Tutorial questions and solutions

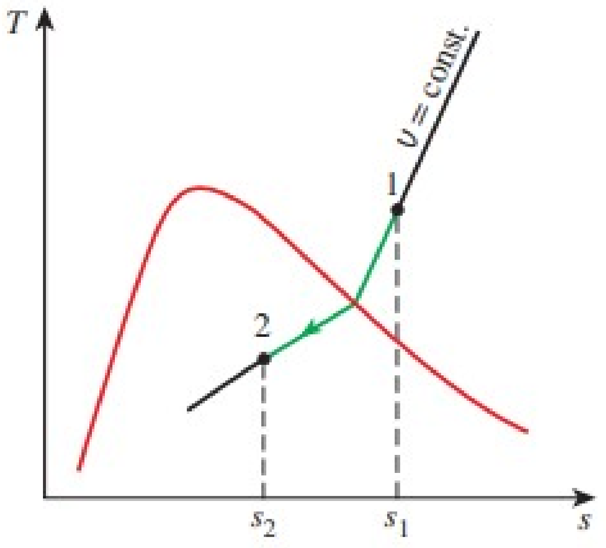
EGR3030
Energy Systems and Conversion
Lecture 2: Gas laws and entropy

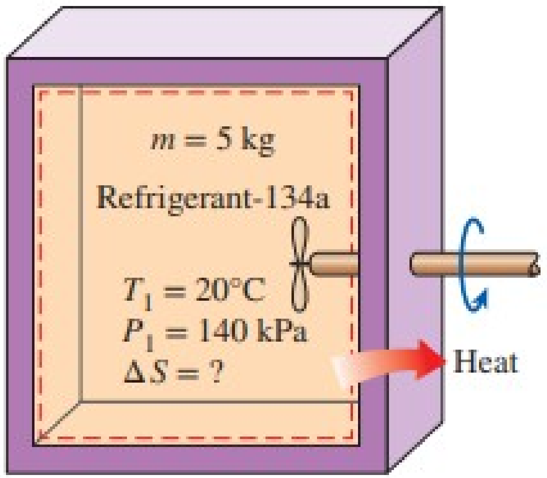
Reference Texts to find solutions and more context:
Fundamentals of Thermal Fluid Sciences Chapter 8 (4th Ed); Thermodynamics: An Engineering Approach Chapter 7 (7th Ed) (Both books by Yunus Cengel, et al.)

Worked example 1

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process

Solution:





Solution 1

Assumptions

- The volume of the tank is constant and thus .
- Refrigerant in the tank is a closed system, no mass crosses boundary.

The initial state of the refrigerant is completely specified.

State 1:

The refrigerant is a saturated liquid–vapor mixture at the final state since at 100 kPa pressure.

Therefore, we need to determine the quality x:

Hence,

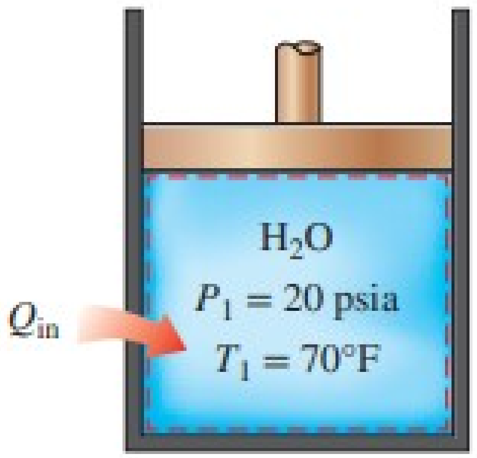
Therefore, the entropy change of the refrigerant during this process is

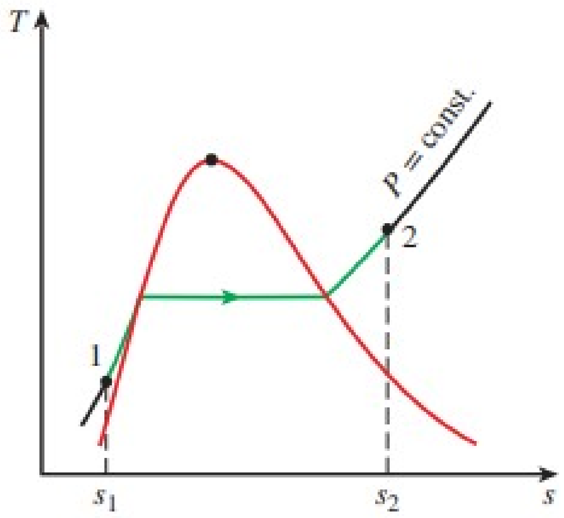
The -ve sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the entropy generation that cannot be negative.

Worked example 2

A piston–cylinder device initially contains 1.5 kg of liquid water at 150 kPa and 20°C. The water is now heated at constant pressure by the addition of 4000 kJ of heat. Determine the entropy change of the water during this process.

Solution:





Solution 2

Assumptions

- The tank is stationary, thus the kinetic and potential energy changes are 0, ΔKE = ΔPE = 0.
- The process is quasi-equilibrium.
- The pressure remains constant during the process and thus P2 = P1

This is a closed system since no mass crosses the system boundary during the process. A piston–cylinder device typically involves a moving boundary and thus work is boundary work . Also, heat is transferred to the system.

Water is a compressed liquid at the initial state P > Psat=2.3392 kPa at 20°C. Approximating the compressed liquid as a saturated liquid at the given temperature, initial state properties are:

State 1:

At the final state, the pressure is still 150 kPa, but we need one more property to fix the state. This property is determined from the energy balance,

But

Since for a constant pressure quasi equilibrium process, then:

Therefore, the entropy change of water during this process is

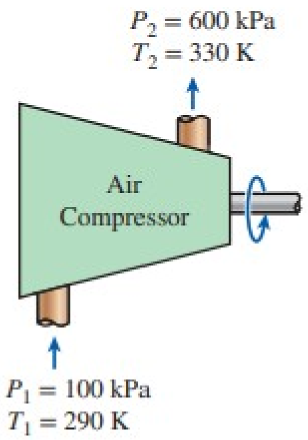
Worked example 3 – ideal gas entropy evaluation

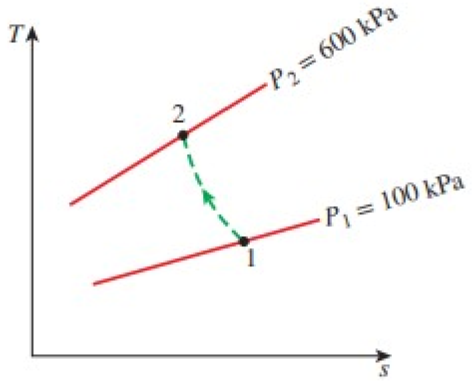
Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change during this compression process by using:

(a) property values from the air table

(b) average specific heats.

Solution:





Worked example 3 – solution

Note that both the initial and the final states of air are completely specified.

(a) The properties of air are given in the air table (Table A–17). Reading s° values at given temperatures and substituting, we find

=

(b) The entropy change of air during this process can also be determined approximately from

by using a Cp value at the average temperature of 37°C (Table A–2b) and treating it as a constant:

=

The two results are almost identical since the change in temperature during this process is relatively small (290 to 330 K). When the temperature change is large, they may differ significantly.

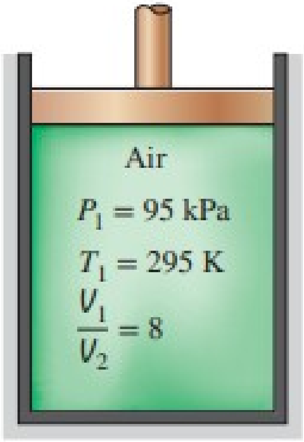
In such cases, method (a) should be used instead of method (b) since it accounts for the variation of specific heats with temperature, rather than an average value between the temperatures.

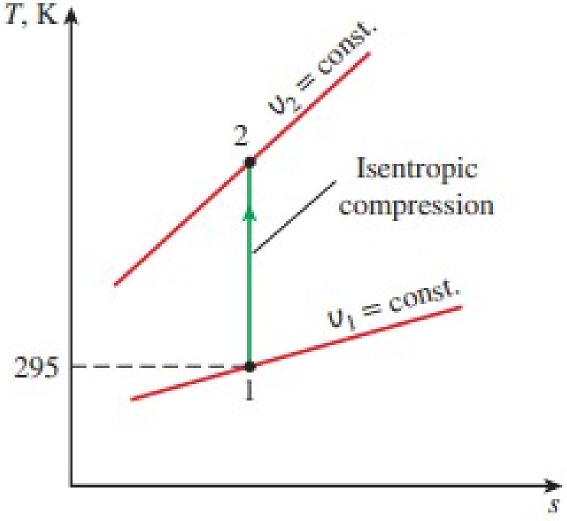
Worked example 4

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio V1/V2 of this engine is 8, determine the final temperature of the air.

Solution:

First, your sketches





Which equation?

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

Which property table? Table A–2b

Worked example 4 – solution

This process is easily recognised as isentropic since it is both reversible and adiabatic. The final temperature can be determined using the 1st isentropic relationship:

Assuming constant specific heats for air. What value of k in Table A2b do we use? The final temperature is not given, and so we cannot determine the average temperature in advance

First guess a k value at initial T or at the anticipated average temperature. Can be refined later, and the calculation repeated, if necessary.

- Air temperature will rise considerably during this adiabatic compression process.
- Guess the average temperature to be ~450 K.
- Determine the k value at this temperature from Table A–2b (k=1.391). The final temperature of air becomes

Average temperature = (295+665.2)/2 = 480.1 K. Hence, we may not repeat the calculation by using a k value at 480.1 K.

Homework

1. Steam turbine isentropic efficiency

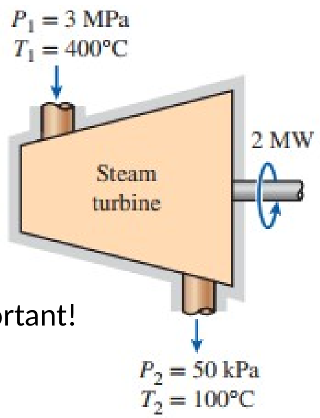
Steam enters an adiabatic turbine steadily at 2 MPa and 350°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.

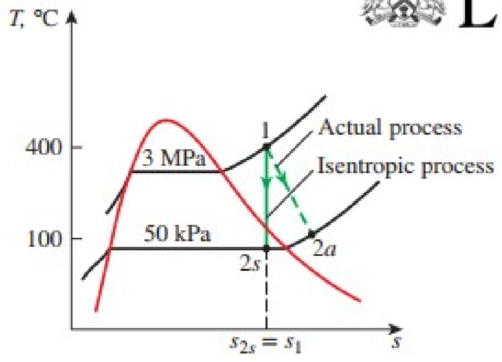
2. Compressor outlet temperature and power input

Air is compressed by an adiabatic compressor from 150 kPa and 20°C to a pressure of 700 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 75 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.

Homework solution guide t
Q1

Please use the numbers in the questions, the method is the same as given here.
First, draw your schematic and superimpose all known quantities. Then your phase diagram – very important!





(*a*) The enthalpies at various states are

State 1: $\left.\begin{array}{l} P_1 = 3 \text{ MPa} \\ T_1 = 400°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3231.7 \text{ kJ/kg} \\ s_1 = 6.9235 \text{ kJ/kg·K} \end{array}$ (Table A–6)

State 2a: $\left.\begin{array}{l} P_{2a} = 50 \text{ kPa} \\ T_{2a} = 100°\text{C} \end{array}\right\} h_{2a} = 2682.4 \text{ kJ/kg}$ (Table A–6)

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

State 2s: $\begin{array}{l} P_{2s} = 50 \text{ kPa} \\ (s_{2s} = s_1) \end{array} \longrightarrow \begin{array}{l} s_f = 1.0912 \text{ kJ/kg·K} \\ s_g = 7.5931 \text{ kJ/kg·K} \end{array}$ (Table A–5)

Obviously, at the end of the isentropic process steam exists as a saturated mixture since $s_f < s_{2s} < s_g$. Thus, we need to find the quality at state 2*s* first:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

and

$$h_{2s} = h_f + x_{2s}h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}$$

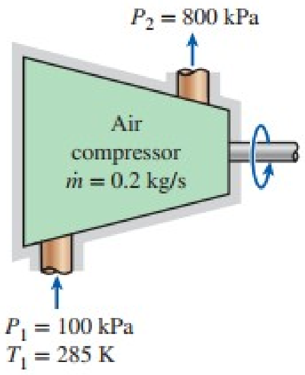
By substituting these enthalpy values into Eq. 7–61, the isentropic efficiency of this turbine is determined to be

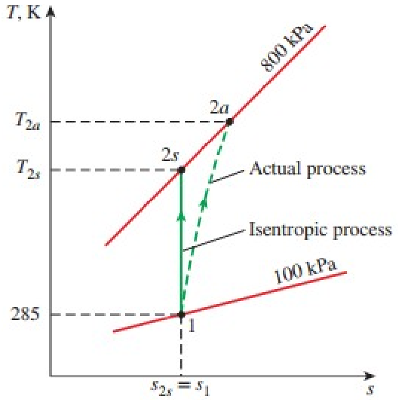
$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667} \text{ (or 66.7\%)}$$

(*b*) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$
$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_{2a}$$
$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_{2a})$$
$$2 \text{ MW}\left(\frac{1000 \text{ kJ/s}}{1 \text{ MW}}\right) = \dot{m}(3231.7 - 2682.4) \text{ kJ/kg}$$
$$\dot{m} = \mathbf{3.64 \text{ kg/s}}$$

Homework solution guide to Q2





SOLUTION Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The changes in kinetic and potential energies are negligible.

Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 7–52.

(*a*) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is h_{2a} since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$T_1 = 285 \text{ K} \rightarrow h_1 = 285.14 \text{ kJ/kg} \qquad \text{(Table A–17)}$$
$$P_{r1} = 1.1584$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$

and

$$P_{r2} = 9.2672 \rightarrow h_{2s} = 517.05 \text{ kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \rightarrow 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

Thus,

$$h_{2a} = 575.03 \text{ kJ/kg} \rightarrow T_{2a} = \mathbf{569.5\ K}$$

(*b*) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$
$$\dot{m}h_1 + \dot{W}_{a,\text{in}} = \dot{m}h_{2a}$$
$$\dot{W}_{a,\text{in}} = \dot{m}(h_{2a} - h_1)$$
$$= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}]$$
$$= \mathbf{58.0\ kW}$$

Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.