DATA MODELLING AND SIMULATION

LECTURE 1:

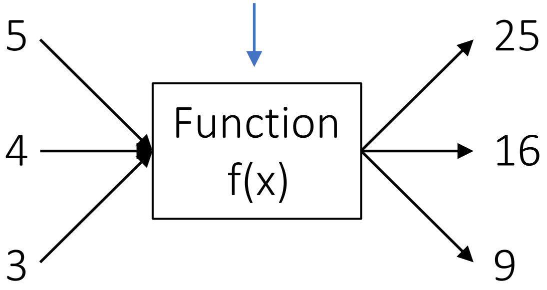
Eigen Vectors & Eigen Values

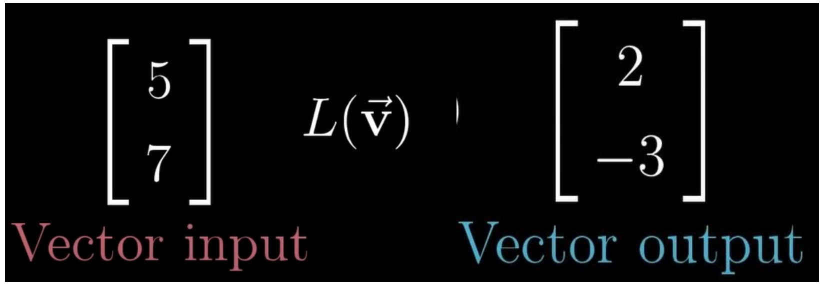
By,
Manu H. Nair
PhD Student, Space Robotics
L-CAS, University of Lincoln

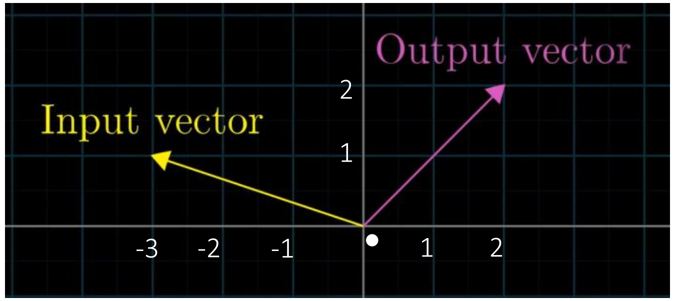
18710796@students.Lincoln.ac.uk
mnair@lincoln.ac.uk

Linear Transformation

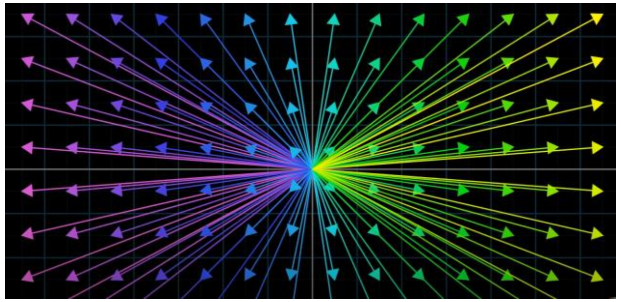
What is a <u>transformation</u>?



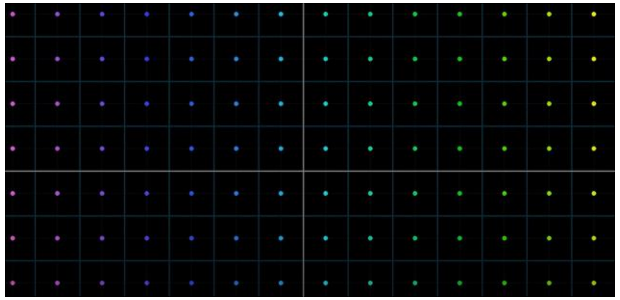




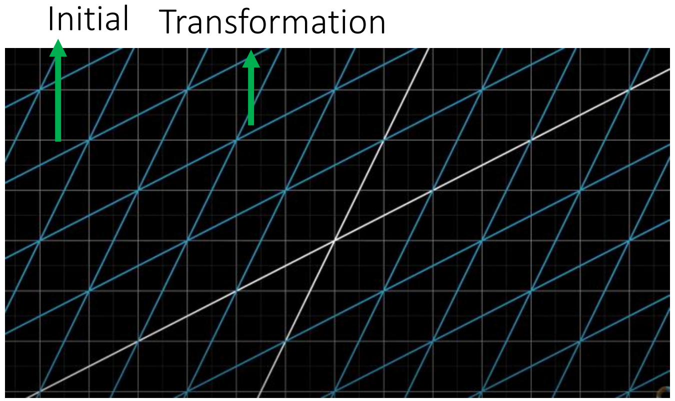
(-3,1) -> (2,2)



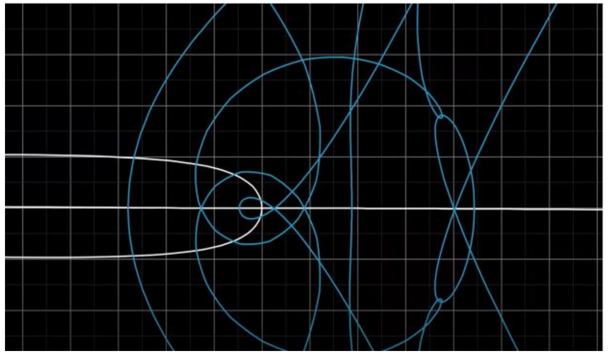
Transformation as a whole
- Too crowded with arrow representation



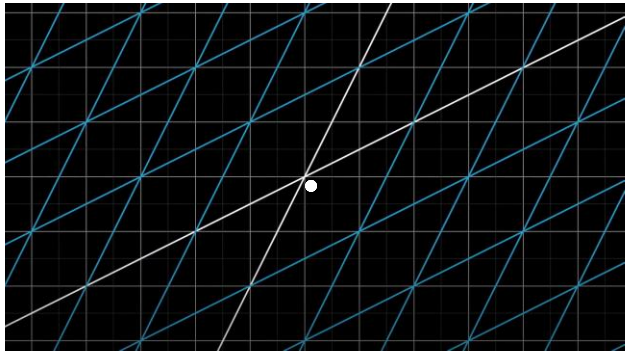
Conceptualise each vector not as an arrow but as a single point



Grid method – Transformation in 2D



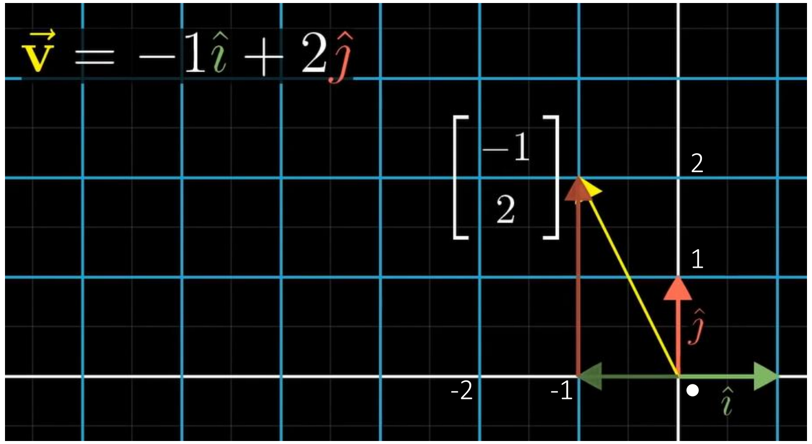
Transformation can get messy



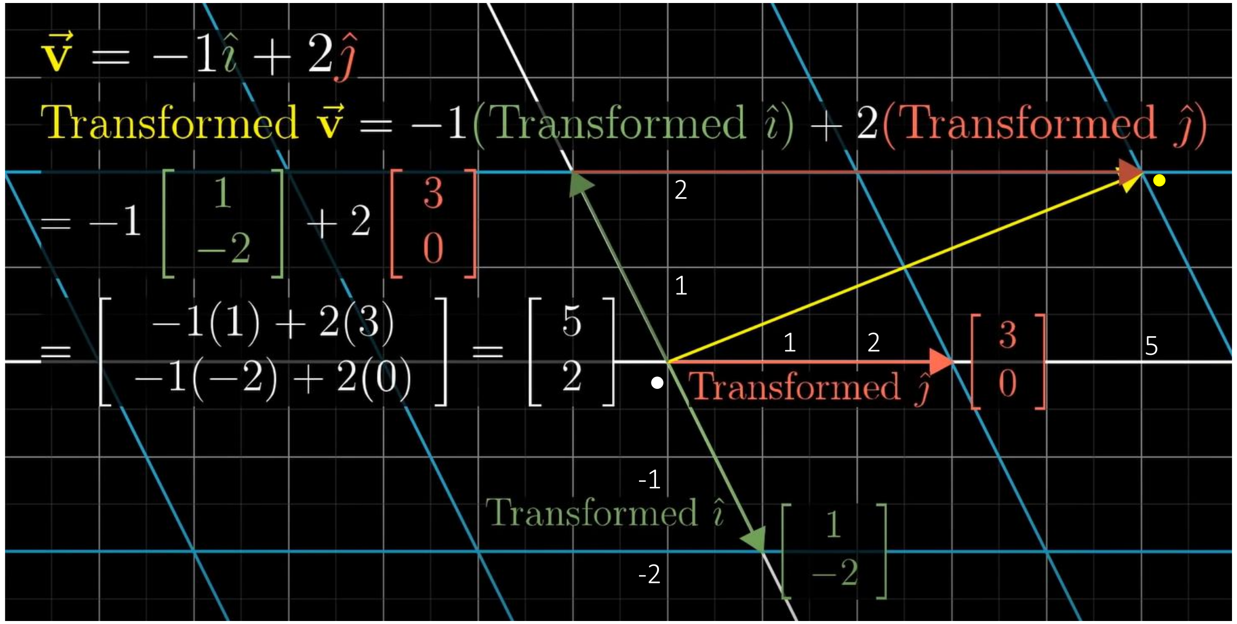
Interest: Linear Transformations

- All Lines Remain Lines
- No curves
- Fixed Origin
- Gride lines remain | | and evenly spaced

Numerical Description



$(\hat{\imath}\ ;\ \hat{\jmath})$ = (0, 0); (0,0)



$$\hat{\imath} \rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \hat{\jmath} \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \underbrace{x \begin{bmatrix} 1 \\ -2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 0 \end{bmatrix}}_{\text{Where all the intuition is}} = \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$

2x2 Matrix

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$$

Where $\hat{\imath}$ lands Where $\hat{\jmath}$ lands

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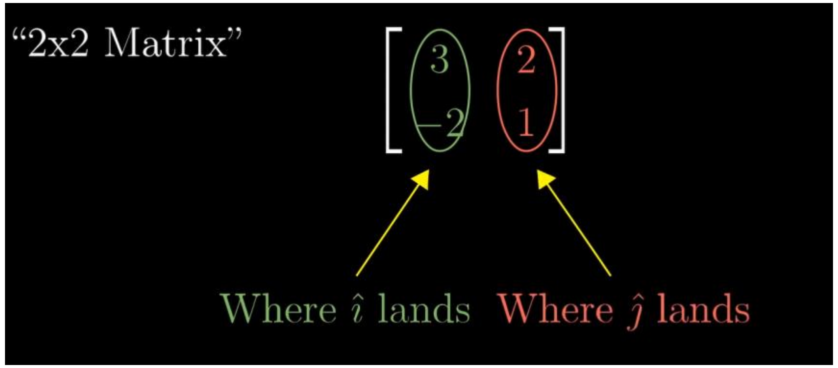
TO-DO LIST

1. Where does the linear transformation take vector b?



$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

Vector b

$$\begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

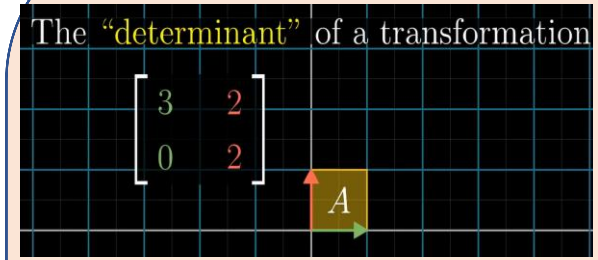
2. Assume a generalised 2x2 matrix with $\hat{\imath}$ (components being a and c) and $\hat{\jmath}$ (components b and d). With a vector [x,y], prove its linear transformation results in a 2x1 matrix as below:

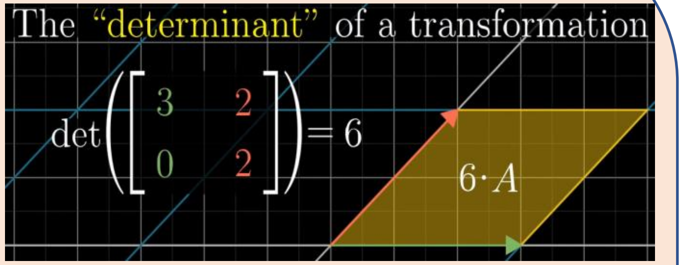
$$\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

Determinant of a Transformation

Factor by which a linear transformation changes any area is called the Determinant of that transformation

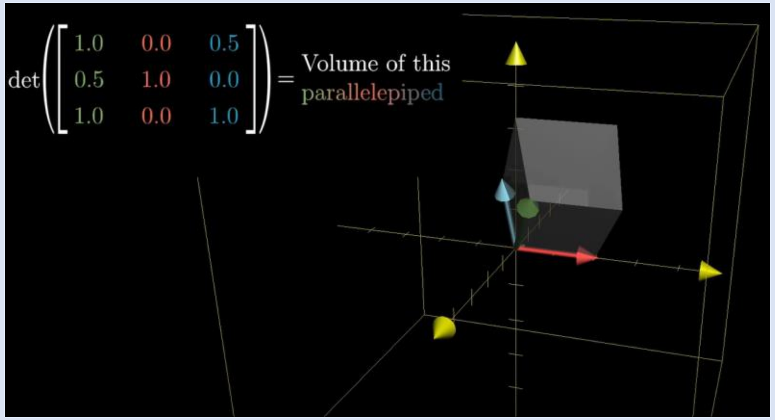
2D





$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

3D



$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a \det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b \det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c \det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

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TO-DO LIST

1. Can determinants be negative? If so what does it mean? How does it affect the area?

2. Compute Determinant of the following matrices:

A. $\begin{bmatrix} 1 & 9 \\ 3 & 8 \end{bmatrix}$

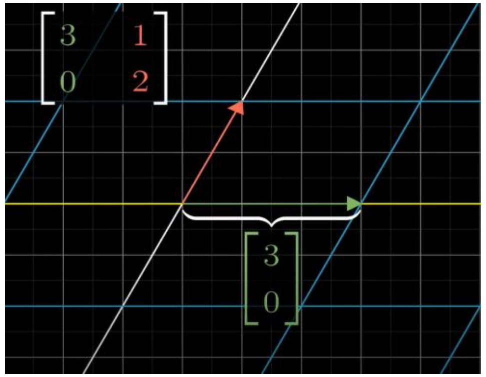
B. $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

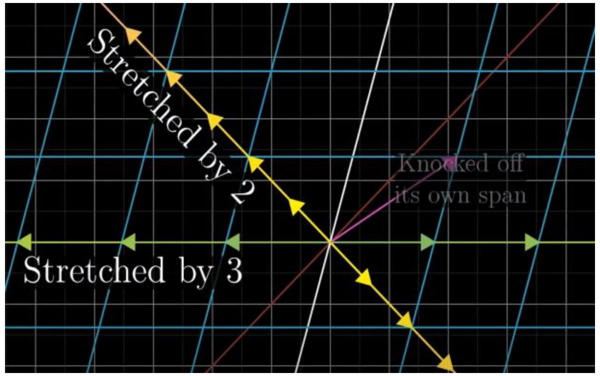
C. $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

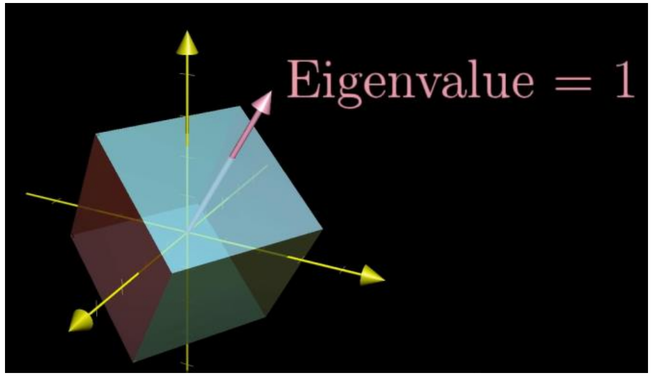
D. $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -1 \end{bmatrix}$

Eigen Vectors and Eigen Values

Special Vectors which remains in its own span after transformation

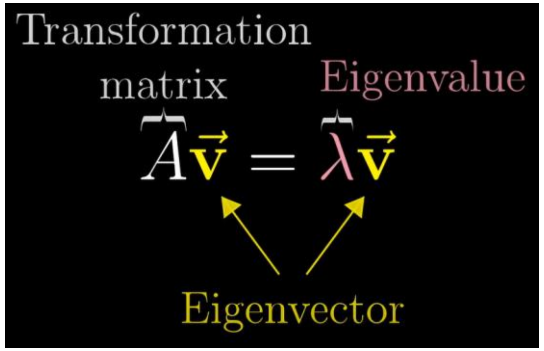


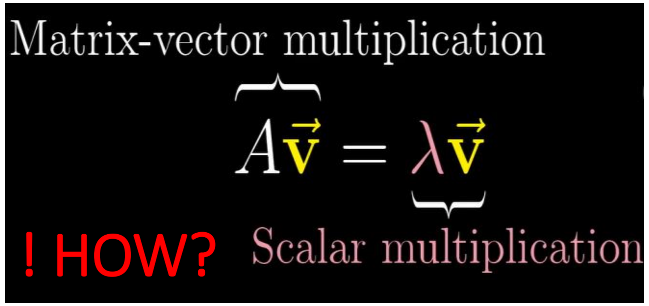


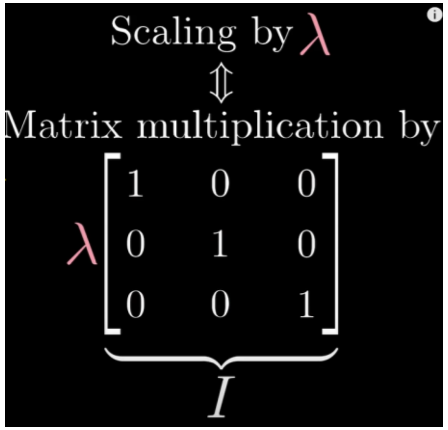


- Remains on x axis
- Any other vector on X-axis gets stretched by a factor of 3
- Factor by which it is stretched, X-axis: 3 = **Eigen value** (λ) of the Eigen Vector ($\vec{v}$)

3D rotation doesn't stretch or squish any vector, so Eigen value remains 1





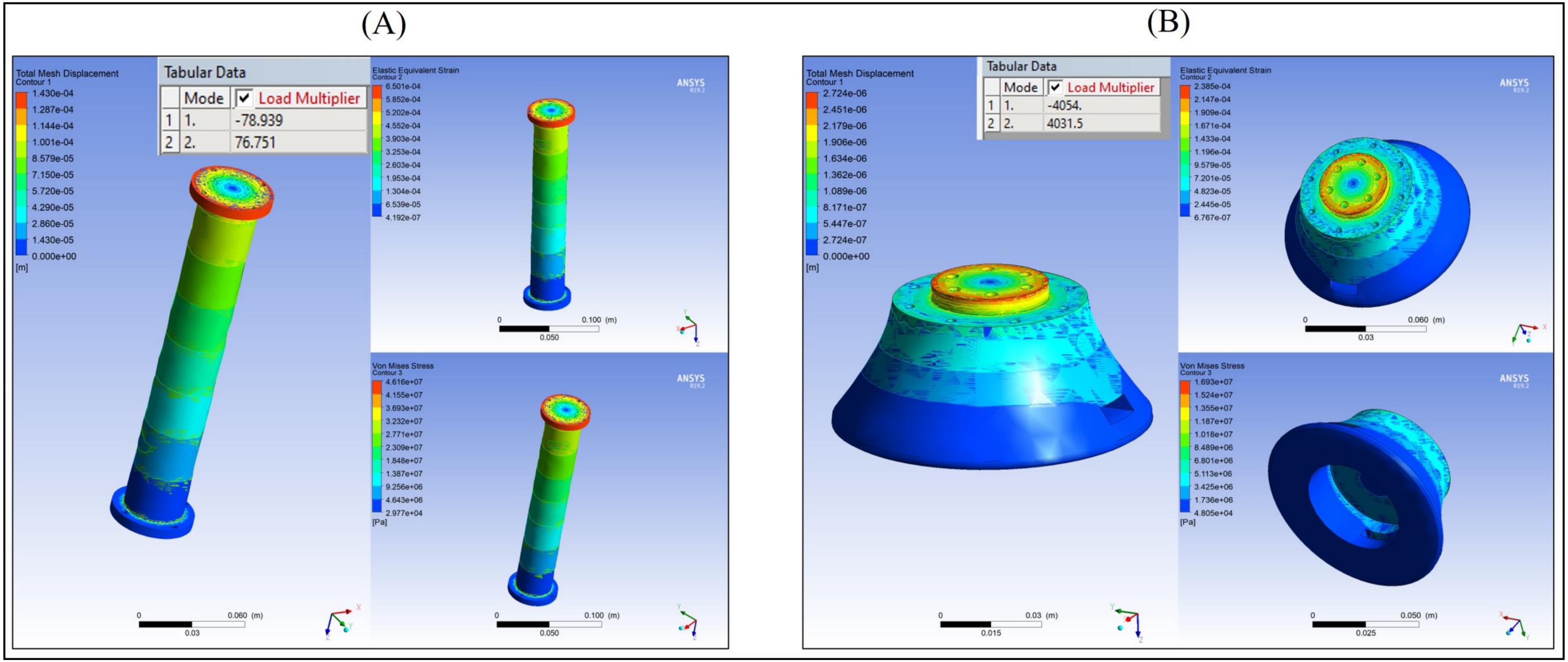


$$(A - \lambda I)\vec{\mathbf{v}} = \vec{0}$$

$$\det(A - \lambda I) = 0$$

There exists a non-zero vector v

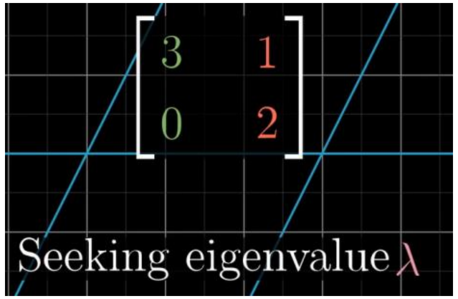
Eigen Value Buckling – Significance and Practical Application of Eigen Vectors and Values

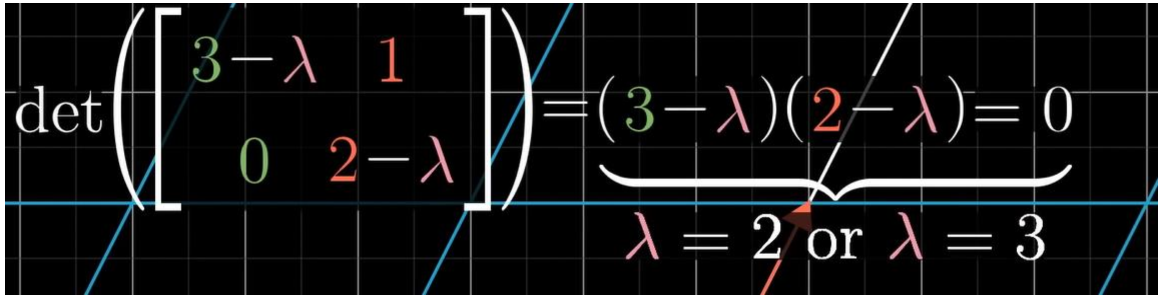


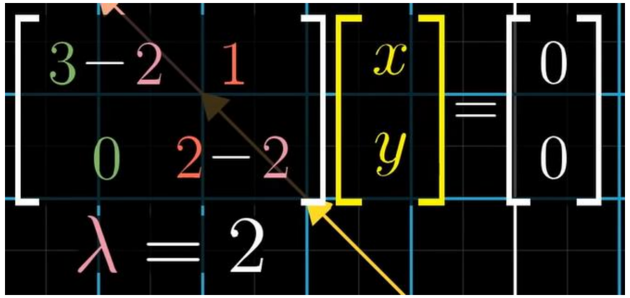
M. H. **Nair**, M. C. Rai and M. Poozhiyil, "Design Engineering a Walking Robotic Manipulator for In-Space Assembly Missions", Frontiers in Robotics and AI: Robotic In-Space Servicing, Assembly and Manufacturing, Sep 2022.

https://www.frontiersin.org/articles/10.3389/frobt.2022.995813/abstract

1. Calculating Eigen values and vectors

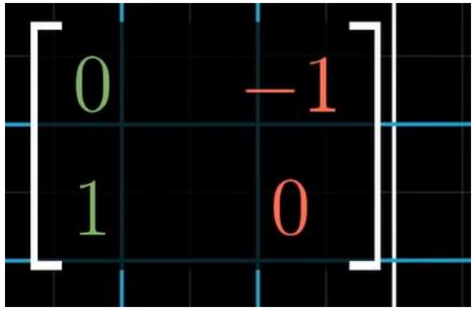


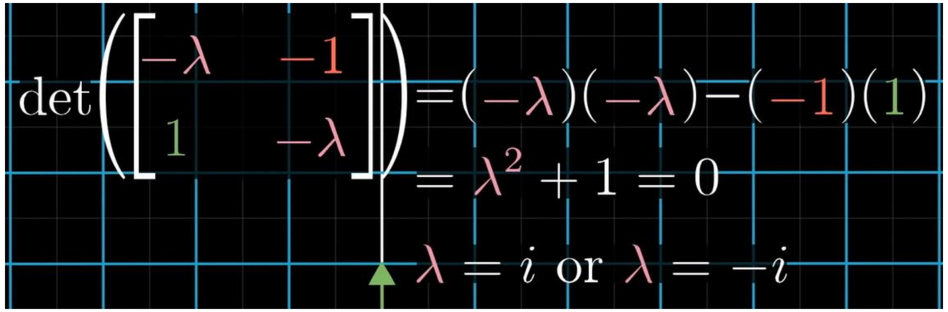




Substitute Eigen Values to compute Eigen Vectors

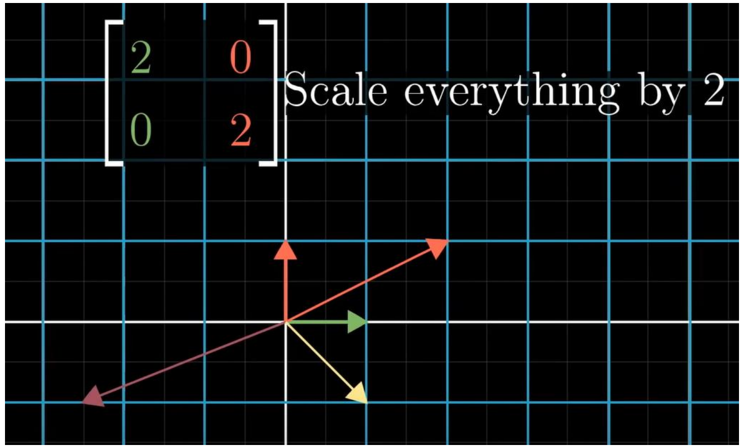
2. There could be no Eigen vectors

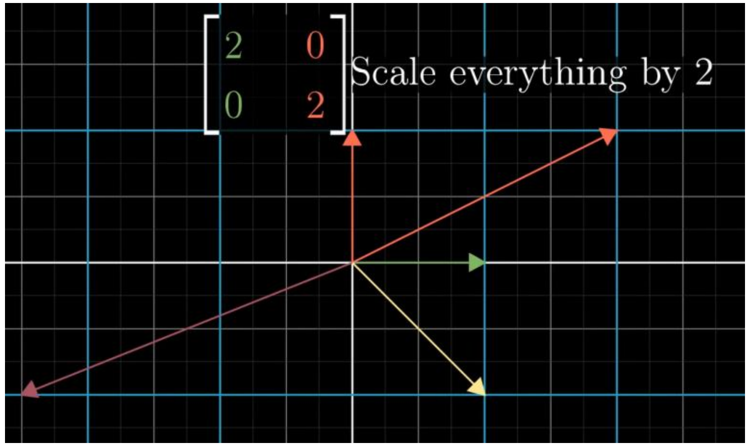




$$\boxed{\det(A - \lambda I) = 0}$$

3. A single Eigen Value can have more than a line full of Eigen Vectors





- Scaling up matrix
- Only Eigen Value = 2
- Every vector in the plane gets to be an Eigen Vector with that Eigen Value

TO-DO LIST

Eigen Values

Example 1. Find the eigenvalues in the system
$$\begin{aligned} x + 4y &= \lambda x \\ 2x + 3y &= \lambda y \end{aligned}$$

Example 2. Find the eigenvalues of A where $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 3 & 2 & -2 \end{pmatrix}$

+ TUTORIAL 1 QUESTIONS – Check Blackboard

Eigen Values + Vectors

Example 1. Find the eigenvectors of $AX = \lambda X$ where $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$

Example 2. Determine the eigenvectors of $\begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 3 & 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \end{pmatrix}$