Solving PDE's by Direct Partial Integration

1) Determine the general solution of $\frac{\partial u}{\partial y} = 4ty$

Answer

$$u = 2ty^2 + f(t)$$

2) Solve
$$\frac{\partial u}{\partial t} = 2t \cos \theta$$
 given that $u = 2t$ when $\theta = 0$

Answer

$$u = t^2(\cos\theta - 1) + 2t$$

3) Solve
$$\frac{\partial^2 u}{\partial x \partial y} = 8e^y \sin 2x$$
 given that at $y = 0$, $\frac{\partial u}{\partial x} = \sin x$ and at $x = \frac{\pi}{2}$, $u = 2y^2$

Answer

$$u = -4e^y \cos 2x - \cos x + 4\cos 2x + 2y^2 - 4e^y + 4$$

4) Solve
$$\frac{\partial^2 u}{\partial x^2} = y(4x^2 - 1)$$
 given that at $x = 0$, $u = \sin y$ and $\frac{\partial u}{\partial x} = \cos 2y$

Answer

$$u = y\left(\frac{x^4}{3} - \frac{x^2}{2}\right) + x\cos 2y + \sin y$$