UNIVERSITY OF LINCOLN
UNITED KINGDOM

**College of Science**
**School of Engineering**

**Time Constrained Assessment**

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| **Module Title** | **Data Modelling and Simulation** |
| **Module Code** | **EGR2010M** |
| **Module Coordinator** | **TBA** |
| **Duration of Assessment** | **4.5 hours** |
| **Date** | **16/05/2023** |
| **Release Time** | **11:30 - British Summer Time (BST)** |
| **Submission Time** | **16:00 - British Summer Time (BST)** |

**General Instructions to Candidates.**

1. You **must** submit your answers to TurnItIn on Blackboard **before** the submission time: failure to do so will be classified as misconduct in examinations. We strongly recommend you submit 15 minutes prior to the deadline.
2. You **must** also send a copy of your work to: soesubmissions@lincoln.ac.uk at the same time. You must place the Module Code and your Student Id in the Subject Field of the Mail.
3. For students who choose to word process their answers, hand-written notes or diagrams, **must** be photographed (preferably using Microsoft Lens which is available as part of your Office 365 package) and inserted into the Word Document as an image.
4. This assessment is an open resource format: you may use online resources, lecture and seminar notes, text books and journals.
5. All work will be subject to plagiarism and academic integrity checks. In submitting your assessment you are claiming that it is your own original work; if standard checks suggest otherwise, Academic Misconduct Regulations will be applied.
6. The duration of the Time Constrained Assessment will vary for those students with Learning Support Plans (LSPs). Extensions do not apply, but Extenuating Circumstances can be applied for in the normal way.

**Module Specific Instructions to Candidates**

1. Answer **ALL** questions. All the four questions carry equal marks.
2. No marks will be awarded if only the final answers are provided. If you carry out the calculation by hand, please explain the derivation step by step. If you use MATLAB to get the answer, please provide the MATLAB code and explain the answer(s).

**Question 1**

(a) Prove that the eigenvalues and eigenvectors of the system $AX = \lambda X$ are
$\lambda_1 = 7, \lambda_2 = -1$ and $X_1 = \begin{pmatrix} 1 \\ 1/3 \end{pmatrix}, X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
Where matrix $A = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$.

[9 marks]

(b) Find the general and particular solutions of the following system of differential equations.

$$x_1' = 5x_1 + 6x_2$$

$$x_2' = 2x_1 + x_2$$

where $x(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

[8 marks]

(c) Let $\lambda$ be an eigenvalue of the $n \times n$ matrix $C$. Show that $\lambda^2$ is an eigenvalue of $C^2$

[8 marks]

**Question 2**

(a) An elastic string is stretched between two points 100cm apart. Its centre is displaced by 3cm from its resting position and then released with zero velocity. Sketch the graph, show the boundary conditions and only determine the general solution that represents the subsequent motion $u(x,t)$ in terms of $A_n$ and $B_n$ using the wave equation, where $c^2 = 9$.

[10 marks]

(b) Find the particular solution of the partial differential equation,

$$\frac{\partial^2 u}{\partial x \partial y} = \cos x \cos y$$

Given the conditions that when $y = \pi, \dfrac{\partial u}{\partial x} = x$, and when $x = \pi, u = 2\cos(y)$

[7 marks]

(c) In a galvanometer the deflection $\theta$ satisfies the differential equation:

$$\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + \theta = 4$$

Solve the equation for $\theta$ given that when $t = 0, \theta = o\ and \frac{d\theta}{dt} = 0$

[8 marks]

**Question 3**

(a) Consider the following system of differential equations:

$$\dot{y}_1(t) = 4y_1(t) + 3y_2(t) - y_1(t)y_2^2(t) = g_1\big(y_1(t), y_2(t)\big)$$
$$\dot{y}_2(t) = 6y_1(t) - y_2(t) - y_2^3(t) = g_2\big(y_1(t), y_2(t)\big)$$

Show that $(y_1\,(t), y_2\,(t)) = (0,0)$ is a fixed point and find the stability of the system at this point using Jacobian matrix.

[13 marks]

(b) Use the Runge-Kutta method to solve the differential equation:

$$\frac{dy}{dx} = 2y - 4x$$

in the range $0(0.5)2.0$, given the initial conditions that at $y(0) = 1$ to find the first two terms $(y_1\ and\ y_2)$. Work to an accuracy of 6 decimal places.

[12 marks]

**Question 4**

(a) Sketch and find the Fourier Transform of the rectangular pulse:

$$p(t) = \begin{cases} 6 & -2a < t < 2a \\ 0 & otherwise \end{cases}$$

[5 marks]

(b) Find the co-ordinates of the stationary points on the surface

$$z = x^3 - 6x^2 - 8y^2$$

and distinguish between them using Taylor's Theorem.

[10 marks]

(c) Find the co-ordinates of the stationary points on the surface

$$z = x^3 - x + y^3 - y$$

and distinguish between them using Hessian matrix.

[10 marks]

**[End of Time Constrained Assessment]**