

Use Euler's method to find the solution to the differential equation $dy/dx=3x+4y$ at $x=1$ with the initial condition $y(0)=0$ and step size $h=0.25$.

Euler's method uses iterative equations to find a numerical solution to a differential equation. The following equations

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + h * f(x_n, y_n)$$

are solved starting at the initial condition and ending at the desired value. $f(x_n, y_n)$ is the solution to the differential equation.

In this problem,

$$f(x, y) = \frac{dy}{dx} = 3x + 4y$$

$$h = 0.25$$

Starting at the initial point $(x_0, y_0) = (0, 0)$

$$x_1 = x_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + h * f(x_0, y_0) = 0 + 0.25 * (3 * 0 + 4 * 0) = 0$$

$$(x_1, y_1) = (0.25, 0)$$

$$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$$

$$y_2 = y_1 + h * f(x_1, y_1) = 0 + 0.25 * (3 * 0.25 + 4 * 0) = 0.1875$$

$$(x_2, y_2) = (0.5, 0.1875)$$

We continue using Euler's method until $x = 1$. The results of Euler's method are in the table below.

n	x_n	y_n
0	0	0
1	0.25	0
2	0.5	0.1875
3	0.75	0.75
4	1	2.0625