Question 1

(a) Answer TRUE or FALSE to the following statements

- A state space model consists of transfer functions and frequency response graphs.

[1 mark]

- The general form of the LTI state equation is $\boldsymbol{y} = \mathbf{C}\boldsymbol{x} + \mathbf{D}u$.

[1 mark]

(b) How many state equations are needed to represent a ninth-order system?

[1 mark]

(c) Consider the system shown in Figure 1. Derive a mathematical model in state space form, showing both state space and output equations.

- Assume a linear spring, and a linear friction model $F_f = \mu\dot{x}$, where μ is a friction coefficient.

- The displacement *x(t)* is output.

[12 marks]

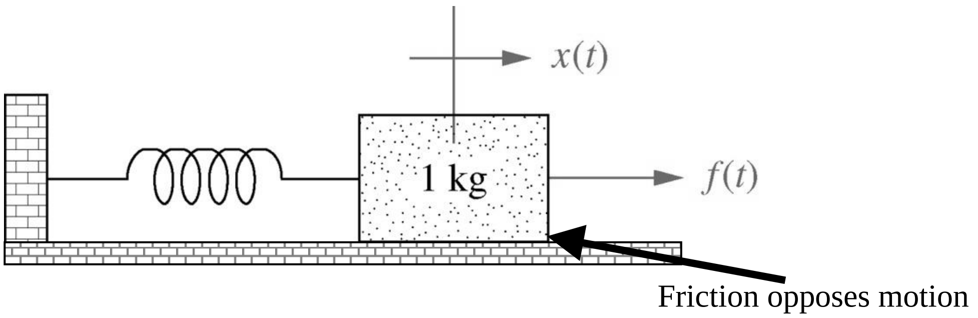

Figure 1: A Simple Mechanical System

(d) Assume that friction coefficient μ=*0.1* Nm/s and spring stiffness *K*=200 N/m. Calculate the poles of the system and plot them on the *s*-plane. Is the system stable?

[10 marks]

Question 2

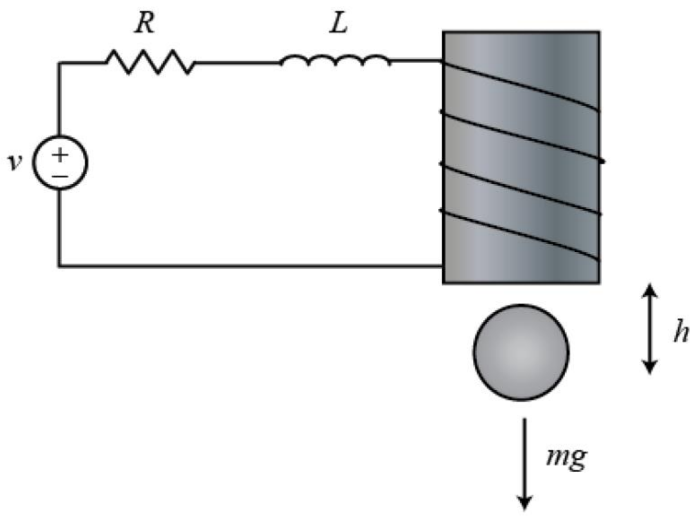

Figure 2: Diagram of a Magnetically Suspended Ball

(a) You are given the following model for a Magnetically Suspended Ball (shown in Figure 3).

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 980 & -2.8 \\ 0 & -100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Where the states are displacement $x_1 = h$ and current $x_2 = i$. Control input is the voltage v(t). You are asked to design a controller for the system.

(i) check the stability of the open-loop system.

[5 marks]

(ii) Show whether the system is controllable.

[3 marks]

(iii) Design a regulator with the following characteristic equation, designed to hold the ball at the reference height h=0:

$$s^2 + 150s + 9000 = 0$$

[10 marks]

(b) Sketch the model as a Block Diagram, including the state feedback controller.

[7 marks]

Question 3

A System contains a 'v^2' damper, shown in Figure 3. Inspecting the free body diagram and summing forces gives:

$$M\ddot{x} + f_D(t) = f(t)$$

Where damping force $f_D(t) = D\dot{x}^2\,(t)$, *D* = *0.1* Nm/s, and applied force $f(t) = 10 + \delta f(t)$

(a) Explain why this model is considered to be nonlinear.

[3 marks]

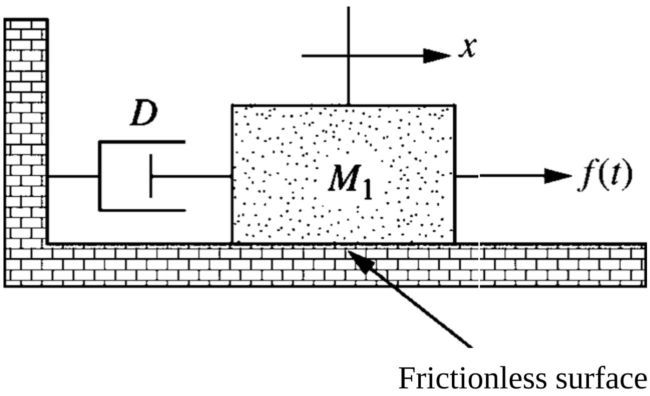

Figure 3: Nonlinear Translational Mechanical System

(b) Linearise the model about the equilibrium point.

[10 marks]

(c) Write the linearised model in LTI state space format

[6 marks]

(i) Can this linearised model be used to assess the system when relatively small forces are applied, causing small deflections of ± 5mm from the equilibrium point? Explain your answer.

[3 marks]

(ii) Can this linearised model be used to assess the system when a large impact is applied, causing the damper to fully compress and extend? Explain your answer.

[3 marks]