5. For the following equation (second order ode)

$$y'' + y' + 2y = 0$$

2

$x_1 = y$

$x_2 = y'$ → $n-1$ highest derivative

$2-1=1$

get deravative of final variable

$x_2 = y' \Rightarrow x_2' = y''$

a) write the system as 1st order differential equations by change variable method.

b) write the first order ode equations of previous part (part a) in matrix form.

$y'' + y' + 2y = 0 \Longrightarrow$ sub new variables x_1 and x_2

$x_2' + x_2 + 2x_1 = 0 \longrightarrow x_2' = -x_2 - 2x_1$

$x_1' = ?$ →

1. $\Rightarrow$ we need to define new equation based on new variable

$x_2' = \quad x_1 \quad x_2$

$\downarrow$

$x_1' = ?$ $\Longrightarrow$ How can find $x_1' = ?$

$$x_1 = y \xrightarrow{\text{deravative}} \begin{cases} x_1' = y' \\ x_2 = y' \end{cases} \Longrightarrow x_1' = x_2$$

$$\Longrightarrow x_1' = 0x_1 + 1x_2$$

$$x_2' = -x_2 - 2x_1 \Longrightarrow x_2' = -2x_1 - x_2$$

$$x_1' = +1x_2 + 0x_1 \Longrightarrow x_1' = 0x_1 + 1x_2$$

are 1st order ode

b) $y' = Ax \rightarrow x_1, x_2$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} A \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Longrightarrow \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$y' + 2\cos 2t = 0$, $y(0) = \frac{2}{\pi}$

$t=0$ boundry condition

How → find particular solution for top equation?

$y' = \frac{dy}{dt} \Rightarrow$ instead of $y' = \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} + 2\cos 2t = 0 \Longrightarrow \frac{dy}{dt} = -2\cos 2t$

$\times$

$$\frac{dy}{} = -2\cos 2t\, dt \Rightarrow \underbrace{\int dy}_{y} = \int -2\cos 2t\, dt + C$$

$$\Rightarrow y = \underbrace{\int -2\cos 2t\, dt}_{-2 \times \frac{1}{2}\sin 2t} + C \Rightarrow \underbrace{y = -\sin 2t + C}_{\text{general solution}}$$

$\Rightarrow$ How can I find C value =? by boundry condition

To find value of c :

$t = 0 \Rightarrow y = \frac{2}{\pi}$

$\Rightarrow y = -\sin 2t + c \xRightarrow{t=0} y(0) = \frac{2}{\pi} = -\underbrace{\sin(2\times 0)}_{\sin(0) = 0} + c$

$\Rightarrow \frac{2}{\pi} = 0 + c \Rightarrow c = \frac{2}{\pi}$

$\Rightarrow y = -\sin 2t + \frac{2}{\pi}$ ← particular solution for our equation

instead of c

4. For the system $AX = \lambda X$ where $A = \begin{matrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{matrix}$

a) Find the eigenvalues of matrix A.

b) Find the eigenvectors of matrix A.

$\Longrightarrow$ eigenvalues of our matrix

1. $\det(A - \lambda I) = 0$

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

$I =$

if $A_{2\times 2} \Rightarrow I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (a_{11}, a_{22})

if $A_{3\times 3} \Rightarrow I_{3\times 3}$

$$I_{3\times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a_{11}, a_{22}, a_{33}

$$\underbrace{\lambda I}_{\checkmark} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}, \quad A_{matrix}$$

$\Rightarrow$ next step is $\underbrace{A - \lambda I}$

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \end{array}$$

$$\underbrace{\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}}_{A} - \underbrace{\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}}_{\lambda I} = \begin{pmatrix} 8-\lambda & -6-0 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix}$$

$\Rightarrow$ get $\det(A - \lambda I) =$

$$A - \lambda I = \begin{pmatrix} 8-\lambda & -6-0 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix} \Longrightarrow \det(A - \lambda I)$$

$$= (8-\lambda) \times \begin{vmatrix} 7-\lambda & -4 \\ -4 & 3-\lambda \end{vmatrix} - (-6) \times \begin{vmatrix} -6 & -4 \\ 2 & 3-\lambda \end{vmatrix} + 2 \times \begin{vmatrix} -6 & 7-\lambda \\ 2 & -4 \end{vmatrix}$$

$\longrightarrow (7-\lambda)(3-\lambda) - (-4)(-4)$

$(-6)(3-\lambda) - (2)(-4)$

$(-6)(-4) - (7-\lambda) \times (2)$

$\Longrightarrow$ when we $\det(A - \lambda I) = \lambda^3 - 18\lambda^2 + 45\lambda$

next step is $\det(A - \lambda I) = 0$

$$\Longrightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0 \longrightarrow \text{roots} = 0, 3, 15$$

$\downarrow$

take factor of λ $\Longrightarrow$ $\lambda(\lambda^2 - 18\lambda + 45) = 0$

one of roots is $\lambda = 0$

$$\lambda^2 - 18\lambda + 45 = 0$$

$\hookrightarrow (\lambda - 3)(\lambda - 15) = 0$

$\hookrightarrow \lambda = 3, 15$

b) find eigenvectors

$\downarrow$

$\lambda = 0, 3, 15 \implies$ for each eigenvalue we need to find eigenvector

$\lambda = 0 \implies$ eigenvectors

$\lambda = 3 \implies$

$\lambda = 15$

Eigen Vectors :

variable

$(A - \lambda I) X = 0$

3x3

it depends on dimension of A matrix

X must be 3 variable $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$\lambda = 0$

$\lambda = 0$

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

$$\left(\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \lambda \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\lambda I = 0 \Rightarrow \text{bcz } \lambda = 0} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$A - \lambda I = A \Rightarrow AX = 0$$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

1. $8x_1 - 6x_2 + 2x_3 = 0$
2. $-6x_1 + 7x_2 - 4x_3 = 0$
3. $2x_1 - 4x_2 + 3x_3 = 0$

1. $8x_1 - 6x_2 + 2x_3 = 0$

2. $-6x_1 + 7x_2 - 4x_3 = 0$

3. $2x_1 - 4x_2 + 3x_3 = 0$

eigenvector

$\Longrightarrow$ example $\Longrightarrow \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

From First equation $\Longrightarrow 2x_3 = 6x_2 - 8x_1$

$\Longrightarrow x_3 = \frac{6x_2}{2} - \frac{8x_1}{2} \Longrightarrow x_3 = 3x_2 - 4x_1$

I must sub new x_3 into 2, 3 $\Longrightarrow \begin{cases} -6x_1 + 7x_2 - 12x_2 + 16x_1 \\ 2x_1 - 4x_2 + 9x_2 - 12x_1 \end{cases}$

$$\begin{cases} -6x_1 + 7x_2 - 12x_2 + 16x_1 = 0 \Rightarrow 10x_1 - 5x_2 = 0 \Rightarrow 10x_1 = 5x_2 \\ 2x_1 - 4x_2 + 9x_2 - 12x_1 = 0 \Rightarrow -10x_1 + 5x_2 = 0 \Rightarrow -10x_1 = -5x_2 \end{cases}$$

$$10x_1 = 5x_2 \Rightarrow x_2 = 2x_1 \Rightarrow x_1 = t \Rightarrow x_2 = 2t,\ x_3 = 2t$$

$$-10x_1 = -5x_2 \Rightarrow x_2 = 2x_1$$

$\underbrace{x_3 = 3x_2}_{A} - 4x_1 \Rightarrow$ in order to find $x_3 \Rightarrow$ sub x_1, x_2 into (A) $\Rightarrow$ $x_3 = 3 \times (2t) - 4(t) = 6t - 4t = 2t$

$x_1 = t$

$x_2 = 2t$

$x_3 = 2t$

eigenvector for $\lambda = 0$

$$\Longrightarrow X = \begin{pmatrix} t \\ 2t \\ 2t \end{pmatrix}$$

Find the solution 3 equation by calculator

$$\longrightarrow \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

✓ it is eigenvector.

eigenvectors for $\lambda = 0$

$\downarrow$

you must continue it for $\begin{cases} \lambda = 3 \\ \lambda = 15 \end{cases}$

$\Rightarrow \lambda = 3 \Rightarrow A - \lambda I$, $\lambda = 3$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \underline{\underline{\lambda I}} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$\lambda = 3$

$\longrightarrow A - \lambda I = B \Rightarrow BX = 0 \Rightarrow \ldots$