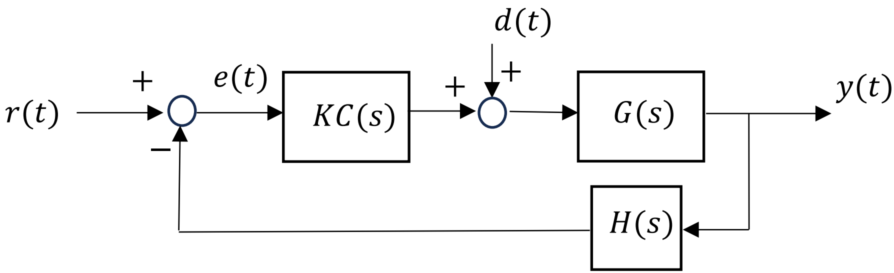
Week 12 REVISION – including TCA examples

Review of basics – live in the class



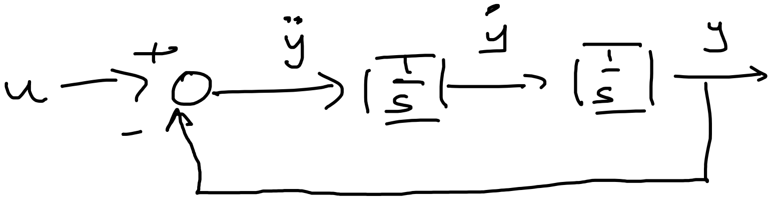
$L = KCGH$

$$T = \frac{Y(s)}{R(s)} = \frac{KC(s)G(s)}{1 + L(s)} \approx 1$$

$$S = \frac{1}{1+L}$$

1. Overview SIMULINK

2. ODE $\frac{d^2y}{dt^2} + y = u \Rightarrow \ddot{y} = u - y$



$s =$ diff operator

$$\frac{Y}{U} = G(s)$$

$$(s^2 + 1)Y = U$$

$$\frac{Y}{U} = \frac{1}{s^2+1}$$

3 $\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 u$

$y_{ss} = u_{ss}$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$G(0) = 1$

ss gain

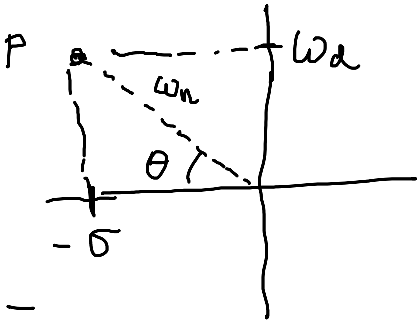
Laplace $\mathcal{L}\left[1(t)\right] = \frac{1}{s}$

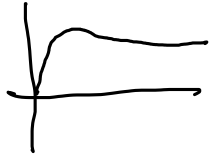
ODE $\rightarrow$ - - -

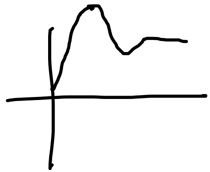
$$Y(s) = \frac{a}{s-p_1} + \frac{b}{s-p_2} + \frac{c}{s}$$

$$y(t) = \underline{a\,e}^{p_1 t} + \underline{b\,e}^{p_2 t} + c$$

④ poles/zeros







e^{-at}

$t_s = \frac{3}{a}$

$\cos\theta = \zeta$

$\bar{p}$ dominant

poles $\quad s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$\zeta < 1$

5

$$t_s = \frac{3}{\sigma}$$

$$(\sigma = \zeta \omega_n)$$

$$\theta \approx 55^\circ$$

$-\sigma$

$$G = \frac{s+1}{s^2+3s+2}, \qquad C(s) = 2s+1$$

$$T = \frac{(2s+1)(s+1)}{(s^2+3s+2)} \Big/ 1 + \frac{(\quad)(\quad)}{(s^2+3s+2)}$$

$$\frac{(2s+1)(s+1)}{s^2+3s+2+(2s+1)(s+1)}$$

type 0

6 steady-state error

$$r(t) \longrightarrow \boxed{T(s)} \longrightarrow y(t)$$

$T(0) \longrightarrow$ ss gain for unit step

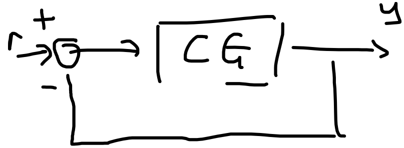
order

type min power of s in $D(s)$

$$\frac{s+2}{s^3+2s^2}$$

type 2

$$K_p = \lim_{s \to 0} C(s)\,G(s)$$



$$K_v = \lim_{s \to 0} s \cdot C(s) \cdot G(s)$$

$$K_a = \lim_{s \to 0} s^2 \cdot C(s) \cdot G(s)$$

7 root locus

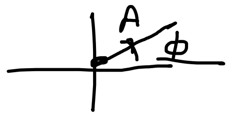
8 root locus +

Ziegler Nichols

9 frequency response

$$H(\omega) = G(j\omega)$$

$$Ae^{j\phi}$$



gain margin , phase margin

phase = -180^0 gain = 1 (0 dB)

$C(s)G(s)$

10 stability tests [Nyquist]

Routh-Hurwitz

CLTF

$$T(s) = \frac{\sim\sim}{s^4 + 3s^3 + 2s^2 + s + K}$$

$$
\begin{array}{c|ccc}
s^4 & 1 & 2 & 5 \\
s^3 & 3 & 1 & 0 \\
s^2 & a & b & \\
s^1 & c & & \\
s^0 & & &
\end{array}
$$

$$a = \frac{1\times 1 - 2\times 3}{-3}$$

$$b = \frac{1\times 0 - 5\times 3}{-3}$$

11 [sensitivity function, linearization]

block diagram <u>reduction</u>

Question One:

1. Derive the transfer function of the whole system shown in Figure 1. **(10 points)**
2. What is the order of **(5 points)**
   2.1. Each sub-system in the dotted boxes?
   2.2. The whole system?
3. Calculate the steady-state error of the whole system for a unit step input. **(10 points)**

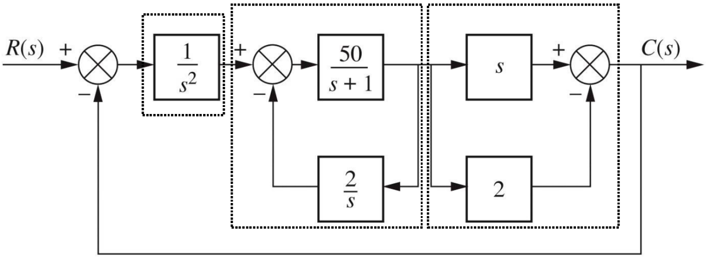
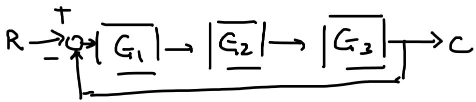


Fig. 1

of form



$$G_1 = \frac{1}{s^2}, \quad G_2\,(\text{feedback}) = \frac{50/(s+1)}{1 + \frac{50}{s+1}\cdot\frac{2}{s}} = \dots = \frac{50s}{s^2 + s + 100}$$

$$G_3 = s - 2$$

total system $\frac{C}{R} = \frac{G_1G_2G_3}{1+G_1G_2G_3}$, $G_1G_2G_3 = \frac{50s(s-2)}{s^2(s^2+s+100)}$

$= \frac{50(s-2)}{s(s^2+s+100)}$

$$\frac{C}{R} = \frac{\frac{50(s-2)}{s(s^2+s+100)}}{1+\frac{50(s-2)}{s(s^2+s+100)}} = \ldots = \frac{50(s-2)}{s^3+s^2+150s-100}$$

(2)

TF	G_1	G_2	G_3	C/R
order	2	2	0	3

(3) For unit step, ss error $= \frac{1}{1+K_p}$

$K_p = \lim_{s\to 0} G(s)$ where G = open loop TF $= G_1G_2G_3$

as $s \to 0$, $G \to \frac{-100}{0} \to \infty$

$\therefore$ ss error $\to \frac{1}{1+\infty} = 0$ //

OR from $\frac{C}{R}$ let $s \rightarrow 0$ for steady-state gain $= \frac{-100}{-100} = 1$

$\therefore$ error $= 0$ for step input.

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Question Two:

For the following system shown in Figure 2,

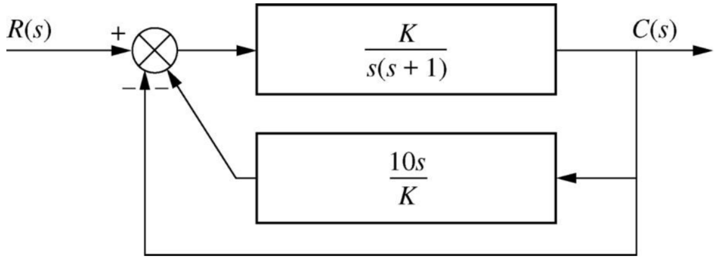


Fig. 2

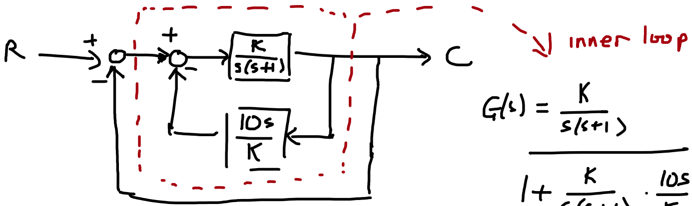
a. Find the system transfer function. **(5 points)**

b. Design the value of K so that for an input 100tu(t), there will be a 1% steady-state error. **(7 points)**

c. Does the calculated value for K meets the criteria for stability? **(6 points)**

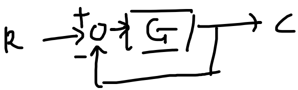
d. Find the steady-state error in terms of K for the following inputs: $100r(t)$, $100t^2r(t)$ where r(t) is a step input and comment on the results. **(7 points)**

a) re-draw to separate the connections



$$G(s) = \frac{\frac{K}{s(s+1)}}{1 + \frac{K}{s(s+1)} \cdot \frac{10s}{K}} = \ldots = \frac{K}{s(s+11)}$$

hence



$$T = \frac{C}{R} = \frac{G}{1+G} = \ldots = \frac{K}{s^2+11s+K} \quad //$$

b) For ramp input, ss error $= \frac{1}{K_v}$ (= fraction of input amplitude error) $= 1\% = 0.01$

$\therefore \quad K_v = 100 = \lim_{s \to 0} s\,G(s)$ <u>note</u> use open-loop TF

$$= \lim_{s \to 0} s \cdot \frac{K}{s(s+11)} = \frac{K}{11} \qquad \therefore \quad \underline{\underline{K = 1100}}$$

c) set $K = 1100$ in the closed-loop TF and find poles

$$T = \frac{1100}{s^2 + 11s + 1100}$$

poles : $s^2 + 11s + 1100 = 0$

$s = -5.5 \pm 32.7j$ <u>stable</u>

d) Use open-loop TF $G = \dfrac{K}{s(s+11)}$

type 1 system $\Rightarrow$ ss error $= \underline{\underline{0}}$ for $1(t)$ (or $100\ 1(t)$)

ss error $= 1/K_v$ for $t.1(t)$ – see above

ss error $= \underline{\underline{\infty}}$ for $t^2 1(t)$ – in other words the error keeps increasing

<u>Question Three</u>: **(25 points)**

It is desired to develop a policy for drug delivery to maintain the virus count at prescribed levels. For the purpose of obtaining an appropriate u_1, the feedback shown in Figure 3 will be used. As a first approach, consider $G(s) = K$, a constant to be selected. Use the Routh – Hurwitz criterion to find the range of the gain K to keep the closed loop system stable. The HIV (AIDS) linearized model can be shown to have the following transfer function:

$$P(s) = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

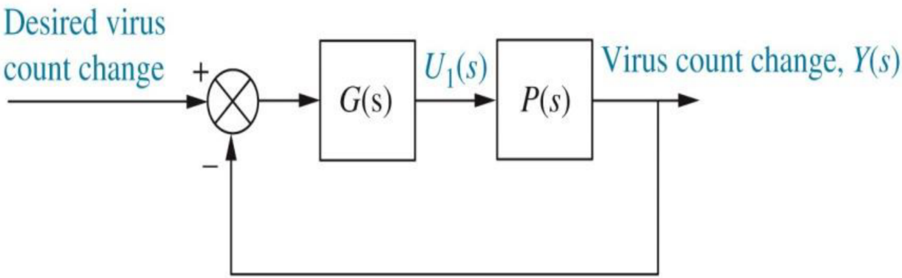


Fig. 3

With $E(s) = K$, CLTF $T(s) = \frac{KP}{1+KP} = \frac{KN/D}{1+KN/D} = \frac{KN}{D+KN}$

where $P = \frac{N}{D}$, $N = -520s - 10.3844$, $D = s^3 + 2.6817s^2 + 0.11s + 0.0126$

characteristic equation $D + KN = 0$

$$\Rightarrow s^3 + 2.6817s^2 + (0.11 - 520K)s + (0.0126 - 10.3844K) = 0$$

Routh Array

s^3	1	$(0.11-520K)$
s^2	2.6817	$(0.0126-10.3844K)$
s^1	a	0
s^0	b	0

$$a = \frac{0.0126 - 10.3844K - 2.6817(0.11 - 520K)}{-2.6817}$$

$$= 0.1053 - 516.1K$$

$$b = \frac{2.6817 \times 0 - (0.0126 - 10.3844K) \times a}{-a}$$

$$= 0.0126 - 10.3844K$$

Required for stability: $0.11 - 520K > 0$, $a > 0$, $b > 0$

$\Rightarrow$ ---- $\underline{K < 0.000204}$

<u>Question Four</u>:

For the system shown in Figure 4,

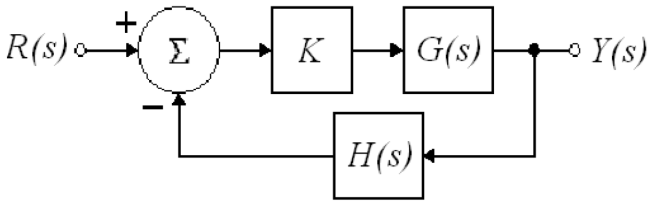


Fig. 4

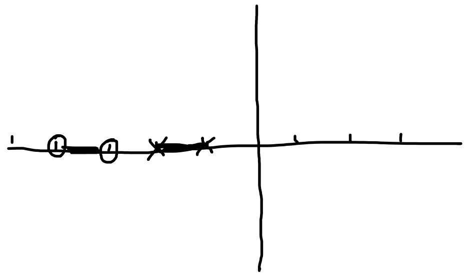
The function $G(s)H(s)$ is given as:

$$G(s)H(s) = \frac{(S+4)(S+3)}{(S+1)(S+2)}$$

- **a.** Sketch the root locus of the closed-loop system **(10 points)**
- **b.** State all root locus rules you used in solving this problem. **(5 points)**
- **c.** What is the value for the control gain K that keeps the system stable? **(10 points)**

It's good to use Matlab (if available) to solve the problem first!
(but don't use the result in the official solution)

SKETCH : open-loop poles and zeros, real axis rule



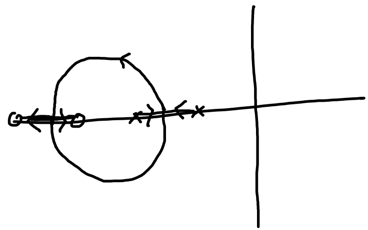
poles at $-1, -2$

zeros at $-3, -4$

$n - m = 0$

∴ no asymptotes

complete the sketch with break-out and break-in



Confirm break-out and break-in points using "$N'D = ND'$" rule

where $N = (s+3)(s+4) = s^2+7s+12$, $N' = 2s+7$

$D = (s+1)(s+2) = s^2+3s+2$, $D' = 2s+3$

substitute and simplify $\longrightarrow$ $s^2 + 5s + 5.5 = 0$

solve $\longrightarrow$ $s = -1.634, -3.366$

The system is stable for all values of K, since the root locus always has negative real parts