

$$\text{Ref } \ddot{\theta}_h + k_h \dot{\theta}_h = 0 \rightarrow u_r$$

$$\dot{h} = \tau_{\text{total}}$$

$$L\ddot{\theta} - g \sin \theta = \tau$$

$$\ddot{\theta} = \left( \frac{g}{L} \sin \theta + \frac{\tau}{L} \right)$$

$$\Rightarrow \ddot{\theta} = \omega_h^2 \sin \theta + u_r + \delta u$$

Goal  $\boxed{\theta - \theta_h \rightarrow 0} \mid \boxed{\dot{\theta} - \dot{\theta}_h \rightarrow 0}$

$$V(s\theta, s\dot{\theta}) = \frac{1}{2} s\dot{\theta}^2 + \frac{1}{2} k s\theta^2$$

$$\dot{V}(s\theta, s\dot{\theta}) = \left( s\dot{\theta} \right) (s\ddot{\theta} + k s\theta) = \boxed{-c s\dot{\theta}^2}$$

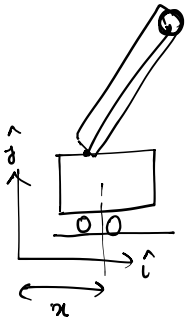
$$s\dot{\theta} (s\ddot{\theta} + k s\theta + c s\dot{\theta}) = 0$$

$$\begin{aligned} s\ddot{\theta} + k s\theta + c s\dot{\theta} &= 0 \\ s\ddot{\theta} - \ddot{\theta}_h + k s\theta + c s\dot{\theta} &= 0 \\ s\ddot{\theta} - \ddot{\theta}_h + (-k_r \theta_r) + k s\theta &= 0 \\ s\ddot{\theta} + \omega_h^2 \sin \theta + u_r &= (-k_r \theta_r) - \omega_h^2 \sin \theta - u_r \\ \delta u &= -k_r \theta_r - c s\dot{\theta} - k s\theta \end{aligned}$$

$$\dot{V}(s\theta, s\dot{\theta}) = \underline{\underline{-c s\dot{\theta}^2}} \rightarrow \boxed{\leq 0} \rightarrow s\dot{\theta} = 0 - \Omega$$

$$\ddot{V}(s\dot{\theta}=0, s\theta) = -2 s\dot{\theta} s\ddot{\theta} = \cancel{0}$$

$$\ddot{V}(s\dot{\theta}=0, s\theta) = (s\theta) \rightarrow < 0$$



$$\theta_i = k_i \dot{\theta}_i$$

$$\boxed{Q_{\max}}$$

$$\textcircled{1} \quad |k_i \dot{\theta}_i| \leq Q_{\max}$$

Controller performance  
Not optimal

$$\textcircled{2} \quad \boxed{\dot{\theta}_i = -k \text{sgn}(\dot{\theta}_i) \dot{\theta}_i}$$

Optimal

$$\textcircled{3} \quad k_i \dot{\theta}_i \quad Q < Q_{\max}$$

Suboptimal

$$Q_i = -Q_{\max} \text{sgn}(\dot{\theta}_i) \quad Q \geq Q_{\max}$$