EGR3032 Tutorial 1 – Introduction and Fundamentals

1. A SISO system is modelled by the differential equations

$$\frac{d^2 y}{d t^2}+2\frac{dy}{d t}+2\,y=u$$

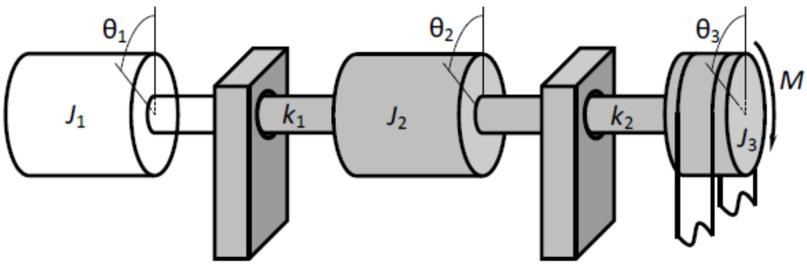
(i) find the transfer function $Y(s)/U(s)$. Find the system poles and decide whether the plant is stable.

(ii) find the state-space equations, using the definition $x_1=y\,,x_2=\dot{y}$.

2. A state-space system has one input and two outputs:

$$\dot{x}_1=x_1+x_2\dot{x}_2=x_3\dot{x}_3=x_1-x_2+u y_1=x_1 y_2=x_1+x_2+u\;A,B,C,D\cdot$$

3. A mechanical system comprises three rotors mounted on a flexible shaft and supported by two bearings.



The shaft is driven by a belt, exerting moment $M(t)$ on the third rotor. How many degrees of freedom does the system have? How many state variables are required to model the system? Assuming all angles are measured, what are the dimensions of the four state-space matrices?   [There is no need to derive the equations of motion!]

4. [Use Matlab for this question]. In Example 1 it was found that proportional control was not effective in improving the speed of response of the manipulator. Use transfer function analysis (Matlab commands ‘step’ and ‘feedback’ to design a PID controller to try to improve the step response. Create a Matlab script, then iterate on the PID gains in a systematic way to determine whether classical control is capable of improving response speed, while maintaining overshoot no worse than 10%.

5. Recall that the eigenvalues of a square matrix A are found from the determinant equation $|A-\lambda I|=0$. Find the eigenvalues of matrix A from question 1 and comment. (This should be done using ‘pen and paper’, but you can check your result using Matlab).

6. A DC electric motor is driven by an input voltage $v_a(t)$. Two outputs are measured, the rotor (armature) angle $\theta(t)$ and the angular velocity $\omega(t)=\dot{\theta}(t)$. The armature generates a ‘back-emf’ (voltage drop) $v_b=K_b\,\omega$. The motor has inductance L and resistance R. This

leads to a voltage balance given by:

$$L\frac{d\,i_a}{dt}+R\,i_a+K_b\,\omega=v_a \qquad (1)$$

where i_a is the armature current. Motor torque is proportional to this current:

$$T_m=K_m\,i_a \qquad (2)$$

which in turn generates an angular acceleration of the rotor:

$$J\frac{d\omega}{dt}+c\,\omega=T_m \qquad (3)$$

where c represents viscous damping at the bearings and J is moment of inertia.

Use equations (1)-(3) to find a state-space model of the DC motor using variables $u=v_a, x_1=\theta, x_2=\omega, x_3=i_a, y_1=\theta, y_2=\omega$. Find the elements of the four state-space matrices in terms of the parameters mentioned.