Tutorial Sheet 2 – Solving Systems of ODE's

1. Find the general solutions to the following 1st order systems of differential equations:

(a)
$$\begin{aligned} x_1' &= 2x_1 - x_2 \\ x_2' &= 3x_2 \end{aligned}$$

(b)
$$\begin{aligned} x_1' &= 7x_1 - 2x_2 \\ x_2' &= x_1 + 4x_2 \end{aligned}$$

(c)
$$\begin{aligned} x_1' &= x_1 - x_2 + 2x_3 \\ x_2' &= -3x_1 - 2x_2 + 3x_3 \\ x_3' &= 2x_1 - x_2 + x_3 \end{aligned}$$

2. Find the particular solutions to the following systems of differential equations:

(a)
$$\begin{aligned} x_1' &= 2x_1 - x_2 \\ x_2' &= 3x_2 \end{aligned} \quad \text{when } x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(b)
$$\begin{aligned} x_1' &= 7x_1 - 2x_2 \\ x_2' &= x_1 + 4x_2 \end{aligned} \quad \text{when } x(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

(c)
$$\begin{aligned} x_1' &= x_1 - x_2 + 2x_3 \\ x_2' &= -3x_1 - 2x_2 + 3x_3 \\ x_3' &= 2x_1 - x_2 + x_3 \end{aligned} \quad \text{when } x(0) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

3. Write the following 2nd order differential equations in 1st order form:

(a) $x'' - 10x = 0$

(b) $5x'' - 3x' + 6x = 0$

(c)
$$\begin{aligned} 3x_1'' - 2x_2' + 4x_1 &= 0 \\ 6x_2'' - 4x_1' + 6x_2 - 5x_1 &= 0 \end{aligned}$$

(d)
$$\begin{aligned} 5x_1'' + 6x_2 - 7x_1' - 6x_1 + 5x_2' &= 0 \\ 4x_2'' - 6x_2 + 5x_1 - 4x_1' - 3x_2' &= 0 \end{aligned}$$

Answers

1.

(a) $x(t) = a_1\begin{pmatrix}0\\0\end{pmatrix}e^{2t} + a_2\begin{pmatrix}1\\-1\end{pmatrix}e^{3t}$ (b) $x(t) = a_1\begin{pmatrix}1\\1\end{pmatrix}e^{5t} + a_2\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix}e^{6t}$

(c) $x(t) = a_1\begin{pmatrix}1\\5\\1\end{pmatrix}e^{-2t} + a_2\begin{pmatrix}1\\12\\5\end{pmatrix}e^{-t} + a_3\begin{pmatrix}1\\0\\1\end{pmatrix}e^{3t}$

2.

(a) $x(t) = 1\begin{pmatrix}1\\-1\end{pmatrix}e^{3t} \quad (a_1 = 0,\ a_2 = 1)$

(b) $x(t) = -1\begin{pmatrix}1\\1\end{pmatrix}e^{5t} + 4\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix}e^{6t} \quad (a_1 = -1,\ a_2 = 4)$

(c) $x(t) = -0.8\begin{pmatrix}1\\5\\1\end{pmatrix}e^{-2t} + 0.5\begin{pmatrix}1\\12\\5\end{pmatrix}e^{-t} + 1.3\begin{pmatrix}1\\0\\1\end{pmatrix}e^{3t} \quad (a_1 = -0.8,\ a_2 = 0.5,\ a_3 = 1.3)$

3.

(a)
$$y_1' = y_2$$
$$y_2' = 10y_1$$

(b)
$$y_1' = y_2$$
$$y_2' = \frac{3}{5}y_2 - \frac{6}{5}y_1$$

(c)
$$y_1' = y_3$$
$$y_2' = y_4$$
$$y_3' = \frac{2}{3}y_4 - \frac{4}{3}y_1$$
$$y_4' = \frac{2}{3}y_3 - y_2 + \frac{5}{6}y_1$$

(d) $y_1' = y_3$

$y_2' = y_4$

$y_3' = -\frac{6}{5}y_2 + \frac{7}{5}y_3 + \frac{6}{5}y_1 - y_4$

$y_4' = -\frac{3}{2}y_2 - \frac{5}{4}y_1 + y_3 + \frac{3}{4}y_4$