(a) From $y_1 + y_2 - y_1^2 = 0$. we have

$$y_2 = y_1^2 - y_1$$

Then. $2y_1 - (y_1^2 - y_1) + y_1(y_1^2 - y_1) = 0$

$$y_1(y_1^2 - 2y_1 + 3) = 0$$

$y_1 = 0$ or $y_1 = \dfrac{2 \pm \sqrt{-8}}{2}$

complex number. skip it.

fixed point $(0, 0)$.

Jacobian Matrix

$$\begin{bmatrix} 1 - 2y_1 & 1 \\ 2 + y_2 & -1 + y_1 \end{bmatrix}$$

substituting $y_1 = 0$ & $y_2 = 0$. we have

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$(1-\lambda)(-1-\lambda) - 2 = 0$$

$$\lambda^2 = 3$$

$\lambda = \pm\sqrt{3}$ unstable.

(b) From $-6y_1 + 4y_2 + y_2^3 = 0$. we have.

$$y_1 = \frac{1}{6}(4y_2 + y_2^3)$$

$$0 = \frac{2}{3}(4y_2 + y_2^3) + 3y_2 - \frac{1}{6}y_2^2(4y_2 + y_2^3)$$

$$\frac{17}{3}y_2 - \frac{y_2^5}{6} = 0$$

$$34y_2 - y_2^5 = 0$$

$$y_2(y_2^4 - 34) = 0$$

$y_2 = 0$ or $y_2^2 = \pm\sqrt{34}$

$y_2 = \pm\sqrt[4]{34}$

$y_2 = \pm\sqrt{\sqrt{34}}\, j$

skip

fixed points $(0, 0)$

$$\left(\frac{2}{3}\cdot\sqrt[4]{34} + \frac{1}{6}(\sqrt[4]{34})^3, \sqrt[4]{34}\right)$$

$$\left(-\frac{2}{3}\sqrt[4]{34} - \frac{1}{6}(\sqrt[4]{34})^3, -\sqrt[4]{34}\right)$$

For $(0, 0)$

$$\begin{bmatrix} 4 - y_2^2 & 3 - 2y_1 y_2 \\ -6 & 4 + 3y_2^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 3 \\ -6 & 4 \end{bmatrix}$$

$$(4-\lambda)^2 + 18 = 0$$

$$\lambda - 4 = \pm\sqrt{-18}$$

$$\lambda = 4 \pm \sqrt{-18}$$

$\text{real}(\lambda) = 4 > 0$ unstable.

(c) From $2y_1+3y_2-y_1^3=0$, we have

$$y_2=\frac{1}{3}(y_1^3-2y_1)$$

Then. $-4y_1+2(y_1^3-2y_1)+y_1^2(y_1^3-2y_1)=0$

$$y_1(y_1^4-8)=0$$

$y_1=0$ or $y_1^2=\pm\sqrt{8}$

$y_1=\pm\sqrt[4]{8}$ $\quad y_1=\pm\sqrt[4]{8}$

skip

fixed points $(0,0)$

$$\left(\sqrt[4]{8},\ \frac{1}{3}(\sqrt[4]{8})^3-\frac{2}{3}\sqrt[4]{8}\right)$$

$$\left(-\sqrt[4]{8},\ -\frac{1}{3}(\sqrt[4]{8})^3+\frac{2}{3}\sqrt[4]{8}\right)$$

For the point $(0,0)$

$$\begin{bmatrix} 2-3y_1^2 & 3 \\ -4+6y_1y_2 & 6+3y_1^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ -4 & 6 \end{bmatrix}$$

$(2-\lambda)(6-\lambda)+12=0$ $\quad \lambda^2-8\lambda+24=0$

$$\lambda = \frac{8 \pm \sqrt{64 - 24 \times 4}}{2}$$

$\text{real}(\lambda) = 4 > 0$

unstable.