

$$\boxed{\frac{dy}{dx} = y - x}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$u \frac{dv}{dx} + v \frac{du}{dx} = y - x = uv - x$$

$$\begin{cases} u \left( \frac{dv}{dx} - v \right) = 0 & \textcircled{1} \\ v \frac{du}{dx} = -x & \textcircled{2} \end{cases}$$

$$\textcircled{1} \Rightarrow \frac{dv}{dx} = v$$

$$v = e^x + C_1$$

$$C_1 = 0 \Rightarrow v = e^x$$

$$\textcircled{2} \Rightarrow \frac{du}{dx} = -\frac{x}{v} = -\frac{x}{e^x} = -xe^{-x}$$

$$u = \int -xe^{-x} dx = \int xe^{-x} d(-x)$$

$$= \int x de^{-x} = xe^{-x} - \int e^{-x} dx$$

$$= xe^{-x} + e^{-x} + C_2$$

$$y = uv = (xe^{-x} + e^{-x} + C_2)e^x$$

$$= x + 1 + C_2 e^x$$

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