1)

$$u = 4t \cdot \frac{y^2}{2} + f(t)$$

$$= 2ty^2 + f(t)$$

2) $$u = 2 \cdot \frac{t^2}{2} \cos\theta + f(t)$$

$$= t^2 \cos\theta + f(t)$$

$\theta = 0$, $u = 2t$ $\Rightarrow$ $f(t) = 2t - t^2$

$$\therefore \quad u = t^2(\cos\theta - 1) + 2t$$

3) $$\frac{\partial u}{\partial x} = 8e^y \sin 2x + f_1(x)$$

$y = 0$, $\frac{\partial u}{\partial x} = \sin x$ $\Rightarrow$ $f_1(x) = \sin x - 8\sin 2x$

$$\frac{\partial u}{\partial x} = (8e^y - 8)\sin 2x + \sin x$$

$$u = 8(e^y - 1)\left(-\frac{1}{2}\cos 2x\right) - \cos x + f_2(y)$$

$x = \frac{\pi}{2}$, $u = 2y^2 \Rightarrow f_2(y) = 2y^2 - 4(e^y - 1)$

$u = -4e^y \cos 2x + 4\cos 2x - \cos x$

$+ 2y^2 - 4e^y + 4$

4) $\frac{\partial u}{\partial x} = y\left(\frac{4}{3}x^3 - x\right) + f_1(y)$

$f_1(y) = \cos 2y$

$u = y\left(\frac{1}{3}x^4 - \frac{1}{2}x^2\right) + x\cos 2y + f_2(y)$

$f_2(y) = \sin y$

$\therefore u = y\left(\frac{x^4}{3} - \frac{x^2}{2}\right) + x\cos 2y + \sin y$