UNIVERSITY OF
LINCOLN
UNITED KINGDOM

College of Science
School of Engineering

Time Constrained Assessment

Module Title	**Control Systems**
Module Code	**EGR2006**
Module Coordinator	**Prof. Tim Gordon**
Duration of Assessment	**4 hours**
Date	**15/01/2024**
Release Time	**13:30 GMT**
Submission Time	**17:30 GMT**

General Instructions to Candidates.

1. You **must** submit your answers to TurnItIn on Blackboard **before** the submission time: failure to do so will be classified as misconduct in examinations. It is strongly recommended you submit at least 15 minutes prior to the deadline.
2. You **must** also send a copy of your work to: soesubmissions@lincoln.ac.uk at the same time. You must place the Module Code and your Student ID in the Subject Field of the Mail.
3. For students who choose to word process their answers, hand-written notes or diagrams **must** be photographed (preferably using Microsoft Lens which is available as part of your Office 365 package) and inserted into the Word Document as an image.
4. This assessment is an open resource format: you may use online resources, lecture and seminar notes, text books and journals. All sources must be correctly attributed or referenced.
5. All work will be subject to plagiarism and academic integrity checks. In submitting your assessment, you are certifying that this is entirely your own work, without input from either commercial or non-commercial writers or editors, or advanced technologies such as Artificial Intelligence services. If standard checks suggest otherwise, Academic Misconduct Regulations will be applied.
6. The duration of the Time Constrained Assessment will vary for those students with Personalised Academic Study Support (PASS) plan. Extensions do not apply, but Extenuating Circumstances can be applied for in the normal way.

Module Specific Instructions to Candidates

1. Answer FOUR questions
2. All questions carry equal marks
3. No further marks will be awarded for answers to a fifth question

Question 1

Given the plant transfer function

$$G(s) = \frac{s+1}{s^3 + s^2 + s}$$

(a) State the system order and type [2 marks]

(b) Find the system poles and zeros [5 marks]

(c) The plant is to be controlled using a proportional controller, as in Figure Q1c. Determine

(i) the closed-loop transfer function [4 marks]

(ii) the steady step error in case $r(t) = 1(t)$ [2 marks]

(iii) the steady step error in case $r(t) = t\ 1(t)$ [4 marks]

(iv) the settling time and overshoot for a unit step reference input, when the proportional gain is set to K=20. Sketch the likely step response and hence comment on whether this is a suitable gain for this control system. [8 marks]

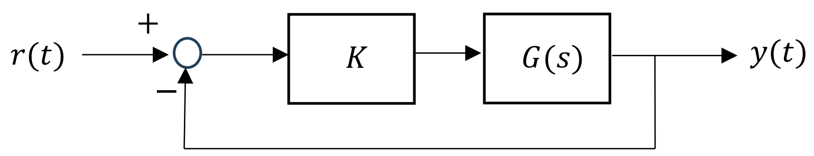

Figure Q1c

Question 2

Given the system shown in Figure Q2, with

$$G(s) = \frac{s+2}{s^3 + 5s^2 + 11s + 15}$$

(a) Find the open-loop poles and zeros, including both controller and plant

[6 marks]

(b) Sketch the root locus, explaining all key features. Note, calculating the angle of departure or arrival is not required. [16 marks]

(c) Determine the range of values of K for which the closed-loop system is stable. [3 marks]

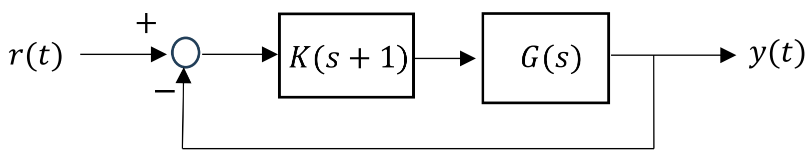

Figure Q2

Question 3

Figure Q3 shows a system modelled as mass-spring-damper, where an upper support is moved vertically to control the motion of the lower mass. The input is the vertical displacement of the support, $y_0(t)$ and the output is the resulting vertical displacement of the mass, $y(t)$.

(a) Draw a free-body diagram and derive the following equation of motion for the system as:

$$\ddot{y} = 100y_0 + 10\dot{y}_0 - 100y - 10\dot{y}$$

[5 marks]

(b) Find the transfer function $G(s) = Y(s)/Y_0(s)$ and determine both the damped natural frequency and damping ratio. [6 marks]

(c) A control system is to be implemented to move the mass between different desired reference positions. This requires an actuator, which has transfer function $A(s) = \frac{1}{1+0.2s}$.

The signal $u(t)$ is the command input for the actuator, and proportional control is to be applied in the form $u(t) = K(r(t) - y(t))$ with the support position given by $Y_0(s) = A(s)U(s)$.

(i) draw a block diagram of the overall system, labelling clearly all sub-systems and signals. [6 marks]

(ii) Determine the closed-loop transfer function $Y(s)/R(s)$. [6 marks]

(iii) Write down the closed-loop characteristic equation and mention a technique that could be used to determine the stability of the system as K is adjusted. [2 marks]

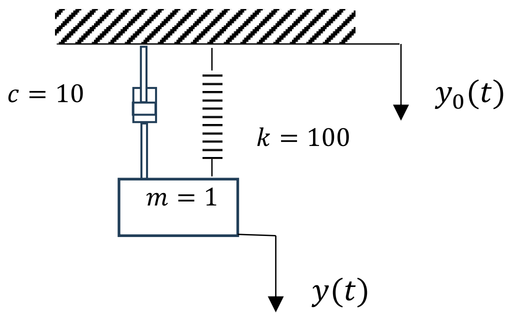

Figure Q3

Question 4

Consider the feedback system shown in Figure 4.

(a) Find the transfer function $T(s) = Y(s)/R(s)$ [8 marks]

(b) Determine the poles of $T(s)$ and hence show that the system is stable. [*Note: you may use Matlab to find the poles and zeros, but show your method clearly*]. [4 marks]

(c) Identify the pair of dominant poles in part (b) and hence estimate the overshoot and settling time for the system. What deficiencies do you notice for this control system? [5 marks]

(d) A sinusoidal reference signal is applied, $r(t) = \cos \omega t$ with $\omega = 2$ rad/s. Determine the output signal in the form

$$y(t) = A\cos(\omega t - \phi)$$

assuming sufficient time has elapsed for initial transients to decay to zero. [8 marks]

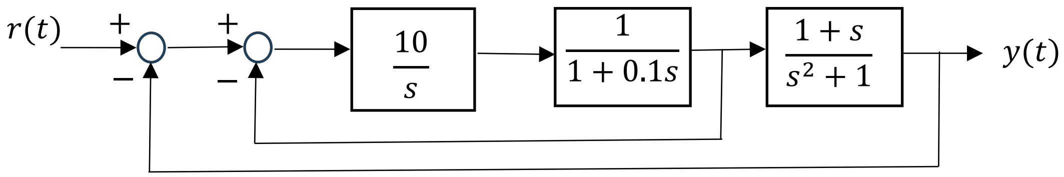

Figure Q4

Question 5

Figure Q5 represents a unity feedback control system which includes a PD controller with overall gain K.

(a) Determine the open-loop transfer function, together with its poles and zeros. [5 marks]

(b) Determine the closed-loop transfer function [4 marks]

(c) For the purpose of closed-loop stability analysis, to which of the above transfer functions would you apply the following analysis techniques?

(i) root locus [2 marks]

(ii) gain margin [2 marks]

(iii) Routh array [2 marks]

(d) Using the Routh array to find the range of values of K for which the closed-loop system is stable. [10 marks]

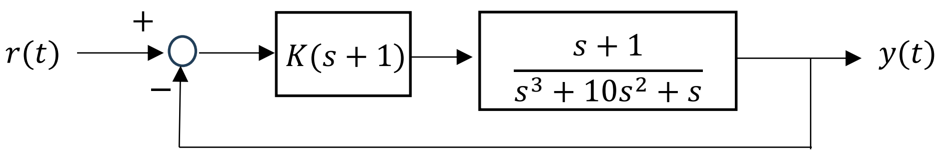

Figure Q5