Tutorial Sheet 3 – Stability analysis of ODE's

1. Examine the stability of all points in the following systems using Jacobian matrix

(a)

$$\dot{y}_1 = y_1 + y_2 - {y_1}^2 = F_1(y_1, y_2)$$

$$\dot{y}_2 = 2y_1 - y_2 + y_1 y_2 = F_2(y_1, y_2)$$

(b)

$$\dot{y}_1 = 4y_1 + 3y_2 - y_1 {y_2}^2 = F_1(y_1, y_2)$$

$$\dot{y}_2 = -6y_1 + 4y_2 + {y_2}^3 = F_2(y_1, y_2)$$

(c)

$$\dot{y}_1 = 2y_1 + 3y_2 - {y_1}^3 = F_1(y_1, y_2)$$

$$\dot{y}_2 = -4y_1 + 6y_2 + 3{y_1}^2 y_2 = F_2(y_1, y_2)$$

Answers

1. (a) $(y_1, y_2) = (0,0)$ Unstable

   (b) $(y_1, y_2) = (0,0)$ Unstable

   $(y_1, y_2) = (3.9565,2.4147)$ Unstable

   $(y_1, y_2) = (-3.9565, -2.4147)$ Unstable

   (c) $(y_1, y_2) = (0,0)$ Unstable

   $(y_1, y_2) = (1.6818\ , 0.04644)$ Unstable

   $(y_1, y_2) = (-1.6818\ , -0.04644)$ Unstable