UNIVERSITY OF
LINCOLN
UNITED KINGDOM

College of Health & Science
School of Engineering & Physical Sciences

Time Constrained Assessment

Module Title	**State Space Control**
Module Code	**EGR3032**
Module Coordinator	**Dr Yaxing Ren & Dr Aliyu Aliyu**
Duration of Assessment	**4 hours**
Date	**14/01/2025**
Release Time	**09:00**
Submission Time	**13:00**
	PASS 14:20

General Instructions to Candidates.

1. You **must** submit your answers to TurnItIn on Blackboard **before** the submission time: failure to do so will be classified as misconduct in examinations. It is strongly recommended you submit at least 15 minutes prior to the deadline.
2. You **must** also send a copy of your work to: sepssubmissions@lincoln.ac.uk at the same time. You must place the Module Code and your Student ID in the Subject Field of the Mail.
3. For students who choose to word process their answers, hand-written notes or diagrams **must** be photographed (preferably using Microsoft Lens which is available as part of your Office 365 package) and inserted into the Word Document as an image.
4. This assessment is an open resource format: you may use online resources, lecture and seminar notes, text books and journals. All sources must be correctly attributed or referenced.
5. All work will be subject to plagiarism and academic integrity checks. In submitting your assessment, you are certifying that this is entirely your own work, without input from either commercial or non-commercial writers or editors or advanced technologies such as artificial intelligence services unless explicitly allowed and referenced. If standard checks suggest otherwise, Academic Misconduct Regulations will be applied.
6. The duration of the Time Constrained Assessment will vary for those students with Personalised Academic Study Support (PASS) plan. Extensions do not apply, but Extenuating Circumstances can be applied for in the normal way.

Module Specific Instructions to Candidates

1. Answer 4 out of 5 questions
2. All questions carry EQUAL marks
3. No further marks will be awarded for answers to a fifth question

Question 1

(a) In each case determine the state space matrices A, B, C, D for the LTI system model

(i) The SISO system with transfer function shown (use any state variables but clearly show they are chosen)

$$u(t) \longrightarrow \boxed{\frac{1}{s^3 + 3s^2 + s}} \longrightarrow y(t)$$

[8 marks]

(ii) A two-input system has differential equations

$$\dot{\mathbf{x}} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$y = x_1 + x_2$$

The state vector is $\mathbf{x} = (x_1 \quad x_2)^T$, y is the output and $(u \ \ v)^T$ is a two-component control input. The two control signals are coupled so that

$$u + v = 0$$

yielding a SISO system.

[8 marks]

(b) For the system presented in part (a)(i), decide whether it is

(i) controllable

[2 marks]

(ii) observable

[2 marks]

(c) Briefly explain the meaning of controllability. What are the implications for the control engineer if a system is not fully controllable.

[5 marks]

Total 25 marks

[End of Question 1]

Question 2.

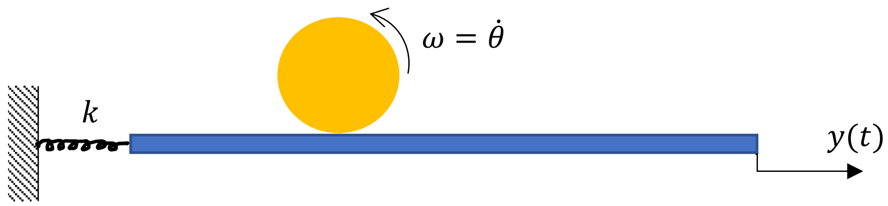
Consider the following position control system: a motor with angular velocity $\omega(t)$ and output rotor with radius r controls the linear motion of a slider of mass m constrained by a spring of stiffness k. The input signal is a voltage v_a applied to the motor armature and the output signal is the linear displacement of the slider, $y(t)$.

The equation of motion for the motor is

$$\tau\dot{\omega} + \omega = K_0 v_a + K_1 T_l$$

where T_l is the torque reaction (load) reacted by the slider on the motor drive gear. With non-sliding contact between the drive gear and the slider it follows that

$$y = r\theta$$



Using state variables $\mathbf{x} = (\theta \quad \dot{\theta})^T$ and the data given below

(a) Determine the state-space equations of the system

[15 marks]

(b) Use the Ackerman formula to determine the feedback gains required to place both closed-loop poles at $s = -2 \pm 4j$.

[10 marks]

DATA (in SI units): $m = 0.4,\ \tau = 0.15,\ K_0 = 1.6,\ K_1 = 2.3, r\ = 0.15, k = 200$.

Total 25 marks

[End of Question 2]

Question 3.

A state-space system has the following matrices

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 0 & -6 & 3 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0], \quad D = [1]$$

We seek to design a full-state feedback controller for the system

(a) Determine whether the system is completely state controllable and/or completely state observable.

[8 marks]

(b) Find *by hand* the eigenvalues of the A matrix and determine whether the open-loop system is stable.

[5 marks]

(c) Determine the transfer function $G(s) = Y(s)/U(s)$ where $Y(s)$ is the Laplace transform of the output and $U(s)$ is the Laplace transform of the input. Explain why the transfer function is not ‘strictly proper’ – the numerator has the same highest power of s as the denominator.

[8 marks]

(d) Show that the poles of $G(s)$ are equal to the eigenvalue of A for <u>any</u> SISO state-space system.

[4 marks]

Total 25 marks

[End of Question 3]

Question 4.

A nonlinear system has the following differential equations:

$$\dot{x}_1 = -x_1 + 2x_2^2$$

$$\dot{x}_2 = 1 - x_1 + ux_2$$

(a) Show that the open-loop system ($u = 0$) has <u>two</u> possible equilibrium states.

[5 marks]

(b) Linearize about each of these two equilibrium states and find the state space matrices, given that x_1 is the measured output variable.

[15 marks]

(c) Determine the stability of the two equilibrium points.

[5 marks]

Total 25 marks

[End of Question 4]

Question 5

Consider the state-space system with state equations:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

for which the output signal is defined by $y(t) = x_1(t) - u(t)$.

(a) Determine whether the system is stable.

[4 marks]

(b) Obtain by hand the transfer function of the system and compare the result to that of ss2tf in MATLAB.

[8 marks]

(c) Explain why the results are the same but that for a problem involving tf2ss this may not be the case.

[3 marks]

(d) Design a state-feedback controller with poles at $s = -2 \pm j$, showing your method clearly.

[10 marks]

Total 25 marks

[End of Question 5]

[End of Time Constrained Assessment]