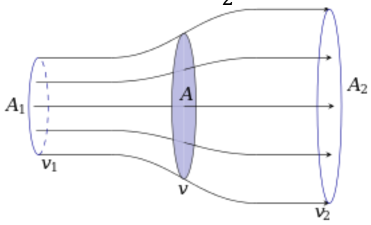
EGR3030 2023-24 TCA solutions

Question 1

a. Gearbox /other mechanical energy transmission
   Icing
   Leading edge erosion to blades (1 mark each for any mentioned. 5 will give full marks)

b. Starting from $P = \frac{1}{2}m(v_1^2 - v_2^2)$, and



Start from: $P = \frac{1}{2}m(v_1^2 - v_2^2)$, and substitute for m=pSv, then simplify.
(3 marks if working shown)

By differentiating P with respect to v_2/v_1 for a given fluid speed v_1 and a given area S,

one finds the *maximum* or *minimum* value for P. The result is that P reaches maximum value when $v_2/v_1 = 1/3$. (4 marks if differentiated and working to the 1/3 ratio shown)

Substituting this value results in $P_{max} = \frac{16}{27} \times \frac{1}{2}\rho S v_1^3$ (2 marks if substitution and working shown)

➔16/27 = 0.593 or 59.3%, then judge the engineer's remarks against this. (2 marks if explained)

C. ii. At a height of 65 m, the wind speed is estimated using:

$$U(z) = U(z_r)\left(\frac{Z}{Z_r}\right)^a$$

$$U(@35m) = U(@2m)\left(\frac{z(50m)}{z_r(2m)}\right)^a = 5\left(\frac{50}{2}\right)^{0.2} = 9.5\ m/s$$ (2 marks if working shown)

$$P_{theor} = 0.5C_p\rho u^3 S$$

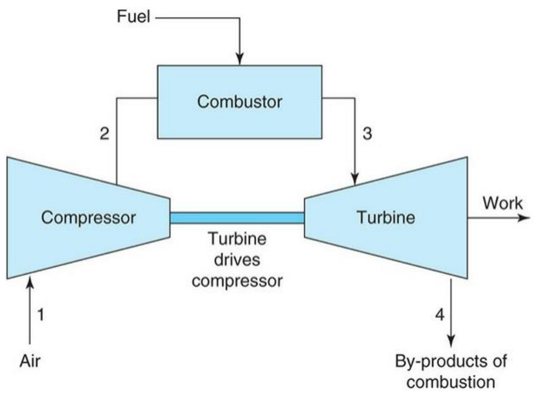
$$P_{actual} = 0.5 \times 0.593 \times 1.225 \times (8.9)^3 \times 706.9 = 221{,}408W$$ (2 marks)

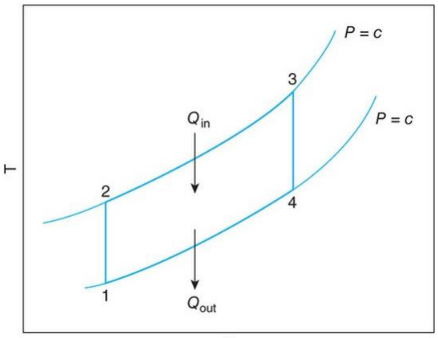
Even at maximum efficiency, only 221kW is produced. Hence, the engineer's claim of 250kW is false. (1 mark)

Question 2

a. i. Any 2 mentioned will give the full 2 marks.
   ii. Rationalising both scenario will give 3 full marks

b. i. annotated schematics





(1 mark) (1 mark)

1-2 Air from the atmosphere is drawn into the system and compressed to the maximum system pressure. (1 mark)
2-3 Fuel is injected into the compressed air, and the mixture is combusted at constant pressure, heating it to the system maximum temperature. (1 mark)
3-4 The combustion products are expanded through a turbine, creating the work output in the form of the spinning turbine shaft. (1 mark)

ii. From the air tables, the values for enthalpy and relative pressure at states 1 and 3 are known, that is, h1 = 295.2 kJ/kg, pr1 = 1.3068, h3 = 1161.1 kJ/kg, pr3 = 167.1. (2 marks)

In order to solve for heat added in the combustor, work output from the turbine, and work required for the compressor, we need to solve for h2 and h4.

Taking h2 first, it is observed that since the compression ratio is 6:1, the relative pressure leaving the compressor is pr2 =pr1(6) = 7.841. The value of h2 can then be obtained by interpolation in the air tables. From the tables, air at T = 490 K has a relative pressure of pr = 7.824, or approximately the same as pr2, so h2 is approximately the enthalpy at T = 490 K from the table, or h2 = 492.7 kJ/kg. (2 marks)
For h4, it is necessary to interpolate between values in the table, as follows. First calculate pr4 =pr3/6 = 27.85. From the tables, air at T = 690 K has a relative pressure of pr = 27.29, and air at T = 700 K has a relative pressure of pr = 28.8, so we calculate an interpolation factor f:

$$f = \frac{27.85 - 27.29}{28.8 - 27.29} = 0.37$$

From the tables, enthalpy values at T = 690 and 700 K are 702.5 and 713.3 kJ/kg, respectively, so h4 can be obtained:

$h4 = 702.5 + 0.37(713.3 - 702.5) = 706.5\ kJ/kg$ (2 marks)

Linear interpolation is commonly used in this way to obtain values not given directly in the tables. It is now possible to calculate heat input, turbine work, and compressor work:

$$q_{in} = h3 - h2 = 1161.1 - 492.7 = 668.4\ kJ/kg \text{ (1 mark)}$$
$$w_{turbine} = h3 - h4 = 1161.1 - 706.5 = 454.6\ kJ/kg \text{ (1 mark)}$$
$$w_{compressor} = h2 - h1 = 492.7 - 295.2 = 197.5\ kJ/kg \text{ (1 mark)}$$

The overall cycle efficiency is then

$\eta_{th} = (w_{turbine} - w_{compressor})/q_{in}$ = 0.43 (1 mark)

Discussion The value of the compressor work in this ideal cycle is 43% of the total turbine work. Furthermore, these ideal compressor and turbine components do not take into account losses that would occur in real-world equipment (1 mark). Losses in the compressor increase the amount of work required to achieve the targeted compression ratio, while losses in the turbine reduce the work output (1 mark). Thus, the actual percentage would be significantly higher than 43% (1 mark). This calculation shows the importance of maximising turbine output (1 mark) and minimising compressor losses (1 mark) in order to create a technically and economically viable gas turbine technology.

Question 3

b. Combustion products whose composition is given are cooled to 25°C. The AF, the percent theoretical air used, and the fraction of water vapor that condenses are to be determined.

Assumptions: Combustion gases are ideal gases.

Properties: The saturation pressure of water at 25°C is 3.1698.

Analysis: Note that we know the relative composition of the products, but we do not know how much fuel or air is used during the combustion process. However, they can be determined from mass balances. The H2O in the combustion gases will start condensing when the temperature drops to the dew-point temperature. For ideal gases, the volume fractions are equivalent to the mole fractions.

i. Considering 100 kmol of dry products for convenience, the combustion equation can be written as:

xC 8H 18 + a (O_2 + 3.76N_2) → 10.02CO_2 + 0.88CO + 5.62O_2 + 83.48N_2 + bH_2O

ii.

The unknown coefficients x, a, and b are determined from mass balances

N_2: 3.76a = 83.48 → a = 22.20

C: 8x = 10.02 + 0.88 → x = 1.36

H: 18x = 2b → b = 12.24

O_2: a = 10.02 + 0.44 + 5.62 + b/2 → 22.20 = 22.20

The air–fuel ratio is determined by taking the ratio of the mass of the air to the mass of the fuel

AF = m_{air}/m_{fuel} = [(16.32 × 4.76 kmol)(29 kg/kmol)] / [(8 kmol)(12 kg/kmol) + (9 kmol)(2 kg/kmol)] = 19.76 kg air/kg fuel [full 5 marks if working shown if only answer step, 3 marks]

iii. To find the percentage of theoretical air used, we need to know the theoretical amount of air, which is determined from the theoretical combustion equation of the fuel,

C_8H_{18} + a_{th} (O_2 + 3.76 N_2) → 8 CO_2 + 9H_2O + 3.76 $a_{th}N_2$

O_2 balance: a_{th} = 8 + 4.5 → a_{th} = 12.5

Then, Percentage of theoretical air = $m_{air,act}$ /$m_{air,th}$ = $N_{air,act}$ /$N_{air,th}$

= [(16.32)(4.76) kmol] /[(12.50)(4.76) kmol]

= 131% [5 marks]

iv. For each kmol of fuel burned, 7.37 + 0.65 + 4.13 + 61.38 + 9 = 82.53 kmol of products are formed, including 9 kmol of H2O. Assuming that the dew-point temperature of the products is above 25°C, some of the water vapor will condense as the products are cooled to 25°C. If Nw kmol of H2O condenses, there will be (9 – Nw) kmol of water vapor left in the products. The mole number of the products in the gas phase will also decrease to 82.53 – Nw as a result. By treating the product gases (including the remaining water vapor) as ideal gases, Nw is determined by equating the mole fraction of the water vapor to its pressure fraction

$N_v/N_{prod,gas}$ = Pv/Pprod

(9 – Nw)/(82.53 – Nw) = 3.1698 kPa /100 kPa

Solving gives, Nw = 6.59 kmol [full 5 marks if rationalised, else 3 marks]

Question 4

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in Fig. 5–31. (*a*) Compare the magnitudes of Δh, Δke, and Δpe. (*b*) Determine the work done per unit mass of the steam flowing through the turbine. (*c*) Calculate the mass flow rate of the steam.

SOLUTION The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** The system is adiabatic and thus there is no heat transfer.

Analysis We take the *turbine* as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, work is done by the system. The inlet and exit velocities and elevations are given, and thus the kinetic and potential energies are to be considered.

(*a*) At the inlet, steam is in a superheated vapor state, and its enthalpy is (1 mark)

$$\left.\begin{array}{l} P_1 = 2\text{ MPa} \\ T_1 = 400°\text{C} \end{array}\right\} \; h_1 = 3248.4\text{ kJ/kg} \qquad \text{(Table A–6)}$$

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is (1 mark)

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)]\text{ kJ/kg} = 2361.01\text{ kJ/kg}$$ (2 marks)

Then (2 marks)

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4)\text{kJ/kg} = \mathbf{-887.39\ kJ/kg}$$

$$\Delta\text{ke} = \frac{V_2^2 - V_1^2}{2} = \frac{(180\text{ m/s})^2 - (50\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.95\ kJ/kg}$$

(2 marks with discussion below)

$$\Delta\text{pe} = g(z_2 - z_1) = (9.81\text{ m/s}^2)[(6 - 10)\text{ m}]\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{-0.04\ kJ/kg}$$

Discussion Two observations can be made from these results. First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. This is typical for most engineering devices. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high. Yet the change in kinetic energy is a small fraction of the change in enthalpy (less than 2 percent in our case) and is therefore often neglected.

(*b*) The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} \overset{\nearrow 0 \text{ (steady)}}{} = 0$$ (1 mark)

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{W}_{out} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) \qquad (\text{since } \dot{Q} = 0)$$ (1 mark)

Dividing by the mass flow rate $\dot{m}$ and substituting, the work done by the turbine per unit mass of the steam is determined to be

$$w_{out} = -\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = -(\Delta h + \Delta ke + \Delta pe)$$ (2 marks)

$$= -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = \mathbf{872.48 \text{ kJ/kg}}$$ (1 mark)

(*c*) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{out}}{w_{out}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = \mathbf{5.73 \text{ kg/s}}$$ (2 marks)

4. d.

A sketch of each highlighting any major differences will give 2 marks each.

One rationale is: In the combined cycle systems, two different types of turbine technologies, namely, gas and steam, are used for the single purpose of generating electricity. Low grade steam which is the output of the steam turbine will be at saturated conditions.

Any two rationales provided will give 3 marks each.

Question 5

5a. any 3 differences between energy and exergy will give full marks (2 each). Note that just stating the equations alone is one difference.

5.b. Show that:

$$X_2 - X_1 = \int_1^2 \left(1 - \frac{T_o}{T_b}\right)\delta Q - [W - P_o(V_2 - V_1)] - T_o S_{gen}$$

Analysis We consider a general closed system (a fixed mass) that is free to exchange heat and work with its surroundings (Fig. 8–34). The system undergoes a process from state 1 to state 2. Taking the positive direction of heat transfer to be *to* the system and the positive direction of work transfer to be *from* the system, the energy and entropy balances for this closed system can be expressed as

Energy balance: $E_{in} - E_{out} = \Delta E_{system} \rightarrow Q - W = E_2 - E_1$ (1 mark)

Entropy balance: $S_{in} - S_{out} + S_{gen} = \Delta S_{system} \rightarrow \int_1^2 \left(\frac{\delta Q}{T}\right)_{boundary} + S_{gen} = S_2 - S_1$ (1 mark)

Multiplying the second relation by T_0 and subtracting it from the first one gives

$$Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_{boundary} - W - T_0 S_{gen} = E_2 - E_1 - T_0(S_2 - S_1)$$ (2 marks)

However, the heat transfer for the process 1-2 can be expressed as $Q = \int_1^2 \delta Q$ and the right side of the preceding equation is, from Eq. 8–17, $(X_2 - X_1) - P_0(V_2 - V_1)$. Thus, (1 mark)

$$\int_1^2 \delta Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_{boundary} - W - T_0 S_{gen} = X_2 - X_1 - P_0(V_2 - V_1)$$ (1 mark)

Letting T_b denote the boundary temperature and rearranging give

$$X_2 - X_1 = \int_1^2 \left(1 - \frac{T_o}{T_b}\right)\delta Q - [W - P_o(V_2 - V_1)] - T_o S_{gen}$$

a. i. Given the initial data, partial pressures for each of the components of the reaction are $PH_2 = 1$, $PO_2 = 0.21$, and $PH_2O = 0.3$. Substituting into

$$E = E^0 + \frac{RT}{2F}\ln\left(\frac{P_{H_2}P_{O_2}{}^{0.5}}{P_{H_2O}}\right)$$ (2 marks)

gives:

$$E = 1.23 + \frac{8.314(298)}{2(96400)}\ln\left(\frac{1(0.21)^{0.5}}{0.3}\right)$$ (2 marks)

$$= 1.23 + 0.0054 = 1.24 \text{ V}$$

(1 marks)

ii. If the fuel cell is operated with pure oxygen, $PO_2 = 1$. Thus $\beta_{new} = 1$ while $\beta_{old} = 0.21$. Rewriting

$$\Delta E = E^{new} - E^{old} = E(\alpha^{new}, \beta^{new}, \delta^{new}, p^{new}) - E(\alpha^{old}, \beta^{old}, \delta^{old}, p^{old})$$ (2 marks)

to include only nonzero terms gives:

$$\Delta E = E^{new} - E^{old} = \frac{RT}{2F}\{\ln[(\beta^{new})^{0.5}] - \ln[(\beta^{old})^{0.5}]\}$$ (2 marks)

$$= (0.5)\frac{RT}{2F}[\ln(1) - \ln(0.21)] = 0.01$$ (2 marks)

Thus $E_{new} = E_{old} + \Delta E = 1.24 + 0.01 = 1.25$ V (2 marks)