State-Space Control (EGR3032)

Week 1

Learning Objectives

- ✓ **LO1** Apply relevant principles and techniques in continuous-time in state-space methodologies
- ✓ **LO2** Use engineering software to fit models to data and design software representation of systems
- ✓ **LO3** Adopt a systems approach to the solution of engineering problems, find technical information and acquire experimental data
- ✓ **LO4** Present, defend and justify a solution

Assessments

Coursework:

Modelling and Control of an Active Vehicle Suspension

Weight: **50%**

Final Exam:

Time Constrained Assessment

4 out of 5 Questions

Weight: **50%**

Recommended Readings

Textbooks

- Linear State-Space Control Systems (2007)

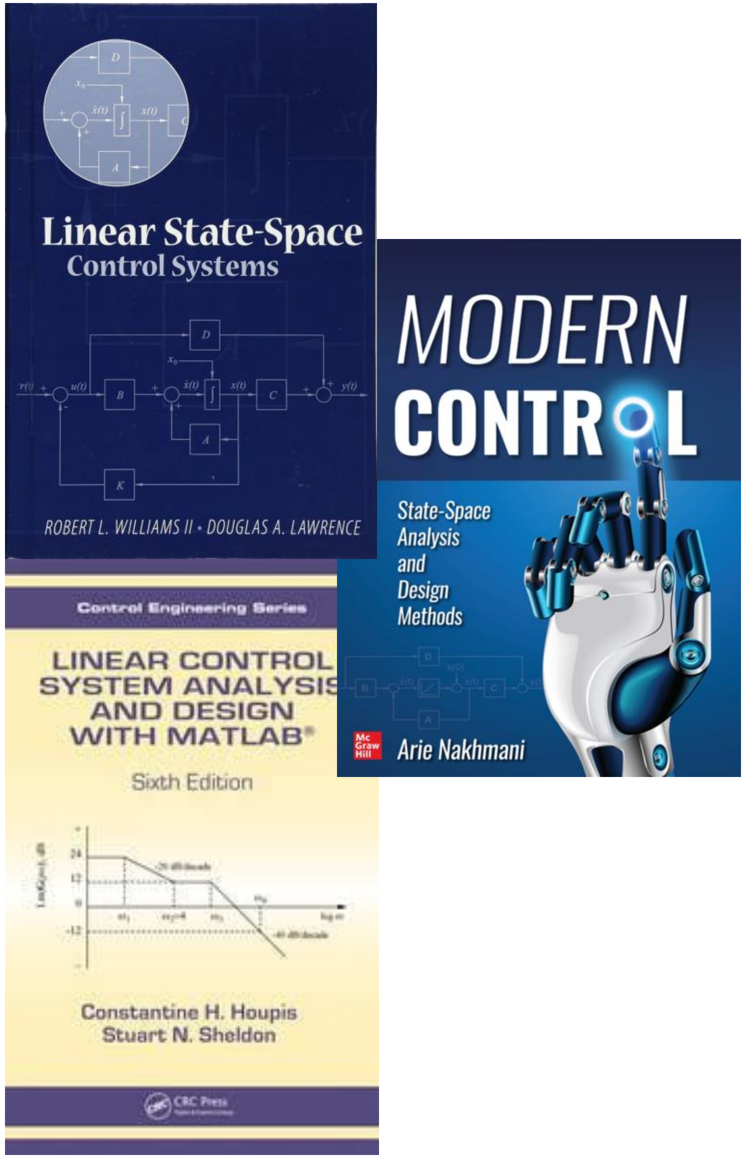
  — by *Robert L. Williams*
  & Douglas A. Lawrence

- Modern Control: State-Space Analysis and Design Methods (2020)

  — by *Arie Nakhmani*

- Linear Control System Analysis and Design with MATLAB (2013)

  — by *Constantine H. Houpis*
  & Stuart N. Sheldon



What is State-Space Control?

- The **state** of a system describes enough about the system to *determine* its *future behaviour* in the absence of any external forces affecting the system.

- *Control systems engineering* activities focus on implementation of control systems mainly derived by *mathematical modelling* of a diverse range of systems.

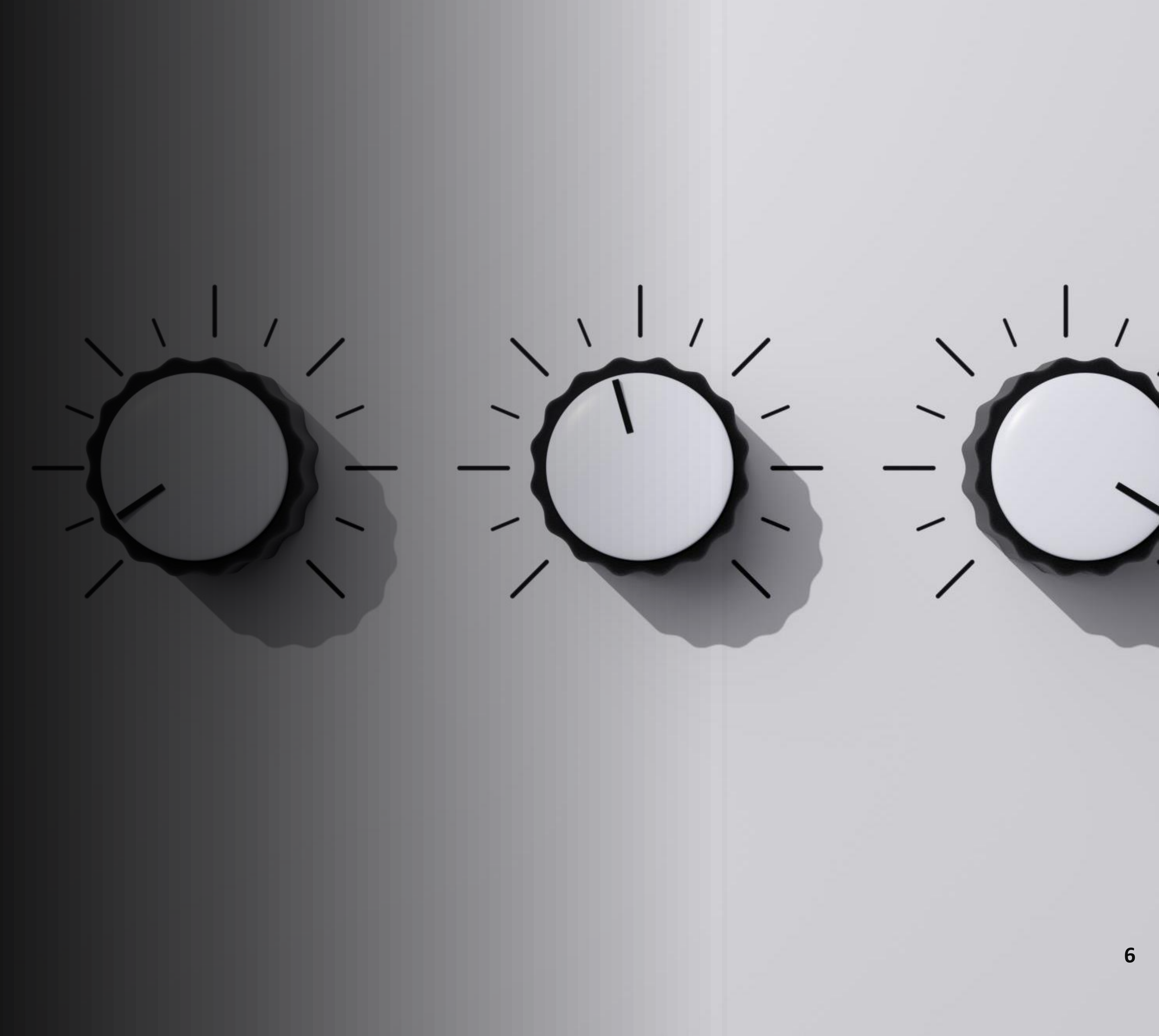
STATE SPACE CONTROL

- A *state variable* is one of the set of variables that are used to *describe* the mathematical "state" of a *dynamical system*.

- The *set* of possible combinations of *state variable* values is called the **State-Space** of the system.

- The **State-Space Control** method takes the *differential equations* that describe the time domain of the system and analyses them in *vector form* using state variables.

Control systems

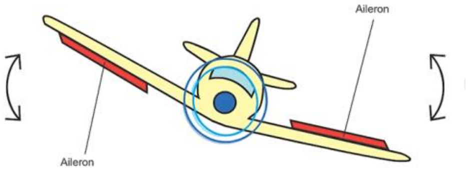


Control System Engineering

Control systems

Science and industry

- from steering ships and planes to guiding missiles, …
- our homes: toaster, washing machine, microwave, …



Nature

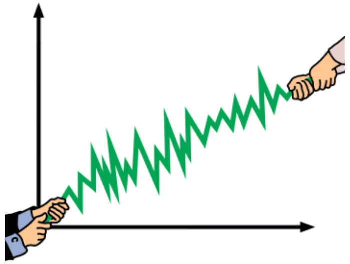
Hummingbird staying fixed in the air, …

Human body

Pancreas, eyes, hand, …

economic and psychological system

Control of Investment, stabilise the Foreign Exchange Rates , …



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What is the purpose of designing a controller?

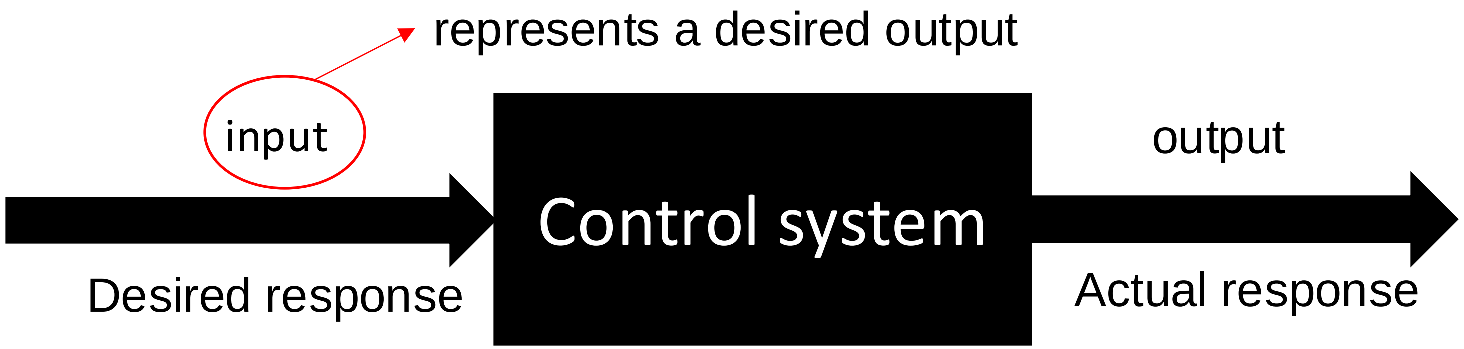
Control system design aim: Reach a target (desired value)

A control system consists of interconnected components to achieve a desired purpose

Target
- $r = cte$ (regulator problem)
- $r = r(t)$ (tracking problem)

Regulation example: **Hummingbird**

https://www.youtube.com/watch?v=jhl892dHqfA



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Hummingbird regulation control

Control engineering

- Deals with the design & implementation of control systems using *linear, time-invariant* (LTI) mathematical models representing actual physical *nonlinear, time-varying* systems with parameter uncertainties in the presence of external disturbances.

- A challenge for control engineers today is to be able to create simple, yet reliable and accurate mathematical models of many of our modern, complex, interrelated, and interconnected systems.

Control System Engineering

- Focuses on the modelling of a wide assortment of physical systems and using those models to design controllers that will cause the *closed-loop systems* to possess desired performance characteristics:
  - Stability
  - Steady-state tracking with prescribed maximum errors
  - Transient tracking (percent overshoot, settling time, rise time, and time to peak)
  - Rejection of external disturbances, and robustness to modelling uncertainties.

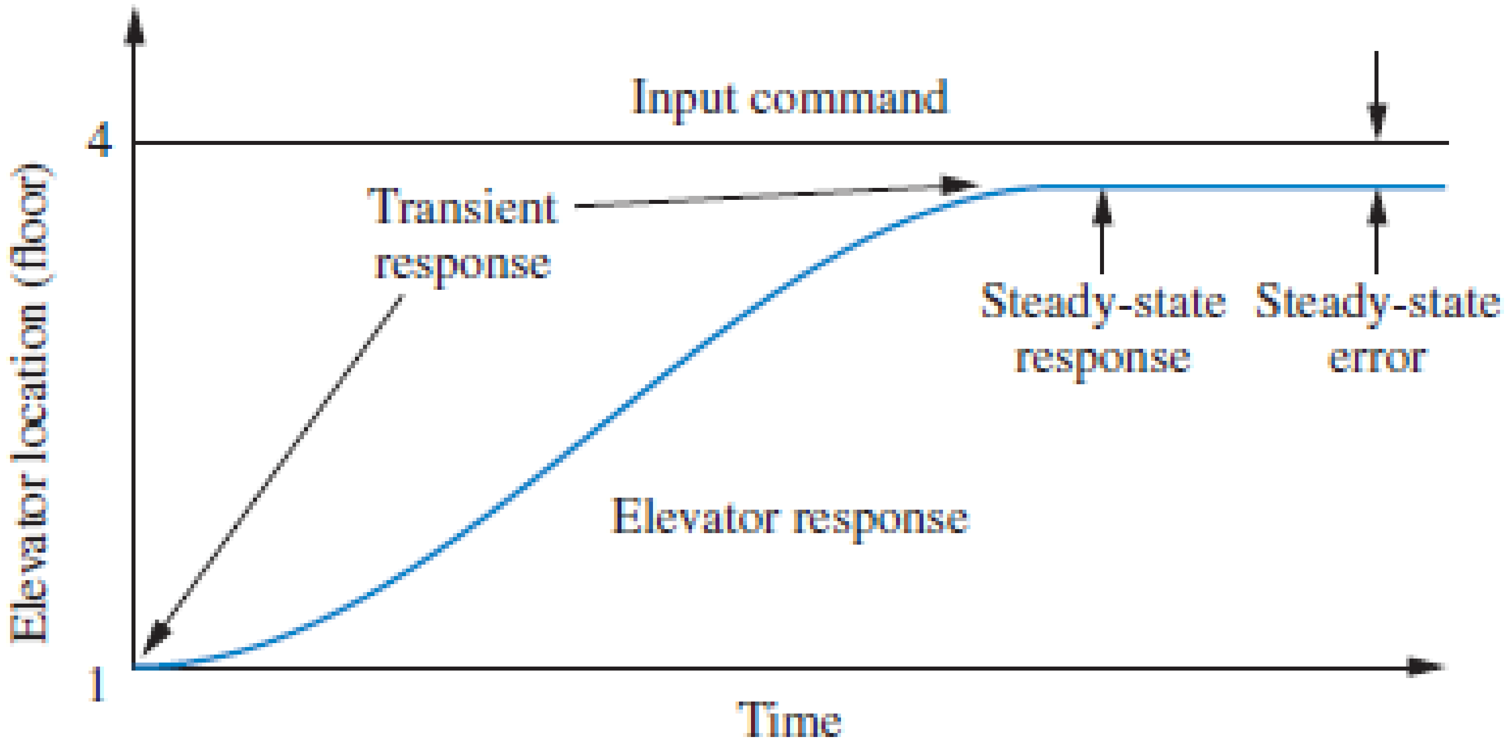
Control systems analysis and design focuses on three primary objectives:

1. Producing the desired transient response (speed of the control system)

2. Reducing steady-state errors (accuracy of the control system)

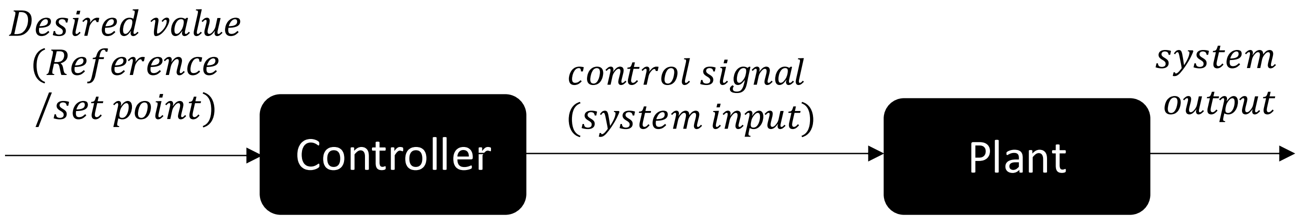
3. Achieving stability

example: **elevator**



Open-loop control systems

- Systems in which the output has no effect on the control action
- The output is neither measured nor fed back for comparison with the input.
- Practical example:
  - Washing machine (Soaking, washing, and rinsing in the washer operate on a time basis. The machine does not measure the output signal, that is, the cleanliness of the clothes.)
  - Toaster

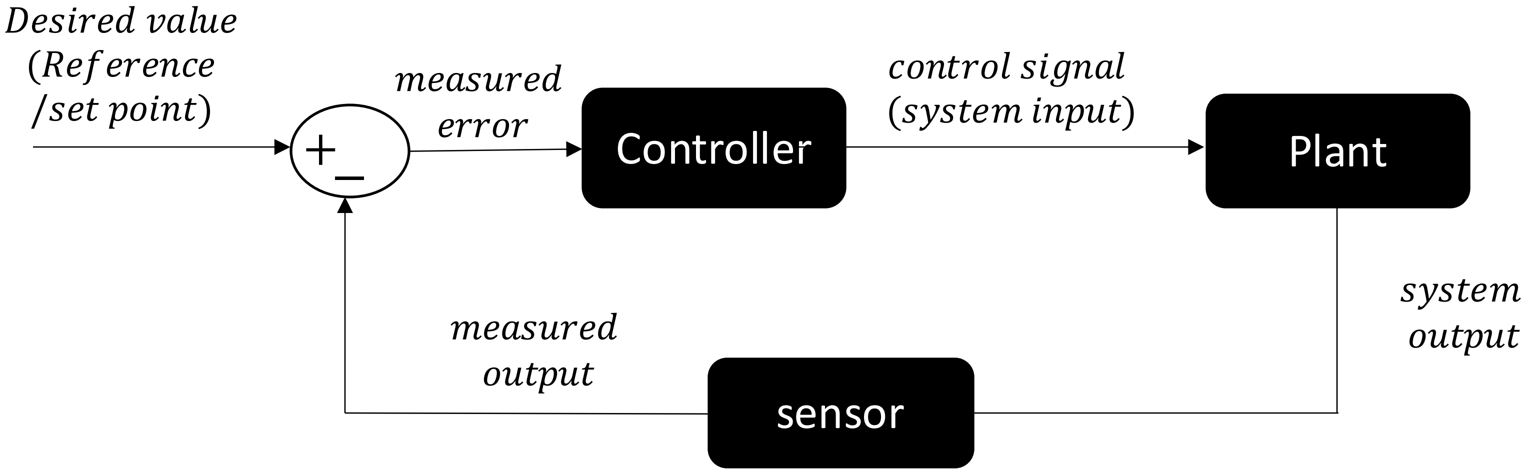


Systems in which the output quantity has no effect upon the input quantity are called *open-loop control systems.*

An open-loop control system utilizes an actuating device to control the process directly without using feedback

Closed-loop control systems

- Feedback control systems are often referred to as *closed-loop control* systems.
- The term closed-loop control always implies the use of feedback control action in order to reduce system error.



home thermostat

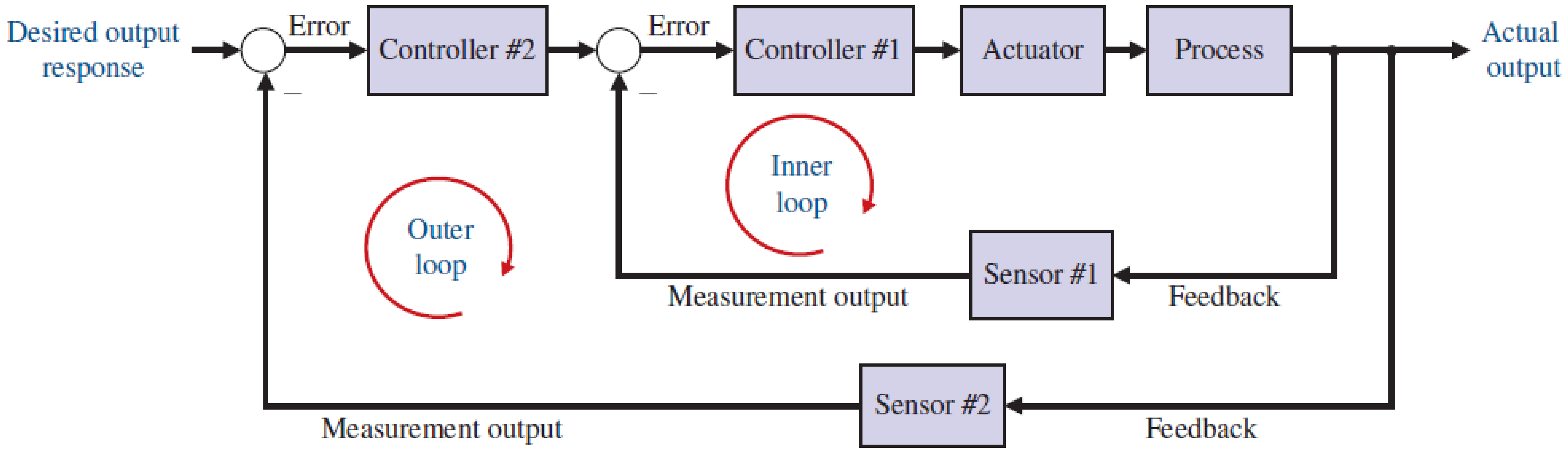
Closed-loop control systems:

A closed-loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output (reference or command).

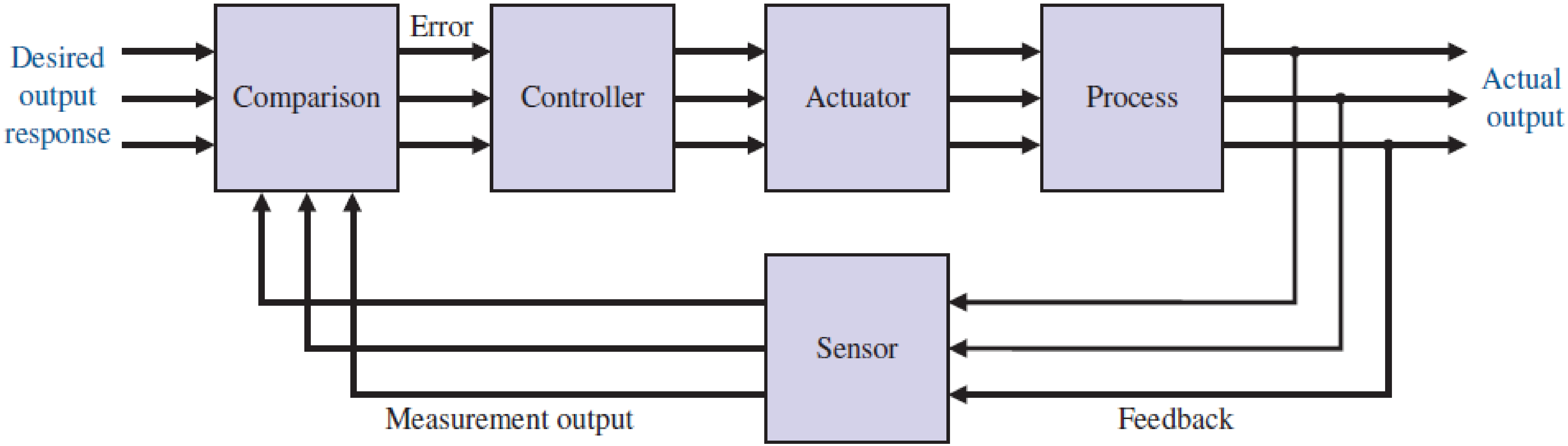
A closed-loop control has many advantages over open-loop control:

- the ability to reject external disturbances
- improve measurement noise attenuation.

Multiloop feedback system with an inner loop and an outer loop



Multivariable control system

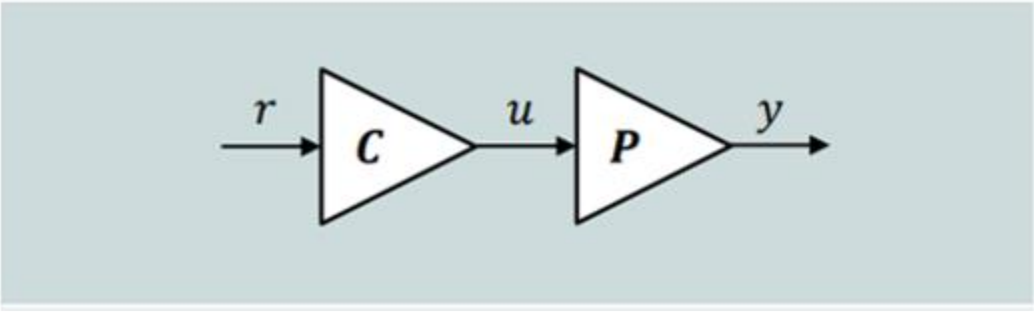


Feedback Miracle:

Consider simple system with constant gain P

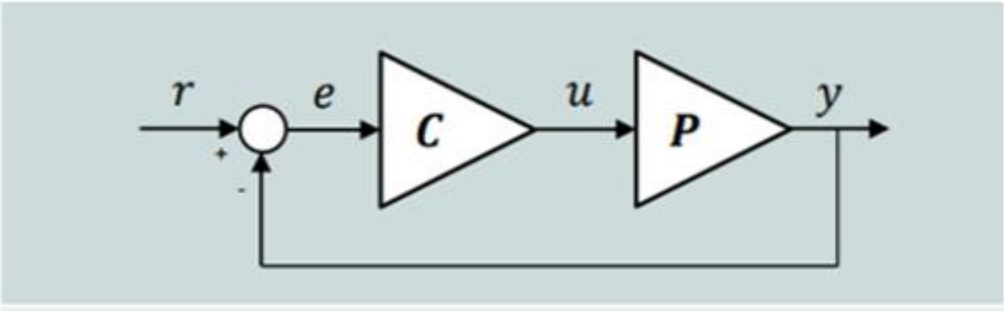
Open-loop vs closed-loop control systems

Open-loop



$$y = Pu, \qquad u = Cr, \qquad y = CPr$$

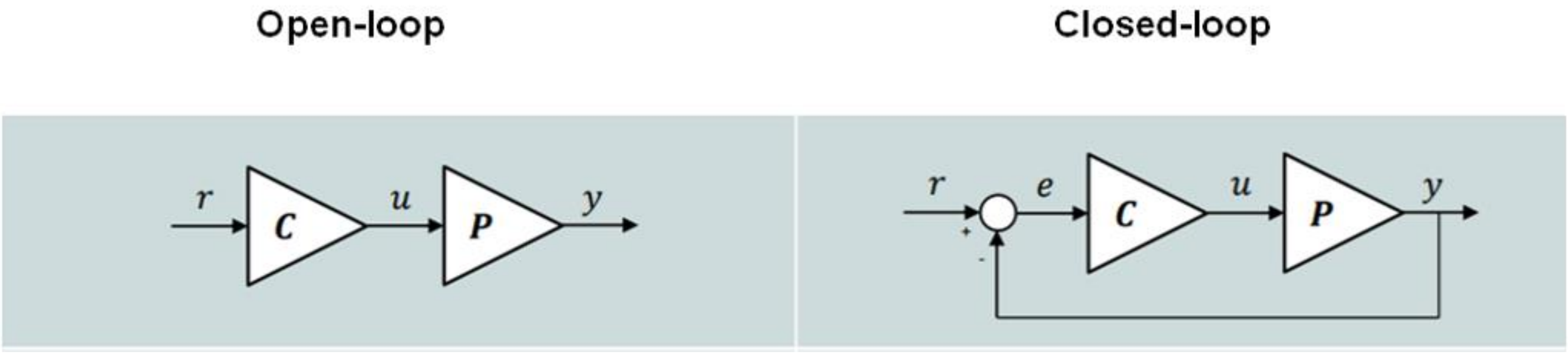
Closed-loop



$$e = r - y, \qquad y = CPe = CP(r - y)$$

$$y(1 + CP) = CPr \rightarrow y = \frac{CP}{1 + CP} r$$

First aim: *Regulation or tracking* $(\boldsymbol{y} \approx \boldsymbol{r})$



First aim: *Regulation or tracking* $(\boldsymbol{y} \approx \boldsymbol{r})$

We want $\boldsymbol{y} \approx \boldsymbol{r}$

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Second aim: *Compensation for disturbances while* $\boldsymbol{y \approx r}$

Open-loop | **Closed-loop**

Open-loop:

$$y = d + Pu, \qquad u = Cr, \qquad y = d + CPr$$

Closed-loop:

$$e = r - y, \qquad y = d + CPe = d + CP(r - y)$$

$$y(1 + CP) = d + CPr \rightarrow y = \frac{CP}{1 + CP} r + \frac{1}{1 + CP} d$$

Control System Design

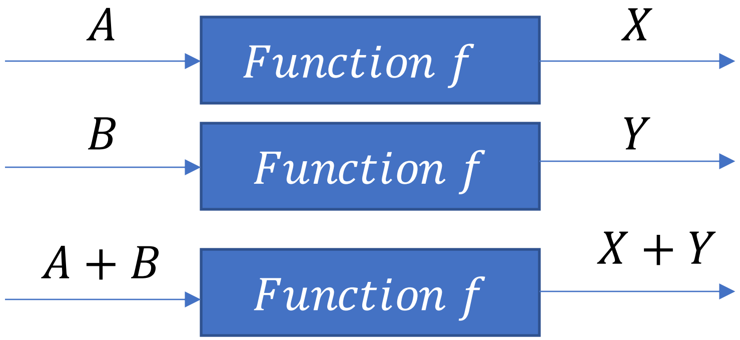
1. Establishment of goals, control variables, and specification

2. System definition and modelling

3. Control system design, simulation, and analysis

Linear Systems Properties

- system is called linear if the ***principle of superposition*** applies.



A function that satisfies the superposition principle is called a linear function.

Superposition can be defined by two simpler properties

Additivity $f(x_1 + x_2) = f(x_1) + f(x_2)$

Homogeneity $f(ax) = af(x)$

Examples.

Recap:

Superposition can be defined by two simpler properties

Additivity $f(x_1 + x_2) = f(x_1) + f(x_2)$

Homogeneity $f(ax) = af(x)$

Prove if the following systems are linear functions?

- $f(x) = 2x$
- $f(x) = 3x^2$

$f(x_1) = 2x_1,\ f(x_2) = 2x_2$

$f(x_1) + f(x_2) =$?

$f(ax_1) =$?

$f(x_1) = 3x_1^2,\ f(x_2) = 3x_2^2$

$f(x_1) + f(x_2)$ = ?

$f(ax_1)$ = ?

State-space control (Modern Control) vs Classical control

Classical control (PID control)

- linear time invariant
- single-input, single-output systems
- frequency-domain approach

Modern Control (State-space control)

- multiple-input, multiple-output
- linear or nonlinear
- time invariant or time varying
- time-domain approach

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