1. Let $f(x) = x,\ 0 \le x \le 1$
   a) Find the even extension Fourier series of $f(x)$
   b) Find the value of the following series based on the even extension of $f(x)$:

$$\sum_{n=odd\ numbers}^{\infty} \frac{1}{(n)^2}$$

2. Let $f(x) = |x|\,,\ -1 \le x \le 1$
   c) Find Fourier series of $f(x)$
   d) Find the value of the following series based on the Fourier series of $f(x)$:

$$\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{(n\pi)^2}$$

3. Determine the value of $y(0.3)$ using three different numerical methods for the initial value problem $y' = 1 - y$ and initial condition $x_0 = 0, y_0 = 0$ with h=0.1:

   a) Runge-Kutta method

      1. Identify x_0, y_0 and h, and values of x_1, x_2, x_3, ...
      2. Evaluate $k_1 = f(x_n, y_n)$ starting with $n = 0$
      3. Evaluate $k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_1\right)$
      4. Evaluate $k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_2\right)$
      5. Evaluate $k_4 = f\,(x_n + h, y_n + hk_3)$
      6. Use the values determined from steps 2 to 5 to evaluate: $y_{n+1} = y_n + \frac{h}{6}\{k_1 + 2k_2 + 2k_3 + k_4\}$

   b) Euler method

4. For the system AX = λX where $A = \begin{matrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{matrix}$
   a) Find the eigenvalues of matrix A.
   b) Find the eigenvectors of matrix A.

5. For the following equation (second order ode)

$$\boldsymbol{y'' + y' + 2y = 0}$$

a) write the system as 1st order differential equations by change variable method.

b) write the first order ode equations of previous part (part a) in matrix form.

6. Solve the differential equation $\frac{\partial^2 u}{\partial x \partial y} = \sqrt{x+y}$*given the boundary conditions that at* $y = 0$, $\frac{\partial u}{\partial x} = \pi$ and at $x = 0$, $u = 0$

7. Determine whether the following equations are linear or nonlinear.

$$\boldsymbol{y''' + y'' = x}$$

$$\boldsymbol{y''' + x^2 y'' = x}$$

$$\boldsymbol{y''} + x\boldsymbol{y'} = x$$