The SYK Model and Non-Fermi Liquids

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Fermi-Liquid Theory

- Originally posited by Landau in 60s.
- Gave a pretty accurate description of a broad range of materials via the introduction of quasiparticles.
- Theory built upon interacting Fermi gases and the use of Pauli exclusion principle to posit that momentum states may be re-normalized to reflect new values for important observables such as mass, etc.
- Important systems such as liquid ^{3}He and most non-superconducting metals can be described by it.

Non-Fermi Liquids

- We find that some recent system such as $La_{2-x}Sr_xCuO_4$ [4]and $BaFe_2(As_{1-x}P_x)_2$[3] have measuremetns that deviate from Fermi liquid theory. In particular, at low temperatures and low energy - the temperature to resistivity scaling.

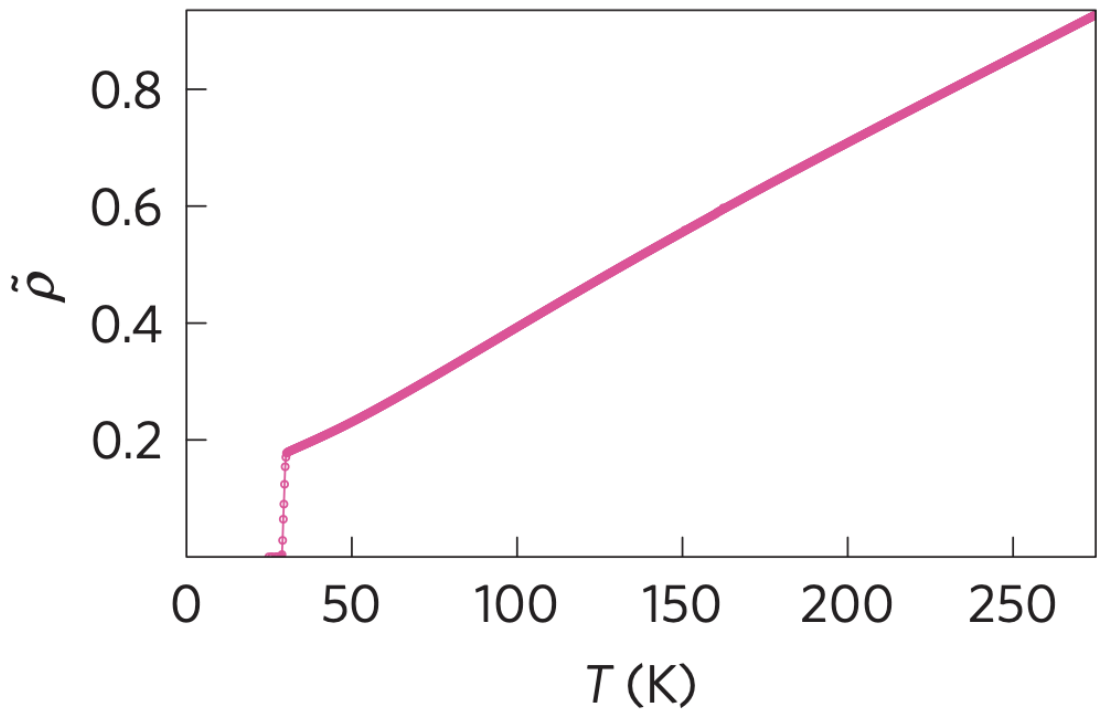

Figure: From [3]. Measurement of $BaFe_2(As_{1-x}P_x)_2$ which deviates from Fermi liquid theory.

Non-Fermi Liquids (cont.)

- Some other clues to Fermi-liquid theory breaking down are in strange metals and low energy symmetry breaking, measured in certain quantum spin liquids [5].

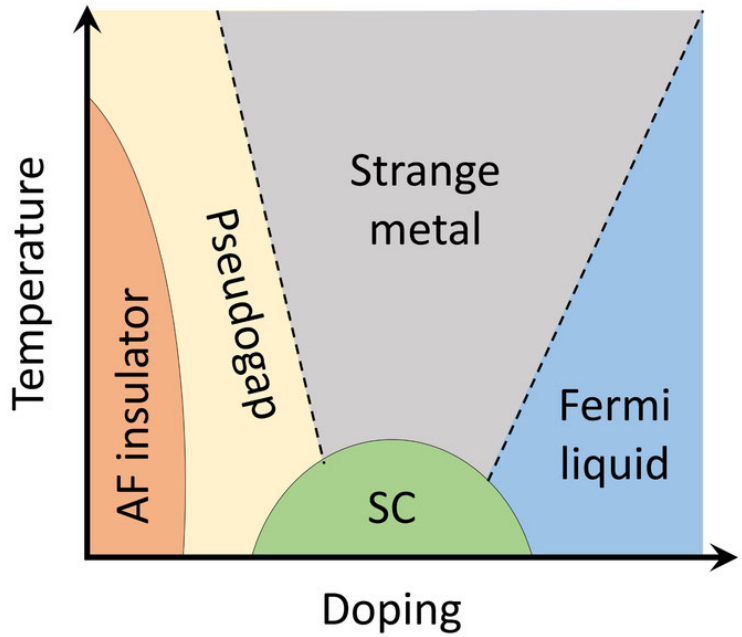

Figure: Informal strange metal diagram

Random Matrix Model

- So we have some clues to look for when considering alternative models. Let's take a look at the random matrix model first.

$$H_2 = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i \tag{1}$$

- Shows promise initially, but at closer look it just gets us back to a Fermi-liquid type explanation for exotic phenomenon. Quasiparticles are an indication of this.

RMM (cont.)

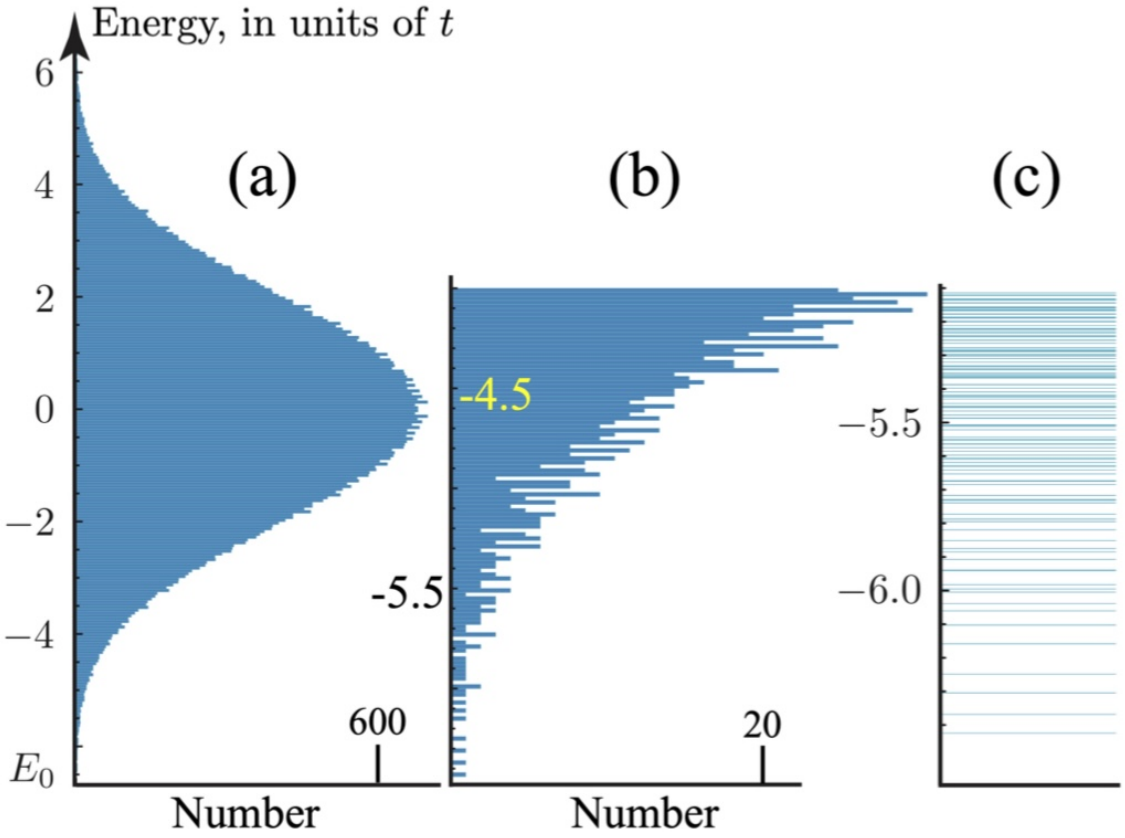

Figure: From [2], we have the many-body eigenvalues for N = 32 random matrix model. $\mathcal{N}$(E) is graphed out in (a) and (b) while (c) is the individual energy levels.

SYK Model

$$H_4 = \frac{1}{(2N)^{3/2}} \sum_{ijkl=1}^{N} U_{ijkl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_i c_i^\dagger c_i \tag{2}$$

- This model looks more promising from the spectral analysis. It also does not have quasiparticles, meaning it is not constrained by the limitations of a model that would.

SYK Model (cont.)

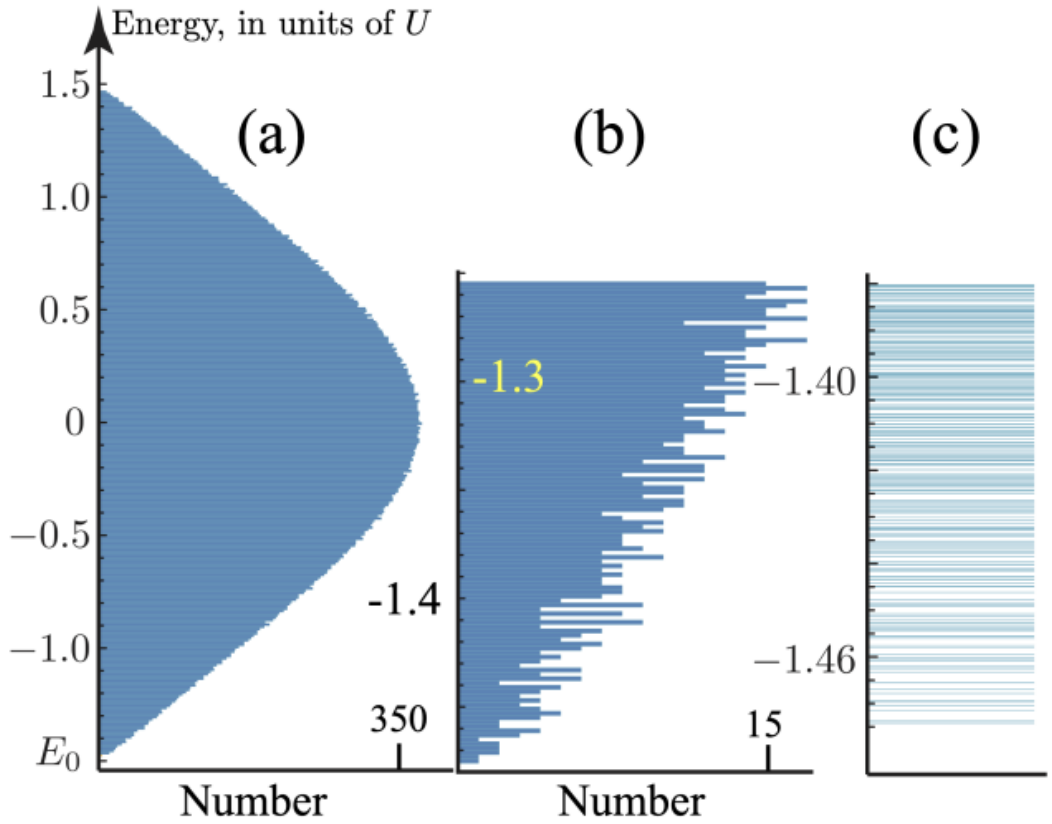

Figure: From [2]. The many-body eigenvalues of a $N = 32$ Majorana SYK Hamiltonian. $\mathcal{N}(E)$ is plotted in (a) and (b) while (c) shows the band energies.

Low Energy Limit of SYK

- Low energy limit immediately informs us model's utility via a numerical analysis.
- There are several iterations of the SYK model at low energies (e.g. SUSY versions, double-scaled version, etc.) but all of them maintain same key assumptions and same general structure of the Hamiltonian.

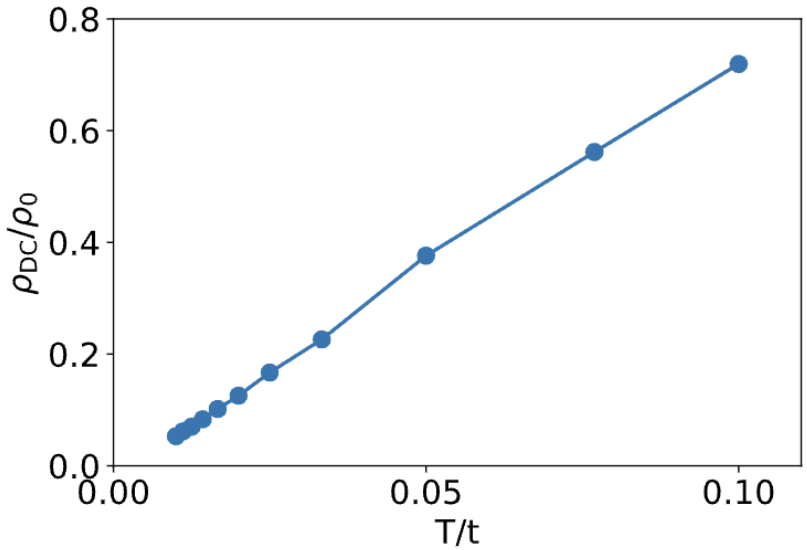

Figure: From [1]. Resistivity ρ_{DC}/ρ_0 vs. temperature T/t computed via analytic continuation of Green's function.

$N \to \infty$ Limit of SYK

- This is a more theoretical limit and lends itself to the holographic duality connection as well as the SYK model as a proxy for modeling black holes.
- However, it is also in this limit that a SYK lattice can be constructed such that it can be used as a solvable model of a strange metal.

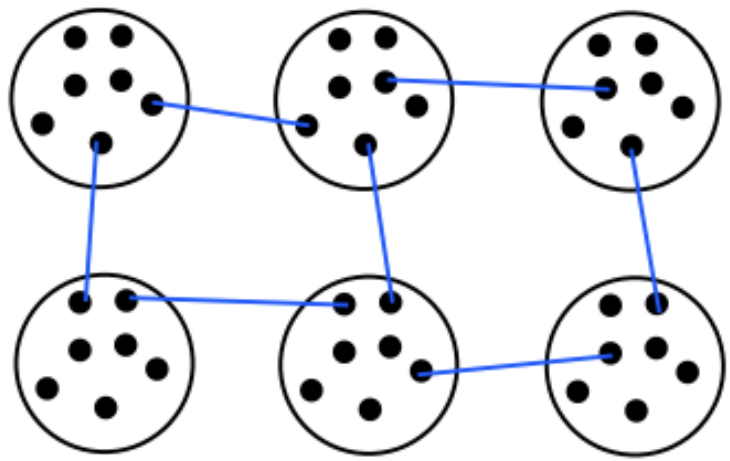

Figure: Simple representation of the way a lattice may be constructed via the SYK model. The lattice is made up of SYK quantum dots with random interactions between dots.

Conclusion

- Fermi-liquid theory's use of quasiparticles hold it back from modeling several disordered and strongly correlated systems including non-Fermi liquids and strange metals.
- The random matrix model is able to reconstruct much of Fermi-liquid theory's range of interaction description without the explicit use of quasiparticles via random Fermionic couplings.
- The SYK model can go beyond the random matrix model with the use of random interactions and couplings with a quartic Majorana fermion model. The SYK model does not have quasiparticles.
- The low-energy limit of the SYK model recovers non-Fermi behavior such as linear resistivity.
- The high energy limit links the SYK model to black holes and thermofield calculations but also allows for the SYK model to be constucted as a working example of a strange metal.

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