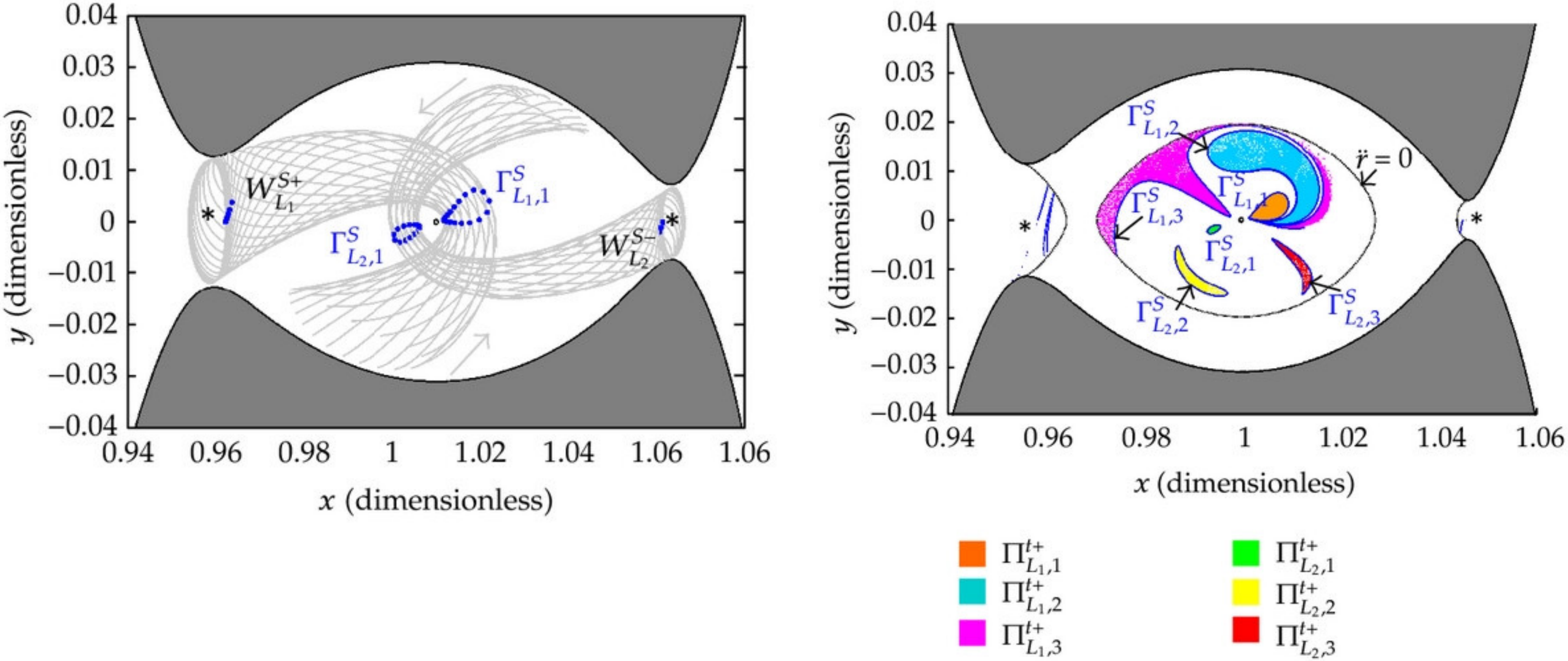


Image credit: Howell, K.C., Davis, D.C., Haapala, A.F., "Application of Periapse Maps for the Design of Trajectories Near the Smaller Primary in Multi-Body Regimes" Mathematical Methods Applied to the Celestial Mechanics of Artificial Satellites, 2011

Example 3: Collinear Eq. Pt. Neighborhood

Poincaré map representation of intersections of bounded motion near L_1 with a surface of section defined as $\Sigma : z = 0$

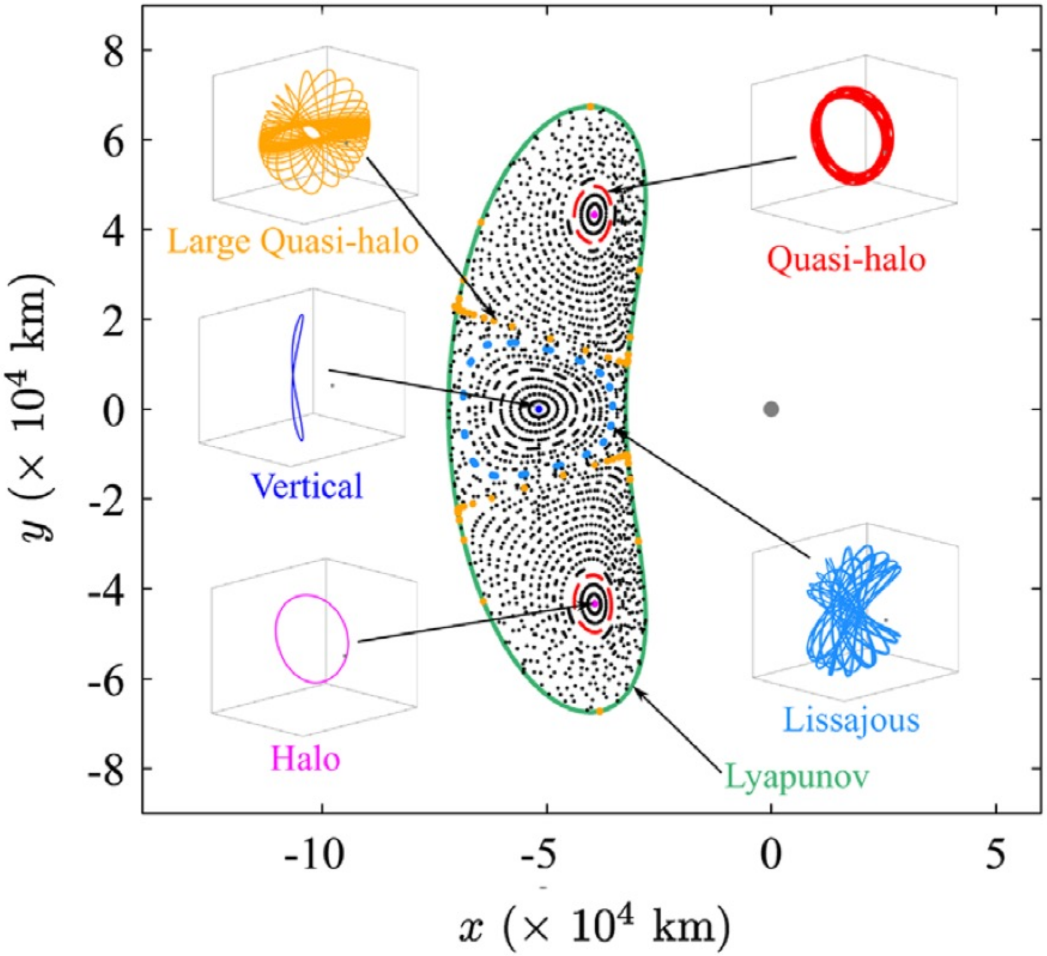


Image credit: Folta, D.C., et al. "Earth–Moon Libration Point Orbit Stationkeeping: Theory, Modeling, And Operations," Acta Astronautica, Vol. 94, No. 1, 2014, pp. 421-433

Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., "patched CR3BP"

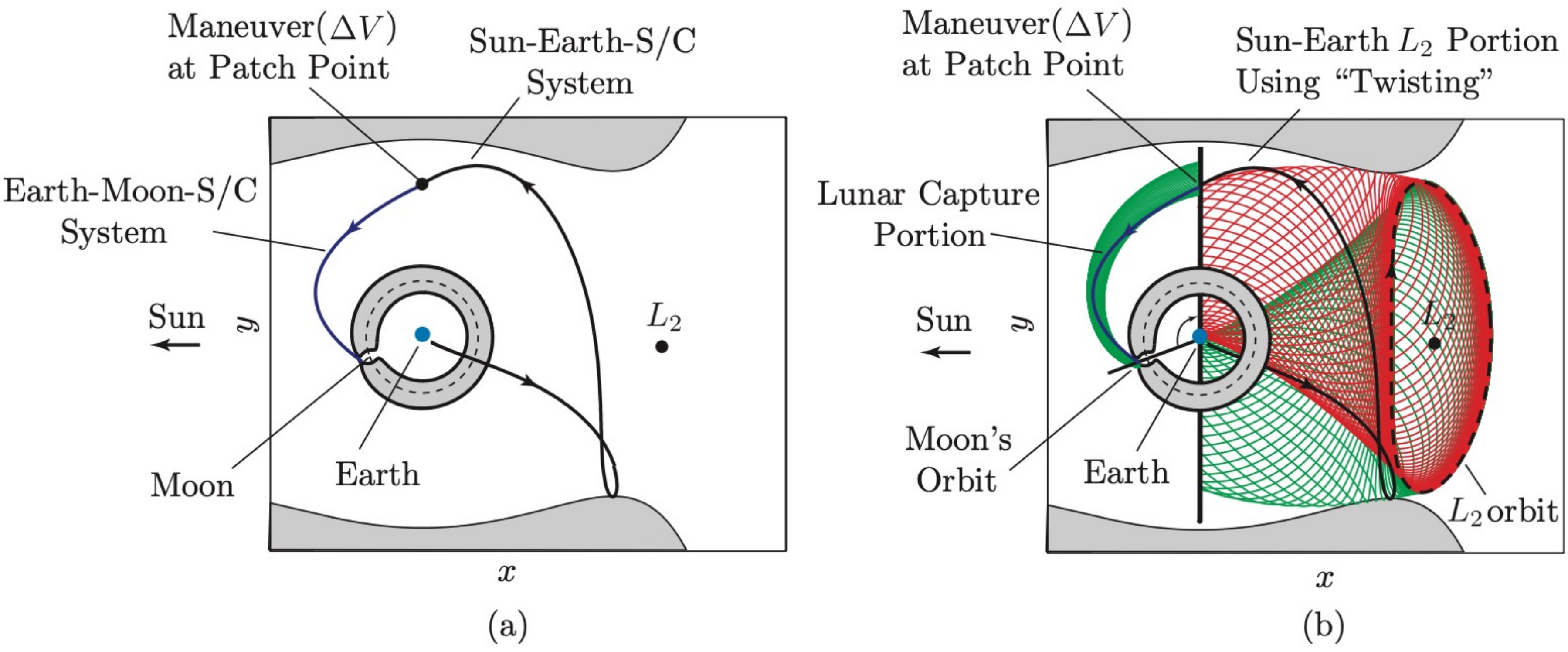
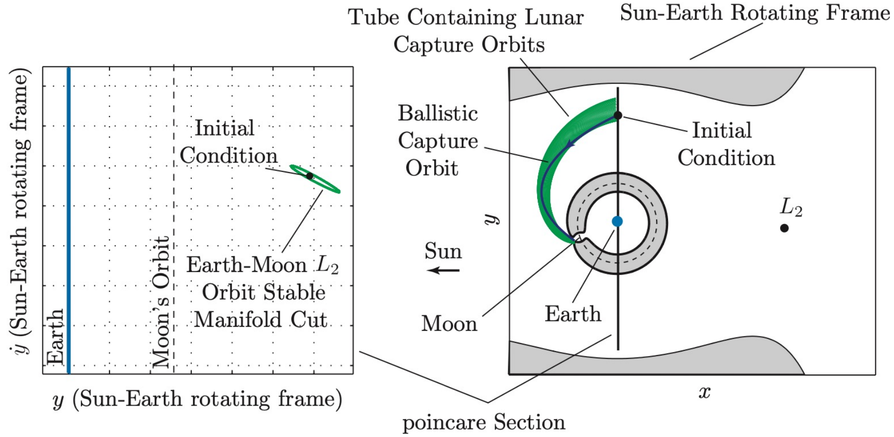


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, "Dynamical Systems, the Three-Body Problem and Space Mission Design"

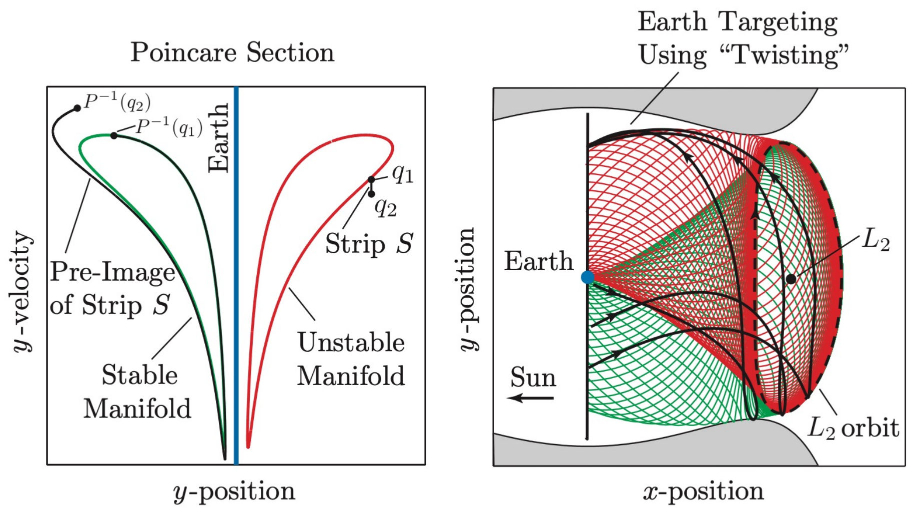
Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., "patched CR3BP"

Earth-Moon CR3BP:



Sun-Earth CR3BP:



(a) (b)

Image credits: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, "Dynamical Systems, the Three-Body Problem and Space Mission Design"

Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., "patched CR3BP"

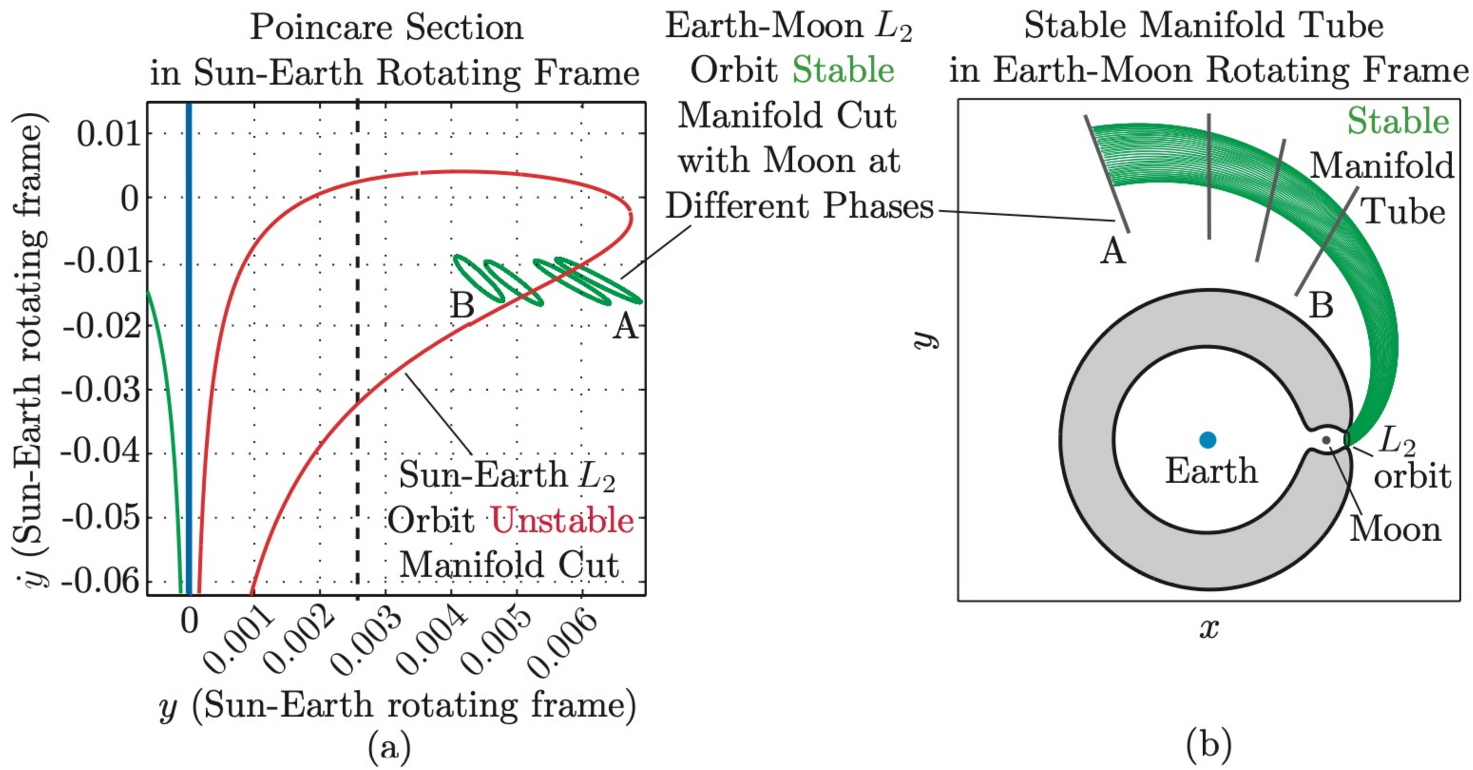


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, "Dynamical Systems, the Three-Body Problem and Space Mission Design"

Example 4: Patched CR3BP

Identifying overlapping fundamental solutions between two CR3BP, i.e., "patched CR3BP"

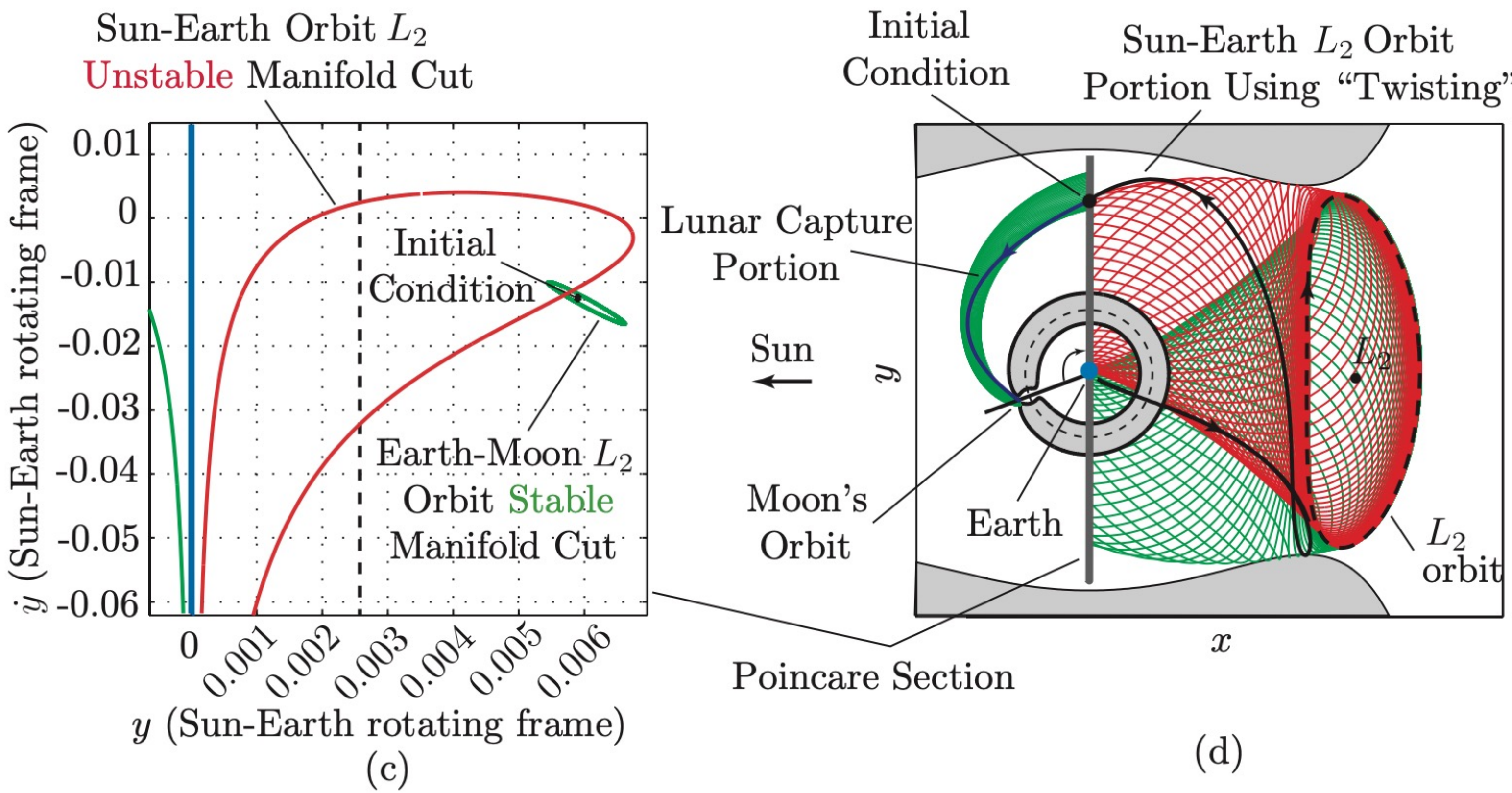
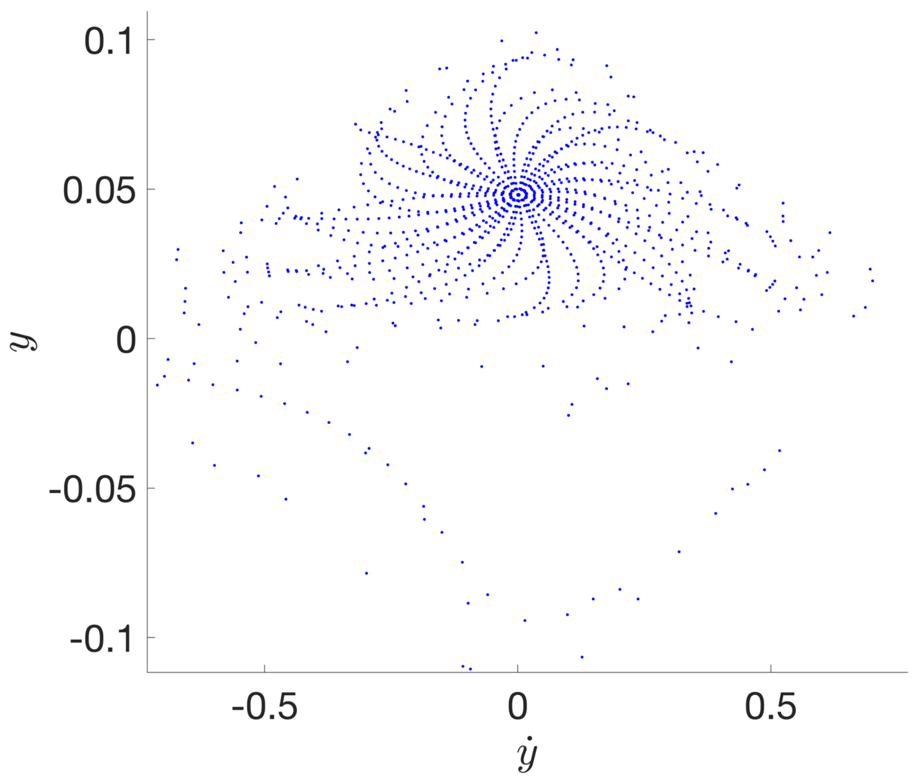


Image credit: Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D., 2011, "Dynamical Systems, the Three-Body Problem and Space Mission Design"

Question 1

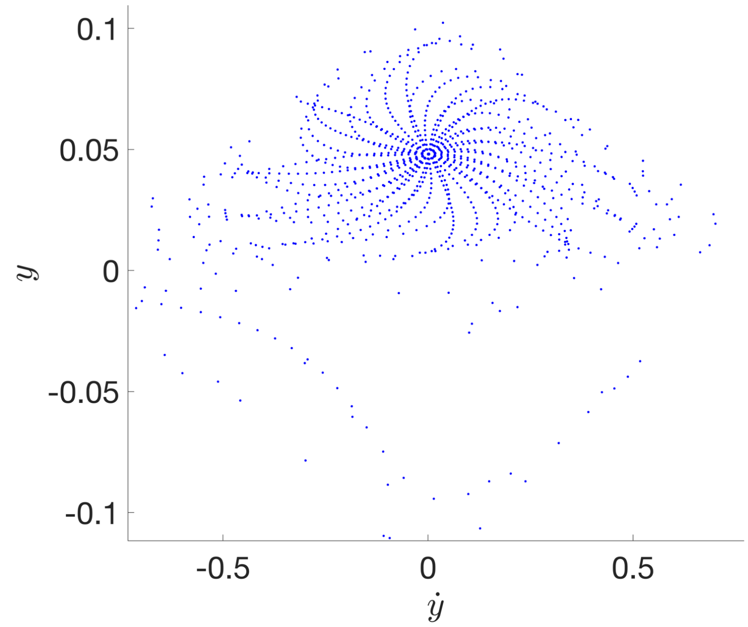
Question 1: Your colleague has created the following Poincaré map and has asked you if it makes sense. What questions would you ask your colleague to help you respond? What are your initial thoughts in response to their question?



Question 1

Question 1: Group Brainstorming:

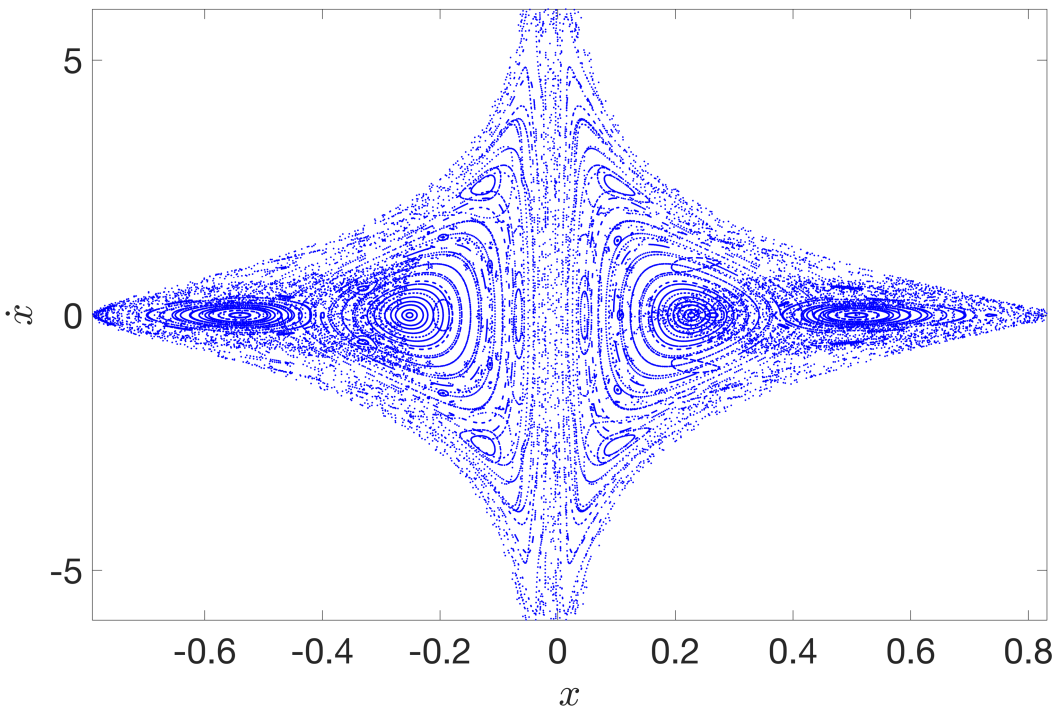
- How many crossings are there? What happens when we add more?
- How many distinct trajectories are here?
- What is the surface of section?
- How are ICs defined?
    - What is the direction of the velocity vector
    - How many?
    - How calculated?
    - What ranges of state components are they sampled from?
    - Do all trajectories exist at one CJ?
- What are the stopping conditions?



Answer: this is an accurate map, just need to add more crossings to see curves fill out

Question 2

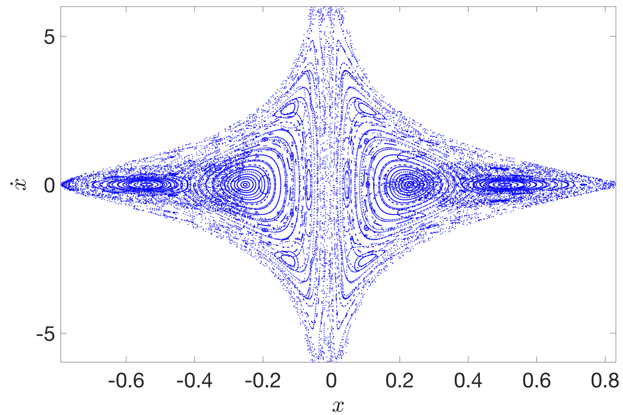
Question 2: Your colleague has created the following Poincaré map and has asked you if it makes sense. What questions would you ask your colleague to help you respond? What are your initial thoughts in response to their question?



Question 2

Question 2: Group Brainstorming:

- New parameters being displayed: x, xdot
- Not a x=1-mu surface of section anymore
- In previous examples, we saw patterns on one side of a primary but not both
    - What are the ICs used here?
- Is this a 2-sided or 1-sided map?
- What is the CJ of the trajectories?



ANSWER: This is an accurate map, it is just 2-sided. The nonuniqueness of map crossings makes it difficult to analyze

Question 3

Question 3: When generating a Poincaré map representation of trajectories, what can you do to check if the results are accurate?

- Checking the value of the Jacobi constant throughout the trajectory and the ICs – do they match our desired value?
- Only if you know a priori that QPOs exist in the trajectory set, do you see their closed curves?
- Do the values of coordinates lie within reasonable bounds?
  - Do y values lie within ZVCs?
  - What is the bound on the speed that also bounds the velocity values for a specific CJ value?

Question 4

Question 4: Consider the following three nearby initial conditions:

$$\bar{x}_{0,1} = [0.3693,0,0,0,1.4772,0]$$

$$\bar{x}_{0,2} = [0.3670,0,0,0,1.4865,0]$$

$$\bar{x}_{0,3} = [0.3640,0,0,0,1.4994,0]$$

Our goal today is to describe the type of trajectories associated with these initial conditions in as much detail as possible.

a) Devise a plan for how you could use the tools and concepts we have covered in class to perform this assessment.

b) Implement your plan and describe the types of these 3 trajectories

Question 4

Question 4a): Devise a plan for how you could use the tools and concepts we have covered in class to perform this assessment.

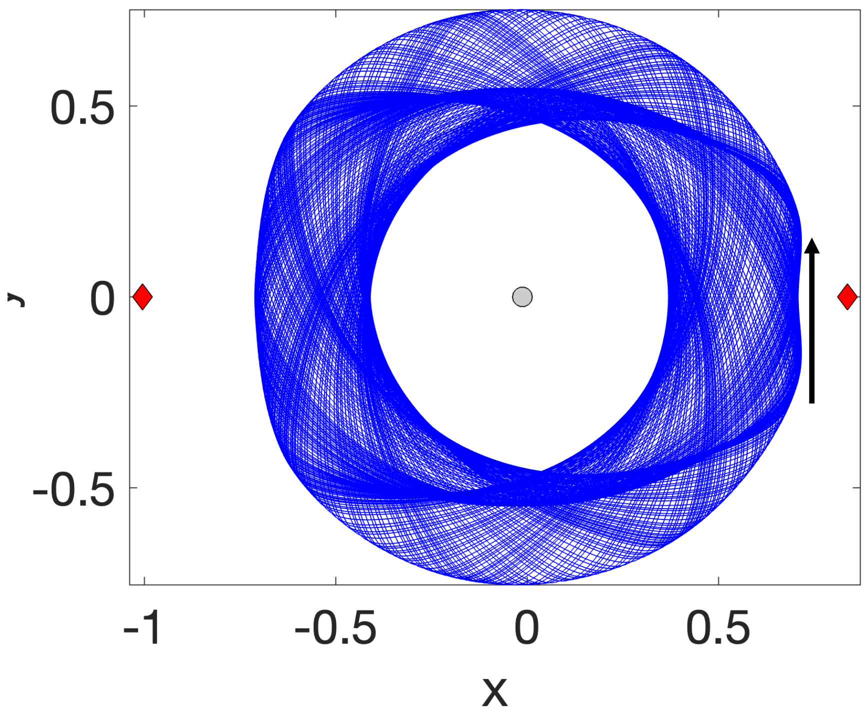
Group Brainstorming:

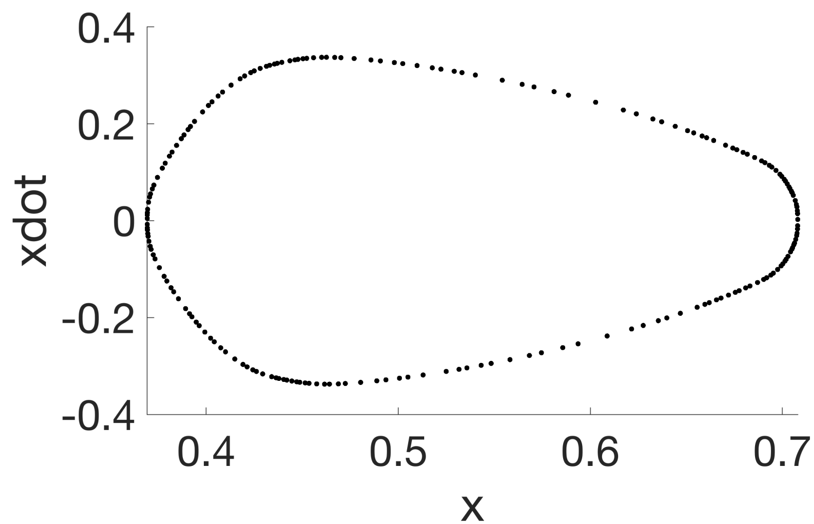
- Propagate the trajectory (for how long?) and plot it?
  - Iteratively select this time frame
- Create a Poincaré map for each trajectory and analyze the structure of the crossings

Question 4

Question 4: Describe the trajectories associated with the three provided initial conditions.

IC 1: 200 returns to map with y=0 surface of section



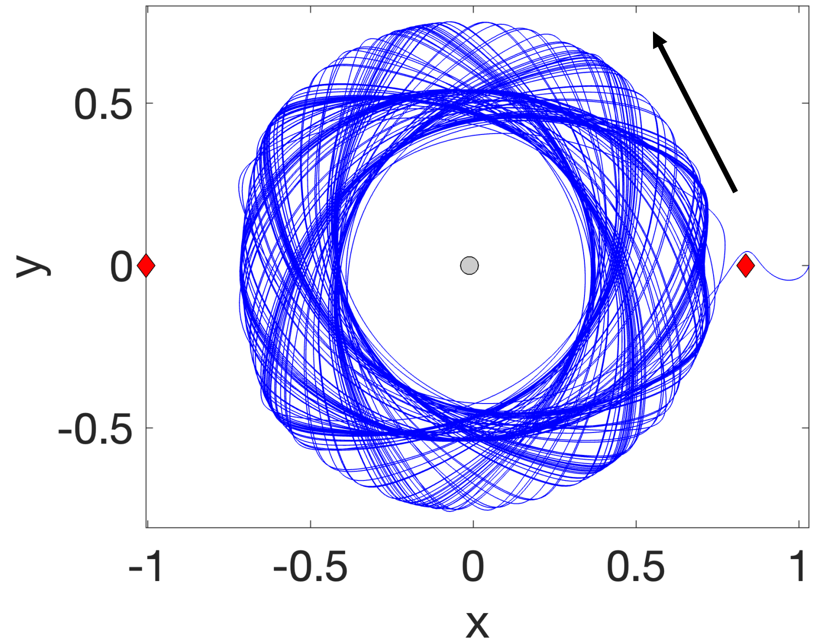


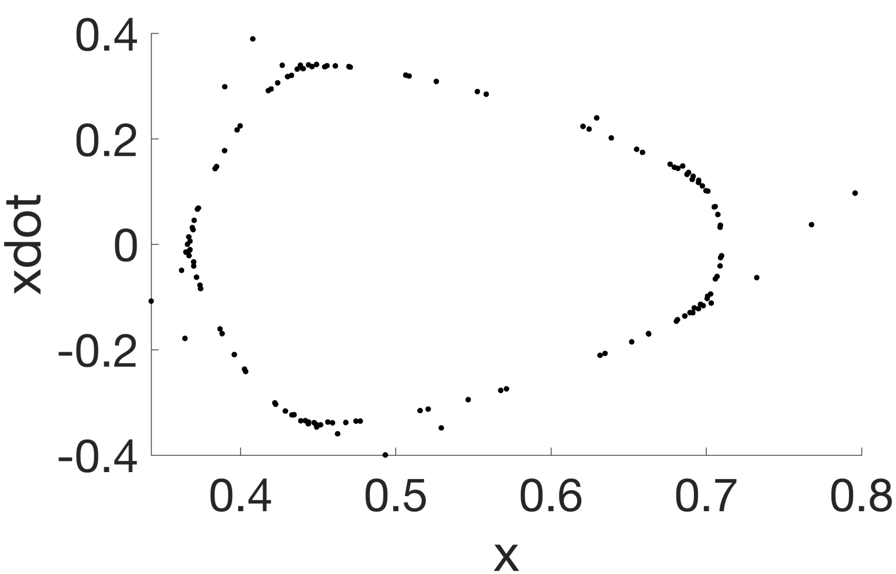
Path appears to be a quasi-periodic orbit, filling out a closed curve over this time interval

Question 4

Question 4: Describe the trajectories associated with the three provided initial conditions.

IC 2: 132 returns to map with y=0 surface of section before leaving through L1 gateway



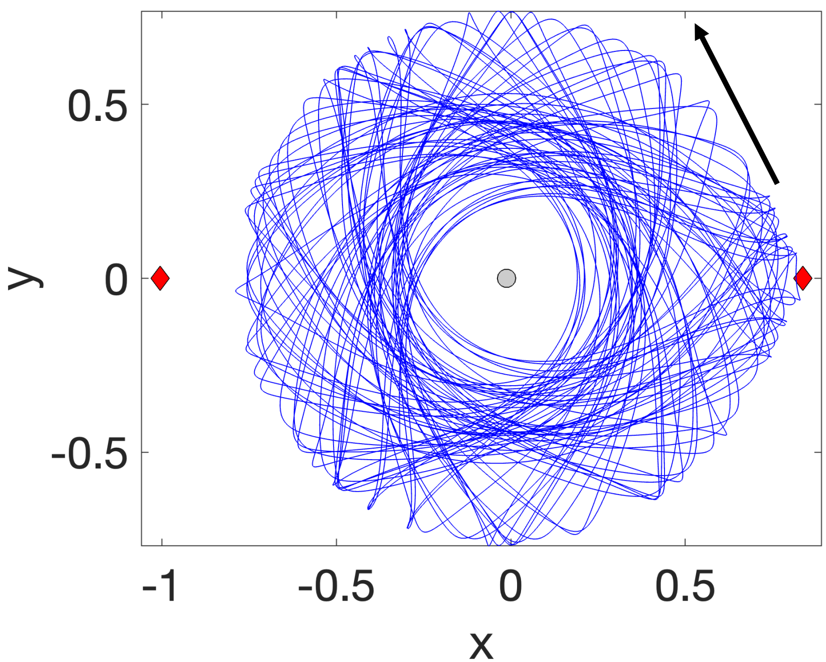


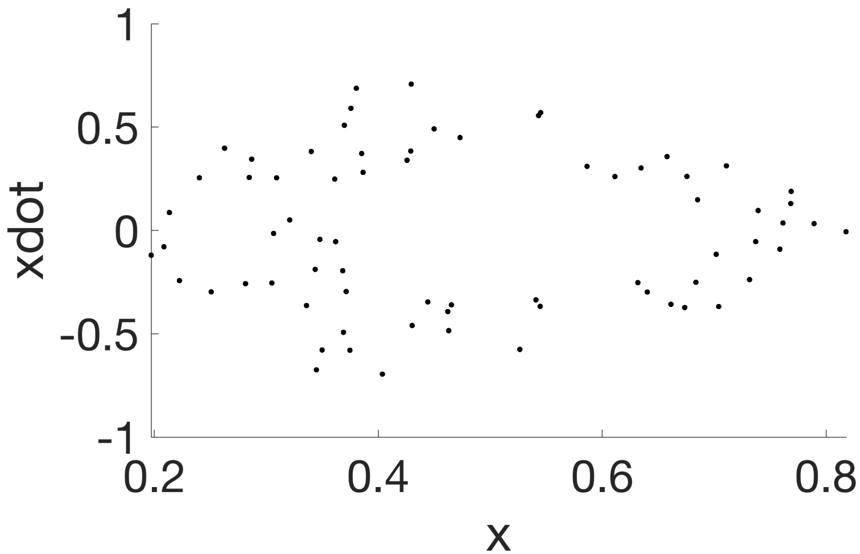
Path appears to lie nearby a quasi-periodic orbit for a little while but then departs its vicinity – need to integrate for long enough to see this

Question 4

Question 4: Describe the trajectories associated with the three provided initial conditions.

IC 3: 71 returns to map with y=0 surface of section before leaving through L1 gateway





Path appears to be a chaotic trajectory

Question 4