thursday – Friday –

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13:00 – 14:30

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TCA – pre

791098

what time ?

7. Determine whether the following equations are linear or nonlinear.

1. $y''' + y'' = x$ → Linear

2. $y''' + x^2 y'' = x$ → nonlinear.

3. $y'' + xy' = x$

→

$(x+1)y'' + (2x+1)y' = x$

6. Solve the differential equation $\frac{\partial^2 u}{\partial x \partial y} = \sqrt{x+y}$ *given the boundary conditions that at* $y = 0$, $\frac{\partial u}{\partial x} = \pi$ and at $x = 0$, $u = 0$

$$\frac{\partial^2 u}{\partial x \partial y} = (x+y)^{\frac{1}{2}}$$

$$u^n \Longrightarrow \int u^n \, du = \frac{1}{n+1} u^{n+1}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \sqrt{x+y} \Rightarrow \quad y = 0, \ \frac{\partial u}{\partial x} = \pi$$

$$x = 0, \ u = 0$$

$\rightarrow$ First integration start from y

C

$$\int \frac{\partial^2 u}{\partial x \partial y} dy = \frac{\partial u}{\partial x} = \int \sqrt{x+y}\, dy + F(x)$$

$$f(x) = x$$

$$\Rightarrow \int x \, dx + C$$

$$\Rightarrow f(x,y) = xy + 1 \Rightarrow \frac{\partial f}{\partial x} = y + 0$$

$$\Rightarrow$$

$$\int (x+y)^{\frac{1}{2}}\, dy + C = \frac{\partial u}{\partial x}$$

$$\downarrow$$

$$\int u^n \, du = \frac{1}{n+1} u^{n+1} + C$$

$$\downarrow$$

$$\frac{1}{\frac{1}{2}+1}(x+y)^{\frac{1}{2}+1} + C = \frac{2}{3}(x+y)^{\frac{3}{2}} + C$$

$$\frac{\partial u}{\partial x} = \frac{2}{3}(x+y)^{\frac{3}{2}} + C$$

$y=0, \ \frac{\partial u}{\partial x} = \pi$

$\Longrightarrow$ Sub into top equation

$$\pi = \frac{2}{3}(x+0)^{\frac{3}{2}} + C \Longrightarrow C = \pi - \frac{2}{3}(x)^{\frac{3}{2}}$$

$$\frac{\partial u}{\partial x} = \frac{2}{3}(x+y)^{\frac{3}{2}} + \pi - \frac{2}{3}x^{\frac{3}{2}}$$

$\downarrow$

next step $\Rightarrow$ integration with to $\underline{\underline{x}}$

$$\int \frac{\partial u}{\partial x}\, dx = u$$

$\Longrightarrow$ other side of equation —

$$u = \int \frac{2}{3}(x+y)^{\frac{3}{2}} + \left(\pi - \frac{2}{3}x^{\frac{3}{2}}\right) dx$$

$$\int u^n \, du = \frac{1}{n+1} u^{n+1}$$

$$\frac{2}{3} \times \frac{1}{\frac{3}{2}+1} (x+y)^{\frac{3}{2}+1} + \pi x - \frac{2}{3} \times \frac{1}{\frac{3}{2}+1} x^{\frac{3}{2}+1} + C$$

$$u = \frac{2}{3} \times \frac{2}{5} (x+y)^{\frac{5}{2}} + \pi x - \frac{2}{3} \times \frac{2}{5} x^{\frac{5}{2}} + C$$

$\downarrow$

boundry condition: $x = 0, u = 0$

$\Rightarrow$ Sub into the equation:

$$0 = \frac{2}{3} \times \frac{2}{5} (0 + y)^{\frac{5}{2}} + \pi (0) - \frac{2}{3} \times \frac{2}{5} \times (0)^{\frac{5}{2}} + C$$

$$\Rightarrow 0 = \frac{4}{15} y^{\frac{5}{2}} + 0 - 0 + C \Rightarrow C = -\frac{4}{15} y^{\frac{5}{2}}$$

$$u = \frac{4}{15}(x+y)^{\frac{5}{2}} + \pi x - \frac{4}{15}x^{\frac{5}{2}} - \frac{4}{15}y^{\frac{5}{2}}$$

How did the integration of $\int (2x+y)^{\frac{1}{2}}\,dx =$

$$\int (2x+y)^{\frac{1}{2}}\,dy =$$

$$\int (2x+y)^{\frac{1}{2}}\,dx =$$

$$\downarrow$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1}$$

constant

instead of y

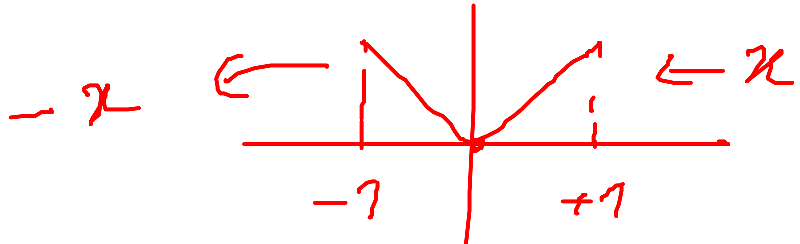
$$\int (2x+y)^{\frac{1}{2}}\,dx = \int (2x+a)^{\frac{1}{2}}\,dx = \frac{1}{2}(2x+y)^{\frac{1}{2}+1} \times \frac{1}{\frac{1}{2}+1}$$

$$= \frac{1}{2} \times \frac{1}{\frac{1}{2}+1}(2x+y)^{\frac{1}{2}+1}$$

2. Let $f(x) = |x|, \quad -1 \le x \le 1$

c) Find Fourier series of $f(x)$

d) Find the value of the following series based on the Fourier series of $f(x)$:



$$\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{(n\pi)^2}$$

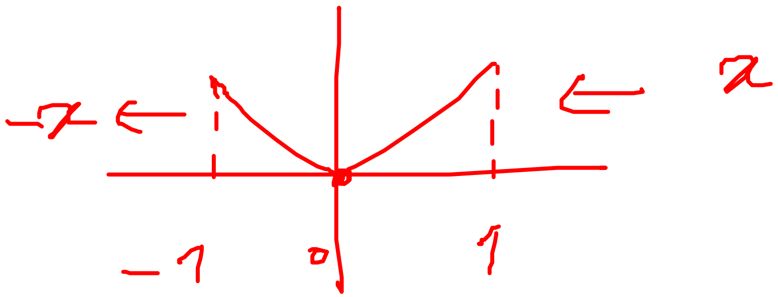
a_0, a_n, b_n

$$a_0 = \frac{2}{T}\int_0^T f(t)\, dt$$

$$a_n = \frac{2}{T}\int_0^T f(t) \cos \frac{2\pi n t}{T}\, dt$$

$$b_n = \frac{2}{T}\int_0^T f(t) \sin \frac{2\pi n}{T} t\, dt$$

$$f(x) = |x| = \begin{cases} -x & x < 0 \\ +x & x > 0 \end{cases} \qquad T = 2$$



$$\Longrightarrow a_0 = \frac{2}{2}\Big(\underbrace{\int_{-1}^{0} -x \, dx}_{-\frac{x^2}{2}} + \underbrace{\int_{0}^{1} x \, dx}_{\frac{x^2}{2}} \Big) = -\frac{x^2}{2}\Big]_{-1}^{0} + \frac{x^2}{2}\Big]_{0}^{1}$$

$$a_n = \frac{2}{T}\int_0^T f(x)\cos\frac{2\pi n}{T}x\,dx$$

$$\begin{cases} -x & x<0 \\ +x & x>0 \end{cases}$$

$$\Longrightarrow a_n = \frac{2}{2}\left(\int_{-1}^{0} -x\cos\frac{2\pi n}{2}x\,dx + \int_0^1 x\cos\frac{2\pi n}{2}x\,dx\right)$$

$$\int \frac{1}{n\pi}\sin n\pi x\,dx = \frac{-1}{n\pi}\times\frac{1}{n\pi}\cos n\pi x$$

$$\int x\,\underbrace{\cos\pi n x\,dx} = uv - \int v\,du = \frac{x}{n\pi}\sin n\pi x + \frac{1}{(n\pi)^2}\cos n\pi x$$

$$dv = \cos\pi n x\,dx \Longrightarrow v = \frac{1}{n\pi}\sin n\pi x$$

$$u = x \longrightarrow du = dx$$

$$\int x \cos n\pi x \, dx = \frac{x}{n\pi} \sin n\pi x + \frac{1}{(n\pi)^2} \cos n\pi x$$

$\downarrow$

$$a_n = \underbrace{\int_{-1}^{0} -x \cos n\pi x \, dx}_{\downarrow} + \underbrace{\int_0^1 x \cos n\pi x \, dx}_{\left. \frac{x}{n\pi} \sin n\pi x + \frac{1}{(n\pi)^2} \cos n\pi x \right]_0^1}$$

$$\left. -\frac{x}{n\pi} \sin n\pi x - \frac{1}{(n\pi)^2} \cos n\pi x \right]_{-1}^{0}$$

$$a_n = \overbrace{-\frac{x}{n\pi}\sin n\pi x - \frac{1}{(n\pi)^2}\cos n\pi x\Big]_{-1}^{0}}^{A} +$$

$$\underbrace{\frac{x}{n\pi}\sin n\pi x + \frac{1}{(n\pi)^2}\cos n\pi x\Big]_{0}^{1}}_{B}$$

$$= -\frac{1}{(n\pi)^2}$$

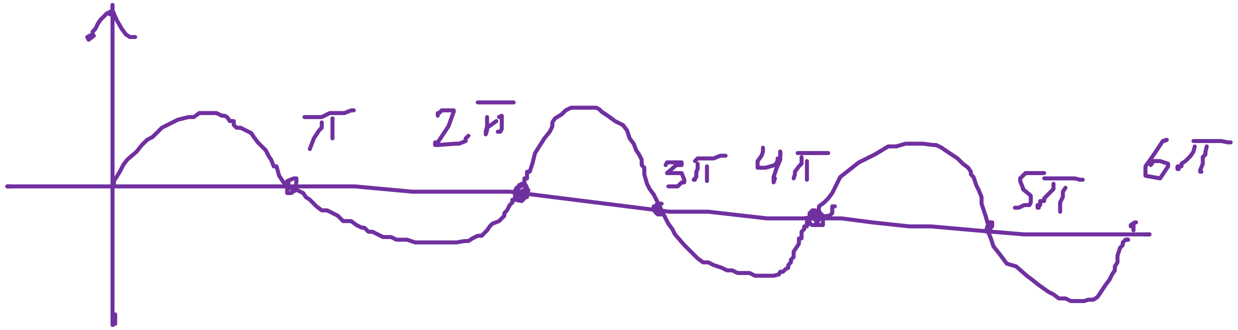
$$\sin n\pi = 0$$

$$A = \frac{-0}{n\pi} \times \overbrace{\sin n\pi(0)}^{=0} - \frac{1}{(n\pi)^2}\cos n\pi(0) - \left(-\frac{(-1)}{n\pi}\sin n\pi(-1)\right.$$

$$\left. - \frac{1}{(n\pi)^2}\cos n\pi(-1)\right) = -\frac{1}{(n\pi)^2} + \frac{1}{(n\pi)^2} \times (-1)^n$$

Why $\sin n\pi = 0$

$\cos n\pi = (-1)^n$



$\rightarrow \sin \pi = 0$

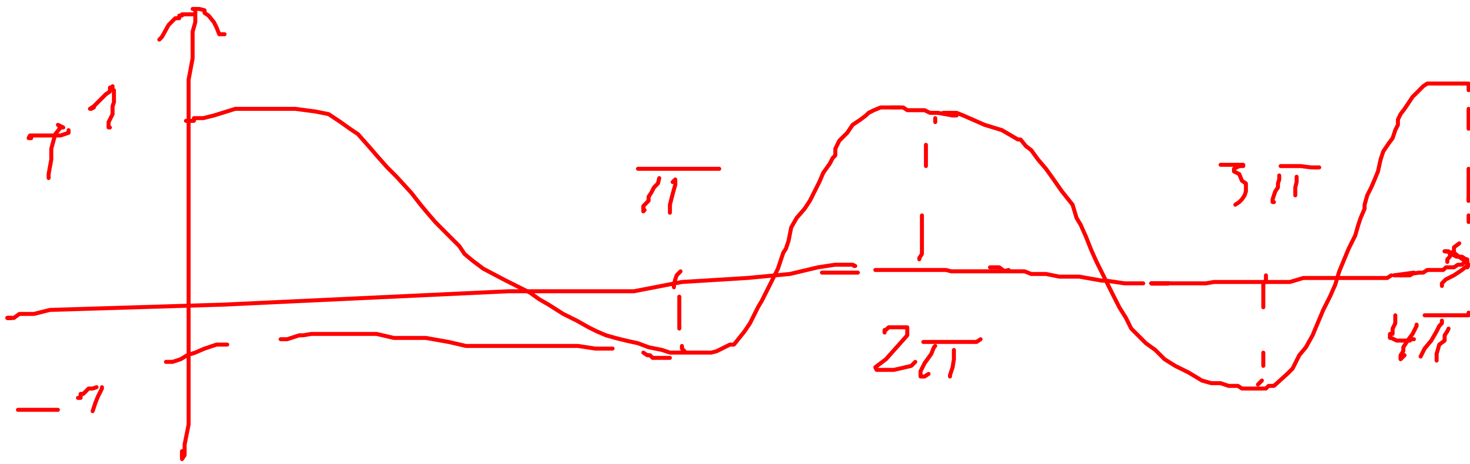
$\sin 2\pi = 0$

$\sin 3\pi = 0$

$\sin 4\pi = 0$

based on this graph

$$\cos n\pi = (-1)^n$$



$$\cos \pi = -1$$

$$\cos 2\pi = +1$$

$$\cos 3\pi = -1$$

$$\cos 4\pi = +1$$

$$\Longrightarrow \quad \cos n\pi = (-1)^n$$

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$$B = \left[\frac{x}{n\pi}\sin n\pi x + \frac{1}{(n\pi)^2}\cos n\pi x\right]_0^1$$

$$\hookrightarrow \frac{1}{n\pi}\underbrace{\sin n\pi 1}_{\sin n\pi = 0} + \frac{1}{(n\pi)^2}\overbrace{\cos n\pi \times 1}^{(-1)^n} - \left(\underbrace{\frac{0}{n\pi}\sin n\pi \times 0}_{=0} + \frac{1}{(n\pi)^2}\overbrace{\cos n\pi \times 0}^{1}\right)$$

$$\Rightarrow B = \frac{1}{(n\pi)^2}\times(-1)^n - \frac{1}{(n\pi)^2}\times 1$$

$$a_n = A + B = \frac{1}{(n\pi)^2}(-1)^n - \frac{1}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{1}{(n\pi)^2}(-1)^n$$

$$a_n = \frac{-2}{(n\pi)^2} + \frac{2\times(-1)^n}{(n\pi)^2}$$

$\Rightarrow$ next step to find $b_n = ?$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi n}{2}t\,dt$$

$$\Rightarrow b_n = \frac{2}{2}\left(\int_{-1}^{0} -x\sin n\pi x\,dx + \int_0^1 x\sin n\pi x\,dx\right)$$

$\rightarrow \int -\frac{1}{n\pi}\cos n\pi x\,dx = -\frac{1}{n\pi}\times\frac{1}{n\pi}\sin n\pi x$

$$\int x\,\sin n\pi x\,dx = uv - \int v\,du = -\frac{x}{n\pi}\cos n\pi x + \frac{1}{(n\pi)^2}\sin n\pi x$$

$u = x$
$du = dx$

$\int dv = \int \sin n\pi x\,dx = \frac{-1}{n\pi}\cos n\pi x$

$$\int x \sin n\pi x \, dx = \frac{-x}{n\pi} \cos n\pi x + \frac{1}{(n\pi)^2} \sin n\pi x$$

$$\downarrow$$

$$b_n = \underbrace{\int_{-1}^{0} -x \sin n\pi x \, dx}_{} + \underbrace{\int_{0}^{1} x \sin n\pi x \, dx}_{}$$

$$\underbrace{-\left[\left(\frac{-x}{n\pi} \cos n\pi x + \frac{1}{(n\pi)^2} \sin n\pi x\right)\right]_{-1}^{0}}_{A} \qquad \Rightarrow \underbrace{\left[\frac{-x}{n\pi} \cos n\pi x + \frac{1}{(n\pi)^2} \sin n\pi x\right]_{0}^{1}}_{B}$$

$$\Rightarrow \quad b_n = A + B$$

$$A = \frac{x}{n\pi}\cos n\pi x - \frac{1}{(n\pi)^2}\sin n\pi x \Big]_{-1}^{0}$$

$$= \Big(\underbrace{\frac{0x}{n\pi}\cos n\pi \times 0}_{=0} - \underbrace{\frac{1}{(n\pi)^2}\sin n\pi \times 0}_{=0}\Big) - \Big(\frac{-1}{n\pi}\cos(n\pi \times -1)$$

$$- \frac{1}{(n\pi)^2}\underbrace{\sin(n\pi \times -1)}_{=0}\Big) \Rightarrow A = \frac{1}{n\pi}(-1)^n$$

$*$ $\cos n\pi = ?$ and $\cos -n\pi = ?$

$$\Rightarrow \cos(n\pi) = \cos(-n\pi) = (-1)^n$$

$\sin n\pi = 0$, $\sin(-n\pi) = 0$ $\Rightarrow \sin n\pi = \sin(-n\pi) = 0$

$$B = \left[\frac{-x}{n\pi} \cos n\pi x + \frac{1}{(n\pi)^2} \sin n\pi x \right]_0^1$$

$$\downarrow$$

$$B = \left(\frac{-1}{n\pi} \underbrace{\cos(n\pi \times 1)}_{\to (-1)^n} + \frac{1}{(n\pi)^2} \underbrace{\sin n\pi}_{=0} \right) - \left(\underbrace{\frac{-0 \times}{n\pi} \cos n\pi \times 0}_{=0} + \right.$$

$$\left. \underbrace{\frac{1}{(n\pi)^2} \sin n\pi \times 0}_{=0} \right) \Longrightarrow B = \frac{-1}{n\pi} (-1)^n$$

$$\Longrightarrow b_n = A + B = \frac{1}{n\pi} (-1)^n - \frac{1}{n\pi} (-1)^n = 0$$

a_0 , a_n , b_n

$$\Rightarrow f(x) = \frac{a_0}{2} + \sum \underline{a_n} \cos n\pi x + \underline{b_n} \sin n\pi x$$

$$\downarrow$$

$$b_n = 0$$

$$a_n = -\frac{2}{(n\pi)^2} + \frac{2}{(n\pi)^2}(-1)^n$$

$$\Rightarrow f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} - \frac{2 + 2 \times (-1)^n}{(n\pi)^2} \cos n\pi x$$

d)
$$\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{(n\pi)^2} = ?$$

$$f(x) = \frac{a_0}{2} + \sum \underbrace{\frac{-2 + 2 \times (-1)^n}{(n\pi)^2}}_{\text{take factor } 2} \cos n\pi x$$

$x = 0 \Longrightarrow \cos n\pi \times 0 = 1$

$$2 \sum \frac{-1 + (-1)^n}{(n\pi)^2} \cos n\pi x$$

$$\Rightarrow f(0) = \frac{a_0}{2} + 2 \sum \frac{-1 + (-1)^n}{(n\pi)^2} \times 1 \Rightarrow f(0) - \frac{a_0}{2} = 2\sum$$

1. Let $f(x) = x,\ 0 \leq x \leq 1$

   a) Find the even extension Fourier series of $f(x)$

   b) Find the value of the following series based on the even extension of $f(x)$:

$$\sum_{n=odd\ numbers}^{\infty} \frac{1}{(n)^2}$$

5. For the following equation (second order ode)

$$y'' + y' + 2y = 0$$

$y = x_1$

$y' = x_2$

highest value of derivative

$(n-1)$

$x_2' + x_2 + 2x_1 = 0$

$x_1' = ?$

derivative of $y' = x_2$

$y'' = x_2'$

a) write the system as 1st order differential equations by change variable method.

b) write the first order ode equations of previous part (part a) in matrix form.